$\mathcal{CP}$ violation from strings

Michael Ratz

FLASY 19

Based on:

- **M.–C. Chen**, M. Fallbacher, K.T. Mahanthappa, M.R. & A. Trautner
  Nucl. Phys. **B883**, 267–305 (2014)
- H.P. Nilles, A. Trautner, M.R. & P. Vaudrevange, Phys. Lett. **B786**, 283–287 (2018)

Overview

☞ string theory is the arguably most promising candidate for a unified description of Nature

Overview

- ☞ string theory is the arguably most promising candidate for a unified description of Nature
- ☞ it has been notoriously hard to establish $\mathcal{CP}$ violationg in string theory

  e.g. Dine, Leigh & MacIntire (1992)

Overview

- string theory is the arguably most promising candidate for a unified description of Nature
- it has been notoriously hard to establish $\mathcal{CP}$ violationg in string theory
  e.g. Dine, Leigh & MacIntire (1992)
- however, string compactifications give rise to non–Abelian discrete flavor symmetries
  e.g. Kobayashi, Nilles, Plöger, Raby & M.R. (2007)

Overview

- ☞ string theory is the arguably most promising candidate for a unified description of Nature
- ☞ it has been notoriously hard to establish $\mathcal{CP}$ violationg in string theory
  e.g. Dine, Leigh & MacIntire (1992)
- ☞ however, string compactifications give rise to non–Abelian discrete flavor symmetries
  e.g. Kobayashi, Nilles, Plöger, Raby & M.R. (2007)
- ☞ already the very first stringy standard models turn out to have a $\Delta(54)$ flavor symmetry
  Ibáñez, Kim, Nilles & Quevedo (1987)

Overview

- ☞ string theory is the arguably most promising candidate for a unified description of Nature
- ☞ it has been notoriously hard to establish $\mathcal{CP}$ violationg in string theory
  e.g. Dine, Leigh & MacIntire (1992)
- ☞ however, string compactifications give rise to non–Abelian discrete flavor symmetries
  e.g. Kobayashi, Nilles, Plöger, Raby & M.R. (2007)
- ☞ already the very first stringy standard models turn out to have a $\Delta(54)$ flavor symmetry
  Ibáñez, Kim, Nilles & Quevedo (1987)
- ☞ some of these flavor symmetries clash with $\mathcal{CP}$
  Chen & Mahanthappa (2009) ,Chen, Fallbacher, Mahanthappa, M.R. & Trautner (2014)

Overview

- string theory is the arguably most promising candidate for a unified description of Nature
- it has been notoriously hard to establish $\mathcal{CP}$ violationg in string theory
  e.g. Dine, Leigh & MacIntire (1992)
- however, string compactifications give rise to non–Abelian discrete flavor symmetries
  e.g. Kobayashi, Nilles, Plöger, Raby & M.R. (2007)
- already the very first stringy standard models turn out to have a $\Delta(54)$ flavor symmetry
  Ibáñez, Kim, Nilles & Quevedo (1987)
- some of these flavor symmetries clash with $\mathcal{CP}$
  Chen & Mahanthappa (2009) ,Chen, Fallbacher, Mahanthappa, M.R. & Trautner (2014)

purpose of this talk:

show that $\mathcal{CP}$ is violated in many potentially realistic string compactifications without further ado

$\mathcal{CP}$ violation

from finite groups

$\mathcal{CP}$ violation in Nature

☞ $\not{\mathcal{CP}}$ so far only observed in flavor sector

$\mathcal{CP}$ violation in Nature

☞ $\not{\mathcal{CP}}$ so far only observed in flavor sector

➥ it appears natural to seek connection between flavor physics & $\not{\mathcal{CP}}$

$\mathcal{CP}$ violation in Nature

- ☞ $\not{\mathcal{CP}}$ so far only observed in flavor sector
- ➥ it appears natural to seek connection between flavor physics & $\not{\mathcal{CP}}$
- ☞ flavor structure may be partially explained by (non–Abelian discrete) flavor symmetries

huge literature

$\mathcal{CP}$ violation in Nature

- ☞ $\not{\mathcal{CP}}$ so far only observed in flavor sector
- ➥ it appears natural to seek connection between flavor physics & $\not{\mathcal{CP}}$
- ☞ flavor structure may be partially explained by (non–Abelian discrete) flavor symmetries

huge literature

here:

non–Abelian discrete (flavor) symmetry $G \leftrightarrow \not{\mathcal{CP}}$

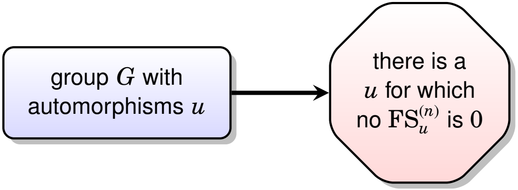
Three types of groups

Chen, Fallbacher, Mahanthappa, M.R. & Trautner (2014)

group G with automorphisms u

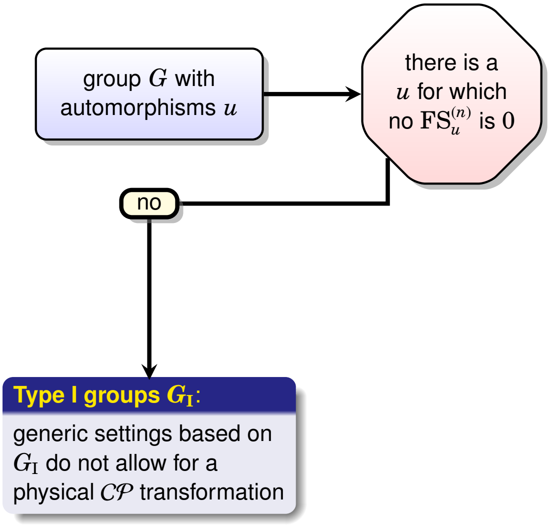
Three types of groups

Chen, Fallbacher, Mahanthappa, M.R. & Trautner (2014)



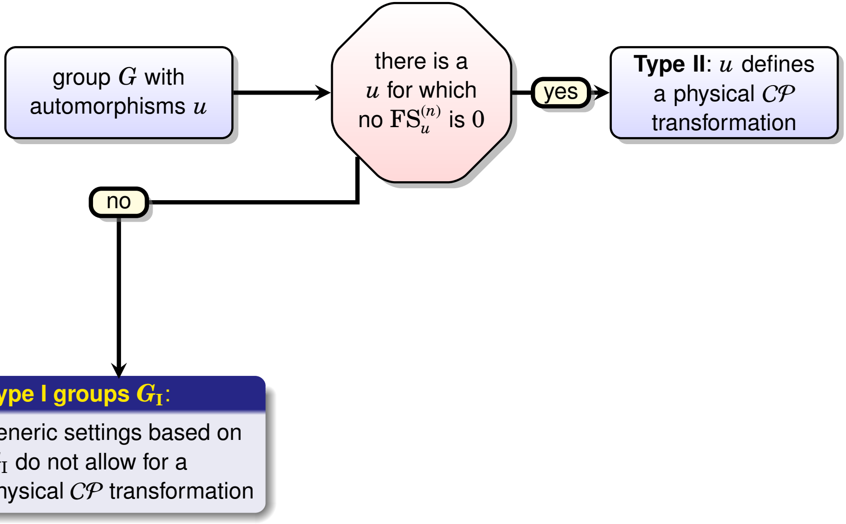
Three types of groups

Chen, Fallbacher, Mahanthappa, M.R. & Trautner (2014)



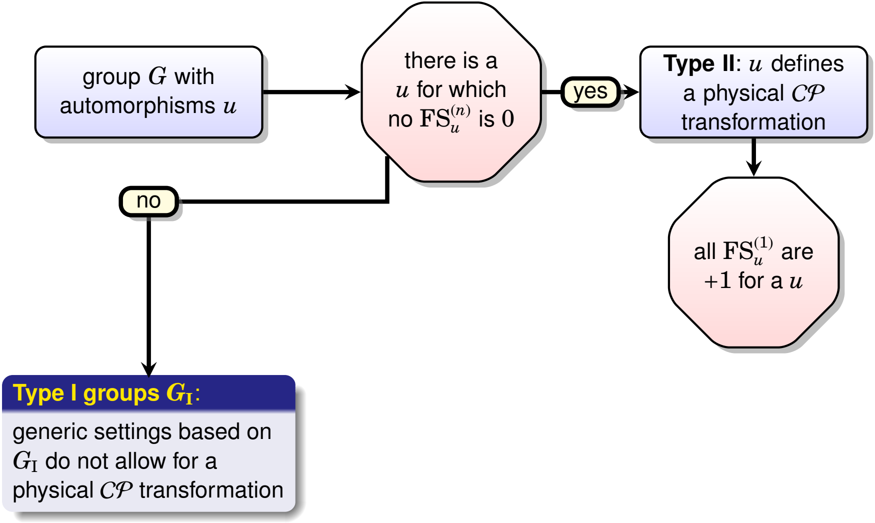
Three types of groups

Chen, Fallbacher, Mahanthappa, M.R. & Trautner (2014)



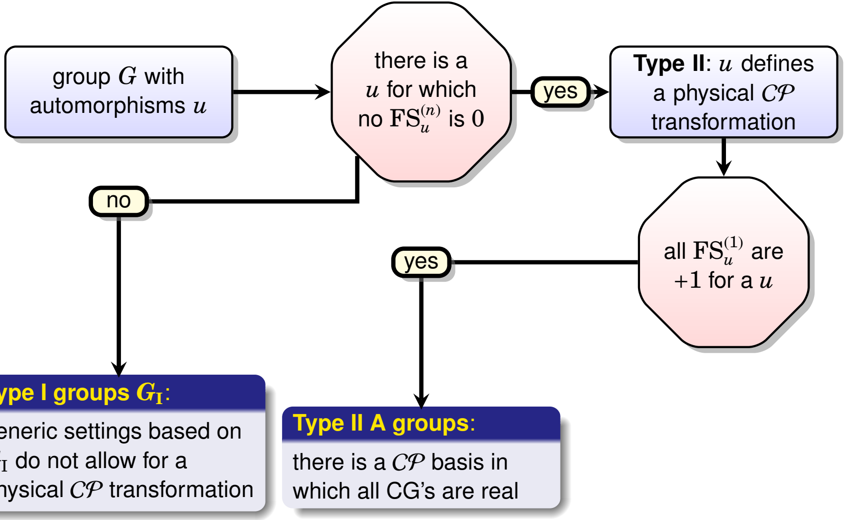
Three types of groups

Chen, Fallbacher, Mahanthappa, M.R. & Trautner (2014)



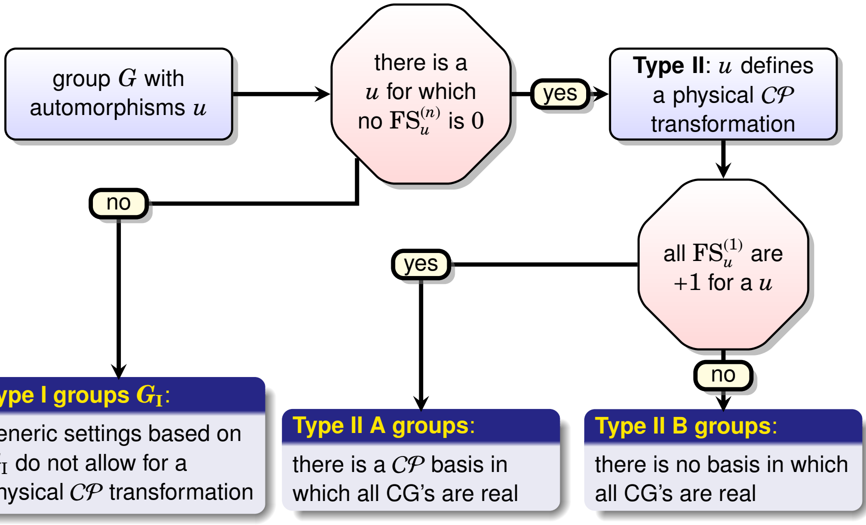
Three types of groups

Chen, Fallbacher, Mahanthappa, M.R. & Trautner (2014)



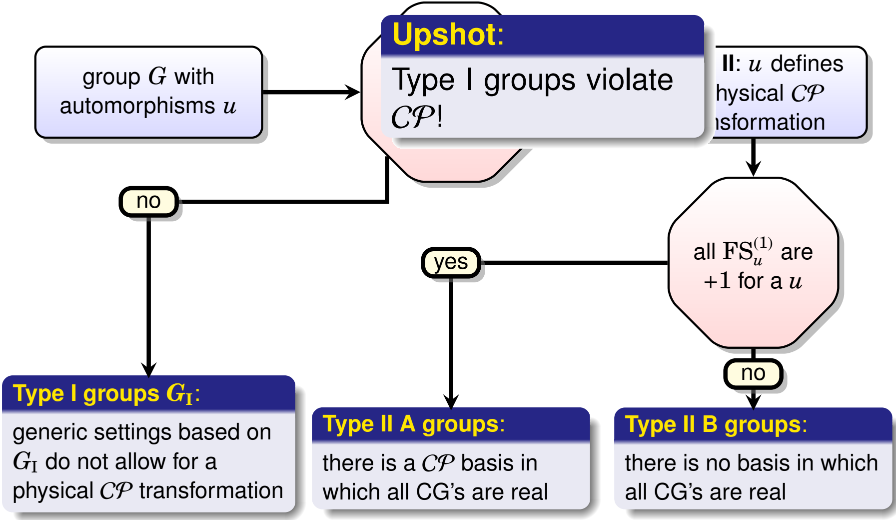
Three types of groups

Chen, Fallbacher, Mahanthappa, M.R. & Trautner (2014)



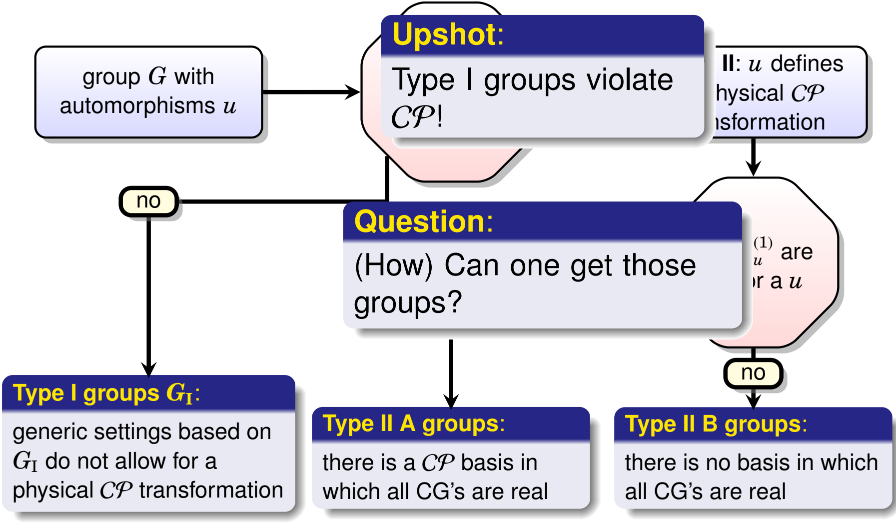
Three types of groups

Chen, Fallbacher, Mahanthappa, M.R. & Trautner (2014)



