Effective field theory approach to lepton number violation

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New physics beyond SM: $m_\nu \neq 0$ and lepton mixing

- Single, definite evidence for phys beyond SM on particle phys side.
- Extremely tiny $m_\nu < 1\ \mathrm{eV}$ calls for understanding of its origin.
- Being neutral, ν can be either Dirac or Majorana type.

Why Majorana?

- Not forbidden by conservation law –
  Lepton number L conservation as accidental symmetry in SM.
- ν easily appears as Majorana type in underlying theory for m_ν –
  Special arrangements required to guarantee Dirac type.
  3 canonical seesaws plus many others.
- Bonus: lepton number violation (LNV) → BAU via leptogenesis

SM as effective field theory (EFT)

SM has been very successful though fundamental questions remain to be answered:

Mechanism for electroweak symmetry breaking?

Flavor puzzle? *Unification of gauge couplings?* ……

From modern point of view of QFT, what we have verified is that

- SM is a very good EFT at energies below ~100 GeV;
- All predicted particles discovered, no additional particles of mass below ~100 GeV;
- Potential effects from New Phys on SM particles' interactions appear as suppressed effective interactions – not yet in sight beyond $m_\nu \neq 0$.

Central issue: what else LNV effects to expect for Majorana ν besides $m_\nu \neq 0$?

Approaches to LNV signals

Experimental

High-energy frontier:

Discover new particles/interactions at colliders – like-charge multileptons

High-intensity frontier:

Search for forbidden processes with large samples/extreme precision – $0\nu\beta\beta$, $M_1^- \to M_2^+ \ell_\alpha^- \ell_\beta^-$, $\tau^- \to M_1^- M_2^- \ell^+$

Theoretical

Top-down approach:

Study signals model by model

Many parameters, but once fixed, def. answer

Bottom-up approach:

Work with EFT, not specific for one model, but possibly for a class of them

Uncertainties due to hadronic/nuclear phys

Both approaches are necessary and complementary.

EFT offers a means *via a series of EFTs from Λ_{NP} to Λ_χ to nuclear scale*,

to connect high and low energy processes, and

to organize all data in a coherent manner.

Flow chart of EFTs for LNV phys

Matching at scales	Series of EFTs	Interactions
	Underlying NP for LNV and m_ν	Fundamental theory/EFT
$\Lambda_{\rm NP} \gg \Lambda_{\rm EW}$	SMEFT	SM + high-dim operators
$\Lambda_{\rm EW} \sim 100$ GeV	LEFT	QCD + QED + high-dim operators
$\Lambda_\chi \sim$ GeV	Chiral effective theory	Interactions of mesons/nucleons
Nuclear$\sim$ MeV	Nuclear theory	Nuclear many body

Experimental bounds on LNV: collider

Bounds from LHC:

Typical signal – like-charge multi-leptons;

Depending on theoretical assumptions about parameters and decay Brs;

Typically, new particles > few 100 GeV or TeV.

Skipped here.

Experimental bounds on LNV: low energy processes

- Rich data from intense experimental activities.
- Most extensively studied theoretically and experimentally:

  Nuclear neutrinoless double-beta decay ($0\nu\beta\beta$);

  LNV meson and τ decays.

Experimental bounds on LNV $0\nu\beta\beta$ decays

Current limits on lifetime (and effective ν mass $m_{\beta\beta}$ assuming m_ν dominance)

