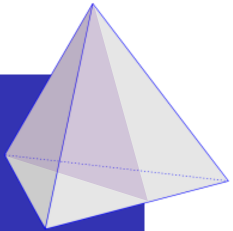
FLASY2019: 8th Workshop on Flavour Symmetries and Consequences in Accelerators and Cosmology

USTC Hefei, 24-26 July 2019



S_4 modular symmetry and lepton masses and mixing

in collaboration with S.T. Petcov [1806.03203, NPB 939 (2019) 292], A.V. Titov and P.P. Novichkov [1811.04933, JHEP 1904 (2019) 005]

Déjà vu

João Penedo
(CFTP, Lisbon)

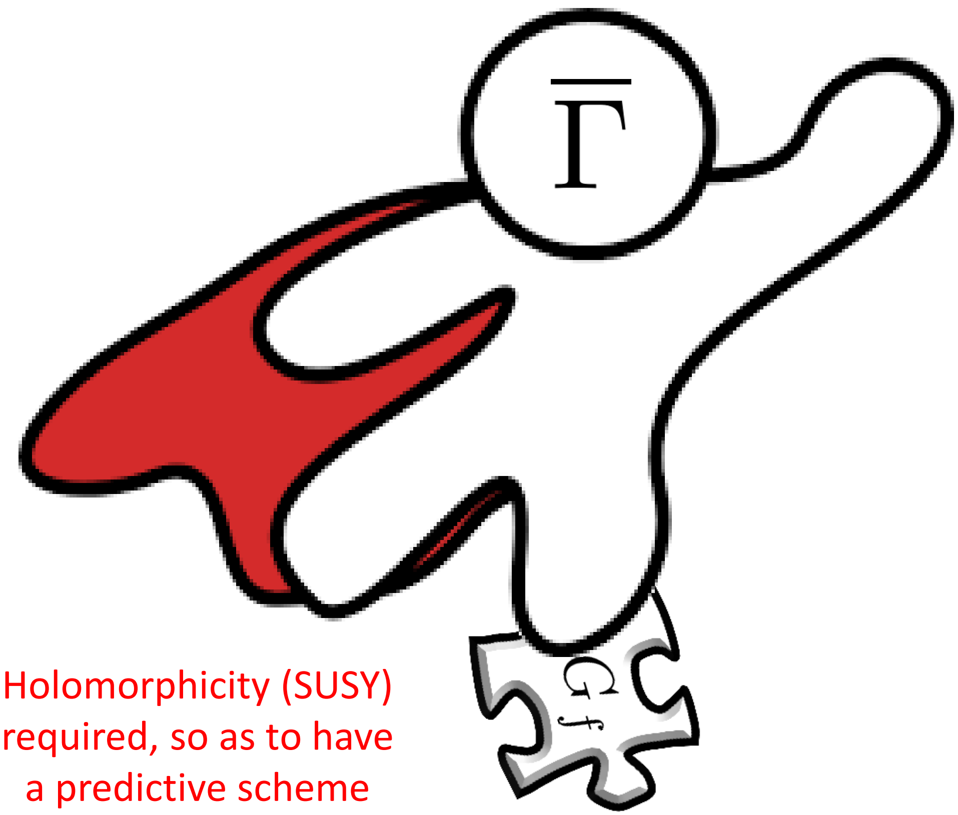
Plan

- The modular symmetry framework (recall)
- Modular S_4 in some detail
- Model building and predictions

The modular symmetry framework

(recall)

recall: Modular symmetry as Flavour symmetry



Holomorphicity (SUSY) required, so as to have a predictive scheme

Feruglio, 1706.08749

can constrain all:

neutrino masses, mixing, Dirac and Majorana phases

recall: Modular symmetry

$$\overline{\Gamma} \simeq \mathrm{PSL}(2,\mathbb{Z})$$

$$\tau \to \frac{a\tau + b}{c\tau + d}$$

$$a,b,c,d \in \mathbb{Z} \qquad \begin{vmatrix} a & b \\ c & d \end{vmatrix} = 1$$

$$S^2 = (ST)^3 = 1 \quad \begin{cases} S: \; \tau \to -1/\tau \,, & S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \\ T: \; \tau \to \tau + 1 \,, & T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \end{cases}$$

τ is a dimensionless spurion, **parameterises all** modular sym. breaking

recall: Modular symmetry

Quotient behaves like a flavour group

$$\underbrace{\overline{\Gamma} \Big/ \overline{\Gamma}(N)}_{\Gamma_N}$$

Bottom-up approach

We will choose N & scan τ

For top-down, see e.g.:

Kobayashi et al., 1804.06644
Kobayashi, Tamba, 1811.11384
de Anda et al., 1812.05620
Baur et al., 1901.03251
Kariyazono et al., 1904.07546

$$\overline{\Gamma}(N) \equiv \left\{ \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \,\middle|\, \gamma \in \overline{\Gamma} \ \wedge \ (\gamma = \mathbb{1}) \bmod N \right\}$$

recall: Modular symmetry

Quotient behaves like a flavour group

$$\overline{\Gamma} \Big/ \overline{\Gamma}(N)$$

$$\underbrace{\overline{\Gamma} / \overline{\Gamma}(N)}_{\Gamma_N}$$

$$S^2 = (ST)^3 = T^N = 1$$

$\Gamma_2 \simeq S_3$

Kobayashi et al., 1803.10391 (+A_4)
Kobayashi et al., 1812.11072 (+A_4)
Kobayashi et al., 1906.10341
Okada, Orikasa, 1907.04716

$\Gamma_3 \simeq A_4$

Feruglio, 1706.08749
Feruglio, Criado, 1807.01125
Kobayashi et al., 1808.03012
Okada, Tanimoto, 1812.09677
Novichkov et al., 1812.11289
Nomura, Okada, 1904.03937
Okada, Tanimoto, 1905.13421
Nomura, Okada, 1906.03927

$\Gamma_4 \simeq S_4$

JP, Petcov, 1806.11040
Novichkov et al., 1811.04933
Kobayashi et al., 1907.09141

$\Gamma_5 \simeq A_5$

Novichkov et al., 1812.02158
Ding et al., 1903.12588

Modular forms: the stars of the show

Transformation of superfields:

$$\psi \to (c\tau + d)^{-k_\psi} \underbrace{\rho_{\mathbf{r}}(\gamma)}_{\Gamma_N,\ \gamma \in \overline{\Gamma}} \psi$$

our theory is invariant under the full modular group!

