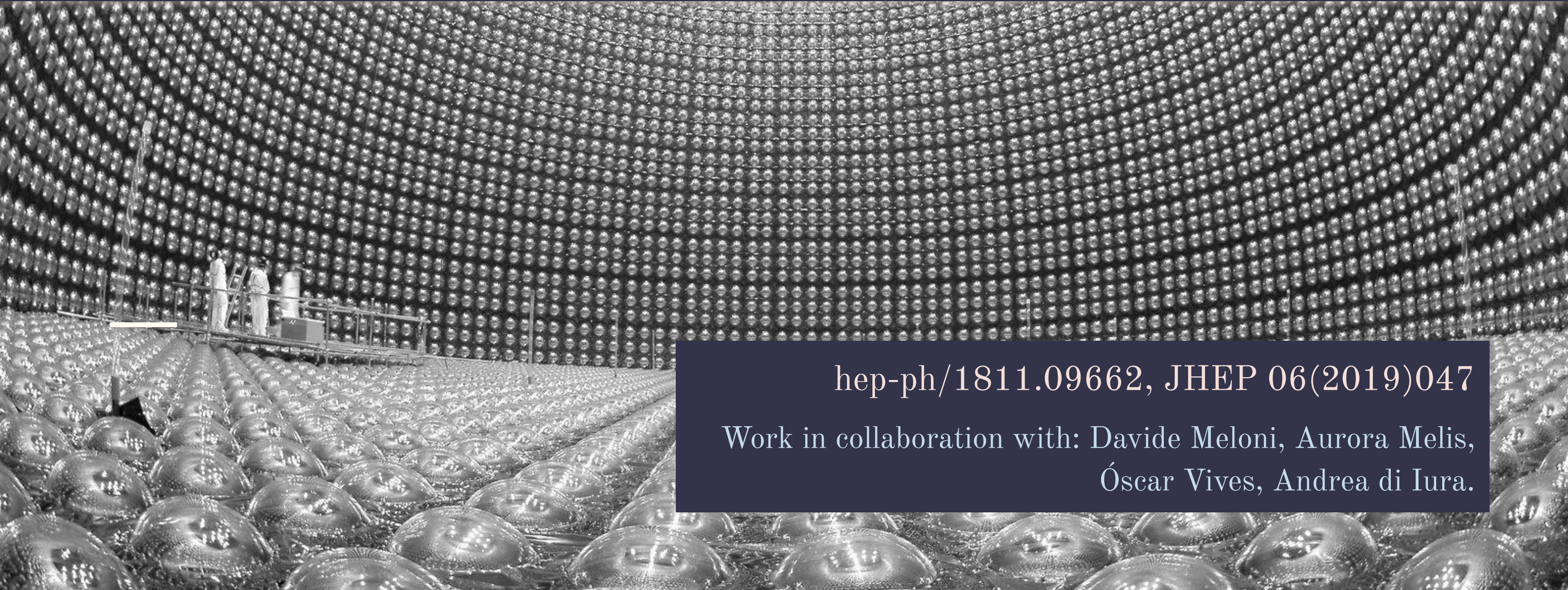
Lepton Phenomenology from A_5 and CP

FLASY 2019

· María Luisa López-Ibáñez · (ITP-CAS, Beijing) ·



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Work in collaboration with: Davide Meloni, Aurora Melis, Óscar Vives, Andrea di Iura.

Outline

- Flavour symmetries?
  - The SM flavour puzzle.
  - Residual symmetries.
    Generalised CP Symmetries.
- A_5 and generalised CP
  - Lepton Mixing. Neutrino Masses.
- FLASY + BSM
  - Example: supergravity!
  - Slepton Bounds. Interplay neutrino-charged leptons.
- Conclusions

Why flavour symmetries?

Why Flavour Symmetries?

The Flavour Puzzle

u c t

d s b

e μ τ

ν_e ν_μ ν_τ

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The Flavour Puzzle

1. Why are charged-fermions masses so hierarchical? Why up-type quarks the most hierarchical? Why small mixing among generations?

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4. What's the origin of the three families?
5. What's the physics behind CP violation?

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Why Flavour Symmetries?

The Flavour Puzzle

Is there any symmetry behind this?

u	c	t
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Flavour Symmetries

and generalised CP transformations

Froggatt-Nielsen Mechanism

Flavour Symmetries

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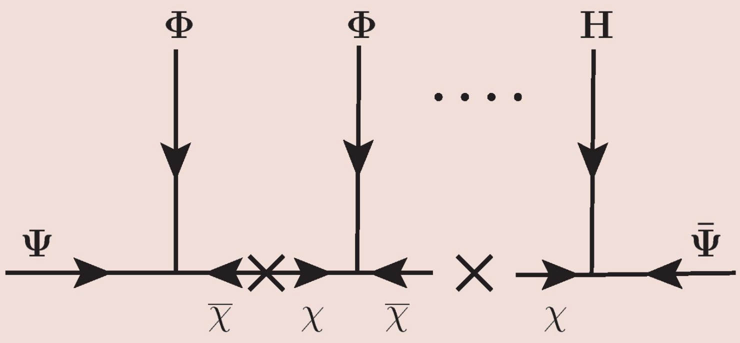

Froggatt-Nielsen Mechanism

Yukawa couplings may be effectively generated after the spontaneous breaking of a flavour symmetry by some scalar fields called flavons.

$$L_Y = Y_{ij} \Psi_i \Psi_j H$$

$$Y_{ij} = \left(\frac{\langle \Phi \rangle}{M_\chi}\right)^{n_{ij}} \equiv \epsilon^{n_{ij}} \ll 1$$

Neutrino Textures

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Bimaximal Mixing (S_4)

$$\sin^2\theta_{12} = 1/2 \quad \sin^2\theta_{23} = 1/2$$

$$U^0_{PMNS} \sim \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

TriBimaximal Mixing (S_4 A_4 $\Delta(27)$)

$$\sin^2\theta_{12} = 1/\sqrt{3} \quad \sin^2\theta_{23} = 1/2$$

$$\begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Golden Ratio Mixing (A_5)

$$\tan\theta_{12} = 1/\varphi \quad \sin^2\theta_{23} = 1/2$$

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Majorana neutrino mass matrix is the most general matrix symmetric under $\mathcal{K} = Z_2^U \times Z_2^S$.

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$$M_\nu \sim \begin{pmatrix} x & y & y \\ y & z & w \\ y & w & z \end{pmatrix}$$

Residual Symmetries

Majorana neutrino mass matrix is the most general matrix symmetric under $\mathcal{K} = Z_2^U \times Z_2^S$. Then, one may assume the existence of residual symmetries: Klein group for neutrinos

Neutrinos

$$S^T M_\nu S = M_\nu \quad \oplus \quad U^T M_\nu U = M_\nu$$

$$\mathcal{K} = \{S, U, US, E\}$$

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Charged Leptons

$$Q^\dagger m_e^\dagger m_e Q = m_e^\dagger m_e \longrightarrow Z_N \subset A_5$$

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Solutions: NLO corrections to previous patterns, non diagonal charged leptons, decreasing the symmetry neutrino sector ~~Klein group~~

$$\downarrow$$

$$\mathcal{G}_\nu = Z_2 \times CP$$

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$$U_{PMNS} = \Omega R_\theta K_\nu$$

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$$U_{PMNS} = \Omega\, R_\theta\, K_\nu$$

$$R_\theta = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$\sin\theta_{13} \neq 0$$

A_5 and CP

Lepton Mixing

$\mathcal{A}_5$ and CP $\longrightarrow \mathcal{G}_\nu, \mathcal{G}_e$

A. Di Iura, C. Hagedorn, D. Meloni
hep-ph/1503.04140

Residual symmetries that reproduce U_{PMNS}?

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Residual Symmetry in the **neutrino** sector is $\mathcal{G}_\nu = Z_2 \times CP$

All possibilities for CP transformations: $X = V X_0, \; V \in Z_2 \times Z_2$

All possibilities for **charged leptons**: $Z_3, Z_2 \times Z_2, Z_5$

All possible row and column permutations for $U_{PMNS} = \Omega R_\theta K_\nu$

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Only 4 patterns

at 3σ or better

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	Case I		Case II		Case III		Case IV-P1		Case IV-P2	
(Q_i, Z, X)	$(T, T^2ST^3ST^2, SX_0)$		(T, ST^2ST, X_0)		$(T^2ST^2, ST^2ST^3S, X_0)$		$(\{S, T^2ST^3ST^2\}, ST^2ST, X_0)$			
	NO	IO	NO	IO	NO	IO	NO	IO	NO	IO
$\chi^2_{\rm min}$	5.64	3.46	4.04	7.74	8.84	12.56	4.48	11.80	6.19	6.43
$\theta_{\rm bf}$	0.174	2.967	$\{0.175,\ 2.967\}$		$\{0.604,\ 0.967\}$	$\{0.603,\ 0.967\}$	0.254	0.258	0.255	0.254
$\sin^2\theta_{12}$	0.283	0.283	0.283	0.283	0.341	0.341	0.331	0.330	0.331	0.331
$\sin^2\theta_{13}$	0.0217	0.0219	0.0218	0.0220	0.0217	0.0218	0.0219	0.0225	0.0220	0.0218
$\sin^2\theta_{23}$	0.408	0.592	0.5		0.5		0.475	0.478	0.524	0.525
J_{CP}	0	0	∓ 0.0325	∓ 0.0326	± 0.0342	± 0.0342	0	0	0	0
$\sin\delta$	0	0	∓ 1		± 1		0	0	0	0

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Lepton Masses

$\mathcal{A}_5$ and CP $\longrightarrow \mathcal{G}_\nu\,,\ \mathcal{G}_e$

$$\mathsf{U}_{\mathsf{PMNS}} = \Omega\, \mathsf{R}_\theta\, \mathsf{K}_\nu$$

Case

$$\sin^2\theta_{23} = 1/2 \qquad \sin^2\theta_{12} = \frac{\sin^2\varphi}{\cos^2\theta_{13}} \qquad \sin^2\theta_{13} = \frac{2+\varphi}{5}\sin^2\theta\ \left\{\theta_{\text{bf}} = 0.175\right\}$$

$(Z_5\,, Z_2\,, X_0)$ II

Lepton Masses

$\mathcal{A}_5$ and CP $\longrightarrow \mathcal{G}_\nu\,,\ \mathcal{G}_e$

$\langle\phi_\nu\rangle = \mathsf{X}_0\,\langle\phi_\nu\rangle^\star$

$\langle\phi_\nu\rangle = \mathsf{Z}\,\langle\phi_\nu\rangle$

$\mathsf{U}_{\mathsf{PMNS}} = \Omega\,\mathsf{R}_\theta\,\mathsf{K}_\nu$

Case II

$(\mathrm{Z}_5\,,\mathrm{Z}_2\,,\mathrm{X}_0)$

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$\langle\phi_\nu\rangle = \mathsf{X}_0 \langle\phi_\nu\rangle^\star \qquad \langle\phi_\nu\rangle = \mathsf{Z}\langle\phi_\nu\rangle$

$$\langle\phi_{\nu,\mathbf{1}}\rangle = \upsilon_1 \qquad \langle\phi_{\nu,\mathbf{3}}\rangle = \upsilon\begin{pmatrix}-\sqrt{2}\varphi^{-1}\\1\\1\end{pmatrix} \qquad \langle\phi_{\nu,\mathbf{3'}}\rangle = \omega\begin{pmatrix}\sqrt{2}\varphi\\1\\1\end{pmatrix}$$

$$\langle\phi_{\nu,\mathbf{4}}\rangle^{\mathrm{T}} = (y_r - iy_i,\ (1+2\varphi)y_r - iy_i,\ (1+2\varphi)y_r + iy_i,\ y_r + iy_i)$$

$$\langle\phi_{\nu,\mathbf{5}}\rangle^{\mathrm{T}} = -\left(\sqrt{\frac{2}{3}}(x_r + x_{r,2}),\ -x_r + i\varphi x_i,\ x_{r,2} - ix_i,\ x_{r,2} + ix_i,\ x_r + i\varphi x_i\right)$$

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Case $(\mathrm{Z}_5, \mathrm{Z}_2, \mathrm{X}_0)$ II

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Lepton Masses

$\mathcal{A}_5$ and CP $\longrightarrow \mathcal{G}_\nu,\ \mathcal{G}_e$

$$X_0^T\, M_\nu\, X_0 = M_\nu^\star \qquad Z^T\, M_\nu\, Z = M_\nu$$

$$\langle\phi_\nu\rangle = X_0\,\langle\phi_\nu\rangle^\star \qquad \langle\phi_\nu\rangle = Z\,\langle\phi_\nu\rangle$$

$$\langle\phi_{\nu,\mathbf{1}}\rangle = v_1 \qquad \langle\phi_{\nu,\mathbf{3}}\rangle = v\begin{pmatrix}-\sqrt{2}\varphi^{-1}\\ 1\\ 1\end{pmatrix} \qquad \langle\phi_{\nu,\mathbf{3'}}\rangle = \omega\begin{pmatrix}\sqrt{2}\varphi\\ 1\\ 1\end{pmatrix}$$

$$\langle\phi_{\nu,\mathbf{4}}\rangle^{\mathrm{T}} = (y_r - i y_i,\ (1+2\varphi)y_r - i y_i,\ (1+2\varphi)y_r + i y_i,\ y_r + i y_i)$$

$$\langle\phi_{\nu,\mathbf{5}}\rangle^{\mathrm{T}} = -\left(\sqrt{\frac{2}{3}}(x_r + x_{r,2}),\ -x_r + i\varphi x_i,\ x_{r,2} - i x_i,\ x_{r,2} + i x_i,\ x_r + i\varphi x_i\right)$$

$$U_{PMNS} = \Omega\, R_\theta\, K_\nu$$

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(Z_5, Z_2, X_0)

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Lepton Masses

$\mathcal{A}_5$ and CP $\longrightarrow \mathcal{G}_\nu, \mathcal{G}_e$

$$X_0^T M_\nu X_0 = M_\nu^\star \qquad Z^T M_\nu Z = M_\nu$$

$$M_\nu = m_0 \begin{pmatrix} s+x+z & \frac{3}{2\sqrt{2}}(z+i\varphi y) & \frac{3}{2\sqrt{2}}(z-i\varphi y) \\ \frac{3}{2\sqrt{2}}(z+i\varphi y) & \frac{3}{2}(x+iy) & s-\frac{x+z}{2} \\ \frac{3}{2\sqrt{2}}(z-i\varphi y) & s-\frac{x+z}{2} & \frac{3}{2}(x-iy) \end{pmatrix}$$

$$\langle\phi_\nu\rangle = X_0 \langle\phi_\nu\rangle^\star \qquad \langle\phi_\nu\rangle = Z\langle\phi_\nu\rangle$$

$$\langle\phi_{\nu,\mathbf{1}}\rangle = v_1 \qquad \langle\phi_{\nu,\mathbf{3}}\rangle = v\begin{pmatrix} -\sqrt{2}\varphi^{-1} \\ 1 \\ 1\end{pmatrix} \qquad \langle\phi_{\nu,\mathbf{3'}}\rangle = \omega\begin{pmatrix} \sqrt{2}\varphi \\ 1 \\ 1\end{pmatrix}$$

$$\langle\phi_{\nu,\mathbf{4}}\rangle^T = (y_r - iy_i,\ (1+2\varphi)y_r - iy_i,\ (1+2\varphi)y_r + iy_i,\ y_r + iy_i)$$

$$\langle\phi_{\nu,\mathbf{5}}\rangle^T = -\left(\sqrt{\frac{2}{3}}(x_r + x_{r,2}),\ -x_r + i\varphi x_i,\ x_{r,2} - ix_i,\ x_{r,2} + ix_i,\ x_r + i\varphi x_i\right)$$

$$U_{PMNS} = \Omega\, R_\theta\, K_\nu$$

Case

$$\sin^2\theta_{23} = 1/2 \qquad \sin^2\theta_{12} = \frac{\sin^2\varphi}{\cos^2\theta_{13}} \qquad \sin^2\theta_{13} = \frac{2+\varphi}{5}\sin^2\theta \left\{ \theta_{bf} = 0.175 \right\}$$

