FLASY2019: 8th Workshop on Flavour Symmetries and Consequences in Accelerators and Cosmology

USTC Hefei, 24-26 July 2019

gCP symmetry in modular- -invariant models of flavour

in collaboration with S.T. Petcov, A.V. Titov and P.P. Novichkov [1905.11970, accepted in JHEP]

INFN
SISSA
FCT Fundação para a Ciência e a Tecnologia
DF Departamento de Física Técnico Lisboa
CFTP Centro de Física Teórica de Partículas
European Union
QREN Quadro de Referência Estratégico Nacional
COMPETE

João Penedo
(CFTP, Lisbon)

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in collaboration with S.T. Petcov, A.V. Titov and P.P. Novichkov [1905.11970, accepted in JHEP]

$\nexists$ 

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3ν flavour paradigm


Recall e.g. talk by J. Valle

Masses: ordering

$$\frac{\Delta m^2_\odot}{|\Delta m^2_A|} \sim \frac{1}{30}$$

Normal ordering (NO)

$m_1 < m_2 < m_3$

vs.

Inverted ordering (IO)

$m_3 < m_1 < m_2$

Mixing: parameterisation

$c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$

$$U_{\rm PMNS} = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\alpha_{21}/2} & \\ & & e^{i\alpha_{31}/2} \end{pmatrix}$$

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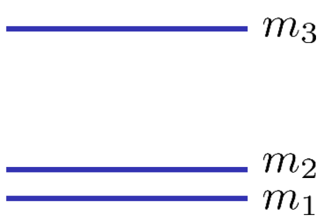

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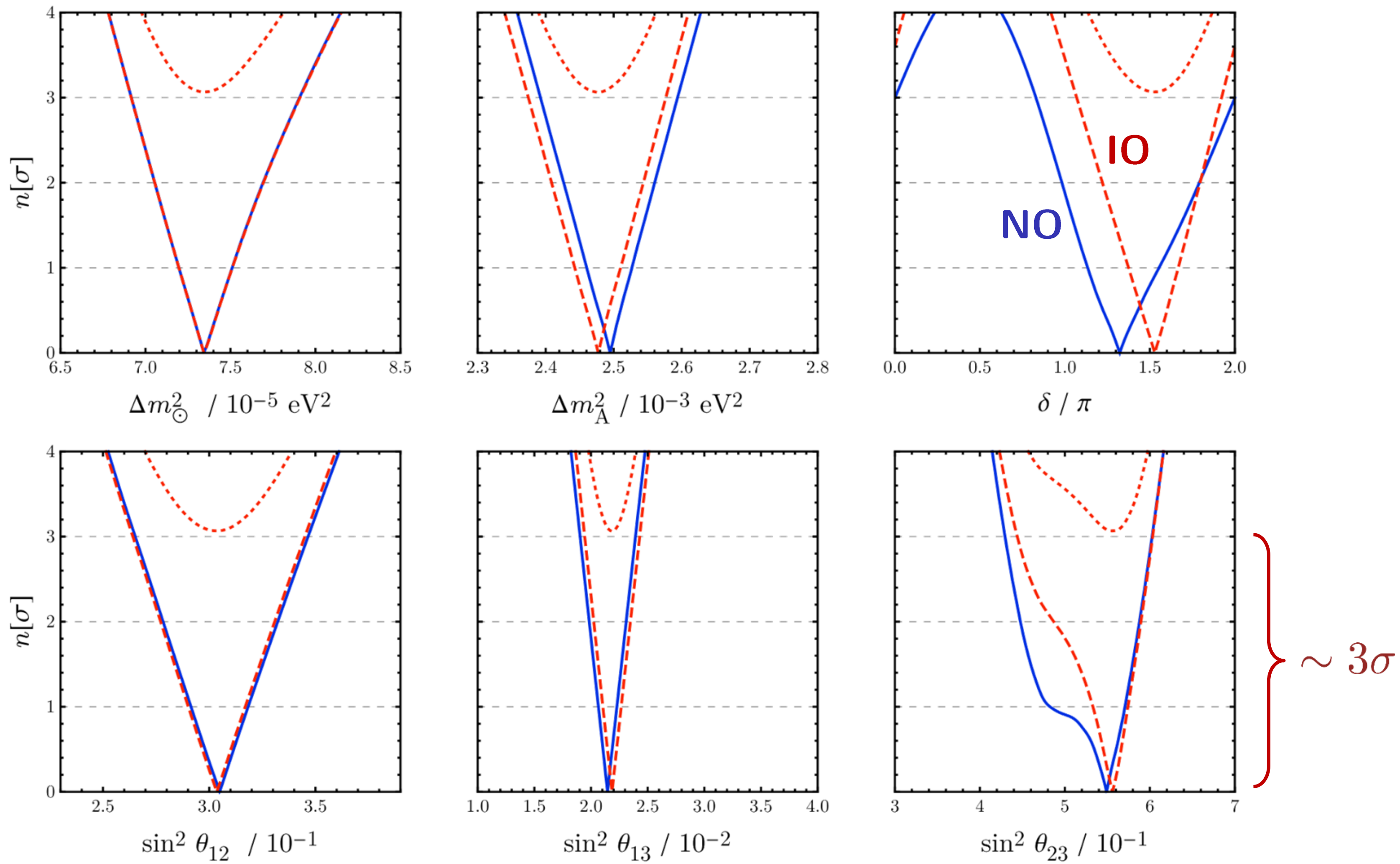
3ν flavour paradigm (cont.)

Capozzi et al., 1804.09678,
see also Esteban et al., 1811.05487



3ν flavour paradigm (cont.)

Capozzi et al., 1804.09678,
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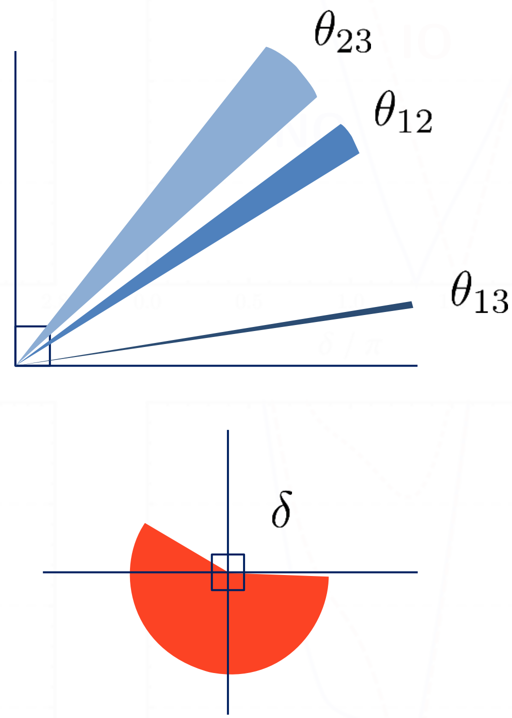
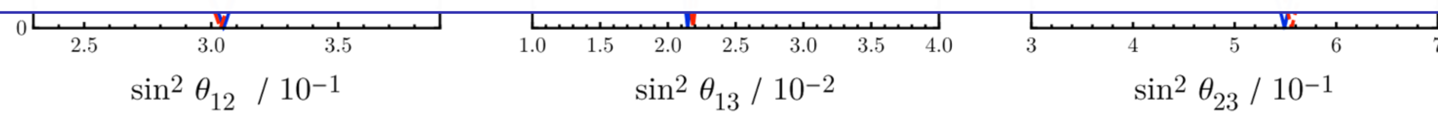
For a spectrum with NO:

Parameter	Best-fit value
$\Delta m^2_\odot$	7.34×10^{-5} eV^2
$\lvert\Delta m^2_\mathrm{A}\rvert$	2.49×10^{-3} eV^2
$\sin^2\theta_{12}$	0.304
$\sin^2\theta_{13}$	0.0214
$\sin^2\theta_{23}$	0.551
δ	$1.32\,\pi$

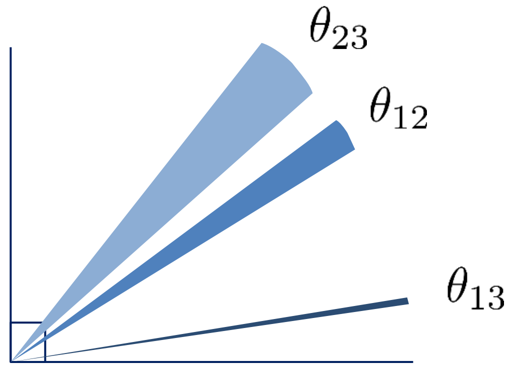


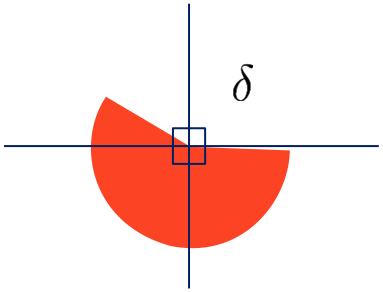
$\sim 3\sigma$



Is there an organizing principle behind this?

