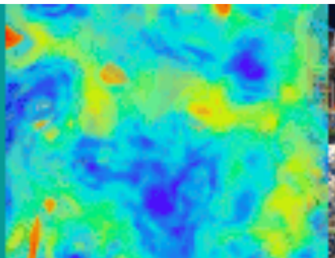
Origin of non-Abelian discrete flavour symmetries

Ye-Ling Zhou(周也铃), Southampton U., 2019-07-26

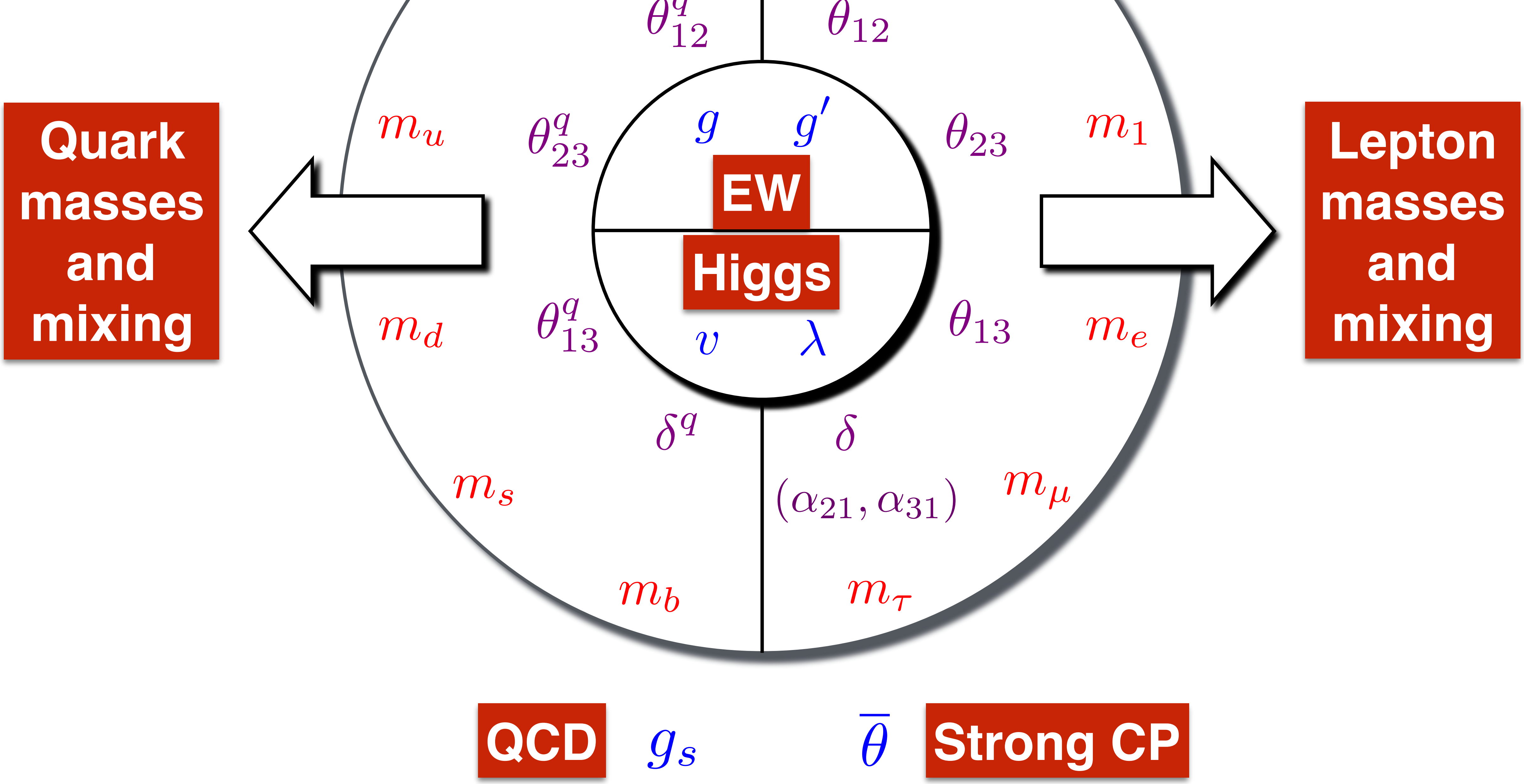


Motivation of non-Abelian discrete flavour symmetries

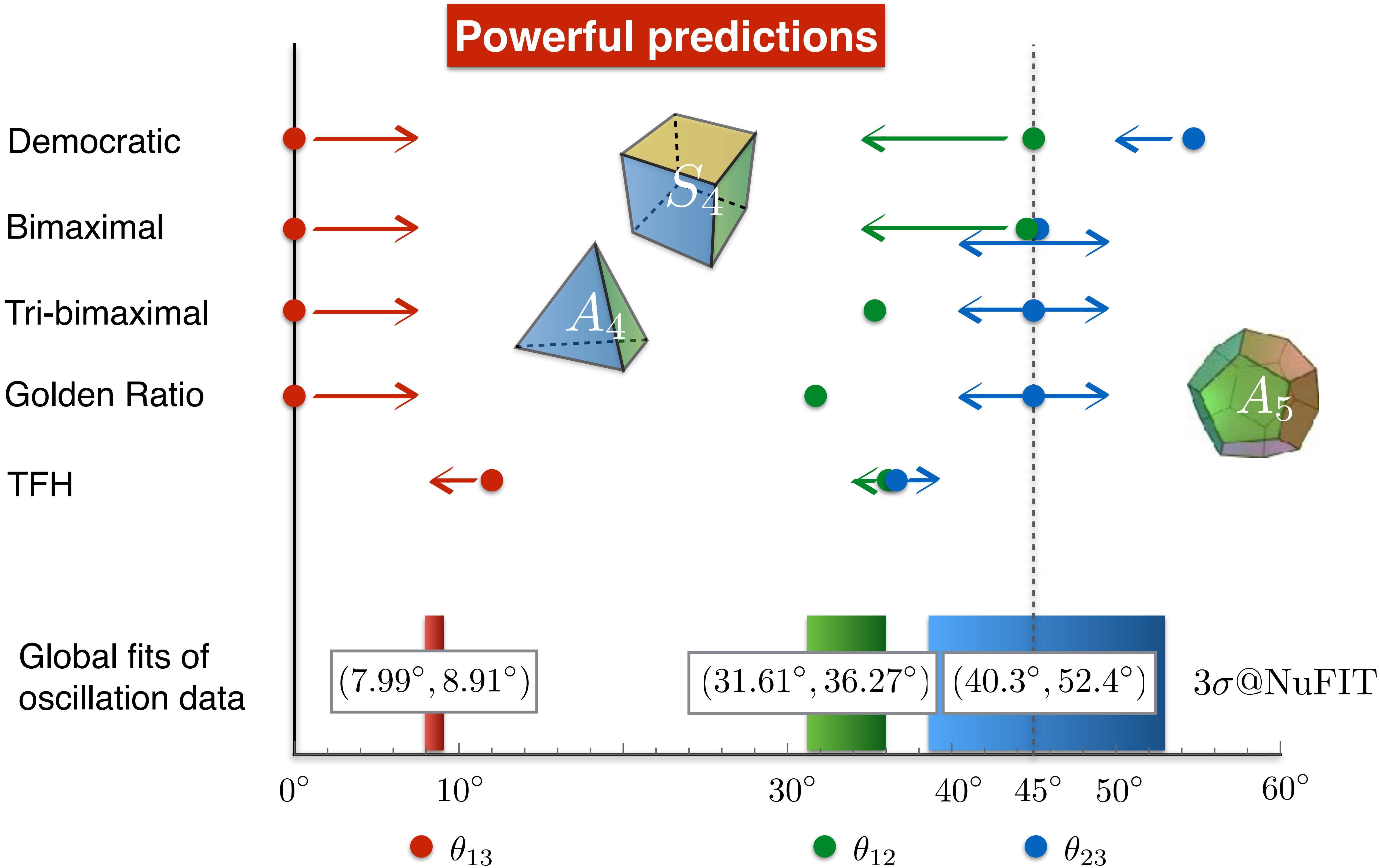
SM + massive neutrinos

Neutrino may take
Dirac or Majorana masses

24(26)+2 free parameters



Motivation of non-Abelian discrete flavour symmetries



Anything behind non-Abelian discrete symmetries?

- A fundamental symmetry?
- A consequence of some more fundamental physics?