Isotope	$T^{0\nu}_{1/2}$ ($\times 10^{25}$ y)	$\langle m_{\beta\beta}\rangle$ (eV)	Experiment
^{48}Ca	$> 5.8 \times 10^{-3}$	$< 3.5 - 22$	ELEGANT-IV
^{76}Ge	> 8.0	$< 0.12 - 0.26$	GERDA
	> 1.9	$< 0.24 - 0.52$	MAJORANA DEMONSTRATOR
^{82}Se	$> 3.6 \times 10^{-2}$	$< 0.89 - 2.43$	NEMO-3
^{96}Zr	$> 9.2 \times 10^{-4}$	$< 7.2 - 19.5$	NEMO-3
^{100}Mo	$> 1.1 \times 10^{-1}$	$< 0.33 - 0.62$	NEMO-3
^{116}Cd	$> 1.0 \times 10^{-2}$	$< 1.4 - 2.5$	NEMO-3
^{128}Te	$> 1.1 \times 10^{-2}$	—	—
^{130}Te	> 1.5	$< 0.11 - 0.52$	CUORE
^{136}Xe	> 10.7	$< 0.061 - 0.165$	KamLAND-Zen
	> 1.8	$< 0.15 - 0.40$	EXO-200
^{150}Nd	$> 2.0 \times 10^{-3}$	$< 1.6 - 5.3$	NEMO-3

Lifetime expected to be pushed up further by $10^{1\sim 2}$ in future at

EXO-200 (4yr), nEXO, KamLAND-Zen (300 kg, 3 yr), GERDA II, CUORE (5yr), SNO+, SuperNEMO, NEXT, ⋯

Experimental bounds on LNV meson decays

Current limits on Brs from Belle, BarBar, LHCb

Modes for $\ell_\alpha\ell_\beta =$	ee	$e\mu$	$\mu\mu$
$K^- \to \pi^+\ell_\alpha^-\ell_\beta^-$	6.4×10^{-10}	5.0×10^{-10}	1.1×10^{-9}
$D^- \to \pi^+\ell_\alpha^-\ell_\beta^-$	1.1×10^{-6}	2.0×10^{-6}	2.2×10^{-8}
$D^- \to K^+\ell_\alpha^-\ell_\beta^-$	9.0×10^{-7}	1.9×10^{-6}	1.0×10^{-5}
$B^- \to \pi^+\ell_\alpha^-\ell_\beta^-$	2.3×10^{-8}	1.5×10^{-7}	4.0×10^{-9}
$B^- \to K^+\ell_\alpha^-\ell_\beta^-$	3.0×10^{-8}	1.6×10^{-7}	4.1×10^{-8}
……	……	……	……

Expected to be improved by $10^{1\sim2}$ in future at

LHCb run2, NA62 (TUM), Belle 2, and possibly at FCC-ee, Super-τ-Charm, etc?

Experimental bounds on LNV τ decays

Current limits on Brs from Belle

Modes for $\ell =$	e	μ
$\tau^- \to \pi^-\pi^-\ell^+$	2.0×10^{-8}	3.9×10^{-8}
$\tau^- \to \pi^-K^-\ell^+$	3.2×10^{-8}	4.8×10^{-8}
$\tau^- \to K^-K^-\ell^+$	3.3×10^{-8}	4.7×10^{-8}

Back to theory

- Start from EFT above electroweak scale Λ_{EW},
  go downwards crossing scales $\Lambda_{\text{EW}}, m_b, \cdots$,
  till EFT at scale of process under consideration.

- This requires to study a sequence of EFTs.

- Essential for an EFT:
  Dynamical degrees of freedom for which EFT is constructed;
  Symmetries as guiding principle for dynamics;
  Power counting rule: what is more important.

What to do with an EFT

effective interaction = Wilson coefficient/low-energy constants (LECs) × operators

- Find out a basis of complete and independent operators, *necessary*
  to catch all possible phys,
  for being renormalizable in EFT, and
  for correct theory-experiment comparison.

- Renormalize operators
  to improve naïve perturbation theory by summing up large $\log(M/m)$
  M: where effective interaction is generated
  m: where matrix elements of effective interaction are evaluated
  or effective interaction is further matched to next EFT
  This is done by RGE analysis.

What to do with an EFT

- Matching calculation at the boundary of 2 EFTs

  to link LECs from EFT at high scale to EFT at scale for interested process.

  Essentially trivial for perturbative theory.

  Difficult when nonperturbative effects are involved, e.g.,

  Λ_χ: Nambu-Goldstone bosons born out of strong quark dynamics;

  $\Lambda_{\rm N}$: nucleus formed out of nucleons.