(Z_5, Z_2, X_0) II

Lepton Masses

$\mathcal{A}_5$ and CP $\longrightarrow \mathcal{G}_\nu\,,\ \mathcal{G}_e$

$$X_0^T\, M_\nu\, X_0 = M_\nu^\star \qquad Z^T\, M_\nu\, Z = M_\nu$$

$$M_\nu = m_0 \begin{pmatrix} s+x+z & \frac{3}{2\sqrt{2}}(z+i\varphi y) & \frac{3}{2\sqrt{2}}(z-i\varphi y) \\ \frac{3}{2\sqrt{2}}(z+i\varphi y) & \frac{3}{2}(x+iy) & s-\frac{x+z}{2} \\ \frac{3}{2\sqrt{2}}(z-i\varphi y) & s-\frac{x+z}{2} & \frac{3}{2}(x-iy) \end{pmatrix}$$

$$\langle\phi_\nu\rangle = X_0\,\langle\phi_\nu\rangle^\star \qquad \langle\phi_\nu\rangle = Z\,\langle\phi_\nu\rangle$$

$$\langle\phi_{\nu,\mathbf{1}}\rangle = v_1 \qquad \langle\phi_{\nu,\mathbf{3}}\rangle = v\begin{pmatrix} -\sqrt{2}\varphi^{-1} \\ 1 \\ 1\end{pmatrix} \qquad \langle\phi_{\nu,\mathbf{3'}}\rangle = \omega\begin{pmatrix} \sqrt{2}\varphi \\ 1 \\ 1\end{pmatrix}$$

$$\langle\phi_{\nu,\mathbf{4}}\rangle^{\mathrm{T}} = (y_r - iy_i,\ (1+2\varphi)y_r - iy_i,\ (1+2\varphi)y_r + iy_i,\ y_r + iy_i)$$

$$\langle\phi_{\nu,\mathbf{5}}\rangle^{\mathrm{T}} = -\left(\sqrt{\frac{2}{3}}(x_r + x_{r,2}),\ -x_r + i\varphi x_i,\ x_{r,2} - ix_i,\ x_{r,2} + ix_i,\ x_r + i\varphi x_i\right)$$

$$U_{PMNS} = \Omega\, R_\theta\, K_\nu$$

Case II

$$\sin^2\theta_{23} = 1/2 \qquad \sin^2\theta_{12} = \frac{\sin^2\varphi}{\cos^2\theta_{13}} \qquad \sin^2\theta_{13} = \frac{2+\varphi}{5}\sin^2\theta \left\{\theta_{bf} = 0.175\right\}$$

$$(Z_5\,,\, Z_2\,,\, X_0)$$

Lepton Masses

$\mathcal{A}_5$ and CP $\longrightarrow \mathcal{G}_\nu,\ \mathcal{G}_e$

$$X_0^T\, M_\nu\, X_0 = M_\nu^\star \qquad Z^T\, M_\nu\, Z = M_\nu$$

$$M_\nu = m_0 \begin{pmatrix} s+x+z & \frac{3}{2\sqrt{2}}(z+i\varphi y) & \frac{3}{2\sqrt{2}}(z-i\varphi y) \\ \frac{3}{2\sqrt{2}}(z+i\varphi y) & \frac{3}{2}(x+iy) & s-\frac{x+z}{2} \\ \frac{3}{2\sqrt{2}}(z-i\varphi y) & s-\frac{x+z}{2} & \frac{3}{2}(x-iy) \end{pmatrix}$$

$$\langle\phi_\nu\rangle = X_0\, \langle\phi_\nu\rangle^\star \qquad \langle\phi_\nu\rangle = Z\, \langle\phi_\nu\rangle$$

$$\langle\phi_{\nu,\mathbf{1}}\rangle = v_1 \qquad \langle\phi_{\nu,\mathbf{3}}\rangle = v \begin{pmatrix} -\sqrt{2}\varphi^{-1} \\ 1 \\ 1 \end{pmatrix} \qquad \langle\phi_{\nu,\mathbf{3'}}\rangle = \omega \begin{pmatrix} \sqrt{2}\varphi \\ 1 \\ 1 \end{pmatrix}$$

$$\tan 2\theta = \frac{2\sqrt{7+11}y}{2x(\varphi+1)+z(2\varphi+1)}$$

$$\langle\phi_{\nu,\mathbf{4}}\rangle^{\mathrm{T}} = (y_r - iy_i,\ (1+2\varphi)y_r - iy_i,\ (1+2\varphi)y_r + iy_i,\ y_r + iy_i)$$

$$\langle\phi_{\nu,\mathbf{5}}\rangle^{\mathrm{T}} = -\left(\sqrt{\frac{2}{3}}(x_r + x_{r,2}),\ -x_r + i\varphi x_i,\ x_{r,2} - ix_i,\ x_{r,2} + ix_i,\ x_r + i\varphi x_i\right)$$

$$U_{PMNS} = \Omega\, R_\theta\, K_\nu$$

Case II

$$\sin^2\theta_{23} = 1/2 \qquad \sin^2\theta_{12} = \frac{\sin^2\varphi}{\cos^2\theta_{13}} \qquad \sin^2\theta_{13} = \frac{2+\varphi}{5}\sin^2\theta \; \{\theta_{bf} = 0.175\}$$

$$(Z_5,\, Z_2,\, X_0)$$

Lepton Masses

$\mathcal{A}_5$ and CP $\longrightarrow \mathcal{G}_\nu,\ \mathcal{G}_e$

$$X_0^T\, M_\nu\, X_0 = M_\nu^\star \qquad Z^T\, M_\nu\, Z = M_\nu$$

$$M_\nu = m_0 \begin{pmatrix} s+x+z & \frac{3}{2\sqrt{2}}(z+i\varphi y) & \frac{3}{2\sqrt{2}}(z-i\varphi y) \\ \frac{3}{2\sqrt{2}}(z+i\varphi y) & \frac{3}{2}(x+iy) & s-\frac{x+z}{2} \\ \frac{3}{2\sqrt{2}}(z-i\varphi y) & s-\frac{x+z}{2} & \frac{3}{2}(x-iy) \end{pmatrix}$$

$$\langle\phi_\nu\rangle = X_0\,\langle\phi_\nu\rangle^\star \qquad \langle\phi_\nu\rangle = Z\,\langle\phi_\nu\rangle$$

$$\langle\phi_{\nu,\mathbf{3}}\rangle = v\begin{pmatrix} -\sqrt{2}\varphi^{-1} \\ 1 \\ 1\end{pmatrix} \qquad \langle\phi_{\nu,\mathbf{3'}}\rangle = \omega\begin{pmatrix} \sqrt{2}\varphi \\ 1 \\ 1\end{pmatrix}$$

$$\langle\phi_{\nu,\mathbf{4}}\rangle^{\mathrm{T}} = (y_r - iy_i,\ (1+2\varphi)y_r - iy_i,\ (1+2\varphi)y_r + iy_i,\ y_r + iy_i)$$

$$\langle\phi_{\nu,\mathbf{5}}\rangle^{\mathrm{T}} = -\left(\sqrt{\frac{2}{3}}(x_r + x_{r,2}),\ -x_r + i\varphi x_i,\ x_{r,2} - ix_i,\ x_{r,2} + ix_i,\ x_r + i\varphi x_i\right)$$

$|y| \ll |x|, |z|$

$$\tan 2\theta = \frac{2\sqrt{7+11}y}{2x(\varphi+1)+z(2\varphi+1)}$$

$$U_{PMNS} = \Omega\, R_\theta\, K_\nu$$

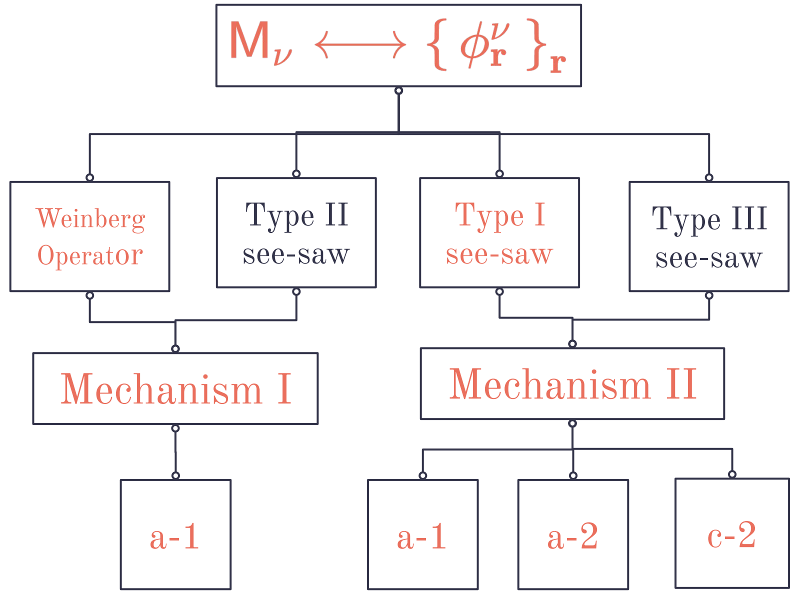
Case II

(Z_5, Z_2, X_0)

$$\sin^2\theta_{23} = 1/2 \qquad \sin^2\theta_{12} = \frac{\sin^2\varphi}{\cos^2\theta_{13}} \qquad \sin^2\theta_{13} = \frac{2+\varphi}{5}\sin^2\theta \left\{ \theta_{bf} = 0.175 \right\}$$

Lepton Masses

$\mathcal{A}_5$ and CP $\longrightarrow \mathcal{G}_\nu\,,\ \mathcal{G}_e$



$$U_{PMNS} = \Omega\, R_\theta\, K_\nu$$

$|y| \ll |x|, |z|$

Case II

$(Z_5\,, Z_2\,, X_0)$

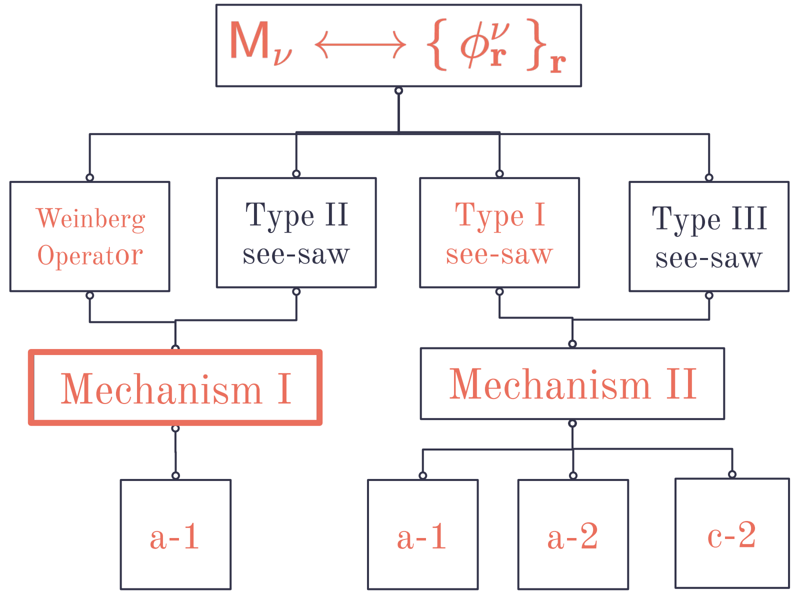
$$\sin^2\theta_{23} = 1/2 \qquad \sin^2\theta_{12} = \frac{\sin^2\varphi}{\cos^2\theta_{13}} \qquad \sin^2\theta_{13} = \frac{2+\varphi}{5}\sin^2\theta \left\{\theta_{bf} = 0.175\right\}$$

Lepton Masses

$\mathcal{A}_5$ and CP $\longrightarrow \mathcal{G}_\nu, \mathcal{G}_e$

$$\mathcal{L}^{\rm eff} = y_1 \frac{[\,L\tilde{H}\,]_1^2}{\Lambda^2}\phi_1 + y_5 \frac{[\,L\tilde{H}\,]_5^2}{\Lambda^2}\phi_5$$

$L \sim \mathbf{3} \longleftrightarrow \mathbf{3}'$ ✓



Case II

$(Z_5\,, Z_2\,, X_0)$

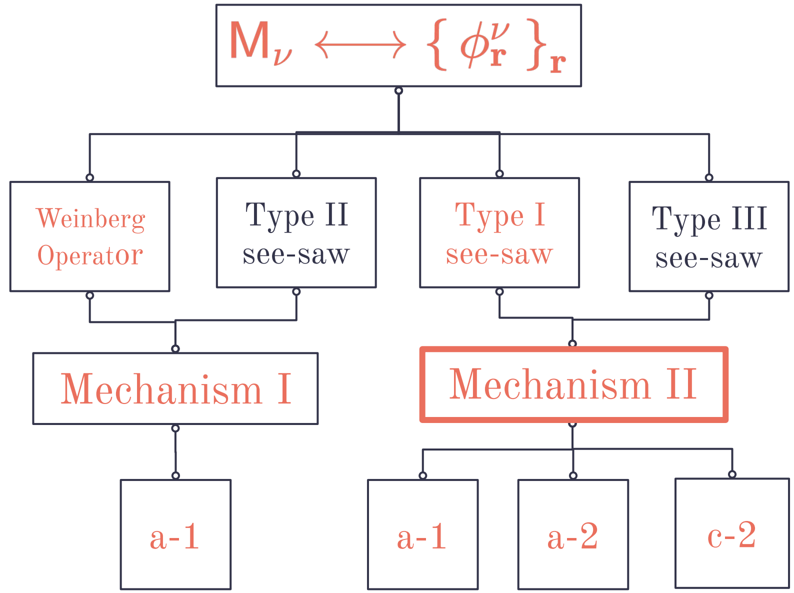
$$U_{\rm PMNS} = \Omega\, R_\theta\, K_\nu$$

$|y| \ll |x|, |z|$

$\sin^2\theta_{23} = 1/2 \qquad \sin^2\theta_{12} = \frac{\sin^2\varphi}{\cos^2\theta_{13}} \qquad \sin^2\theta_{13} = \frac{2+\varphi}{5}\sin^2\theta \; \{\theta_{\rm bf} = 0.175\}$