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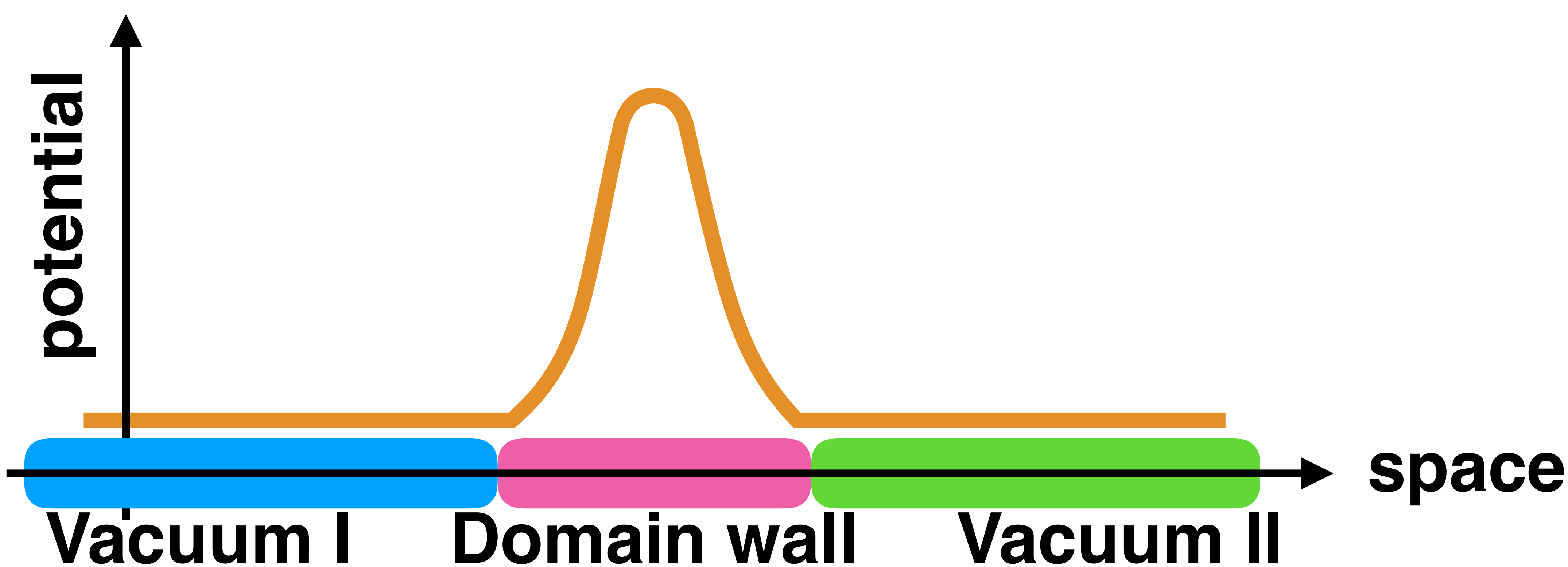
Domain wall problem

Zeldovich, Kobzarev, Okun, 74;
Kibble, 76; Vilenkin, 85



Fundamental problems of non-Abelian discrete symmetries

Anything behind non-Abelian discrete symmetries?

- A fundamental symmetry?
- A consequence of some more fundamental physics?

Is non-Abelian discrete symmetry

a fundamental symmetry of spacetime

or

an effective symmetry after a (gauge) continuous symmetry breaking

?