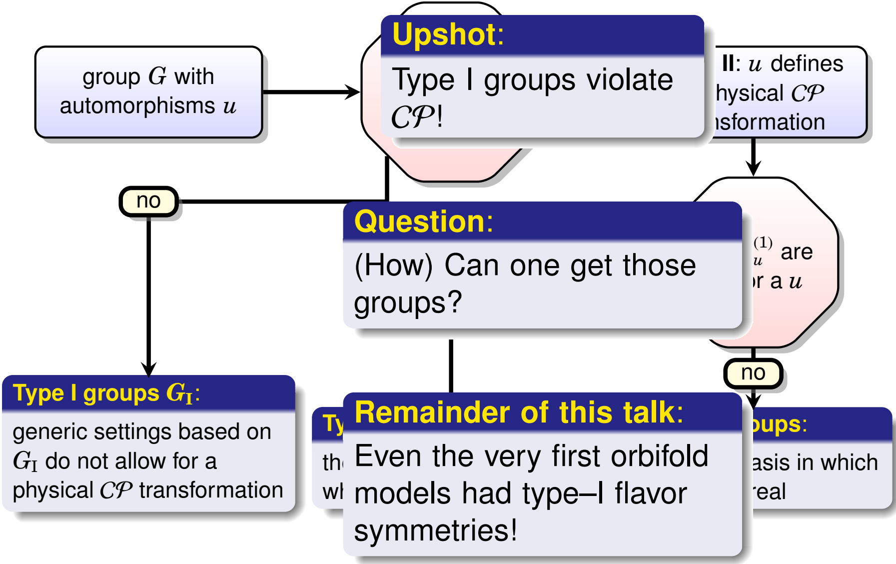
Three types of groups

Chen, Fallbacher, Mahanthappa, M.R. & Trautner (2014)



Three types of groups

Chen, Fallbacher, Mahanthappa, M.R. & Trautner (2014)