Lepton Masses

$\mathcal{A}_5$ and CP $\longrightarrow \mathcal{G}_\nu, \mathcal{G}_e$



$$U_{PMNS} = \Omega\, R_\theta\, K_\nu$$

$|y| \ll |x|, |z|$

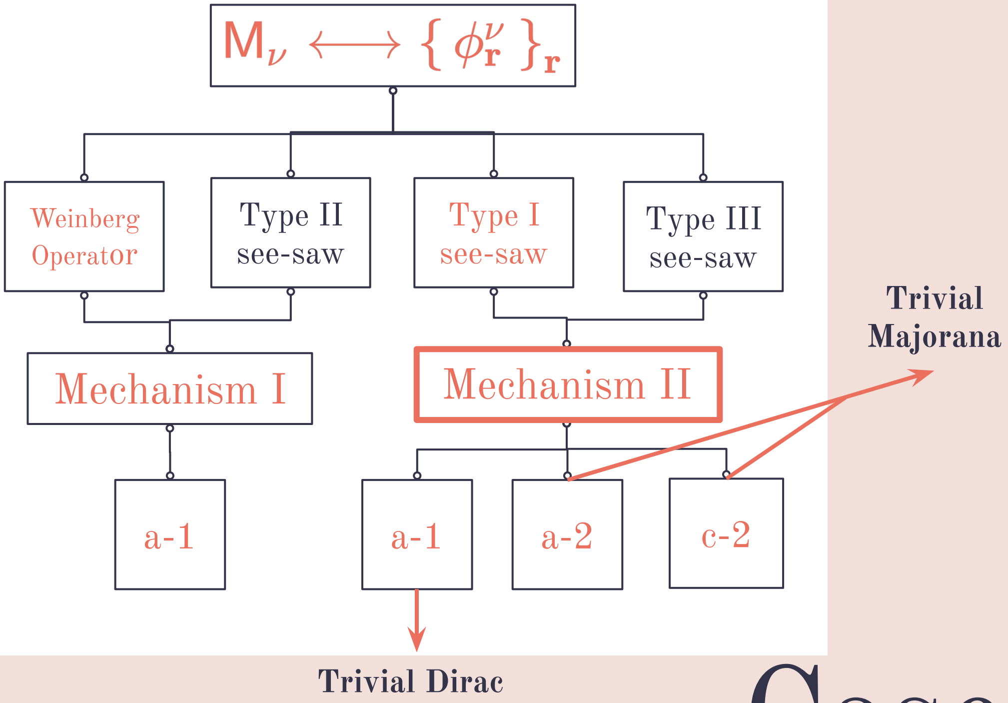
Case II

(Z_5, Z_2, X_0)

$$\sin^2\theta_{23} = 1/2 \qquad \sin^2\theta_{12} = \frac{\sin^2\varphi}{\cos^2\theta_{13}} \qquad \sin^2\theta_{13} = \frac{2+\varphi}{5}\sin^2\theta \ \{\theta_{bf} = 0.175\}$$

Lepton Masses

$\mathcal{A}_5$ and CP $\longrightarrow \mathcal{G}_\nu, \mathcal{G}_e$



$|y| \ll |x|, |z|$

$U_{PMNS} = \Omega\, R_\theta\, K_\nu$

Case II

$\sin^2\theta_{23} = 1/2$ $\quad \sin^2\theta_{12} = \frac{\sin^2\varphi}{\cos^2\theta_{13}}$ $\quad \sin^2\theta_{13} = \frac{2+\varphi}{5}\sin^2\theta \left\{\theta_{bf} = 0.175\right\}$

(Z_5, Z_2, X_0)

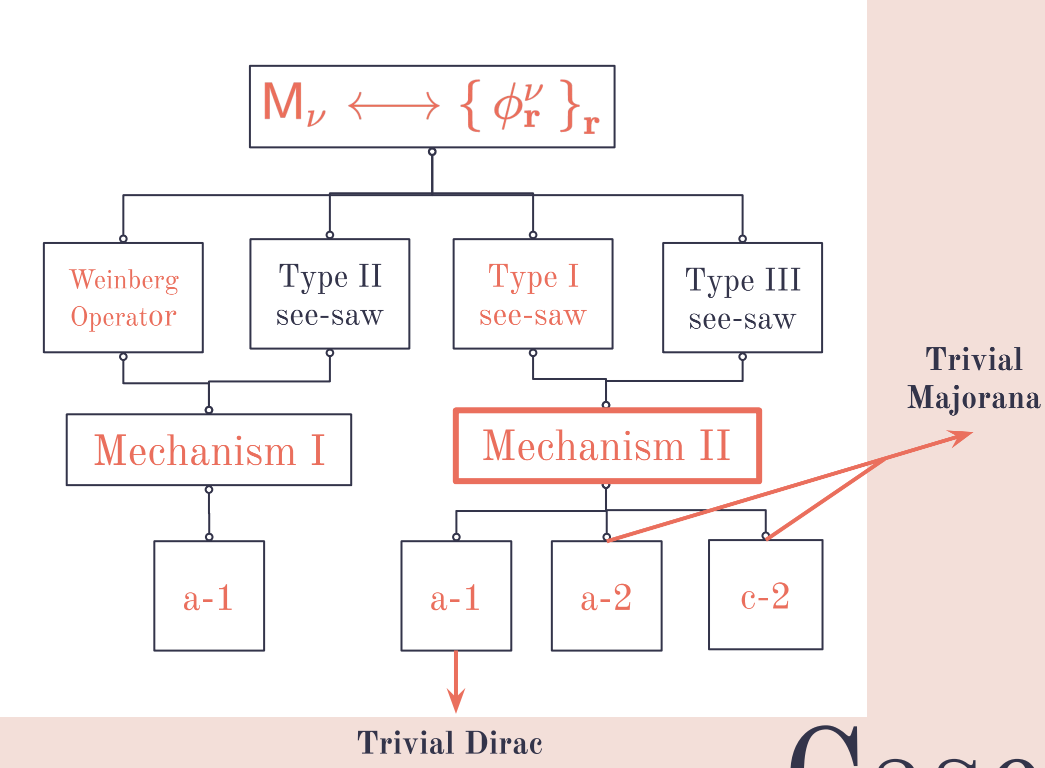
Lepton Masses

$\mathcal{A}_5$ and CP $\longrightarrow \mathcal{G}_\nu, \mathcal{G}_e$

a-1: RH and LH fields same 3 rep.

a-2: RH and LH same 3 rep.

c-2: RH and LH different 3 reps.



$|y| \ll |x|, |z|$

$U_{PMNS} = \Omega\, R_\theta\, K_\nu$

Case II

$\sin^2\theta_{23} = 1/2 \qquad \sin^2\theta_{12} = \frac{\sin^2\varphi}{\cos^2\theta_{13}} \qquad \sin^2\theta_{13} = \frac{2+\varphi}{5}\sin^2\theta \;\{\theta_{bf} = 0.175\}$

(Z_5, Z_2, X_0)

Results

hep-ph/1811.09662

- Three free parameters: setting one or two vevs equal to zero.

Mechanism I		VEVs	$[\alpha,\beta]$	Mechanism II a-1		VEVs	$[\alpha,\beta]$
$z=0$	NO	$y\simeq\pm0.19\,x$ $s\simeq-20\,x$	$[0,\,0]$	$Z=0$	NO	$Y\simeq\pm0.19\,X$ $S\simeq X,\,44X$	$[0,\,\pi/0]$
$x=0$	IO	$y\simeq\pm0.16\,z$ $s\simeq-0.3\,z$	$[\pi,\,\pi]$	$X=0$	IO	$Y\simeq\pm0.16\,X$ $S\simeq-Z/4,\,Z/3,\,2Z/3$	$[\pi,\,\pi/\pi/0]$
$s=0$	IO	$y\simeq\pm0.10\,z$ $s\simeq-0.3\,z$	$[\pi,\,0]$	$S=0$	IO	✗	✗

Mechanism II a-2

$g=0$		VEVs	$h_i=0$		VEVs
$f=0$	NO IO	$g\simeq\pm h_{r,2}/10$, $h_r\simeq0$ $g\simeq\pm h_{r,2}/2$, $h_r\simeq-4h_{r,2}$	$f=0$	NO IO	$h_i\simeq\pm h_{r,2}/5$, $h_r\simeq0$ $h_i\simeq\pm2h_{r,2}/5$, $h_r\simeq-4h_{r,2}$
$h_r=0$	NO	$g\simeq\pm h_{r,2}/100$ $f\simeq-7h_{r,2}/20$	$h_r=0$	NO	$h_i\simeq\pm0.19h_{r,2}$ $f\simeq20h_{r,2}$
$h_{r,2}=0$	IO	$g\simeq\pm7\sqrt{6}h_r/40$ $f\simeq h_r/(2\sqrt{6})$	$h_{r,2}=0$	IO	$h_i\simeq\pm0.16h_r$ $f\simeq h_r/(2\sqrt{6})$

Mechanism II c-2

$f_r=0$		VEVs	$h_i=0$		VEVs
$f_i=0$	NO IO	$f_r\simeq\pm h_r/25$, $h_r\simeq-h_{r,2}$ $f_r\simeq\pm h_r/10$, $h_r\simeq h_{r,2}$	$f_i=0$	NO IO	$h_i\simeq\pm h_r/10$, $h_r\simeq-h_{r,2}$ $h_i\simeq\pm h_r/10$, $h_r\simeq h_{r,2}$
$h_r=0$	NO	$f_r\simeq\pm h_{r,2}/50$ $f_i\simeq-h_{r,2}/(2\sqrt{6})$	$h_r=0$	NO	$h_i\simeq\pm0.03h_{r,2}$ $f_i\simeq-h_{r,2}/(2\sqrt{6})$
$h_{r,2}=0$	IO	$f_r\simeq\pm h_r/100$ $f_i\simeq h_r/(2\sqrt{6})$	$h_{r,2}=0$	IO	$h_i\simeq\pm h_r/10$ $f_i\simeq h_r/(2\sqrt{6})$

Results

hep-ph/1811.09662

- Three free parameters: setting one or two vevs equal to zero.
- Allowed ordering

Mechanism I		VEVs	$[\alpha,\beta]$	Mechanism II a-1		VEVs	$[\alpha,\beta]$
$z=0$	NO	$y\simeq\pm0.19\,x$ $s\simeq-20\,x$	$[0,\,0]$	$Z=0$	NO	$Y\simeq\pm0.19\,X$ $S\simeq X,\,44X$	$[0,\,\pi/0]$
$x=0$	IO	$y\simeq\pm0.16\,z$ $s\simeq-0.3\,z$	$[\pi,\,\pi]$	$X=0$	IO	$Y\simeq\pm0.16\,X$ $S\simeq-Z/4,\,Z/3,\,2Z/3$	$[\pi,\,\pi/\pi/0]$
$s=0$	IO	$y\simeq\pm0.10\,z$ $s\simeq-0.3\,z$	$[\pi,\,0]$	$S=0$	IO	✗	✗

Mechanism II a-2

$g=0$		VEVs	$h_i=0$		VEVs
$f=0$	NO IO	$g\simeq\pm h_{r,2}/10\quad h_r\simeq0$ $g\simeq\pm h_{r,2}/2\quad h_r\simeq-4h_{r,2}$	$f=0$	NO IO	$h_i\simeq\pm h_{r,2}/5\quad h_r\simeq0$ $h_i\simeq\pm2h_{r,2}/5\quad h_r\simeq-4h_{r,2}$
$h_r=0$	NO	$g\simeq\pm h_{r,2}/100$ $f\simeq-7h_{r,2}/20$	$h_r=0$	NO	$h_i\simeq\pm0.19h_{r,2}$ $f\simeq20h_{r,2}$
$h_{r,2}=0$	IO	$g\simeq\pm7\sqrt{6}h_r/40$ $f\simeq h_r/(2\sqrt{6})$	$h_{r,2}=0$	IO	$h_i\simeq\pm0.16h_r$ $f\simeq h_r/(2\sqrt{6})$

Mechanism II c-2

$f_r=0$		VEVs	$h_i=0$		VEVs
$f_i=0$	NO IO	$f_r\simeq\pm h_r/25\quad h_r\simeq-h_{r,2}$ $f_r\simeq\pm h_r/10\quad h_r\simeq h_{r,2}$	$f_i=0$	NO IO	$h_i\simeq\pm h_r/10\quad h_r\simeq-h_{r,2}$ $h_i\simeq\pm h_r/10\quad h_r\simeq h_{r,2}$
$h_r=0$	NO	$f_r\simeq\pm h_{r,2}/50$ $f_i\simeq-h_{r,2}/(2\sqrt{6})$	$h_r=0$	NO	$h_i\simeq\pm0.03h_{r,2}$ $f_i\simeq-h_{r,2}/(2\sqrt{6})$
$h_{r,2}=0$	IO	$f_r\simeq\pm h_r/100$ $f_i\simeq h_r/(2\sqrt{6})$	$h_{r,2}=0$	IO	$h_i\simeq\pm h_r/10$ $f_i\simeq h_r/(2\sqrt{6})$

Results

hep-ph/1811.09662

➔ Three free parameters: setting one or two vevs equal to zero.

➔ Allowed ordering and Majorana phases.

$[\alpha,\ \beta] = [0,\ 0]$

Mechanism I		VEVs	$[\alpha,\ \beta]$	Mechanism II a-1		VEVs	$[\alpha,\ \beta]$
$z = 0$	NO	$y \simeq \pm 0.19\, x$ $s \simeq -20\, x$	$[0,\ 0]$	$Z = 0$	NO	$Y \simeq \pm 0.19\, X$ $S \simeq X,\ 44X$	$[0,\ \pi/0]$
$x = 0$	IO	$y \simeq \pm 0.16\, z$ $s \simeq -0.3\, z$	$[\pi,\ \pi]$	$X = 0$	IO	$Y \simeq \pm 0.16\, X$ $S \simeq -Z/4,\ Z/3,\ 2Z/3$	$[\pi,\ \pi/\pi/0]$
$s = 0$	IO	$y \simeq \pm 0.10\, z$ $s \simeq -0.3\, z$	$[\pi,\ 0]$	$S = 0$	IO	✗	✗

Mechanism II a-2

$g = 0$		VEVs	$h_i = 0$		VEVs
$f = 0$	NO IO	$g \simeq \pm h_{r,2}/10 \quad h_r \simeq 0$ $g \simeq \pm h_{r,2}/2 \quad h_r \simeq -4h_{r,2}$	$f = 0$	NO IO	$h_i \simeq \pm h_{r,2}/5 \quad h_r \simeq 0$ $h_i \simeq \pm 2h_{r,2}/5 \quad h_r \simeq -4h_{r,2}$
$h_r = 0$	NO	$g \simeq \pm h_{r,2}/100$ $f \simeq -7h_{r,2}/20$	$h_r = 0$	NO	$h_i \simeq \pm 0.19 h_{r,2}$ $f \simeq 20 h_{r,2}$
$h_{r,2} = 0$	IO	$g \simeq \pm 7\sqrt{6} h_r/40$ $f \simeq h_r/(2\sqrt{6})$	$h_{r,2} = 0$	IO	$h_i \simeq \pm 0.16 h_r$ $f \simeq h_r/(2\sqrt{6})$

Mechanism II c-2

$f_r = 0$		VEVs	$h_i = 0$		VEVs
$f_i = 0$	NO IO	$f_r \simeq \pm h_r/25 \quad h_r \simeq -h_{r,2}$ $f_r \simeq \pm h_r/10 \quad h_r \simeq h_{r,2}$	$f_i = 0$	NO IO	$h_i \simeq \pm h_r/10 \quad h_r \simeq -h_{r,2}$ $h_i \simeq \pm h_r/10 \quad h_r \simeq h_{r,2}$
$h_r = 0$	NO	$f_r \simeq \pm h_{r,2}/50$ $f_i \simeq -h_{r,2}/(2\sqrt{6})$	$h_r = 0$	NO	$h_i \simeq \pm 0.03 h_{r,2}$ $f_i \simeq -h_{r,2}/(2\sqrt{6})$
$h_{r,2} = 0$	IO	$f_r \simeq \pm h_r/100$ $f_i \simeq h_r/(2\sqrt{6})$	$h_{r,2} = 0$	IO	$h_i \simeq \pm h_r/10$ $f_i \simeq h_r/(2\sqrt{6})$

Results

hep-ph/1811.09662

- **Three free parameters:** setting one or two vevs equal to zero.
- **Allowed ordering and Majorana phases.**
- **Correlations between vevs:** ratio mass splittings and reactor angle.