$$\frac{\Delta m_\odot^2}{|\Delta m_A^2|} \sim \frac{1}{30}$$





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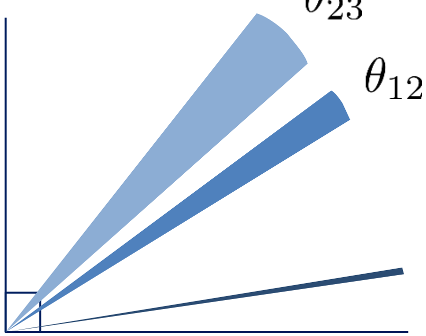
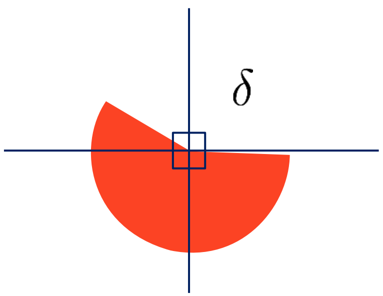
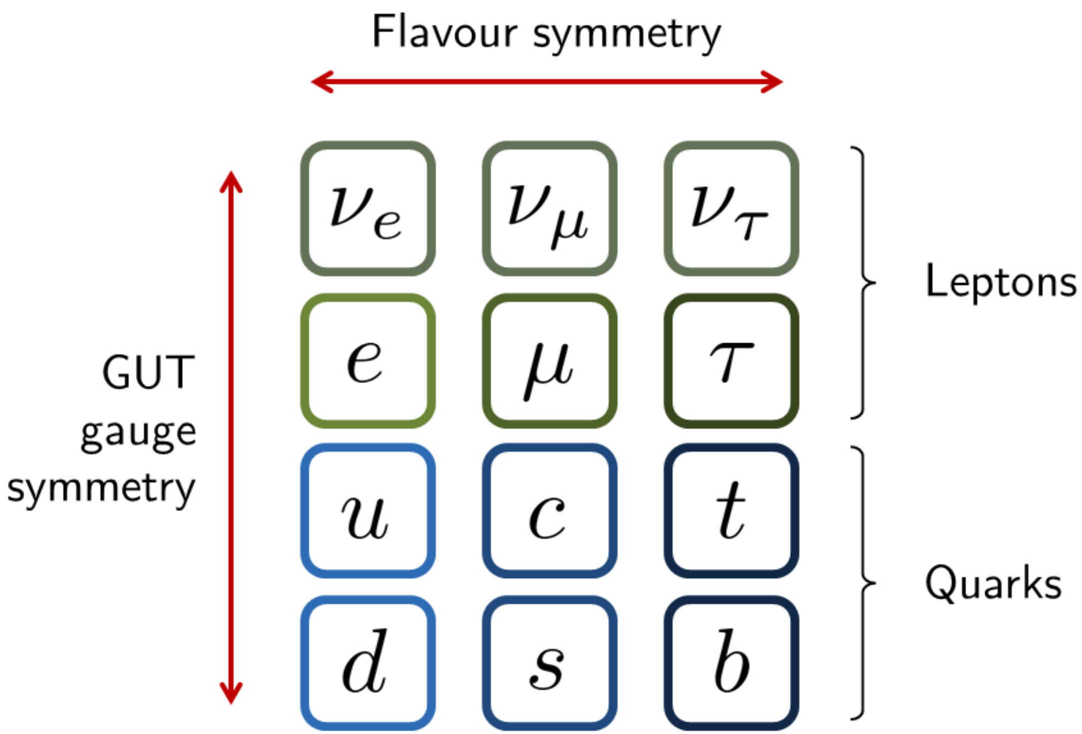




Flavour symmetries

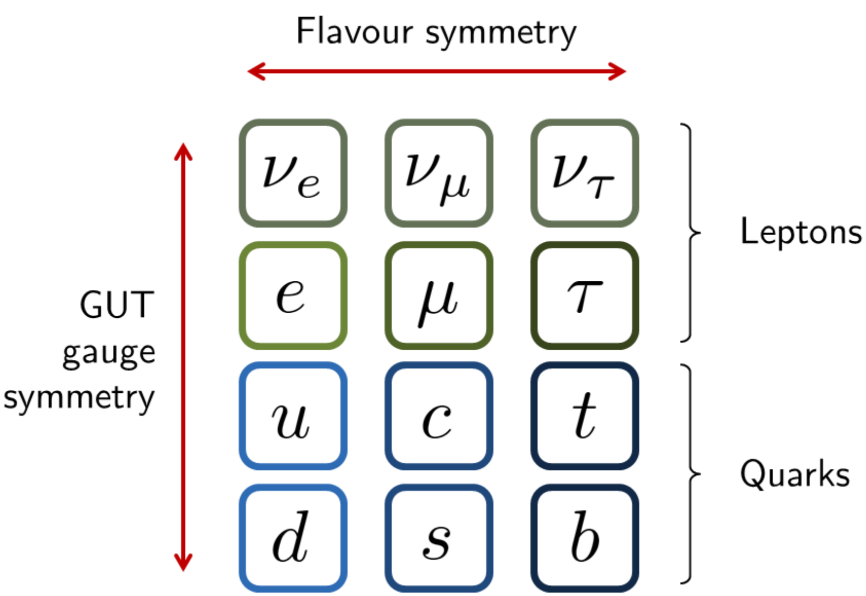


For reviews, see: Altarelli and Feruglio (2010), Ishimori et al. (2010), King and Luhn (2013), Petcov (2017)

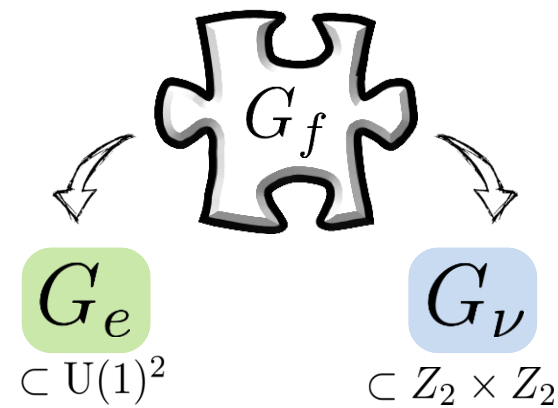
Flavour symmetries

For the lepton sector, at low energy and in some flavour basis:

$$\mathcal{L}_\ell = -\left(M_e\right)_{ij} \overline{\ell_{iL}} \, \ell_{jR} - \frac{1}{2} \left(M_\nu\right)_{ij} \overline{\nu^C_{iR}} \, \nu_{jL} + \text{h.c.}$$



Non-Abelian discrete flavour symmetries



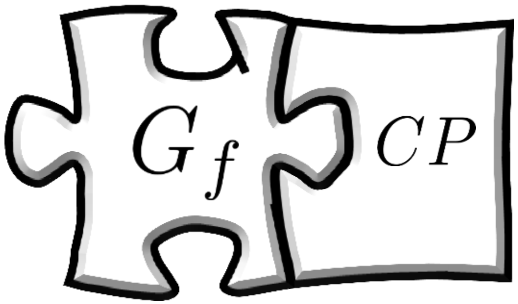
constrain mixing and Dirac phase

Flavour symmetries + gCP

$$\psi(x) \to \rho_{\mathbf{r}}(g)\,\psi(x) \qquad\qquad \psi(x) \to X_{\mathbf{r}}^{\mathrm{CP}}\,\overline{\psi}(x_{\mathrm{P}})$$

Branco, Lavoura, Rebelo (1986), Harrison, Scott (2002), Grimus, Lavoura (2003), Farzan, Smirnov (2006), Ferreira, Grimus, Lavoura, Ludl (2012) , ...

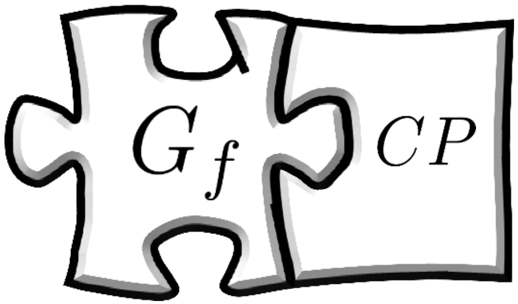
Flavour symmetries + gCP



constrain mixing, Dirac and Majorana phases

Feruglio, Hagedorn, Ziegler (2012),
Holthausen, Lindner, Schmidth (2013),
Chen, Fallbacher, Mahanthappa, Ratz, Trautner (2014), ...