Origin I

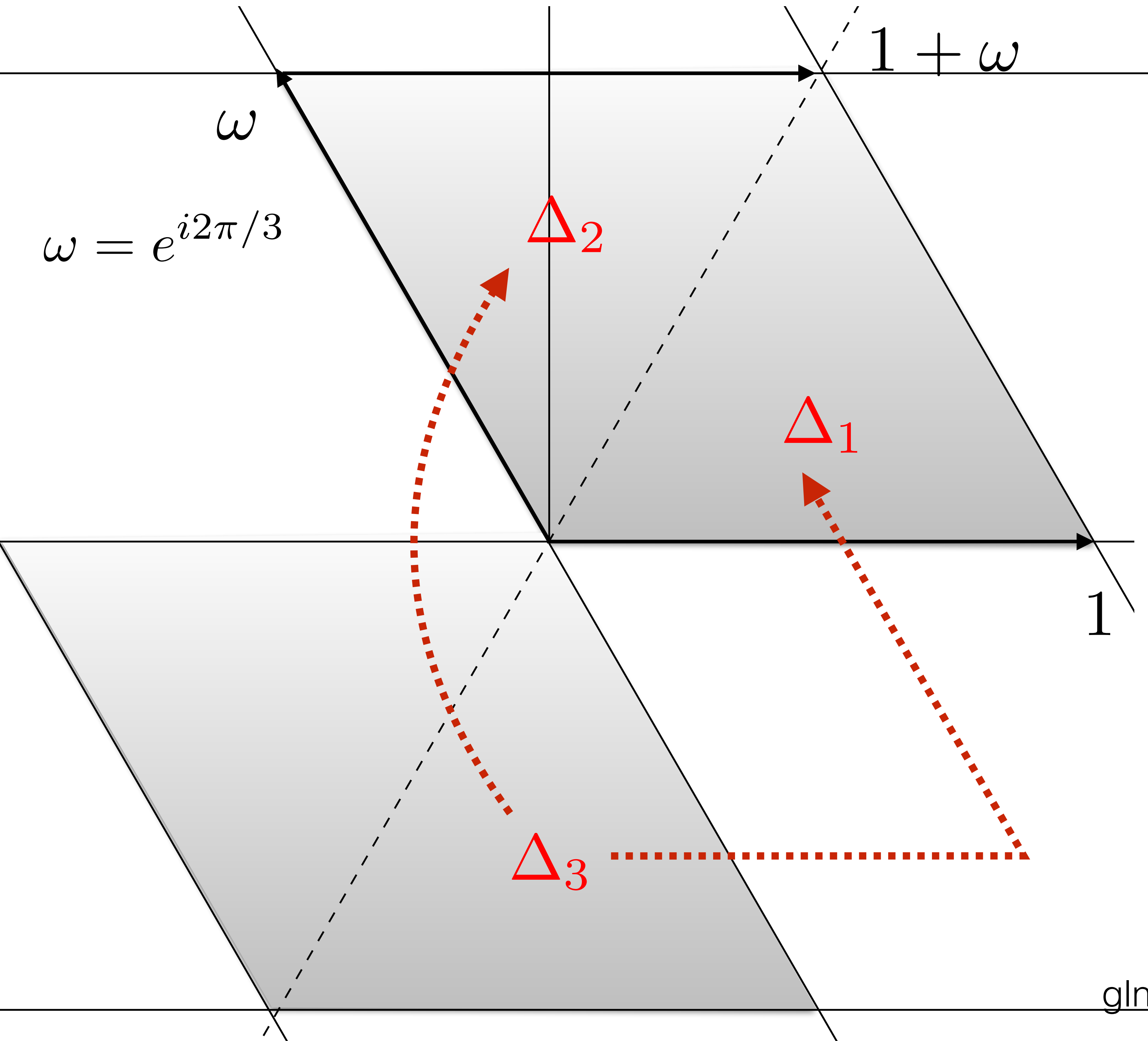
The non-Abelian discrete symmetry as

a <u>fundamental</u> symmetry of spacetime

Extra dimensions with orbifolding

- Twisted torus with a Z_2 parity

Altarelli, Feruglio, Lin, 0610165;
de Anda, King, Perdomo, 1812.05620

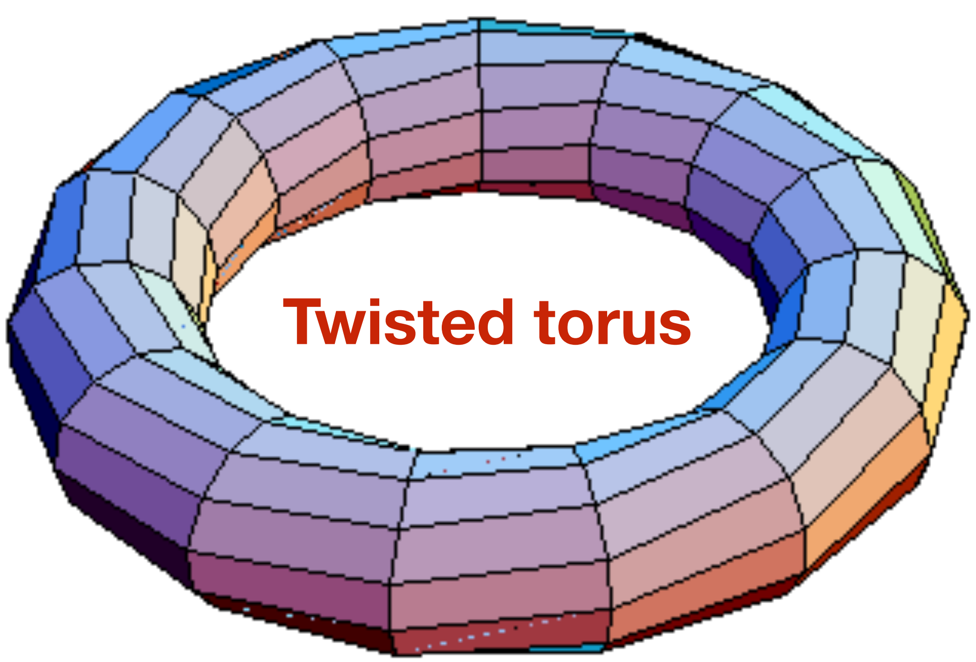


$$z = x_5 + i x_6$$

$$T^2 \begin{cases} z \to z+1 \\ z \to z+\omega \end{cases}$$

$$Z_2 \quad z \to -z$$

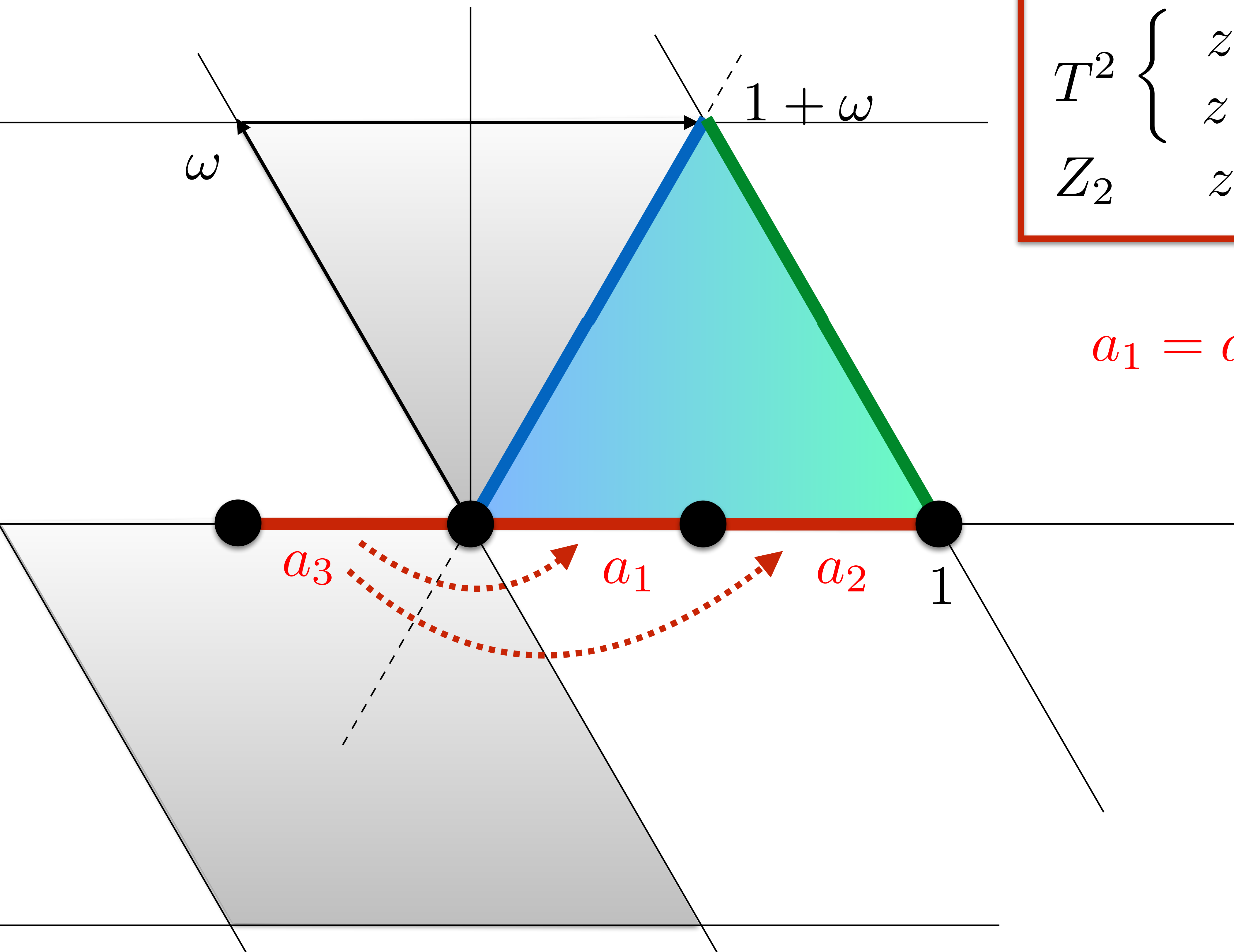
$$\Delta_1 = \Delta_3 = \Delta_2$$



credit: http://www.geocities.ws/glnarasimham/TorusMoebiusMorph.htm

Extra dimensions with orbifolding

- Twisted torus with a Z_2 parity



$$z = x_5 + ix_6$$

$$T^2 \begin{cases} z \to z+1 \\ z \to z+\omega \end{cases}$$

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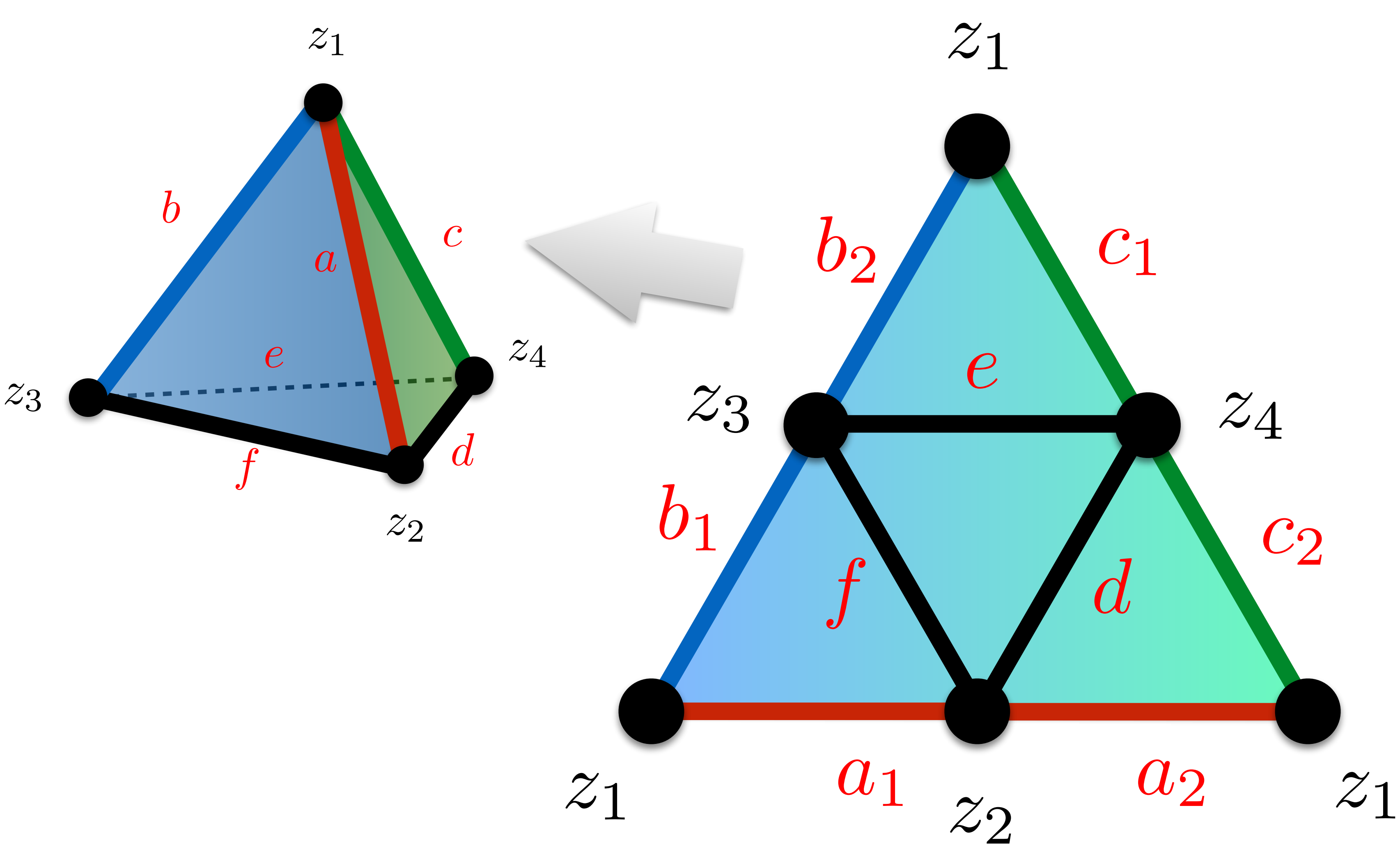
$$a_1 = a_3 = a_2 \equiv a$$

Extra dimensions with orbifolding

- Twisted torus with a Z_2 parity

$$z_1 = 0$$
$$z_2 = \frac{1}{2}$$
$$z_3 = \frac{\omega}{2}$$
$$z_4 = 1 + \frac{\omega}{2}$$

Fixed Points



$$a_1 = a_3 = a_2 \equiv a$$
$$b_1 = b_2 = b$$
$$c_1 = c_2 = c$$

- Flavour symmetries from different orbifoldings

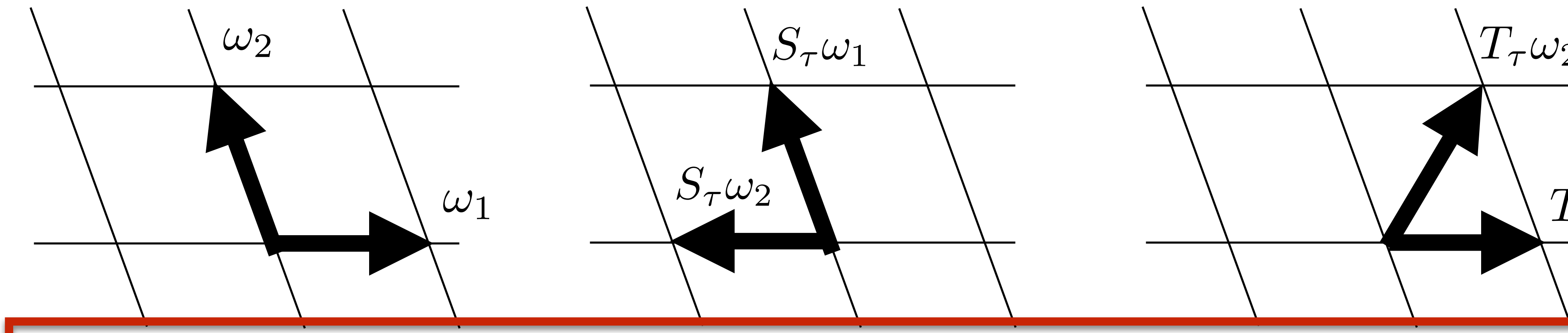
Kobayashi, Nilles, Ploger, Raby, Ratz, 0611020; Adulpravitchai, Blum, Lindner, 0906.0468; Burrows, King, 0909.1433; 1007.2310; Adulpravitchai, de Anda, King, 1803.04978…

Modular symmetry as origin of flavour symmetry

- Modular symmetry

Ferrara, Lust, Theisen, 89

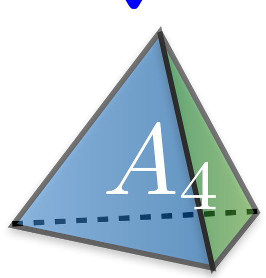
generated by two independent lattice transformations.