  Appeals to symmetry arguments, *ab initial* calculations, etc

What to do with an EFT

We outline EFT approach by examples.

Start with SMEFT between Λ_{NP} and Λ_{EW}:

- Dynamical degrees of freedom restricted to SM fields;
- Symmetries – $SU(3)_C \times SU(2)_L \times U(1)_Y$, no B or L conservation etc;
- Power counting – expansion in p/Λ_{NP}.

SMEFT is an infinite tower of effective interactions involving higher and higher dimensional operators:

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{\mathrm{SM}} + \mathscr{L}_5 + \mathscr{L}_6 + \mathscr{L}_7 + \cdots$$

SMEFT: dim-5

Unique Weinberg operator for Majorana m_ν, $\Delta L = 2$ Weinberg 1979

$$\epsilon_{ij}\epsilon_{mn}(L_p^i C L_r^m) H^j H^n$$

L : LH lepton doublet
H : Higgs doublet
i, j, m, n : SU(2) indices
p, r, s, t : flavor indices

1-loop RGE Babu et al 1993, Antusch et al 2001

Nothing interesting beyond m_ν.

SMEFT: dim-6

- Long history on basis of operators.

  Started with Buchmuller-Wyler 1986,

  Corrected and improved by efforts by many groups,

  Culminated with Warsaw basis Grzadkowski et al 2010 –

- 63 operators $\begin{cases} 59:\ \Delta B = \Delta L = 0, \text{rich pheno} \\ 4:\ \Delta B = \Delta L = 1,\ p \to e^{+}\pi^{0}, \text{etc} \end{cases}$

  Without counting flavors (easy with trivial flavor relations) and Hermitian conjugate.

- 1-loop RGE, complicated, by UC San Diego group in 2013, 2014 Barcelona group in 2013

SMEFT: dim-7

- Early partial analysis by Weinberg 1980 Weldon-Zee 1980
- 1st systematic analysis by Lehman 2014
- Final answer by Liao-Ma 2016:

$$\text{18 operators} \begin{cases} 12\text{: } \Delta B = 0,\ \Delta L = 2, \\ \ \ 6\text{: } -\Delta B = \Delta L = 1, p \to \nu\pi^+, \text{etc} \end{cases}$$

All violating L and thus non-Hermitian, Hermitian conjugate not counted.

- Consistent with independent counting by Hilbert series approach Henning et al 2015.