Modular forms: the stars of the show

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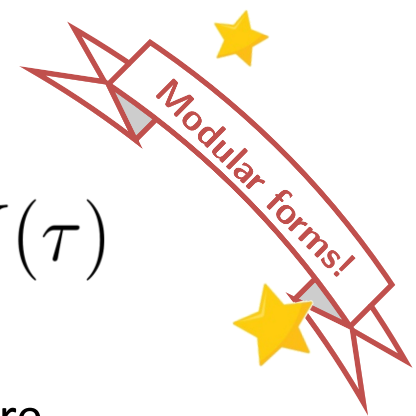
<u>Invariance of superpotential requires functions:</u>



$$Y(\tau) \to (c\tau + d)^{k_Y} \, \rho_{\mathbf{r}_Y}(\gamma) \, Y(\tau)$$

Play the role of flavons, but structures are **completely fixed** given the modulus VEV

Modular forms: the stars of the show

Transformation of superfields:

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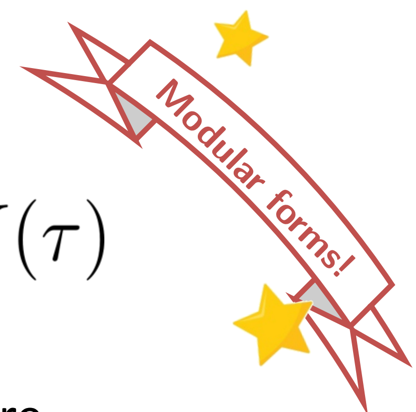
Invariance of superpotential requires functions:

$$Y(\tau) \xrightarrow{\text{Why?}} (c\tau + d)^{k_Y} \rho_{\mathbf{r}_Y}(\gamma)\, Y(\tau)$$



Play the role of flavons, but structures are **completely fixed** given the modulus VEV

Modular-invariant SUSY actions

Ferrara et al, '89

$$W(\psi;\tau) = \sum_{n} \sum_{\{i_1,\ldots,i_n\}} \sum_{s} g_{i_1\ldots i_n,s} \left(Y_{i_1\ldots i_n,s}(\tau)\, \psi_{i_1}\ldots\psi_{i_n}\right)_{\mathbf{1},s}$$

$$\mathcal{S} = \int d^4x\, d^2\theta\, d^2\overline{\theta}\; K(\psi,\overline{\psi};\tau,\overline{\tau}) + \int d^4x\, d^2\theta\; W(\psi;\tau) + \text{h.c.}$$

Modular-invariant SUSY actions

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$$\begin{cases} \tau \to \gamma\tau = \dfrac{a\tau+b}{c\tau+d} \\ \psi_i \to (c\tau+d)^{-k_i}\,\rho_i(\gamma)\,\psi_i \\ Y(\tau) \to ? \end{cases} \qquad \text{with } \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Feruglio, 1706.08749

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Modular-invariant SUSY actions

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$Y(\tau)$ are **modular forms** obeying $\begin{cases} k_Y = k_{i_1} + \ldots + k_{i_n} \\ \rho_Y \otimes \rho_{i_1} \otimes \ldots \otimes \rho_{i_1} \supset \mathbf{1} \end{cases}$

Modular-invariant SUSY actions

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weights (k_i, k_Y)

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$$\begin{cases} \tau \to \gamma\tau = \dfrac{a\tau + b}{c\tau + d} \\ \psi_i \to (c\tau + d)^{-k_i} \rho_i(\gamma)\, \psi_i \\ Y(\tau) \to Y(\gamma\tau) = (c\tau + d)^{k_Y} \rho_Y(\gamma)\, Y(\tau) \end{cases} \quad \text{with } \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

weights (k_i, k_Y)

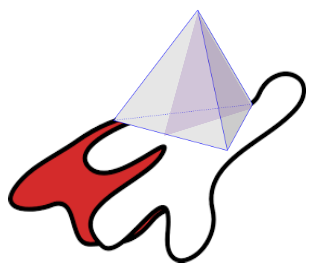
k_Y even, positive

$Y(\tau)$ are **modular forms** obeying
$$\begin{cases} k_Y = k_{i_1} + \ldots + k_{i_n} \\ \rho_Y \otimes \rho_{i_1} \otimes \ldots \otimes \rho_{i_1} \supset \mathbf{1} \end{cases}$$

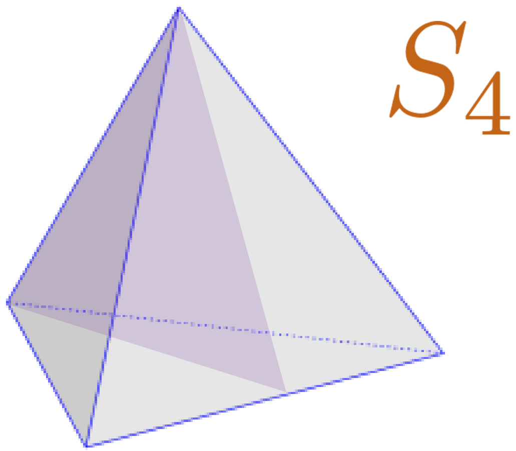
How to build them?

Modular S_4

(in some detail)



S_4 as the flavour group



rotations + **reflection**
$\Leftrightarrow$ **permutation of vertices**

Group presentations

3 generators, S, T, and U, obeying:

$$\tilde{S}^2 = \tilde{T}^3 = \tilde{U}^2 = (\tilde{S}\tilde{T})^3$$
$$= (\tilde{S}\tilde{U})^2 = (\tilde{T}\tilde{U})^2 = (\tilde{S}\tilde{T}\tilde{U})^4 = 1$$

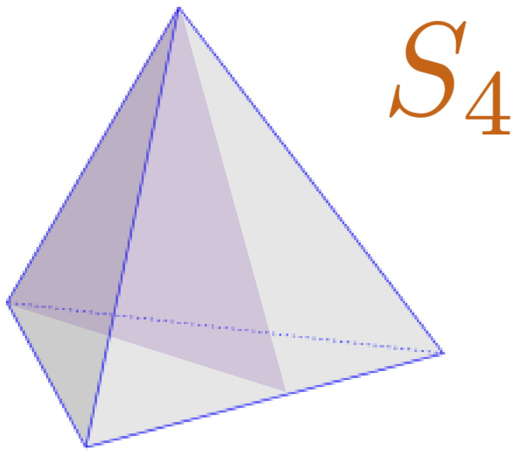
vs.

2 generators, S and T, obeying:

$$S^2 = (ST)^3 = T^4 = 1$$

For reviews on the discrete sym. approach to flavour, see e.g.: Altarelli, Feruglio, 1002.0211; Ishimori et al, 1003.3552; King et al, 1402.4271; Petcov, 1711.10806.

S_4 as the flavour group



rotations + **reflection**
$\Leftrightarrow$ **permutation of vertices**

Group presentations

$$\begin{cases} S = \tilde{S}\tilde{T}\tilde{S}\tilde{U}\,, \\ T = \tilde{T}^2\tilde{S}\tilde{T}\tilde{U}\,, \end{cases}$$

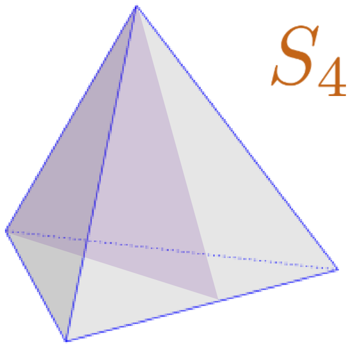
$$\Leftrightarrow$$

$$\begin{cases} \tilde{S} = T^2\,, \\ \tilde{T} = ST\,, \\ \tilde{U} = ST^2ST^3\,. \end{cases}$$

For reviews on the discrete sym. approach to flavour, see e.g.: Altarelli, Feruglio, 1002.0211; Ishimori et al, 1003.3552; King et al, 1402.4271; Petcov, 1711.10806.

