University of Southampton School of Physics and Astronomy

elusives invisiblesPlus

neutrinos, dark matter & dark energy physics

Recent Developments in Neutrino Models

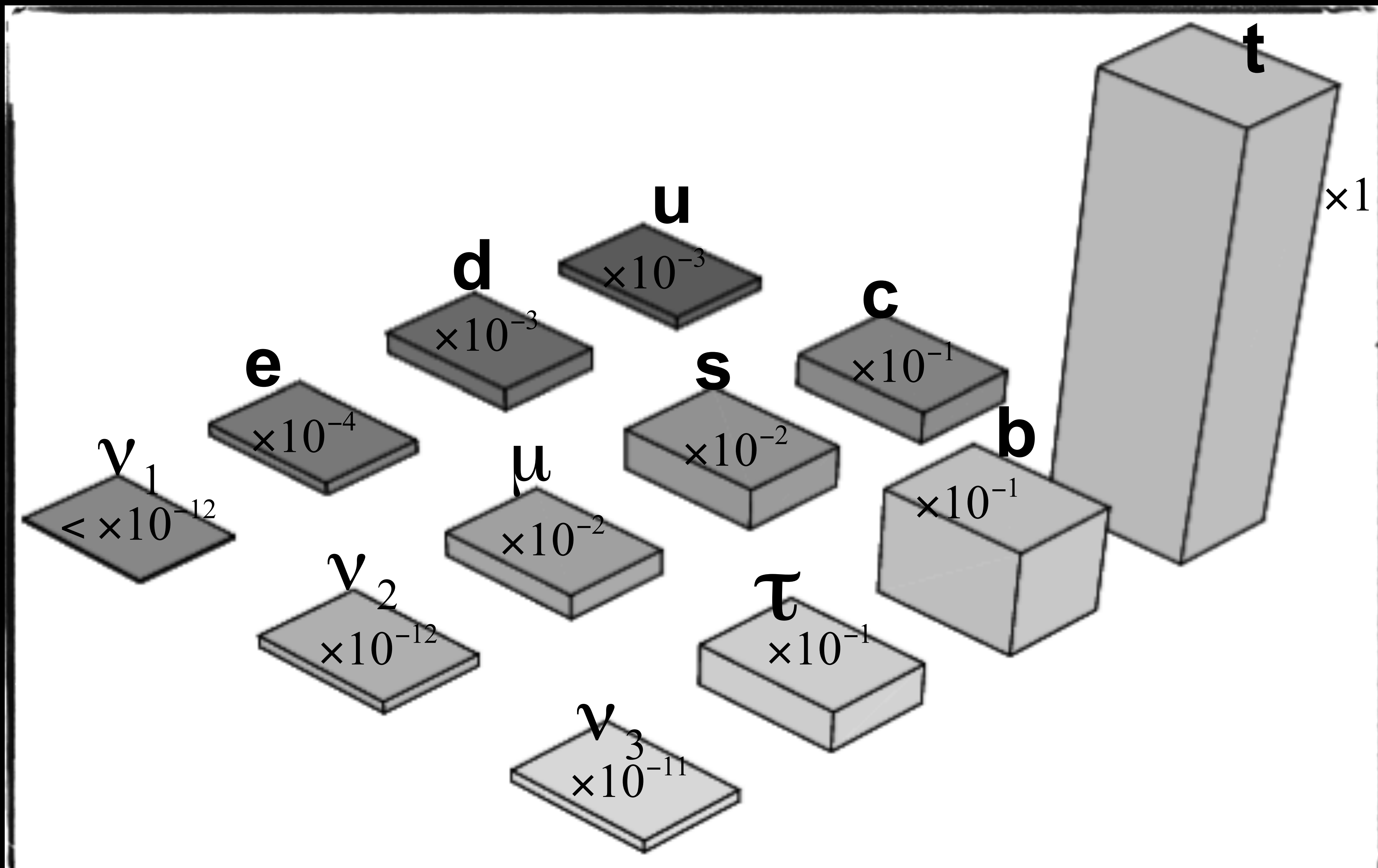
with F. de Anda, E.Perdomo, S. Molina Sedgewick, S. Rowley, Y.L.Zhou, J.Hernandez-Garcia, C.C.Nishi, G.J.Ding, C.C.Li, P.T.Chen, X.G.Liu, A. Carcamo Hernandez

Steve King, 27th July, 2019

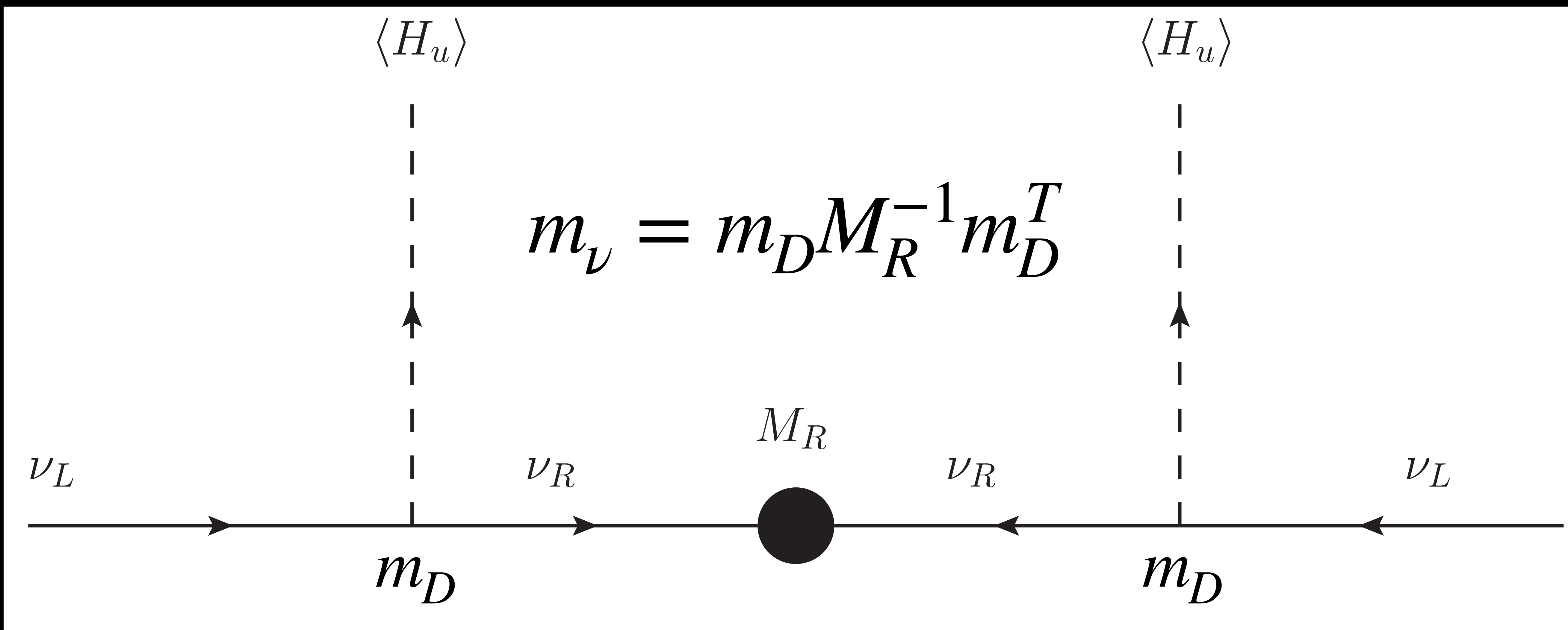
USTC, Hefei, P.R. China

FLASY2019: 8th Workshop on Flavor Symmetries and Consequences in Accelerators and Cosmology

Why nu mass small?