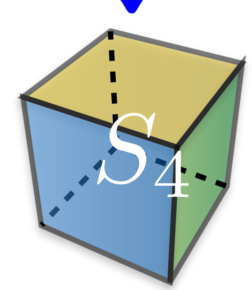


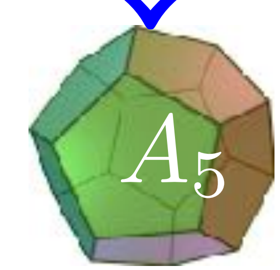
$\tau = \omega_2/\omega_1$ $\qquad S_\tau : -1/\tau$ $\qquad T_\tau : \tau \to \tau + 1$

- Finite modular symmetries $\quad S_\tau^2 = (S_\tau T_\tau)^3 = \mathbf{1}$

$T_\tau^3 = 1$ $\qquad T_\tau^4 = 1$ $\qquad T_\tau^5 = 1$

$\Gamma_3 \simeq$ 

$\Gamma_4 \simeq$ 

$\Gamma_5 \simeq$ 

de Adelhart Toorop, Feruglio and Hagedorn, 1112.1340

Modular symmetry as direct origin of flavour mixing

- A "classical" flavour transformation

$$\psi \to \rho_I(\gamma)\psi \qquad\qquad Y(\varphi_1, \varphi_2, ...) \to \rho_{I_Y}(\gamma) Y(\varphi_1, \varphi_2, ...)$$

- A modular transformation

$$\gamma : \tau \to \gamma\tau = \frac{a\tau + b}{c\tau + d}$$ in modular space τ with Im(τ) > 0

$$\psi \to (c\tau + d)^{2k} \rho_I(\gamma)\psi \qquad\qquad Y(\tau) \to (c\tau + d)^{2k_Y} \rho_{I_Y}(\gamma) Y(\tau)$$

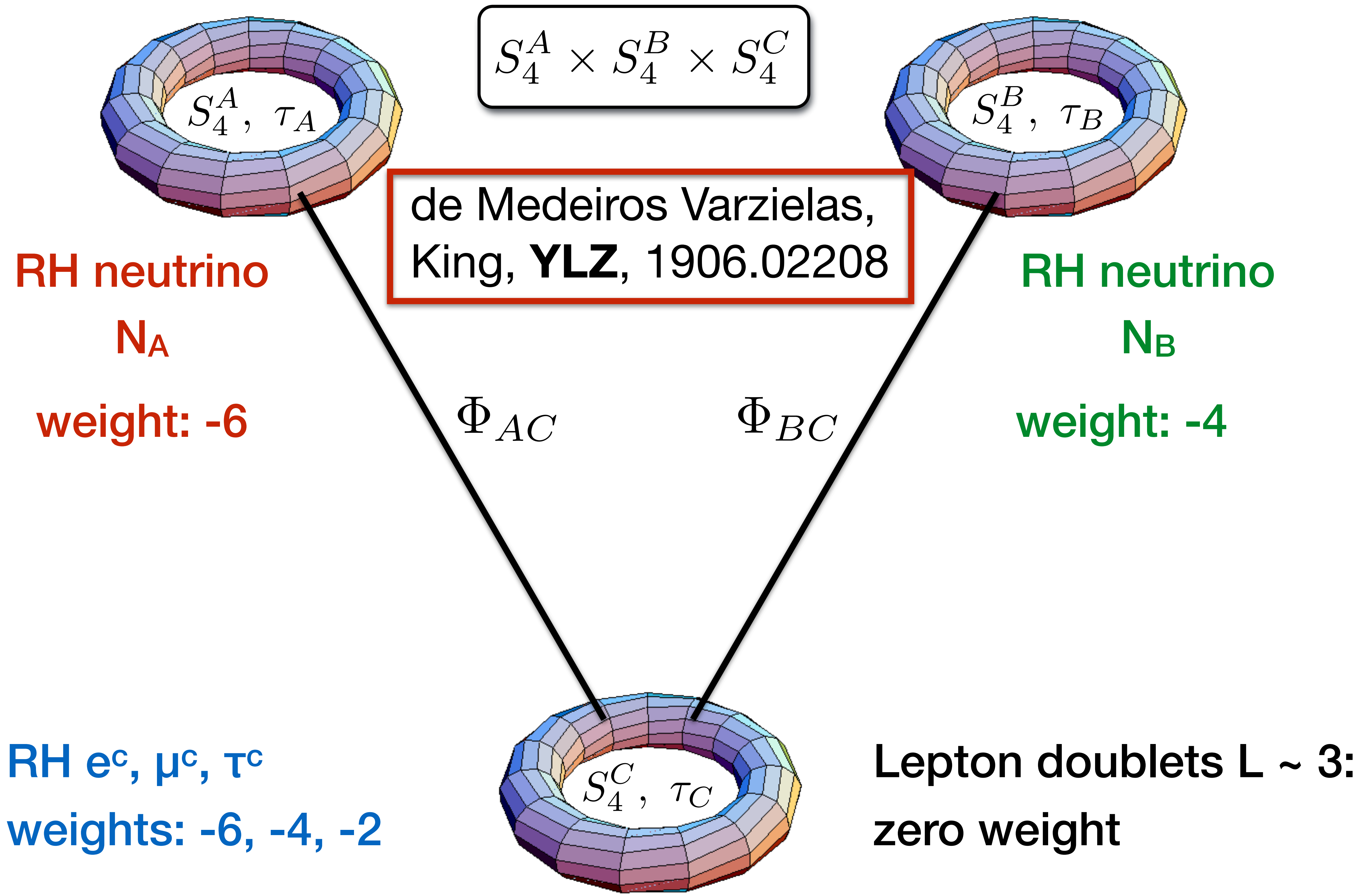
- A modular symmetry as the direct origin of flavour mixing

Proposed by Feruglio in 1706.08749, studied by …

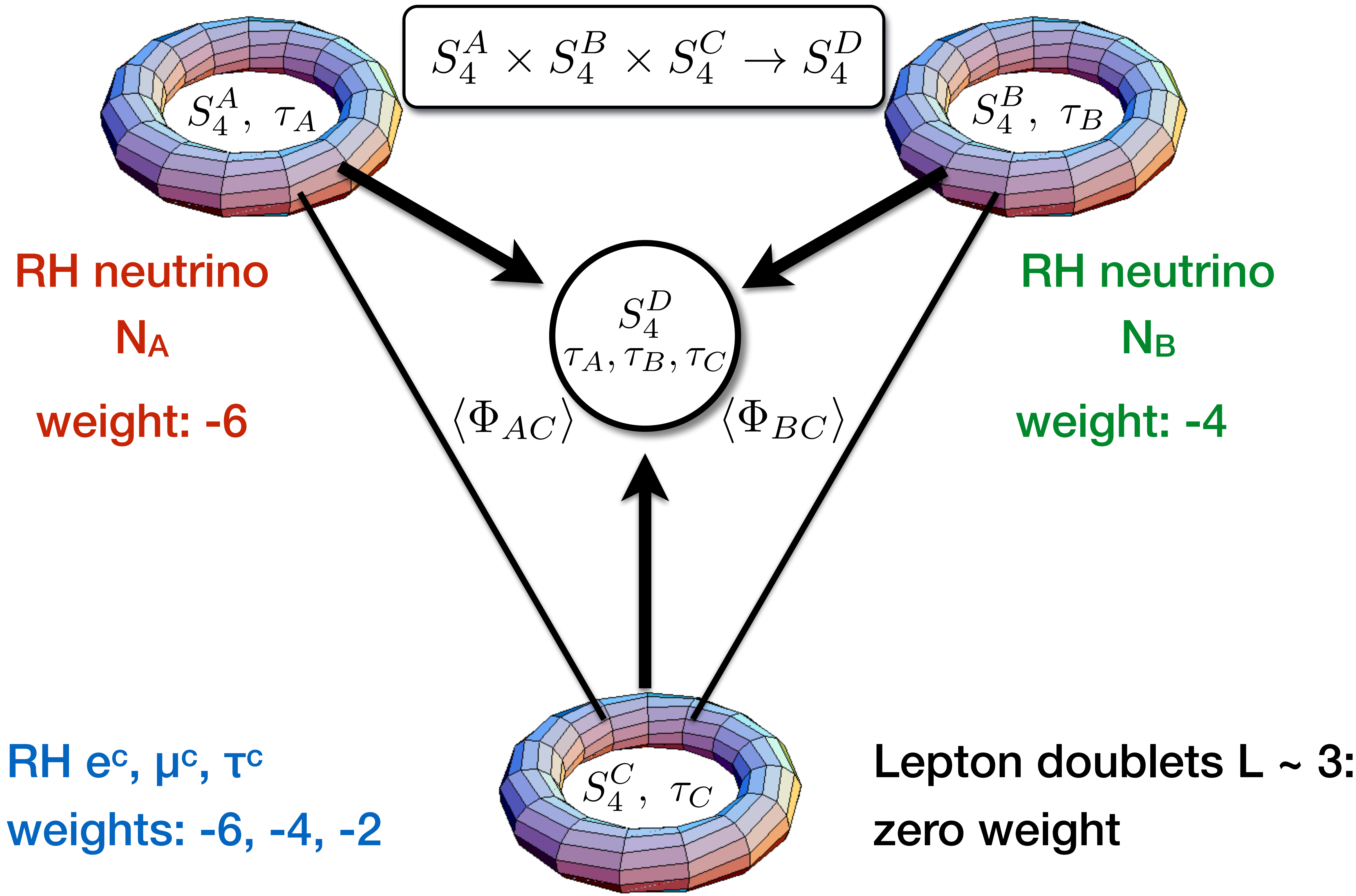
see talks by Feruglio, King, Penedo, Tanimoto, Titov