S_4 in the symmetric basis (remember?)

1905.11970



$$S^2 = (ST)^3 = T^4 = 1$$

$$\mathbf{1}: \quad \rho(S) = 1, \quad \rho(T) = 1\,,$$

$$\mathbf{1}': \quad \rho(S) = -1, \quad \rho(T) = -1\,,$$

$$\mathbf{2}: \quad \rho(S) = \frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}, \quad \rho(T) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

$$\mathbf{3}: \quad \rho(S) = -\frac{1}{2}\begin{pmatrix} 0 & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & -1 & 1 \\ \sqrt{2} & 1 & -1 \end{pmatrix}, \quad \rho(T) = -\begin{pmatrix} 1 & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & -i \end{pmatrix},$$

$$\mathbf{3}': \quad \rho(S) = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & -1 & 1 \\ \sqrt{2} & 1 & -1 \end{pmatrix}, \quad \rho(T) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & -i \end{pmatrix}.$$

see, e.g., Altarelli, Feruglio, Merlo, 0903.1940

S_4 in the symmetric basis (remember?)

1905.11970

S_4

$S^2 = (ST)^3 = T^4 = 1$

“And now for something completely different”

see, e.g., Altarelli, Feruglio, Merlo, 0903.1940

The Dedekind eta function

Useful to build the sought-out modular forms

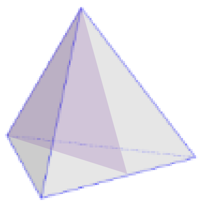
$$\eta(\tau) \equiv q^{1/24} \prod_{k=1}^{\infty} (1 - q^k), \quad \text{with } q = e^{2\pi i \tau}$$

why this function?

$S : \tau \to -1/\tau$ $\Rightarrow$ $\eta(-1/\tau) = \sqrt{-i\tau}\,\eta(\tau)$

$T : \tau \to \tau + 1$ $\Rightarrow$ $\eta(\tau + 1) = e^{i\pi/12}\,\eta(\tau)$

Action of S_4 generators

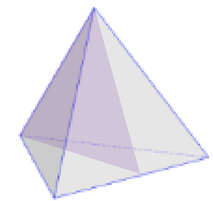


$$S^2 = (ST)^3 = T^4 = 1$$

Set of 'seed' functions

$$\{\eta_i\} = \left\{\eta\left(\tau + \frac{1}{2}\right), \eta\left(4\tau\right), \eta\left(\frac{\tau}{4}\right), \eta\left(\frac{\tau+1}{4}\right), \eta\left(\frac{\tau+2}{4}\right), \eta\left(\frac{\tau+3}{4}\right)\right\}$$

Action of S_4 generators



$$S^2 = (ST)^3 = T^4 = 1$$

Set of 'seed' functions

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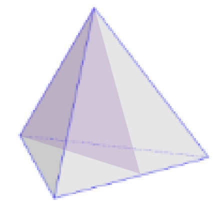
up to multiplicative factors

Action of S_4 generators



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Set of 'seed' functions

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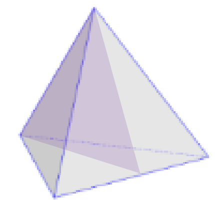
$$\eta\left(\tau + \frac{1}{2}\right) \xrightarrow{S} \eta\left(-\frac{1}{\tau} + \frac{1}{2}\right) = (\ldots) \times \eta\left(\frac{\tau+2}{4}\right)$$

up to multiplicative factors

Action of S_4 generators



$$S^2 = (ST)^3 = \underline{T^4 = 1}$$

<u>Set of 'seed' functions</u>

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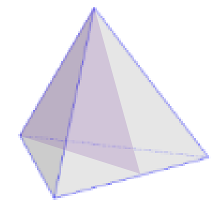
T :

up to multiplicative factors

Action of S_4 generators



$$S^2 = (ST)^3 = T^4 = 1$$

Set of 'seed' functions

S :

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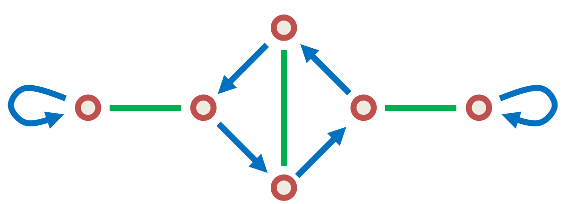
T :



up to multiplicative factors

Building lowest-weight forms

$$Y(a_1, \ldots, a_6|\tau) \equiv \frac{d}{d\tau}\left(\sum_{i=1}^{6} a_i \log \eta_i(\tau)\right)$$

$$\sum_i a_i = 0$$

$$S: \; Y(a_1, \ldots, a_6|\tau) \to Y(a_1, a_2, a_3, a_4, a_5, a_6|-1/\tau) = \tau^2 Y(a_5, a_3, a_2, a_6, a_1, a_4|\tau)$$

$$T: \; Y(a_1, \ldots, a_6|\tau) \to Y(a_1, a_2, a_3, a_4, a_5, a_6|\tau + 1) = Y(a_1, a_2, a_6, a_3, a_4, a_5|\tau)$$

Building lowest-weight forms

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$$\sum_i a_i = 0$$

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$$T: \; Y(a_1,\ldots,a_6|\tau) \to Y(a_1,a_2,a_3,a_4,a_5,a_6|\tau+1) = Y(a_1,a_2,a_6,a_3,a_4,a_5|\tau)$$

$$Y(\tau) \to (c\tau+d)^{k_Y}\,\rho_Y(\gamma)\,Y(\tau) \implies$$

Modular forms of weight 2

Correct dimension! (5)

Building lowest-weight forms: multiplets

$$Y(a_1,\ldots,a_6|\tau) \equiv \frac{d}{d\tau}\left(\sum_{i=1}^{6} a_i \log \eta_i(\tau)\right)$$

$$\boxed{\sum_i a_i = 0}$$

Lowest weight forms arrange into:

$$Y_{\mathbf{2}}(\tau) = i\begin{pmatrix} Y(1,1,-1/2,-1/2,-1/2,-1/2|\tau) \\ Y(0,0,\sqrt{3}/2,-\sqrt{3}/2,\sqrt{3}/2,-\sqrt{3}/2|\tau) \end{pmatrix} \equiv \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$$ **doublet 2**

$$Y_{\mathbf{3'}}(\tau) = i\begin{pmatrix} Y(1,-1,0,0,0,0|\tau) \\ Y(0,0,-1/\sqrt{2},i/\sqrt{2},1/\sqrt{2},-i/\sqrt{2}|\tau) \\ Y(0,0,-1/\sqrt{2},-i/\sqrt{2},1/\sqrt{2},i/\sqrt{2}|\tau) \end{pmatrix} \equiv \begin{pmatrix} Y_3 \\ Y_4 \\ Y_5 \end{pmatrix}$$ **triplet 3'**