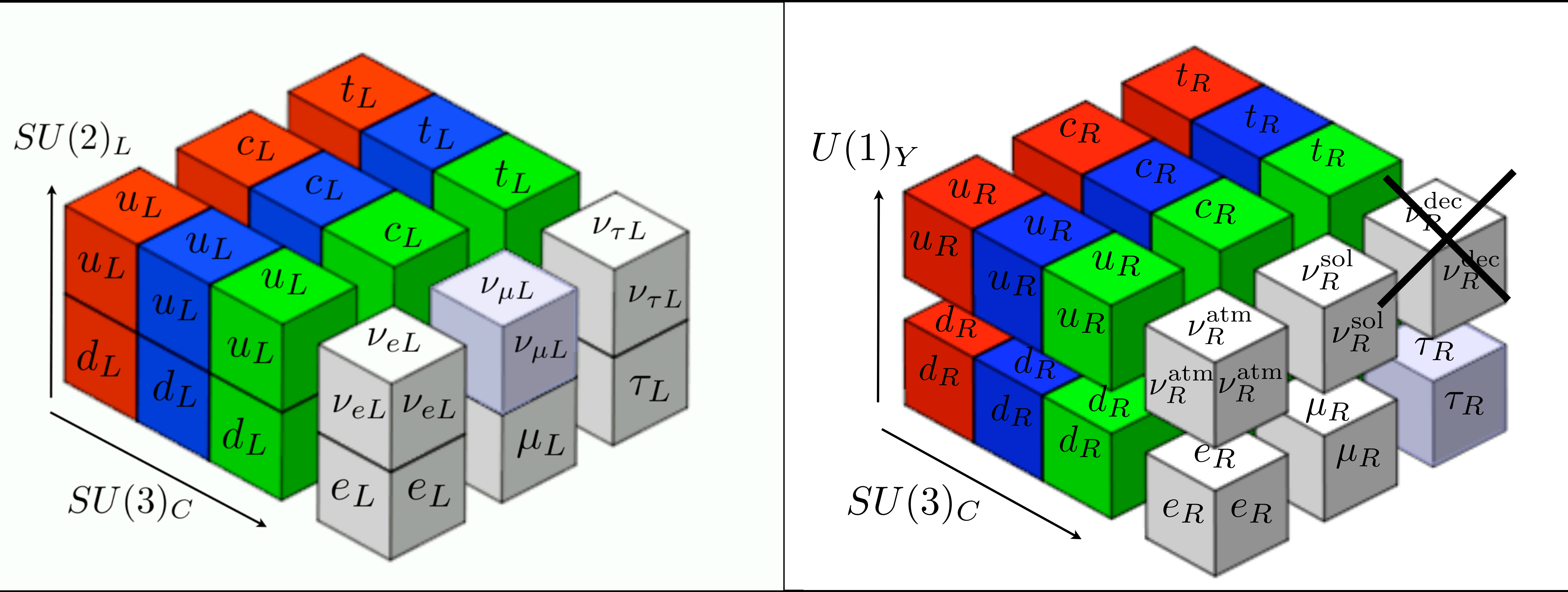


Type I seesaw



Minkowski,Yanagida,Gell-Mann,Slansky,Ramond,Mohapatra,Senjanovic,Schecter,Valle

Minimal Type I seesaw



Type I seesaw with two RHNs — S.F.K, hep-ph/9912492

Either one Dirac texture zero (NO) — S.F.K, hep-ph/0204360

Or two Dirac texture zeros (IO) — Frampton, Glashow, Yanagida, hep-ph/0208157

Littlest Seesaw

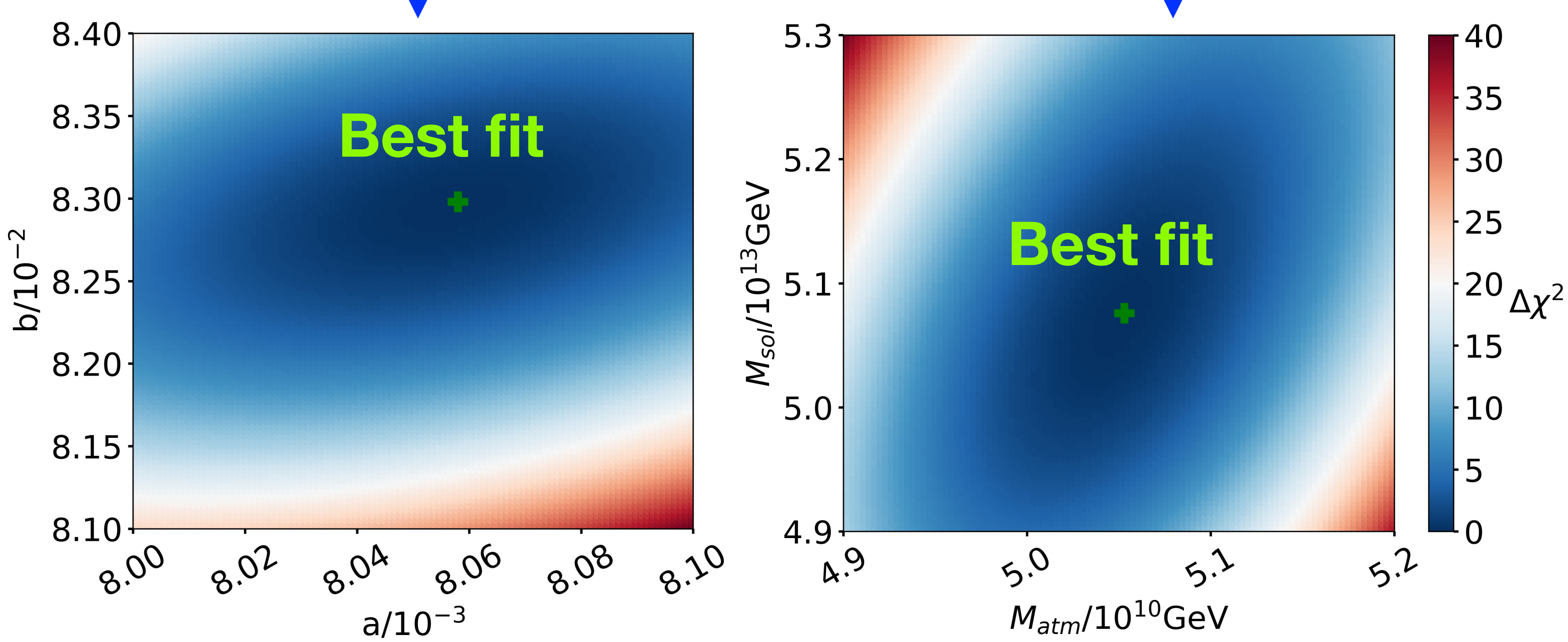
SFK 1304.6264; 1512.07531

SFK, Molina Sedgwick, Rowley, 1808.01005

Dirac texture zero

$$m_D = \begin{pmatrix} 0 & be^{i\pi/3} \\ a & 3be^{i\pi/3} \\ a & be^{i\pi/3} \end{pmatrix} \qquad M_R = \begin{pmatrix} M_{\rm atm} & 0 \\ 0 & M_{\rm sol} \end{pmatrix}$$

4 real input parameters



Fit includes effects of RG corrections

Describes:

- 3 neutrino masses (m_1=0),
- 3 mixing angles,
- 1 Dirac CP phase,
- 2 Majorana phases (1 zero)
- 1 BAU parameter Y_B

= 10 observables

of which 7 are constrained

Predictions	$1\,\sigma$ range
$\theta_{12}/^\circ$	$34.254 \to 34.350$
$\theta_{13}/^\circ$	$8.370 \to 8.803$
$\theta_{23}/^\circ$	$45.405 \to 45.834$
$\Delta m_{12}{}^2/10^{-5}\mathrm{eV}^2$	$7.030 \to 7.673$
$\Delta m_{31}{}^2/10^{-3}\mathrm{eV}^2$	$2.434 \to 2.561$
$\delta/^\circ$	$-88.284 \to -86.568$
$Y_B/10^{-10}$	$0.839 \to 0.881$

Littlest Seesaw

SFK 1304.6264; 1512.07531

SFK, Molina Sedgwick, Rowley, 1808.01005

Seesaw formula $M_\nu = m_D M_R^{-1} m_D^T \longrightarrow (M_\nu)_{ij}\nu^c_{iL}\nu^c_{jL} = (M^*_\nu)_{ij}\nu_{iL}\nu_{jL}$

$$\text{Case I}: \quad \mathrm{M}_\nu^{\mathrm{I}} = \omega \mathrm{m_a} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + \mathrm{m_s} \begin{pmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{pmatrix}$$

$$\text{Case II}: \quad \mathrm{M}_\nu^{\mathrm{II}} = \omega^2 \mathrm{m_a} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + \mathrm{m_s} \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & 3 \\ 3 & 3 & 9 \end{pmatrix}$$

Fits neutrino data with $m_a/m_s=10$

$$\omega = e^{i2\pi/3}$$

Special case $m_a/m_s=11$ gives Littlest mu-tau seesaw

$$\text{Case I:} \quad M_\nu = m_s \begin{pmatrix} 1 & 3 & 1 \\ 3 & 9+11\omega & 3+11\omega \\ 1 & 3+11\omega & 1+11\omega \end{pmatrix},$$

$$\text{Case II:} \quad M_\nu = m_s \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1+11\omega^2 & 3+11\omega^2 \\ 3 & 3+11\omega^2 & 9+11\omega^2 \end{pmatrix}.$$

Maximal atmospheric

Maximal CPV

S.F.K. and C.C.Nishi,1807.00023; S.F.K. and Y.L.Zhou,1901.06877

Littlest mu-tau Seesaw

$m_a/m_s=11$

$$M_\nu = m_s \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1+11\omega^2 & 3+\omega^2 \\ 3 & 3+11\omega^2 & 9+11\omega^2 \end{pmatrix} \qquad \omega = e^{i2\pi/3}$$

unequal

S.F.K. and C.C.Nishi,1807.00023; S.F.K. and Y.L.Zhou,1901.06877

Littlest mu-tau Seesaw

$m_a/m_s=11$

$$M_\nu = m_s \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1+11\omega^2 & 3+\omega^2 \\ 3 & 3+11\omega^2 & 9+11\omega^2 \end{pmatrix}$$

$$\omega = e^{i2\pi/3}$$

unequal

$$H_\nu = M_\nu^\dagger M_\nu = 11\,|m_s|^2 \begin{pmatrix} 1 & -1-2i\sqrt{3} & 1-2i\sqrt{3} \\ -1+2i\sqrt{3} & 19 & 17+4i\sqrt{3} \\ 1+2i\sqrt{3} & 17-4i\sqrt{3} & 19 \end{pmatrix}$$

equal

S.F.K. and C.C.Nishi,1807.00023; S.F.K. and Y.L.Zhou,1901.06877

Littlest mu-tau Seesaw

$m_a/m_s=11$

$$M_\nu = m_s \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1+11\omega^2 & 3+\omega^2 \\ 3 & 3+11\omega^2 & 9+11\omega^2 \end{pmatrix}$$

$$\omega = e^{i2\pi/3}$$

unequal

$$H_\nu = M_\nu^\dagger M_\nu = 11\,|m_s|^2 \begin{pmatrix} 1 & -1-2i\sqrt{3} & 1-2i\sqrt{3} \\ -1+2i\sqrt{3} & 19 & 17+4i\sqrt{3} \\ 1+2i\sqrt{3} & 17-4i\sqrt{3} & 19 \end{pmatrix}$$

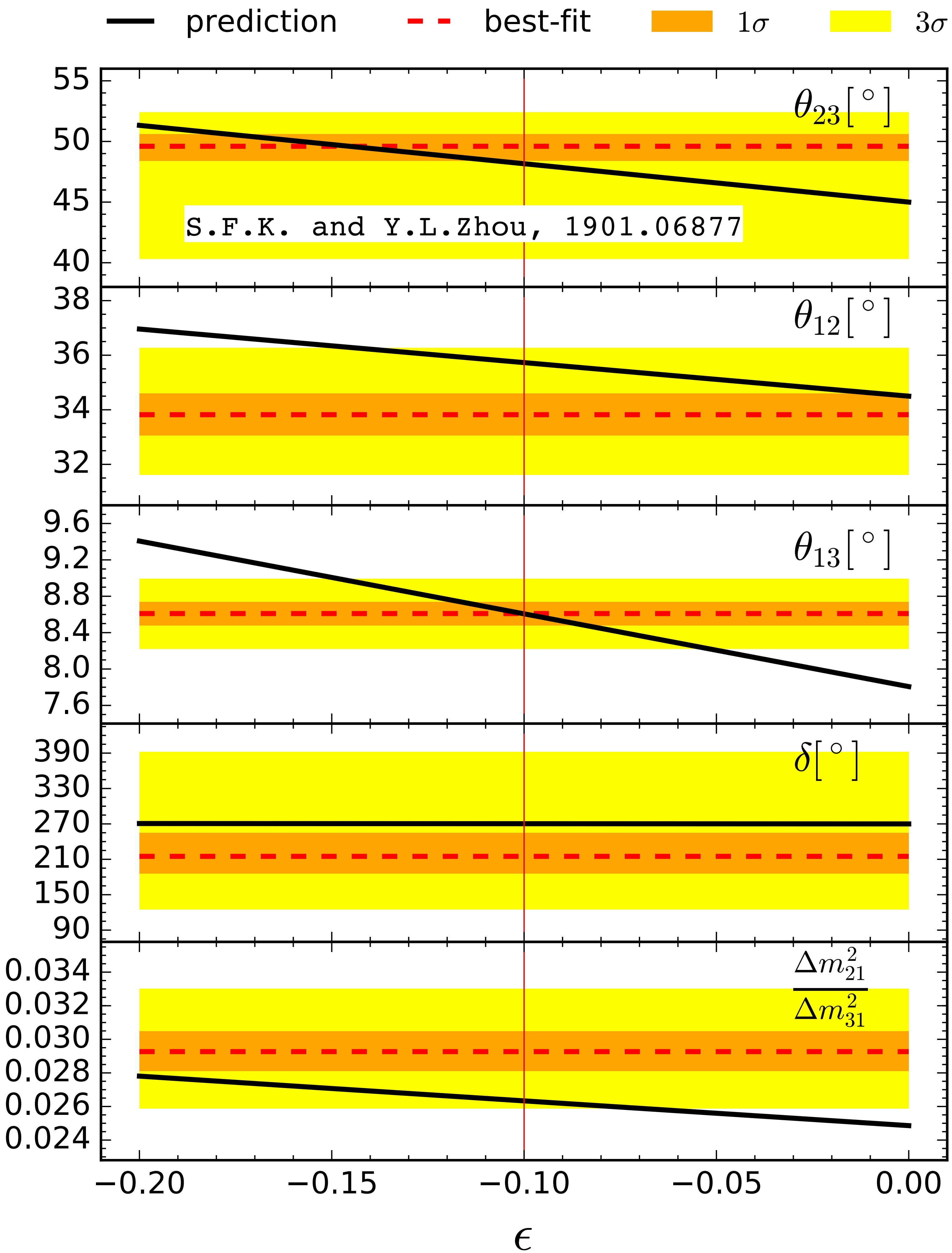
equal

$$U = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{c_+}{\sqrt{6}} & \frac{c_-}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} & -\frac{c_+}{\sqrt{6}} - i\frac{c_-}{2} & -\frac{c_-}{\sqrt{6}} + i\frac{c_+}{2} \\ \frac{1}{\sqrt{6}} & -\frac{c_+}{\sqrt{6}} + i\frac{c_-}{2} & -\frac{c_-}{\sqrt{6}} - i\frac{c_+}{2} \end{pmatrix}$$

