

Sequentially loop-generated pattern of quark and lepton masses in models with extended symmetries.

Antonio Enrique Cárcamo Hernández

Universidad Técnica Federico Santa María

FLASY 2019 Conference, Hefei, China, 25th July of 2019.

Based on: A. E. Cárcamo Hernández, S. Kovalenko, I. Schmidt,
arxiv:hep-ph/1611.09797, JHEP **1702** (2017) 125.

A. E. Cárcamo Hernández, S. Kovalenko, R. Pasechnik, I. Schmidt,
JHEP **1906**, 056 (2019) and EPJC **C79** (2019) no.7, 610

- 1 Introduction
- 2 A toy model: Generating $m_b \neq 0$ at one loop level with $m_d = m_s = 0$.
- 3 The Cárcamo Hernández-Kovalenko-Schmidt (CKS) mechanism
- 4 An extended IDM with sequentially loop-generated fermion mass hierarchies.
- 5 Conclusions

Introduction

The origin of fermion masses and mixings is not explained by the SM.

FERMIONS

matter constituents

spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c ²	Electric charge
ν_L lightest neutrino*	$(0-0.13)\times 10^{-9}$	0
e electron	0.000511	-1
ν_M middle neutrino*	$(0.009-0.13)\times 10^{-9}$	0
μ muon	0.106	-1
ν_H heaviest neutrino*	$(0.04-0.14)\times 10^{-9}$	0
τ tau	1.777	-1

Quarks spin = 1/2

Flavor	Approx. Mass GeV/c ²	Electric charge
u up	0.002	2/3
d down	0.005	-1/3
c charm	1.3	2/3
s strange	0.1	-1/3
t top	173	2/3
b bottom	4.2	-1/3

$$\sqrt{|\Delta m_{13}^2|} \sim \lambda^{20} m_t, \quad \sqrt{\Delta m_{12}^2} \sim \lambda^{21} m_t,$$

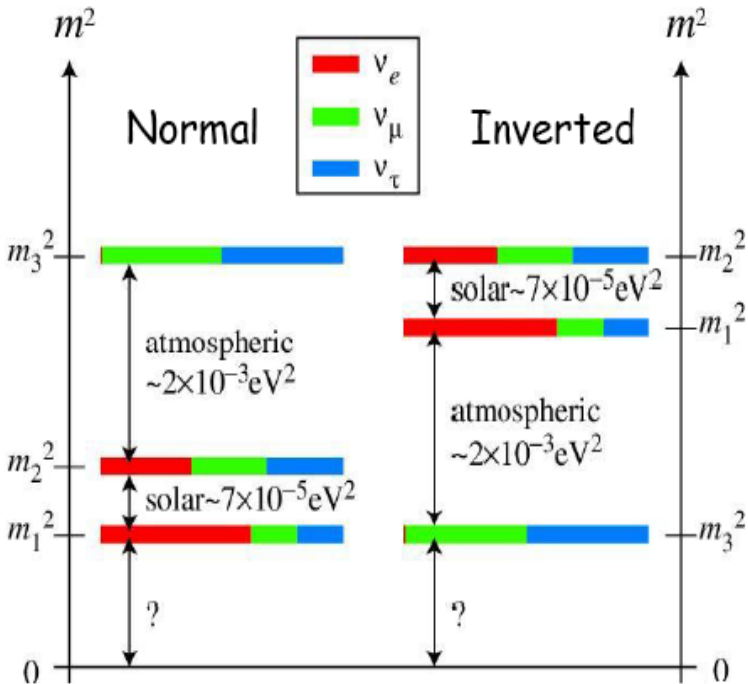
$$m_e \sim \lambda^9 m_t, \quad m_u \sim m_d \sim \lambda^8 m_t,$$

$$m_s \sim m_\mu \sim \lambda^5 m_t, \quad \lambda = 0.225,$$

$$m_c \sim \lambda^4 m_t, \quad m_b \sim m_\tau \sim \lambda^3 m_t,$$

$$\sin \theta_{12}^{(q)} \sim \lambda, \quad \sin \theta_{23}^{(q)} \sim \lambda^2, \quad \sin \theta_{13}^{(q)} \sim \lambda^4,$$

$$\sin \theta_{12}^{(l)} \sim \sqrt{\frac{1}{3}}, \quad \sin \theta_{23}^{(l)} \sim \sqrt{\frac{1}{2}}, \quad \sin \theta_{13}^{(l)} \sim \frac{\lambda}{\sqrt{2}}.$$



Some mechanisms to describe the SM charged fermion mass hierarchy are:

- 1 Spontaneously broken abelian symmetries as originally proposed by [Froggatt and Nielsen in NPB, 1979](#).
- 2 Universal Seesaw mechanism as originally proposed by [Davidson and Wali in PRL, 1987](#)
- 3 Localization of the profiles of the fermionic zero modes in extradimensions as proposed by [Dvali and Schifman in PLB, 2000](#).
- 4 Combining spontaneous breaking of discrete symmetries with radiative seesaw processes as in [A.E. Cárcamo Hernández, EPJC, 2016](#) and [C. Arbeláez, A.E. Cárcamo Hernández, S. Kovalenko and I. Schmidt, EPJC, 2017](#).
- 5 [CKS mechanism](#) of sequential loop suppression proposed by [A.E. Cárcamo Hernández, S. Kovalenko and I. Schmidt in JHEP, 2017](#).
- 6 Combining Universal Seesaw with spontaneous breaking of discrete symmetries as in [A.E. Cárcamo Hernández, S. Kovalenko, J. W. F. Valle and C. A. Vaquera-Araujo JHEP, 2017, 2019](#), [A.E. Cárcamo Hernández, Juan Marchant González, U. J. Saldana-Salazar, 2019](#), [A.E. Cárcamo Hernández, Yocelyne Hidalgo Velásquez and Nicolás A. Pérez-Julve, 2019](#)