Flavour symmetries + gCP

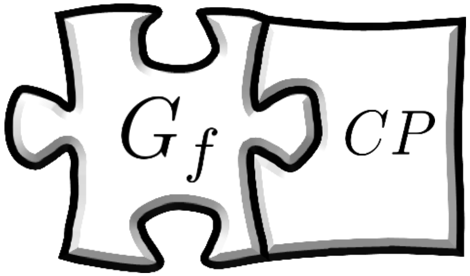


$$X_{\mathbf{r}}^{\mathrm{CP}}\,\rho_{\mathbf{r}}^{*}(g)\,\left(X_{\mathbf{r}}^{\mathrm{CP}}\right)^{-1}=\rho_{\mathbf{r}}(u(g))$$

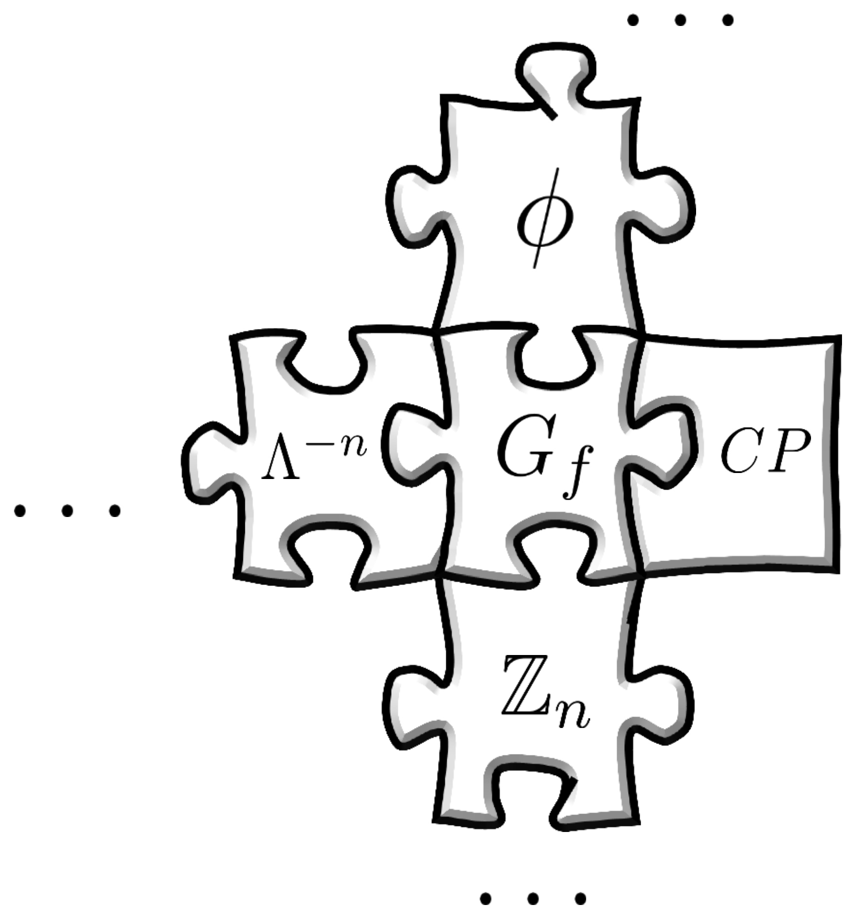
Consistency condition [Feruglio, et al., Holthausen et al. (2012)]

Class-inverting outer automorphism [Chen et al. (2014)]

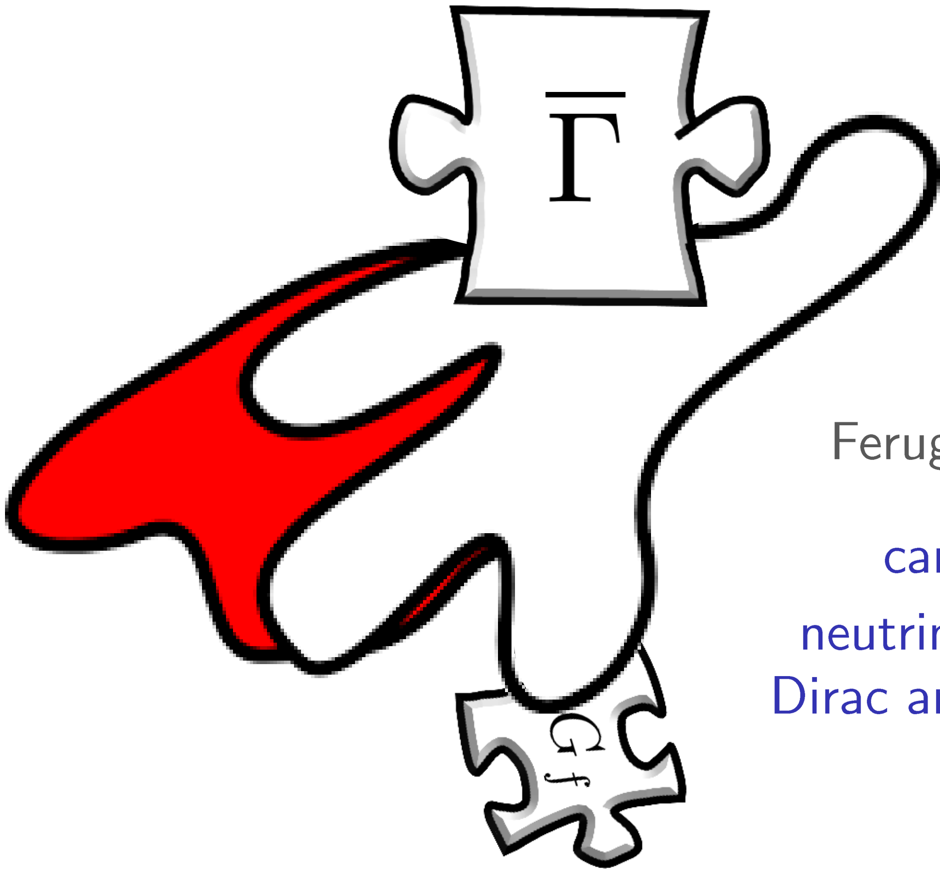
Problems with the usual approach



Problems with the usual approach



Modular symmetry to the rescue!



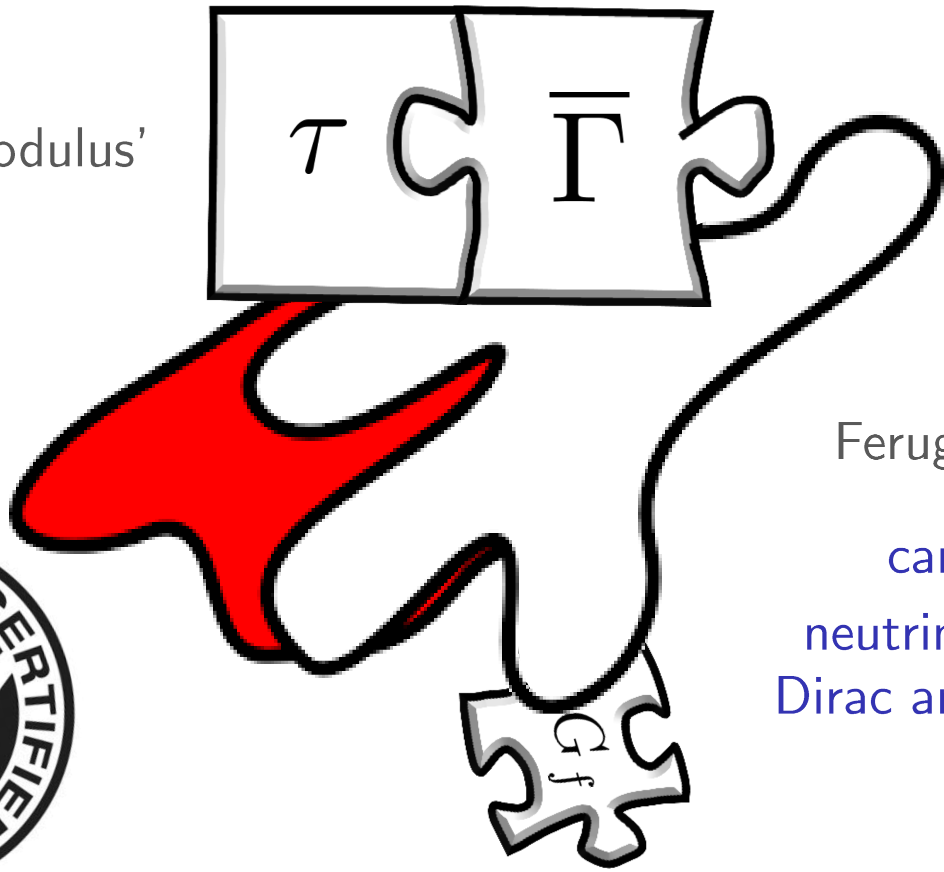
Feruglio, 1706.08749

can constrain all:

neutrino masses, mixing,
Dirac and Majorana phases

Modular symmetry to the rescue!