TM1

Mu-tau reflection symmetry

$\theta_{23} = 45^\circ,\ \delta = -\pi/2$



Littlest mu-tau seesaw

$$m_1 = 0$$

$$U = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{c_+}{\sqrt{6}} & \frac{c_-}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} & -\frac{c_+}{\sqrt{6}} - i\frac{c_-}{2} & -\frac{c_-}{\sqrt{6}} + i\frac{c_+}{2} \\ \frac{1}{\sqrt{6}} & -\frac{c_+}{\sqrt{6}} + i\frac{c_-}{2} & -\frac{c_-}{\sqrt{6}} - i\frac{c_+}{2} \end{pmatrix}$$

$$c_\pm = \sqrt{1 \pm \frac{11}{3\sqrt{17}}}$$

Renormalisation Group Corrections

$$\theta_{13} \approx 7.807^\circ - 8.000^\circ \epsilon\,,$$

$$\theta_{12} \approx 34.50^\circ - 12.30^\circ \epsilon\,,$$

$$\theta_{23} \approx 45.00^\circ - 31.64^\circ \epsilon\,,$$

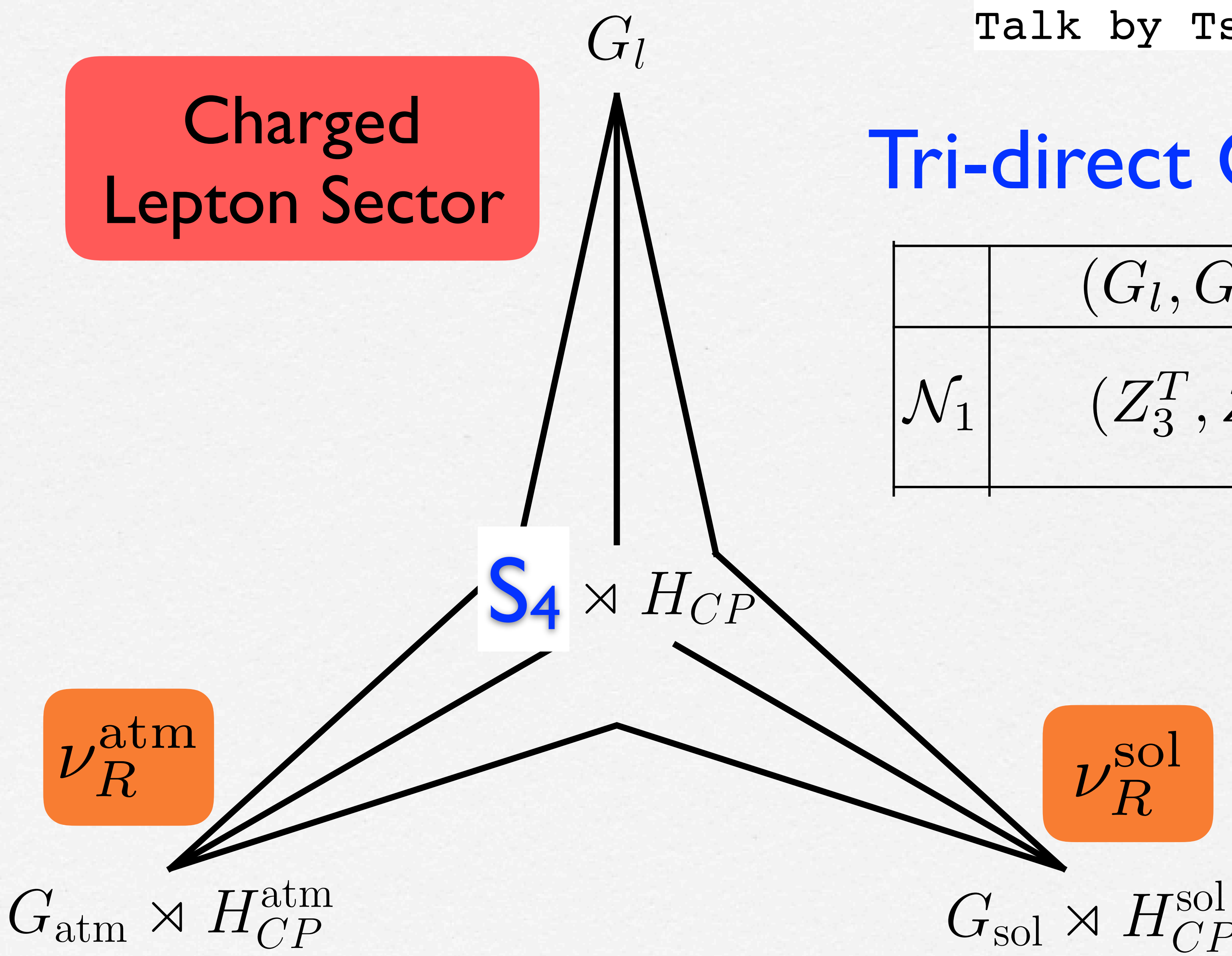
$$\delta \approx 270.00^\circ + 3.23^\circ \epsilon\,,$$

$$\Delta m^2_{21}/\Delta m^2_{31} \approx 0.0247 - 0.0147\epsilon$$

G.J.Ding, S.F.K. and C.C.Li, 1807.07538, 1811.12340

Littlest Seesaw from S_4

Talk by TseChun Wang



Tri-direct CP with S_4

	$(G_l, G_{\rm atm}, G_{\rm sol})$
$\mathcal{N}_1$	(Z_3^T, Z_2^U, Z_2^{SU})

P.T.Chen, G.J.Ding, S.F.K. and C.C.Li, 1906.11414

Littlest Seesaw from S_4

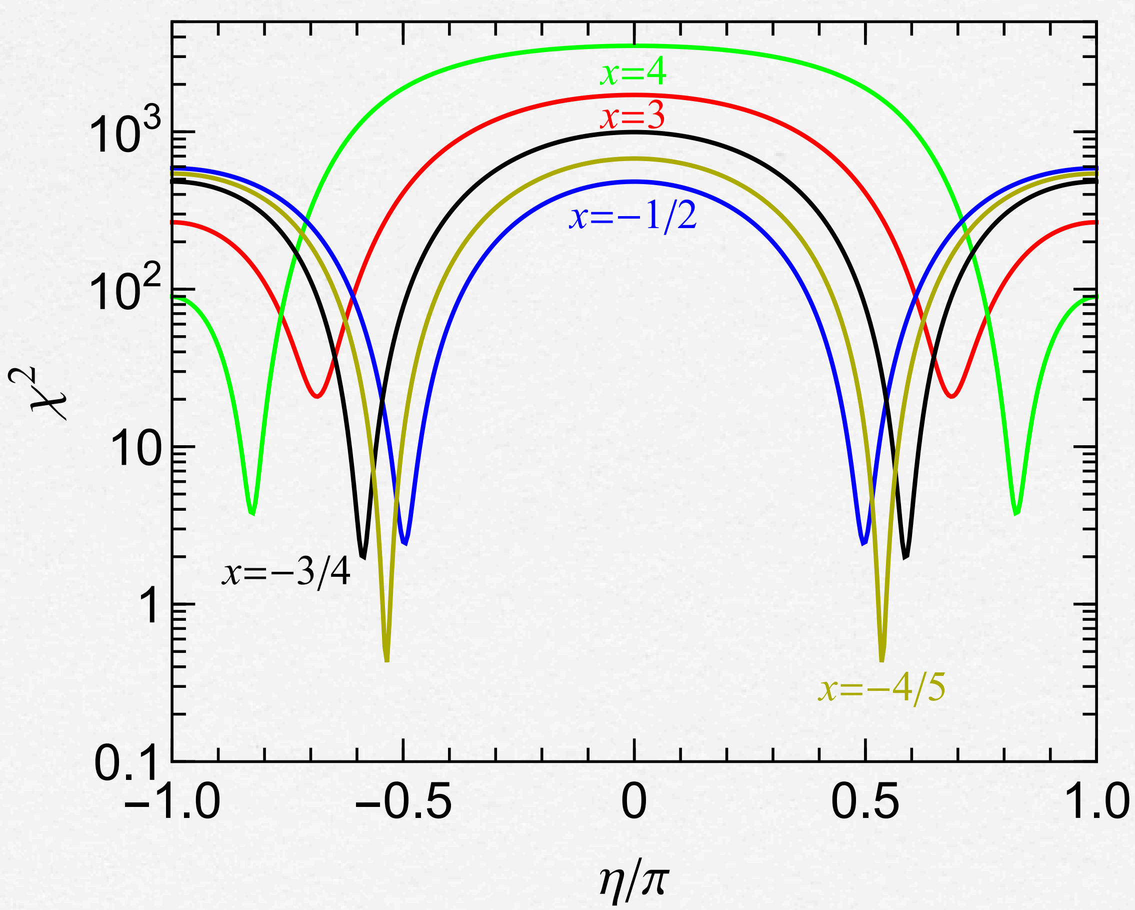
Tri-direct CP with S_4 gives the structure

$$m_\nu = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} + m_s e^{i\eta} \begin{pmatrix} 1 & 2-x & x \\ 2-x & (x-2)^2 & (2-x)x \\ x & (2-x)x & x^2 \end{pmatrix}$$