Several mechanisms to generate light active neutrino masses are: Weinberg Operator, type I seesaw, type II seesaw, type III seesaw, double seesaw, linear seesaw, inverse seesaw (IS), radiative seesaw at one, two, three or four loop level. Some low scale seesaw models with discrete symmetries are:

- 1 Low scale type I seesaw model with A_4 , A.E. Cárcamo Hernández, Marcela González, Nicolás A. Neill, 2019, thanks to suppressed Dirac Yukawa terms.
- 2 IS model with S_4 , A.E. Cárcamo Hernández, S. F. King, 2019 and with A_4 A.E. Cárcamo Hernández, Juan Marchant González, U. J. Saldana-Salazar, 2019.
- 3 $331 T'$ model with IS, A.E. Cárcamo Hernández, Yocelyne Hidalgo Velásquez and Nicolás A. Pérez-Julve, 2019.
- 4 Pati-Salam and LR symmetric theories with $\Delta(27)$ symmetry and IS, A.E. Cárcamo Hernández, S. Kovalenko, J. W. F. Valle and C. A. Vaquera-Araujo, JHEP, 2017, 2019.

Some examples of radiative seesaw models are:

- 1 Non renormalizable one loop radiative seesaw $\Delta(27)$ model Nicolás Bernal, A. E. Cárcamo Hernández, Ivo de Medeiros Varzielas and Sergey Kovalenko, JHEP 2018.
- 2 Two loop radiative seesaw model A. E. Cárcamo Hernández, S. Kovalenko, H. N. Long, I. Schmidt, JHEP 2018
- 3 Three loop radiative seesaw model A.E. Cárcamo Hernández, EPJC, 2016, A. E. Cárcamo Hernández, S. Kovalenko, R. Pasechnik, I. Schmidt, JHEP 2019
- 4 Four loop radiative seesaw model A.E. Cárcamo Hernández, S. Kovalenko and I. Schmidt in JHEP, 2017

The S_3 discrete group

The S_3 is the smallest non-abelian group having a doublet and two singlet irreducible representations. The S_3 group has three irreducible representations: $\mathbf{1}$, $\mathbf{1}'$ and $\mathbf{2}$. Denoting the basis vectors for two S_3 doublets as $(x_1, x_2)^T$ and $(y_1, y_2)^T$ and y' a non trivial S_3 singlet, the S_3 multiplication rules are (Ishimori, et al, Prog. Theor. Phys. Suppl 2010):

$$\begin{aligned} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_2 \otimes \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}_2 &= (x_1 y_1 + x_2 y_2)_{\mathbf{1}} + (x_1 y_2 - x_2 y_1)_{\mathbf{1}'} \\ &+ \begin{pmatrix} x_2 y_2 - x_1 y_1 \\ x_1 y_2 + x_2 y_1 \end{pmatrix}_2, \end{aligned} \quad (1)$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_2 \otimes (y')_{\mathbf{1}'} = \begin{pmatrix} -x_2 y' \\ x_1 y' \end{pmatrix}_2, \quad (x')_{\mathbf{1}'} \otimes (y')_{\mathbf{1}'} = (x' y')_{\mathbf{1}}. \quad (2)$$

A toy model: Generating $m_b \neq 0$ at one loop level with $m_d = m_s = 0$.

To get massless d , s and b quarks at tree level, we forbid the operators

$$\bar{q}_{iL}\phi d_{jR}, \quad i, j = 1, 2, 3, \quad (3)$$

To this end, we consider the following S_3 assignments:

$$q_{iL} \sim \mathbf{1}, \quad d_{iR} \sim \mathbf{1}', \quad \phi \sim \mathbf{1} \quad (4)$$

We assume S_3 softly broken and we add gauge singlet scalars η_k ($k = 1, 2$) and vector like down type quarks B_k ($k = 1, 2$) grouped in S_3 doublets as follows:

$$\eta = (\eta_1, \eta_2) \sim \mathbf{2}, \quad B_{L,R} \sim \mathbf{2} \quad (5)$$

Cárcamo Hernández-Kovalenko-Schmidt (CKS) mechanism

In the CKS mechanism the SM fermion mass hierarchy is explained by a sequential loop suppression, so that the masses are generated according to:

$$t\text{-quark} \rightarrow \text{tree-level mass from } \bar{q}_{jL}\tilde{\phi}u_{3R}, \quad (8)$$

$$b, c, \tau, \mu \rightarrow \text{1-loop mass; tree-level} \quad (9)$$

suppressed by a *symmetry*.

$$s, u, d, e \rightarrow \text{2-loop mass; tree-level \& 1-loop} \quad (10)$$

suppressed by a *symmetry*.

$$\nu_i \rightarrow \text{4-loop mass; tree-level \& lower loops} \quad (11)$$

suppressed by a *symmetry*.