'modulus'



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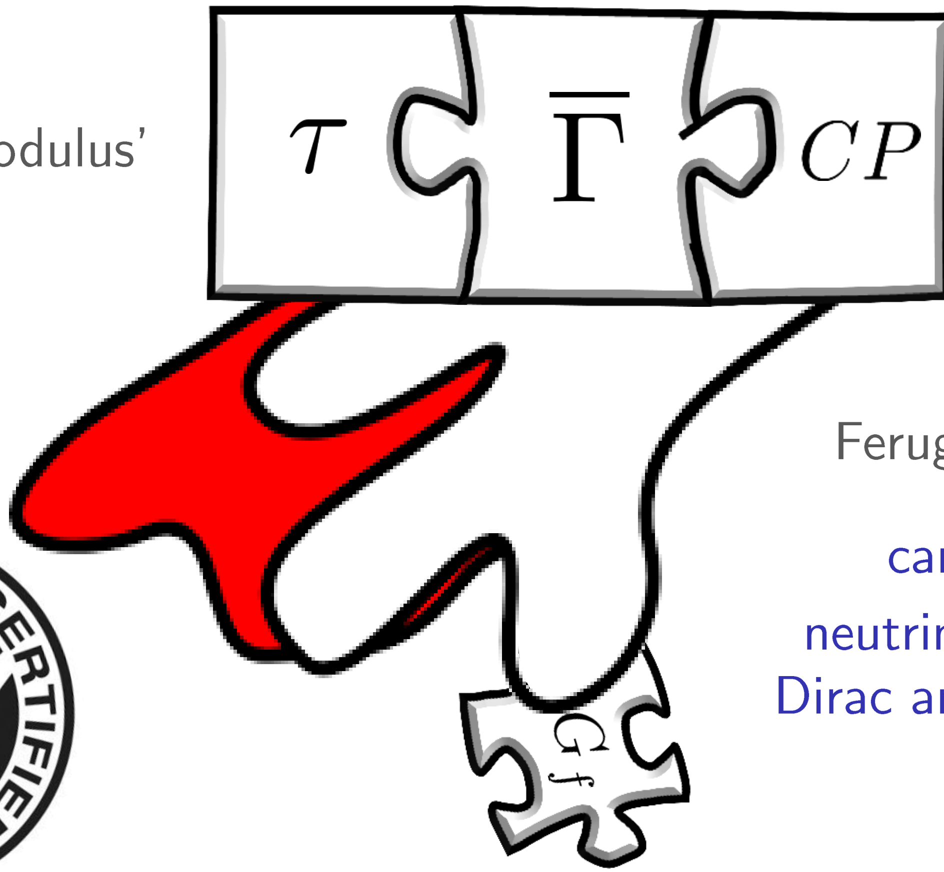
Modular symmetry to the rescue!

This work, 1905.11970

‘modulus’



Feruglio, 1706.08749

can constrain all:

neutrino masses, mixing,
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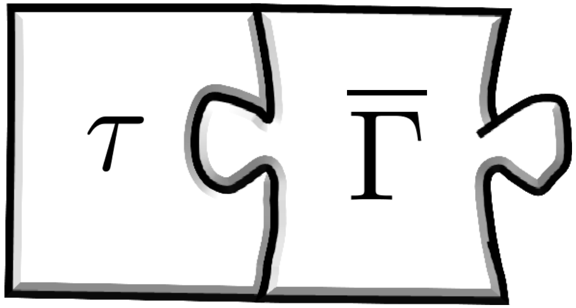
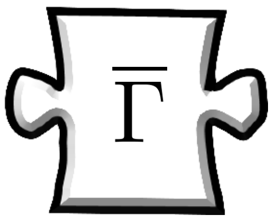
How does this work?



Modular symmetry

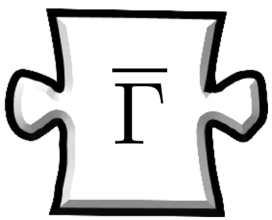


$$\overline{\Gamma} \simeq \mathrm{PSL}(2,\mathbb{Z})$$

$$\tau \to \frac{a\tau + b}{c\tau + d}$$

$$a, b, c, d \in \mathbb{Z} \qquad \begin{vmatrix} a & b \\ c & d \end{vmatrix} = 1$$

Modular symmetry



$$\overline{\Gamma} \simeq \mathrm{PSL}(2,\mathbb{Z})$$

$$\tau \to \frac{a\tau + b}{c\tau + d}$$

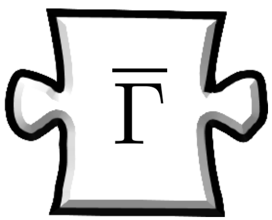
$$a, b, c, d \in \mathbb{Z} \qquad \begin{vmatrix} a & b \\ c & d \end{vmatrix} = 1$$

$$S^2 = (ST)^3 = 1 \quad \begin{cases} S: \ \tau \to -1/\tau\,, & S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \\ T: \ \tau \to \tau + 1\,, & T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \end{cases}$$

For recent generalisations, see Varzielas et al., 1906.02208; Liu, Ding, 1907.01488

Modular symmetry

Quotient behaves like a flavour group:



$$\underbrace{\overline{\Gamma} \,/\, \overline{\Gamma}(N)}_{\Gamma_N}$$

Bottom-up approach

For top-down, see e.g.:

Kobayashi et al., 1804.06644
Kobayashi, Tamba, 1811.11384
de Anda et al., 1812.05620
Baur et al., 1901.03251
Kariyazono et al., 1904.07546

Modular symmetry

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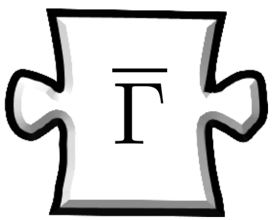


$$/\ \overline{\Gamma}(N)$$

$$\Gamma_N$$

$$S^2 = (ST)^3 = T^N = 1$$

$\Gamma_2 \simeq S_3$

Kobayashi et al., 1803.10391 (+A_4)
Kobayashi et al., 1812.11072 (+A_4)
Kobayashi et al., 1906.10341
Okada, Orikasa, 1907.04716

$\Gamma_3 \simeq A_4$

Feruglio, 1706.08749
Feruglio, Criado, 1807.01125
Kobayashi et al., 1808.03012
Okada, Tanimoto, 1812.09677
Novichkov et al., 1812.11289
Nomura, Okada, 1904.03937
Okada, Tanimoto, 1905.13421
Nomura, Okada, 1906.03927

$\Gamma_4 \simeq S_4$

JP, Petcov, 1806.11040
Novichkov et al., 1811.04933
Kobayashi et al., 1907.09141

$\Gamma_5 \simeq A_5$

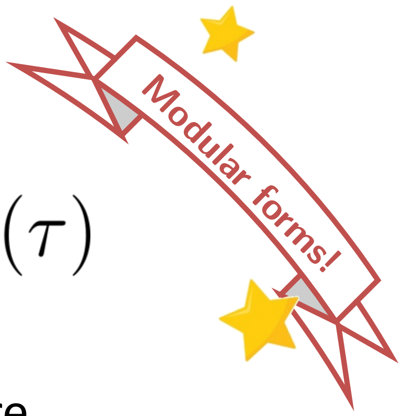
Novichkov et al., 1812.02158
Ding et al., 1903.12588

Modular forms: the stars of the show

Transformation of superfields:

$$\psi \to (c\tau + d)^{-k_\psi} \underbrace{\rho_{\mathbf{r}}(\gamma)}_{\Gamma_N,\ \gamma \in \overline{\Gamma}} \psi$$

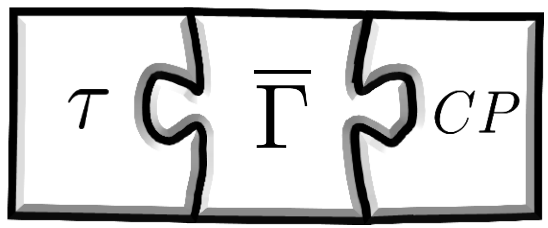
Invariance of superpotential requires functions:



$$Y(\tau) \to (c\tau + d)^{k_Y} \rho_{\mathbf{r}_Y}(\gamma)\, Y(\tau)$$

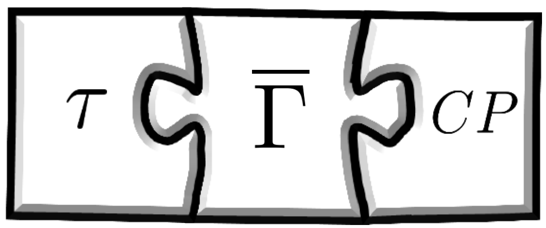
Play the role of flavons, but structures are **completely fixed** given the modulus VEV