Mechanism I		VEVs	$[\alpha,\ \beta]$	Mechanism II a-1		VEVs	$[\alpha,\ \beta]$
$z = 0$	NO	$y \simeq \pm 0.19\,x$ $s \simeq -20\,x$	$[0,\ 0]$	$Z = 0$	NO	$Y \simeq \pm 0.19\,X$ $S \simeq X,\ 44X$	$[0,\ \pi/0]$
$x = 0$	IO	$y \simeq \pm 0.16\,z$ $s \simeq -0.3\,z$	$[\pi,\ \pi]$	$X = 0$	IO	$Y \simeq \pm 0.16\,X$ $S \simeq -Z/4,\ Z/3,\ 2Z/3$	$[\pi,\ \pi/\pi/0]$
$s = 0$	IO	$y \simeq \pm 0.10\,z$ $s \simeq -0.3\,z$	$[\pi,\ 0]$	$S = 0$	IO	✗	✗

Mechanism II a-2

$g = 0$		VEVs	$h_i = 0$		VEVs
$f = 0$	NO IO	$g \simeq \pm h_{r,2}/10 \quad h_r \simeq 0$ $g \simeq \pm h_{r,2}/2 \quad h_r \simeq -4h_{r,2}$	$f = 0$	NO IO	$h_i \simeq \pm h_{r,2}/5 \quad h_r \simeq 0$ $h_i \simeq \pm 2h_{r,2}/5 \quad h_r \simeq -4h_{r,2}$
$h_r = 0$	NO	$g \simeq \pm h_{r,2}/100$ $f \simeq -7h_{r,2}/20$	$h_r = 0$	NO	$h_i \simeq \pm 0.19 h_{r,2}$ $f \simeq 20 h_{r,2}$
$h_{r,2} = 0$	IO	$g \simeq \pm 7\sqrt{6} h_r/40$ $f \simeq h_r/(2\sqrt{6})$	$h_{r,2} = 0$	IO	$h_i \simeq \pm 0.16 h_r$ $f \simeq h_r/(2\sqrt{6})$

Mechanism II c-2

$f_r = 0$		VEVs	$h_i = 0$		VEVs
$f_i = 0$	NO IO	$f_r \simeq \pm h_r/25 \quad h_r \simeq -h_{r,2}$ $f_r \simeq \pm h_r/10 \quad h_r \simeq h_{r,2}$	$f_i = 0$	NO IO	$h_i \simeq \pm h_r/10 \quad h_r \simeq -h_{r,2}$ $h_i \simeq \pm h_r/10 \quad h_r \simeq h_{r,2}$
$h_r = 0$	NO	$f_r \simeq \pm h_{r,2}/50$ $f_i \simeq -h_{r,2}/(2\sqrt{6})$	$h_r = 0$	NO	$h_i \simeq \pm 0.03 h_{r,2}$ $f_i \simeq -h_{r,2}/(2\sqrt{6})$
$h_{r,2} = 0$	IO	$f_r \simeq \pm h_r/100$ $f_i \simeq h_r/(2\sqrt{6})$	$h_{r,2} = 0$	IO	$h_i \simeq \pm h_r/10$ $f_i \simeq h_r/(2\sqrt{6})$

Results

hep-ph/1811.09662

- **Three free parameters:** setting one or two vevs equal to zero.
- **Allowed ordering and Majorana phases.**
- **Correlations between vevs:** ratio mass splittings and reactor angle.
- **Perturbation Theory:** sum rules, total sum masses, $m_{\beta\beta}$ and m_β.

$|y| \ll |x|, |z|$

Mechanism I		VEVs	$[\alpha,\beta]$	Mechanism II a-1		VEVs	$[\alpha,\beta]$
$z=0$	NO	$y \simeq \pm 0.19\,x$ $s \simeq -20\,x$	$[0,0]$	$Z=0$	NO	$Y \simeq \pm 0.19\,X$ $S \simeq X, 44X$	$[0, \pi/0]$
$x=0$	IO	$y \simeq \pm 0.16\,z$ $s \simeq -0.3\,z$	$[\pi,\pi]$	$X=0$	IO	$Y \simeq \pm 0.16\,X$ $S \simeq -Z/4, Z/3, 2Z/3$	$[\pi, \pi/\pi/0]$
$h_{r,2}=0$	IO	$f_i \simeq h_r/(2\sqrt{6})$		$h_{r,2}=0$	IO	$f_i \simeq h_r/(2\sqrt{6})$	

Mechanism I	$\Sigma(\tilde m_1, \tilde m_2, \tilde m_3)$	Mechanism II a-1	$\Sigma(\tilde m_1, \tilde m_2, \tilde m_3)$
$z=0$	$\tilde m_1 - \tilde m_2 + (3-\varphi)(\tilde m_3 - \tilde m_2)\sin^2\theta_{13}$	$Z=0$	$\frac{1}{\tilde m_1} - \frac{1}{\tilde m_2} + (3-\varphi)\left(\frac{1}{\tilde m_3} - \frac{1}{\tilde m_2}\right)\sin^2\theta_{13}$
$x=0$	$\tilde m_1 + (\varphi+1)\tilde m_2 - (\varphi+2)\tilde m_3$	$X=0$	$\frac{1}{\tilde m_1} + \frac{\varphi+1}{\tilde m_2} - \frac{\varphi+2}{\tilde m_3}$
$s=0$	$\tilde m_1 + \tilde m_2 + \tilde m_3$	$S=0$	✗

Mechanism II a-2	$\Sigma(\tilde m_1, \tilde m_2, \tilde m_3)$
$f=0=(g, h_i)$	$(\tilde m_1 + \tilde m_2 - \tilde m_3)^2 - 4\tilde m_1 \tilde m_2$
$h_r=0=(g, h_i)$	$(\tilde m_1 - \tilde m_2)^2 + 2(3-\varphi)(\tilde m_1 - 3\tilde m_2)(\tilde m_1 - \tilde m_2)\sin^2\theta_{13}$ $+20(\varphi-2)(\tilde m_1 - 2\tilde m_2 + \tilde m_3)\tilde m_2 \sin^4\theta_{13}$
$h_{r,2}=0=(g, h_i)$	$\left(\tilde m_1 + (3\varphi+2)\tilde m_2 - 5(\varphi+1)\tilde m_3\right)^2 - 4(3\varphi+2)\tilde m_1\tilde m_2$

Mechanism II c-2	$\Sigma(\tilde m_1, \tilde m_2, \tilde m_3)$
$f_i=0=(f_r, h_i)$	$(\tilde m_1 + \tilde m_2 - \tilde m_3)^2 - 4\tilde m_1 \tilde m_2$
$h_r=0=(f_r, h_i)$	$\left(\tilde m_1 + (21\varphi+13)\tilde m_2 - 5(3\varphi+2)\tilde m_3\right)^2 - (84\varphi+52)\tilde m_1\tilde m_2$
$h_{r,2}=0=(f_r, h_i)$	$\left(\tilde m_1 + (34-21\varphi)\tilde m_2 + 5(3\varphi-5)\tilde m_3\right)^2 + (84-136)\tilde m_1\tilde m_2$

Results

hep-ph/1811.09662

- Three free parameters: setting one or two vevs equal to zero.
- Allowed ordering and Majorana phases.
- Correlations between vevs: ratio mass splittings and reactor angle.
- Perturbation Theory: sum rules, total sum masses, $m_{\beta\beta}$ and m_β.

$|y| \ll |x|, |z|$

$$\left\{\sum_j m_j,\, m_\beta,\, m_{\beta\beta}\right\} \equiv \sqrt{(-1)^{\ell+1}\,\Delta m^2_{3\ell}}\,\left[a + b\,\sin^2\theta_{13} + \mathcal{O}(\sin^4\theta_{13})\right]$$

Mechanism I	$\sum m_j$	m_β	$m_{\beta\beta}$	Mechanism II a-1		$\sum m_j$	m_β	$m_{\beta\beta}$
$z=0$	[3.8, 5.2]	[1.1, −1.6]	[1.1, −1.6]	$Z=0$	$S\simeq X$	[2.3, 5.2]	[0.6, −0.2]	[0.6, 2.8]
					$S\simeq 44X$	[8, −10]	[2.6, −1.8]	[2.5, −1.8]
$x=0$	[3.8, 5.2]	[1.1, −1.6]	[1.1, −1.6]	$X=0$	$S\simeq -Z/4$	[2.3, 5.2]	[0.6, −0.2]	[0.6, 2.8]
					$S\simeq Z/3$	[8, −10]	[2.6, −1.8]	[2.5, −1.8]
					$S\simeq 2Z/3$	[8, −10]	[2.6, −1.8]	[2.5, −1.8]
$s=0$	[3.8, 5.2]	[1.1, −1.6]	[1.1, −1.6]	$S=0$	✗	✗	✗	✗

Mechanism II a-2

$g=0$		$\sum m_j$	m_β	$m_{\beta\beta}$	$h_i=0$		$\sum m_j$	m_β	$m_{\beta\beta}$
$f=0$	NO	[1.6, −1.9]	[0.3, 1.0]	[0.3, −0.2]	$f=0$	NO	[1.6, −1.9]	[0.3, 1.0]	[0.3, −0.2]
	IO	[2.0, 2.8]	[1.0, 1.5]	[1.0, 1.0]		IO	[2.0, 2.8]	[1.0, 1.0]	[1.0, 1.5]
$h_r=0$		[4.7, 0.04]	[1.5, 0.05]	[1.5, 0.05]	$h_r=0$		[4.2, 0.06]	[1.3, −1.7]	[1.3, −1.8]
$h_{r,2}=0$		[2.3, 6.8]	[1.0, 2.8]	[1.0, 2.5]	$h_{r,2}=0$		[2.3 − 0.2]	[1.0, −0.8]	[1.0, 0.5]

Mechanism II c-2

$f_r=0$		$\sum m_j$	m_β	$m_{\beta\beta}$	$h_i=0$		$\sum m_j$	m_β	$m_{\beta\beta}$
$f_i=0$	NO	[1.6, −2.6]	[0.3, 0.7]	[0.3, −0.5]	$f_i=0$	NO	[1.6, −0.5]	[0.3, 1.7]	[0.3, 0.6]
	IO	[2.0, 1.6]	[1.0, 0.7]	[1.0, 0.2]		IO	[2, 11]	[1.0, 7.4]	[1.0, 6.9]
$h_r=0$		[2.5, −3.9]	[0.7, −0.9]	[0.7, −1.1]	$h_r=0$		[2.5, −2.0]	[0.7, −0.3]	[0.7, −0.3]
$h_{r,2}=0$		[2.5, −7.6]	[0.7, −2.3]	[0.7, −2.5]	$h_{r,2}=0$		[2.5 − 3.3]	[0.7, −2.3]	[0.7, −2.5]

$h_{r,2}=0$ IO $f_i \simeq h_r/(2\sqrt{6})$ $h_{r,2}=0$ IO $f_i \simeq h_r/(2\sqrt{6})$

Results

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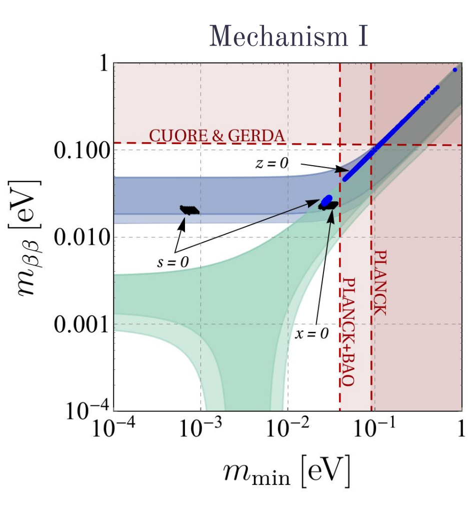
Mechanism I	$m_{\beta\beta}$ [meV]	Mechanism II a-1		$m_{\beta\beta}$ [meV]
$z=0$	55	$Z=0$	$S \simeq \pm X$	29
			$S \simeq \pm 44X$	125
$x=0$	23	$X=0$	$S \simeq -Z/4$	10
			$S \simeq Z/3$	0.4
			$S \simeq 2Z/3$	0.6
$s=0$	20	$S=0$	✗	✗

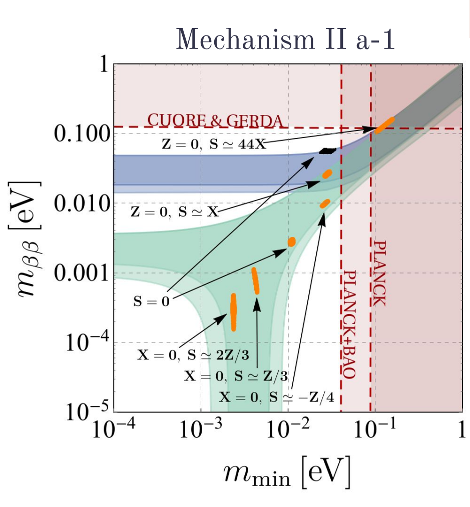
Mechanism II a-2

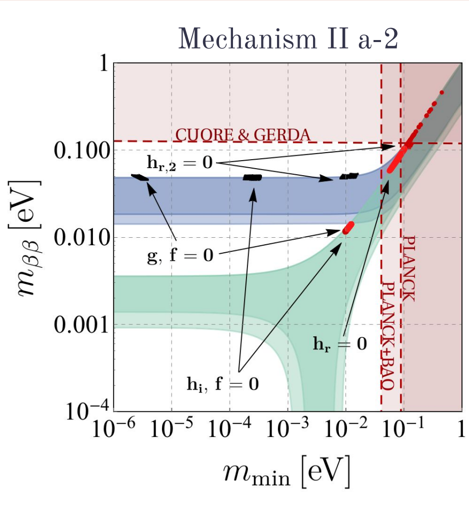
$g=0$		$m_{\beta\beta}$ [meV]	$h_i=0$		$m_{\beta\beta}$ [meV]
$f=0$	NO	14	$f=0$	NO	13
	IO	51		IO	51
$h_r=0$		62	$h_r=0$		74
$h_{r,2}=0$		53	$h_{r,2}=0$		50

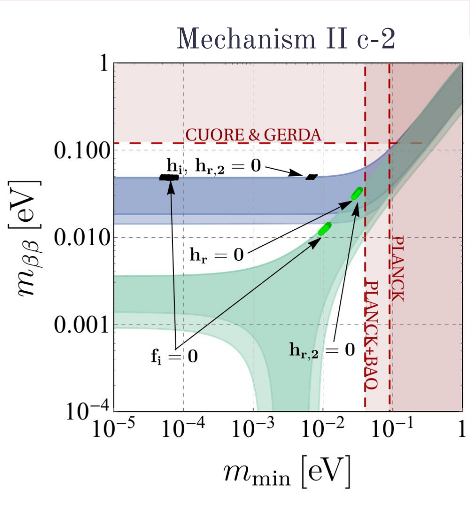
Mechanism II c-2

$f_r=0$		$m_{\beta\beta}$ [meV]	$h_i=0$		$m_{\beta\beta}$ [meV]
$f_i=0$	NO	12	$f_i=0$	NO	13
	IO	50		IO	57
$h_r=0$		32	$h_r=0$		33
$h_{r,2}=0$		30	$h_{r,2}=0$		32