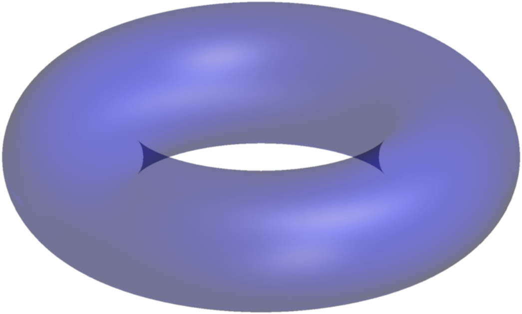


heterotic

orbifolds

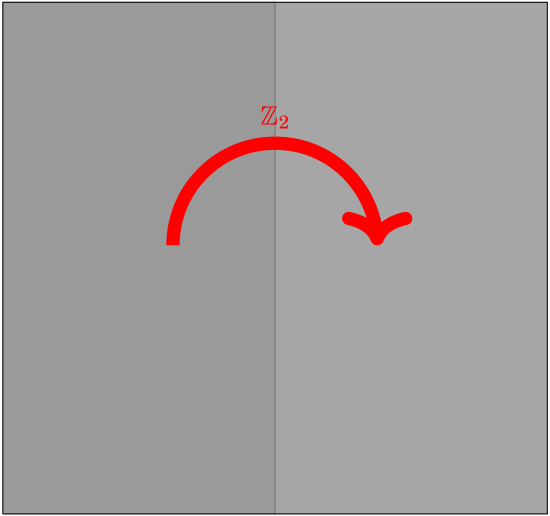
$\mathbb{Z}_2$ orbifold pillow

☞ Starting point: torus

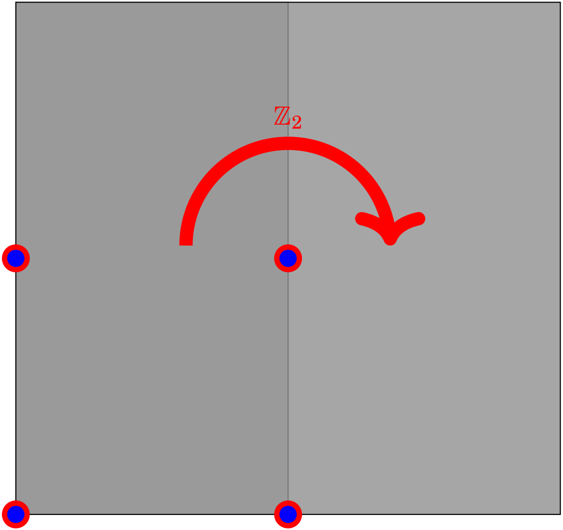


$\mathbb{Z}_2$ orbifold pillow

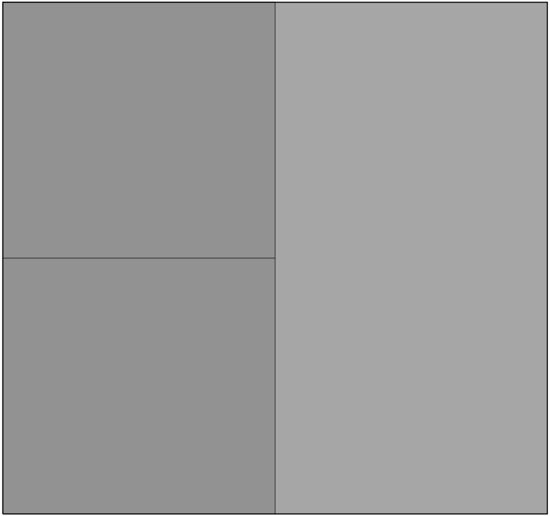
$\mathbb{Z}_2$ orbifold pillow



$\mathbb{Z}_2$ orbifold pillow

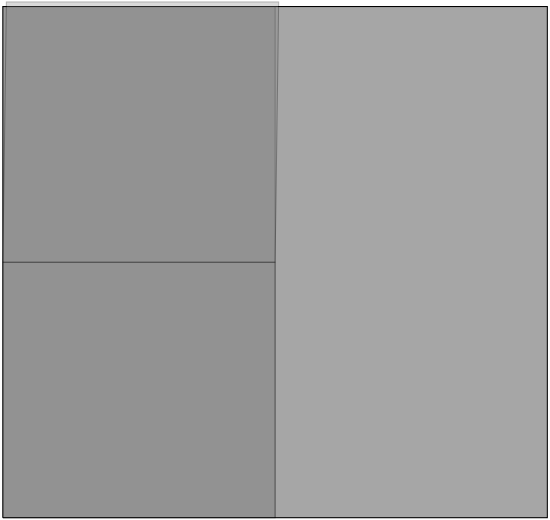


$\mathbb{Z}_2$ orbifold pillow



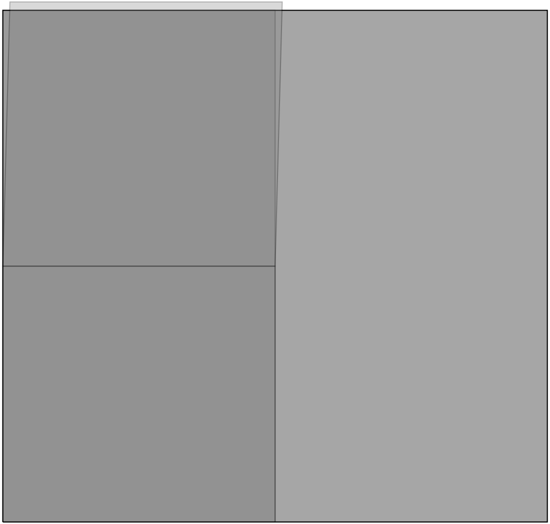
back

$\mathbb{Z}_2$ orbifold pillow



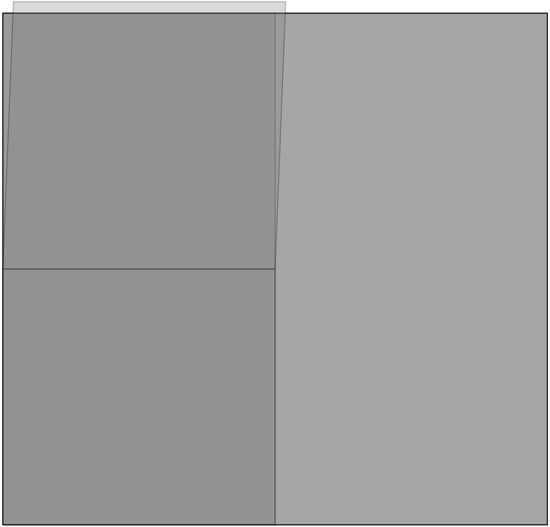
back

$\mathbb{Z}_2$ orbifold pillow



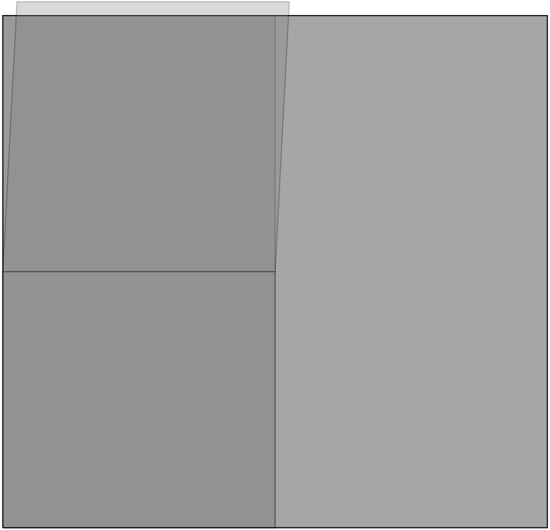
▸ back

$\mathbb{Z}_2$ orbifold pillow



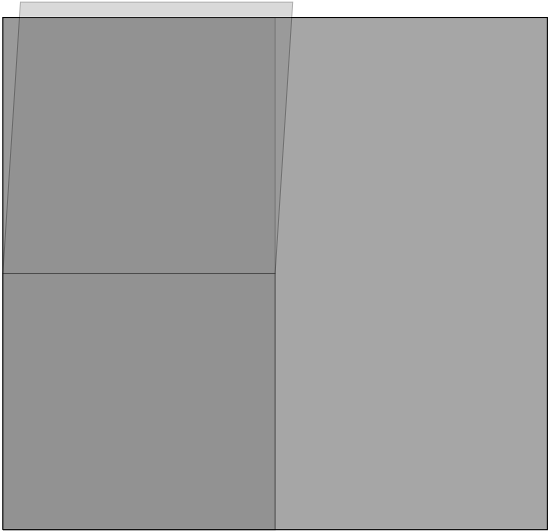
▸ back

$\mathbb{Z}_2$ orbifold pillow



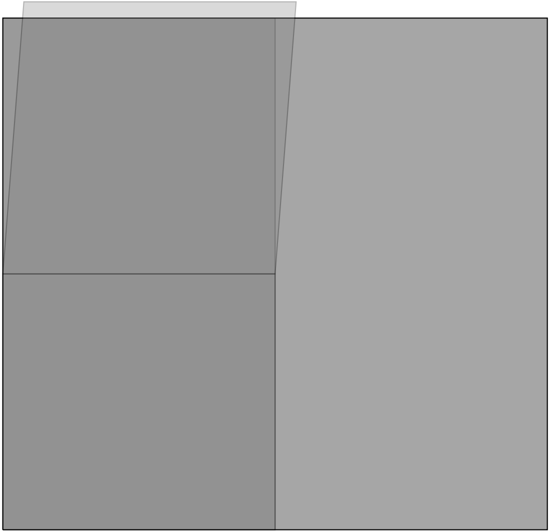
back

$\mathbb{Z}_2$ orbifold pillow



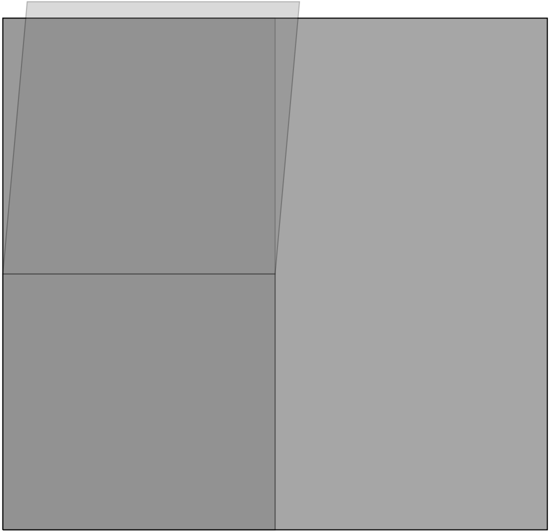
back

$\mathbb{Z}_2$ orbifold pillow



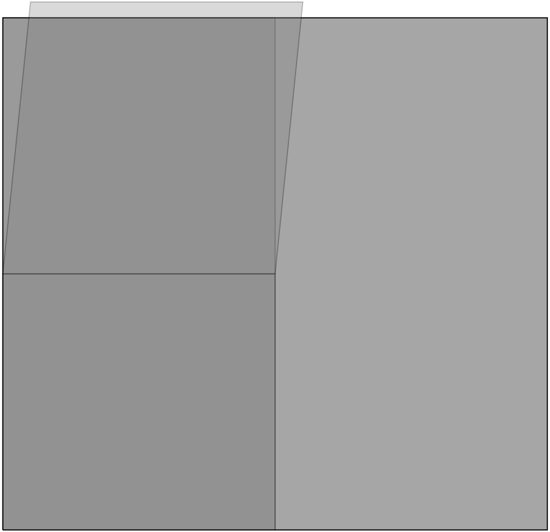
back

$\mathbb{Z}_2$ orbifold pillow



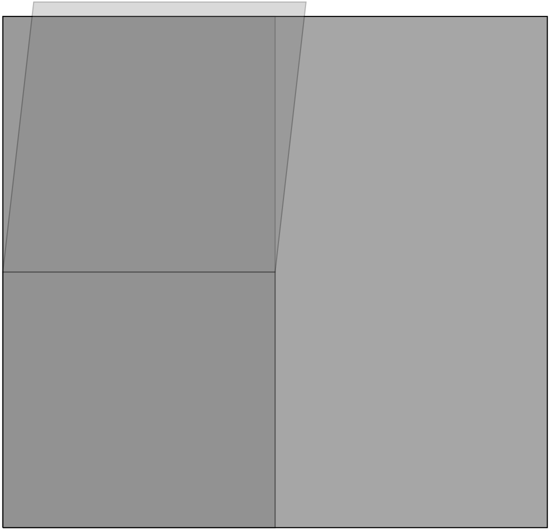
back

$\mathbb{Z}_2$ orbifold pillow



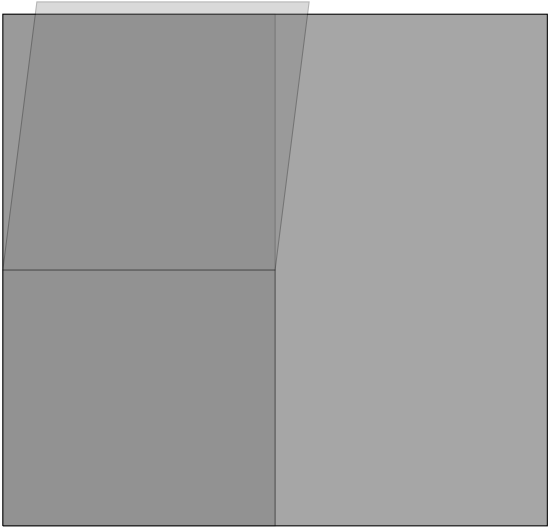
back

$\mathbb{Z}_2$ orbifold pillow



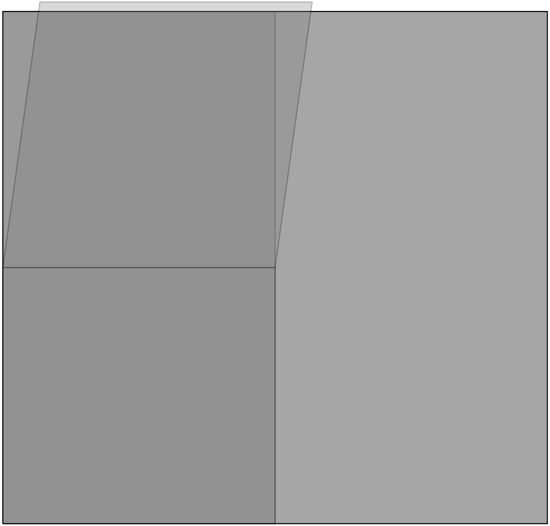
back

$\mathbb{Z}_2$ orbifold pillow



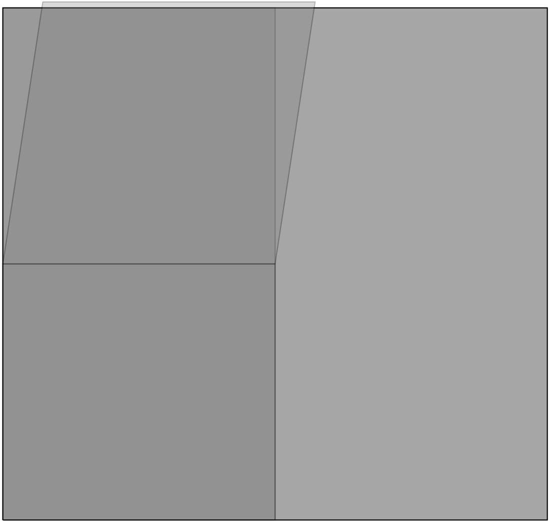
back

$\mathbb{Z}_2$ orbifold pillow



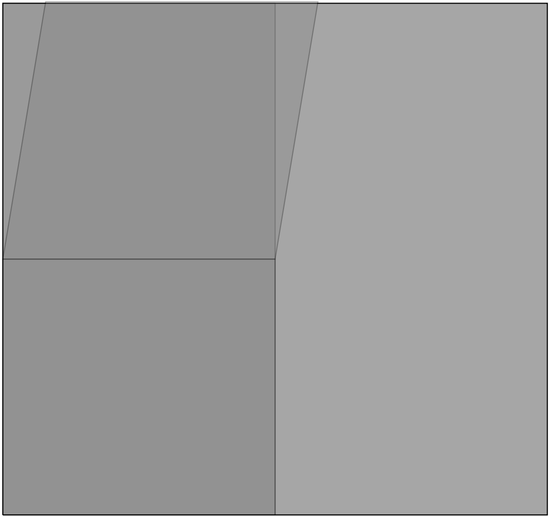
back

$\mathbb{Z}_2$ orbifold pillow



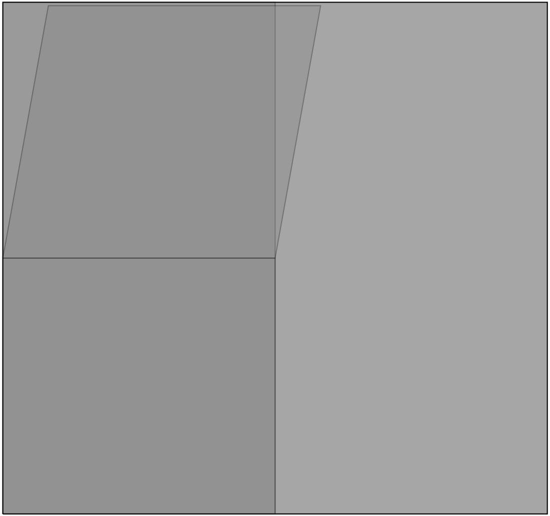
back

$\mathbb{Z}_2$ orbifold pillow



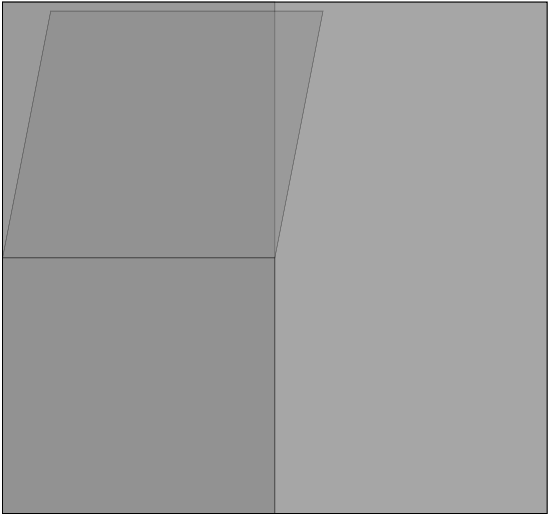
back

$\mathbb{Z}_2$ orbifold pillow



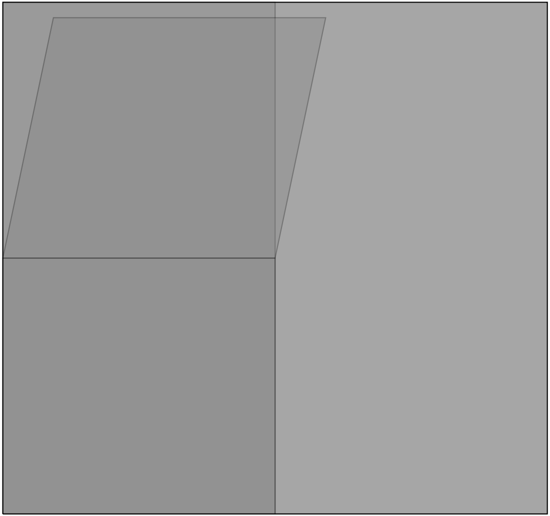
back

$\mathbb{Z}_2$ orbifold pillow



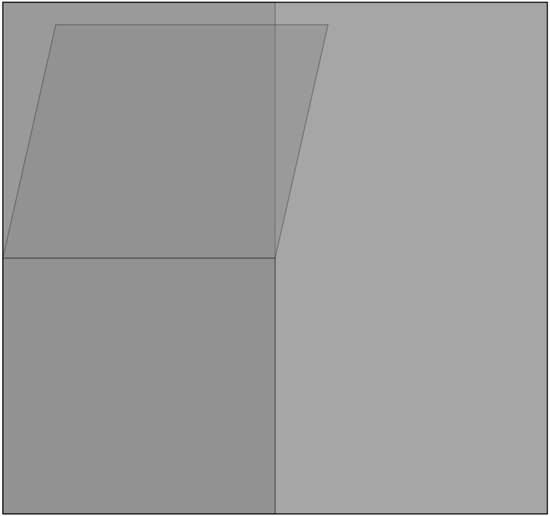
back

$\mathbb{Z}_2$ orbifold pillow



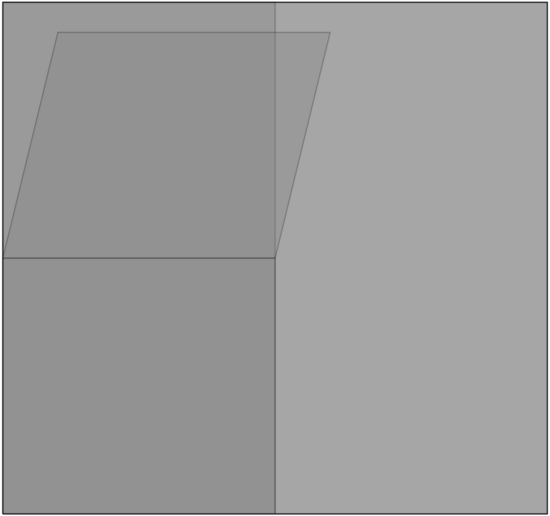
back

$\mathbb{Z}_2$ orbifold pillow



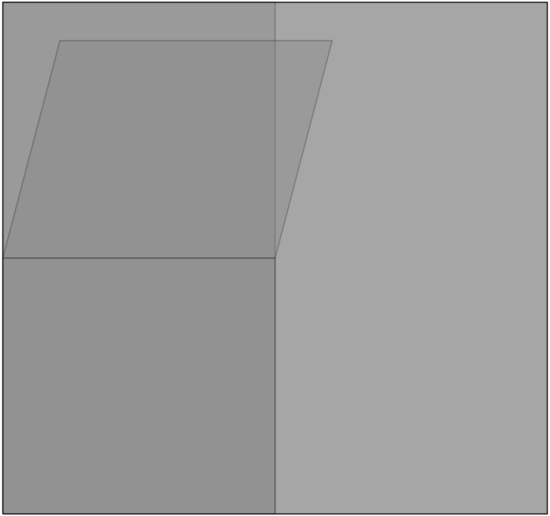
back

$\mathbb{Z}_2$ orbifold pillow



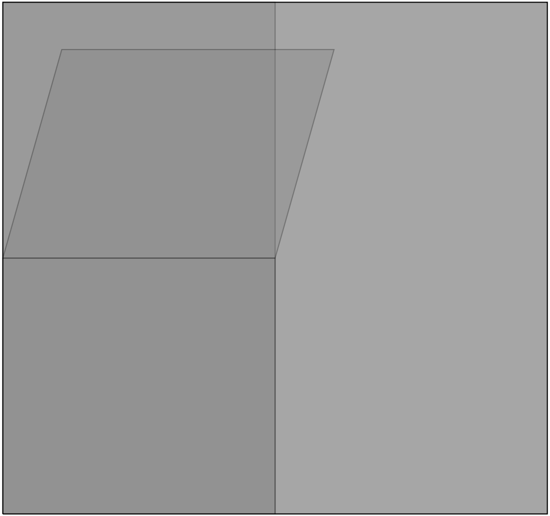
back

$\mathbb{Z}_2$ orbifold pillow



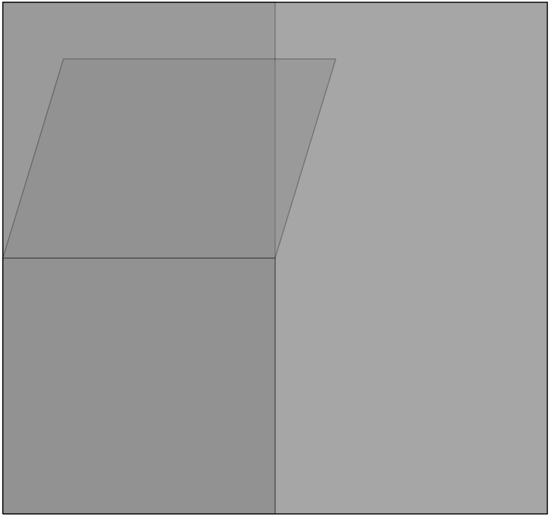
▶ back

$\mathbb{Z}_2$ orbifold pillow



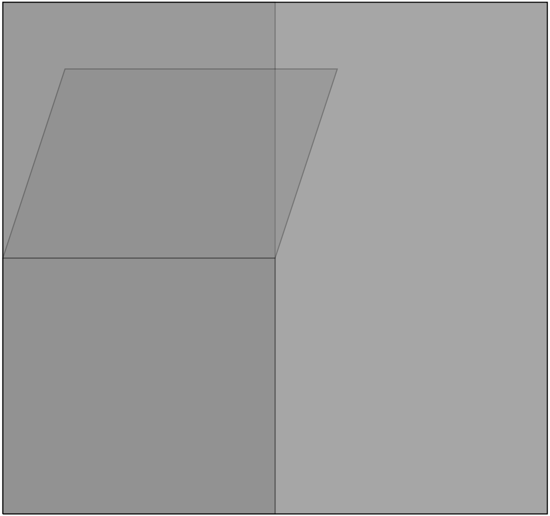
▸ back

$\mathbb{Z}_2$ orbifold pillow



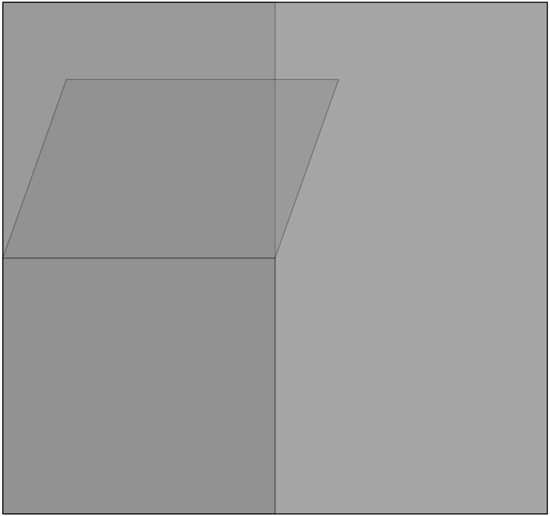
back

$\mathbb{Z}_2$ orbifold pillow



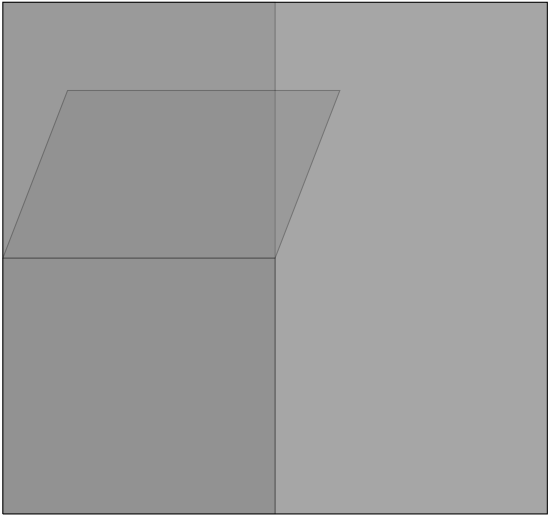
back

$\mathbb{Z}_2$ orbifold pillow



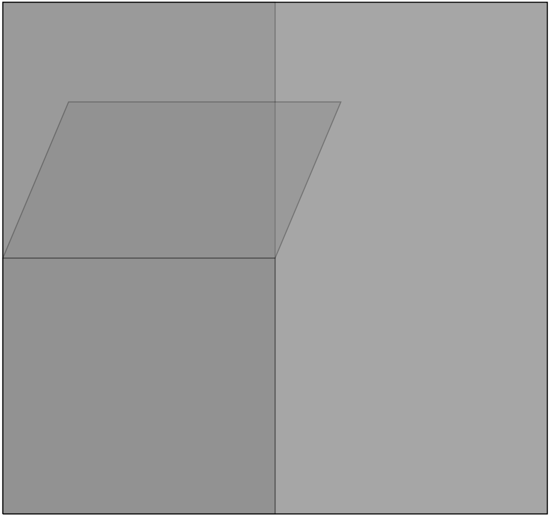
back

$\mathbb{Z}_2$ orbifold pillow



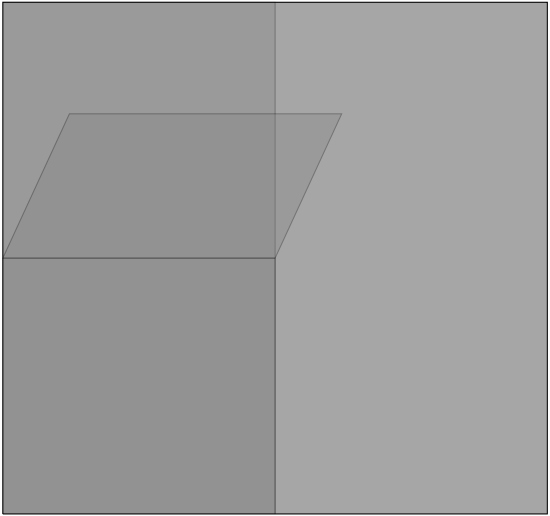
▸ back

$\mathbb{Z}_2$ orbifold pillow



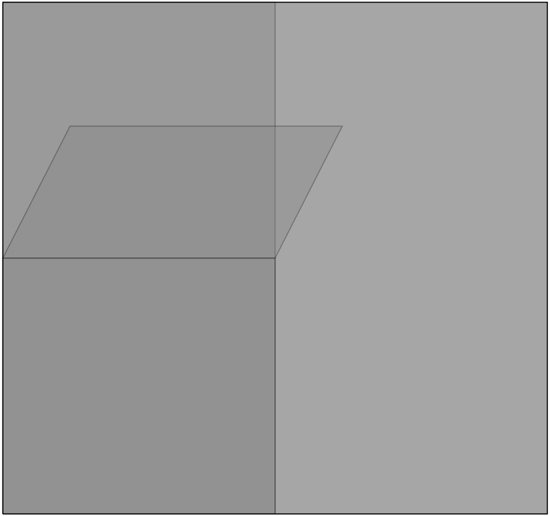
back

$\mathbb{Z}_2$ orbifold pillow



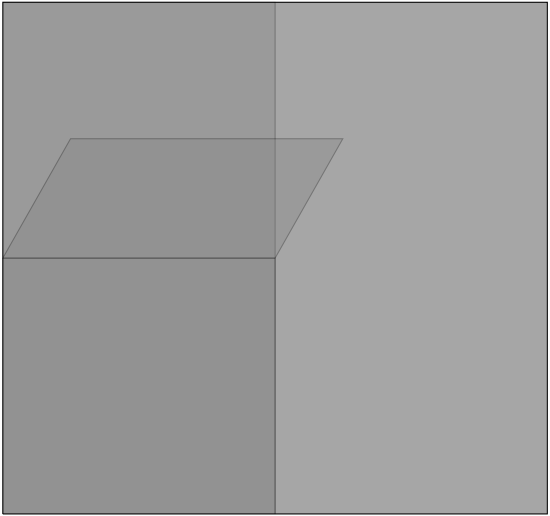
▶ back

$\mathbb{Z}_2$ orbifold pillow



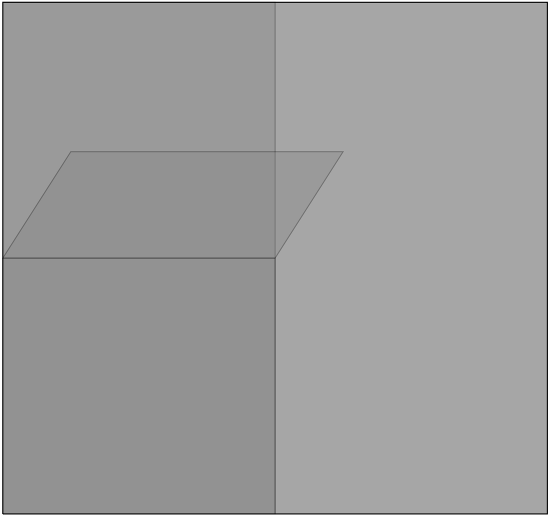
back

$\mathbb{Z}_2$ orbifold pillow



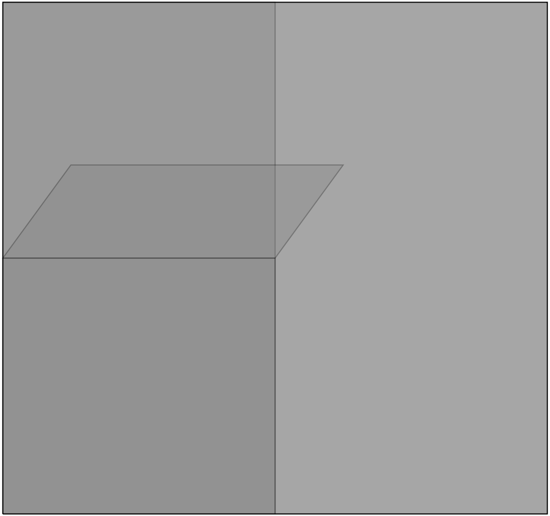
back

$\mathbb{Z}_2$ orbifold pillow



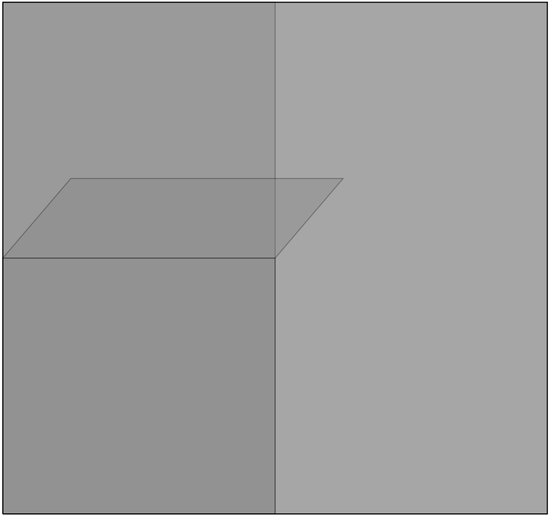
▶ back

$\mathbb{Z}_2$ orbifold pillow



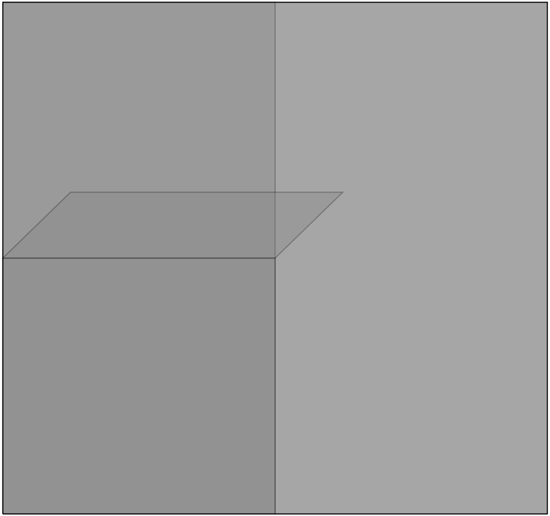
▸ back

$\mathbb{Z}_2$ orbifold pillow



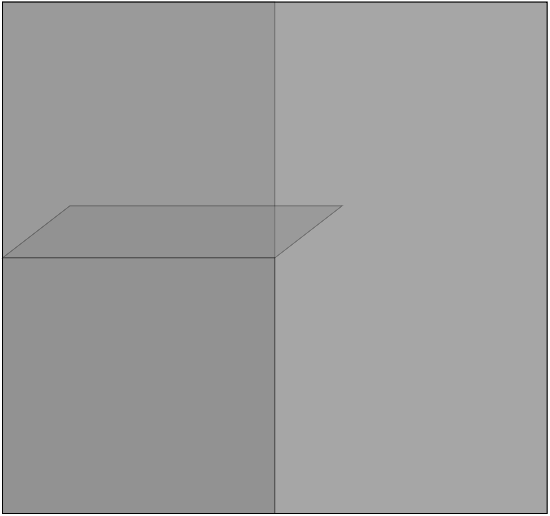
▸ back

$\mathbb{Z}_2$ orbifold pillow



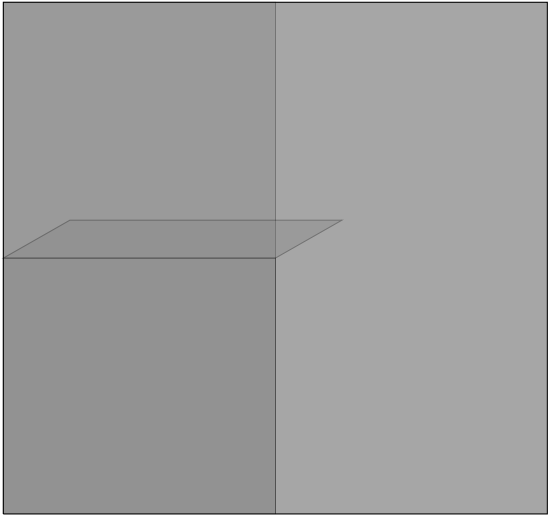
back

$\mathbb{Z}_2$ orbifold pillow



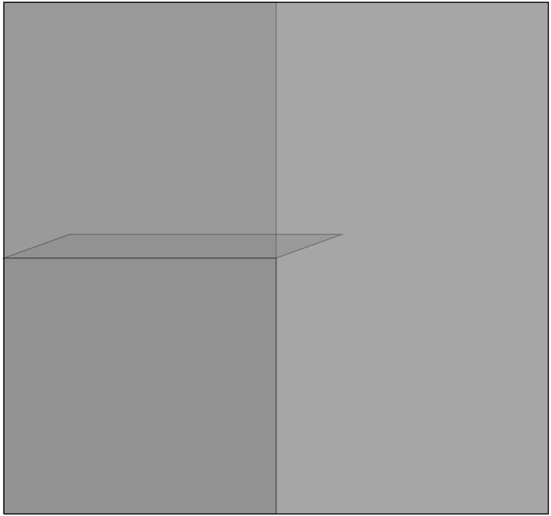
back

$\mathbb{Z}_2$ orbifold pillow



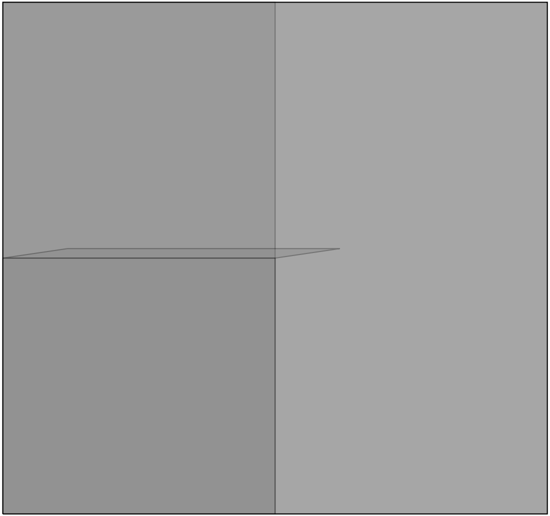
back

$\mathbb{Z}_2$ orbifold pillow



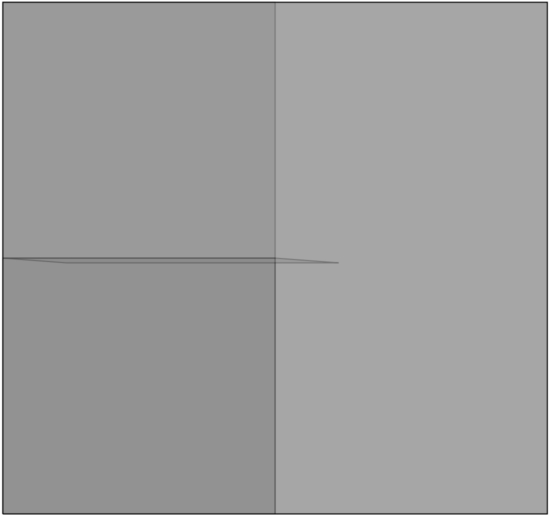
back

$\mathbb{Z}_2$ orbifold pillow



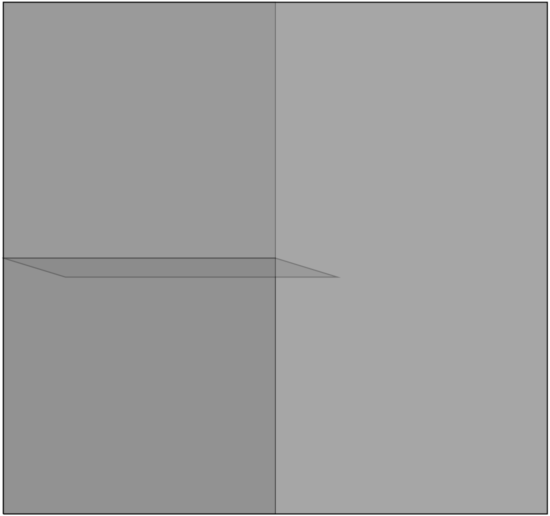
back

$\mathbb{Z}_2$ orbifold pillow



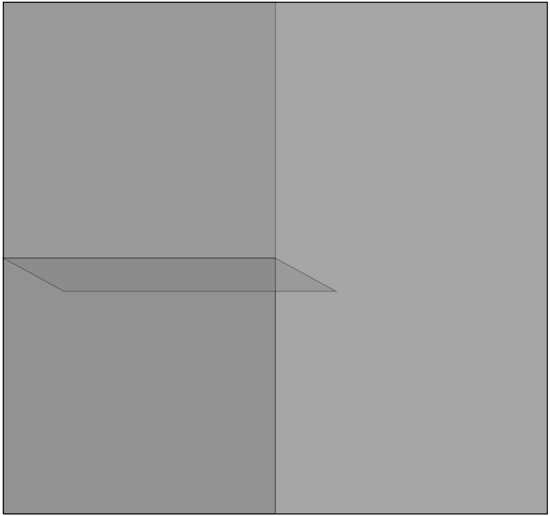
back

$\mathbb{Z}_2$ orbifold pillow



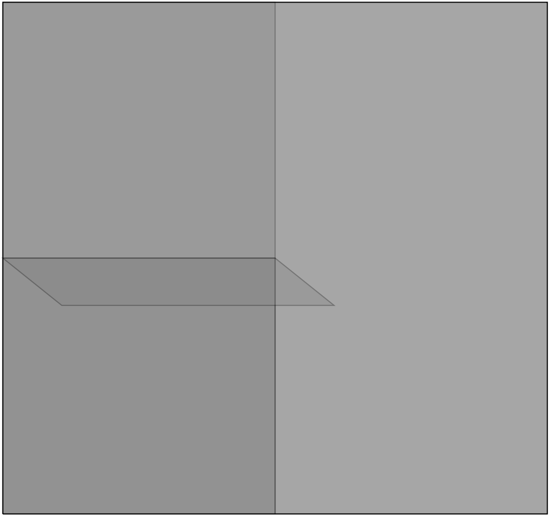
back

$\mathbb{Z}_2$ orbifold pillow



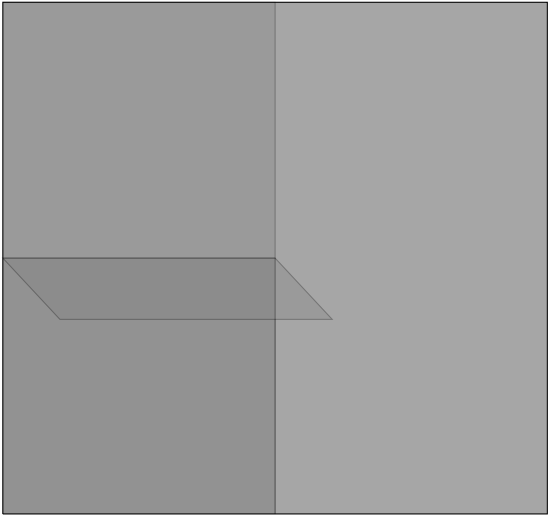
back

$\mathbb{Z}_2$ orbifold pillow



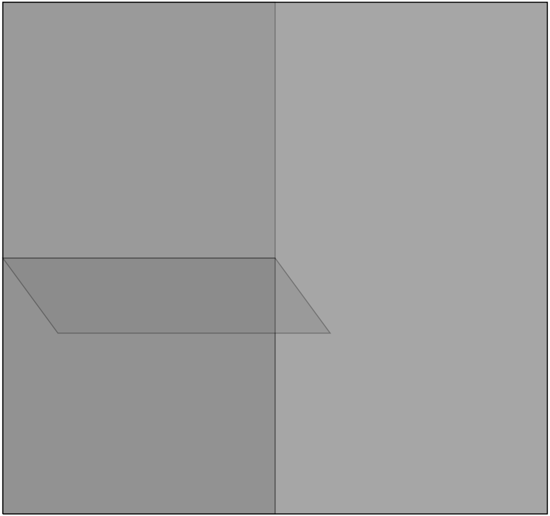
back

$\mathbb{Z}_2$ orbifold pillow



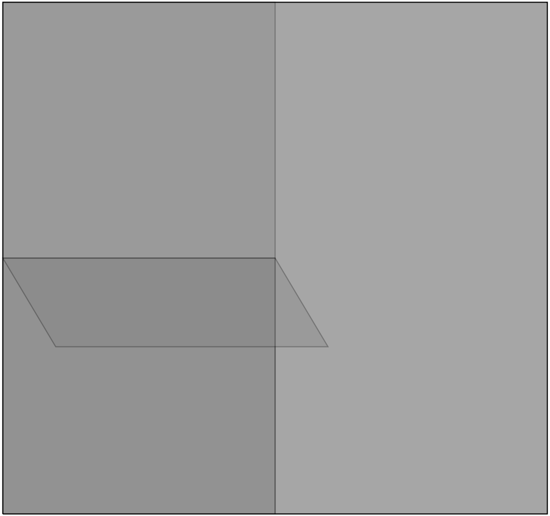
back

$\mathbb{Z}_2$ orbifold pillow



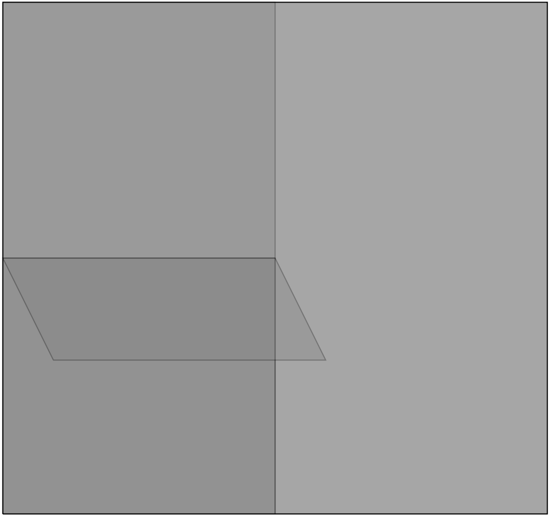
back