The $S_3 \times Z_2$ particle assignments of the model are:

	ϕ	σ	η
S_3	1	2	1
Z_2	1	1	-1

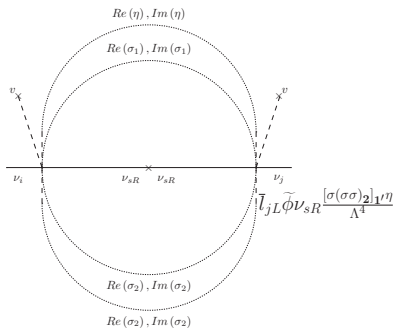
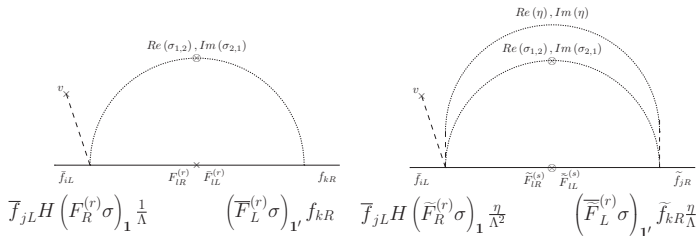
	q_{iL}	u_{1R}	u_{2R}	u_{3R}	d_{1R}	d_{2R}	d_{3R}	l_{iL}	l_{1R}	l_{2R}	l_{3R}
S_3	1	1'	1'	1	1'	1'	1'	1	1'	1'	1'
Z_2	1	-1	1	1	-1	-1	1	1	-1	1	1

	ν_{sR}	T_L	T_R	\tilde{T}_L	\tilde{T}_R	B_L	B_R	$\tilde{B}_L^{(s)}$	$\tilde{B}_R^{(s)}$	$E_L^{(s)}$	$E_R^{(s)}$	\tilde{E}_L	\tilde{E}_R
S_3	1'	2	2	2	2	2	2	2	2	2	2	2	2
Z_2	-1	1	1	1	-1	1	1	1	-1	1	1	1	-1

ϕ is the SM Higgs doublet.

The scalar fields σ and η and all exotic fermions are $SU(2)_L$ singlets.

The $S_3 \times Z_2$ discrete group is assumed to be softly broken.



The mass matrices $M_{U,D}$ of up and down quarks, $M_{l,\nu}$, of charged leptons and light active neutrinos

$$M_U = \begin{pmatrix} \tilde{\varepsilon}_{11}^{(u)} & \varepsilon_{12}^{(u)} & \kappa_{13}^{(u)} \\ \tilde{\varepsilon}_{12}^{(u)} & \varepsilon_{22}^{(u)} & \kappa_{23}^{(u)} \\ \tilde{\varepsilon}_{13}^{(u)} & \varepsilon_{32}^{(u)} & \kappa_{33}^{(u)} \end{pmatrix} \frac{v}{\sqrt{2}}, \quad M_D = \begin{pmatrix} \tilde{\varepsilon}_{11}^{(d)} & \tilde{\varepsilon}_{12}^{(d)} & \varepsilon_{13}^{(d)} \\ \tilde{\varepsilon}_{21}^{(d)} & \tilde{\varepsilon}_{22}^{(d)} & \varepsilon_{23}^{(d)} \\ \tilde{\varepsilon}_{31}^{(d)} & \tilde{\varepsilon}_{32}^{(d)} & \varepsilon_{33}^{(d)} \end{pmatrix} \frac{v}{\sqrt{2}},$$

$$M_l = \begin{pmatrix} \tilde{\varepsilon}_{11}^{(l)} & \varepsilon_{12}^{(l)} & \varepsilon_{13}^{(l)} \\ \tilde{\varepsilon}_{21}^{(l)} & \varepsilon_{22}^{(l)} & \varepsilon_{23}^{(l)} \\ \tilde{\varepsilon}_{31}^{(l)} & \varepsilon_{32}^{(l)} & \varepsilon_{33}^{(l)} \end{pmatrix} \frac{v}{\sqrt{2}}, \quad M_\nu = \begin{pmatrix} \varepsilon_{11}^{(\nu)} & \varepsilon_{12}^{(\nu)} & \varepsilon_{13}^{(\nu)} \\ \varepsilon_{12}^{(\nu)} & \varepsilon_{22}^{(\nu)} & \varepsilon_{23}^{(\nu)} \\ \varepsilon_{13}^{(\nu)} & \varepsilon_{23}^{(\nu)} & \varepsilon_{33}^{(\nu)} \end{pmatrix} \frac{v^2}{\sqrt{2} \Lambda},$$

their entries are generated at different loop-levels:

$$\kappa_{j3}^{(u)} \rightarrow \text{tree-level} \quad (12)$$

$$\varepsilon_{j2}^{(u)}, \varepsilon_{j3}^{(d)}, \varepsilon_{j2}^{(l)}, \varepsilon_{j3}^{(l)} \rightarrow \text{1-loop-level} \quad (13)$$

$$\tilde{\varepsilon}_{j1}^{(u)}, \tilde{\varepsilon}_{j1}^{(d)}, \tilde{\varepsilon}_{j2}^{(d)}, \tilde{\varepsilon}_{j1}^{(l)} \rightarrow \text{2-loop-level} \quad (14)$$

$$\varepsilon_{jk}^{(\nu)} \rightarrow \text{4-loop-level}, \quad (15)$$

where $j, k = 1, 2, 3$.

$$m_b \sim \frac{y_b^2}{16\pi^2} f_1 \frac{v}{\Lambda} \frac{\mu_{12}}{M} \mu_{12}, \quad (16)$$

$$m_s \sim \frac{y_s^2}{(16\pi^2)^2} f_2 \frac{v}{M} \frac{\mu_{12}^3}{\Lambda^3} \mu_{12}, \quad (17)$$

Assuming $y_b^2 f_1 \sim y_s^2 f_2 \sim 1$ and $\mu_{12} \sim M$, we find a rough estimate

$$\Lambda \sim 10v \sim 2.5\text{TeV} \quad (18)$$

for the correct order of magnitude of m_b and m_s .