Results

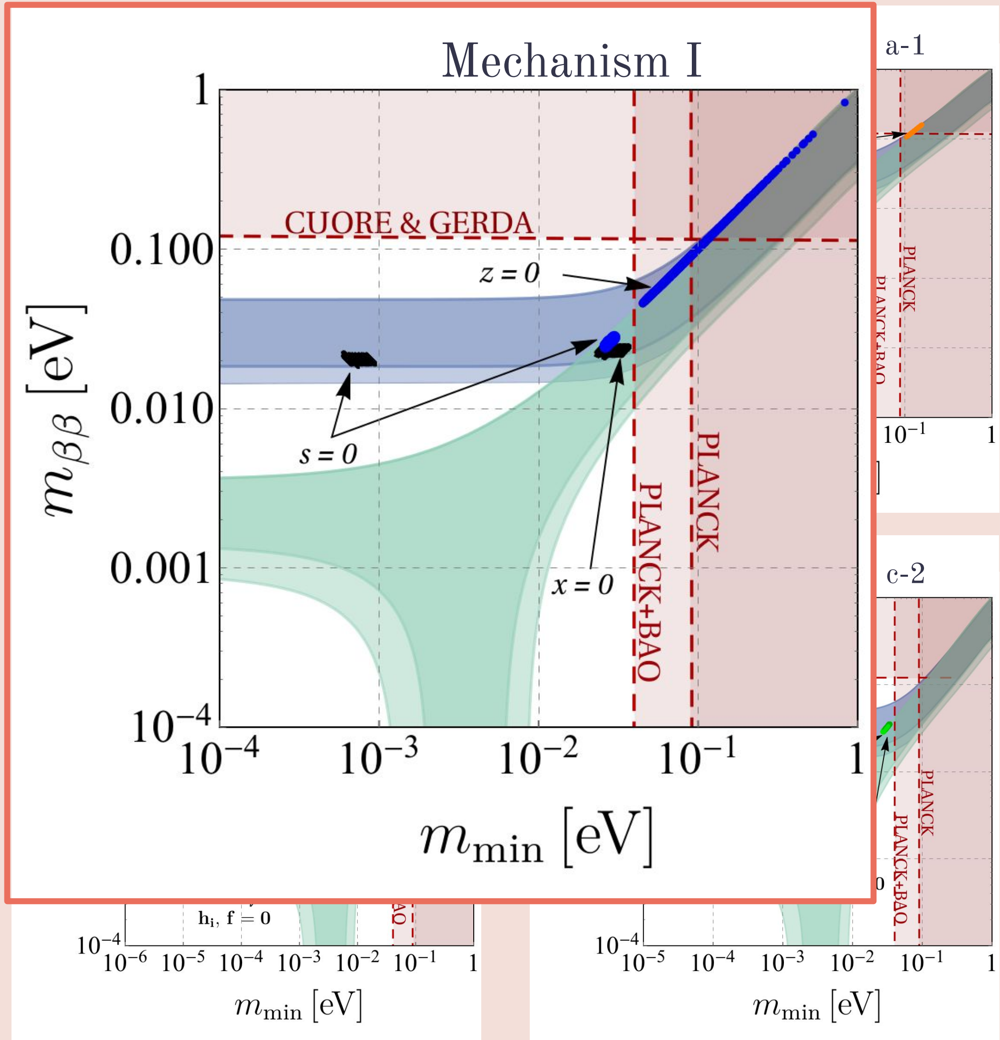
hep-ph/1811.09662

Mechanism I	$m_{\beta\beta}$ [meV]
$z = 0$	55
$x = 0$	23
$s = 0$	20

	$m_{\beta\beta}$ [meV]
	29
	125
	10
	0.4
	0.6
	✗

	$m_{\beta\beta}$ [meV]
	13
	51
	74
	50

Mechanism II c-2					
$f_r = 0$		$m_{\beta\beta}$ [meV]	$h_i = 0$		$m_{\beta\beta}$ [meV]
$f_i = 0$	NO	12	$f_i = 0$	NO	13
	IO	50		IO	57
$h_r = 0$		32	$h_r = 0$		33
$h_{r,2} = 0$		30	$h_{r,2} = 0$		32



Results

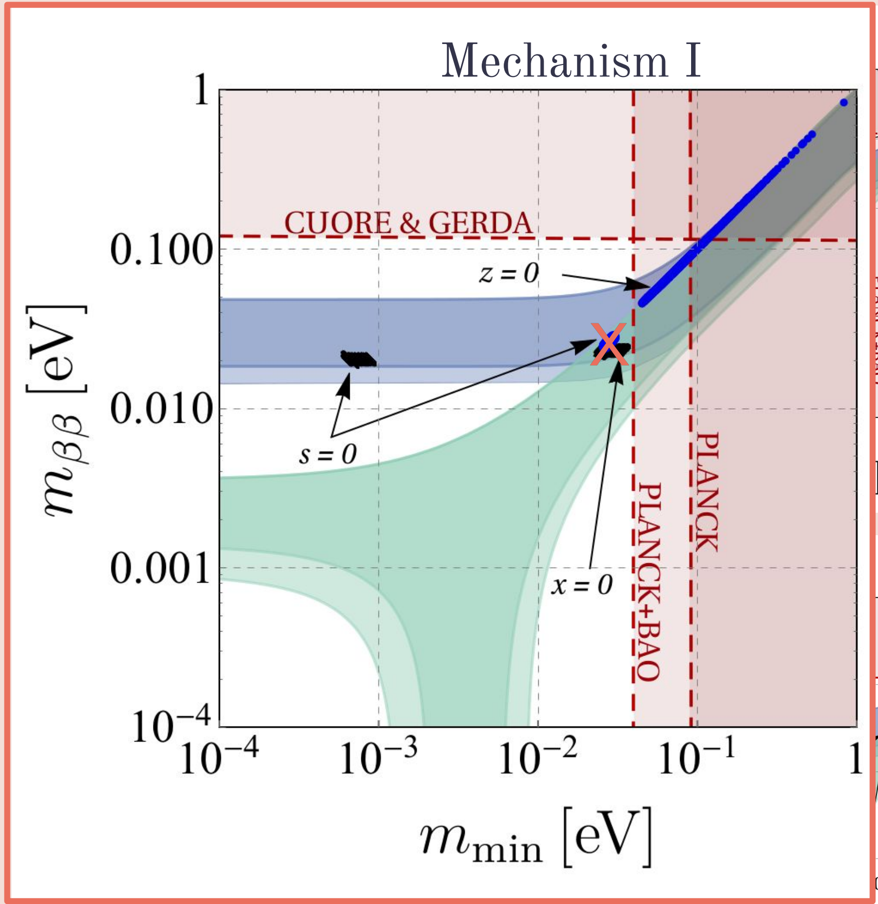
hep-ph/1811.09662

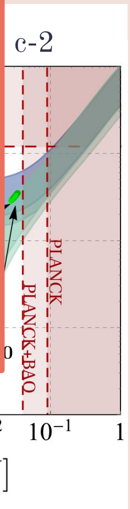
Mechanism I	$m_{\beta\beta}$ [meV]
$z = 0$	55
$x = 0$	23
$s = 0$	20

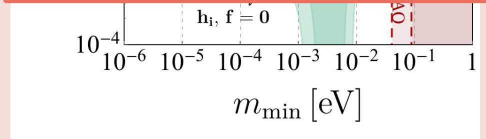
	$m_{\beta\beta}$ [meV]
	29
	125
	10
	0.4
	0.6
	✗

	$m_{\beta\beta}$ [meV]
	13
	51
	74
	50

Mechanism II c-2					
$f_r = 0$		$m_{\beta\beta}$ [meV]	$h_i = 0$		$m_{\beta\beta}$ [meV]
$f_i = 0$	NO	12	$f_i = 0$	NO	13
	IO	50		IO	57
$h_r = 0$		32	$h_r = 0$		33
$h_{r,2} = 0$		30	$h_{r,2} = 0$		32







Results

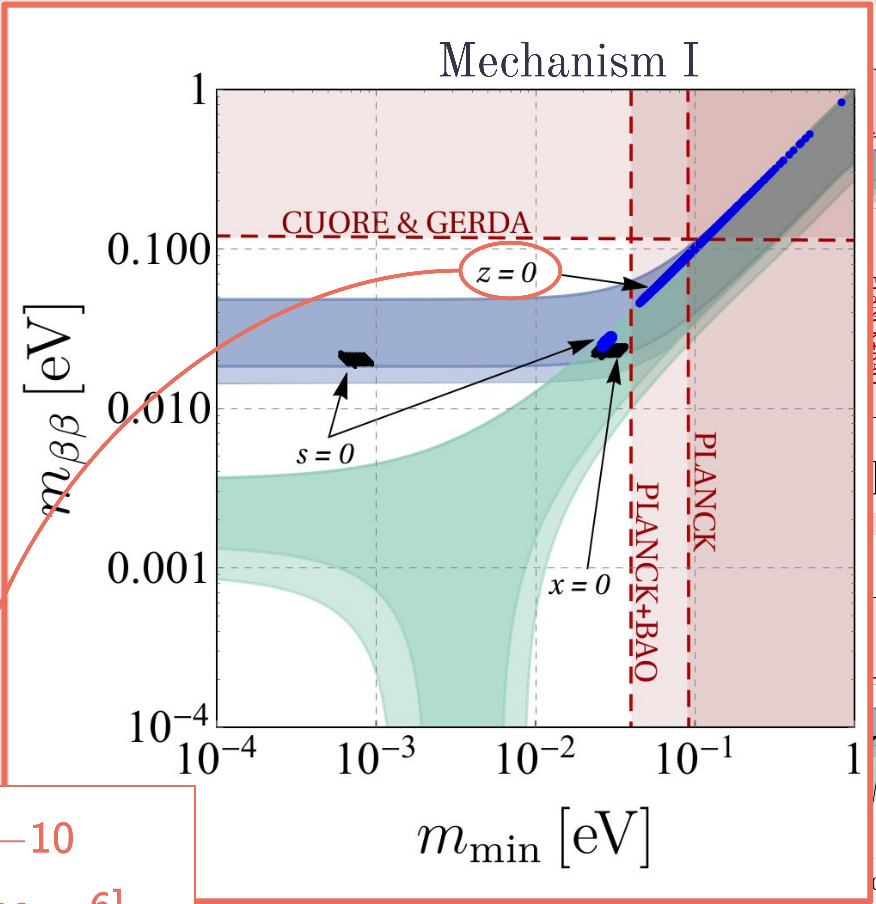
hep-ph/1811.09662

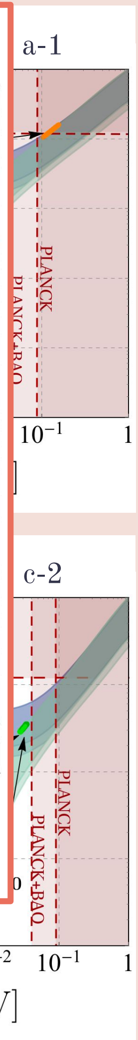
Mechanism I	$m_{\beta\beta}$ [meV]
$z = 0$	55
$x = 0$	23
$s = 0$	20

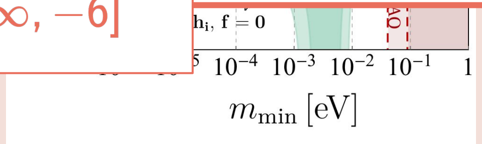
	$m_{\beta\beta}$ [meV]
	29
	125
	10
	0.4
	0.6
	✗

	$m_{\beta\beta}$ [meV]
	13
	51
	74

Mechanism II c-2			
$f_r = 0$	$m_{\beta\beta}$ [meV]	$h_i =$	
$f_i = 0$ NO	12	$f_i =$	
$f_i = 0$ IO	50		
$h_r = 0$	32	$h_r = 0$	33
$h_{r,2} = 0$	30	$h_{r,2} = 0$	32







$s/x = -10$

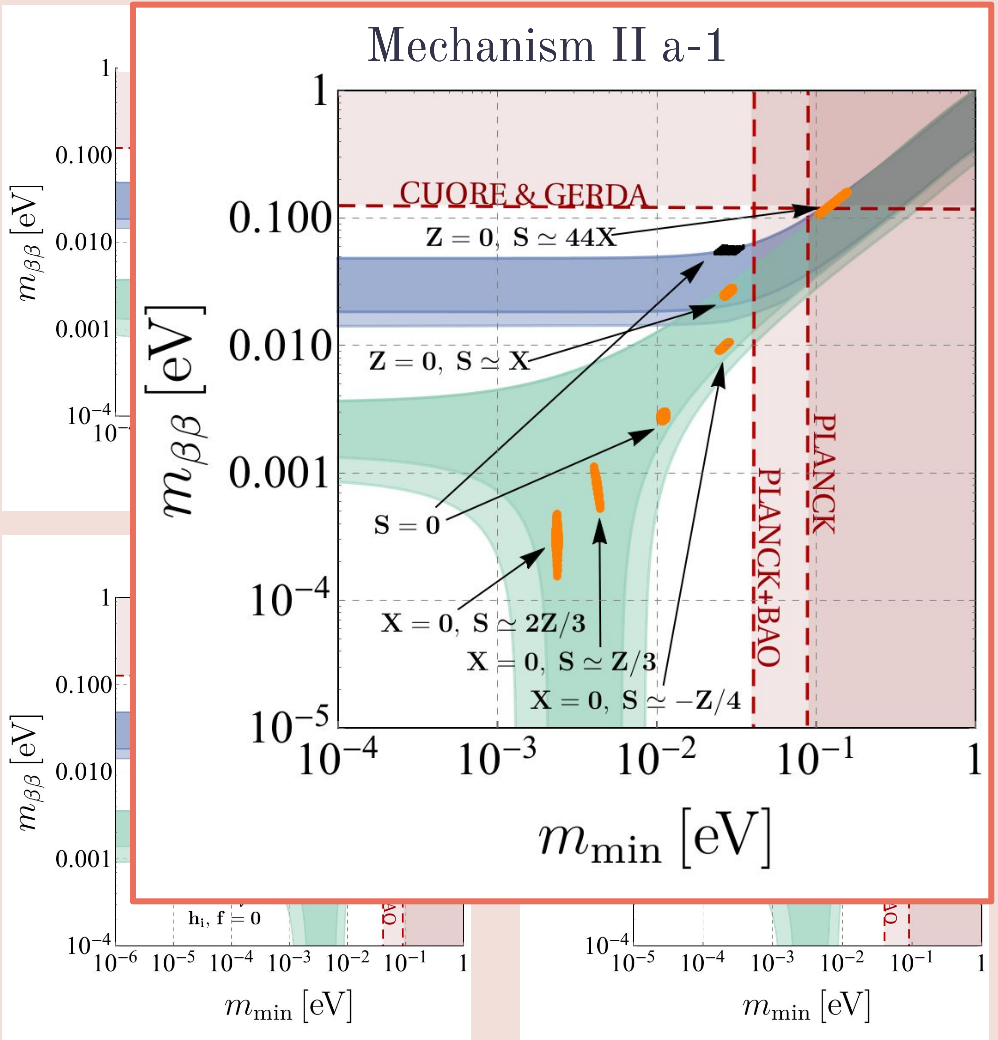
$s/x \in [-\infty, -6]$

Results

hep-ph/1811.09662

Mechanism II a-1		$m_{\beta\beta}$ [meV]
$Z = 0$	$S \simeq \pm X$	29
	$S \simeq \pm 44X$	125
$X = 0$	$S \simeq -Z/4$	10
	$S \simeq Z/3$	0.4
	$S \simeq 2Z/3$	0.6
$S = 0$	✗	✗

