Fermion mass ratios from non-renormalizable Yukawa operators in $\mathrm{SO}(10)$

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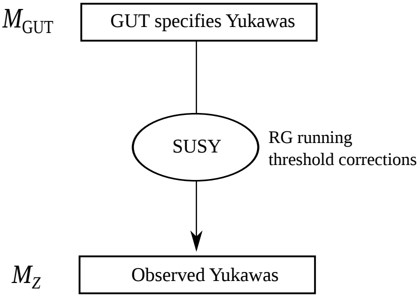
Outline

- Introduction
  - Flavor GUT models.
  - Usual $\mathrm{SO}(10)$ Yukawa operators.
- A class of non-renormalizable $\mathrm{SO}(10)$ Yukawa operators.
  - discrete & continuous VEV alignements
- A simple predictive setup using these opeartors.
- Conclusions

Flavor GUT models

- Flavor GUT models: relate Yukawas in different sectors at M_{GUT}.
- Connecting Yukawas at low and high scale:



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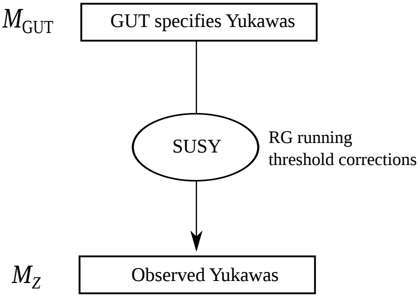

- Examples of ratios in $\mathrm{SU}(5)$:
  - $m_b = m_\tau$: "b-τ unification"
  - Georgi-Jarlskog: $m_e = m_d/3$, $m_\mu = 3m_s$
  - $y_\tau = \pm\frac{3}{2}y_b$, $y_\mu = 6y_s$, $y_\mu = \frac{9}{2}y_s$

Reminder on $\mathrm{SO}(10)$ group

Dimension: 45

Maximal subgroups:
$\mathrm{SU}(5) \times \mathrm{U}(1)$
$\mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$

Fermions:
spinorial 16
All sectors join: u, d, e, ν

Selection of irreducible representations:
10, 45, 54, 120, 210 (real)
16, $\overline{16}$, 126, $\overline{126}$ (complex)

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MSSM fields		SU(5)	SO(10)
Q	$\sim (3, 2, +1/6)$	10	16
u^c	$\sim (\overline{3}, 1, -4/6)$	10	16
e^c	$\sim (1, 1, +6/6)$	10	16
d^c	$\sim (\overline{3}, 1, +2/6)$	$\overline{5}$	16
L	$\sim (1, 2, -3/6)$	$\overline{5}$	16
ν^c	$\sim (1, 1, +0/6)$	1	16

$H_u \sim (1, 2, +3/6)$
$H_d \sim (1, 2, -3/6)$

Renormalizable operators

- Fermions: $3 \times 16_F$
  $\rightarrow 16 \otimes 16 = 10 \oplus 126 \oplus 120$

- Yukawa operator:

$$W = 16 \cdot 16 \cdot (\mathbf{Y}_{10}\, 10 + \mathbf{Y}_{\overline{126}}\, \overline{126} + \mathbf{Y}_{120}\, 120)$$

- 3 independent matrices: 2 symmetric, 1 antisymmetric
- MSSM H_u and H_d are present in all reps 10, $\overline{126}$ (126), 120
- Well investigated in the literature[1]

[1] see e.g. arXiv:1612.04329, arXiv:1306.4468, arXiv:1811.02895

Effective operators

- Non-renormalizable Yukawa operators (partly in hep-ph/9308333):

$$W_{Yuk} \propto (\prod_{\alpha}^{n_1} 45_\alpha \cdot \prod_{\beta}^{m_1} 210_\beta \cdot 16_I) \cdot H \cdot (\prod_{\alpha'}^{n_2} 45_{\alpha'} \cdot \prod_{\beta'}^{m_2} 210_{\beta'} \cdot 16_J)$$

H: 10, $\overline{126}$ or 120

GUT scale VEVs
$\langle 45 \rangle$ and $\langle 210 \rangle$
in SM singlet direction

45: 2 SM singlets
210: 3 SM singlets

32×32 matrices,
diagonal with "charges"