TM1

NO

$m_1=0$



Original Littlest Seesaw

$$(x,\eta) = (3, 2\pi/3),\ (-1, -2\pi/3)$$

$$\sin^2\theta_{23} \approx 0.5 \quad \delta_{CP} \approx -\pi/2$$

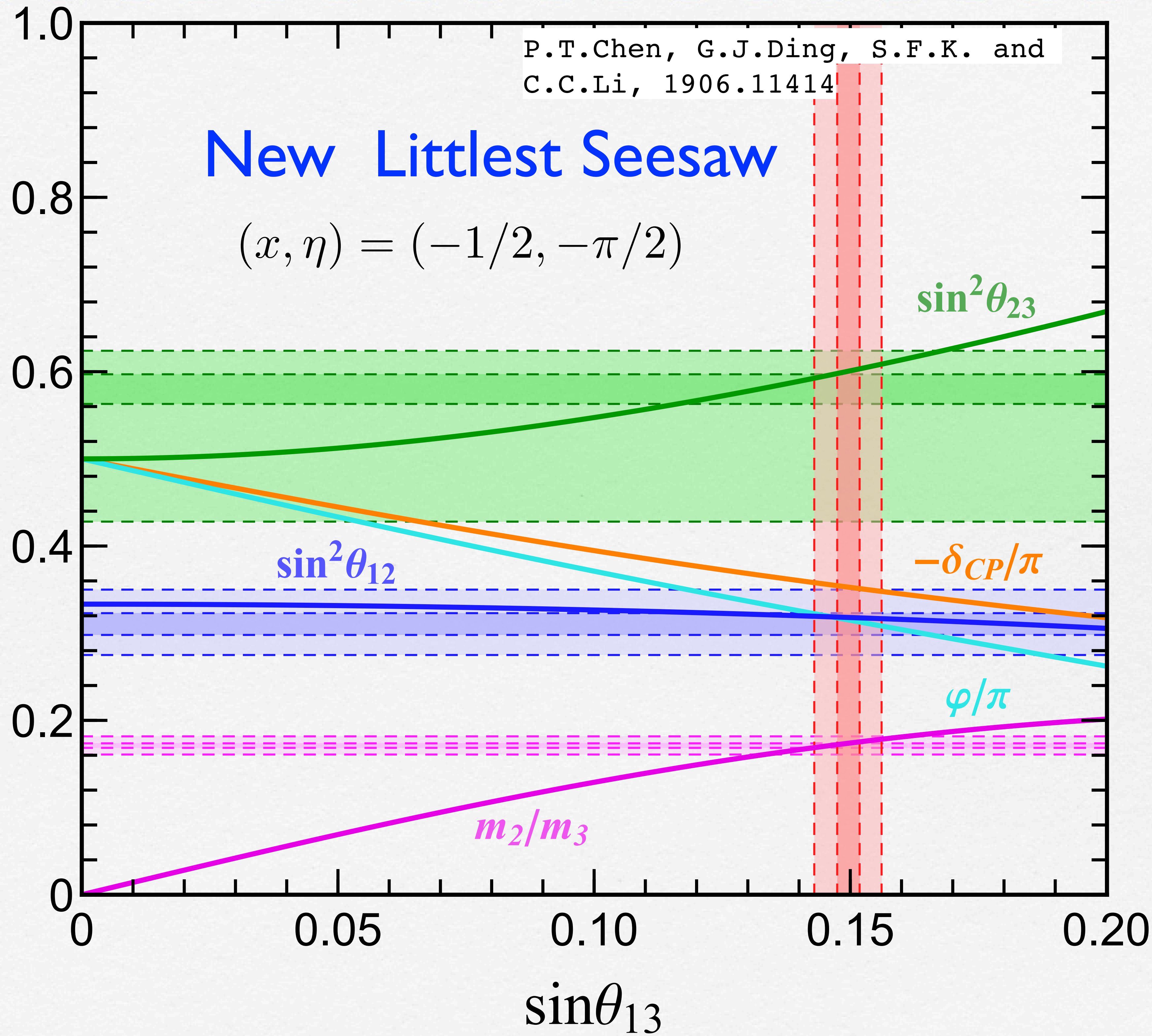
New Littlest Seesaw

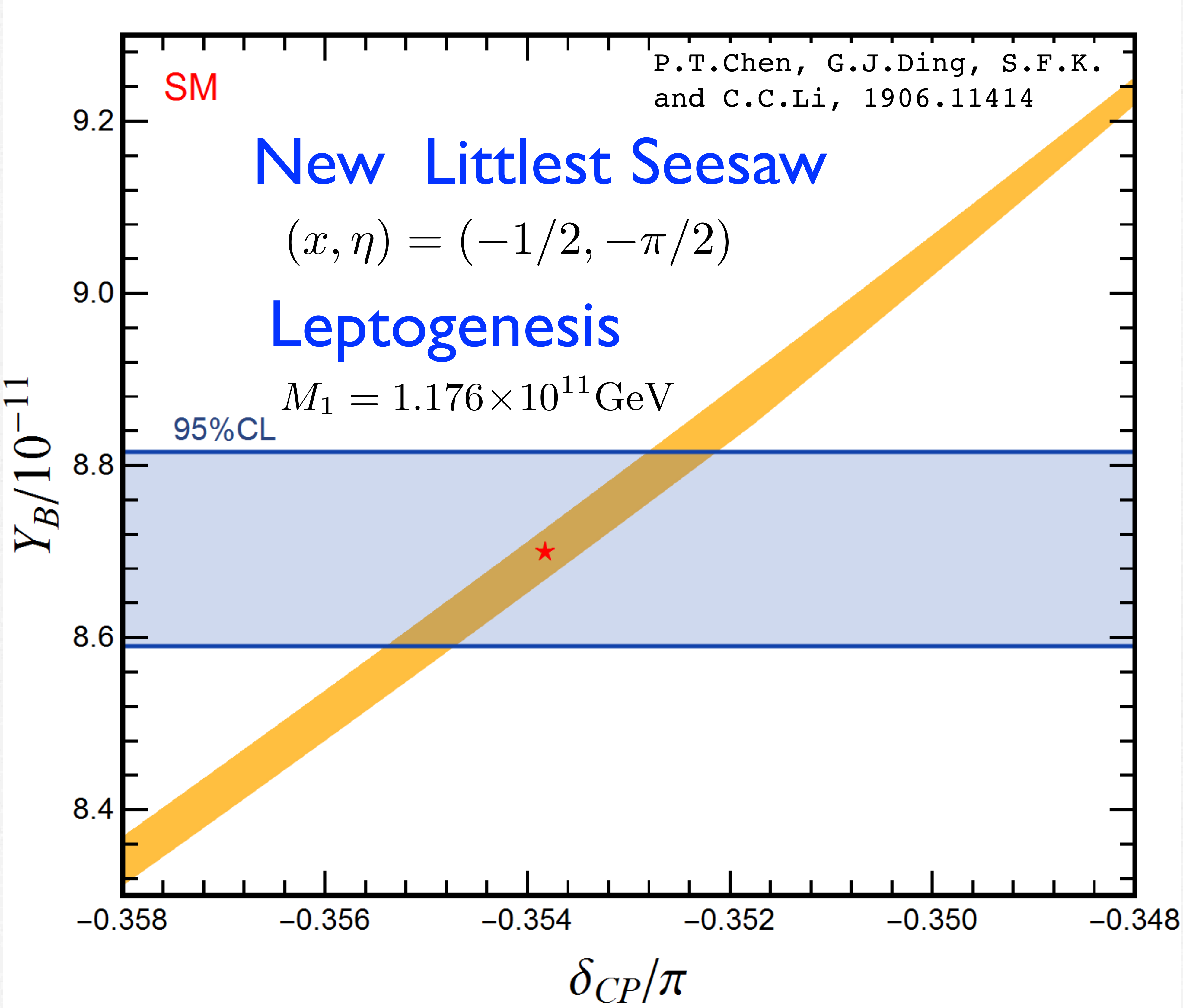
$$(x,\eta) = (-1/2, -\pi/2)$$

$$0.593 \le \sin^2\theta_{23} \le 0.609$$

$$-0.358 \le \delta_{CP}/\pi \le -0.348$$

UO


P.T.Chen, G.J.Ding, S.F.K. and C.C.Li, 1906.11414
New Littlest Seesaw
$(x,\eta) = (-1/2, -\pi/2)$
$\sin^2\theta_{23}$
$-\delta_{CP}/\pi$
$\sin^2\theta_{12}$
φ/π
m_2/m_3
1.0
0.8
0.6
0.4
0.2
0
0
0.05
0.10
0.15
0.20
$\sin\theta_{13}$



P.T.Chen, G.J.Ding, S.F.K.
and C.C.Li, 1906.11414
SM
New Littlest Seesaw
$(x,\eta) = (-1/2, -\pi/2)$
Leptogenesis
$M_1 = 1.176\times10^{11}\text{GeV}$
95%CL
9.2
9.0
8.8
8.6
8.4
$Y_B/10^{-11}$
−0.358
−0.356
−0.354
−0.352
−0.350
−0.348
δ_{CP}/π

A.E.Cárcamo Hernández and S.F.King,1903.02565

Littlest Inverse Seesaw

Another Possibility

$$M_\nu = \begin{pmatrix} 0_{3\times3} & m_D & 0_{3\times2} \\ m_D^T & 0_{2\times2} & M \\ 0_{2\times3} & M^T & \mu \end{pmatrix}$$

cLFV, collider…

Talk by Antusch

$$m_D \sim \begin{pmatrix} 0 & b \\ a & 3b \\ a & b \end{pmatrix}, \qquad M \sim \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \mu \sim \begin{pmatrix} 1 & 0 \\ 0 & \omega \end{pmatrix}, \qquad \omega = e^{\frac{2\pi i}{3}}.$$

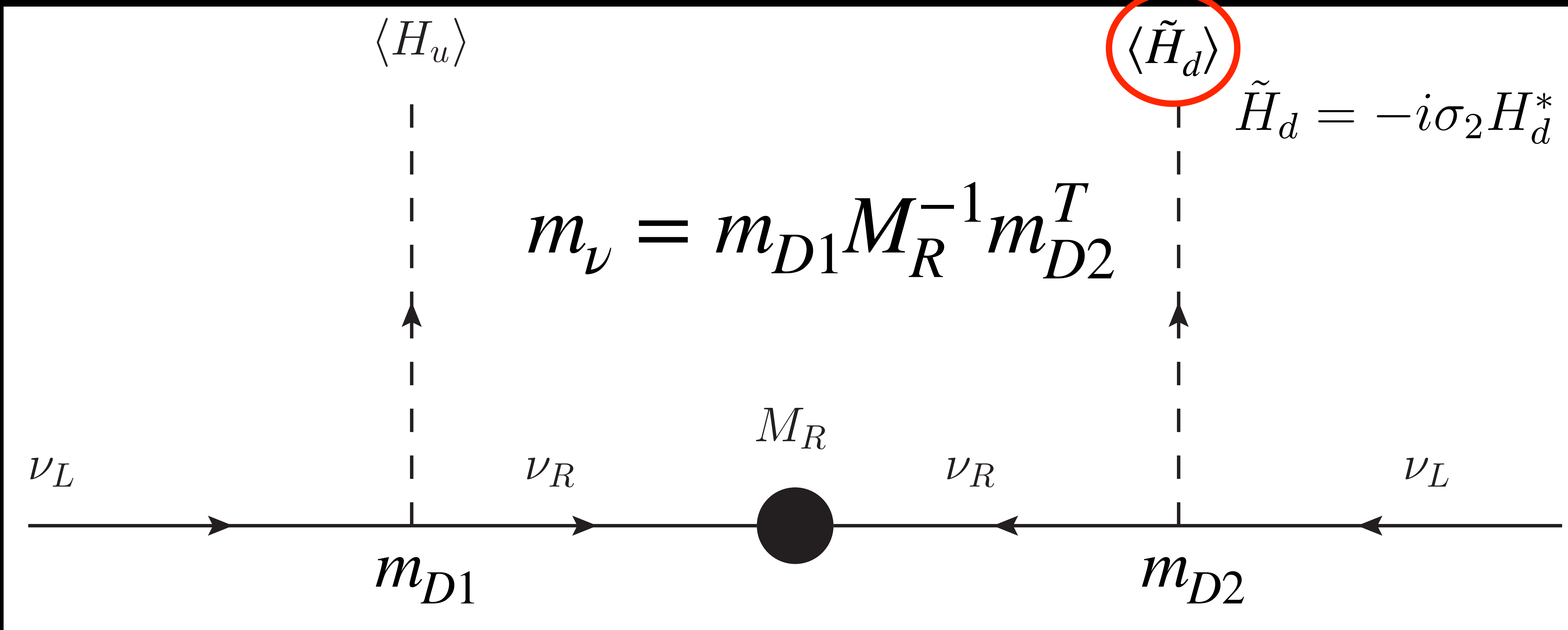
$$m_\nu = -m_D (M^T)^{-1} \mu M^{-1} m_D^T$$

Talk by Valle

Same low energy matrix

$$m_\nu = m_{\nu a} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_{\nu b}\,\omega \begin{pmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{pmatrix}$$

Type 1b seesaw



Hernandez-Garcia and SFK 1903.01474

Effective Weinberg operators for 2HDM in J.F.Oliver,A.Santamaria,hep-ph/0108020

Hernandez-Garcia and SFK 1903.01474

Minimal Type Ib seesaw

Z'

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)'$
Q_i	$\mathbf{3}$	$\mathbf{2}$	$1/6$	0
u_i^c	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	0
d_i^c	$\overline{\mathbf{3}}$	$\mathbf{1}$	$1/3$	0
L_i	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	0
e_i^c	$\mathbf{1}$	$\mathbf{1}$	1	0
ν^c	$\mathbf{1}$	$\mathbf{1}$	0	1
$\overline{\nu^c}$	$\mathbf{1}$	$\mathbf{1}$	0	-1
ϕ	$\mathbf{1}$	$\mathbf{1}$	0	1
H_u	$\mathbf{1}$	$\mathbf{2}$	$1/2$	-1
H_d	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	-1

$$y_i^\nu H_u L_i \nu^c + \epsilon_1 y_i^{\nu\prime} \tilde{H}_d L_i \overline{\nu^c}$$