$f_r = 0$		$m_{\beta\beta}$ [meV]	$h_i = 0$		$m_{\beta\beta}$ [meV]
$f_i = 0$	NO	12	$f_i = 0$	NO	13
	IO	50		IO	57
$h_r = 0$		32	$h_r = 0$		33
$h_{r,2} = 0$		30	$h_{r,2} = 0$		32



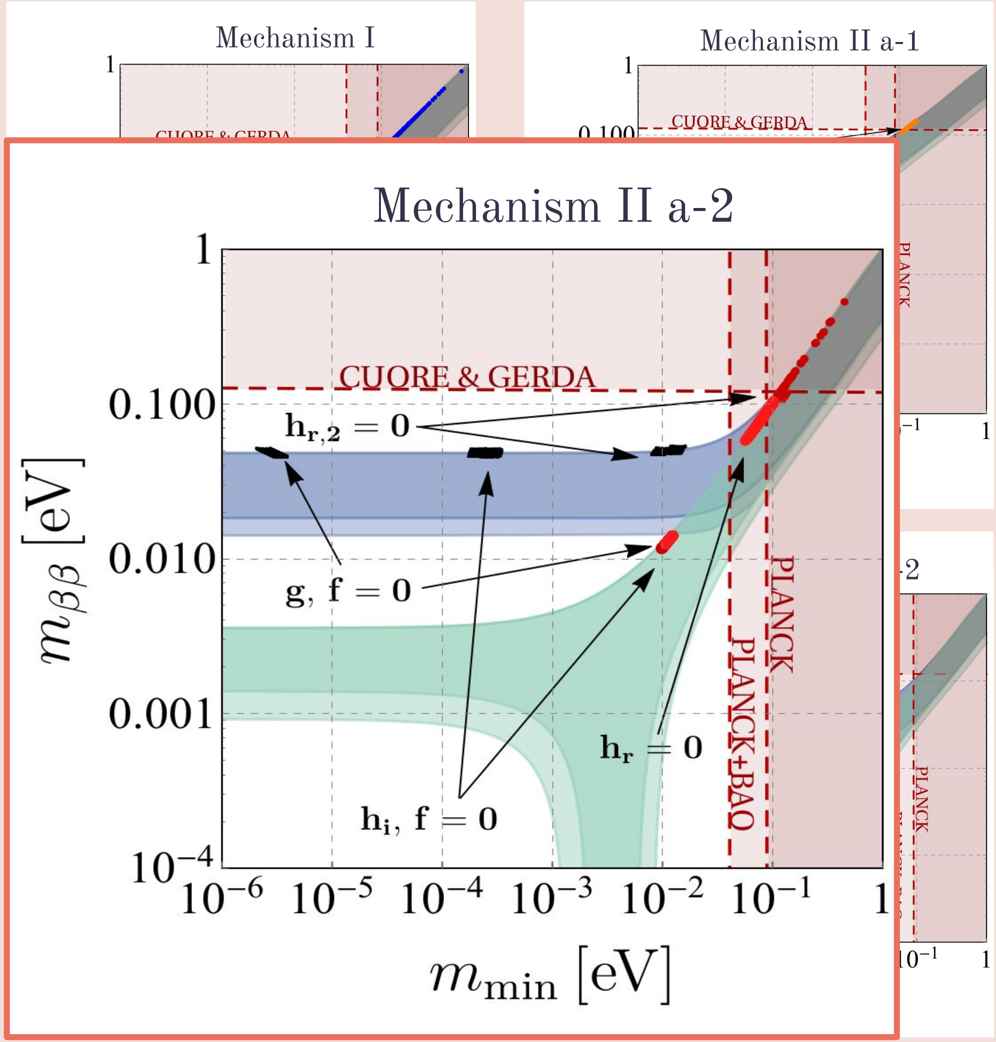
Results

hep-ph/1811.09662

Mechanism I	$m_{\beta\beta}$ [meV]	Mechanism II a-1		$m_{\beta\beta}$ [meV]
$z = 0$	55	$Z = 0$	$S \simeq \pm X$	29
			$S \simeq \pm 44X$	125
$x = 0$	23	$X = 0$	$S \simeq -Z/4$	10
			$S \simeq Z/3$	0.4
			$S \simeq 2Z/3$…	0.…

Mechanism II a-2					
$g = 0$		$m_{\beta\beta}$ [meV]	$h_i = 0$		$m_{\beta\beta}$ [meV]
$f = 0$	$h_r \simeq 0$	14	$f = 0$	$h_r \simeq 0$	13
	$h_r \simeq -4h_{r,2}$	51		$h_r \simeq -4h_{r,2}$	51
$h_r = 0$		62	$h_r = 0$		74
$h_{r,2} = 0$		53	$h_{r,2} = 0$		50

Mechanism II c-2					
$f_r = 0$		$m_{\beta\beta}$ [meV]	$h_i = 0$		$m_{\beta\beta}$ [meV]
$f_i = 0$	NO	12	$f_i = 0$	NO	13
	IO	50		IO	57
$h_r = 0$		32	$h_r = 0$		33
$h_{r,2} = 0$		30	$h_{r,2} = 0$		32



Results

hep-ph/1811.09662

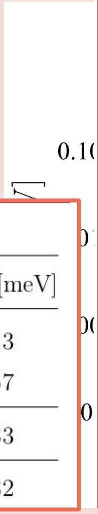
Mechanism I	$m_{\beta\beta}$ [meV]	Mechanism II a-1		$m_{\beta\beta}$ [meV]
$z = 0$	55	$Z = 0$	$S \simeq \pm X$	29
			$S \simeq \pm 44X$	125
$x = 0$	23	$X = 0$	$S \simeq -Z/4$	10
			$S \simeq Z/3$	0.4
			$S \simeq 2Z/3$	0.6
$s = 0$	20	$S = 0$	✗	✗

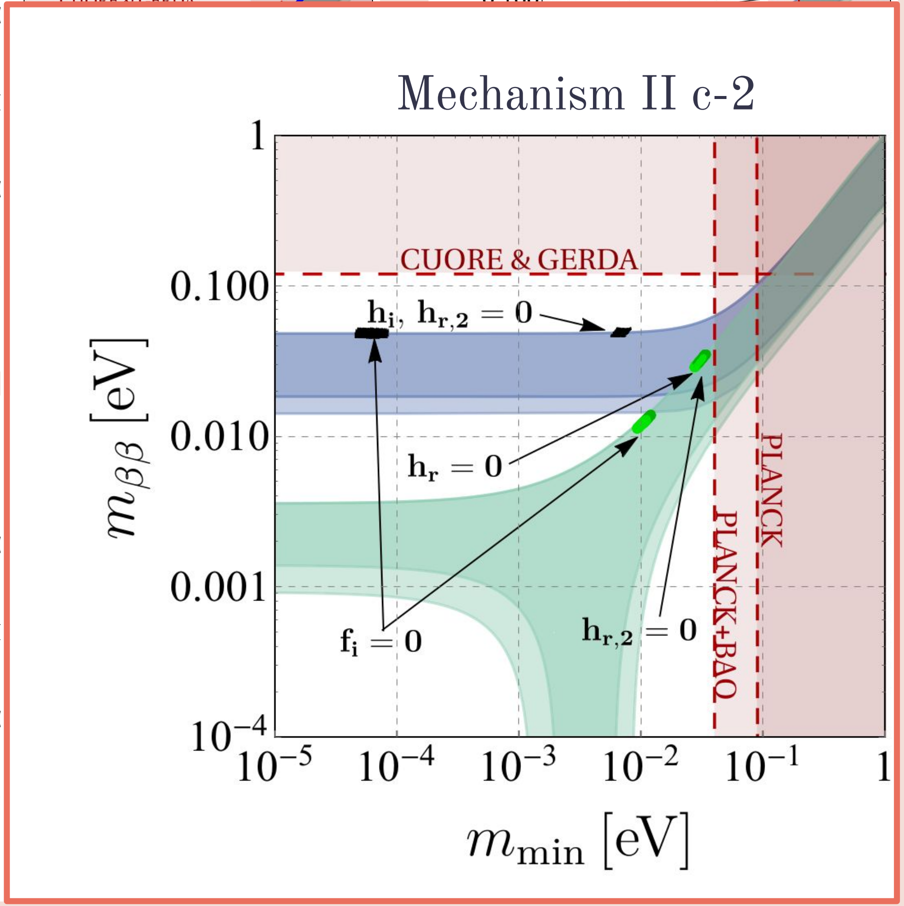
Mechanism II a-2

$g = 0$		$m_{\beta\beta}$ [meV]	$h_i = 0$		$m_{\beta\beta}$ [meV]
$f = 0$	NO	14	$f = 0$	NO	13

Mechanism II c-2

$f_r = 0$		$m_{\beta\beta}$ [meV]	$h_i = 0$		$m_{\beta\beta}$ [meV]
$f_i = 0$	$h_r \simeq -h_{r,2}$	12	$f_i = 0$	$h_r \simeq -h_{r,2}$	13
	$h_r \simeq +h_{r,2}$	50		$h_r \simeq +h_{r,2}$	57
$h_r = 0$		32	$h_r = 0$		33
$h_{r,2} = 0$		30	$h_{r,2} = 0$		32





BSM + FLASY

Flavour Symmetries

in BSM models.

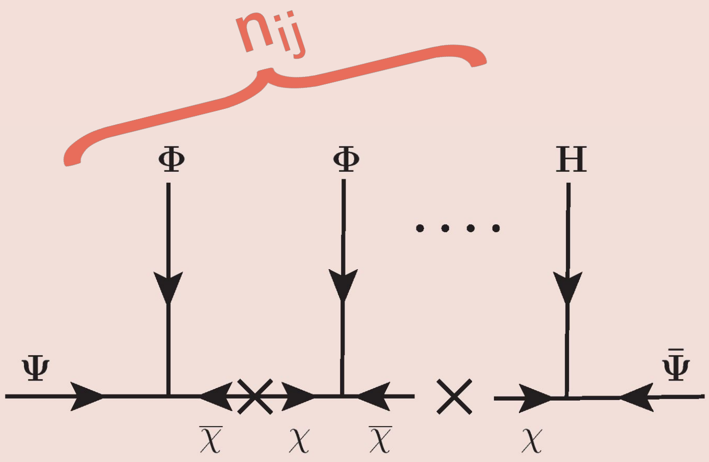
one example!
SUGRA + FLASY

Froggatt-Nielsen Mechanism

Flavour Symmetries
in BSM models.

one example!
SUGRA + FLASY

Froggatt-Nielsen Mechanism

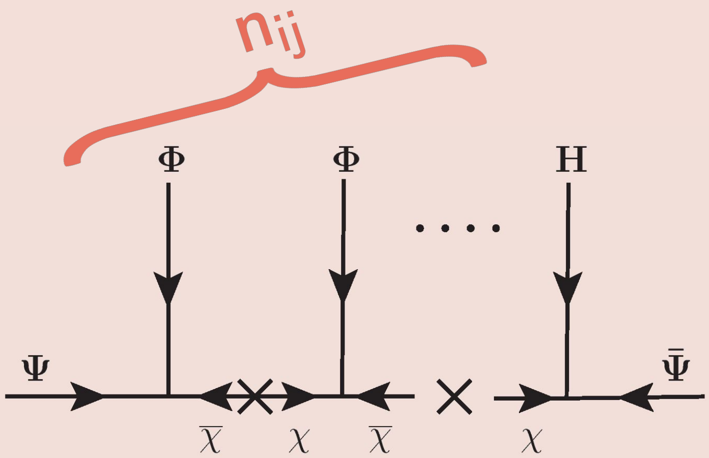


for Yukawas

$$Y_{ij} = \left(\frac{\langle \Phi \rangle}{M_\chi}\right)^{n_{ij}} \equiv \epsilon^{n_{ij}} \ll 1$$

Flavour Symmetries
in BSM models.

$$\mathsf{L}_{\text{soft}} \supseteq \frac{\langle X \rangle}{M_{\text{Planck}}} \times W_{\text{MSSM}}$$



one example!
SUGRA + FLASY

Froggatt-Nielsen Mechanism

for Yukawas

$$Y_{ij} = \left(\frac{\langle \Phi \rangle}{M_\chi}\right)^{n_{ij}} \equiv \epsilon^{n_{ij}} \ll 1$$

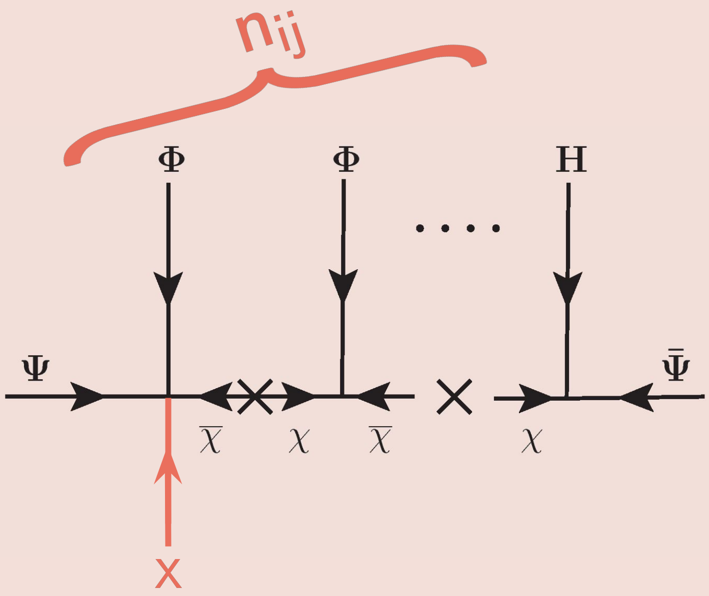
for trilinear couplings

$$A_{ij} \propto a_0 \frac{\langle X \rangle}{M_{\text{Planck}}} Y_{ij}$$

Flavour Symmetries *in BSM models.*

one example!
SUGRA + FLASY

$$L_{soft} \supseteq \frac{\langle X \rangle}{M_{Planck}} \times W_{MSSM}$$



Froggatt-Nielsen Mechanism

for Yukawas

$$Y_{ij} = \left(\frac{\langle \Phi \rangle}{M_\chi}\right)^{n_{ij}} \equiv \epsilon^{n_{ij}} \ll 1$$

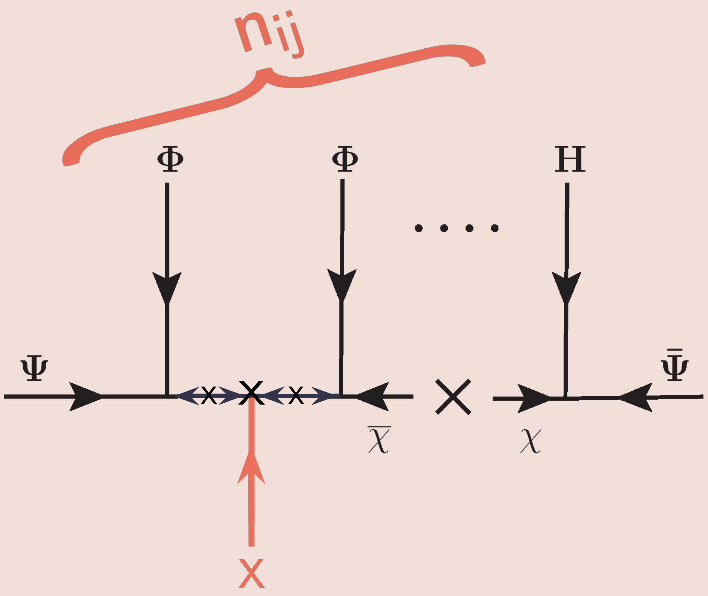
for trilinear couplings

$$A_{ij} \propto a_0 \frac{\langle X \rangle}{M_{Planck}} Y_{ij}$$

Flavour Symmetries *in BSM models.*

one example! SUGRA + FLASY

$$L_{soft} \supseteq \frac{\langle X \rangle}{M_{Planck}} \times W_{MSSM}$$



Froggatt-Nielsen Mechanism

for Yukawas

$$Y_{ij} = \left(\frac{\langle \Phi \rangle}{M_\chi}\right)^{n_{ij}} \equiv \epsilon^{n_{ij}} \ll 1$$