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45:	$\langle 1 \rangle$	$\langle 24 \rangle$	T_R^3	$B-L$
Q	1	1	0	1
u^c	1	−4	1	−1
d^c	−3	2	−1	−1
L	−3	−3	0	−3
e^c	1	6	−1	3
ν^c	5	0	1	3

210:	$\langle 1 \rangle$	$\langle 24 \rangle$	$\langle 75 \rangle$	$(1,1,1)$	$(1,1,15)$	$(1,3,15)$
Q	1	1	1	1	1	0
u^c	1	−4	−1	−1	1	1
d^c	−1	−6	0	−1	1	−1
L	−1	9	0	1	−3	0
e^c	1	6	−3	−1	−3	3
ν^c	−5	0	0	−1	−3	−3

General Yukawa term — discrete directions

General Yukawa term for discrete VEV alignements $\langle 45 \rangle$ and $\langle 210 \rangle$:

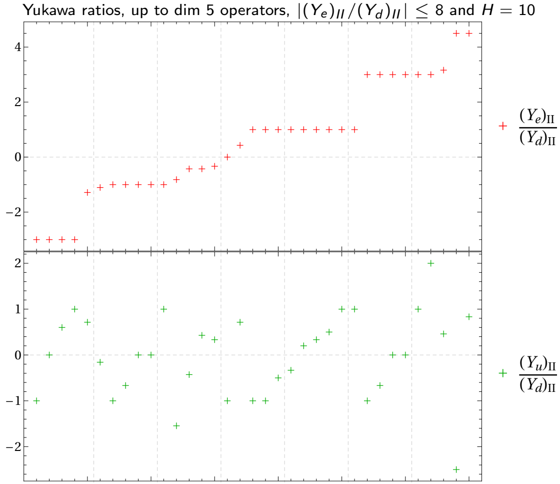
$$
\begin{aligned}
W \propto \Big(& Q_I\, u^c_J\, H^{\mathbf{H}}_u \Big[\quad C^{\mathbf{H}}_{ud} \prod_{\alpha,\beta,\alpha'\beta'} q_\alpha(Q)\, q_\beta(Q)\, q_{\alpha'}(u^c)\, q_{\beta'}(u^c) \Big] \\
& + Q_J\, u^c_I\, H^{\mathbf{H}}_u \Big[s^{\mathbf{H}}\, C^{\mathbf{H}}_{ud} \prod_{\alpha,\beta,\alpha'\beta'} q_\alpha(u^c)\, q_\beta(u^c)\, q_{\alpha'}(Q)\, q_{\beta'}(Q) \Big] \\
& + Q_I\, d^c_J\, H^{\mathbf{H}}_d \Big[\quad C^{\mathbf{H}}_{ud} \prod_{\alpha,\beta,\alpha'\beta'} q_\alpha(Q)\, q_\beta(Q)\, q_{\alpha'}(d^c)\, q_{\beta'}(d^c) \Big] \\
& + Q_J\, d^c_I\, H^{\mathbf{H}}_d \Big[s^{\mathbf{H}}\, C^{\mathbf{H}}_{ud} \prod_{\alpha,\beta,\alpha'\beta'} q_\alpha(d^c)\, q_\beta(d^c)\, q_{\alpha'}(Q)\, q_{\beta'}(Q) \Big] \\
& + L_I\, e^c_J\, H^{\mathbf{H}}_e \Big[\quad C^{\mathbf{H}}_{e\nu} \prod_{\alpha,\beta,\alpha'\beta'} q_\alpha(L)\, q_\beta(L)\, q_{\alpha'}(e^c)\, q_{\beta'}(e^c) \Big] \\
& + L_J\, e^c_I\, H^{\mathbf{H}}_e \Big[s^{\mathbf{H}}\, C^{\mathbf{H}}_{e\nu} \prod_{\alpha,\beta,\alpha'\beta'} q_\alpha(e^c)\, q_\beta(e^c)\, q_{\alpha'}(L)\, q_{\beta'}(L) \Big] \\
& + L_I\, \nu^c_J\, H^{\mathbf{H}}_\nu \Big[\quad C^{\mathbf{H}}_{e\nu} \prod_{\alpha,\beta,\alpha'\beta'} q_\alpha(L)\, q_\beta(L)\, q_{\alpha'}(\nu^c)\, q_{\beta'}(\nu^c) \Big] \\
& + L_J\, \nu^c_I\, H^{\mathbf{H}}_\nu \Big[s^{\mathbf{H}}\, C^{\mathbf{H}}_{e\nu} \prod_{\alpha,\beta,\alpha'\beta'} q_\alpha(\nu^c)\, q_\beta(\nu^c)\, q_{\alpha'}(L)\, q_{\beta'}(L) \Big] \Big)
\end{aligned}
$$