Assume Hd couplings small

$$M^\nu = \begin{array}{c} \\ \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu^c \\ \overline{\nu^c} \end{array} \begin{array}{c} \begin{array}{ccccc} \nu_1 & \nu_2 & \nu_3 & \nu^c & \overline{\nu^c} \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & y_1^\nu v & \epsilon_1 y_1^{\nu\prime} v' \\ 0 & 0 & 0 & y_2^\nu v & \epsilon_1 y_2^{\nu\prime} v' \\ 0 & 0 & 0 & y_3^\nu v & \epsilon_1 y_3^{\nu\prime} v' \\ y_1^\nu v & y_2^\nu v & y_3^\nu v & 0 & M^\nu \\ \epsilon_1 y_1^{\nu\prime} v' & \epsilon_1 y_2^{\nu\prime} v' & \epsilon_1 y_3^{\nu\prime} v' & M^\nu & 0 \end{pmatrix} \end{array}$$

c.f. Antusch talk

Light effective neutrino matrix

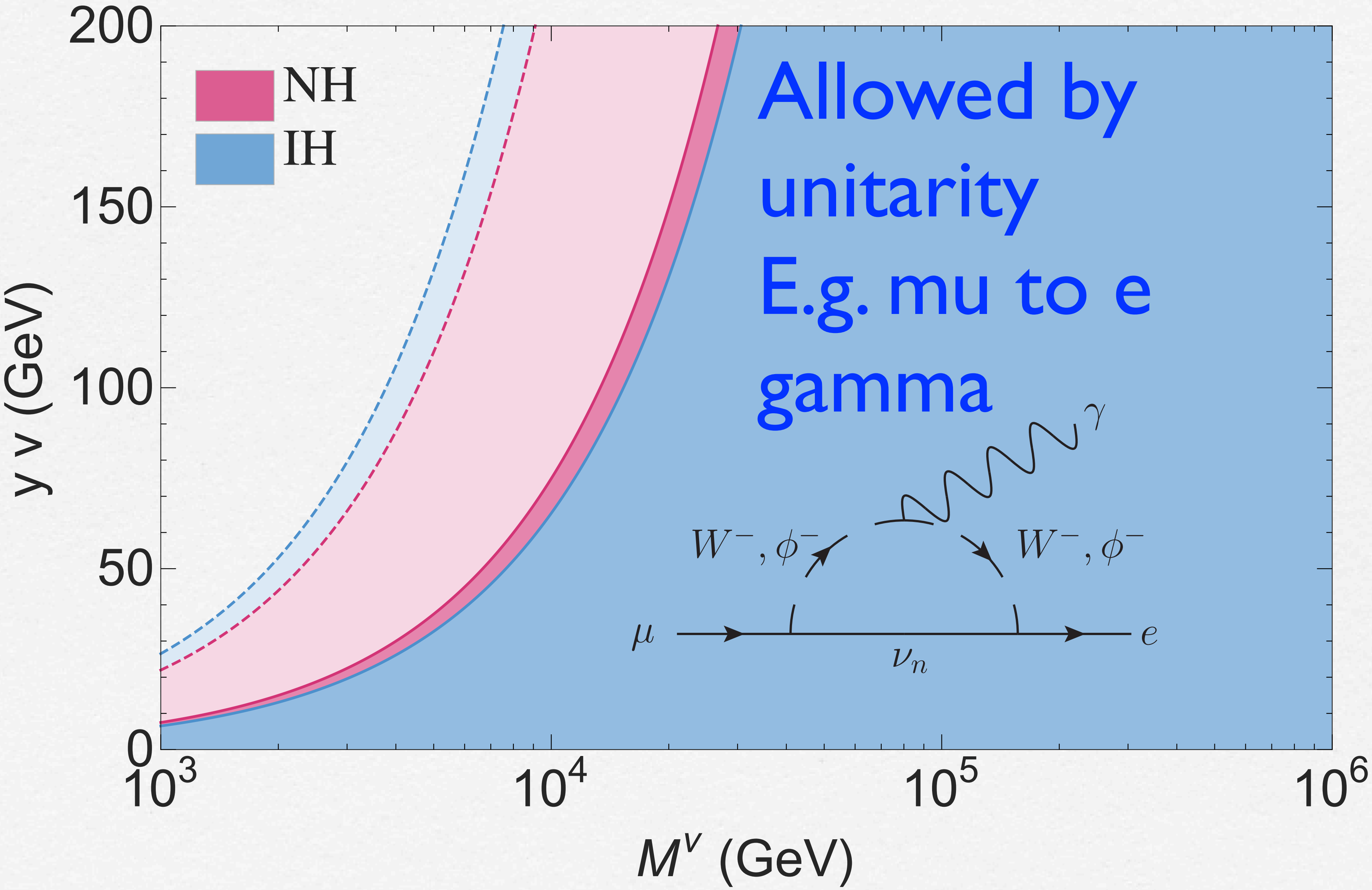
$$\hat{m}_{ij} = \frac{\epsilon_1 v v'}{M^\nu} \left(y_i^\nu y_j^{\nu\prime} + y_i^{\nu\prime} y_j^\nu \right)$$

Unitarity violation due to large y

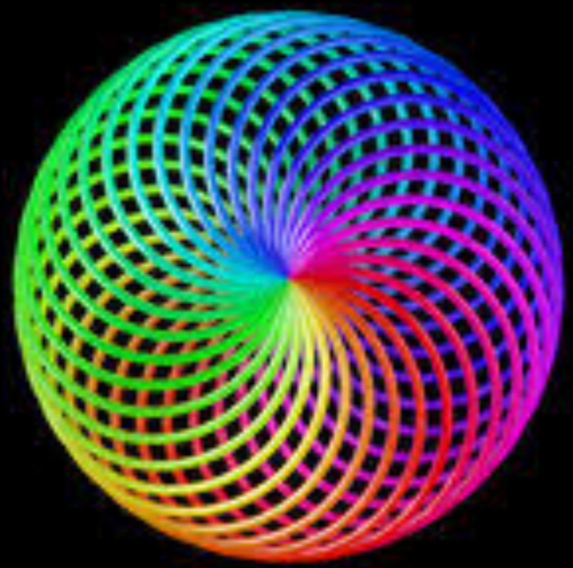
$$\eta_{ij} = \frac{1}{2M^{\nu 2}} \left(v^2 y_i^{\nu *} y_j^\nu + \epsilon_1^2 v'^2 y_i^{\nu\prime *} y_j^{\nu\prime} \right) \simeq \frac{v^2}{2M^{\nu 2}} y_i^{\nu *} y_j^\nu$$

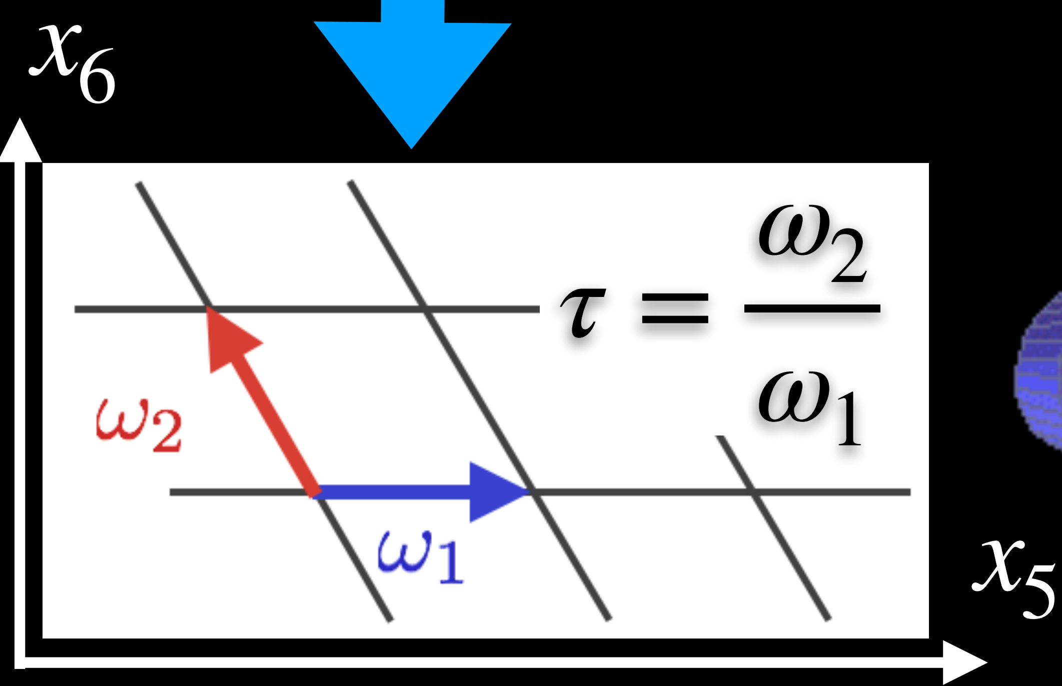
Hernandez-Garcia and SFK 1903.01474

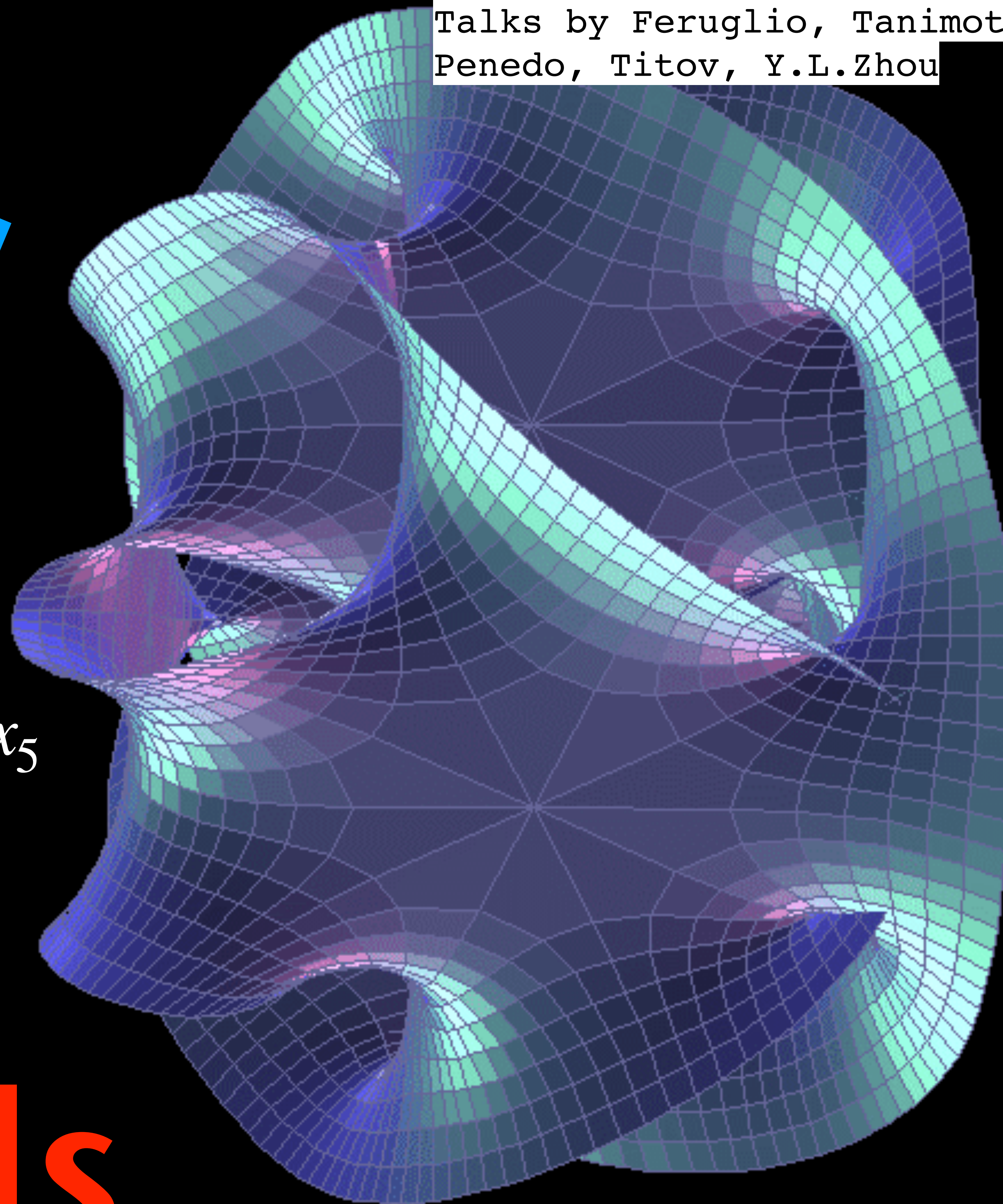
Minimal Type Ib seesaw



Talks by Feruglio, Tanimoto, Penedo, Titov, Y.L.Zhou







6d models

F.Feruglio, 1706.08749
Also Ferruccio's talk

Modular Forms

Yukawa couplings involving twisted states whose modular weights do not add up to zero are modular forms