Building lowest-weight forms: multiplets

$$Y(a_1,\ldots,a_6|\tau) \equiv \frac{d}{d\tau}\left(\sum_{i=1}^{6} a_i \log \eta_i(\tau)\right)$$

$$\boxed{\sum_i a_i = 0}$$

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Tensor products generate higher weight forms

Building higher-weight forms

At weight 4

$$Y_{\mathbf{1}}^{(4)} = Y_1 Y_2 \sim \mathbf{1}$$

$$Y_{\mathbf{2}}^{(4)} = (Y_2^2,\, Y_1^2)^T \sim \mathbf{2}$$

$$Y_{\mathbf{3}}^{(4)} = (Y_1 Y_4 - Y_2 Y_5,\, Y_1 Y_5 - Y_2 Y_3,\, Y_1 Y_3 - Y_2 Y_4)^T \sim \mathbf{3}$$

$$Y_{\mathbf{3'}}^{(4)} = (Y_1 Y_4 + Y_2 Y_5,\, Y_1 Y_5 + Y_2 Y_3,\, Y_1 Y_3 + Y_2 Y_4)^T \sim \mathbf{3'}$$

$\mathbf{1'}$ arises at weight 6

Constraints

guarantee correct dimensionality

$$\frac{1}{3}\left(Y_3^2 + 2Y_4Y_5\right) = Y_1Y_2\,, \qquad -\frac{1}{\sqrt{3}}\left(Y_3^2 - Y_4Y_5\right) = Y_1Y_4 - Y_2Y_5\,,$$
$$\frac{1}{3}\left(Y_4^2 + 2Y_3Y_5\right) = Y_2^2\,, \qquad -\frac{1}{\sqrt{3}}\left(Y_5^2 - Y_3Y_4\right) = Y_1Y_5 - Y_2Y_3\,,$$
$$\frac{1}{3}\left(Y_5^2 + 2Y_3Y_4\right) = Y_1^2\,, \qquad -\frac{1}{\sqrt{3}}\left(Y_4^2 - Y_3Y_5\right) = Y_1Y_3 - Y_2Y_4\,.$$

Generators of modular forms: *q*-expansions

$$Y_{\mathbf{2}}(\tau) \equiv \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}$$ **doublet 2**

$$Y_{\mathbf{3'}}(\tau) \equiv \begin{pmatrix} Y_3(\tau) \\ Y_4(\tau) \\ Y_5(\tau) \end{pmatrix}$$ **triplet 3'**

$$\begin{aligned}
-\frac{8}{3\pi} Y_1(\tau) &= 1 + 24\, q_4^4 + 24\, q_4^8 + 96\, q_4^{12} + 24\, q_4^{16} + \ldots \\
-\frac{1}{3\sqrt{3}\pi} Y_2(\tau) &= q_4^2 + 4\, q_4^6 + 6\, q_4^{10} + 8\, q_4^{14} + 13\, q_4^{18} + \ldots \\
\frac{4}{\pi} Y_3(\tau) &= 1 - 8\, q_4^4 + 24\, q_4^8 - 32\, q_4^{12} + 24\, q_4^{16} + \ldots \\
-\frac{1}{\sqrt{2}\pi} Y_4(\tau) &= q_4 + 6\, q_4^5 + 13\, q_4^9 + 14\, q_4^{13} + 18\, q_4^{17} + \ldots \\
-\frac{1}{4\sqrt{2}\pi} Y_5(\tau) &= q_4^3 + 2\, q_4^7 + 3\, q_4^{11} + 6\, q_4^{15} + 5\, q_4^{19} + \ldots
\end{aligned}$$

Extremely constrained functions, with nice properties $\qquad q_4 = e^{\pi i \tau/2}$

Model building and predictions

Guidelines for model building

Using minimality as a guiding principle...

- **No flavons** are introduced,
- Higgs multiplets transform trivially,
- Lepton doublets transform as an S_4 **triplet**,
- Lepton singlets transform as S_4 **singlets**, and
- Lowest possible weights are chosen such that all charged leptons are massive

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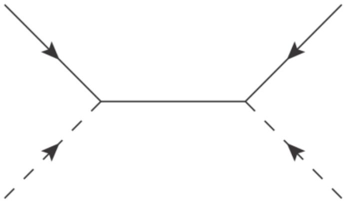

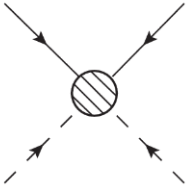

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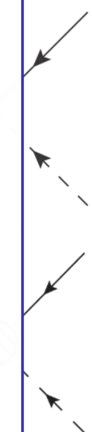


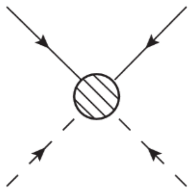
- **RGEs** need to be considered, depend on tan β
- **SUSY-breaking** corrections can be made negligible via separation of scales (power counting argument)

Feruglio and Criado, 1807.01125

Lepton masses and mixing from the Weinberg operator

JP, Petcov, 1806.03203

$$W = \sum_i \alpha_i \left[E_i^c\, L\, H_d\, f_i(Y)\right]_{\mathbf{1}} + \frac{g}{\Lambda}\left[L\, H_u\, L\, H_u\, f_W(Y)\right]_{\mathbf{1}}$$



- Study models systematically by increasing weight of L
- Minimal working model ($k_L = 2$) has **7** real parameters, predicting 9 observables in the neutrino sector
- Correlations between observables are expected

Lepton masses and mixing from the Weinberg operator: a benchmark

	H_u	H_d	L	E_1^c	E_2^c	E_3^c
ρ_i	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$ $\mathbf{3'}$	$\mathbf{1'}$ $\mathbf{1}$	$\mathbf{1}$ $\mathbf{1'}$	$\mathbf{1'}$ $\mathbf{1}$
k_i	0	0	2	0	2	2

NO spectrum

$$\frac{m_e}{m_\mu} \simeq 0.0048\,,\ \sin^2\theta_{12} \simeq 0.292\,,\quad \delta \simeq 1.64\pi\,,$$

$$\frac{m_\mu}{m_\tau} \simeq 0.0560\,,\ \sin^2\theta_{13} \simeq 0.021\,,\ \alpha_{21} \simeq 0.10\pi\,,$$