$s^{\mathbf{H}}$, $C^{\mathbf{H}}$: group theoretical factors (depending on the rep **H**)

q: "charges"

Discrete directions and new ratios for model building

- If $\mathrm{SU}(5)$ or PS discrete VEV alignements of 45 or 210
  New possible Yukawa predictions at M_{GUT}.



analogous plot for $(Y_\nu)_{II}/(Y_d)_{II}$

Effective operators — arbitrary directions

- Arbitrary alignments of $\langle 45 \rangle$ and $\langle 210 \rangle$ ($\rightarrow$ no analog in $\mathrm{SU}(5)$)
  - 45: 2 SM singlets $\rightarrow$ 1 ratio $\kappa \in \mathbb{C}$
  - 210: 3 SM singlets $\rightarrow$ 2 ratios $\kappa_1, \kappa_2 \in \mathbb{C}$

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  } additional model parameters

- General Yukawa term for a single field 45 and 210:

$$
\begin{aligned}
W \propto \Big(& Q_I\, u^c_J\, H^{\mathbf{H}}_u \Big[\quad C^{\mathbf{H}}_{ud}\ P_Q(\kappa)^{n_1}\ R_Q(\kappa_1,\kappa_2)^{m_1}\ P_{u^c}(\kappa)^{n_2}\ R_{u^c}(\kappa_1,\kappa_2)^{m_2} \Big] \\
& + Q_J\, u^c_I\, H^{\mathbf{H}}_u \Big[s^{\mathbf{H}}\ C^{\mathbf{H}}_{ud}\ P_{u^c}(\kappa)^{n_1}\ R_{u^c}(\kappa_1,\kappa_2)^{m_1}\ P_Q(\kappa)^{n_2}\ R_Q(\kappa_1,\kappa_2)^{m_2} \Big] \\
& + Q_I\, d^c_J\, H^{\mathbf{H}}_d \Big[\quad C^{\mathbf{H}}_{ud}\ P_Q(\kappa)^{n_1}\ R_Q(\kappa_1,\kappa_2)^{m_1}\ P_{d^c}(\kappa)^{n_2}\ R_{d^c}(\kappa_1,\kappa_2)^{m_2} \Big] \\
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& + L_I\, e^c_J\, H^{\mathbf{H}}_e \Big[\quad C^{\mathbf{H}}_{e\nu}\ P_L(\kappa)^{n_1}\ R_L(\kappa_1,\kappa_2)^{m_1}\ P_{e^c}(\kappa)^{n_2}\ R_{e^c}(\kappa_1,\kappa_2)^{m_2} \Big] \\
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\end{aligned}
$$

P, R: polynomials of κ, κ_1, κ_2 (dependent on the "charges" q)

Doublet-triplet splitting

- Weak doublets D_i come along with color triplets T_i in GUT irreps:
  - Linear combinations $H_u = \sum_i a_i D_i$, $H_d = \sum_i b_i \bar{D}_i$ light.
  - All other masses at M_{GUT}.
  - Possible solution: fine-tuning
- Multiple doublets:
  - Yukawa operators dependant on a_i, b_i.
  - May give extra freedom to $\mathrm{SO}(10)$ constraints.

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  - May give extra freedom to $\mathrm{SO}(10)$ constraints.
- Example models of Higgs sectors with predictive (anti)doublet VEVs
  - $W_{\text{Higgs}} = \frac{1}{2} m_{10}\, 10 \cdot 10 - \sqrt{\frac{5}{3}} \lambda\, 10 \cdot 10 \cdot 54$
    $\to$ fine tuning condition: $\lambda = -\frac{m_{10}}{\langle 54 \rangle}$
  - $W_{\text{Higgs}} = \frac{1}{2} m_{120}\, 120 \cdot 120 - 2\sqrt{15} \lambda\, 120 \cdot 120 \cdot 54$
    $\to$ fine tuning condition: $\lambda = -\frac{m_{120}}{6\langle 54 \rangle}$

Simple model

- No 210, only one 45 with arbitrary direction κ.
  - The 10 is used for H_u and H_d.
- Only 3rd and 2nd family, neglect mixing, no neutrino masses:
- Single operator dominance:
  $\rightarrow$ Each Yukawa entry comes from a single GUT operator.