$\mathbb{Z}_2$ orbifold pillow



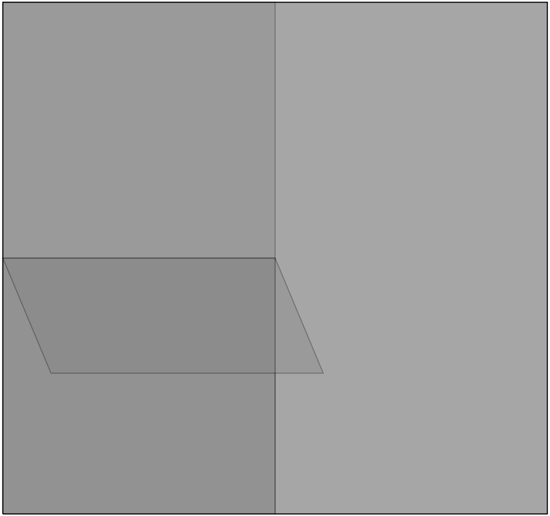
back

$\mathbb{Z}_2$ orbifold pillow



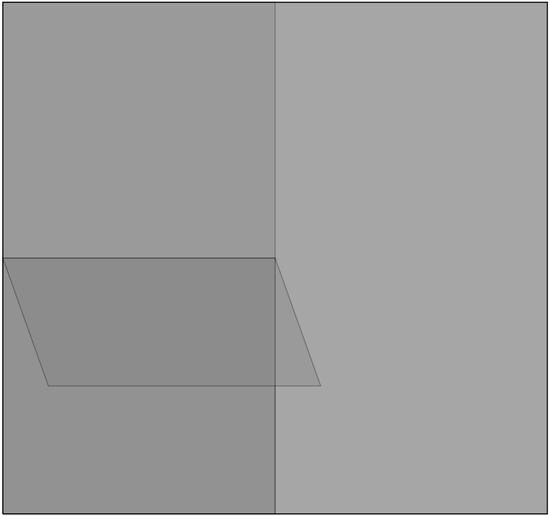
back

$\mathbb{Z}_2$ orbifold pillow



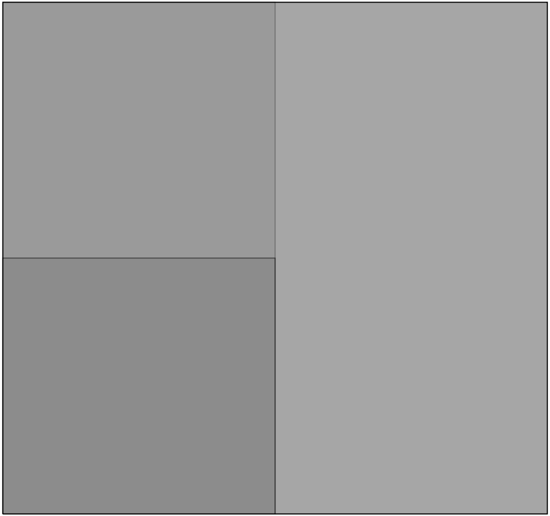
back

$\mathbb{Z}_2$ orbifold pillow



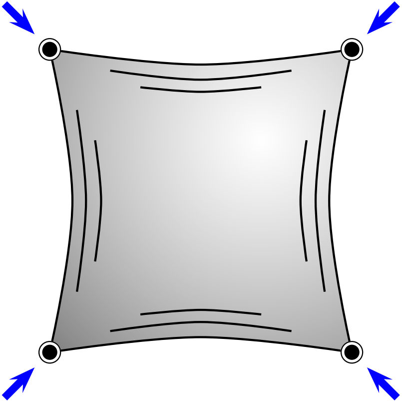
back

$\mathbb{Z}_2$ orbifold pillow



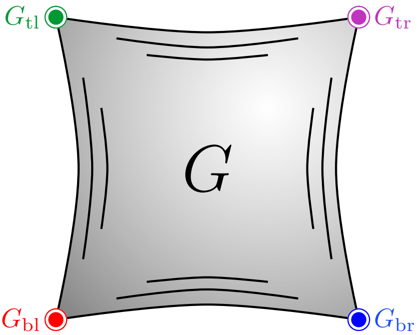
back

What is an orbifold?



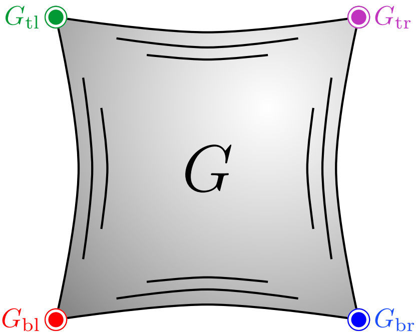
☞ an orbifold is a space which is smooth/flat everywhere except for special (orbifold fixed) points

What is an orbifold?



- an orbifold is a space which is smooth/flat everywhere except for special (orbifold fixed) points
- ‘bulk’ gauge symmetry G is broken to (different) subgroups (local GUTs) at the fixed points

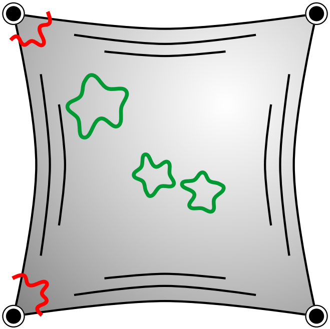
What is an orbifold?



- an orbifold is a space which is smooth/flat everywhere except for special (orbifold fixed) points
- 'bulk' gauge symmetry G is broken to (different) subgroups (local GUTs) at the fixed points
- low–energy gauge group : $G_{\text{low-energy}} = G_{\text{bl}} \cap G_{\text{br}} \cap G_{\text{tl}} \cap G_{\text{tr}}$

Strings on orbifolds

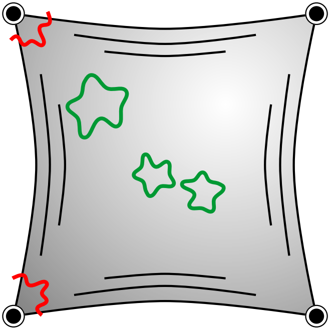
heterotic string	field theory
untwisted sector = strings closed on the torus	extra components of gauge fields
'twisted' sectors = strings which are only closed on the orbifold	'brane fields' (hard to understand in field–theoretical framework)



☞ ('brane') Fields living at a fixed point with a certain symmetry appear as complete multiplet of that symmetry

Strings on orbifolds

heterotic string	field theory
untwisted sector = strings closed on the torus	extra components of gauge fields
'twisted' sectors = strings which are only closed on the orbifold	'brane fields' (hard to understand in field–theoretical framework)



☞ ('brane') Fields living at a fixed point with a certain symmetry appear as complete multiplet of that symmetry

➥ e.g. if the electron lives at a point with SO(10) symmetry also u and d quarks live there

First 3 family models from stringy orbifolds

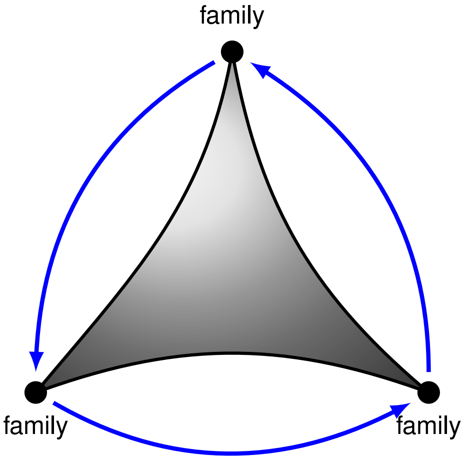
Ibáñez, Kim, Nilles & Quevedo (1987)

☞ Very first stringy model of particle physics based on $\mathbb{Z}_3$ orbifold

First 3 family models from stringy orbifolds

Ibáñez, Kim, Nilles & Quevedo (1987)

- Very first stringy model of particle physics based on $\mathbb{Z}_3$ orbifold
  - three generations may live on equivalent fixed points
  - permutation symmetry of fixed points/families

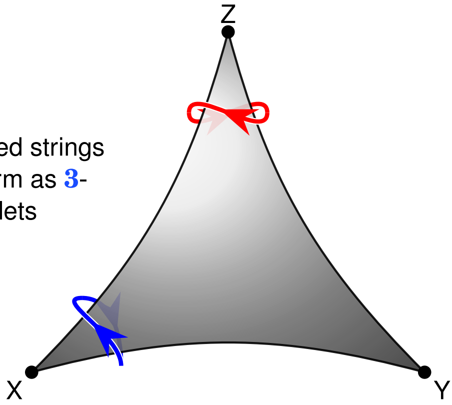


First 3 family models from stringy orbifolds

Ibáñez, Kim, Nilles & Quevedo (1987)

- Very first stringy model of particle physics based on $\mathbb{Z}_3$ orbifold
  - three generations may live on equivalent fixed points
  - permutation symmetry of fixed points/families
  - ➥ flavor/family symmetry

localized strings tansform as $\mathbf{3}$- or $\overline{\mathbf{3}}$–plets



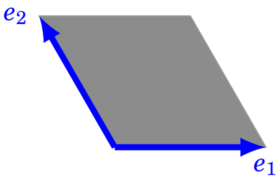
$\Delta(54)$

from a

$\mathbb{Z}_3$ orbifold plane

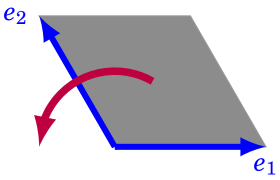
$\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Kobayashi, Nilles, Plöger, Raby & M.R. (2007)



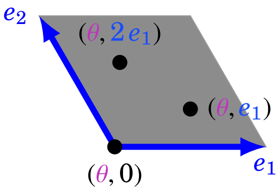
$\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Kobayashi, Nilles, Plöger, Raby & M.R. (2007)



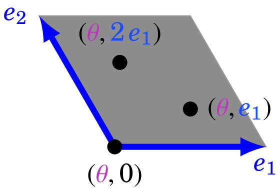
$\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Kobayashi, Nilles, Plöger, Raby & M.R. (2007)



$\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Kobayashi, Nilles, Plöger, Raby & M.R. (2007)



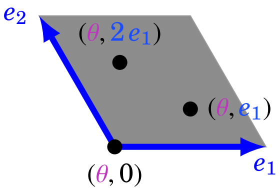
Hamidi & Vafa (1987)
Dixon, Friedan, Martinec & Shenker (1987)

☞ coupling between n localized states $|(\theta, m^{(j)}\,e_1)\rangle$ only allowed if

$$n \;=\; 3\times(\text{integer}) \quad\wedge\quad \sum_{j=1}^{n} m_1^{(j)} \;=\; 0 \mod 3$$