- Motivation for multiple modular symmetries

Multiple modular symmetries as origin of flavour mixing



Multiple modular symmetries as origin of flavour mixing



Multiple modular symmetries as origin of flavour mixing

- Φ_{AC} and Φ_{BC}: bridges to connect different modular symmetries

- VEVs of Φ_{AC} and Φ_{BC} are achieved via the flat directions

bi-triplet contraction $\qquad \Phi_{AC}\Phi_{AC} + \mu_A \Phi_{AC} = 0$

triplet contraction $\qquad \Phi_{AC}\Phi_{AC} = 0$

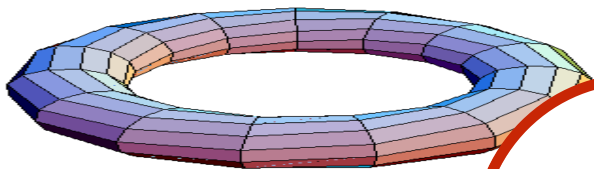
- Theory before and after $S_4^A \times S_4^B \times S_4^C$ breaking

$$
\begin{aligned}
w_\ell \;=\; & \frac{1}{\Lambda}\left[L\Phi_{AC}Y_A(\tau_A)N_A^c + L\Phi_{BC}Y_B(\tau_B)N_B^c\right] H_u \\
& + \left[LY_e(\tau_C)e^c + LY_\mu(\tau_C)\mu^c + LY_\tau(\tau_C)\tau^c\right] H_d \\
& + \frac{1}{2}M_A(\tau_A)N_A^cN_A^c + \frac{1}{2}M_B(\tau_B)N_B^cN_B^c + M_{AB}(\tau_A,\tau_B)N_A^cN_B^c
\end{aligned}
$$

$$
\begin{aligned}
w_\ell^{\text{eff}} \;=\; & \left[\frac{v_{AC}}{\Lambda}LY_A(\tau_A)N_A^c + \frac{v_{BC}}{\Lambda}LY_B(\tau_B)N_B^c\right] H_u \\
& + \left[LY_e(\tau_C)e^c + LY_\mu(\tau_C)\mu^c + LY_\tau(\tau_C)\tau^c\right] H_d \\
& + \frac{1}{2}M_A(\tau_A)N_A^cN_A^c + \frac{1}{2}M_B(\tau_B)N_B^cN_B^c + M_{AB}(\tau_A,\tau_B)N_A^cN_B^c
\end{aligned}
$$

Multiple modular symmetries as origin of flavour mixing

$$S_4^D \to Z_3^T, Z_3^{TS}, T_2^U$$



Z_3^{TS}

$\langle \tau_A \rangle$

N_A Yukawa coupling

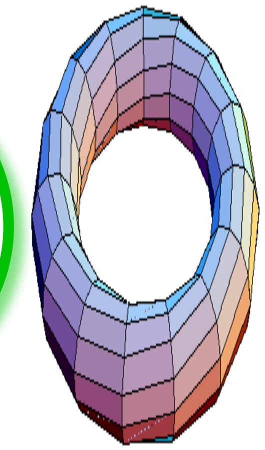
$$Y_A(\langle \tau_A \rangle) = \begin{pmatrix} -1 \\ 2\omega \\ 2\omega^2 \end{pmatrix}$$

S_4^D

τ_A, τ_B, τ_C

$\langle \tau_B \rangle$

Z_2^U



N_B Yukawa coupling

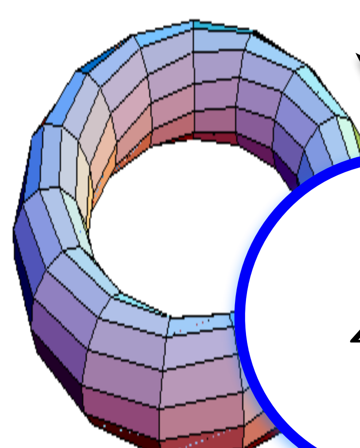
$$Y_B(\langle \tau_B \rangle) = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$\langle \tau_A \rangle = \frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$\langle \tau_B \rangle = \frac{1}{2} + \frac{i}{2}$$

$$\langle \tau_C \rangle = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$\langle \tau_C \rangle$



Z_3^T

$$\begin{aligned} Z_3^{TS} &= \{\mathbf{1}, TS, (TS)^2\} \\ Z_2^U &= \{\mathbf{1}, U\} \\ Z_3^T &= \{\mathbf{1}, T, T^2\} \end{aligned}$$

$$S = T_\tau^2,\ T = S_\tau T_\tau,$$
$$U = T_\tau S_\tau T_\tau^2 S_\tau$$

Diagonal charged lepton Yukawa coupling

Trimaximal mixing TM_1

$$U_{\rm TM_1} = \begin{pmatrix} \frac{2}{\sqrt{6}} & - & - \\ -\frac{1}{\sqrt{6}} & - & - \\ -\frac{1}{\sqrt{6}} & - & - \end{pmatrix}$$

Xing, S. Zhou, 0607302; Lam, hep-ph/0611017; Albright, Rodejohann, 0812.0436.

normal hierarchy

$$0 = m_1 < m_2 < m_3$$

best-fit

BF								
Para.		χ^2	α_1	α_2	θ_R	μ_1	μ_2	
		0.74	64.53°	20.38°	43.01°	0.00633 eV	0.0114 eV	
Obs.	θ_{12}	θ_{13}	θ_{23}	δ	m_2	m_3	m_{ee}	
	34.33°	8.61°	49.6°	290°	0.00860 eV	0.0502 eV	0.00206 eV	

second octant

B1								
Para.		χ^2	α_1	α_2	θ_R	μ_1	μ_2	
		1.6	70.16°	16.62°	43.51°	0.00651 eV	0.0135 eV	
Obs.	θ_{12}	θ_{13}	θ_{23}	δ	m_2	m_3	m_{ee}	
	34.33°	8.62°	48.6°	285°	0.00860 eV	0.0502 eV	0.00188 eV	

first octant

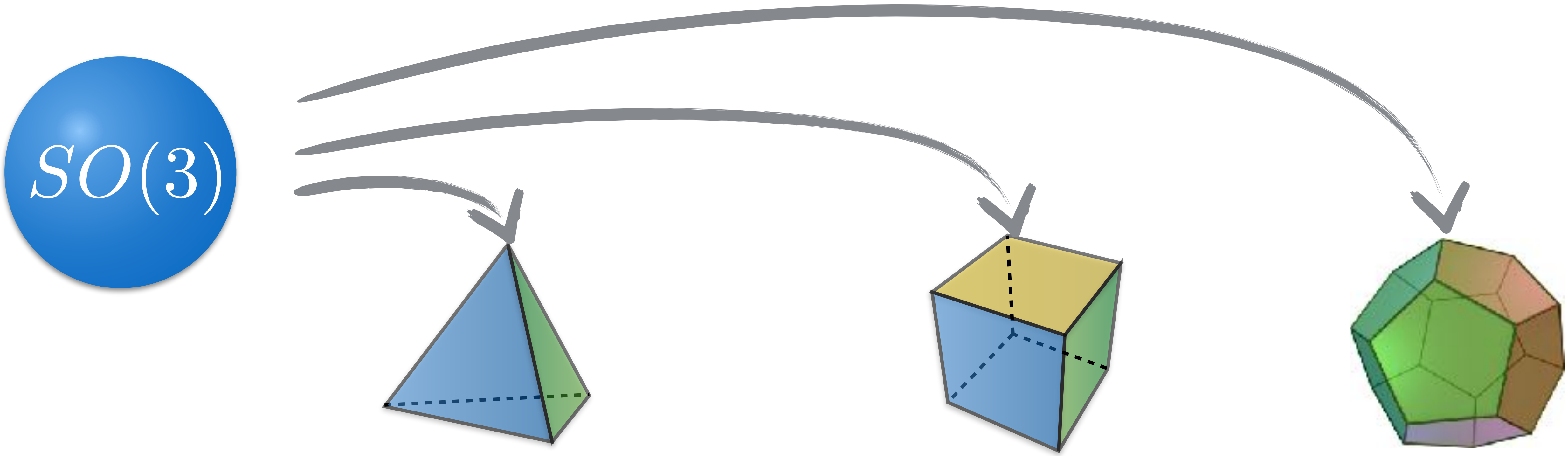
B2								
Para.		χ^2	α_1	α_2	θ_R	μ_1	μ_2	
		55	358.73°	338.89°	24.65°	0.00533 eV	0.0114 eV	
Obs.	θ_{12}	θ_{13}	θ_{23}	δ	m_2	m_3	m_{ee}	
	34.34°	8.56°	41.5°	254°	0.00860 eV	0.0502 eV	0.00319 eV	