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  - The 10 is used for H_u and H_d.
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- $W_{Yuk} = \lambda_2 \mathcal{O}_{22} + \lambda_3 \mathcal{O}_{33}$
  - 3rd family: t-b-τ unification ($n_1 = n_2 = 0$)

$$\mathcal{O}_{33} = 16_3 \cdot 10 \cdot 16_3$$

  - 2nd family: choose $n_1 = 1$, $n_2 = 1$ (two copies of the same 45)

$$\mathcal{O}_{22} = 45 \cdot 16_2 \cdot 10 \cdot 45 \cdot 16_2$$

$$Y_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}, \quad Y_e = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda_2 |y_u/y_s| & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}, \quad Y_u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda_2 |y_c/y_s| & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \quad @ \; M_{GUT}$$

$|y_u/y_s|$ and $|y_c/y_s|$ are polynomials of κ.

- Numerical analysis for CMSSM
  - parameters: $\tan\beta, \underbrace{m_0, M_{1/2}, A_0,}_{\text{soft parameters}} |\kappa|, \arg\kappa, \lambda_3, \lambda_2$ (8) $@M_{GUT}$
  - observables (fit): $y_t, y_b, y_\tau, y_c, y_s, y_\mu, m_h$ (7) $@M_Z$
  - observables (no fit): sparticle spectrum $@M_{\text{SUSY}}$

Short analysis

- Numerical analysis for CMSSM
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  - observables (no fit): sparticle spectrum $@M_{\text{SUSY}}$
- Calculation of RG running, SUSY threshold corrections and Higgs mass:
  SusyTC[2], REAP[3], FeynHiggs[4]
- Best fit: $\chi^2 < 10^{-5}$
  - EW vacuum stability check with Vevacious[5]
  - Bayesian analysis using MCMC

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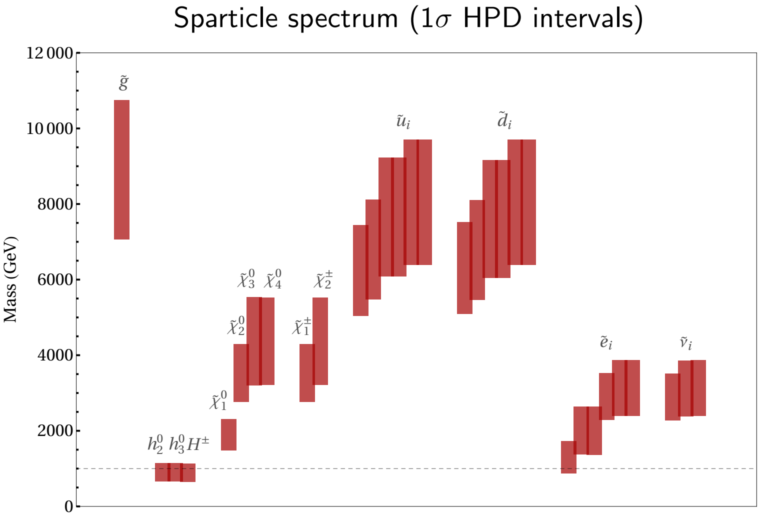
[2] arXiv:1512.06727
[3] hep-ph/0501272
[4] hep-ph/9812320
[5] arXiv:1307.1477

SUSY spectrum



Conclusions

- Alternative class of operators for $\mathrm{SO}(10)$ Yukawa sector:

$$16 \cdot 16 \cdot H \cdot 45^n \cdot 210^m \quad (H = 10,\ \overline{126} \text{ or } 120)$$

- Discrete and continuous direction of $\langle 45 \rangle$ and $\langle 210 \rangle$
  $\rightarrow$ new ratios for flavor GUT model building
- A simple model:
  using only one 45, arbitrary direction
  single operator dominance, Y_{22} and Y_{33} entries
  prediction: sparticle spectrum