SMEFT: 18 dim-7 operators

Liao-Ma 2016

	$\psi^2 H^4$		$\psi^2 H^3 D$
$\mathcal{O}_{LH}$	$\epsilon_{ij}\epsilon_{mn}(L^i C L^m)H^j H^n (H^\dagger H)$	$\mathcal{O}_{LeHD}$	$\epsilon_{ij}\epsilon_{mn}(L^i C\gamma_\mu e)H^j H^m iD^\mu H^n$
	$\psi^2 H^2 D^2$		$\psi^2 H^2 X$
$\mathcal{O}_{LHD1}$ $\mathcal{O}_{LHD2}$	$\epsilon_{ij}\epsilon_{mn}(L^i CD^\mu L^j)H^m(D_\mu H^n)$ $\epsilon_{im}\epsilon_{jn}(L^i CD^\mu L^j)H^m(D_\mu H^n)$	$\mathcal{O}_{LHB}$ $\mathcal{O}_{LHW}$	$\epsilon_{ij}\epsilon_{mn}(L^i Ci\sigma_{\mu\nu}L^m)H^j H^n B^{\mu\nu}$ $\epsilon_{ij}(\tau^I\epsilon)_{mn}(L^i Ci\sigma_{\mu\nu}L^m)H^j H^n W^{I\mu\nu}$
	$\psi^4 D$		$\psi^4 H$
$\mathcal{O}_{\bar{d}uLLD}$ $\mathcal{O}_{\bar{L}QddD}$ $\mathcal{O}_{\bar{e}dddD}$	$\epsilon_{ij}(\bar{d}\gamma_\mu u)(L^i CiD^\mu L^j)$ $(\bar{L}\gamma_\mu Q)(dCiD^\mu d)$ $(\bar{e}\gamma_\mu d)(dCiD^\mu d)$ all non-Herm., breaking L: $\Delta B = -\Delta L = 1$ $\Delta B = 0,\ \Delta L = 2$ flavor/color indices implied $(\psi C\chi) = \overline{\psi^C}\chi,\ (\psi^C)^C = \psi$	$\mathcal{O}_{\bar{e}LLLH}$ $\mathcal{O}_{\bar{d}LQLH1}$ $\mathcal{O}_{\bar{d}LQLH2}$ $\mathcal{O}_{\bar{d}LueH}$ $\mathcal{O}_{\bar{Q}uLLH}$ $\mathcal{O}_{\bar{L}dud\tilde{H}}$ $\mathcal{O}_{\bar{L}dddH}$ $\mathcal{O}_{\bar{e}Qdd\tilde{H}}$ $\mathcal{O}_{\bar{L}dQQ\tilde{H}}$	$\epsilon_{ij}\epsilon_{mn}(\bar{e}L^i)(L^j CL^m)H^n$ $\epsilon_{ij}\epsilon_{mn}(\bar{d}L^i)(Q^j CL^m)H^n$ $\epsilon_{im}\epsilon_{jn}(\bar{d}L^i)(Q^j CL^m)H^n$ $\epsilon_{ij}(\bar{d}L^i)(uCe)H^j$ $\epsilon_{ij}(\bar{Q}u)(LCL^i)H^j$ $(\bar{L}d)(uCd)\tilde{H}$ $(\bar{L}d)(dCd)H$ $\epsilon_{ij}(\bar{e}Q^i)(dCd)\tilde{H}^j$ $\epsilon_{ij}(\bar{L}d)(QCQ^i)\tilde{H}^j$
	redundant operators		
$\mathcal{O}^{(2)}_{\bar{d}uLLD}$	$\epsilon_{ij}(\bar{d}\gamma_\mu u)(L^i C\sigma^{\mu\nu}D_\nu L^j)$	$\mathcal{O}_{\bar{L}dQdD}$	$(\bar{L}iD^\mu d)(QC\gamma_\mu d)$

SMEFT: basis of dim-7 operators

- For physical applications:

  18 complete and independent *structures* are not enough;

  Must specify operators with respect to *fermion flavors* – *B* phys., *K* phys., etc.

- *Trivial* for dim-5 and dim-6 operators: (anti)symmetric in like-charge flavors
- Nontrivial flavor relations first appear at dim 7 – involving Yukawas Liao-Ma 2019

- Count of basis operators including flavors:

  1 family: 15

  3 families: 771

SMEFT: flavor relations of dim-7 operators

Class	Operator	Flavor relations
$\psi^2 H^4$	$\mathcal{O}_{LH}$	$\mathcal{O}_{LH}^{pr} - p \leftrightarrow r = 0$
$\psi^2 H^2 D^2$	$\mathcal{O}_{LHD1}$	$(\mathcal{O}_{LDH1}^{pr} + \mathcal{K}^{pr}) - p \leftrightarrow r = 0$
	$\mathcal{O}_{LHD2}$	$\left[4\mathcal{O}_{LHD2}^{pr} + 2(Y_e)_{rv}\mathcal{O}_{LeHD}^{pv} - \mathcal{O}_{LHW}^{pr} + 2\mathcal{K}^{pr}\right] - p \leftrightarrow r = \mathcal{O}_{LHB}^{pr}$
$\psi^2 H^2 X$	$\mathcal{O}_{LHB}$	$\mathcal{O}_{LHB}^{pr} + p \leftrightarrow r = 0$
$\psi^4 H$	$\mathcal{O}_{\bar{e}LLLH}$	$(\mathcal{O}_{\bar{e}LLLH}^{prst} + r \leftrightarrow t) - r \leftrightarrow s = 0$
$\psi^4 D$	$\mathcal{O}_{\bar{d}uLLD}$	$\left[\mathcal{O}_{\bar{d}uLLD}^{prst} + (Y_d)_{vp}\mathcal{O}_{\bar{Q}uLLH}^{vrst} - (Y_u^\dagger)_{rv}\mathcal{O}_{\bar{d}LQLH2}^{psvt}\right] - s \leftrightarrow t = 0 \Rightarrow$ example
$\psi^4 H$	$\mathcal{O}_{\bar{L}dddH}$	$\mathcal{O}_{\bar{L}dddH}^{prst} + s \leftrightarrow t = 0, \quad \mathcal{O}_{\bar{L}dddH}^{prst} + \mathcal{O}_{\bar{L}dddH}^{pstr} + \mathcal{O}_{\bar{L}dddH}^{ptrs} = 0$
	$\mathcal{O}_{\bar{e}Qdd\tilde{H}}$	$\mathcal{O}_{\bar{e}Qdd\tilde{H}}^{prst} + s \leftrightarrow t = 0$
$\psi^4 D$	$\mathcal{O}_{\bar{L}QddD}$	$\left[\mathcal{O}_{\bar{L}QddD}^{prst} + (Y_u)_{rv}\mathcal{O}_{\bar{L}dud\tilde{H}}^{psvt}\right] - s \leftrightarrow t = -(Y_e^\dagger)_{vp}\mathcal{O}_{\bar{e}Qdd\tilde{H}}^{vrst} - (Y_d)_{rv}\mathcal{O}_{\bar{L}dddH}^{pvst}$
	$\mathcal{O}_{\bar{e}dddD}$	$\mathcal{O}_{\bar{e}dddD}^{prst} - r \leftrightarrow s = (Y_d^\dagger)_{tv}\mathcal{O}_{\bar{e}Qdd\tilde{H}}^{pvrs}$
		$(\mathcal{O}_{\bar{e}dddD}^{prst} + r \leftrightarrow t) - s \leftrightarrow t = (Y_e)_{vp}\mathcal{O}_{\bar{L}dddH}^{vrst}$

$$\mathcal{K}^{pr} = (Y_u)_{vw}\mathcal{O}_{\bar{Q}uLLH}^{vwpr} - (Y_d^\dagger)_{vw}\mathcal{O}_{\bar{d}LQLH2}^{vpwr} - (Y_e^\dagger)_{vw}\mathcal{O}_{\bar{e}LLLH}^{vwpr}.$$

SMEFT: why flavor relations relevant?

- Operators mix under renormalization –
  Redundant operators reappear due to renormalization and
  have to be expressed back in terms of basis operators.

- This reexpression employs flavor relations –
  Must not involve inverse Yukawas to avoid artificial singularities.

- It is possible to choose a good basis without involving inverse Yukawas for which anomalous dimension matrix is well-defined. Liao-Ma 2019

LEFT: Low-energy EFT below Λ_{EW} UC San Diego group 2017

- Dynamical degrees of freedom – SM fields with heavy particles integrated out;
- Symmetries – $SU(3)_C \times U(1)_Q$;
- Power counting – expansion in p/Λ_{EW}.

- Basis of operators up to dim-6, their 1-loop RGE finished.

- Basis of dim-7 operators Liao-Ma *in preparation*

Matching between SMEFT and LEFT at Λ_{EW}

- Important: dims of operators in two EFTs not necessarily comparable
  Guide: *L* and *B* numbers together with dims

- SMEFT operators up to dim-6 and
  LEFT operators up to dim-6 UC San Diego group 2017

- Matching up to dim-7 operators on both sides Liao-Ma *in preparation*

- Comments:
  Whether *b* or *c* is integrated out depends on your processes;
  Dim-9 operators in LEFT studied by several groups aiming at $0\nu\beta\beta$ related processes.
  Savage 1999, Prezeau et al 2003, Cirigliano et al 2017, Graesser 2017, ⋯

LNV *B* and *D* decays

- Status: vacuum insertion, form factors, etc, to calculate matrix elements of LEFT operators

  Drawback: unable to estimate errors

- Better to work with EFT:

  $B^- \to D^+ \ell_\alpha^- \ell_\beta^-$: combine with heavy quark symmetry

  $B^- \to K^+ \ell_\alpha^- \ell_\beta^-$, $D^- \to K^+ \ell_\alpha^- \ell_\beta^-$: ?