An extended IDM with sequentially loop-generated fermion mass hierarchies.

$$\begin{aligned}
 \mathcal{G} &= SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X \times Z_2^{(1)} \times Z_2^{(2)} \\
 &\xrightarrow{v_{\sigma_1}, v_{\rho_3}} SU(3)_C \times SU(2)_L \times U(1)_Y \times Z_2^{(2)} \\
 &\xrightarrow{v} SU(3)_C \times U(1)_{em} \times Z_2^{(2)}, \tag{19}
 \end{aligned}$$

Field	ϕ_1	ϕ_2	σ_1	σ_2	σ_3	ρ_1	ρ_2	ρ_3	η	φ_1^+	φ_2^+	φ_3^+	φ_4^+	φ_5^+
SU_{3c}	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SU_{2L}	2	2	1	1	1	1	1	1	1	1	1	1	1	1
U_{1Y}	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	0	0	0	1	1	1	1	1
U_{1X}	1	2	-1	-1	-2	0	0	0	1	5	2	3	2	3
$Z_2^{(1)}$	1	1	1	1	-1	1	-1	-1	-1	-1	1	1	-1	-1
$Z_2^{(2)}$	1	-1	1	-1	-1	-1	-1	1	-1	1	1	-1	1	1

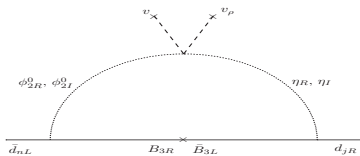
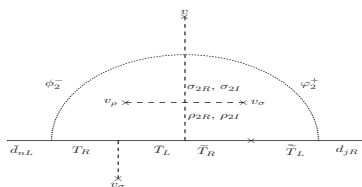
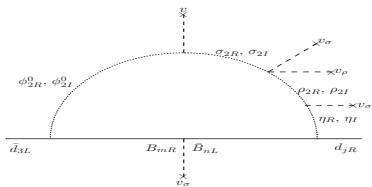
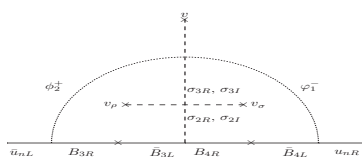
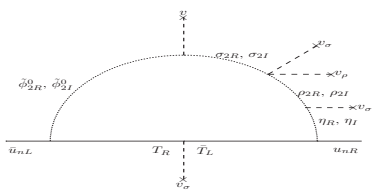
Table: Scalars assignments under the $SU_{3c} \times SU_{2L} \times U_{1Y} \times U_{1X} \times Z_2^{(1)} \times Z_2^{(2)}$ symmetry.

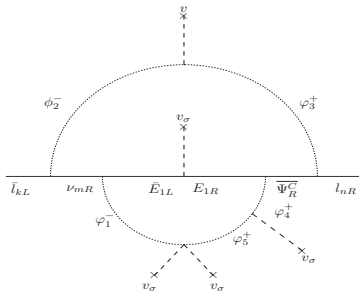
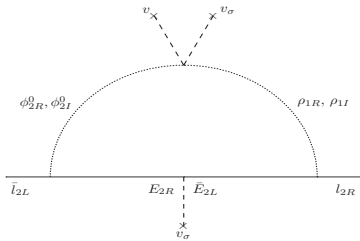
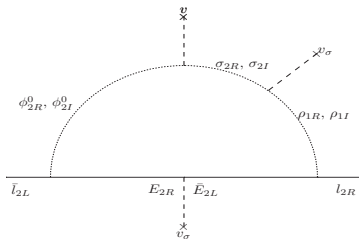
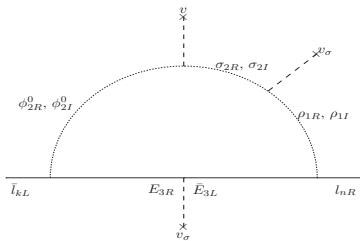
Field	q_{1L}	q_{2L}	q_{3L}	u_{1R}	u_{2R}	u_{3R}	d_{1R}	d_{2R}	d_{3R}	T_L	T_R	\tilde{T}_L	\tilde{T}_R	B_{1L}	B_{1R}	B_{2L}	B_{2R}	B_{3L}	B_{3R}	B_{4L}	B_{4R}	
SU_{3c}	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
SU_{2L}	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U_{1Y}	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
U_{1X}	0	0	1	2	2	2	-1	-1	-1	1	2	1	1	0	-1	0	-1	-2	-2	-3	-3	
$Z_2^{(1)}$	1	1	1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	1	1	1	1	1
$Z_2^{(2)}$	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	1	1	-1	-1	

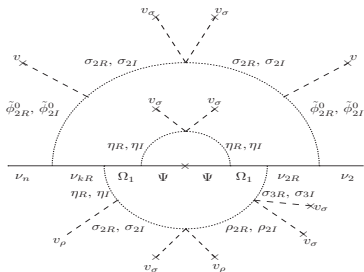
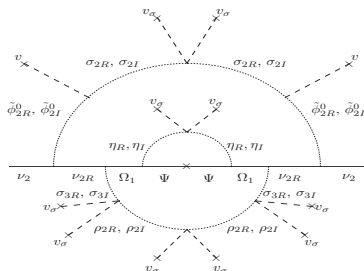
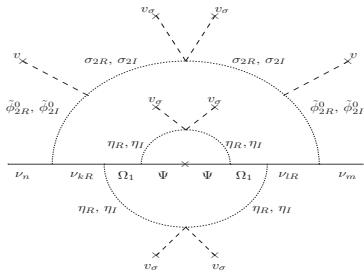
Table: Quark assignments under the $SU_{3c} \times SU_{2L} \times U_{1Y} \times U_{1X} \times Z_2^{(1)} \times Z_2^{(2)}$ symmetry.