Level 3 Weight 2 acts as A4 triplet:

$$Y = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix} = \begin{pmatrix} 1 + 12q + 36q^2 + 12q^3 + 84q^4 + 72q^5 + \dots \\ -6q^{1/3}(1 + 7q + 8q^2 + 18q^3 + 14q^4 + \dots) \\ -18q^{2/3}(1 + 2q + 5q^2 + 4q^3 + 8q^4 + \dots) \end{pmatrix}$$

$$q \equiv e^{i2\pi\tau}$$ ← **free modulus** $\tau = \frac{\omega_2}{\omega_1}$

Weinberg operator $\frac{1}{\Lambda}(H_u H_u \; L L \; Y)$ → $m_\nu = \begin{pmatrix} 2Y_1 & -Y_3 & -Y_2 \\ -Y_3 & 2Y_2 & -Y_1 \\ -Y_2 & -Y_1 & 2Y_3 \end{pmatrix} \frac{v_u^2}{\Lambda}$

A_4: 3 3 3

G.J.Ding, S.F.K. and X.-G.Liu, 19xx.xxxxx
See also talk by Tanimoto

A4 Modular Symmetry

Models		mass matrices	assignment	weight		
			$\rho_{E^c_{1,2,3}}$	$k_{E^c_{1,2,3}}$	k_L	k_{N^c}
Weinberg operator	$\mathcal{A}1$	$W1,\ C1$	$\mathbf{1},\mathbf{1},\mathbf{1}$	$1,3,5$	1	$-$
	$\mathcal{A}2$	$W1,\ C2$	$\mathbf{1}',\mathbf{1}',\mathbf{1}'$	$1,3,5$	1	$-$
	$\mathcal{A}3$	$W1,\ C3$	$\mathbf{1}'',\mathbf{1}'',\mathbf{1}''$	$1,3,5$	1	$-$
	$\mathcal{A}4$	$W1,\ C4$	$\mathbf{1},\mathbf{1},\mathbf{1}'$	$1,3,1$	1	$-$
	$\mathcal{A}5$	$W1,\ C5$	$\mathbf{1},\mathbf{1},\mathbf{1}''$	$1,3,1$	1	$-$
	$\mathcal{A}6$	$W1,\ C6$	$\mathbf{1}',\mathbf{1}',\mathbf{1}$	$1,3,1$	1	$-$
	$\mathcal{A}7$	$W1,\ C7$	$\mathbf{1}'',\mathbf{1}'',\mathbf{1};$	$1,3,1$	1	$-$
	$\mathcal{A}8$	$W1,\ C8$	$\mathbf{1}'',\mathbf{1}'',\mathbf{1}'$	$1,3,1$	1	$-$
	$\mathcal{A}9$	$W1,\ C9$	$\mathbf{1}',\mathbf{1}',\mathbf{1}''$	$1,3,1$	1	$-$
	$\mathcal{A}10$	$W1,\ C10$	$\mathbf{1},\mathbf{1}'',\mathbf{1}'$	$1,1,1$	1	$-$
Type I see-saw	$\mathcal{B}1(\mathcal{C}1)[\mathcal{D}1]$	$S1(S2)[S3],\ C1$	$\mathbf{1},\mathbf{1},\mathbf{1}$	$0(3)[1],2(5)[3],4(7)[5]$	$2(-1)[1]$	$0(1)[1]$
	$\mathcal{B}2(\mathcal{C}2)[\mathcal{D}2]$	$S1(S2)[S3],\ C2$	$\mathbf{1}',\mathbf{1}',\mathbf{1}'$	$0(3)[1],2(5)[3],4(7)[5]$	$2(-1)[1]$	$0(1)[1]$
	$\mathcal{B}3(\mathcal{C}3)[\mathcal{D}3]$	$S1(S2)[S3],\ C3$	$\mathbf{1}'',\mathbf{1}'',\mathbf{1}''$	$0(3)[1],2(5)[3],4(7)[5]$	$2(-1)[1]$	$0(1)[1]$
	$\mathcal{B}4(\mathcal{C}4)[\mathcal{D}4]$	$S1(S2)[S3],\ C4$	$\mathbf{1},\mathbf{1},\mathbf{1}'$	$0(3)[1],2(5)[3],0(3)[1]$	$2(-1)[1]$	$0(1)[1]$
	$\mathcal{B}5(\mathcal{C}5)[\mathcal{D}5]$	$S1(S2)[S3],\ C5$	$\mathbf{1},\mathbf{1},\mathbf{1}''$	$0(3)[1],2(5)[3],0(3)[1]$	$2(-1)[1]$	$0(1)[1]$
	$\mathcal{B}6(\mathcal{C}6)[\mathcal{D}6]$	$S1(S2)[S3],\ C6$	$\mathbf{1}',\mathbf{1}',\mathbf{1}$	$0(3)[1],2(5)[3],0(3)[1]$	$2(-1)[1]$	$0(1)[1]$
	$\mathcal{B}7(\mathcal{C}7)[\mathcal{D}7]$	$S1(S2)[S3],\ C7$	$\mathbf{1}',\mathbf{1}',\mathbf{1}''$	$0(3)[1],2(5)[3],0(3)[1]$	$2(-1)[1]$	$0(1)[1]$
	$\mathcal{B}8(\mathcal{C}8)[\mathcal{D}8]$	$S1(S2)[S3],\ C8$	$\mathbf{1}'',\mathbf{1}'',\mathbf{1}$	$0(3)[1],2(5)[3],0(3)[1]$	$2(-1)[1]$	$0(1)[1]$
	$\mathcal{B}9(\mathcal{C}9)[\mathcal{D}9]$	$S1(S2)[S3],\ C9$	$\mathbf{1}'',\mathbf{1}'',\mathbf{1}'$	$0(3)[1],2(5)[3],0(3)[1]$	$2(-1)[1]$	$0(1)[1]$
	$\mathcal{B}10(\mathcal{C}10)[\mathcal{D}10]$	$S1(S2)[S3],\ C10$	$\mathbf{1},\mathbf{1}'',\mathbf{1}'$	$0(3)[1],0(3)[1],0(3)[1]$	$2(-1)[1]$	$0(1)[1]$

Comprehensive study of 40 simplest cases without flavons

Models	Ordering		Models	Ordering		Models	Ordering		Models	Ordering	
	NO	IO		NO	IO		NO	IO		NO	IO
$\mathcal{A}1$	✘	✘	$\mathcal{B}1$	✔	✔	$\mathcal{C}1$	✘	✘	$\mathcal{D}1$	✔	✔
$\mathcal{A}2$	✘	✘	$\mathcal{B}2$	✔	✔	$\mathcal{C}2$	✘	✘	$\mathcal{D}2$	✔	✔
$\mathcal{A}3$	✘	✘	$\mathcal{B}3$	✔	✔	$\mathcal{C}3$	✘	✘	$\mathcal{D}3$	✔	✔
$\mathcal{A}4$	✘	✘	$\mathcal{B}4$	✘	✘	$\mathcal{C}4$	✘	✘	$\mathcal{D}4$	✘	✔
$\mathcal{A}5$	✘	✘	$\mathcal{B}5$	✘	✘	$\mathcal{C}5$	✘	✘	$\mathcal{D}5$	✔	✘
$\mathcal{A}6$	✘	✘	$\mathcal{B}6$	✘	✔	$\mathcal{C}6$	✘	✘	$\mathcal{D}6$	✔	✘
$\mathcal{A}7$	✘	✘	$\mathcal{B}7$	✘	✘	$\mathcal{C}7$	✘	✘	$\mathcal{D}7$	✔	✔
$\mathcal{A}8$	✘	✘	$\mathcal{B}8$	✘	✘	$\mathcal{C}8$	✘	✘	$\mathcal{D}8$	✔	✔
$\mathcal{A}9$	✘	✘	$\mathcal{B}9$	✔	✔	$\mathcal{C}9$	✘	✘	$\mathcal{D}9$	✔	✔
$\mathcal{A}10$	✘	✘	$\mathcal{B}10$	✔	✔	$\mathcal{C}10$	✘	✘	$\mathcal{D}10$	✔	✔

Minimal Models:

$$\mathcal{B}_9,\ \mathcal{B}_{10},\ \mathcal{D}_5 \sim \mathcal{D}_{10}$$

8 inputs for 12 observables

(6 lepton masses, 6 PMNS)

→ Large nu mass, deltaCP

G.J.Ding, S.F.K. and X.-G.Liu, 1903.12588
See also talk by Titov

A5 Modular Symmetry

Models		mass matrices	assignment $(\rho_{E^c}, \rho_L, \rho_{N^c})$	weight $k_{E_{1,2,3}}$	k_L	k_{N^c}
With flavons	$\mathcal{A}1$	$W1$	$(\mathbf{1}, \mathbf{3}, -)$	$-$	1	$-$
	$\mathcal{A}2$	$W2$	$(\mathbf{1}, \mathbf{3}', -)$	$-$	1	$-$
	$\mathcal{A}3$	$S1$	$(\mathbf{1}, \mathbf{3}, \mathbf{3})$	$-$	2	0
	$\mathcal{A}4$	$S2$	$(\mathbf{1}, \mathbf{3}, \mathbf{3})$	$-$	-1	1
	$\mathcal{A}5$	$S3$	$(\mathbf{1}, \mathbf{3}', \mathbf{3})$	$-$	2	0
	$\mathcal{A}6$	$S4$	$(\mathbf{1}, \mathbf{3}, \mathbf{3}')$	$-$	2	0
	$\mathcal{A}7$	$S5$	$(\mathbf{1}, \mathbf{3}', \mathbf{3}')$	$-$	2	0
	$\mathcal{A}8$	$S6$	$(\mathbf{1}, \mathbf{3}', \mathbf{3}')$	$-$	-1	1
Without flavons	$\mathcal{B}1$	$C1$, $W1$	$(\mathbf{1}, \mathbf{3}, -)$	$1, 3, 5$	1	$-$
	$\mathcal{B}2$	$C2$, $W2$	$(\mathbf{1}, \mathbf{3}', -)$	$1, 3, 5$	1	$-$
	$\mathcal{B}3$	$C1$, $S1$	$(\mathbf{1}, \mathbf{3}, \mathbf{3})$	$0, 2, 4$	2	0
	$\mathcal{B}4$	$C1$, $S2$	$(\mathbf{1}, \mathbf{3}, \mathbf{3})$	$3, 5, 7$	-1	1
	$\mathcal{B}5$	$C2$, $S3$	$(\mathbf{1}, \mathbf{3}', \mathbf{3})$	$0, 2, 4$	2	0
	$\mathcal{B}6$	$C1$, $S4$	$(\mathbf{1}, \mathbf{3}, \mathbf{3}')$	$0, 2, 4$	2	0
	$\mathcal{B}7$	$C2$, $S5$	$(\mathbf{1}, \mathbf{3}', \mathbf{3}')$	$0, 2, 4$	2	0
	$\mathcal{B}8$	$C2$, $S6$	$(\mathbf{1}, \mathbf{3}', \mathbf{3}')$	$3, 5, 7$	-1	1