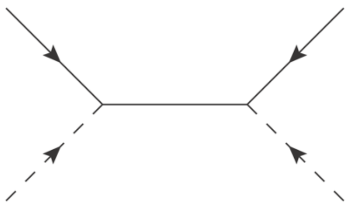
$$r \simeq 0.0298\,,\ \sin^2\theta_{23} \simeq 0.493\,,\ \alpha_{31} \simeq 1.10\pi\,.$$

$$|\langle m \rangle| \simeq 0.042\ \text{eV}$$

Lepton masses and mixing from Seesaw type I

Novichkov, JP, Petcov, Titov, 1811.04933

$$W = \sum_i \alpha_i \left[E_i^c\, L\, f_i(Y)\right]_{\mathbf{1}} H_d + g\left[N^c\, L\, f_N(Y)\right]_{\mathbf{1}} H_u + \Lambda\left[N^c\, N^c\, f_M(Y)\right]_{\mathbf{1}}$$



- UV completion, more predictive
- Minimal working models have **5** parameters (vs. 9 observables)
- Parameter space fully scanned and **correlations** studied in detail
- In viable models, heavy singlets can be integrated out

$$\Lambda\left[N^c\,N^c\,f_M(Y)\right]_{\mathbf{1}}$$

$$k_{N^c} = 0$$

$$2\,\Lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\Lambda \left[N^c N^c f_M(Y) \right]_{\mathbf{1}}$$

$$k_{N^c} = 1$$