Field	l_{1L}	l_{2L}	l_{3L}	l_{1R}	l_{2R}	l_{3R}	E_{1L}	E_{1R}	E_{2L}	E_{2R}	E_{3L}	E_{3R}	ν_{1R}	ν_{2R}	ν_{3R}	Ω_{1R}	Ω_{2R}	Ψ_R
SU_{3c}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SU_{2L}	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U_{1Y}	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0
U_{1X}	0	-3	0	-3	-6	-3	-3	-2	-6	-5	-3	-2	2	-1	2	-1	1	0
$Z_2^{(1)}$	1	-1	1	1	-1	1	-1	-1	-1	-1	1	1	1	-1	1	-1	-1	1
$Z_2^{(2)}$	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	-1	1	1

Table: Lepton assignments under the $SU_{3c} \times SU_{2L} \times U_{1Y} \times U_{1X} \times Z_2^{(1)} \times Z_2^{(2)}$ symmetry.





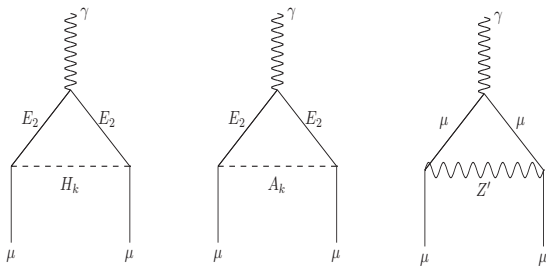


Fitting $R_K = \frac{Br(B \rightarrow K \mu^+ \mu^-)}{Br(B \rightarrow K e^+ e^-)}$ at 1σ and 2σ yields the constraints:

$$14 \text{ TeV} < \frac{M_{Z'}}{g_X} < 20 \text{ TeV} \text{ at } 1\sigma, \quad 13 \text{ TeV} < \frac{M_{Z'}}{g_X} < 26 \text{ TeV} \text{ at } 2\sigma.$$

The $e^+e^- \rightarrow \mu^+\mu^-$ measurement at LEP imposes the following limit :

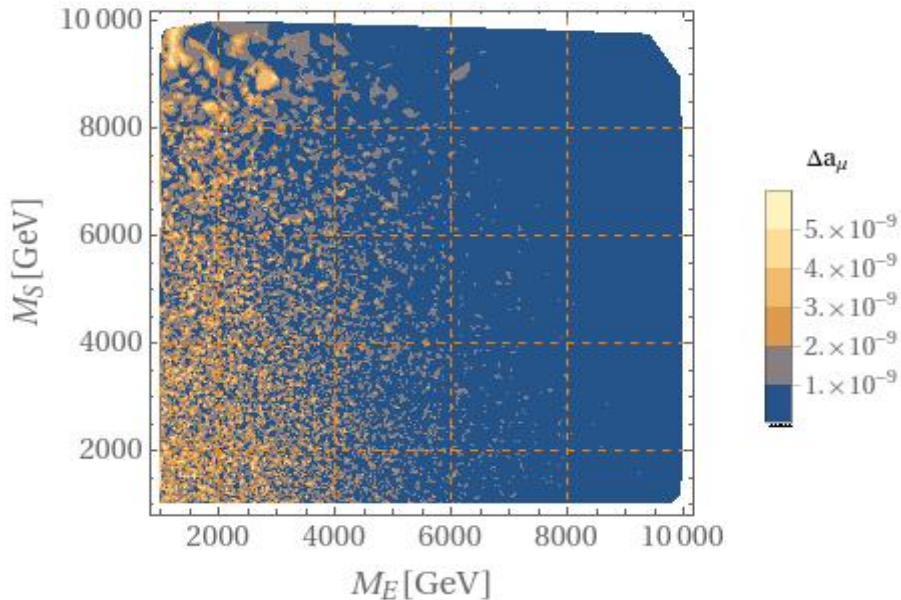
$$\frac{M_{Z'}}{g_X} > 12 \text{ TeV}. \quad (20)$$

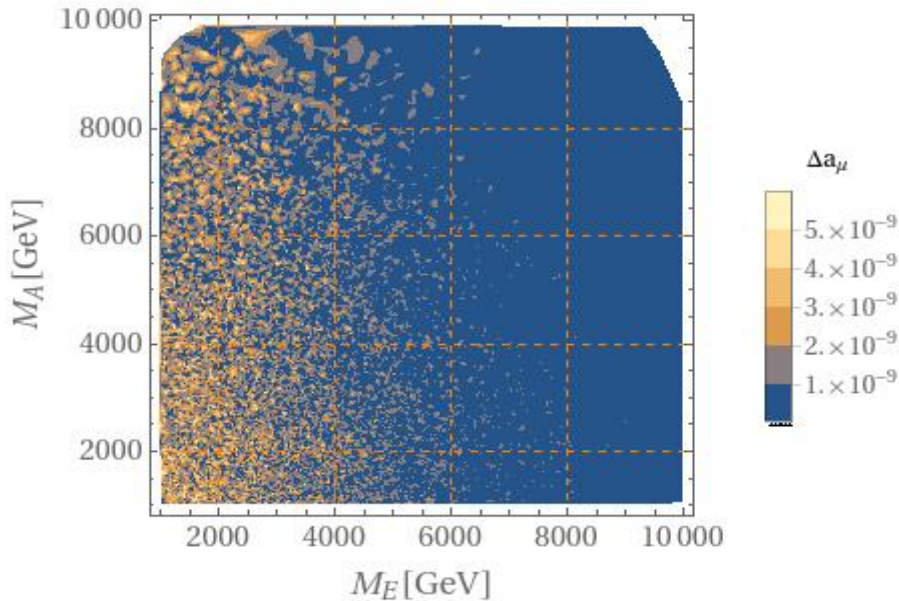


Here H_k and A_k ($k = 1, 2$) are the physical CP even and CP odd states built from ρ_2 and ϕ_2^0 .