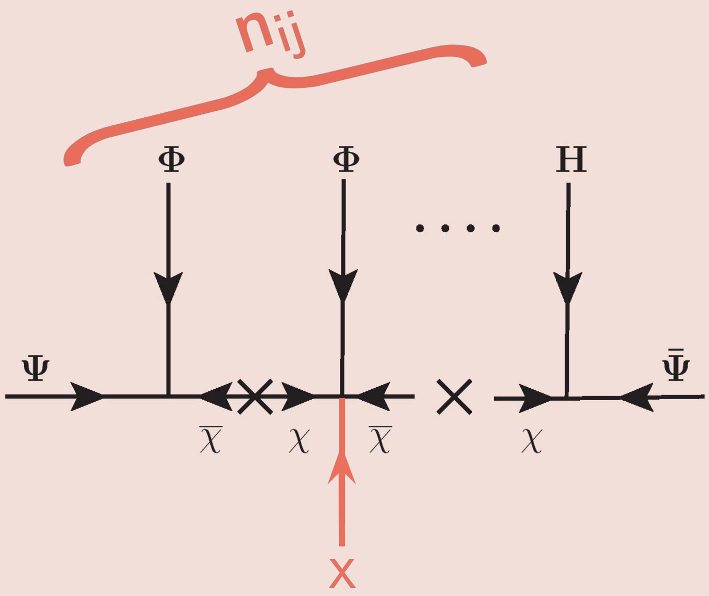
for trilinear couplings

$$A_{ij} \propto a_0 \frac{\langle X \rangle}{M_{Planck}} Y_{ij}$$

Flavour Symmetries
in BSM models.

one example!
SUGRA + FLASY

$$L_{soft} \supseteq \frac{\langle X \rangle}{M_{Planck}} \times W_{MSSM}$$



Froggatt-Nielsen Mechanism

for Yukawas

$$Y_{ij} = \left(\frac{\langle \Phi \rangle}{M_\chi}\right)^{n_{ij}} \equiv \epsilon^{n_{ij}} \ll 1$$

for trilinear couplings

$$A_{ij} \propto a_0 \frac{\langle X \rangle}{M_{Planck}} Y_{ij}$$

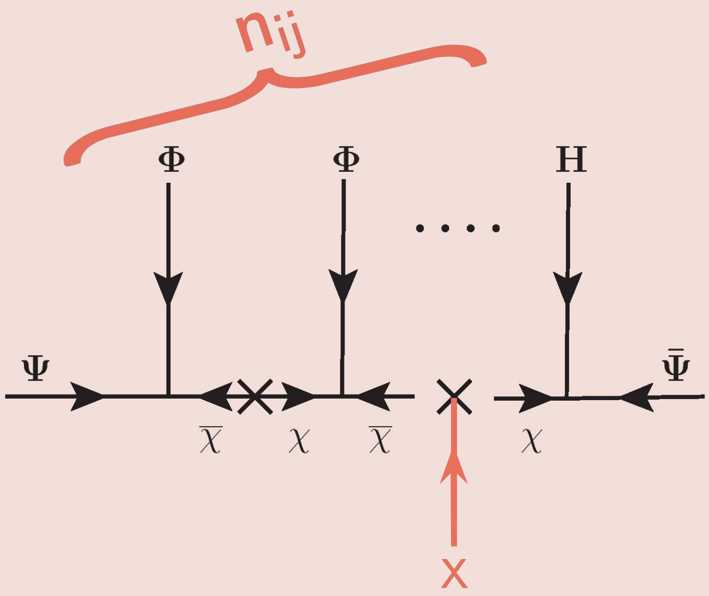
Flavour Symmetries

in BSM models.

one example!

SUGRA + FLASY

$$\mathcal{L}_{soft} \supseteq \frac{\langle X \rangle}{M_{Planck}} \times W_{MSSM}$$



Froggatt-Nielsen Mechanism

for Yukawas

$$Y_{ij} = \left(\frac{\langle \Phi \rangle}{M_\chi}\right)^{n_{ij}} \equiv \epsilon^{n_{ij}} \ll 1$$

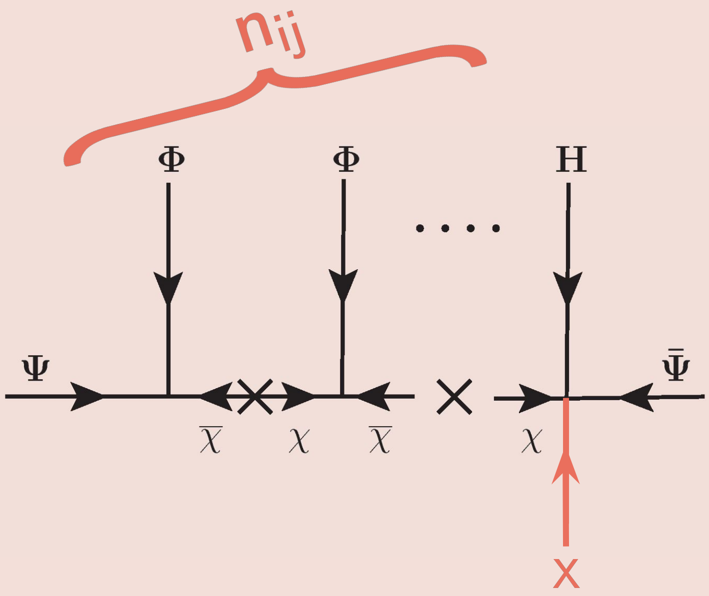
for trilinear couplings

$$A_{ij} \propto a_0 \frac{\langle X \rangle}{M_{Planck}} Y_{ij}$$

Flavour Symmetries
in BSM models.

one example!
SUGRA + FLASY

$$L_{soft} \supseteq \frac{\langle X \rangle}{M_{Planck}} \times W_{MSSM}$$



Froggatt-Nielsen Mechanism

for Yukawas

$$Y_{ij} = \left(\frac{\langle \Phi \rangle}{M_\chi}\right)^{n_{ij}} \equiv \epsilon^{n_{ij}} \ll 1$$

for trilinear couplings

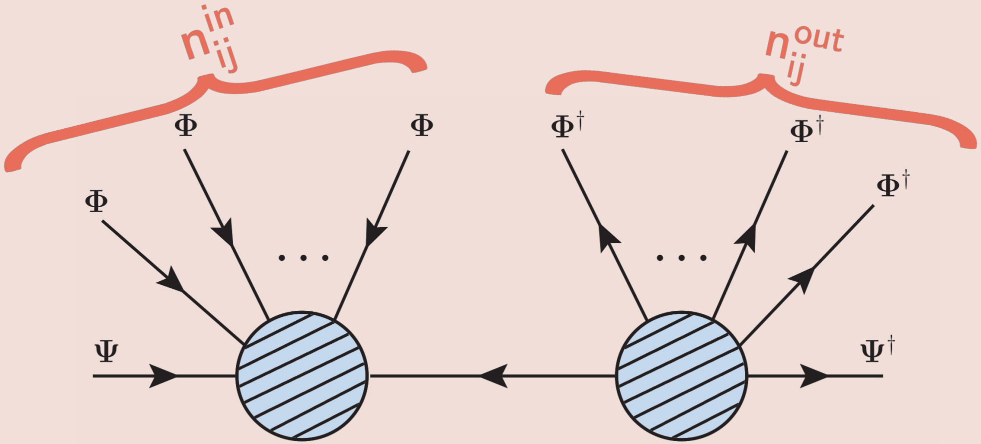
$$A_{ij} \propto a_0 \frac{\langle X \rangle}{M_{Planck}} Y_{ij}$$

$$c_{ij} = 2n_{ij} + 1$$

Flavour Symmetries

in BSM models.

one example!
SUGRA + FLASY



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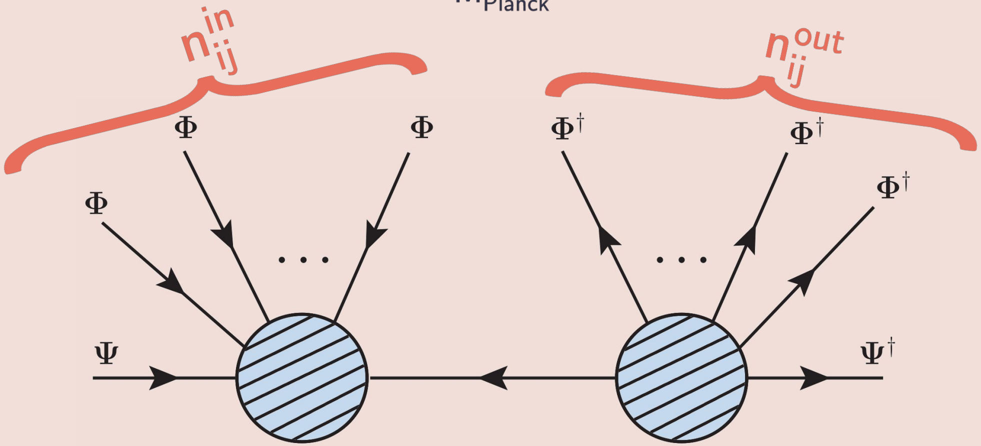

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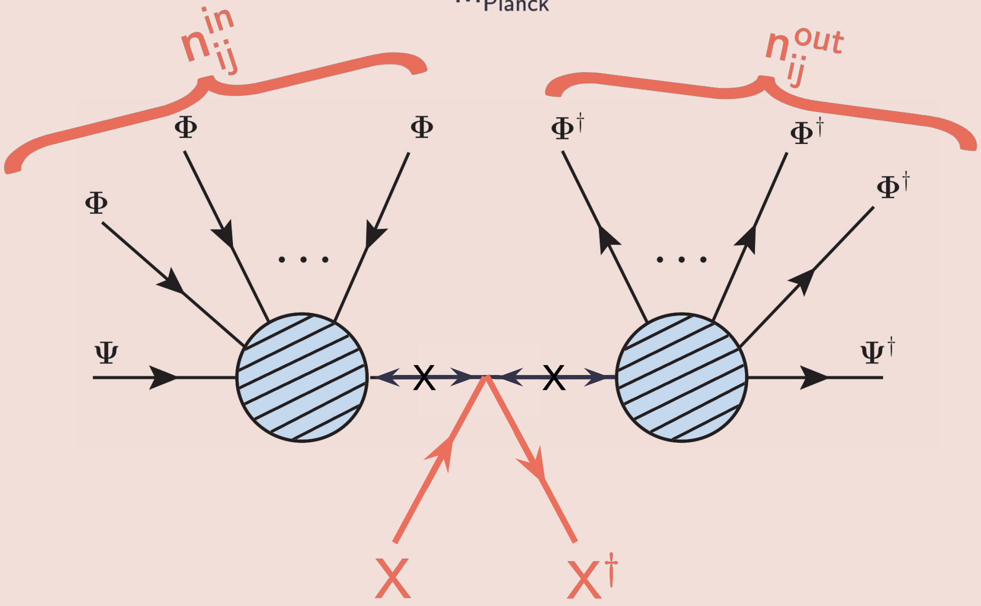

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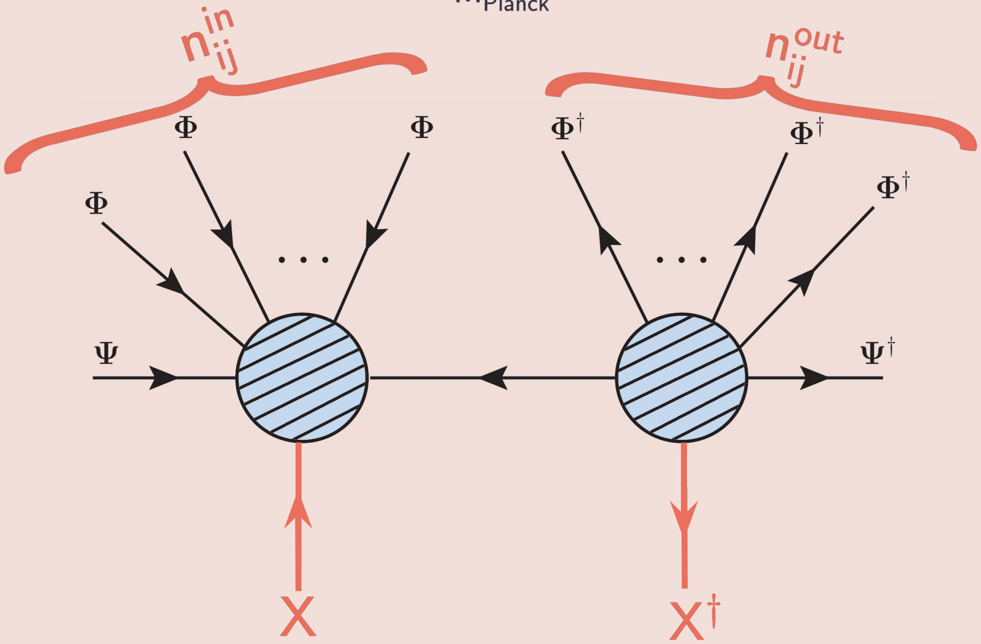

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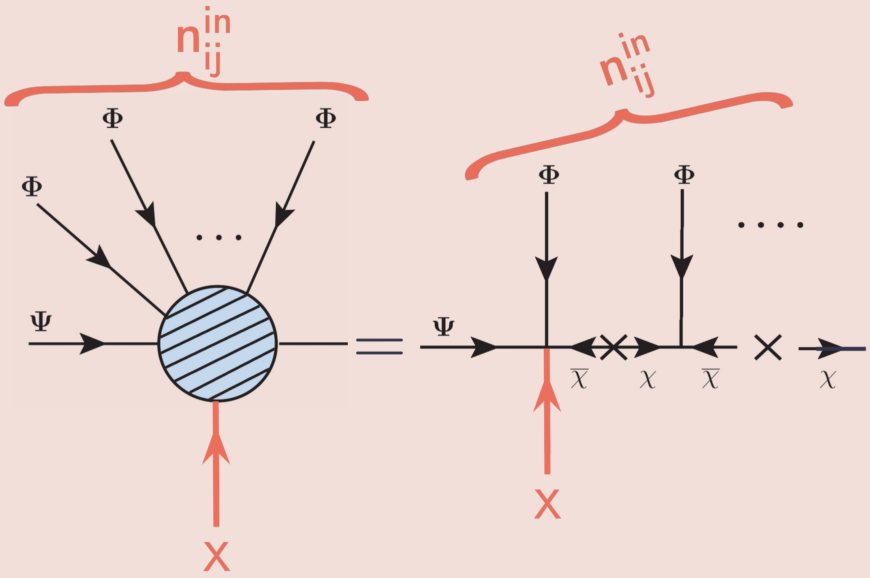