LNV K, τ, and $0\nu\beta\beta$ decays

- Matching LEFT with Chiral EFT $\begin{cases}\text{without nucleons N}\\ \text{with nucleons N}\end{cases}$
- Without N:

  $K^- \to \pi^+ \ell_\alpha^- \ell_\beta^-$, $\tau^- \to \pi^- K^- \ell^+$ Liao-Ma-Wang *in preparation*

  $\pi^- \pi^- \to e^- e^-$ (for $0\nu\beta\beta$) Savage 1999, Cirigliano et al 2017,

  Nicholson et al 2018, Feng et al 2019

- With N: aiming at nucleon potential for $0\nu\beta\beta$ Cirigliano et al 2017, 2018, Horoi-Neacsu 2017 to be applied to many-body systems of nuclei

Dim-7 in SMEFT: RGE effects on $0\nu\beta\beta$ Liao-Ma 2019

- Status on $0\nu\beta\beta$:

  Experiment – best half lifetime for $^{136}\mathrm{Xe} > 1.07\times 10^{26}\ \mathrm{yr}$ KamLAND-Zen 2016

  Theory – most recent and comprehensive analysis, with hadronic+nuclear uncertainties of a factor of few, under control Cirigliano et al 2017, 2018, Horoi-Neacsu 2017

- Complete theory analysis involves a sequence of EFTs:

  LNV underlying theory/EFT $\to$ SMEFT $\to$ LEFT $\to$ chiral EFT $\to$ nuclear theory

Dim-7 in SMEFT: RGE effects on $0\nu\beta\beta$

Our slight improvement focuses on complete 1-loop RGE in SMEFT while employing many shortcuts by other groups:

Bounds on couplings($\mu = m_{\rm p}$) in LEFT by Horoi-Neacsu 2017
(LEFT $\leftarrow$ chiral EFT $\leftarrow$ nuclear theory)

$\Rightarrow$ Bounds on couplings($\mu = \Lambda_{\rm EW}$) in LEFT using QCD-RGE of Cirigliano et al 2017
$\Rightarrow$ Matching LEFT to SMEFT at tree level at $\mu = \Lambda_{\rm EW}$ Liao-Ma 2019
$\Rightarrow$ Bounds on couplings($\Lambda_{\rm NP} > \mu > \Lambda_{\rm EW}$) in SMEFT Liao-Ma 2019

RGE effects on $0\nu\beta\beta$: numbers

Bounds on couplings($\mu = \Lambda_{\mathrm{EW}}$) in SMEFT, in units of $(100\ \mathrm{TeV})^{-3}$, $^{136}\mathrm{Xe}$ data