We have fixed $\tan \theta = \frac{v}{v_{\sigma}}$, $M_{Z'} = 1.5$ TeV and $g_X = 0.1$, in consistency with the 2.6σ R_K anomaly. Considering that the muon anomalous magnetic moment is constrained to be in the range:

$$(\Delta a_{\mu})_{\text{exp}} = (26.1 \pm 8) \times 10^{-10}, \quad (21)$$





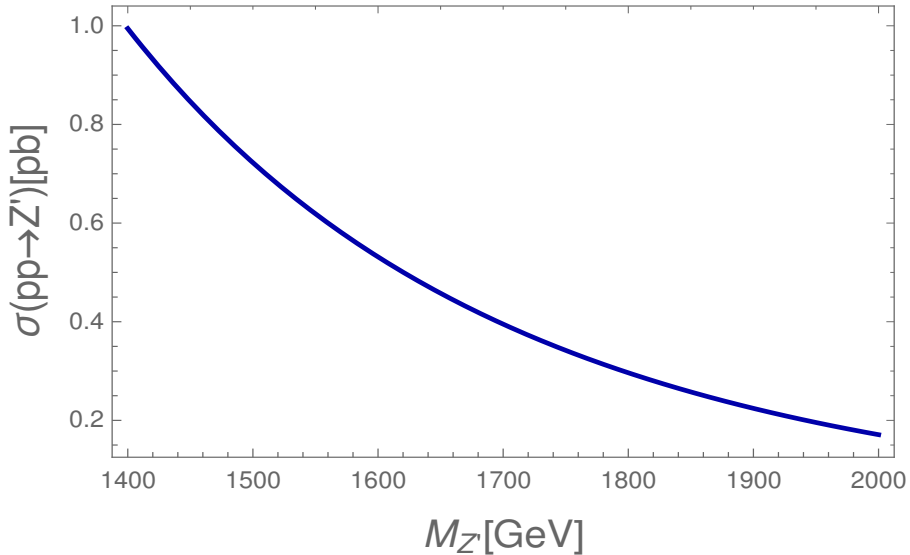


Figure: The total Z' production cross section via the DY mechanism at the LHC for $\sqrt{S} = 13$ TeV and $g_{\chi} = 0.1$ as a function of the Z' mass.

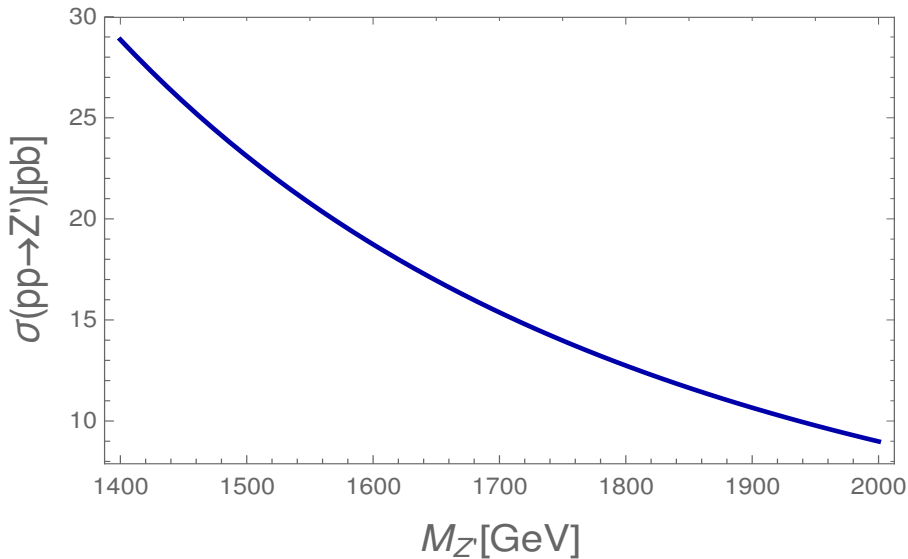


Figure: The total Z' production cross section via the DY mechanism at a future pp collider for $\sqrt{S} = 100$ TeV and $g_X = 0.1$ as a function of the Z' mass.

For $m_{\Phi_{DM}}^2 \gg v^2$, with $v = 246$ GeV, one has the estimate:

$$\langle \sigma v \rangle \simeq \frac{\gamma^2}{128\pi m_{\Phi_{DM}}^2}, \quad (22)$$

which results in a DM relic abundance

$$\frac{\Omega_{DM} h^2}{0.12} = \frac{0.1 \text{ pb}}{0.12 \langle \sigma v \rangle} \simeq \left(\frac{1}{\gamma} \right)^2 \left(\frac{m_{\Phi_{DM}}}{1.1 \text{ TeV}} \right)^2, \quad (23)$$

In the scenario with a fermionic DM candidate, when $m_{\Omega_{1R}}^2 \ll m_{\eta_R}^2 \sim m_{\eta_I}^2 \sim m_{\eta}^2$, one has:

$$\langle \sigma v \rangle \simeq \frac{9y_{\Omega}^4 m_{\Omega}^2}{16\pi m_{\eta}^4}. \quad (24)$$

Then, the DM relic abundance is

$$\frac{\Omega_{DM} h^2}{0.12} = \frac{0.1 \text{ pb}}{0.12 \langle \sigma v \rangle} \simeq \left(\frac{1}{y_{\Omega}} \right)^4 \left(\frac{400 \text{ GeV}}{m_{\Omega}} \right)^2 \left(\frac{m_{\eta}}{1.9 \text{ TeV}} \right)^4. \quad (25)$$

For the $S_3 \times Z_2$ flavor model:

- The SM fermion mass hierarchy is generated by the loops.
- The cutoff scale is $\Lambda \sim 2.5$ TeV.
- The model predicts one massless and two non-zero mass neutrinos.
- The mass scale of the non-SM particles are of the order of 1 TeV.
- The model possesses DM particle candidates.