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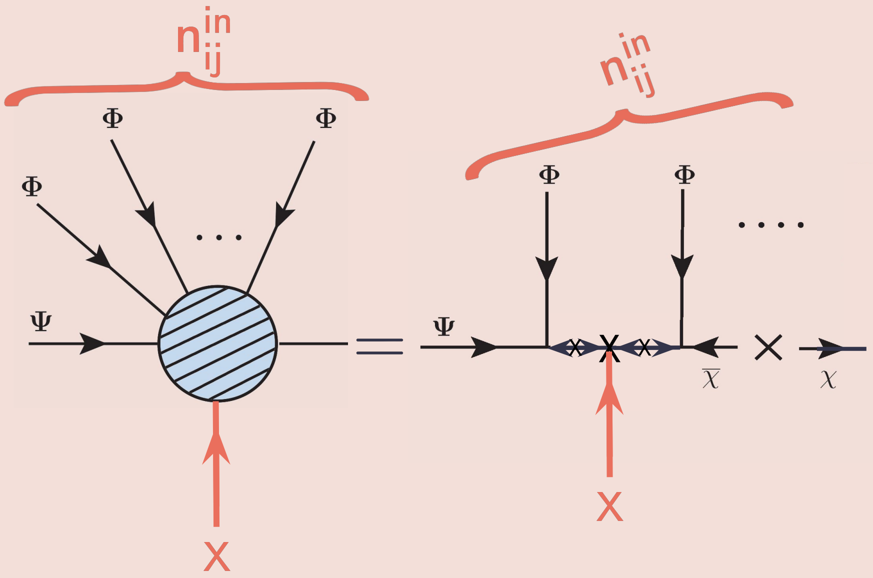

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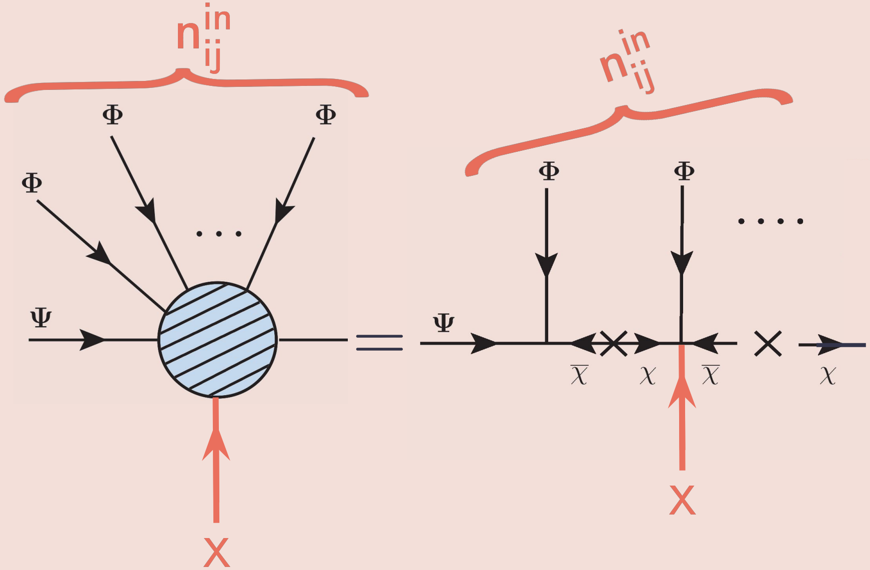

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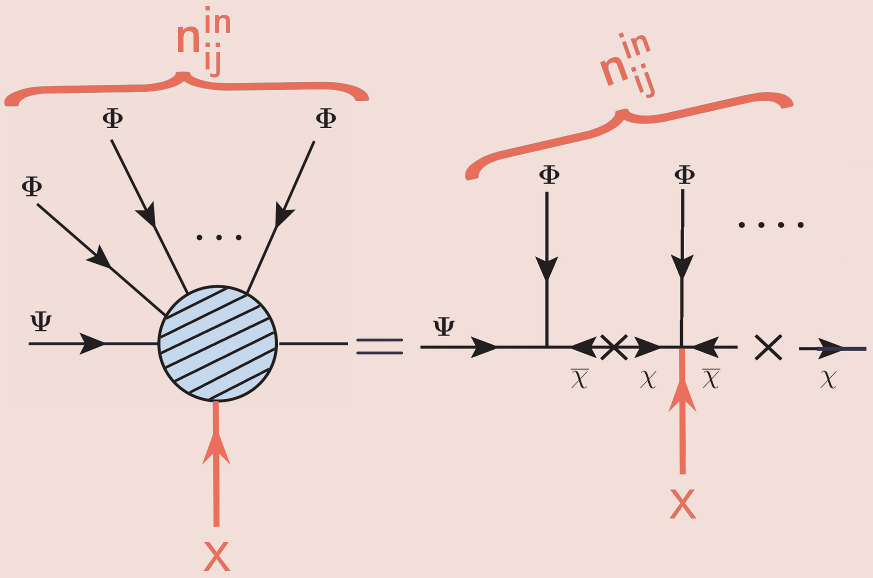

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for soft-mass terms

$$\widetilde{m}^2_{ij} \propto \frac{\langle X\rangle\langle X^\dagger\rangle}{\Lambda^2} K_{ij}$$

$$f_{ij} = (2n^{in}_{ij} - 1)(2n^{out}_{ij} - 1) + 1$$

Flavour Symmetries

in BSM models.

one example!
SUGRA + FLASY

$$Y = \begin{pmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{pmatrix}$$

$$A = a_0\, m_0 \begin{pmatrix} c_{11}\, y_{11} & c_{12}\, y_{12} & c_{13}\, y_{13} \\ c_{21}\, y_{21} & c_{22}\, y_{22} & c_{23}\, y_{23} \\ c_{31}\, y_{31} & c_{32}\, y_{32} & c_{33}\, y_{33} \end{pmatrix}$$

$$K = \begin{pmatrix} K_{11} & K_{12} & K_{13} \\ K^\star_{12} & K_{22} & K_{23} \\ K^\star_{13} & K^\star_{23} & K_{33} \end{pmatrix}$$

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$$\widetilde{m}^2_{ij} = f_{ij} \frac{\langle X \rangle \langle X^\dagger \rangle}{\Lambda} K_{ij}$$

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Froggatt-Nielsen Mechanism

Physical Basis

Diagonal Yukawas
Kähler metric = Identity

Off-diagonal terms SURVIVE in the soft-terms

NEW EFFECTS CAN BE TESTED IN
FLAVOUR VIOLATING PROCESSES

Flavour Symmetries

in BSM models.

SUGRA + A_5 and CP

$$K_L = \left[\mathbf{1} + \frac{1}{M_\chi^2} \sum_r \left(\phi_r^{e\,\dagger} \phi_r^e + \phi_r^{\nu\dagger} \phi_r^\nu \right) \right] \times$$
$$\left[L^\dagger L + \frac{1}{M_\chi^2} \sum \left(\left[L^\dagger \phi_r^{e\dagger} \right]_1 \left[L \phi_r^e \right]_1 + \left[L^\dagger \phi_r^{\nu\dagger} \right]_1 \left[L \phi_r^\nu \right]_1 \right) + \ldots \right]$$

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Froggatt-Nielsen Mechanism

$\{ \phi_1^e, \phi_3^e, \phi_5^e \}$ → Diagonal Yukawas and Kähler

~~trilinears~~

~~soft masses~~

$\{ \phi_1^\nu, \phi_3^\nu, \phi_{3'}^\nu, \phi_4^\nu, \phi_5^\nu \}$ → Off-diagonal terms in Kähler!

soft masses!

Flavour Symmetries
in BSM models.

SUGRA + $\mathcal{A}_5$ and CP

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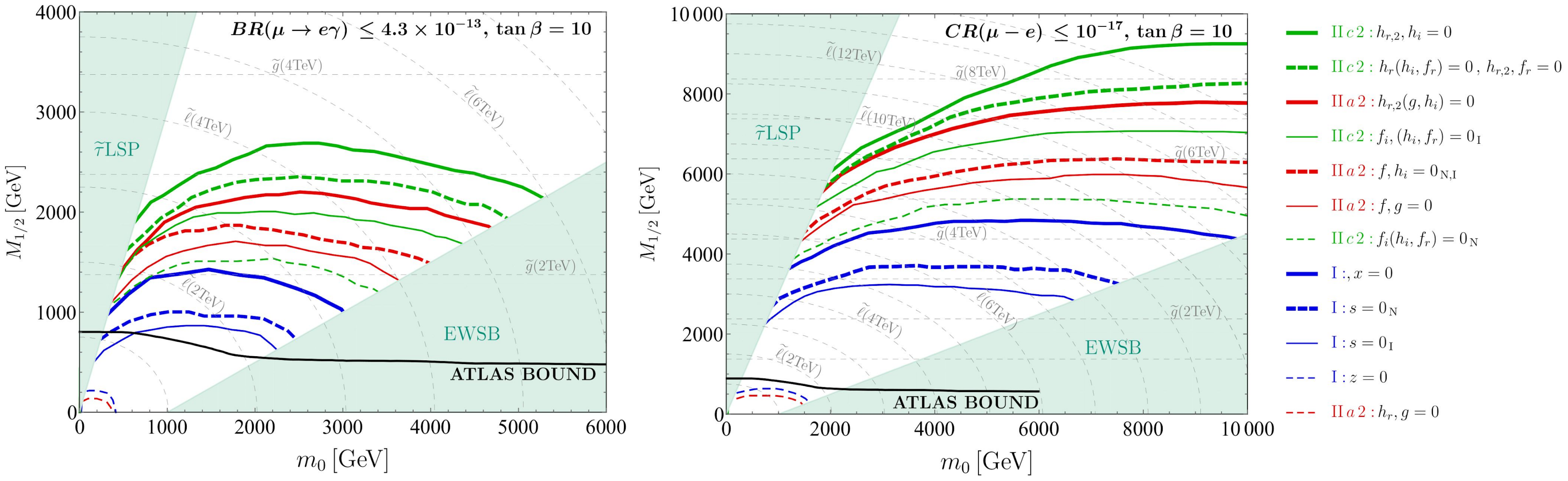
soft masses!

Results

JHEP 06 (2019) 047

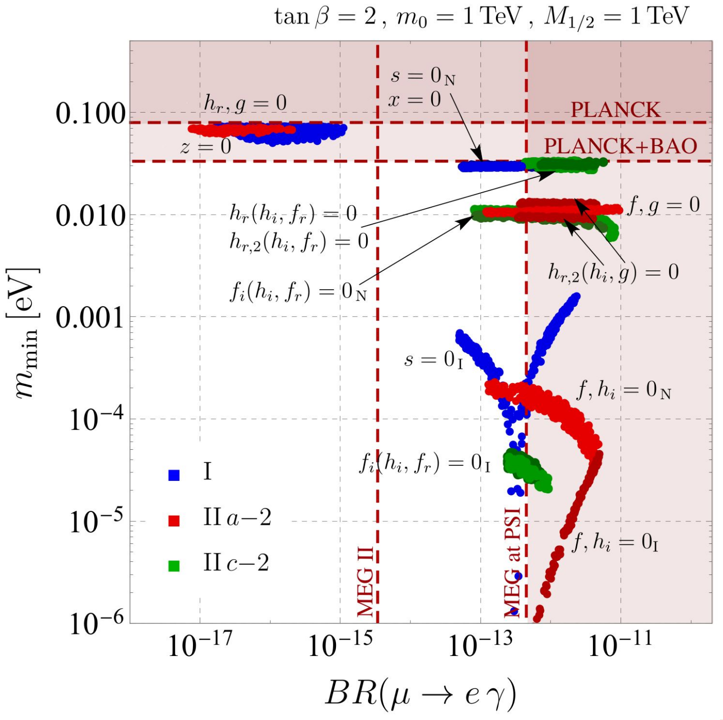
Flavour violating effects from soft terms can set bounds over the SUSY spectrum, even in the most conservative scenario where SUSY breaking is due to one universal spurion field. Condition: $\Lambda_{SUSY} > \Lambda_f$

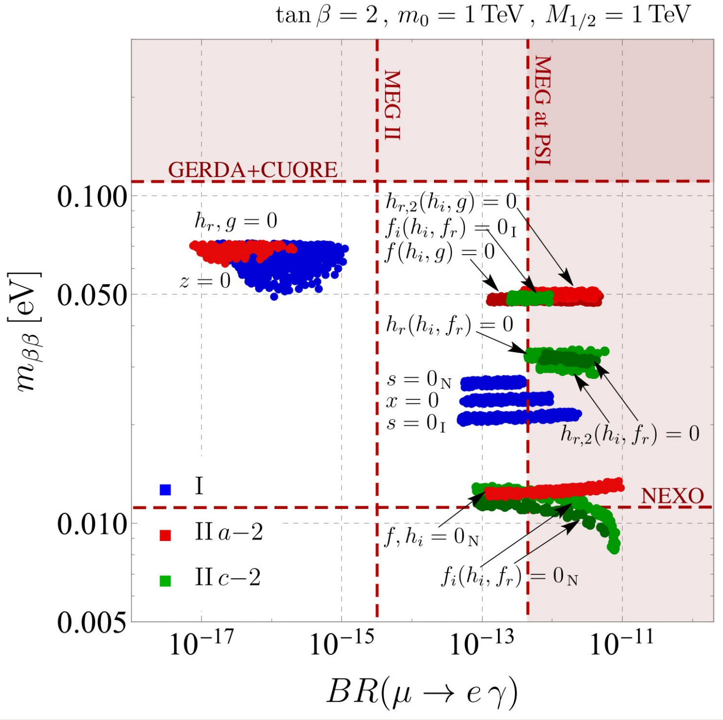


Results

JHEP 06 (2019) 047

Relations among LFV observables in the charged sector and the neutrino mass observables.
This allows to disentangle cases that initially were not distinguishable.





Conclusions

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- The flavour sector remains one of the most puzzling legacies of the SM. Family symmetries may help to disentangle the origin masses and mixing.

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- In **SUSY**, the introduction of family symmetries may help to explore the susy mass spectrum **beyond LHC sensitivity** and give related predictions for different sectors.

Conclusions

Thank you!

- The flavour sector remains one of the most puzzling legacies of the SM. Family symmetries may help to disentangle the origin masses and mixing.

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Back Up

Results

JHEP 06 (2019) 047

- SUSY breaking transmission ~ M_{Planck}
- Flavour scale ~ GUT scale ~ $\Lambda_{\not{f}}$

  .
  .
  .

  [evolution ~ RGE] (SARAH, SPheno)

  .
  .

- EW scale ~ observables

LFV process	Current Bound	Future Bound
$BR(\mu \to e\gamma)$	4.2×10^{-13} (MEG at PSI)	6×10^{-14} (MEG II)
$BR(\mu \to eee)$	1.0×10^{-12} (SINDRUM)	10^{-16} (Mu3e)
$CR(\mu - e)_{A_l}$	–	10^{-17} (Mu2e, COMET)
$BR(\tau \to e\gamma)$	3.3×10^{-8} (BaBar)	5×10^{-9} (Belle II)
$BR(\tau \to \mu\gamma)$	4.4×10^{-8} (BaBar)	10^{-9} (Belle II)
$BR(\tau \to eee)$	2.7×10^{-8} (Belle)	5×10^{-10} (Belle II)
$BR(\tau \to \mu\mu\mu)$	2.1×10^{-8} (Belle)	5×10^{-10} (Belle II)