Liao-Ma 2019

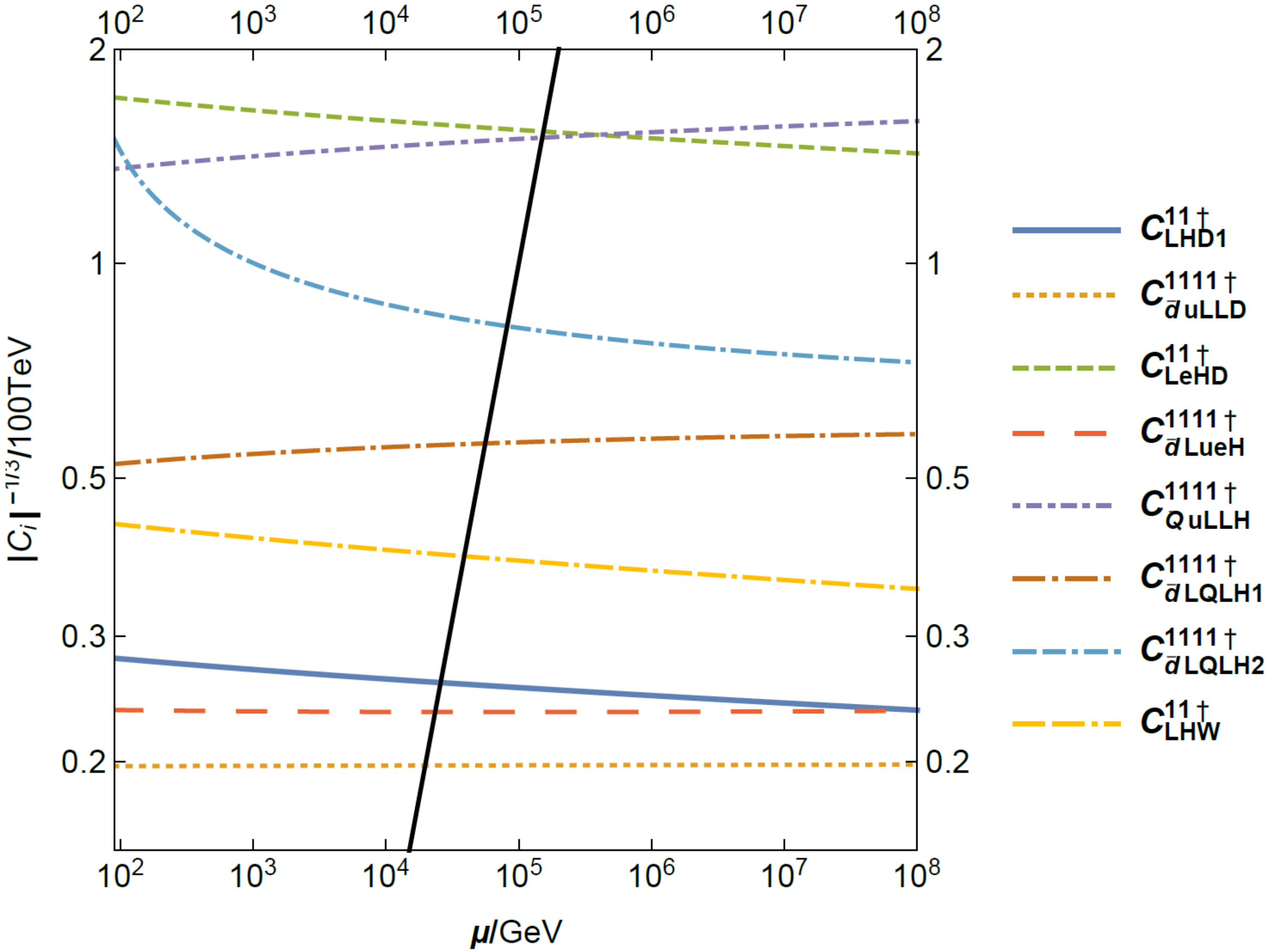
$\lvert C^{11}_{LHD1}\rvert$	$\lvert C^{1111}_{\bar{d}uLLD}$	$\lvert C^{11}_{LeHD}\rvert$	$\lvert C^{1111}_{\bar{d}LueH}$	$\lvert C^{1111}_{\bar{Q}uLLH}$	$\lvert C^{1111}_{\bar{d}LQLH}$	$\lvert C^{1111}_{\bar{d}LQLH}$	$\lvert C^{11}_{LHW}\rvert$
46	131	0.2	76	0.4	0.7	0.3	12