$$2\,\Lambda \begin{pmatrix} 0 & Y_1 & Y_2 \\ Y_1 & Y_2 & 0 \\ Y_2 & 0 & Y_1 \end{pmatrix}$$

$$k_{N^c} = 2$$

$$2\,\Lambda \left[Y_1 Y_2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \frac{\Lambda'}{\Lambda} \begin{pmatrix} 0 & Y_2^2 & Y_1^2 \\ Y_2^2 & Y_1^2 & 0 \\ Y_1^2 & 0 & Y_2^2 \end{pmatrix} \right.$$
$$\left. + \frac{\Lambda''}{\Lambda} \begin{pmatrix} 2\,(Y_1 Y_4 - Y_2 Y_5) & Y_2 Y_4 - Y_1 Y_3 & Y_2 Y_3 - Y_1 Y_5 \\ Y_2 Y_4 - Y_1 Y_3 & 2\,(Y_1 Y_5 - Y_2 Y_3) & Y_2 Y_5 - Y_1 Y_4 \\ Y_2 Y_3 - Y_1 Y_5 & Y_2 Y_5 - Y_1 Y_4 & 2\,(Y_1 Y_3 - Y_2 Y_4) \end{pmatrix} \right]$$

$$Y_i = Y_i(\tau)$$

$$\Lambda\left[N^c N^c f_M(Y)\right]_{\mathbf{1}}$$

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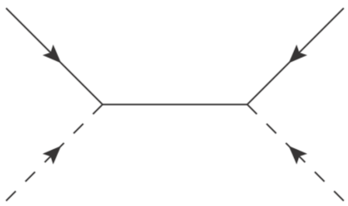

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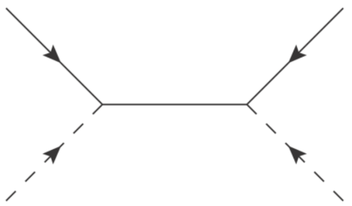
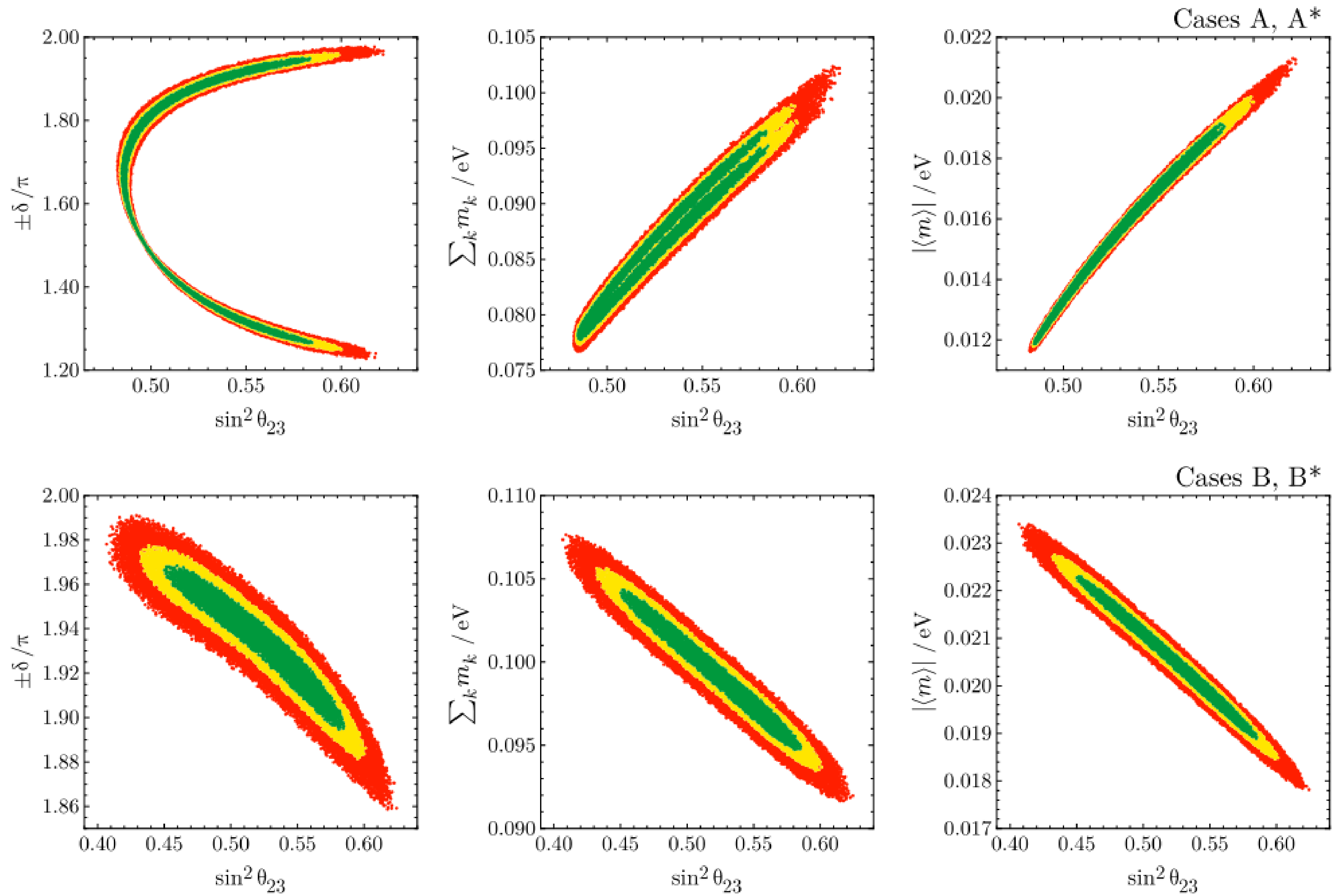
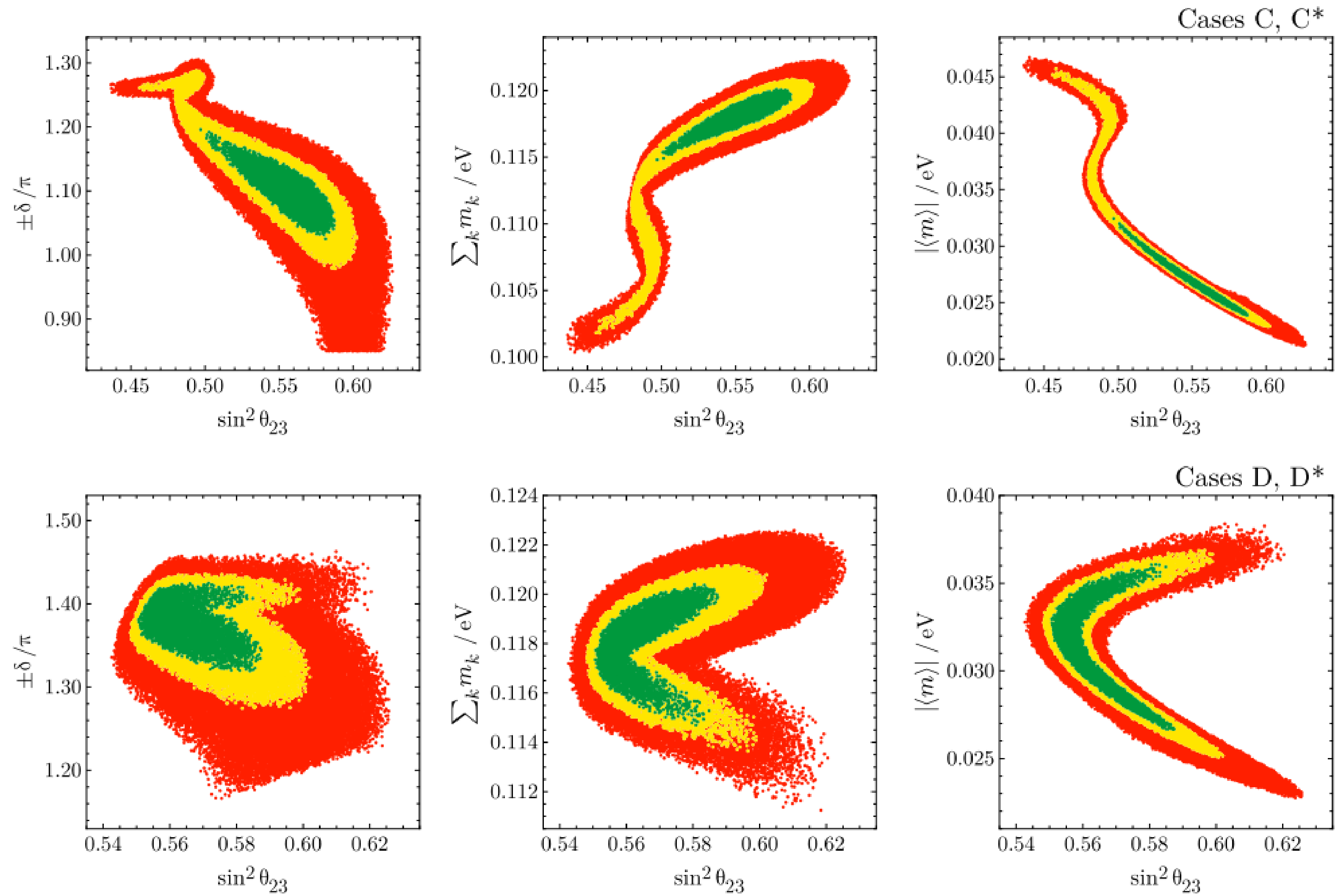