Comprehensive study of simplest cases with and without flavons

Results very dependent on **free modulus**

Models		free input parameters p_i	overall factors
With flavons	$\mathcal{A}1$, $\mathcal{A}2$	$\{\mathrm{Re}\,\tau,\ \mathrm{Im}\,\tau\}$	v_u^2/Λ
	$\mathcal{A}4$, $\mathcal{A}5$, $\mathcal{A}6$, $\mathcal{A}8$	$\{\mathrm{Re}\,\tau,\ \mathrm{Im}\,\tau\}$	$g^2 v_u^2/\Lambda$
	$\mathcal{A}3$, $\mathcal{A}7$	$\{\mathrm{Re}\,\tau,\ \mathrm{Im}\,\tau,\ \lvert g_1/g_2\rvert,\ \mathrm{Arg}(g_1/g_2)\}$	$g_2^2 v_u^2/\Lambda$
Without flavons	$\mathcal{B}1$, $\mathcal{B}2$	$\{\mathrm{Re}\,\tau,\ \mathrm{Im}\,\tau,\ \beta/\alpha,\ \gamma_1/\alpha,\ \lvert\gamma_2/\alpha\rvert,\ \mathrm{Arg}(\gamma_2/\alpha)\}$	αv_d, v_u^2/Λ
	$\mathcal{B}4$, $\mathcal{B}5$, $\mathcal{B}6$, $\mathcal{B}8$	$\{\mathrm{Re}\,\tau,\ \mathrm{Im}\,\tau,\ \beta/\alpha,\ \gamma_1/\alpha,\ \lvert\gamma_2/\alpha\rvert,\ \mathrm{Arg}(\gamma_2/\alpha)\}$	αv_d, $g^2 v_u^2/\Lambda$
	$\mathcal{B}3$, $\mathcal{B}7$	$\{\mathrm{Re}\,\tau,\ \mathrm{Im}\,\tau,\ \beta/\alpha,\ \gamma_1/\alpha,\ \lvert\gamma_2/\alpha\rvert,\ \mathrm{Arg}(\gamma_2/\alpha),\ \lvert g_1/g_2\rvert,\ \mathrm{Arg}(g_1/g_2)\}$	αv_d, $g_2^2 v_u^2/\Lambda$

$$\tau = \frac{\omega_2}{\omega_1}$$

F. De Anda, S.F.K., E.Perdomo,1812.05620

Modular Symmetry and orbifolds

Consider a **finite** modular symmetry

$$\bar{\Gamma}_M \simeq \{S,T|S^2=(ST)^3=T^M=\mathbb{I}\}/\{\pm 1\}$$

Represented by the modular transformations (level M>2)

$$S=\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad T_{(M)}=\begin{pmatrix} e^{-2i\pi/M} & 0 \\ 1 & e^{2i\pi/M} \end{pmatrix}$$

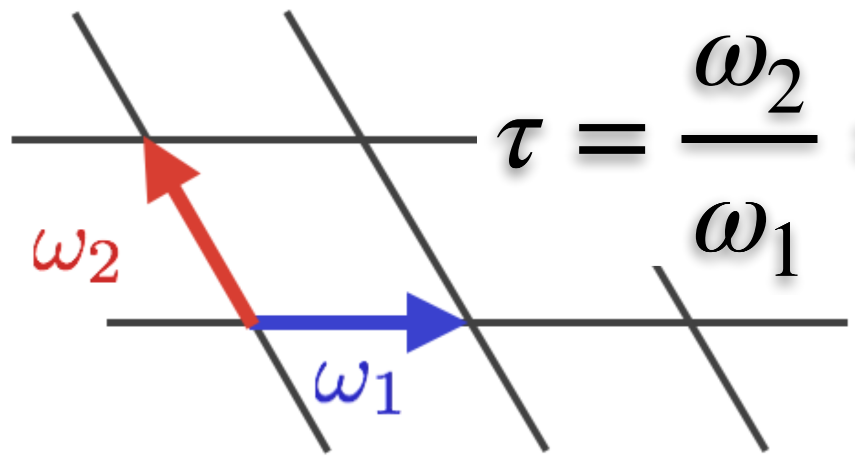
We show that for the orbifold $T^2/\mathbb{Z}_2$
the **fixed points** are only invariant for a particular
level M=3 and **fixed modulus** $\omega = e^{i2\pi/3}$

$$\bar{\Gamma}_3 = A_4 \text{ with } \tau=\omega \text{ or } \tau=\omega+1.$$

F. De Anda, S.F.K., E.Perdomo,1812.05620

The orbifold $T^2/\mathbb{Z}_2$ with $\omega = e^{i2\pi/3}$ and modular A_4 symmetry

$\omega_1 = 1$ and $\omega_2 = e^{i2\pi/3}$

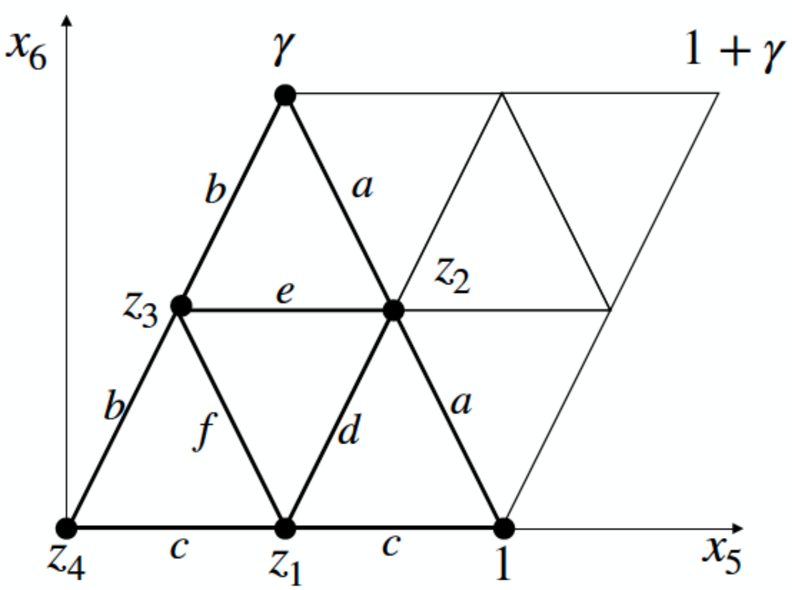


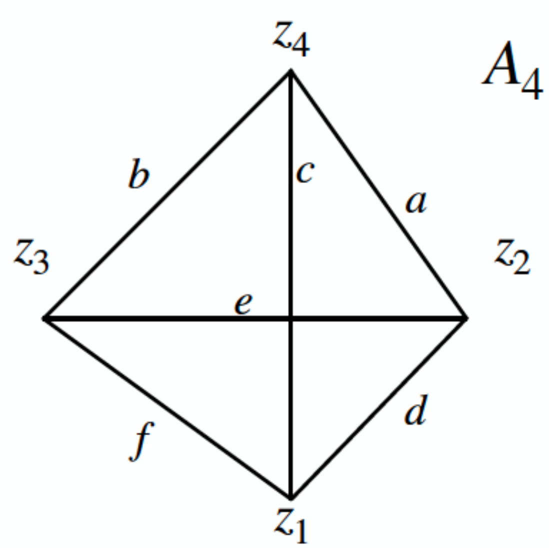
$$\tau = \frac{\omega_2}{\omega_1} = \omega$$

$$z = z + 1,$$
$$z = z + \omega,$$
$$z = -z,$$

$T^2/\mathbb{Z}_2$

Orbifold Fixed Points $\left\{0, \frac{1}{2}, \frac{\omega}{2}, \frac{1+\omega}{2}\right\}$ **Invariant under A4 modular and A4 remnant symmetry:**





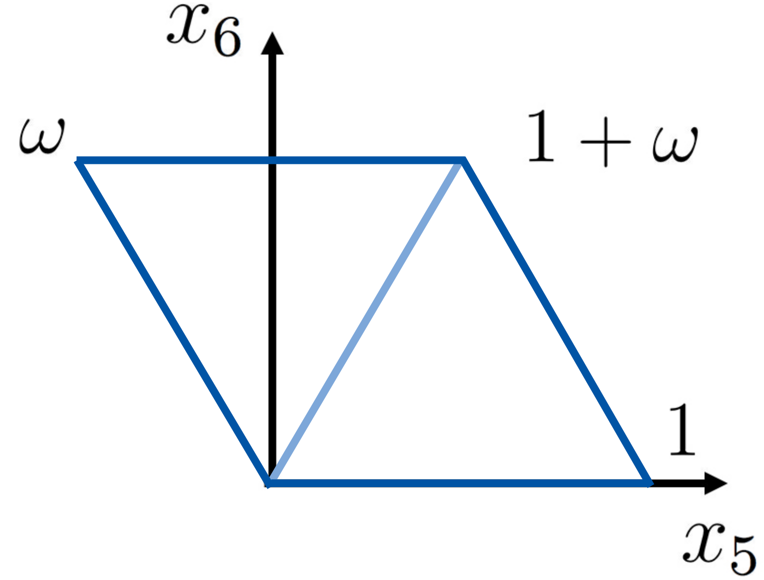
A4 remnant (linear):

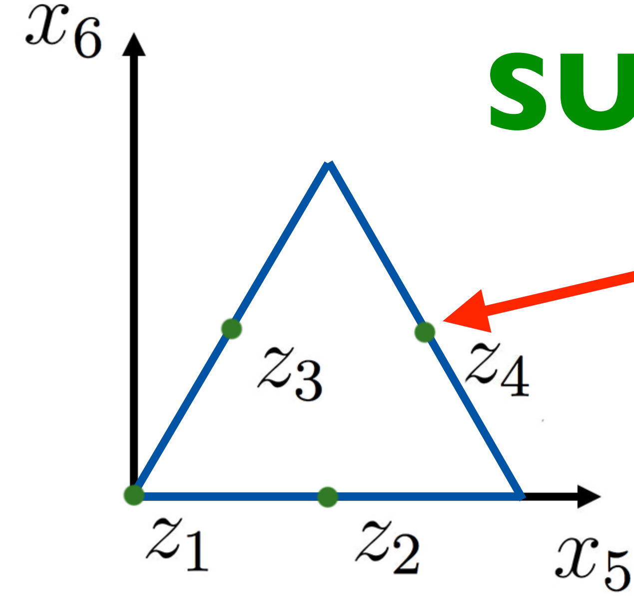
$$\mathcal{S} : (z_1, z_2, z_3, z_4) \to (z_4, z_3, z_2, z_1)$$
$$\mathcal{T} : (z_1, z_2, z_3, z_4) \to (z_2, z_3, z_1, z_4)$$
$$\mathcal{S}^2 = \mathcal{T}^3 = (\mathcal{S}\mathcal{T})^3 = 1$$

(A4 modular="passive", and A4 remnant= "active")

The orbifold $T^2/\mathbb{Z}_2$ with $\omega = e^{i2\pi/3}$ and modular A_4 symmetry

Brane fields have an enhanced Z_2 mu-tau reflection symmetry (arising from remnant $S_4 \simeq A_4 \ltimes \mathbb{Z}_2$)