RGE effects on $0\nu\beta\beta$: numbers

This is how RGEs look like, on dropping small terms: Liao-Ma 2019

$$
\begin{aligned}
\frac{d}{d\ln\mu}C^{11\dagger}_{LHD1} &= \frac{1}{4\pi}\Big(-\frac{9}{10}\alpha_1+\frac{11}{2}\alpha_2+6\alpha_t\Big)C^{11\dagger}_{LHD1}+\frac{1}{4\pi}\Big(-\frac{33}{20}\alpha_1-\frac{19}{4}\alpha_2-2\alpha_\lambda\Big)C^{11\dagger}_{LHD2},\\
\frac{d}{d\ln\mu}C^{1111\dagger}_{\bar{d}uLLD} &= \frac{1}{4\pi}\Big(\frac{1}{10}\alpha_1-\frac{1}{2}\alpha_2\Big)C^{1111\dagger}_{\bar{d}uLLD},\\
\frac{d}{d\ln\mu}C^{11\dagger}_{LeHD} &= \frac{1}{4\pi}\Big(-\frac{9}{10}\alpha_1+6\alpha_\lambda+9\alpha_t\Big)C^{11\dagger}_{LeHD},\\
\frac{d}{d\ln\mu}C^{1111\dagger}_{\bar{d}LueH} &= \frac{1}{4\pi}\Big(-\frac{69}{20}\alpha_1-\frac{9}{4}\alpha_2+3\alpha_t\Big)C^{1111\dagger}_{\bar{d}LueH},\\
\frac{d}{d\ln\mu}C^{1111\dagger}_{\bar{Q}uLLH} &= \frac{1}{4\pi}\Big(\frac{1}{20}\alpha_1-\frac{3}{4}\alpha_2-8\alpha_3+3\alpha_t\Big)C^{1111\dagger}_{\bar{Q}uLLH},\\
\frac{d}{d\ln\mu}C^{1111\dagger}_{\bar{d}LQLH1} &= \frac{1}{4\pi}\Big(\frac{13}{20}\alpha_1+\frac{9}{4}\alpha_2-8\alpha_3+3\alpha_t\Big)C^{1111\dagger}_{\bar{d}LQLH1}+\frac{1}{4\pi}\big(6\alpha_2\big)C^{1111\dagger}_{\bar{d}LQLH2},\\
\frac{d}{d\ln\mu}C^{1111\dagger}_{\bar{d}LQLH2} &= \frac{1}{4\pi}\Big(-\frac{121}{60}\alpha_1-\frac{15}{4}\alpha_2+\frac{8}{3}\alpha_3+3\alpha_t\Big)C^{1111\dagger}_{\bar{d}LQLH2}+\frac{1}{4\pi}\Big(-\frac{4}{3}\alpha_1+\frac{16}{3}\alpha_3\Big)C^{1111\dagger}_{\bar{d}LQLH1},\\
\frac{d}{d\ln\mu}C^{11\dagger}_{LHD2} &= \frac{1}{4\pi}\Big(\frac{12}{5}\alpha_1+3\alpha_2+4\alpha_\lambda+6\alpha_t\Big)C^{11\dagger}_{LHD2}+\frac{1}{4\pi}\big(-8\alpha_2\big)C^{11\dagger}_{LHD1},\\
\frac{d}{d\ln\mu}C^{11\dagger}_{LHW} &= \frac{1}{4\pi}\Big(-\frac{6}{5}\alpha_1+\frac{13}{2}\alpha_2+4\alpha_\lambda+6\alpha_t\Big)C^{11\dagger}_{LHW}+\frac{1}{4\pi}\Big(\frac{5}{8}\alpha_2\Big)C^{11\dagger}_{LHD1}+\frac{1}{4\pi}\Big(-\frac{9}{80}\alpha_1+\frac{11}{16}\alpha_2\Big)C^{11\dagger}_{LHD2}.
\end{aligned}
$$

RGE effects on $0\nu\beta\beta$: numbers



Liao-Ma 2019

Bounds on Λ_{NP}:
~20 – 200 TeV

Conclusions

- Neutrinos as a Majorana fermion are theoretically well motivated.
- Experimental and theoretical investigations on LNV are very active.
- EFT offers a proper means to link pheno from low to high energy processes.
- Theoretical analysis on $0\nu\beta\beta$ remains to be refined on several aspects:

  RGE, matching between EFTs, short-distance vs long-distance physics in hadronic and nuclear physics.
- Theoretical study on LNV meson and τ decays in EFT has just started, much to be done.