Origin II

The non-Abelian discrete symmetry as

an <u>effective</u> symmetry after
a (gauge) continuous symmetry
breaking

SO(3)→A_4, S_4 and A_5



$SO(3)$	A_4	S_4	A_5
$\underline{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\underline{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$
$\underline{5}$	$\mathbf{1'}+\mathbf{1''}+\mathbf{3}$	$\mathbf{2}+\mathbf{3'}$	$\mathbf{5}$
$\underline{7}$	$\mathbf{1}+\mathbf{3}+\mathbf{3}$	$\mathbf{1'}+\mathbf{3}+\mathbf{3'}$	$\mathbf{3'}+\mathbf{4}$
$\underline{9}$	$\mathbf{1}+\mathbf{1'}+\mathbf{1''}+\mathbf{3}+\mathbf{3}$	$\mathbf{1}+\mathbf{2}+\mathbf{3}+\mathbf{3'}$	$\mathbf{4}+\mathbf{5}$
$\underline{11}$	$\mathbf{1'}+\mathbf{1''}+\mathbf{3}+\mathbf{3}+\mathbf{3}$	$\mathbf{2}+\mathbf{3}+\mathbf{3}+\mathbf{3'}$	$\mathbf{3}+\mathbf{3'}+\mathbf{5}$
$\underline{13}$	$\mathbf{1}+\mathbf{1}+\mathbf{1'}+\mathbf{1''}+\mathbf{3}+\mathbf{3}+\mathbf{3}$	$\mathbf{1}+\mathbf{1'}+\mathbf{2}+\mathbf{3}+\mathbf{3'}+\mathbf{3'}$	$\mathbf{1}+\mathbf{3}+\mathbf{4}+\mathbf{5}$

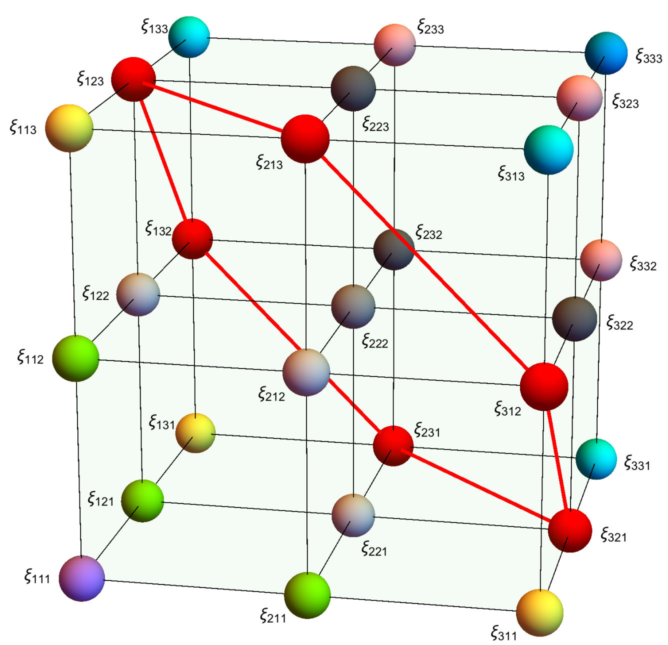
Ovrut, 77; Etesi, 9706029; Berger and Grossman, 0910.4392

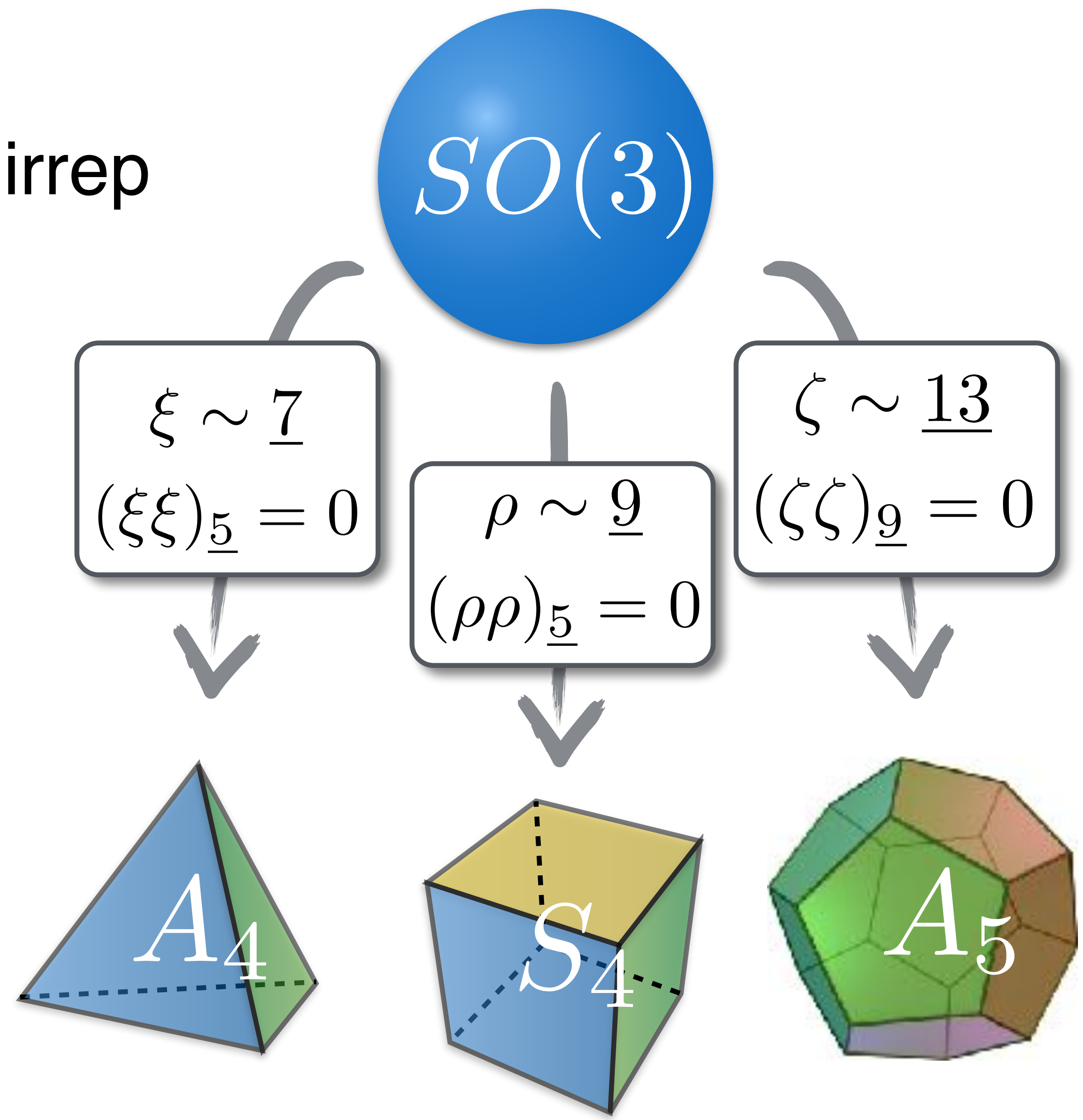
SU(3)→A_4 e.g., Luhn, 1101.2417; Merle, Zwicky, 1110.4891

SO(3) as origin of discrete symmetries

- **How to realise it?**
  ——using high dimensional irrep

$$\xi_{ijk} \sim \underline{7} =$$





- **For the first time, we realised it in SUSY with the help of flat direction**

King, **YLZ**, 1809.10292

A_4 breaking to Z_3 and Z_2

- One way (not unique) to breaking A_4 to Z_3 and Z_2

$$\varphi \sim \underline{3} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix} \qquad \chi_{ij} \sim \underline{5} = \begin{pmatrix} \frac{1}{\sqrt{3}}(\chi' + \chi'') & \frac{1}{\sqrt{2}}\chi_3 & \frac{1}{\sqrt{2}}\chi_2 \\ \frac{1}{\sqrt{2}}\chi_3 & \frac{1}{\sqrt{3}}(\omega\chi' + \omega^2\chi'') & \frac{1}{\sqrt{2}}\chi_1 \\ \frac{1}{\sqrt{2}}\chi_2 & \frac{1}{\sqrt{2}}\chi_1 & \frac{1}{\sqrt{3}}(\omega^2\chi' + \omega\chi'') \end{pmatrix}$$