Modular symmetry + gCP



$\tau \xrightarrow{\text{CP}} ?$ $\psi \xrightarrow{\text{CP}} ?$ $Y(\tau) \xrightarrow{\text{CP}} ?$

Modular symmetry + gCP



$\tau \xrightarrow{\text{CP}} ?$ $\qquad \psi(x) \xrightarrow{\text{CP}} X_{\mathbf{r}}^{\text{CP}}\,\overline{\psi}(x_{\text{P}})$ $\qquad Y(\tau) \xrightarrow{\text{CP}} ?$

Modular symmetry + gCP: the modulus

$$\underbrace{\mathrm{CP} \to \gamma \in \overline{\Gamma} \to \mathrm{CP}^{-1}}_{\gamma' \in \overline{\Gamma}} \implies \tau_{\mathrm{CP}} = n - \tau^*$$

$$(\tau_{\mathrm{CP}^2} = \tau)$$

$n \in \mathbb{Z}$, but can choose n = 0 without loss of generality:

$$\boxed{\tau_{\mathrm{CP}} = -\tau^*}$$

Extended modular group

$$\tau_{\mathrm{CP}} = -\tau^*$$

Extended modular group

$$\tau \xrightarrow{\text{CP}} -\tau^* \xrightarrow{\gamma} -\frac{a\tau^* + b}{c\tau^* + d} \xrightarrow{\text{CP}^{-1}} \frac{a\tau - b}{-c\tau + d}$$

$$u(\gamma) \equiv \text{CP}\,\gamma\,\text{CP}^{-1} = \begin{pmatrix} a & -b \\ -c & d \end{pmatrix}$$

$$\tau_{\text{CP}} = -\tau^*$$

$$\text{CP} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Extended modular group

$$\overline{\Gamma}^* = \left\langle \tau \xrightarrow{T} \tau + 1,\, \tau \xrightarrow{S} -1/\tau,\, \tau \xrightarrow{CP} -\tau^* \right\rangle \simeq \mathrm{PGL}(2, \mathbb{Z})$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \overline{\Gamma}^* : \quad \begin{cases} \tau \to \dfrac{a\tau + b}{c\tau + d} & \text{if} \quad ad - bc = 1 \\ \tau \to \dfrac{a\tau^* + b}{c\tau^* + d} & \text{if} \quad ad - bc = -1 \end{cases}$$

Modular symmetry + gCP: consistency

Déjà vu

$$X_{\mathbf{r}}^{\mathrm{CP}}\, \rho_{\mathbf{r}}^{*}(\gamma)\, \left(X_{\mathbf{r}}^{\mathrm{CP}}\right)^{-1} = \rho_{\mathbf{r}}(u(\gamma))$$

but now there is a unique automorphism

$$u(\gamma) \equiv \mathrm{CP}\,\gamma\,\mathrm{CP}^{-1} = \begin{pmatrix} a & -b \\ -c & d \end{pmatrix} \quad \Rightarrow \quad X_{\mathbf{r}}^{\mathrm{CP}} \text{ uniquely determined}$$

In a symmetric basis:

$\rho_{\mathbf{r}}^{*}(\gamma) = \rho_{\mathbf{r}}(u(\gamma))$

$$X_{\mathbf{r}}^{\mathrm{CP}} = \mathbb{1}_{\mathbf{r}}$$

CP is canonical

Modular symmetry + gCP: the modular forms

$$Y(\tau) \xrightarrow{\text{CP}} ?$$

$$Y(\tau) \xrightarrow{\text{CP}} Y(\tau_{\text{CP}}) = Y(-\tau^*)$$

$$Y(\tau) \sim \psi$$

under the modular group

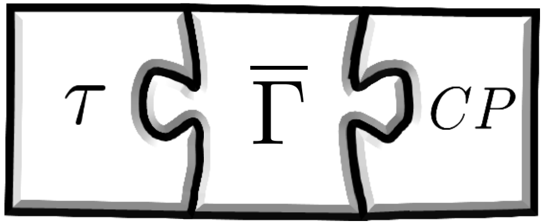
$$Y(-\tau^*) = X_{\mathbf{r}}^{\text{CP}} \, Y^*(\tau)$$

In a sym. basis Y's conjugate, q-expansions have real coeffs

Proven to hold:

- Up to a phase normalisation
- For the groups studied in the literature (N<6)

Modular symmetry + gCP



$$\tau \xrightarrow{\text{CP}} -\tau^* \qquad \psi(x) \xrightarrow{\text{CP}} X_{\mathbf{r}}^{\text{CP}}\,\overline{\psi}(x_{\text{P}}) \qquad Y(\tau) \xrightarrow{\text{CP}} X_{\mathbf{r}}^{\text{CP}}\,Y^*(\tau)$$

Unbroken gCP: couplings

$$W \overset{\text{CP}}{\longleftrightarrow} \overline{W}$$

In a symmetric basis:

$$g(Y\psi\ldots\psi)_{\mathbf{1}} \overset{\text{CP}}{\longleftrightarrow} g^*\overline{(Y\psi\ldots\psi)_{\mathbf{1}}}$$

$$\psi(x) \xrightarrow{\text{CP}} \overline{\psi}(x_{\text{P}})$$

$$Y(\tau) \xrightarrow{\text{CP}} Y^*(\tau)$$

$$g(Y^*\overline{\psi}\ldots\overline{\psi})_{\mathbf{1}}$$

$N < 6$

$$g \in \mathbb{R}$$

Unbroken gCP: observables

In a symmetric basis, gCP implies:

$$M_{e,\nu}(-\tau^*) = M^*_{e,\nu}(\tau)$$

$$M_{e,\nu} \equiv M_{e,\nu}(\underset{\in \mathbb{R}}{g}, \tau)$$

Unbroken gCP: observables

In a symmetric basis, gCP implies:

$$M_{e,\nu}(-\tau^*) = M_{e,\nu}^*(\tau)$$

$$M_{e,\nu} \equiv M_{e,\nu}(\underset{\in \mathbb{R}}{g}, \tau)$$

?

CP-conserving values of the modulus

In a symmetric basis, gCP implies:

$$M_{e,\nu}(-\tau^*) = M^*_{e,\nu}(\tau)$$

$$M_{e,\nu} \equiv M_{e,\nu}(\underbrace{g}_{\in \mathbb{R}}, \tau)$$

?

Naïvely: need $-\tau^* = \tau$ to get real mass matrix and hence

$$\sin\delta = \sin\alpha_{21} = \sin\alpha_{31} = 0$$

CP-conserving values of the modulus

In a symmetric basis, gCP implies:

$$M_{e,\nu}(-\tau^*) = M_{e,\nu}^*(\tau)$$

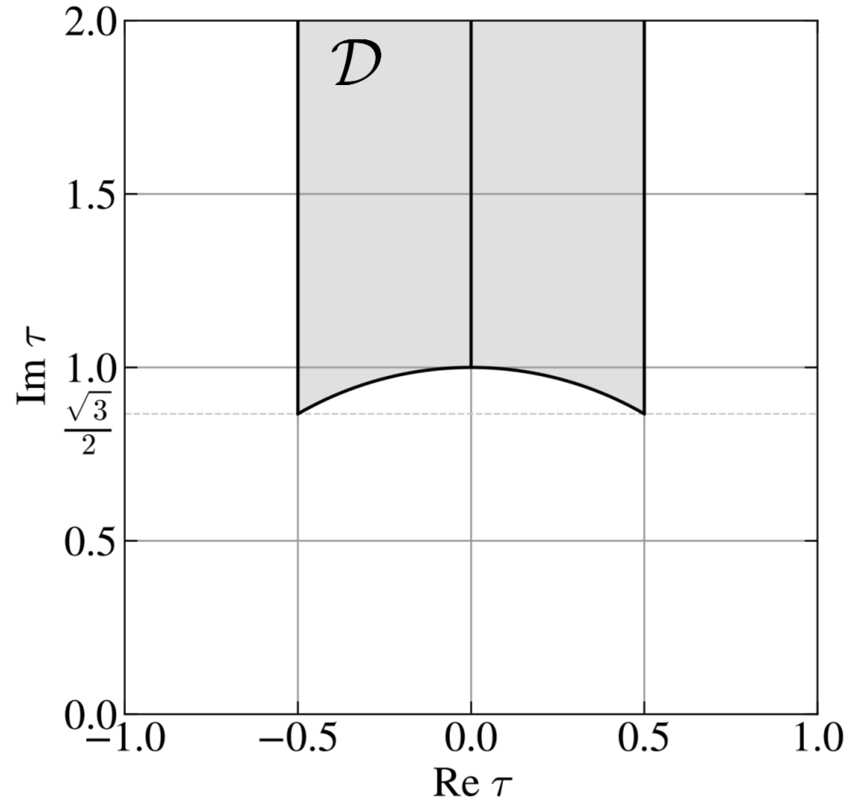
Actually: just need $-\tau^* = \gamma\tau$ to get

$$\sin\delta = \sin\alpha_{21} = \sin\alpha_{31} = 0$$

$$\text{obs}[M_{e,\nu}(\tau)] = \text{obs}[M_{e,\nu}(\gamma\tau)] = \text{obs}[M_{e,\nu}^*(\tau)]$$