$\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Kobayashi, Nilles, Plöger, Raby & M.R. (2007)



$$\begin{pmatrix} |(\theta,0)\rangle \\ |(\theta,e_1)\rangle \\ |(\theta,2e_1)\rangle \end{pmatrix} \to \begin{pmatrix} \omega & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega \end{pmatrix} \begin{pmatrix} c|(\theta,0)\rangle \\ |(\theta,e_1)\rangle \\ |(\theta,2e_1)\rangle \end{pmatrix}$$

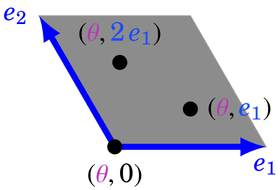
Hamidi & Vafa (1987)
Dixon, Friedan, Martinec & Shenker (1987)

☞ coupling between n localized states $|(\theta, m^{(j)}\,e_1)\rangle$ only allowed if

$$n = 3 \times (\text{integer}) \quad \wedge \quad \sum_{j=1}^{n} m_1^{(j)} = 0 \mod 3$$

$\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Kobayashi, Nilles, Plöger, Raby & M.R. (2007)



$$\begin{pmatrix} |(\theta,0)\rangle \\ |(\theta,e_1)\rangle \\ |(\theta,2e_1)\rangle \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \begin{pmatrix} c|(\theta,0)\rangle \\ |(\theta,e_1)\rangle \\ |(\theta,2e_1)\rangle \end{pmatrix}$$

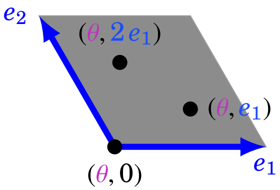
Hamidi & Vafa (1987)
Dixon, Friedan, Martinec & Shenker (1987)

☞ coupling between n localized states $|(\theta, m^{(j)}\,e_1)\rangle$ only allowed if

$$n = 3 \times (\text{integer}) \quad \wedge \quad \sum_{j=1}^{n} m_1^{(j)} = 0 \mod 3$$

$\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Kobayashi, Nilles, Plöger, Raby & M.R. (2007)



Hamidi & Vafa (1987)
Dixon, Friedan, Martinec & Shenker (1987)

☞ coupling between n localized states $|(\theta, m^{(j)}\,e_1)\rangle$ only allowed if

$$n = 3 \times (\text{integer}) \quad \wedge \quad \sum_{j=1}^{n} m_1^{(j)} = 0 \mod 3$$

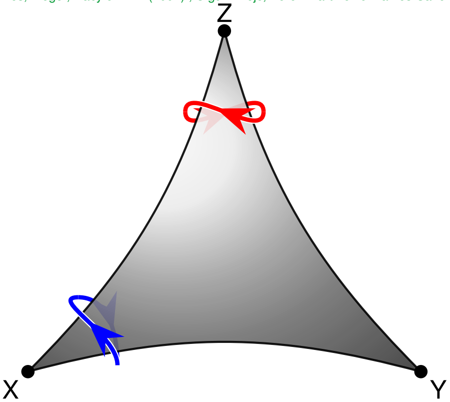
➥ flavor symmetry

$$S_3 \cup (\mathbb{Z}_3 \times \mathbb{Z}_3) = S_3 \ltimes (\mathbb{Z}_3 \times \mathbb{Z}_3) = \Delta(54)$$

$\Delta(54)$ from a $\mathbb{Z}_3$ orbifold plane

- ☞ $\mathbb{Z}_3$ orbifold plane without Wilson lines leads to a $\Delta(54)$ flavor symmetry Kobayashi, Nilles, Plöger, Raby & M.R. (2007) ; Olguin-Trejo, Pérez-Martínez & Ramos-Sánchez (2018)

localized strings tansform as $\mathbf{3}$- or $\overline{\mathbf{3}}$–plets



$\Delta(54)$ from a $\mathbb{Z}_3$ orbifold plane

- $\mathbb{Z}_3$ orbifold plane without Wilson lines leads to a $\Delta(54)$ flavor symmetry
  Kobayashi, Nilles, Plöger, Raby & M.R. (2007) ; Olguin-Trejo, Pérez-Martínez & Ramos-Sánchez (2018)
- explicit model
  Carballo-Perez, Peinado & Ramos-Sánchez (2016)

#	irrep	$\Delta(54)$	label
3	$(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$	$\mathbf{3}_{11}$	Q_i
3	$\left(\overline{\mathbf{3}},\mathbf{1}\right)_{-\frac{2}{3}}$	$\mathbf{3}_{11}$	$\overline{u}_i$
3	$\left(\overline{\mathbf{3}},\mathbf{1}\right)_{\frac{1}{3}}$	$\mathbf{3}_{11}$	$\overline{d}_i$
3	$(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$\mathbf{3}_{11}$	L_i
3	$(\mathbf{1},\mathbf{1})_{1}$	$\mathbf{3}_{11}$	$\overline{e}_i$
3	$(\mathbf{1},\mathbf{1})_{0}$	$\mathbf{3}_{12}$	$\overline{\nu}_i$

$\Delta(54)$ from a $\mathbb{Z}_3$ orbifold plane

- $\mathbb{Z}_3$ orbifold plane without Wilson lines leads to a $\Delta(54)$ flavor symmetry Kobayashi, Nilles, Plöger, Raby & M.R. (2007) ; Olguin-Trejo, Pérez-Martínez & Ramos-Sánchez (2018)
- explicit model Carballo-Perez, Peinado & Ramos-Sánchez (2016)
- quarks and leptons transform as $\mathbf{3}$–plets (or $\overline{\mathbf{3}}$–plets) of $\Delta(54)$

$\Delta(54)$ from a $\mathbb{Z}_3$ orbifold plane

- $\mathbb{Z}_3$ orbifold plane without Wilson lines leads to a $\Delta(54)$ flavor symmetry Kobayashi, Nilles, Plöger, Raby & M.R. (2007) ; Olguin-Trejo, Pérez-Martínez & Ramos-Sánchez (2018)
- explicit model Carballo-Perez, Peinado & Ramos-Sánchez (2016)
- quarks and leptons transform as $\mathbf{3}$–plets (or $\overline{\mathbf{3}}$–plets) of $\Delta(54)$
- $\Delta(54)$ is type I group: $\curvearrowright \mathcal{CP}$ violation for free?