For the IDM model with sequential loop suppression mechanism

- The first renormalizable model of sequential loop suppression mechanism without soft-breaking mass terms.
- Only the top quark and exotic fermions acquire tree-level masses.
- The masses for the bottom, strange and charm quarks, tau and muon leptons are generated at one-loop level, whereas the masses for the up and down quarks as well as the electron mass appear at two-loop level.
- Light active neutrino masses arise at three-loop level.
- The model has DM particle candidates.
- The model successfully accommodates the experimental value of the muon magnetic moment.

Acknowledgements

Thank you very much to all of you for the attention.

A.E.C.H was supported by Fondecyt (Chile), Grant No. 1170803.

Extra Slides

Combining radiative mechanisms with spontaneously broken symmetries.

The S_3 symmetry is softly broken whereas the Z_8 discrete group is broken.

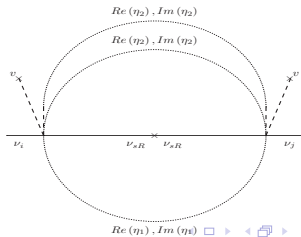
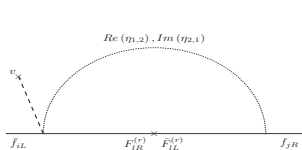
$$\begin{aligned} \phi &\sim (\mathbf{1}, 1), & \eta = (\eta_1, \eta_2) &\sim \left(\mathbf{2}, e^{-\frac{\pi i}{4}} \right), & \chi &\sim (\mathbf{1}, -i), \\ \nu_\chi &= \lambda \Lambda, & \lambda &= 0.225. \end{aligned} \quad (26)$$

$$\begin{aligned} q_{jL} &\sim \left(\mathbf{1}, e^{-\frac{\pi i(3-j)}{2}} \right), & u_{kR} &\sim \left(\mathbf{1}', e^{\frac{\pi i(3-k)}{2}} \right), & u_{3R} &\sim (\mathbf{1}, 1), \\ d_{jR} &\sim \left(\mathbf{1}', e^{\frac{\pi i(3-j)}{2}} \right), & l_{jL} &\sim \left(\mathbf{1}, e^{-\frac{\pi i(3-j)}{2}} \right), & l_{jR} &\sim \left(\mathbf{1}', e^{\frac{\pi i(3-j)}{2}} \right), \\ T_L^{(k)} &\sim \left(\mathbf{2}, e^{-\frac{\pi i}{4}} \right), & T_R^{(k)} &\sim \left(\mathbf{2}, e^{\frac{\pi i}{4}} \right), & k &= 1, 2, \\ B_L^{(j)} &\sim \left(\mathbf{2}, e^{-\frac{\pi i}{4}} \right), & B_R^{(j)} &\sim \left(\mathbf{2}, e^{\frac{\pi i}{4}} \right), & j &= 1, 2, 3, \\ E_L^{(j)} &\sim \left(\mathbf{2}, e^{-\frac{\pi i}{4}} \right), & E_R^{(j)} &\sim \left(\mathbf{2}, e^{\frac{\pi i}{4}} \right), \\ \nu_{kR} &\sim \left(\mathbf{1}', e^{-\frac{\pi i}{4}} \right), & k &= 1, 2. \end{aligned} \quad (27)$$

I use the S_3 discrete group since it is the smallest non-Abelian group.

$$\begin{aligned}
-\mathcal{L}_Y^{(U)} &= \sum_{j=1}^3 \sum_{r=1}^2 y_{jr}^{(u)} \bar{q}_{jL} \tilde{\phi} \left(T_R^{(r)} \eta \right)_1 \frac{\chi^{3-j}}{\Lambda^{4-j}} \\
&+ \sum_{r=1}^2 \sum_{s=1}^2 x_{rs}^{(u)} \left(\bar{T}_L^{(r)} \eta \right)_{1'} u_{sR} \frac{\chi^{3-k}}{\Lambda^{3-k}} \\
&+ \sum_{j=1}^3 y_{j3}^{(u)} \bar{q}_{jL} \tilde{\phi} u_{3R} \frac{\chi^{3-j}}{\Lambda^{3-j}} + \sum_{r=1}^2 y_r^{(T)} \left(\bar{T}_L^{(r)} T_R^{(r)} \right)_1 \chi + h.c.
\end{aligned}$$

$$-\mathcal{L}_Y^{(v)} = \sum_{j=1}^3 \sum_{s=1}^2 y_{js}^{(v)} \bar{l}_{jL} \tilde{\phi} \nu_{sR} \frac{[\eta^* (\eta \eta^*)_2]_{1'}}{\Lambda^{6-j}} \chi^{3-j} + \sum_{s=1}^2 y_s \bar{\nu}_{sR} \nu_{sR}^C \chi + h.c.$$