- UV completion, more predictive
- Minimal working models have **5** parameters (vs. 9 observables) — **4** if gCP imposed
- Parameter space fully scanned and **correlations** studied in detail
- In viable models, heavy singlets can be integrated out

Modular Seesaw correlations

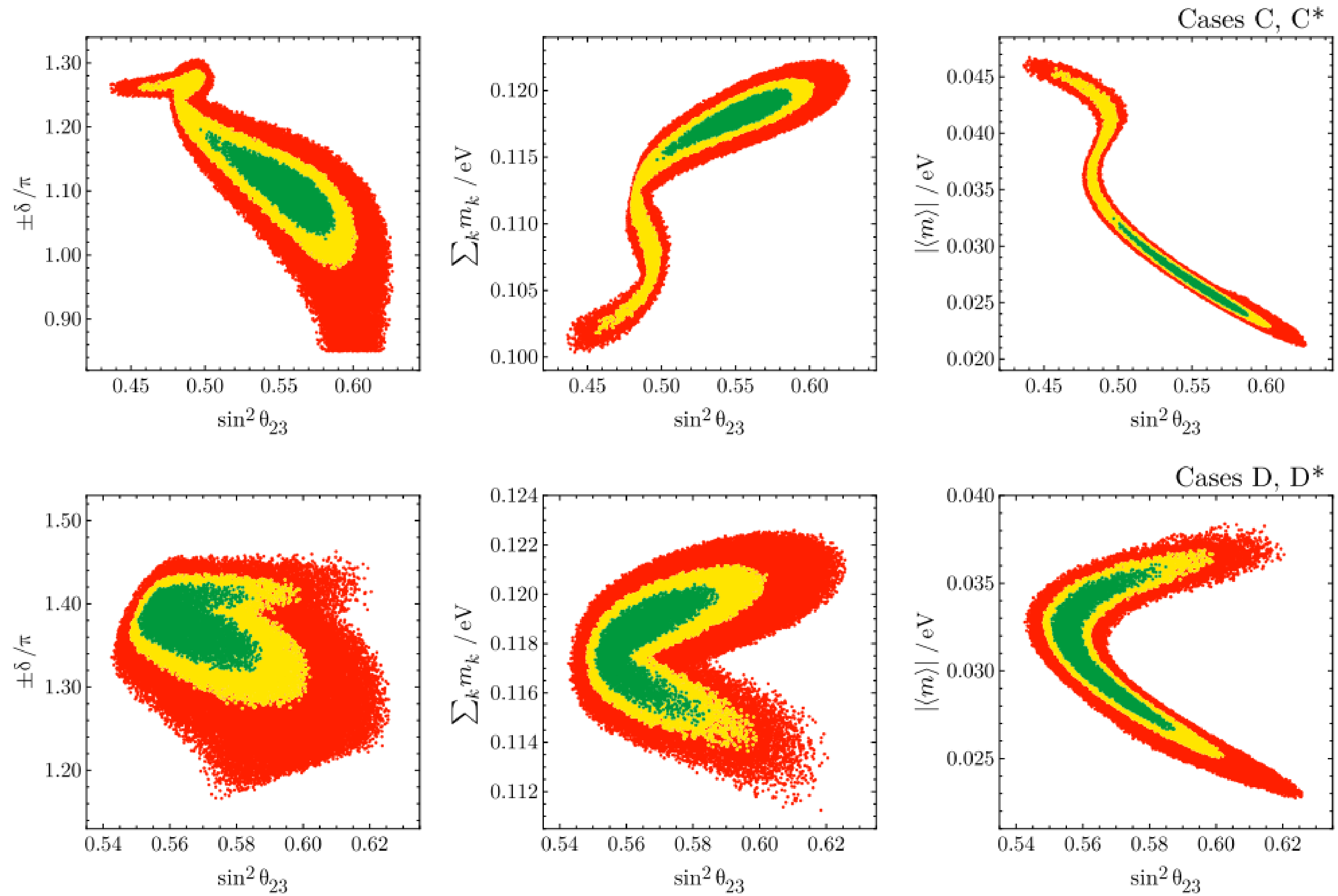


Modular Seesaw correlations

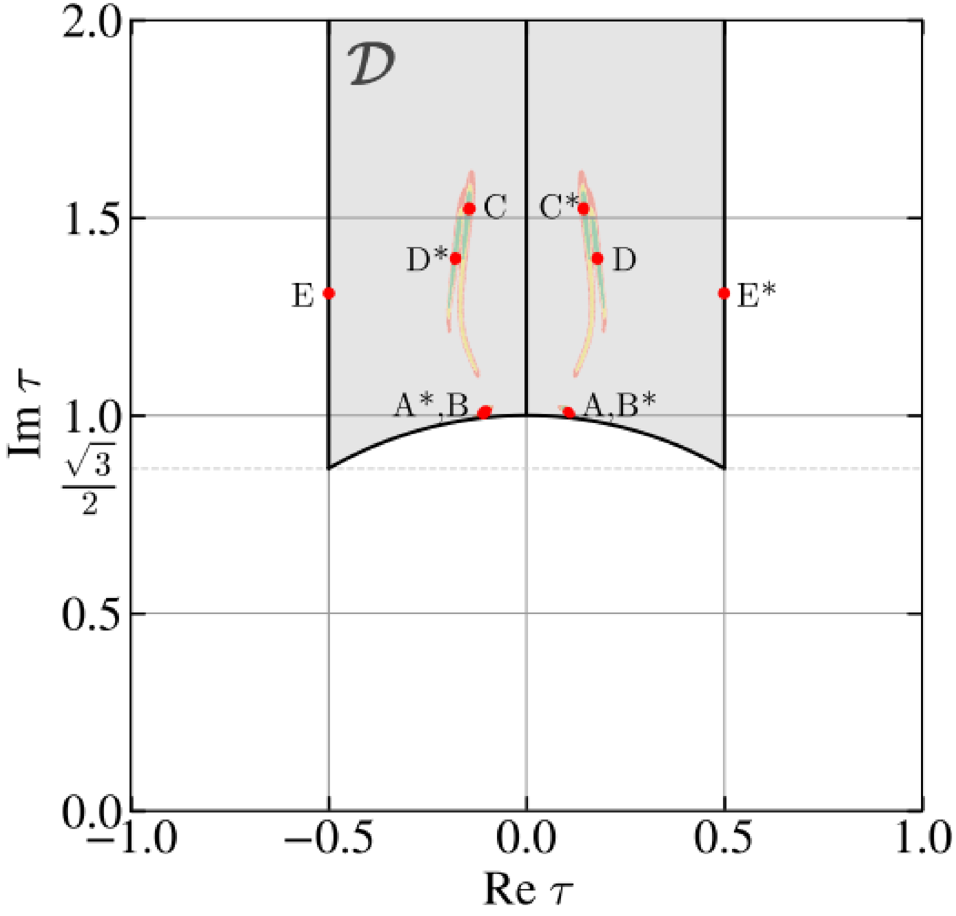


Modular Seesaw correlations

$|\langle m \rangle|_{\beta\beta} > 6$ meV
$\sum_i m_i > 0.07$ eV



Viable τ regions from Seesaw



Novichkov, JP, Petcov, Titov, 1811.04933

NO cases close to top-down minima of Cvetic et al., Nucl. Phys. B361 (1991) 194

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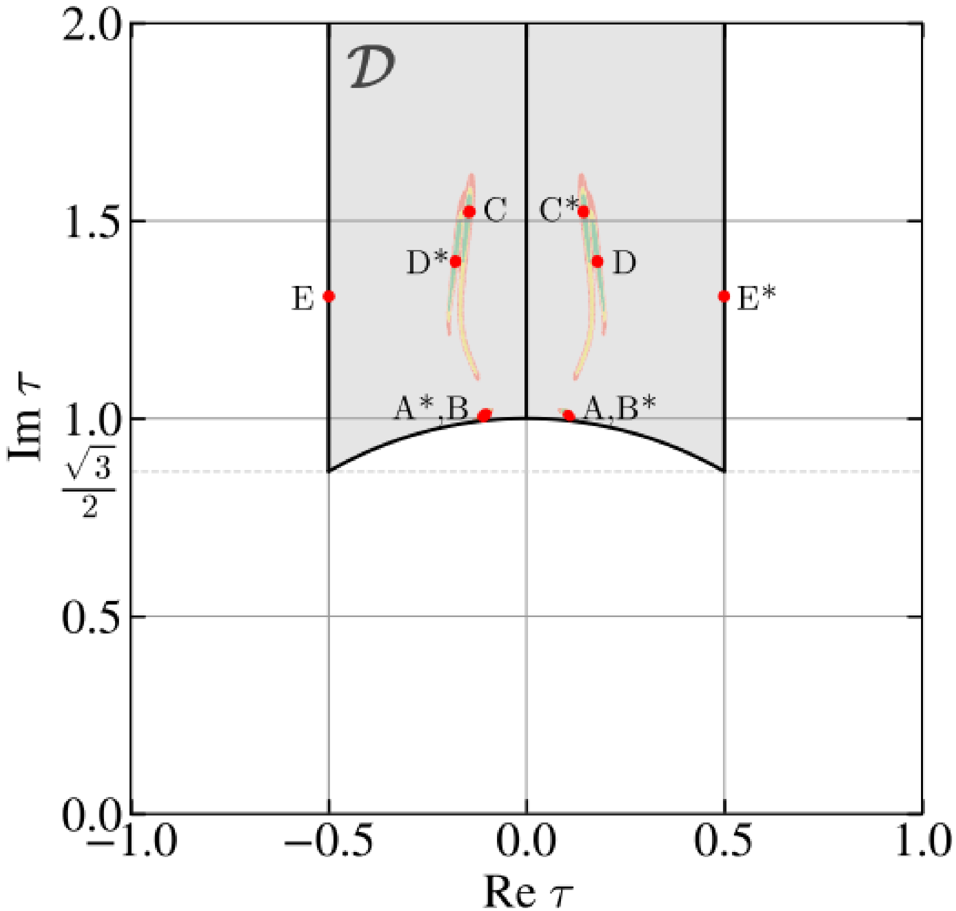

Novichkov, JP, Petcov, Titov, 1811.04933

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Summary and Conclusions (1/2)

- (**Modular symmetry** may strongly constrain masses and mixing.)
- Fields carrying a non-trivial modular weight transform with a scale factor in addition to the usual unitary rotation.
- To build invariants one needs **modular forms.**

$$\mathbf{5} = \mathbf{2} + \mathbf{3'}$$

Summary and Conclusions (2/2)

- We have shown how lepton mixing and Dirac and Majorana CPV phases can be predicted in **minimal modular S4 models**.
- The existence of successful benchmarks and predictive setups warrants further exploration of such an approach.