$\Delta(54)$ from a $\mathbb{Z}_3$ orbifold plane

- $\mathbb{Z}_3$ orbifold plane without Wilson lines leads to a $\Delta(54)$ flavor symmetry Kobayashi, Nilles, Plöger, Raby & M.R. (2007) ; Olguin-Trejo, Pérez-Martínez & Ramos-Sánchez (2018)
- explicit model Carballo-Perez, Peinado & Ramos-Sánchez (2016)
- quarks and leptons transform as $\mathbf{3}$–plets (or $\overline{\mathbf{3}}$–plets) of $\Delta(54)$
- $\Delta(54)$ is type I group: $\curvearrowright$ $\mathcal{CP}$ violation for free?
- not that simple! if the representation content is very special, one *can* impose a $\mathcal{CP}$ transformation

$$\exists\ \text{out}\ :\ \mathbf{3}_i \stackrel{\text{out}}{\longleftrightarrow} \overline{\mathbf{3}}_i \quad \text{and} \quad \mathbf{1}_i \stackrel{\text{out}}{\longleftrightarrow} \overline{\mathbf{1}}_i$$

$\Delta(54)$ from a $\mathbb{Z}_3$ orbifold plane

- $\mathbb{Z}_3$ orbifold plane without Wilson lines leads to a $\Delta(54)$ flavor symmetry Kobayashi, Nilles, Plöger, Raby & M.R. (2007) ; Olguin-Trejo, Pérez-Martínez & Ramos-Sánchez (2018)
- explicit model Carballo-Perez, Peinado & Ramos-Sánchez (2016)
- quarks and leptons transform as $\mathbf{3}$–plets (or $\overline{\mathbf{3}}$–plets) of $\Delta(54)$
- $\Delta(54)$ is type I group: $\curvearrowright$ $\mathcal{CP}$ violation for free?
- not that simple! if the representation content is very special, one *can* impose a $\mathcal{CP}$ transformation
- at the massless level, only 3- and 1–dimensional representations occur $\curvearrowright$ a class–inverting outer automorphism exists $\curvearrowright$ a $\mathcal{CP}$ candidate exists

$\mathcal{CP}$ violation

in the

$\mathbb{Z}_3$ orbifold

$\mathcal{CP}$ violation from strings

☞ however, at the massive level $\Delta(54)$ $\mathbf{2}$–plets arise

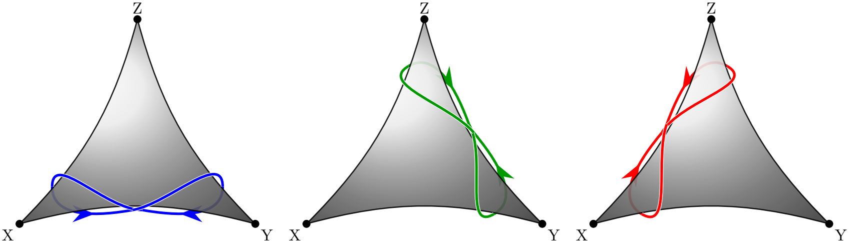
Nilles, M.R., Trautner & Vaudrevange (2018)

$\mathcal{CP}$ violation from strings

- however, at the massive level $\Delta(54)$ $\mathbf{2}$–plets arise

  Nilles, M.R., Trautner & Vaudrevange (2018)

- doublets $\mathbf{2}_1$, $\mathbf{2}_3$ and $\mathbf{2}_4$ correspond to linear combinations of strings that wind around two different fixed points in opposite directions



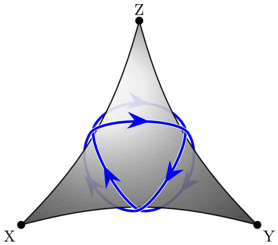
$\mathcal{CP}$ violation from strings

- however, at the massive level $\Delta(54)$ $\mathbf{2}$–plets arise

  Nilles, M.R., Trautner & Vaudrevange (2018)

- doublets $\mathbf{2}_1$, $\mathbf{2}_3$ and $\mathbf{2}_4$ correspond to linear combinations of strings that wind around two different fixed points in opposite directions

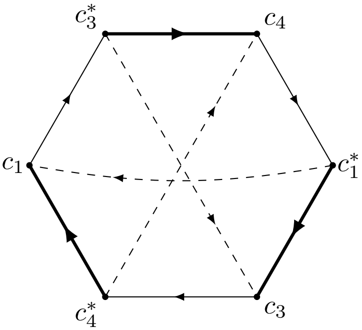
- doublet $\mathbf{2}_2$



$\mathcal{CP}$ violation from strings

☞ doublets save the day

Nilles, M.R., Trautner & Vaudrevange (2018)

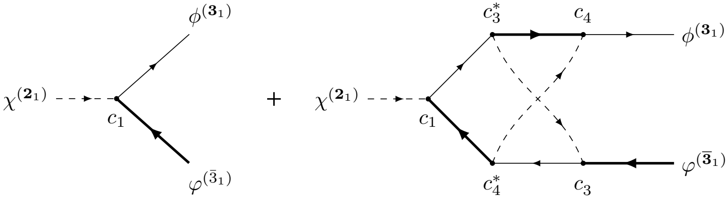


- we follow invariant approach — Bernabéu, Branco & Gronau (1986)
- super powerful tool: `Susyno` — Fonseca (2012)

$\mathcal{CP}$ violation from strings

- doublets save the day
- physical $\not{\mathcal{CP}}$ in doublet decay

Nilles, M.R., Trautner & Vaudrevange (2018)



$\mathcal{CP}$ violation from strings

- ☞ doublets save the day

Nilles, M.R., Trautner & Vaudrevange (2018)

- ☞ physical $\not{\mathcal{CP}}$ in doublet decay
- ☞ phenomenological implications not worked out

$\mathcal{CP}$ violation from strings

- doublets save the day

Nilles, M.R., Trautner & Vaudrevange (2018)

- physical $\not{\mathcal{CP}}$ in doublet decay
- phenomenological implications not worked out

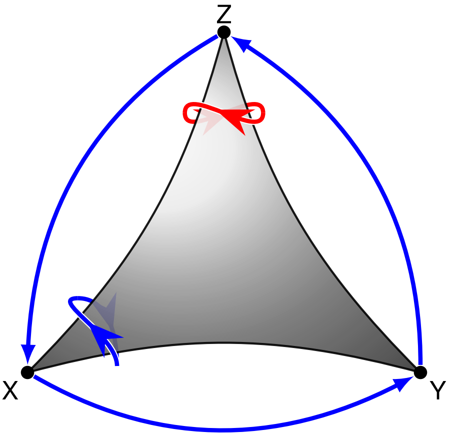
bottom–line:

$\mathcal{CP}$ violation can come from group theory in UV complete settings in which the origin of the flavor group is fully understood

Summary

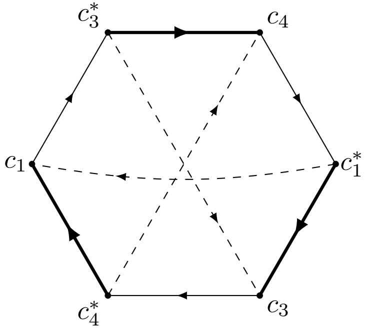
Summary

- string models exhibit flavor symmetries, which have a simple geometric interpretation



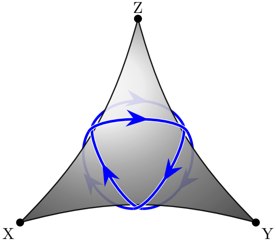
Summary

- string models exhibit flavor symmetries
- even the simplest stringy standard models have built–in $\mathcal{CP}$ violation



Summary

- string models exhibit flavor symmetries
- even the simplest stringy standard models have built–in $\mathcal{CP}$ violation
- amazingly the states that complete the model in the UV are also needed to establish physical $\mathcal{CP}$ violation



Summary

- string models exhibit flavor symmetries
- even the simplest stringy standard models have built–in $\mathcal{CP}$ violation
- amazingly the states that complete the model in the UV are also needed to establish physical $\mathcal{CP}$ violation
- so far no phenomenological implications worked out
  - decay of doublets violates $\mathcal{CP}$
  - integrating out doublets gives rise to $\not{\mathcal{CP}}$ interactions
  - ...

Outlook

☞ new solution of strong $\mathcal{CP}$ problem (?)

Thanks a lot!

$\mathcal{CP}$ violation

with an unbroken

$\mathcal{CP}$ transformation

$\mathcal{CP}$ violation with an unbroken $\mathcal{CP}$ transformation

- type I groups can be embedded in $\mathrm{SU}(N)$

no $\mathcal{CP}$ transformation

has $\mathcal{CP}$ transformation

$\mathcal{CP}$ violation with an unbroken $\mathcal{CP}$ transformation

- ☞ type I groups can be embedded in $\mathrm{SU}(N)$

- ➥ question: at which stage gets $\mathcal{CP}$ broken?

$\mathcal{CP}$ violation with an unbroken $\mathcal{CP}$ transformation

☞ type I groups can be embedded in $\mathrm{SU}(N)$

➥ question: at which stage gets $\mathcal{CP}$ broken?

☞ possible options include:

- $\mathcal{CP}$ gets broken by the VEV that breaks $\mathrm{SU}(N)$ to G

$\mathcal{CP}$ violation with an unbroken $\mathcal{CP}$ transformation

☞ type I groups can be embedded in $\mathrm{SU}(N)$

➥ question: at which stage gets $\mathcal{CP}$ broken?

☞ possible options include:

- $\mathcal{CP}$ gets broken by the VEV that breaks $\mathrm{SU}(N)$ to G
- the resulting setting always has additional symmetries and does not violate $\mathcal{CP}$

$\mathcal{CP}$ violation with an unbroken $\mathcal{CP}$ transformation

☞ type I groups can be embedded in $\mathrm{SU}(N)$

➥ question: at which stage gets $\mathcal{CP}$ broken?

☞ possible options include:

- $\mathcal{CP}$ gets broken by the VEV that breaks $\mathrm{SU}(N)$ to G
- the resulting setting always has additional symmetries and does not violate $\mathcal{CP}$

☞ surprisingly the answer is none of the above

Example: $\mathrm{SU}(3) \to \mathrm{T}_7$

- starting point: $\mathrm{SU}(3)$ gauge theory with

$$\mathscr{L} = \left(D_\mu\,\phi\right)^\dagger (D^\mu\,\phi) - \frac{1}{4}\,G_{\mu\nu}^a\,G^{\mu\nu,a} - \mathscr{V}(\phi)$$

$$D_\mu = \partial_\mu - \mathrm{i}g\,A_\mu$$

field strength

Example: $\mathrm{SU}(3) \to \mathrm{T}_7$

☞ starting point: $\mathrm{SU}(3)$ gauge theory with

$$\mathscr{L} = \left(D_\mu\,\phi\right)^\dagger (D^\mu\,\phi) - \frac{1}{4}\,G^a_{\mu\nu}\,G^{\mu\nu,a} - \mathscr{V}(\phi)$$

$\mathbf{15}$–plet

Example: $\mathrm{SU}(3) \to \mathrm{T}_7$

- starting point: $\mathrm{SU}(3)$ gauge theory with

$$\mathscr{L} \;=\; \left(D_\mu\,\phi\right)^\dagger (D^\mu\,\phi) - \frac{1}{4}\,G^a_{\mu\nu}\,G^{\mu\nu,a} - \mathscr{V}(\phi)$$

- potential: $\mathscr{V}(\phi) \;=\; -\,\mu^2\,\phi^\dagger\phi + \sum_{i=1}^{5}\lambda_i\,\mathcal{I}^{(4)}{}_i(\phi)$

quartic $\mathrm{SU}(3)$ invariants

Example: $\mathrm{SU}(3) \to \mathrm{T}_7$

- starting point: $\mathrm{SU}(3)$ gauge theory with

$$\mathscr{L} = \left(D_\mu \phi\right)^\dagger (D^\mu \phi) - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} - \mathscr{V}(\phi)$$

- potential: $\mathscr{V}(\phi) = -\mu^2 \phi^\dagger \phi + \sum_{i=1}^{5} \lambda_i \mathcal{I}^{(4)}{}_i(\phi)$

- action invariant under $\mathcal{CP}$ transformation

$$A^a_\mu(x) \xmapsto{\mathrm{SU}(3)-\mathcal{CP}} R^{ab} \mathcal{P}_\mu^{\ \nu} A^b_\nu(\mathcal{P}x)$$

$$\phi_i(x) \xmapsto{\mathrm{SU}(3)-\mathcal{CP}} U_{ij} \phi^*_j(\mathcal{P}x)$$

$$\mathcal{P} = \mathrm{diag}(1, -1, -1, -1)$$

Example: $\mathrm{SU}(3) \to \mathrm{T}_7$

- starting point: $\mathrm{SU}(3)$ gauge theory with

$$\mathscr{L} = \left(D_\mu \phi\right)^\dagger (D^\mu \phi) - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} - \mathscr{V}(\phi)$$

- potential: $\mathscr{V}(\phi) = -\mu^2 \phi^\dagger \phi + \sum_{i=1}^{5} \lambda_i \mathcal{I}^{(4)}{}_i(\phi)$

- action invariant under $\mathcal{CP}$ transformation

$$A^a_\mu(x) \xmapsto{\mathrm{SU}(3)-\mathcal{CP}} R^{ab} \mathcal{P}_\mu^{\ \nu} A^b_\nu(\mathcal{P}x)$$

$$\phi_i(x) \xmapsto{\mathrm{SU}(3)-\mathcal{CP}} U_{ij} \phi^*_j(\mathcal{P}x)$$

$\mathrm{SU}(3) \to \mathsf{T}_7$

see e.g. Luhn (2011) & Merle & Zwicky (2012)