CP-conserving values of the modulus

Points outside fundamental domain **physically equivalent** to pts inside

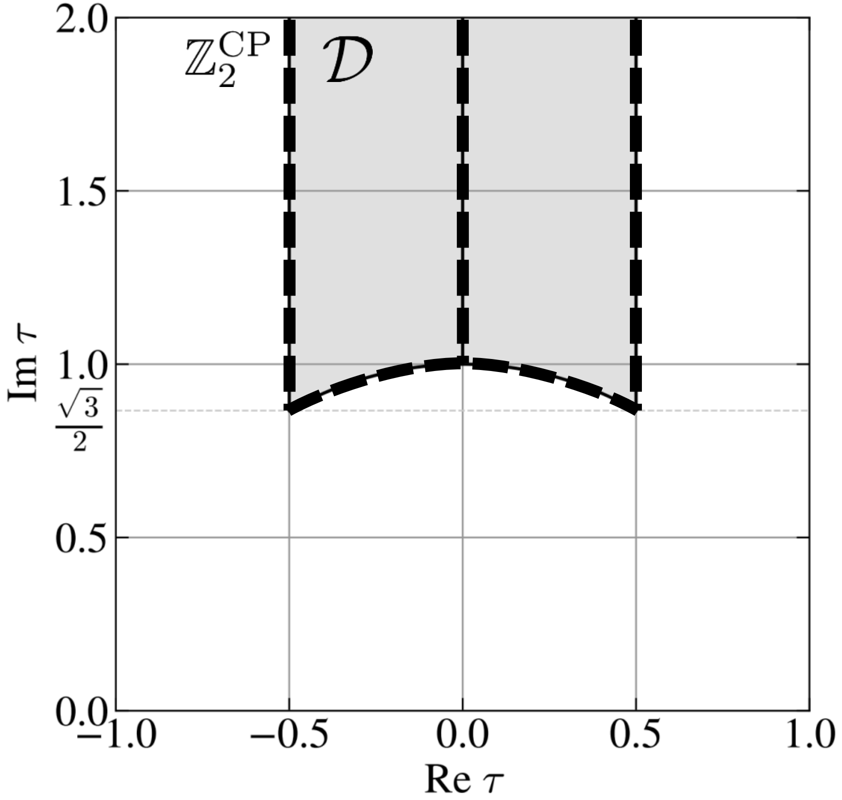


CP-conserving values of the modulus

Points outside fundamental domain **physically equivalent** to pts inside

CPV away from dashed lines

$-\tau^* = \gamma\tau$



$M_{e,\nu} \equiv M_{e,\nu}(g, \tau)$ → can be **only source** of CPV

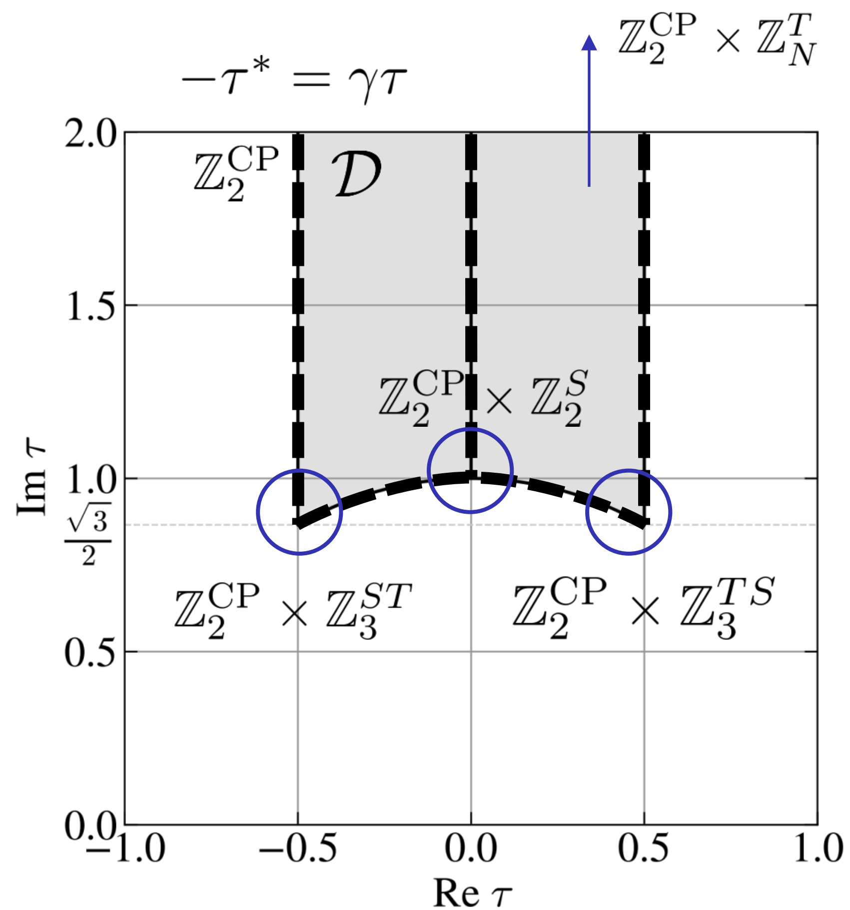
CP-conserving values of the modulus

Points outside fundamental domain **physically equivalent** to pts inside

CPV away from dashed lines

Enhancement by **residual** modular symmetries at certain points

Novichkov et al., 1811.04933
Novichkov et al.', 1812.11289



$M_{e,\nu} \equiv M_{e,\nu}(g, \tau)$ → can be **only source** of CPV, like in...

An (S_4) example, finally!

Ingredients: ***N*, field content, their weights and irreps**

Recipe: find couplings and τ

$$W = \alpha \left(E_1^c L\, Y_{\mathbf{3'}}^{(2)}\right)_{\mathbf{1}} H_d + \beta \left(E_2^c L\, Y_{\mathbf{3}}^{(4)}\right)_{\mathbf{1}} H_d + \gamma \left(E_3^c L\, Y_{\mathbf{3'}}^{(4)}\right)_{\mathbf{1}} H_d + g \left(N^c L\, Y_{\mathbf{2}}^{(2)}\right)_{\mathbf{1}} H_u + g' \left(N^c L\, Y_{\mathbf{3'}}^{(2)}\right)_{\mathbf{1}} H_u + \Lambda \left(N^c N^c\right)_{\mathbf{1}} ,$$

$g' \in \mathbb{C}$

minimal setup

Novichkov, JP, Petcov, Titov, 1811.04933

An (S_4) example, finally!