Thank you / 谢谢

Backup slides

Kähler potential

The minimal choice; a definition of the setup, at this point:

$$K(\chi_i, \overline{\chi}_i; \tau, \overline{\tau}) = -h\,\Lambda_0^2 \log(-i(\tau - \overline{\tau})) + \sum_i \frac{|\chi_i|^2}{(-i(\tau - \overline{\tau}))^{k_i}}$$

$$\mathcal{L} \supset \sum_i \frac{\partial_\mu \overline{\chi}_i\, \partial^\mu \chi_i}{(2\,\mathrm{Im}\langle\tau\rangle)^{k_i}}$$

Under a modular transformation, **invariant up to a Kähler transformation:**

$$K(\chi_i, \overline{\chi}_i; \tau, \overline{\tau}) \to K(\chi_i, \overline{\chi}_i; \tau, \overline{\tau}) + f(\chi_i; \tau) + f(\overline{\chi}_i; \overline{\tau})$$

There may exist potentially dangerous
corrections to the Kähler, to be studied

Mass matrices in Weinberg operator case

$k_L = 1$ ✗

$$M_e^\dagger = v_d \begin{pmatrix} \alpha\, Y_3 & \alpha\, Y_5 & \alpha\, Y_4 \\ \beta\,(Y_1 Y_4 - Y_2 Y_5) & \beta\,(Y_1 Y_3 - Y_2 Y_4) & \beta\,(Y_1 Y_5 - Y_2 Y_3) \\ \gamma\,(Y_1 Y_4 + Y_2 Y_5) & \gamma\,(Y_1 Y_3 + Y_2 Y_4) & \gamma\,(Y_1 Y_5 + Y_2 Y_3) \end{pmatrix}$$

$$M_\nu = \frac{2 g_1 v_u^2}{\Lambda} \begin{pmatrix} 0 & Y_2 & Y_1 \\ Y_2 & Y_1 & 0 \\ Y_1 & 0 & Y_2 \end{pmatrix}$$

$$Y_i \equiv Y_i(\tau)$$

Mass matrices in Weinberg operator case

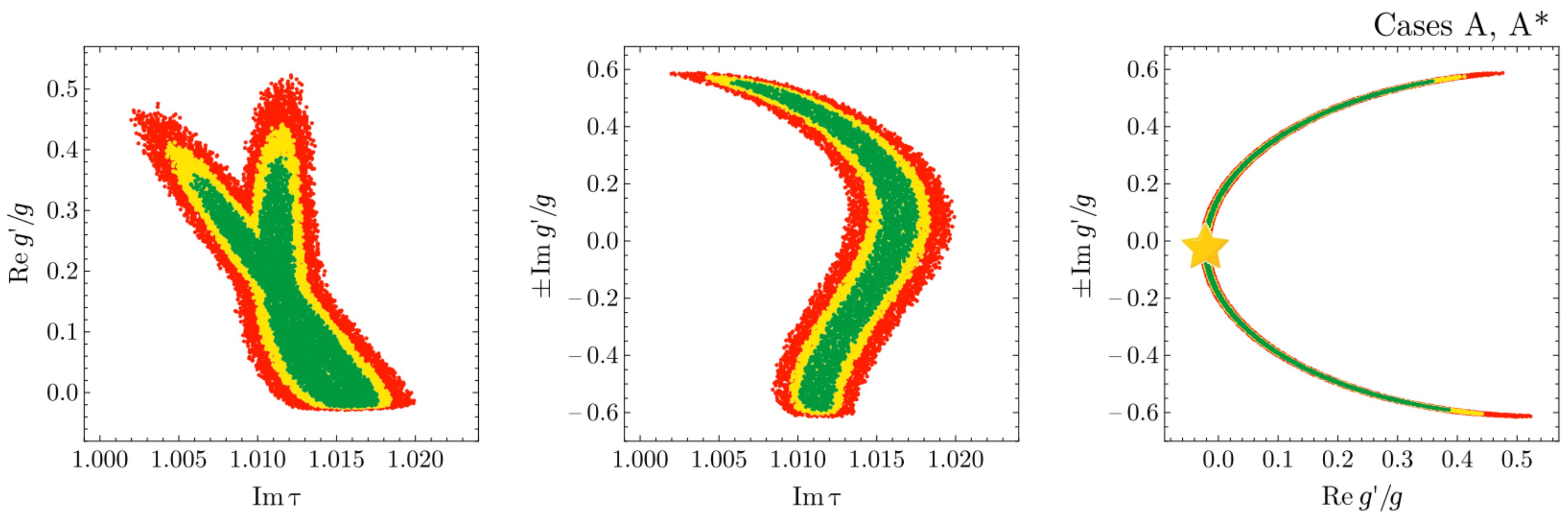
$k_L = 2$ ✓

$$M_e^\dagger = v_d \begin{pmatrix} \alpha\, Y_3 & \alpha\, Y_5 & \alpha\, Y_4 \\ \beta\,(Y_1 Y_4 - Y_2 Y_5) & \beta\,(Y_1 Y_3 - Y_2 Y_4) & \beta\,(Y_1 Y_5 - Y_2 Y_3) \\ \gamma\,(Y_1 Y_4 + Y_2 Y_5) & \gamma\,(Y_1 Y_3 + Y_2 Y_4) & \gamma\,(Y_1 Y_5 + Y_2 Y_3) \end{pmatrix}$$

$$M_\nu = \frac{2 g' v_u^2}{\Lambda} \left[\begin{pmatrix} (g/g') Y_1 Y_2 & Y_2^2 & Y_1^2 \\ Y_2^2 & Y_1^2 & (g/g') Y_1 Y_2 \\ Y_1^2 & (g/g') Y_1 Y_2 & Y_2^2 \end{pmatrix} + \frac{1}{2} \frac{g''}{g'} \begin{pmatrix} 2(Y_1 Y_4 - Y_2 Y_5) & -(Y_1 Y_3 - Y_2 Y_4) & -(Y_1 Y_5 - Y_2 Y_3) \\ -(Y_1 Y_3 - Y_2 Y_4) & 2(Y_1 Y_5 - Y_2 Y_3) & -(Y_1 Y_4 - Y_2 Y_5) \\ -(Y_1 Y_5 - Y_2 Y_3) & -(Y_1 Y_4 - Y_2 Y_5) & 2(Y_1 Y_3 - Y_2 Y_4) \end{pmatrix} \right]$$

$$Y_i \equiv Y_i(\tau)$$

Correlations between parameters



see Novichkov, JP, Petcov, Titov, 1811.04933