☞ $\langle\phi\rangle$ breaks $\mathrm{SU}(3)$ to T_7

$$\mathrm{SU}(3) \rtimes \mathbb{Z}_2 \xrightarrow{\langle\phi\rangle} \mathsf{T}_7 \rtimes \mathbb{Z}_2$$

$\mathrm{SU}(3) \to \mathsf{T}_7$

see e.g. Luhn (2011) & Merle & Zwicky (2012)

☞ $\langle\phi\rangle$ breaks $\mathrm{SU}(3)$ to T_7

$$\mathrm{SU}(3) \rtimes \mathbb{Z}_2 \xrightarrow{\langle\phi\rangle} \mathsf{T}_7 \rtimes \mathbb{Z}_2$$

☞ physical fields before and after symmetry breaking

name	$\mathrm{SU}(3) \xrightarrow{\langle\phi\rangle}$	name	T_7
A_μ	$\mathbf{8}$	Z_μ	$\mathbf{1}_1$
		W_μ	$\mathbf{3}$
ϕ	$\mathbf{15}$	$\mathrm{Re}\,\sigma_0, \mathrm{Im}\,\sigma_0$	$\mathbf{1}_0$
		σ_1	$\mathbf{1}_1$
		τ_1	$\mathbf{3}$
		τ_2	$\mathbf{3}$
		τ_3	$\mathbf{3}$

SU(3) – $\mathcal{CP}$ vs. Out(T_7)

☞ SU(3) – $\mathcal{CP}$ breaks to unique $\mathbb{Z}_2$ outer automorphism of T_7

$$\text{Out}(\mathsf{T}_7)\ :\qquad \mathbf{1_1} \longleftrightarrow \mathbf{1_1}\ ,\quad \overline{\mathbf{1}}_\mathbf{1} \longleftrightarrow \overline{\mathbf{1}}_\mathbf{1}\ ,\quad \mathbf{3} \longleftrightarrow \overline{\mathbf{3}}$$

$\mathrm{SU}(3)-\mathcal{CP}$ vs. $\mathrm{Out}(\mathrm{T}_7)$

- $\mathrm{SU}(3)-\mathcal{CP}$ breaks to unique $\mathbb{Z}_2$ outer automorphism of T_7

$$\mathrm{Out}(\mathrm{T}_7)\ :\qquad \mathbf{1_1} \longleftrightarrow \mathbf{1_1}\ ,\quad \overline{\mathbf{1}}_\mathbf{1} \longleftrightarrow \overline{\mathbf{1}}_\mathbf{1}\ ,\quad \mathbf{3} \longleftrightarrow \overline{\mathbf{3}}$$

- T_7 character table

T_7	C_{1a} e	C_{3a} b	C_{3b} b^2	C_{7a} a	C_{7b} a^3
$\mathbf{1_0}$	1	1	1	1	1
$\underline{\mathbf{1}}_\mathbf{1}$	1	ω	ω^2	1	1
$\mathbf{1_1}$	1	ω^2	ω	1	1
$\underline{\mathbf{3}}$	3	0	0	η	η^*
$\mathbf{3}$	3	0	0	η^*	η

SU(3) – $\mathcal{CP}$ vs. Out(T_7)

- SU(3) – $\mathcal{CP}$ breaks to unique $\mathbb{Z}_2$ outer automorphism of T_7

$$\text{Out}(T_7)\ :\qquad \mathbf{1_1} \longleftrightarrow \mathbf{1_1}\ ,\quad \overline{\mathbf{1}}_\mathbf{1} \longleftrightarrow \overline{\mathbf{1}}_\mathbf{1}\ ,\quad \mathbf{3} \longleftrightarrow \overline{\mathbf{3}}$$

- T_7 character table

T_7	C_{1a} e	C_{3a} b	C_{3b} b^2	C_{7a} a	C_{7b} a^3
$\mathbf{1_0}$	1	1	1	1	1
$\underline{\mathbf{1}}_\mathbf{1}$	1	ω	ω^2	1	1
$\mathbf{1_1}$	1	ω^2	ω	1	1
$\underline{\mathbf{3}}$	3	0	0	η	η^*
$\mathbf{3}$	3	0	0	η^*	η

$\omega := e^{2\pi i/3}$

$\eta = \rho + \rho^2 + \rho^4$ with $\rho := e^{2\pi i/7}$

SU(3) – $\mathcal{CP}$ vs. Out(T_7)

- SU(3) – $\mathcal{CP}$ breaks to unique $\mathbb{Z}_2$ outer automorphism of T_7

$$\text{Out}(\mathrm{T}_7)\ :\qquad \mathbf{1}_1 \longleftrightarrow \mathbf{1}_1\,,\quad \overline{\mathbf{1}}_1 \longleftrightarrow \overline{\mathbf{1}}_1\,,\quad \mathbf{3} \longleftrightarrow \overline{\mathbf{3}}$$

- T_7 character table

T_7	C_{1a} e	C_{3a} b	C_{3b} b^2	C_{7a} a	C_{7b} a^3
$\mathbf{1_0}$	1	1	1	1	1
$\underline{\mathbf{1}}_1$	1	ω	ω^2	1	1
$\mathbf{1}_1$	1	ω^2	ω	1	1
$\underline{\mathbf{3}}$	3	0	0	η	η^*
$\mathbf{3}$	3	0	0	η^*	η

- $\mathbf{1}_1$ and $\overline{\mathbf{1}_1}$ do **not** get swapped!

T_7

- T_7 can be generated by two elements with the presentation

$$\left\langle \mathsf{a}, \mathsf{b} \,\middle|\, \mathsf{a}^7 = \mathsf{b}^3 = \mathsf{e} \,, \mathsf{b}^{-1}\,\mathsf{a}\,\mathsf{b} = \mathsf{a}^4 \right\rangle$$

T_7

- T_7 can be generated by two elements with the presentation

$$\left\langle \mathsf{a}, \mathsf{b} \,\middle|\, \mathsf{a}^7 = \mathsf{b}^3 = \mathsf{e}\,, \mathsf{b}^{-1}\,\mathsf{a}\,\mathsf{b} = \mathsf{a}^4 \right\rangle$$

- triplet representation

$$A = \begin{pmatrix} \rho & 0 & 0 \\ 0 & \rho^2 & 0 \\ 0 & 0 & \rho^4 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

T_7

- T_7 can be generated by two elements with the presentation

$$\left\langle \mathsf{a},\mathsf{b} \,\middle|\, \mathsf{a}^7 = \mathsf{b}^3 = \mathsf{e}\,,\mathsf{b}^{-1}\,\mathsf{a}\,\mathsf{b} = \mathsf{a}^4 \right\rangle$$

- triplet representation

$$A = \begin{pmatrix} \rho & 0 & 0 \\ 0 & \rho^2 & 0 \\ 0 & 0 & \rho^4 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

- embedding into $\mathrm{SU}(3)$

$$X^{(\boldsymbol{r})} = \exp\left(\mathrm{i}\,\alpha_a\,\mathsf{t}_a^{(\boldsymbol{r})}\right)$$

$$\vec{\alpha}^{(A)} = \frac{2\pi}{7}\left(0,0,0,0,0,0,\sqrt{3},5\right)$$

T_7

- T_7 can be generated by two elements with the presentation

$$\left\langle \mathrm{a}, \mathrm{b} \,\middle|\, \mathrm{a}^7 = \mathrm{b}^3 = \mathrm{e}\,, \mathrm{b}^{-1}\,\mathrm{a}\,\mathrm{b} = \mathrm{a}^4 \right\rangle$$

- triplet representation

$$A = \begin{pmatrix} \rho & 0 & 0 \\ 0 & \rho^2 & 0 \\ 0 & 0 & \rho^4 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

- embedding into $\mathrm{SU}(3)$

$$X^{(\boldsymbol{r})} = \exp\left(\mathrm{i}\,\alpha_a\,\mathrm{t}_a^{(\boldsymbol{r})}\right)$$

$$\vec{\alpha}^{(B)} = \frac{4\pi}{3\sqrt{3}}(0,0,1,1,1,0,0,0)$$

T_7

- T_7 can be generated by two elements with the presentation

$$\left\langle \mathsf{a},\mathsf{b}\;\middle|\;\mathsf{a}^7 = \mathsf{b}^3 = \mathsf{e}\,,\mathsf{b}^{-1}\,\mathsf{a}\,\mathsf{b} = \mathsf{a}^4\right\rangle$$

- triplet representation

$$A = \begin{pmatrix}\rho & 0 & 0\\ 0 & \rho^2 & 0\\ 0 & 0 & \rho^4\end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 0\end{pmatrix}$$

- embedding into $\mathrm{SU}(3)$

$$X^{(\boldsymbol{r})} = \exp\left(\mathrm{i}\,\alpha_a\,\mathsf{t}_a^{(\boldsymbol{r})}\right)$$

- work in `SusyNo` basis

Fonseca (2012)

T_7 scalar states

- branchings:

$$\mathbf{8} \to \mathbf{1}_1 \oplus \overline{\mathbf{1}}_1 \oplus \mathbf{3} \oplus \overline{\mathbf{3}}$$

$$\mathbf{15} \to \mathbf{1}_0 \oplus \mathbf{1}_1 \oplus \overline{\mathbf{1}}_1 \oplus \mathbf{3} \oplus \mathbf{3} \oplus \overline{\mathbf{3}} \oplus \overline{\mathbf{3}}$$

T_7 scalar states

- branchings:
$$\mathbf{8} \;\to\; \mathbf{1}_1 \oplus \overline{\mathbf{1}}_1 \oplus \mathbf{3} \oplus \overline{\mathbf{3}}$$
$$\mathbf{15} \;\to\; \mathbf{1}_0 \oplus \mathbf{1}_1 \oplus \overline{\mathbf{1}}_1 \oplus \mathbf{3} \oplus \mathbf{3} \oplus \overline{\mathbf{3}} \oplus \overline{\mathbf{3}}$$

- physical scalar fields (would–be Goldstone bosons subtracted)

$$\phi = \left(v + \phi_1,\, \frac{\phi_2}{\sqrt{2}},\, \frac{\phi_2^*}{\sqrt{2}},\, \phi_4,\, \phi_5,\, \phi_6,\, \frac{\phi_7}{\sqrt{2}},\, \frac{\phi_8}{\sqrt{2}},\, \frac{\phi_9}{\sqrt{2}},\, \phi_{10},\, \phi_{11},\, \phi_{12},\, \frac{\phi_7^*}{\sqrt{2}},\, \frac{\phi_8^*}{\sqrt{2}},\, \frac{\phi_9^*}{\sqrt{2}}\right)$$

T_7 scalar states

- branchings:

$$\mathbf{8} \;\to\; \mathbf{1_1} \oplus \overline{\mathbf{1}}_\mathbf{1} \oplus \mathbf{3} \oplus \overline{\mathbf{3}}$$

$$\mathbf{15} \;\to\; \mathbf{1_0} \oplus \mathbf{1_1} \oplus \overline{\mathbf{1}}_\mathbf{1} \oplus \mathbf{3} \oplus \mathbf{3} \oplus \overline{\mathbf{3}} \oplus \overline{\mathbf{3}}$$

- physical scalar fields (would–be Goldstone bosons subtracted)

$$\phi \;=\; \left(v + \phi_1,\, \frac{\phi_2}{\sqrt{2}},\, \frac{\phi_2^*}{\sqrt{2}},\, \phi_4,\, \phi_5,\, \phi_6,\, \frac{\phi_7}{\sqrt{2}},\, \frac{\phi_8}{\sqrt{2}},\, \frac{\phi_9}{\sqrt{2}},\, \phi_{10},\, \phi_{11},\, \phi_{12},\, \frac{\phi_7^*}{\sqrt{2}},\, \frac{\phi_8^*}{\sqrt{2}},\, \frac{\phi_9^*}{\sqrt{2}}\right)$$

- T_7 representations

$$\phi_1 \;\widehat{=}\; \mathbf{1}_0\;, \qquad \phi_2 \;\widehat{=}\; \mathbf{1}_1\;,$$

$$T_1 \;:=\; (\phi_4,\, \phi_5,\, \phi_6) \;\widehat{=}\; \mathbf{3}\;, \qquad T_2 \;:=\; (\phi_7,\, \phi_8,\, \phi_9) \;\widehat{=}\; \mathbf{3}\;,$$

$$\overline{T}_3 \;:=\; (\phi_{10},\, \phi_{11},\, \phi_{12}) \;\widehat{=}\; \overline{\mathbf{3}}$$

T_7 scalar states

- branchings:

$$\mathbf{8} \to \mathbf{1}_1 \oplus \overline{\mathbf{1}}_1 \oplus \mathbf{3} \oplus \overline{\mathbf{3}}$$

$$\mathbf{15} \to \mathbf{1}_0 \oplus \mathbf{1}_1 \oplus \overline{\mathbf{1}}_1 \oplus \mathbf{3} \oplus \mathbf{3} \oplus \overline{\mathbf{3}} \oplus \overline{\mathbf{3}}$$

- physical scalar fields (would–be Goldstone bosons subtracted)

$$\phi = \left(v + \phi_1, \frac{\phi_2}{\sqrt{2}}, \frac{\phi_2^*}{\sqrt{2}}, \phi_4, \phi_5, \phi_6, \frac{\phi_7}{\sqrt{2}}, \frac{\phi_8}{\sqrt{2}}, \frac{\phi_9}{\sqrt{2}}, \phi_{10}, \phi_{11}, \phi_{12}, \frac{\phi_7^*}{\sqrt{2}}, \frac{\phi_8^*}{\sqrt{2}}, \frac{\phi_9^*}{\sqrt{2}}\right)$$

- T_7 representations
- no physical $\mathcal{CP}$ trafo allowed by T_7!

$$\mathbb{Z}_2 - \mathrm{Out}: \quad \begin{array}{c} \mathbf{15} \to \mathbf{1}_0 \oplus \mathbf{1}_1 \oplus \overline{\mathbf{1}}_1 \oplus \mathbf{3} \oplus \mathbf{3} \oplus \overline{\mathbf{3}} \oplus \overline{\mathbf{3}} \; \& \\ \downarrow \\ \overline{\mathbf{15}} \to \mathbf{1}_0 \oplus \overline{\mathbf{1}}_1 \oplus \mathbf{1}_1 \oplus \overline{\mathbf{3}} \oplus \overline{\mathbf{3}} \oplus \mathbf{3} \oplus \mathbf{3} \end{array}$$

T_7 scalar states

- branchings:

$$\mathbf{8} \;\to\; \mathbf{1}_1 \oplus \overline{\mathbf{1}}_1 \oplus \mathbf{3} \oplus \overline{\mathbf{3}}$$

$$\mathbf{15} \;\to\; \mathbf{1}_0 \oplus \mathbf{1}_1 \oplus \overline{\mathbf{1}}_1 \oplus \mathbf{3} \oplus \mathbf{3} \oplus \overline{\mathbf{3}} \oplus \overline{\mathbf{3}}$$

- physical scalar fields (would–be Goldstone bosons subtracted)

$$\phi \;=\; \left(v + \phi_1,\, \frac{\phi_2}{\sqrt{2}},\, \frac{\phi_2^*}{\sqrt{2}},\, \phi_4,\, \phi_5,\, \phi_6,\, \frac{\phi_7}{\sqrt{2}},\, \frac{\phi_8}{\sqrt{2}},\, \frac{\phi_9}{\sqrt{2}},\, \phi_{10},\, \phi_{11},\, \phi_{12},\, \frac{\phi_7^*}{\sqrt{2}},\, \frac{\phi_8^*}{\sqrt{2}},\, \frac{\phi_9^*}{\sqrt{2}}\right)$$

- T_7 representations
- no physical $\mathcal{CP}$ trafo allowed by T_7!