Ingredients: ***N*, field content, their weights and irreps**

Recipe: **find couplings and τ**

$$W = \alpha \left(E_1^c L\, Y_{\mathbf{3'}}^{(2)}\right)_{\mathbf{1}} H_d + \beta \left(E_2^c L\, Y_{\mathbf{3}}^{(4)}\right)_{\mathbf{1}} H_d + \gamma \left(E_3^c L\, Y_{\mathbf{3'}}^{(4)}\right)_{\mathbf{1}} H_d$$
$$+ g \left(N^c L\, Y_{\mathbf{2}}^{(2)}\right)_{\mathbf{1}} H_u + g' \left(N^c L\, Y_{\mathbf{3'}}^{(2)}\right)_{\mathbf{1}} H_u + \Lambda \left(N^c N^c\right)_{\mathbf{1}} \,,$$

$g' \in \mathbb{C}$

Novichkov, JP, Petcov, Titov, 1811.04933

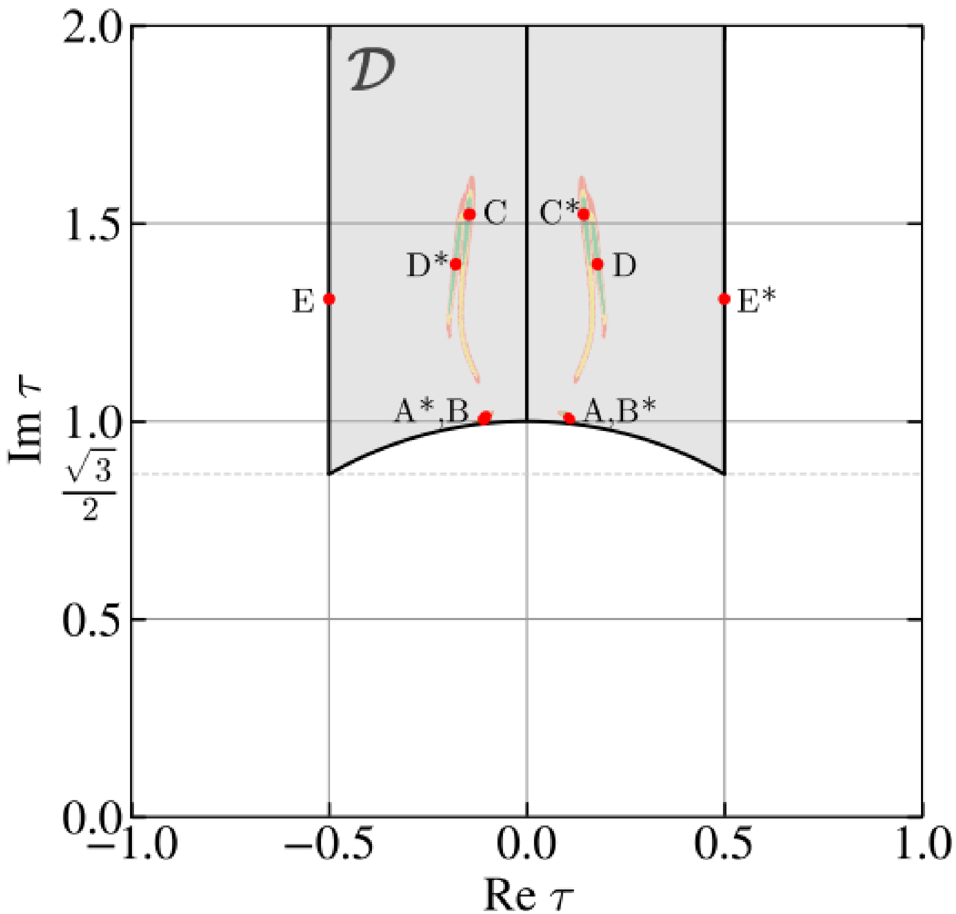
minimal setup
→ even more minimal

$$\text{gCP} \;\Rightarrow\; g' \in \mathbb{R}$$

τ is the only source of CPV

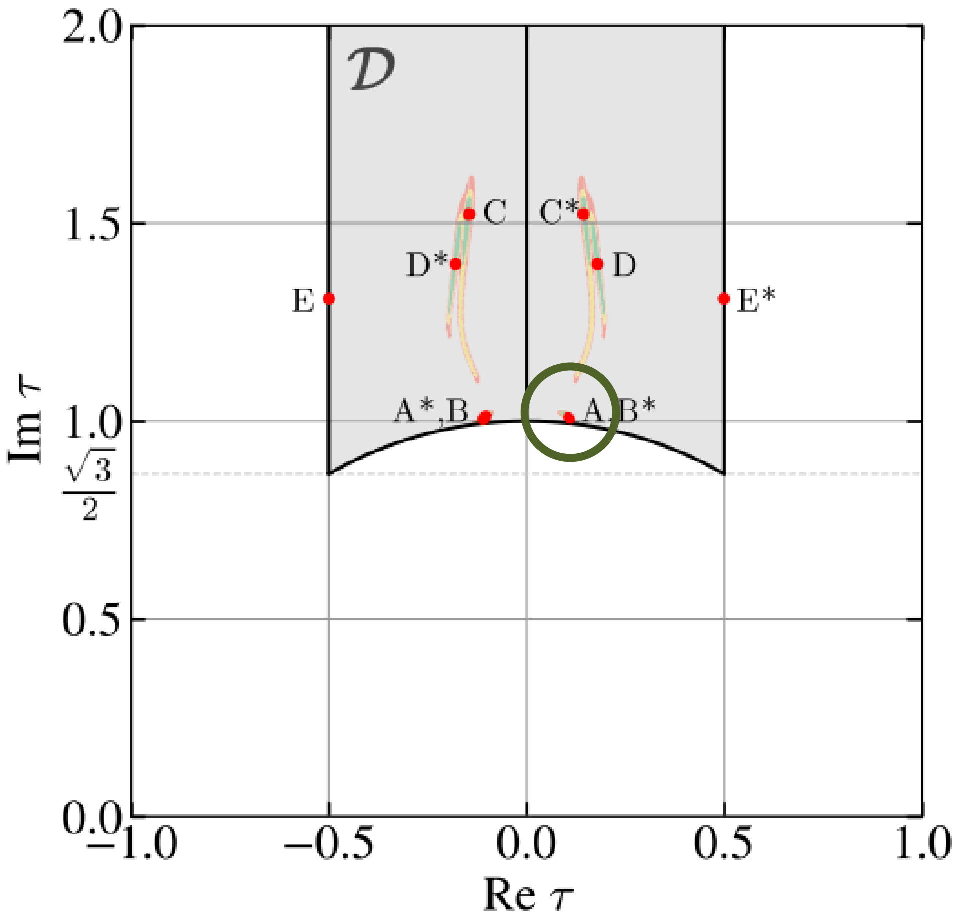
happens accidentally in a model by Feruglio, Criado, 1807.01125

An S_4 example



Novichkov, JP, Petcov, Titov, 1811.04933

An S_4 example



this work/example, 1905.11970

$$\tau \simeq \pm 0.1 + 1.02\, i$$

Novichkov, JP, Petcov, Titov, 1811.04933

An S_4 example

7 (4) parameters

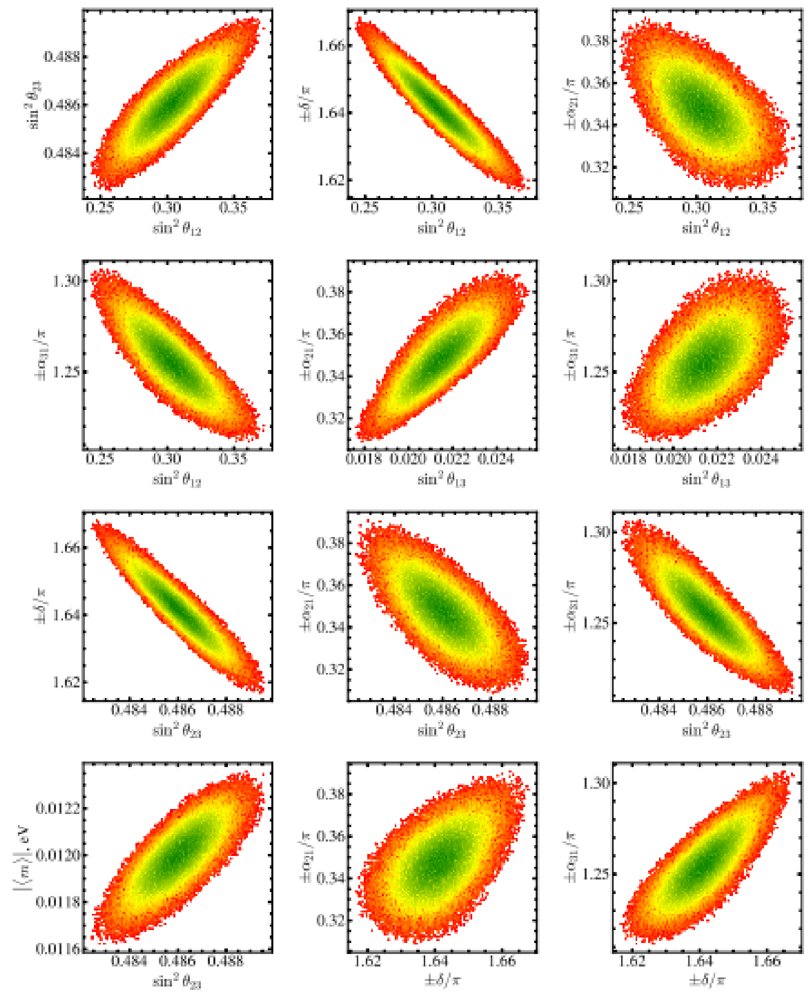
vs.