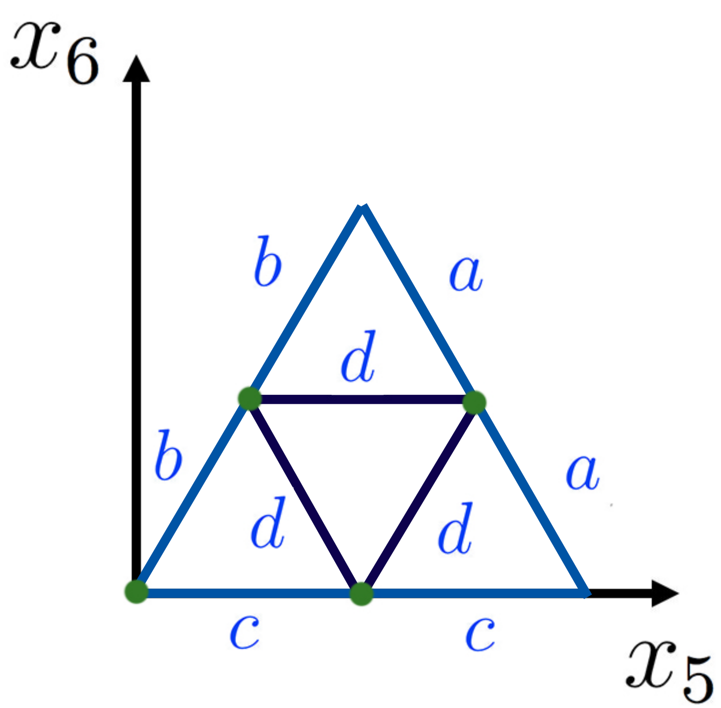


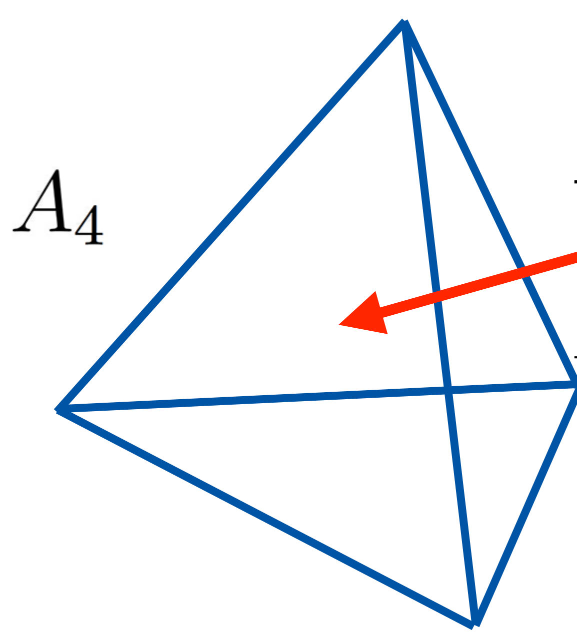


SU(5) GUT Model

Brane fields

Field	Representation		
	$A_4 \ltimes \mathbb{Z}_2$	$SU(5)$	$U(1)$
F	$\mathbf{3}$	$\bar{\mathbf{5}}$	$a+2c$
N_s^c	$\mathbf{1}$	$\mathbf{1}$	a
N_a^c	$\mathbf{1}$	$\mathbf{1}$	$4a$
ξ	$\mathbf{1}$	$\mathbf{1}$	$-2a$





Bulk fields

Field	Representation				Localization		
	A_4	$SU(5)$	$U(1)$	Weight	P_0	$P_{1/2}$	$P_{\omega/2}$
$T_1^{\pm}$	$\mathbf{1''}$	$\mathbf{10}$	$c+4a$	$-\gamma$	$+1$	± 1	± 1
$T_2^{\pm}$	$\mathbf{1'}$	$\mathbf{10}$	$c+2a$	$-\gamma$	$+1$	± 1	± 1
$T_3^{\pm}$	$\mathbf{1}$	$\mathbf{10}$	c	$-\gamma$	$+1$	± 1	± 1
H_5	$\mathbf{1}$	$\mathbf{5}$	$-2c$	$-\alpha$	$+1$	$+1$	$+1$
$H_{\bar{5}}$	$\mathbf{1'}$	$\bar{\mathbf{5}}$	b	$\alpha+\gamma$	$+1$	$+1$	$+1$
ϕ_1	$\mathbf{3}$	$\mathbf{1}$	$-b-a-3c$	$-\alpha$	$+1$	$+1$	-1
ϕ_2	$\mathbf{3}$	$\mathbf{1}$	$-3a$	$\alpha-\beta$	$+1$	-1	$+1$

F. De Anda, S.F.K., E.Perdomo,1812.05620

The orbifold $T^2/\mathbb{Z}_2$ with $\omega = e^{i2\pi/3}$ and modular A_4 symmetry

BC's

$$P_0 = \mathbb{I}_3 \times \mathbb{I}_5,$$
$$P_{1/2} = T_1 \times \mathrm{diag}(-1,-1,-1,1,1),$$
$$P_{\omega/2} = T_2 \times \mathrm{diag}(-1,-1,-1,1,1),$$

$$T_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad T_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = U$$

Breaks A4 and SU(5) with doublet-triplet splitting

$$\langle \phi_1 \rangle = v_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \langle \phi_2 \rangle = v_2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

Vacuum alignment from bc's

$$\begin{aligned} \mathcal{W}_Y = {} & y_s^N \xi N_s^c N_s^c + y_a^N \xi \frac{\xi^3}{\Lambda^3} N_a^c N_a^c \\ & + y_s^\nu \frac{\xi}{\Lambda} F H_5 N_s^c + y_a^\nu \frac{\phi_2 \xi}{\Lambda^2} F H_5 N_a^c \\ & + y_3^e \frac{\phi_1}{\Lambda} F H_{\bar{5}} T_3^+ + y_2^e \frac{\phi_1 \xi}{\Lambda^2} F H_{\bar{5}} T_2^+ + y_1^e \frac{\phi_1 \xi^2}{\Lambda^3} F H_{\bar{5}} T_1^+ \\ & + y_3^d \frac{\phi_1}{\Lambda} F H_{\bar{5}} T_3^- + y_2^d \frac{\phi_1 \xi}{\Lambda^2} F H_{\bar{5}} T_2^- + y_1^d \frac{\phi_1 \xi^2}{\Lambda^3} F H_{\bar{5}} T_1^- \\ & + y_{ij}^u H_5 T_i^+ T_j^- \frac{\xi^{6-i-j}}{\Lambda^{6-i-j}}, \end{aligned}$$

Brane fields

Field	Representation		
	$A_4 \ltimes \mathbb{Z}_2$	$SU(5)$	$U(1)$
F	**3**	$\bar{\mathbf{5}}$	$a+2c$
N_s^c	**1**	**1**	a
N_a^c	**1**	**1**	$4a$
ξ	**1**	**1**	$-2a$

Bulk fields

Field	Representation				Localization		
	A_4	$SU(5)$	$U(1)$	Weight	P_0	$P_{1/2}$	$P_{\omega/2}$
$T_1^\pm$	$\mathbf{1}''$	**10**	$c+4a$	$-\gamma$	+1	±1	±1
$T_2^\pm$	$\mathbf{1}'$	**10**	$c+2a$	$-\gamma$	+1	±1	±1
$T_3^\pm$	**1**	**10**	c	$-\gamma$	+1	±1	±1
H_5	**1**	**5**	$-2c$	$-\alpha$	+1	+1	+1
$H_{\bar{5}}$	$\mathbf{1}'$	$\bar{\mathbf{5}}$	b	$\alpha+\gamma$	+1	+1	+1
ϕ_1	**3**	**1**	$-b-a-3c$	$-\alpha$	+1	+1	-1
ϕ_2	**3**	**1**	$-3a$	$\alpha-\beta$	+1	-1	+1

F. De Anda, S.F.K., E.Perdomo,1812.05620

The orbifold $T^2/\mathbb{Z}_2$ with $\omega = e^{i2\pi/3}$ and modular A_4 symmetry

$$M^d = v_d \begin{pmatrix} y_1^d \tilde{\xi}^2 & 0 & 0 \\ 0 & y_2^d \tilde{\xi} & 0 \\ 0 & 0 & y_3^d \end{pmatrix} \tilde{v}_1,$$

$$M^e = v_d \begin{pmatrix} y_1^e \tilde{\xi}^2 & 0 & 0 \\ 0 & y_2^e \tilde{\xi} & 0 \\ 0 & 0 & y_3^e \end{pmatrix} \tilde{v}_1,$$

$$M_u = v_u \begin{pmatrix} y_{11}^u \tilde{\xi}^4 & y_{12}^u \tilde{\xi}^3 & y_{13}^u \tilde{\xi}^2 \\ y_{21}^u \tilde{\xi}^3 & y_{22}^u \tilde{\xi}^2 & y_{23} \tilde{\xi} \\ y_{31}^u \tilde{\xi}^2 & y_{32}^u \tilde{\xi} & y_{33}^u \end{pmatrix} \tilde{v}_2,$$

α	$(y_s^\nu)_{\mathbf{3}}$
0	0
2	$y\begin{pmatrix} 2 \\ 2\omega \\ -\omega^2 \end{pmatrix}$
4	$y\begin{pmatrix} 2 \\ -\omega \\ 2\omega^2 \end{pmatrix}$
6	$y\begin{pmatrix} -1 \\ 2\omega \\ 2\omega^2 \end{pmatrix}$

Modular Forms

$$\tau = \omega = e^{i2\pi/3}$$

Select

$$\alpha = \beta = 6, \quad \gamma = 7$$

Dirac

$$\alpha_6 = y \begin{pmatrix} -1 \\ 2\omega^2 \\ 2\omega \end{pmatrix}, \quad \beta_6 = \begin{pmatrix} 2y_2 + y_3(2\omega^2 - 2\omega) \\ y_1 + y_2(4\omega + 1) - y_3 \\ y_1 + y_2(4\omega^2 + 1) + y_3 \end{pmatrix},$$

β	$(y_a^\nu \langle \phi_2 \rangle)_3 / v_2$
0	$y_1 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$
2	$y_1 \begin{pmatrix} \omega^2 - 2\omega \\ -2\omega - 2 \\ 4\omega - 2 \end{pmatrix} + y_2 \begin{pmatrix} -\omega^2 - 2\omega \\ -2 \\ 2 \end{pmatrix}$
4	$y_1\omega \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + y_2 \begin{pmatrix} -2\omega^2 + \omega \\ 2\omega^2 - 2 \\ -2\omega^2 - 2 \end{pmatrix} + y_3 \begin{pmatrix} 2\omega^2 + \omega \\ -2 \\ 2 \end{pmatrix}$
6	$y_1 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + y_2 \begin{pmatrix} 2 \\ 4\omega^2 + 1 \\ 4\omega + 1 \end{pmatrix} + y_3 \begin{pmatrix} 2\omega^2 - 2\omega \\ 1 \\ -1 \end{pmatrix}$

Majorana

$$M_R = \langle \xi \rangle \begin{pmatrix} y_a^N \tilde{\xi}^3 & 0 \\ 0 & y_s^N \end{pmatrix}$$

Seesaw

$$m_\nu = \left(\frac{v_u^2}{\langle \xi \rangle} \frac{\tilde{\xi}^2}{y_s^N} \right) \alpha_6 (\alpha_6)^T + \left(\frac{v_u^2}{\langle \xi \rangle} \frac{\tilde{v}_2^2}{\tilde{\xi} y_a^N} \right) \beta_6 (\beta_6)^T$$

mu-tau symmetry

Summary

4d models

- Littlest seesaw fit with RG corrections fixes M_R's
- Littlest mu-tau seesaw…one parameter…wow
- New Littlest seesaw from tri-direct CP symmetry
- Type 1b and Inverse seesaw possibilities

6d models

- A4 and A5 results sensitive to free modulus tau
- Orbifold T2/Z2 suggests A4 with fixed tau = omega
- Explicit A4xSU(5) model with mu-tau symmetry

Thank You!