$A_4 \to Z_3$

$$(\xi(\varphi\varphi)_{\underline{5}})_{\underline{5}} = 0$$

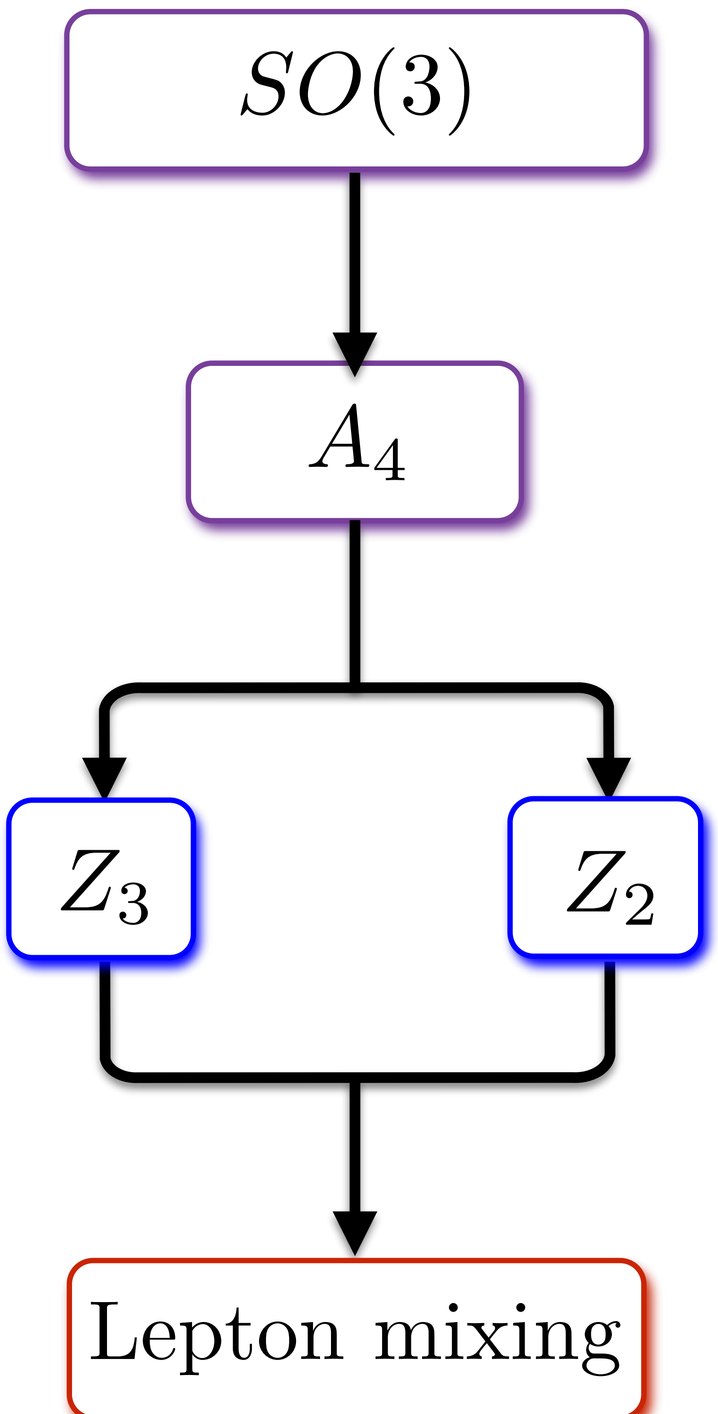
$$\begin{pmatrix} \langle\varphi_1\rangle \\ \langle\varphi_2\rangle \\ \langle\varphi_3\rangle \end{pmatrix} = \pm v_\varphi \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \right\}$$

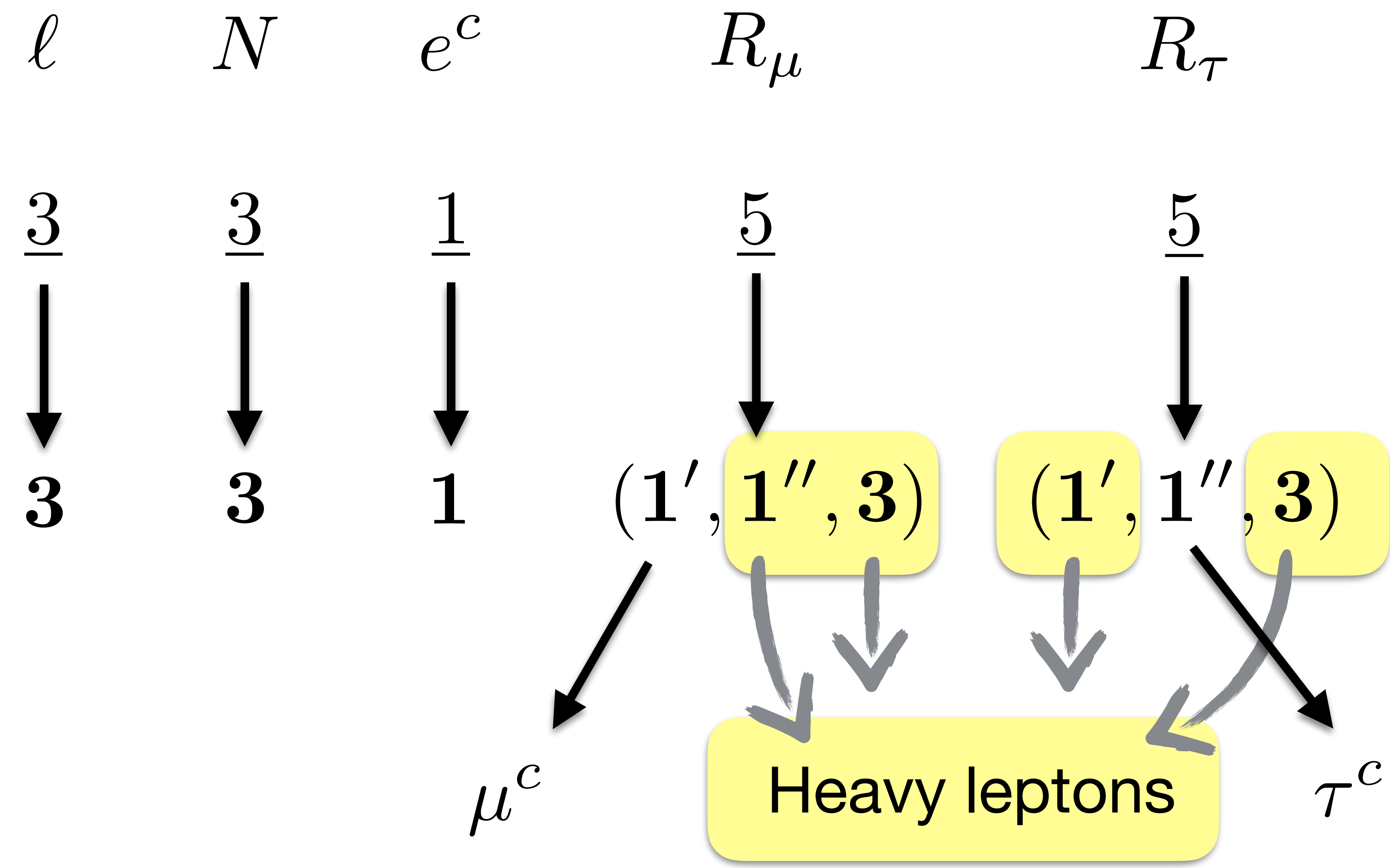
$A_4 \to Z_2$

$$(\xi\chi)_{\underline{5}} = (\xi(\chi\chi)_{\underline{5}})_{\underline{3}} = 0$$

$$\begin{pmatrix} \langle\chi'\rangle \sim \mathbf{1}' \\ \langle\chi''\rangle \sim \mathbf{1}'' \\ \begin{pmatrix} \langle\chi_1\rangle \\ \langle\chi_2\rangle \\ \langle\chi_3\rangle \end{pmatrix} \sim \mathbf{3} \end{pmatrix} = \left\{ \begin{pmatrix} 0 \\ 0 \\ \begin{pmatrix} \pm v_\chi \\ 0 \\ 0 \end{pmatrix} \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ \begin{pmatrix} 0 \\ \pm v_\chi \\ 0 \end{pmatrix} \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ \begin{pmatrix} 0 \\ 0 \\ \pm v_\chi \end{pmatrix} \end{pmatrix} \right\}$$

Framework of model building





$$\langle \zeta \rangle \sim \{v_\zeta,\ 0,\ (0,0,0)\}$$
$$\underline{5} \sim (\mathbf{1}',\ \mathbf{1}'',\ \ \mathbf{3})$$

Lepton masses and mixing

- Charged lepton mass matrices

$$w_e^{\rm eff} = y_e \frac{v_\varphi^3}{\Lambda^3}\, \underset{\underline{3}}{\ell^T} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \underset{\underline{1}}{e^c H_d}$$

$$SO(3)\times U(1) \;\simeq SU(2)\times U(1)$$

How to extract the 1' and 1" of A_4 from the irrep of SO(3)?