12 (9) observables

$$\sin^2\theta_{23} \sim 0.49$$
$$\delta \sim 1.6\pi$$
$$\alpha_{21} \sim 0.3\pi$$
$$\alpha_{31} \sim 1.3\pi$$

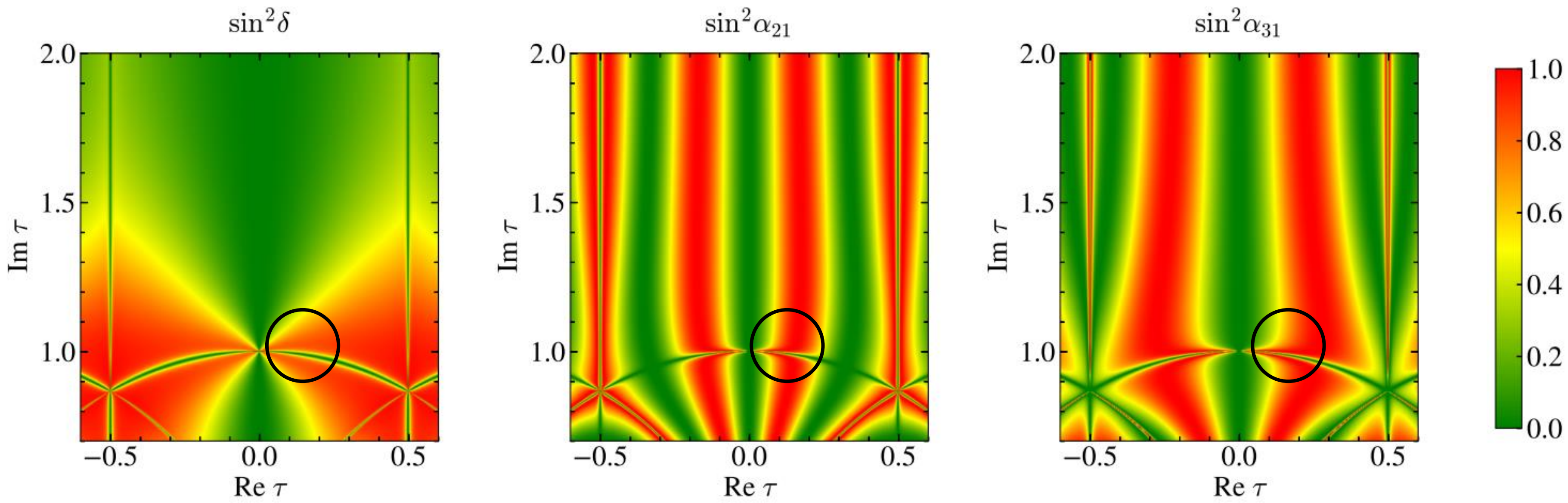
$$|\langle m\rangle|_{\beta\beta} \sim 12\ \text{meV}$$
$$\sum_i m_i \sim 0.08\ \text{eV}$$

$[n\sigma = 1]$



An S_4 example

A check (CP-conserving τ):



A small departure from the lines can already bring about large CPV

Summary

- (Modular symmetry brings advantages over the traditional discrete flavour symmetry approach)
- The modular symmetry can be combined with gCP.

Summary

- (Modular symmetry brings advantages over the traditional discrete flavour symmetry approach)

- *The modular symmetry can be combined with gCP:*

- $\tau \to n - \tau^*$
- Consistency condition w/ unique automorphism
- Extended modular group

(in a symmetric basis:)

- CP canonical
- $Y(\tau_{\mathrm{CP}}) = Y^*(\tau)$
- Real couplings

Summary

- (Modular symmetry brings advantages over the traditional discrete flavour symmetry approach)
- The modular symmetry can be combined with gCP.
- τ can be the only source of CPV. Must be outside 
- We've improved on an S_4 model example, increasing its predictive power.

STAY TUNED...

Fri 26/07 (tomorrow, parallel)

16:20 **A. Titov**: $\Gamma_5 \simeq A_5$

16:40 **J. Penedo**: $\Gamma_4 \simeq S_4$

Thank you / 谢谢

Backup slides

Lowest-weight modular forms: S_4

$$Y(a_1,\ldots,a_6|\tau) \equiv \frac{d}{d\tau}\left(\sum_{i=1}^{6} a_i \log \eta_i(\tau)\right)$$

$$\sum_i a_i = 0$$

Lowest weight forms arrange into:

$$Y_{\mathbf{2}}(\tau) = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}$$ **doublet 2**

$$Y_{\mathbf{3'}}(\tau) = \begin{pmatrix} Y_3(\tau) \\ Y_4(\tau) \\ Y_5(\tau) \end{pmatrix}$$ **triplet 3'**

$$Y_1(\tau) \equiv Y(1,1,\omega,\omega^2,\omega,\omega^2|\tau)$$
$$Y_2(\tau) \equiv Y(1,1,\omega^2,\omega,\omega^2,\omega|\tau)$$
$$Y_3(\tau) \equiv Y(1,-1,-1,-1,1,1|\tau)$$
$$Y_4(\tau) \equiv Y(1,-1,-\omega^2,-\omega,\omega^2,\omega|\tau)$$
$$Y_5(\tau) \equiv Y(1,-1,-\omega,-\omega^2,\omega,\omega^2|\tau)$$

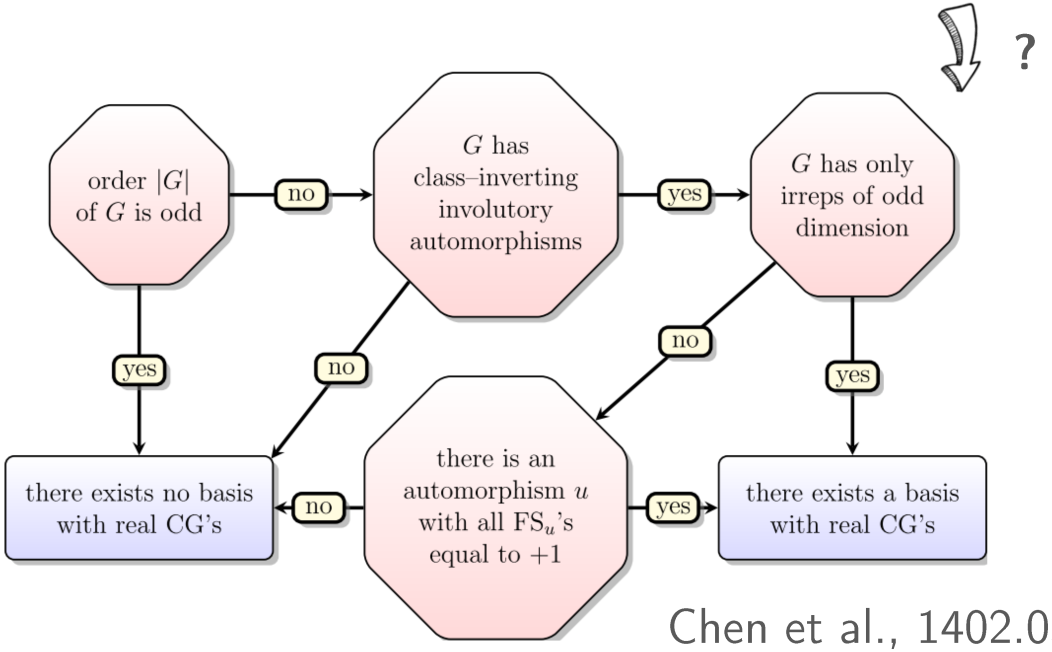
Correct dimension (5)

Products generate higher weight forms

Larger N(>5)

Our conclusions directly apply, provided:

- There is at most one lowest-weight form for each irrep
- There is a symmetric basis with real CGCs



Chen et al., 1402.0507

Kähler potential

The minimal choice; a definition of the setup, at this point:

$$K(\chi_i, \overline{\chi}_i; \tau, \overline{\tau}) = -h\,\Lambda_0^2\,\log(-i(\tau - \overline{\tau})) + \sum_i \frac{|\chi_i|^2}{(-i(\tau-\overline{\tau}))^{k_i}}$$

$$\mathcal{L} \supset \sum_i \frac{\partial_\mu \overline{\chi}_i\,\partial^\mu \chi_i}{(2\,\mathrm{Im}\langle\tau\rangle)^{k_i}}$$

Under a modular transformation, **invariant up to a Kähler transformation:**

$$K(\chi_i, \overline{\chi}_i; \tau, \overline{\tau}) \to K(\chi_i, \overline{\chi}_i; \tau, \overline{\tau}) + f(\chi_i; \tau) + f(\overline{\chi}_i; \overline{\tau})$$

There may exist potentially dangerous corrections to the Kähler, to be studied