$\mathbb{Z}_2 - \mathrm{Out}:$

$$\mathbf{15} \;\to\; \mathbf{1}_0 \oplus \mathbf{1}_1 \oplus \overline{\mathbf{1}}_1 \oplus \mathbf{3} \oplus \mathbf{3} \oplus \overline{\mathbf{3}} \oplus \overline{\mathbf{3}} \;\&$$

$$\mapsto\quad \overline{\mathbf{15}} \;\to\; \mathbf{1}_0 \oplus \overline{\mathbf{1}}_1 \oplus \mathbf{1}_1 \oplus \overline{\mathbf{3}} \oplus \overline{\mathbf{3}} \oplus \mathbf{3} \oplus \mathbf{3}$$

Scalar masses

- VEV

$$|v| \;=\; \mu \times 3\,\sqrt{\frac{7}{2}}\left(-7\,\sqrt{15}\,\lambda_1+14\,\sqrt{15}\,\lambda_2+20\,\sqrt{6}\,\lambda_4+13\,\sqrt{15}\,\lambda_5\right)^{-1/2}$$

Scalar masses

☞ VEV

$$|v| = \mu \times 3\sqrt{\frac{7}{2}}\left(-7\sqrt{15}\,\lambda_1 + 14\sqrt{15}\,\lambda_2 + 20\sqrt{6}\,\lambda_4 + 13\sqrt{15}\,\lambda_5\right)^{-1/2}$$

☞ T_7 $\mathbf{1}$–plet representations

$$\mathrm{Re}\,\sigma_0 = \frac{1}{\sqrt{2}}\left(\phi_1 + \phi_1^*\right) \qquad \mathrm{Im}\,\sigma_0 = -\frac{\mathrm{i}}{\sqrt{2}}\left(\phi_1 - \phi_1^*\right)$$

$$\sigma_1 = \phi_2$$

Scalar masses

- VEV

$$|v| \;=\; \mu \times 3\,\sqrt{\frac{7}{2}}\left(-7\,\sqrt{15}\,\lambda_1 + 14\,\sqrt{15}\,\lambda_2 + 20\,\sqrt{6}\,\lambda_4 + 13\,\sqrt{15}\,\lambda_5\right)^{-1/2}$$

- T_7 $\mathbf{1}$–plet represen

can be eliminated gauging accidental $\mathrm{U}(1)$

$$\mathrm{Re}\,\sigma_0 \;=\; \frac{1}{\sqrt{2}}\left(\phi_1 + \phi_1^*\right) \qquad \mathrm{Im}\,\sigma_0 \;=\; -\frac{\mathrm{i}}{\sqrt{2}}\left(\phi_1 - \phi_1^*\right)$$

$$\sigma_1 \;=\; \phi_2$$

Scalar masses

- VEV

$$|v| = \mu \times 3\sqrt{\frac{7}{2}}\left(-7\sqrt{15}\,\lambda_1 + 14\sqrt{15}\,\lambda_2 + 20\sqrt{6}\,\lambda_4 + 13\sqrt{15}\,\lambda_5\right)^{-1/2}$$

- T_7 $\mathbf{1}$–plet representations

$$\mathrm{Re}\,\sigma_0 = \frac{1}{\sqrt{2}}\left(\phi_1 + \phi_1^*\right) \qquad \mathrm{Im}\,\sigma_0 = -\frac{\mathrm{i}}{\sqrt{2}}\left(\phi_1 - \phi_1^*\right)$$

$$\sigma_1 = \phi_2$$

- masses

$$m^2_{\mathrm{Re}\,\sigma_0} = 2\,\mu^2\,, \qquad m^2_{\mathrm{Im}\,\sigma_0} = 0$$

$$m^2_{\sigma_1} = -\mu^2 + \sqrt{15}\,\lambda_5\,v^2$$

Gauge fields

☞ gauge fields

$$\begin{aligned} Z^\mu &= \frac{1}{\sqrt{2}}\left(A_7^\mu - \mathrm{i}\,A_8^\mu\right) \\ W_1^\mu &= \frac{1}{\sqrt{2}}\left(A_4^\mu - \mathrm{i}\,A_1^\mu\right) \\ W_2^\mu &= \frac{1}{\sqrt{2}}\left(A_5^\mu - \mathrm{i}\,A_2^\mu\right) \\ W_3^\mu &= \frac{\mathrm{i}}{\sqrt{2}}\left(A_6^\mu - \mathrm{i}\,A_3^\mu\right) \end{aligned}$$

Gauge fields

- gauge fields

$$
\begin{aligned}
Z^\mu &= \frac{1}{\sqrt{2}}\left(A_7^\mu - \mathrm{i}\,A_8^\mu\right)\\
W_1^\mu &= \frac{1}{\sqrt{2}}\left(A_4^\mu - \mathrm{i}\,A_1^\mu\right)\\
W_2^\mu &= \frac{1}{\sqrt{2}}\left(A_5^\mu - \mathrm{i}\,A_2^\mu\right)\\
W_3^\mu &= \frac{\mathrm{i}}{\sqrt{2}}\left(A_6^\mu - \mathrm{i}\,A_3^\mu\right)
\end{aligned}
$$

- masses

$$
m_Z^2 = \frac{7}{3}\,g^2\,v^2 \quad \text{and} \quad m_W^2 = g^2\,v^2
$$

Triplet mass eigenstates

- mass eigenstates

$$\begin{pmatrix}\tau_1\\ \tau_2\\ \tau_3\end{pmatrix} = \underbrace{\begin{pmatrix}V_{11} & V_{12} & V_{13}\\ V_{21} & V_{22} & V_{23}\\ V_{31} & V_{32} & V_{33}\end{pmatrix}}_{=V}\begin{pmatrix}T_2\\ \overline{T}^*_3\\ T_1\end{pmatrix}$$

Triplet mass eigenstates

- mass eigenstates

$$\begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} = \underbrace{\begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix}}_{=V} \begin{pmatrix} T_2 \\ \overline{T}_3^* \\ T_1 \end{pmatrix}$$

- masses and mixing matrix depend on potential parameters

T_7 outer automorphism vs. $\mathcal{CP}$

- Out(T_7)

$$\begin{aligned} Z_\mu(x) &\mapsto -\mathcal{P}_\mu^{\ \nu} Z_\nu(\mathcal{P}x)\,, & \sigma_0(x) &\mapsto \sigma_0(\mathcal{P}x)\,, & & \\ W_\mu(x) &\mapsto \mathcal{P}_\mu^{\ \nu} W_\nu^*(\mathcal{P}x)\,, & \sigma_1(x) &\mapsto \sigma_1(\mathcal{P}x)\,, & \tau_i(x) &\mapsto \tau_i^*(\mathcal{P}x) \end{aligned}$$

T_7 outer automorphism vs. $\mathcal{CP}$

- $\mathrm{Out}(\mathsf{T}_7)$

$$\begin{aligned} Z_\mu(x) &\mapsto -\mathcal{P}_\mu^{\ \nu} Z_\nu(\mathcal{P}\,x)\;, & \sigma_0(x) &\mapsto \sigma_0(\mathcal{P}\,x)\;, \\ W_\mu(x) &\mapsto \mathcal{P}_\mu^{\ \nu} W_\nu^*(\mathcal{P}\,x)\;, & \sigma_1(x) &\mapsto \sigma_1(\mathcal{P}\,x)\;, & \tau_i(x) &\mapsto \tau_i^*(\mathcal{P}\,x) \end{aligned}$$

- mode expansion

$$\widehat{\boldsymbol{\sigma}}_1(x) = \int \widetilde{\mathrm{d}p}\, \left\{ \widehat{\boldsymbol{a}}(\vec{p})\, \mathrm{e}^{-\mathrm{i}\,p\,x} + \widehat{\boldsymbol{b}}^\dagger(\vec{p})\, \mathrm{e}^{\mathrm{i}\,p\,x} \right\}$$

T_7 outer automorphism vs. $\mathcal{CP}$

- $\mathrm{Out}(\mathrm{T}_7)$

$$\begin{aligned} Z_\mu(x) &\mapsto -\mathcal{P}_\mu^{\ \nu} Z_\nu(\mathcal{P}x)\,, & \sigma_0(x) &\mapsto \sigma_0(\mathcal{P}x)\,, \\ W_\mu(x) &\mapsto \mathcal{P}_\mu^{\ \nu} W_\nu^*(\mathcal{P}x)\,, & \sigma_1(x) &\mapsto \sigma_1(\mathcal{P}x)\,, & \tau_i(x) &\mapsto \tau_i^*(\mathcal{P}x) \end{aligned}$$

- mode expansion

$$\widehat{\boldsymbol{\sigma}}_1(x) = \int \widetilde{\mathrm{d}p}\, \left\{\widehat{\boldsymbol{a}}(\vec{p})\, \mathrm{e}^{-\mathrm{i}\,p\,x} + \widehat{\boldsymbol{b}}^\dagger(\vec{p})\, \mathrm{e}^{\mathrm{i}\,p\,x}\right\}$$

- outer automorphism of T_7

$$\mathrm{Out}(\mathrm{T}_7)\ :\quad \widehat{\boldsymbol{a}}(\vec{p}) \mapsto \widehat{\boldsymbol{a}}(-\vec{p}) \qquad \text{and} \qquad \widehat{\boldsymbol{b}}^\dagger(\vec{p}) \mapsto \widehat{\boldsymbol{b}}^\dagger(-\vec{p})$$

T_7 outer automorphism vs. $\mathcal{CP}$

- $\mathrm{Out}(\mathrm{T}_7)$

$$\begin{aligned} Z_\mu(x) &\mapsto -\mathcal{P}_\mu^{\ \nu} Z_\nu(\mathcal{P}x)\,, & \sigma_0(x) &\mapsto \sigma_0(\mathcal{P}x)\,, \\ W_\mu(x) &\mapsto \mathcal{P}_\mu^{\ \nu} W_\nu^*(\mathcal{P}x)\,, & \sigma_1(x) &\mapsto \sigma_1(\mathcal{P}x)\,, & \tau_i(x) &\mapsto \tau_i^*(\mathcal{P}x) \end{aligned}$$

- mode expansion

$$\widehat{\boldsymbol{\sigma}}_1(x) = \int \widetilde{\mathrm{d}p}\, \left\{ \widehat{\boldsymbol{a}}(\vec{p})\, \mathrm{e}^{-\mathrm{i}\,p\,x} + \widehat{\boldsymbol{b}}^\dagger(\vec{p})\, \mathrm{e}^{\mathrm{i}\,p\,x} \right\}$$

- outer automorphism of T_7

$$\mathrm{Out}(\mathrm{T}_7)\ :\quad \widehat{\boldsymbol{a}}(\vec{p}) \mapsto \widehat{\boldsymbol{a}}(-\vec{p}) \quad \text{and} \quad \widehat{\boldsymbol{b}}^\dagger(\vec{p}) \mapsto \widehat{\boldsymbol{b}}^\dagger(-\vec{p})$$

- QFT $\mathcal{CP}$ not a symmetry of the action

$$\mathcal{CP}\ :\quad \widehat{\boldsymbol{a}}(\vec{p}) \mapsto \widehat{\boldsymbol{b}}(-\vec{p}) \quad \text{and} \quad \widehat{\boldsymbol{b}}^\dagger(\vec{p}) \mapsto \widehat{\boldsymbol{a}}^\dagger(-\vec{p})$$

$\mathcal{CP}$ violation in the T_7 phase

☞ decay asymmetry

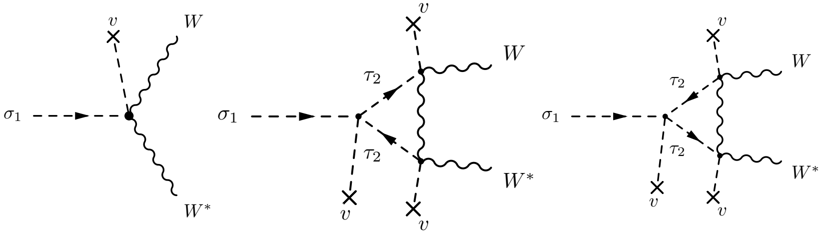
$$\varepsilon_{\sigma_1 \to W\,W^*} := \frac{\left|\mathscr{M}(\sigma_1 \to W\,W^*)\right|^2 - \left|\mathscr{M}(\sigma_1^* \to W\,W^*)\right|^2}{\left|\mathscr{M}(\sigma_1 \to W\,W^*)\right|^2 + \left|\mathscr{M}(\sigma_1^* \to W\,W^*)\right|^2}$$

$\mathcal{CP}$ violation in the T_7 phase

- decay asymmetry

$$\varepsilon_{\sigma_1 \to W\,W^*} \;:=\; \frac{\left|\mathscr{M}(\sigma_1 \to W\,W^*)\right|^2 - \left|\mathscr{M}(\sigma_1^* \to W\,W^*)\right|^2}{\left|\mathscr{M}(\sigma_1 \to W\,W^*)\right|^2 + \left|\mathscr{M}(\sigma_1^* \to W\,W^*)\right|^2}$$

- $\mathcal{CP}$ violation from interference between tree–level and 1–loop



References I

J. Bernabéu, G.C. Branco & M. Gronau. CP RESTRICTIONS ON QUARK MASS MATRICES. *Phys. Lett.*, B169:243–247, 1986. doi: 10.1016/0370-2693(86)90659-3.

Brenda Carballo-Perez, Eduardo Peinado & Saul Ramos-Sánchez. $\Delta(54)$ flavor phenomenology & strings. *JHEP*, 12:131, 2016. doi: 10.1007/JHEP12(2016)131.

Mu-Chun Chen & K.T. Mahanthappa. Group Theoretical Origin of CP Violation. *Phys. Lett.*, B681:444–447, 2009. doi: 10.1016/j.physletb.2009.10.059.

Mu-Chun Chen, Maximilian Fallbacher, K.T. Mahanthappa, Michael Ratz & Andreas Trautner. CP Violation from Finite Groups. *Nucl. Phys.*, B883:267, 2014.

Michael Dine, Robert G. Leigh & Douglas A. MacIntire. Of CP & other gauge symmetries in string theory. *Phys. Rev. Lett.*, 69:2030–2032, 1992. doi: 10.1103/PhysRevLett.69.2030.

References II

Lance J. Dixon, Daniel Friedan, Emil J. Martinec & Stephen H. Shenker. The Conformal Field Theory of Orbifolds. *Nucl. Phys.*, B282:13–73, 1987.

Renato M. Fonseca. Calculating the renormalisation group equations of a SUSY model with Susyno. *Comput. Phys. Commun.*, 183: 2298–2306, 2012. doi: 10.1016/j.cpc.2012.05.017.

Shahram Hamidi & Cumrun Vafa. Interactions on Orbifolds. *Nucl. Phys.*, B279:465, 1987.

Luis E. Ibáñez, Jihn E. Kim, Hans Peter Nilles & F. Quevedo. Orbifold compactifications with three families of SU(3) x SU(2) x U(1)**n. *Phys. Lett.*, B191:282–286, 1987.

Tatsuo Kobayashi, Hans Peter Nilles, Felix Plöger, Stuart Raby & Michael Ratz. Stringy origin of non-Abelian discrete flavor symmetries. *Nucl. Phys.*, B768:135–156, 2007.

References III

Christoph Luhn. Spontaneous breaking of SU(3) to finite family symmetries: a pedestrian's approach. *JHEP*, 1103:108, 2011. doi: 10.1007/JHEP03(2011)108.

Alexander Merle & Roman Zwicky. Explicit & spontaneous breaking of SU(3) into its finite subgroups. *JHEP*, 1202:128, 2012. doi: 10.1007/JHEP02(2012)128.

Hans Peter Nilles, Michael Ratz, Andreas Trautner & Patrick K. S. Vaudrevange. $\mathcal{CP}$ Violation from String Theory. 2018.

Yessenia Olguin-Trejo, Ricardo Pérez-Martínez & Saul Ramos-Sánchez. Charting the flavor landscape of MSSM-like Abelian heterotic orbifolds. 2018.