$$w_{R_\mu}^{\rm eff} = \underset{(\underline{3},\ \underline{1},\ \underline{3})}{(\ell^T, L_{\mu 0}, L_{\mu\mathbf{3}}^T)} \begin{pmatrix} y_{\mu 1}\frac{v_\varphi v_{\bar\eta}}{\sqrt{3}\Lambda^2} V_\omega H_d & y_{\mu 1}\frac{v_\varphi v_{\bar\eta}}{\sqrt{3}\Lambda^2} V_\omega^* H_d & 2\sqrt{3}Y_{\mu 3}\frac{v_\xi}{\Lambda}\ \mathbb{1}_{3\times 3}\ H_d \\ 0 & Y_{\mu 1} v_\zeta & \mathbb{0}_{1\times 3} \\ y_{\mu 2}\frac{v_\varphi v_{\bar\eta}}{\sqrt{3}\Lambda} V_\omega & y_{\mu 2}\frac{v_\varphi v_{\bar\eta}}{\sqrt{3}\Lambda} V_\omega^* & 2\sqrt{3}Y_{\mu 2} v_\xi\ \mathbb{1}_{3\times 3} \end{pmatrix} \underset{\underline{5}}{\begin{pmatrix} \mu^c \\ R''_\mu \\ R_{\mu\mathbf{3}} \end{pmatrix}} \begin{matrix} \mathbf{1}' \\ \mathbf{1}'' \\ \mathbf{3} \end{matrix}$$

$$w_{R_\tau}^{\rm eff} = \underset{(\underline{3},\ \underline{1},\ \underline{3})}{(\ell^T, L_{\tau 0}, L_{\tau\mathbf{3}}^T)} \begin{pmatrix} y_\tau \frac{v_\varphi}{\sqrt{3}\Lambda} V_\omega^* H_d & y_\tau \frac{v_\varphi}{\sqrt{3}\Lambda} V_\omega H_d & \mathcal{O}(y_\tau \frac{v_\varphi}{\sqrt{3}\Lambda}) H_d \\ 0 & Y_{\tau 1}\frac{2 v_\zeta^2}{\sqrt{3}\Lambda} & \mathbb{0}_{1\times 3} \\ \mathbb{0}_{3\times 1} & \mathbb{0}_{3\times 1} & 2\sqrt{3}Y_{\tau 2} v_\xi\ \mathbb{1}_{3\times 3} \end{pmatrix} \underset{\underline{5}}{\begin{pmatrix} \tau^c \\ R'_\tau \\ R_{\tau\mathbf{3}} \end{pmatrix}} \begin{matrix} \mathbf{1}' \\ \mathbf{1}'' \\ \mathbf{3} \end{matrix}$$

After heavy leptons decouple,

$$M_l = \begin{pmatrix} y_e \frac{v_\varphi^3}{\Lambda^3} & y_\mu \frac{v_\varphi v_{\bar\eta}}{\sqrt{3}\Lambda^2} & y_\tau \frac{v_\varphi}{\sqrt{3}\Lambda} \\ y_e \frac{v_\varphi^3}{\Lambda^3} & \omega y_\mu \frac{v_\varphi v_{\bar\eta}}{\sqrt{3}\Lambda^2} & \omega^2 y_\tau \frac{v_\varphi}{\sqrt{3}\Lambda} \\ y_e \frac{v_\varphi^3}{\Lambda^3} & \omega^2 y_\mu \frac{v_\varphi v_{\bar\eta}}{\sqrt{3}\Lambda^2} & \omega y_\tau \frac{v_\varphi}{\sqrt{3}\Lambda} \end{pmatrix} \frac{v_d}{\sqrt{2}} \qquad \delta M_l = \frac{v_\eta v_\chi v_\varphi}{\Lambda^3} \begin{pmatrix} 0 & 0 & 0 \\ 0 & c\omega + d\omega^2 & 0 \\ 0 & c\omega^2 + d\omega & 0 \end{pmatrix} \frac{v_d}{\sqrt{2}}$$

Lepton masses and mixing

- Neutrino mass matrix

$$w_N = y_N(\ell N)_{\underline{1}} H_u + \frac{\lambda_\eta}{\Lambda}\bar{\eta}^2 (NN)_{\underline{1}} + \lambda_\chi \big(\chi (NN)_{\underline{5}}\big)_{\underline{1}}$$

$$M_D = \frac{y_D v_u}{\sqrt{2}} \mathbb{1}_{3\times 3}\,, \qquad M_M = \begin{pmatrix} a & 0 & 0 \\ 0 & a & b \\ 0 & b & a \end{pmatrix}$$

Nothing special, but the Z_2-preserving one.

- Mixing is given by TBM + eμ-mixing correction

$$\sin\theta_{13} = \frac{\sin\theta_{e\mu}}{\sqrt{2}}\,,$$

$$\sin\theta_{12} = \sqrt{\frac{2 - 2\sin 2\theta_{e\mu}\cos\phi_{e\mu}}{3(2-\sin^2\theta_{e\mu})}}\,,$$

$$\sin\theta_{23} = \frac{\cos\theta_{e\mu}}{\sqrt{2-\sin^2\theta_{e\mu}}}\,.$$

first octant

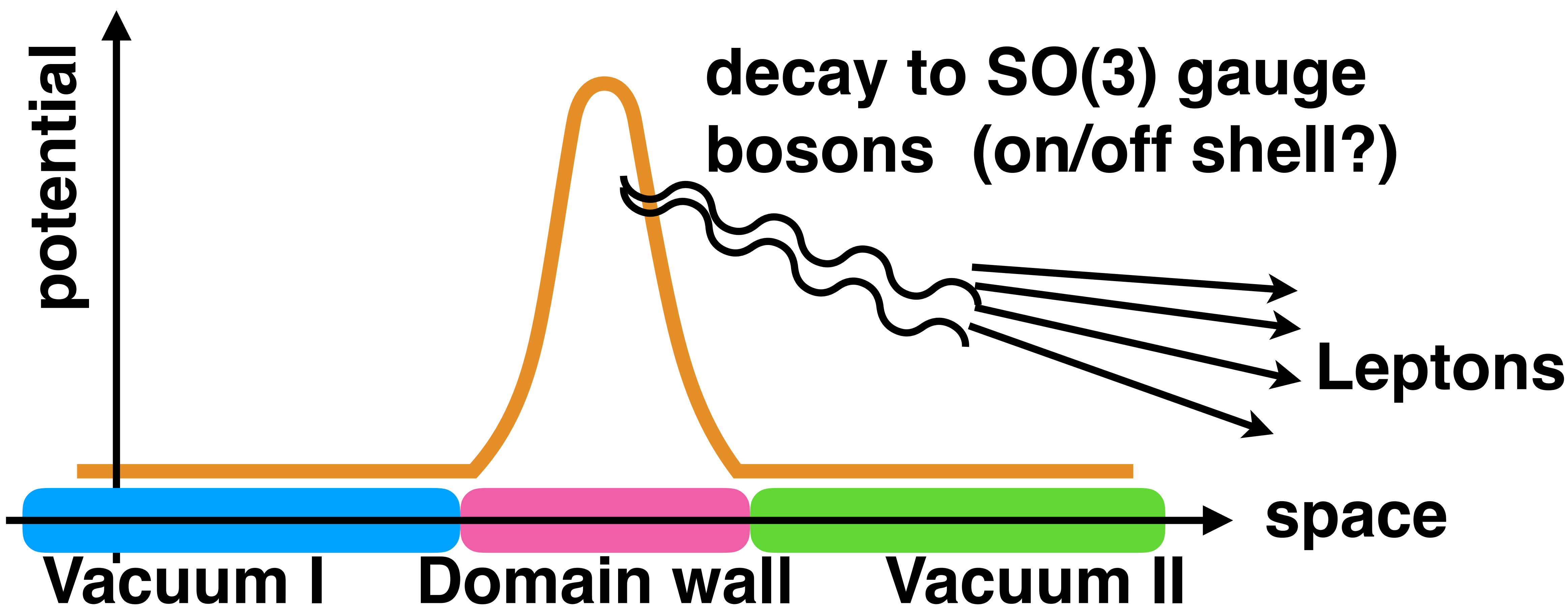
King,0506297;
Antusch and King,0508044;
King and Malinsky, 0608021;
Masina, 0508031;
Antusch, Huber, King, Schwetz, 0702286; Ballett, King, Luhn, Pascoli, Schmidt, 1410.7573;
Girardi, Petcov, Titov, 1410.8056.

$$\delta = \arg\big((3\cos 2\theta_{e\mu} + \cos 4\theta_{e\mu})\cos\phi_{e\mu} - i(\cos 2\theta_{e\mu} + 3)\sin\phi_{e\mu} + \sin 2\theta_{e\mu}\big)$$

The absence of domain wall in our model

- SO(3) x U(1)$\rightarrow A_4$, the breaking of gauge symmetry does not generate domain walls.

- $A_4 \rightarrow Z_2$ and Z_3 , even if the energy gap between different vacuums is generated, it will decay to gauge bosons and finally to leptons. The two vacuums are finally identical with each other via gauge transformation.



Summary

Non-Abelian discrete symmetries from …

extra dimensions

- ☑ A simple example of how to realise A_4 via orbifolding.
- ☑ Multiple modular symmetries as the direct origin of flavour mixing.

a gauge symmetry breaking

- ☑ SO(3) breaking to A_4, S_4 or A_5 via VEV of high irreps.
- ☑ A gauge SO(3) x U(1) flavour model is introduced. Through a two-step symmetry breaking, $SO(3) \to A_4 \to Z_3, Z_2$, flavour mixing is realised, fully consistent with oscillation data.
- ☑ The domain wall problem is absent.

Thank you!