The charged fermion mass matrices are:

$$M_U = \begin{pmatrix} \varepsilon_{11}^{(u)} \lambda^3 & \varepsilon_{12}^{(u)} \lambda^2 & y_{13}^{(u)} \lambda^2 \\ \varepsilon_{21}^{(u)} \lambda^2 & \varepsilon_{22}^{(u)} \lambda & y_{23}^{(u)} \lambda \\ \varepsilon_{31}^{(u)} \lambda & \varepsilon_{32}^{(u)} & y_{33}^{(u)} \end{pmatrix} \frac{v}{\sqrt{2}}, \quad (28)$$

$$M_{D,l} = \begin{pmatrix} \varepsilon_{11}^{(d,l)} \lambda^4 & \varepsilon_{12}^{(d,l)} \lambda^3 & \varepsilon_{13}^{(d,l)} \lambda^2 \\ \varepsilon_{21}^{(d,l)} \lambda^3 & \varepsilon_{22}^{(d,l)} \lambda^2 & \varepsilon_{23}^{(d,l)} \lambda \\ \varepsilon_{31}^{(d,l)} \lambda^2 & \varepsilon_{32}^{(d,l)} \lambda & \varepsilon_{33}^{(d,l)} \end{pmatrix} \frac{v}{\sqrt{2}},$$

where the dimensionless parameters $\varepsilon_{jk}^{(f)}$ ($j, k = 1, 2, 3$) with $f = u, d, l$, are generated at one loop level. The invariance of charged exotic fermion Yukawa interactions under the cyclic symmetry requires to consider the Z_8 instead of the Z_4 discrete symmetry.

$$\begin{aligned}
\mathcal{L}_F = & y_{3j}^{(u)} \bar{q}_{3L} \tilde{\phi}_1 u_{3R} + \sum_{n=1}^2 x_n^{(u)} \bar{q}_{nL} \tilde{\phi}_2 T_R + \sum_{n=1}^2 z_j^{(u)} \bar{T}_L \eta^* u_{nR} + y_T \bar{T}_L \sigma_1 T_R + m_{\tilde{T}} \bar{\tilde{T}}_L \tilde{T}_R + x^{(T)} \bar{T}_L \rho_2 \tilde{T}_R \\
& + \sum_{n=1}^2 x_n^{(d)} \bar{q}_{3L} \phi_2 B_{nR} + \sum_{n=1}^2 \sum_{j=1}^3 y_{nj}^{(d)} \bar{B}_{nL} \eta d_{jR} + \sum_{j=1}^3 z_j^{(d)} \bar{B}_{3L} \eta^* d_{jR} + \sum_{n=1}^2 w_n^{(u)} \bar{B}_{4L} \varphi_1^- u_{nR} \\
& + \sum_{k=3}^4 m_{B_k} \bar{B}_{kL} B_{kR} + \sum_{n=1}^2 x_n^{(d)} \bar{q}_{nL} \phi_2 B_{3R} + \sum_{n=1}^2 \sum_{m=1}^2 y_{nm}^{(B)} \bar{B}_{nL} \sigma_1^* B_{mR} + z^{(B)} \bar{B}_{3L} \sigma_2^* B_{4R} + \sum_{j=1}^3 w_j^{(d)} \bar{\tilde{T}}_L \varphi_2^+ \\
& + \sum_{k=1,3} x_{k3}^{(l)} \bar{l}_{kL} \phi_2 E_{3R} + \sum_{k=1,3} y_{3k}^{(l)} \bar{E}_{3L} \rho_1 l_{kR} + x_{22}^{(l)} \bar{l}_{2L} \phi_2 E_{2R} + y_{22}^{(l)} \bar{E}_{2L} \rho_1 l_{2R} \\
& + \sum_{i=1}^3 y_i^{(E)} \bar{E}_{iL} \sigma_1^* E_{iR} + x_2^{(\nu)} \bar{l}_{2L} \tilde{\phi}_2 \nu_{2R} + \sum_{k=1,3} z_k^{(l)} \bar{\Psi}_R^C \varphi_3^+ l_{kR} + \sum_{k=1,3} z_k^{(\nu)} \bar{E}_{1L} \varphi_1^- \nu_{kR} + z^{(E)} \bar{\Psi}_R^C \varphi_4^+ E_{1R} \\
& + \sum_{k=1,3} \sum_{n=1,3} x_{kn}^{(\nu)} \bar{l}_{kL} \tilde{\phi}_2 \nu_{nR} + \sum_{k=1,3} y_k^{(\Omega)} \bar{\Omega}_{1R}^C \eta^* \nu_{kR} + y^{(\Omega)} \bar{\Omega}_{1R}^C \sigma_3^* \nu_{2R} \\
& + x_1^{(\Psi)} \bar{\Omega}_{1R}^C \eta \Psi_R + x_2^{(\Psi)} \bar{\Omega}_{2R}^C \eta^* \Psi_R + z_{\Omega} \bar{\Omega}_{1R}^C \sigma_2^* \Omega_{2R} + m_{\Psi} \bar{\Psi}_R^C \Psi_R + h.c.,
\end{aligned}$$

Parameter	$\frac{\Delta C_9^{\mu\mu}}{C_9^{SM}}$
Best fit	-0.21
1σ range	-0.27 up to -0.13
2σ range	-0.32 up to -0.08

Table: Constraints on the $C_9^{\mu\mu}$ Wilson coefficient from the LHCb data. Taken from Hurth, et al, 2016.

$$\Delta H_{eff} = -\frac{G_F \alpha_{em} V_{tb} V_{ts}^*}{\sqrt{2}\pi} \sum_{\tilde{l}=e,\mu,\tau} C_9^{\tilde{l}\tilde{l}} (\bar{s}\gamma^\mu P_L b) (\tilde{l}\gamma^\mu \tilde{l}) . \quad (29)$$

$$\Delta C_9^{\mu\mu} = -\frac{9g_X^2}{2M_{Z'}^2} (V_{DL}^*)_{32} (V_{DL})_{33} \frac{\sqrt{2}\pi}{G_F \alpha_{em} V_{tb} V_{ts}^*} \simeq -\frac{9g_X^2}{2M_{Z'}^2} \frac{\sqrt{2}\pi}{G_F \alpha_{em}} . \quad (30)$$