Generalised CP Symmetry and Lepton Mixing

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Collaboration with : Salvador Centelles Chuliá, Gui-Jun Ding, Rahul Srivastava, José W. F. Valle

Outline

- General Discussion
  - Generalized CP Symmetry
  - μ-τ reflection
  - Generalised μ-τ reflection
- Two-Flavor GCP of Charged Leptons
  - Generalized e-μ of charged leptons
  - Generalized e-τ of charged leptons
  - Generalized μ-τ of charged leptons
  - $\Delta(6n^2)$ as an origin of two flavor GCP
- Revamping TBM according to GCP
- Summary

General Discussion

The success of the standard model promotes us to pursue the physic behind it.

The Flavor Puzzle:
- Fermion mass hierarchy
- Flavor mixing structure
- CP violation
- Neutrinos: Dirac vs Majorana
- 3 family ... ...

Flavor Mixing Puzzle:

Flavor mixing structure $\Leftarrow$
- Flavor symmetry : A_4 , S_4 , $\Delta(6n^2)$, ...
- Generalised CP Symmetry : μ-τ Reflection , Generalised μ-τ , ...
- Flavor + GCP : $G_f \rtimes H_{CP}$

Generalised CP Symmetry

Generalised CP(GCP) :

$$\psi \xrightarrow{CP} iX_\psi \gamma^0 C\bar{\psi}^T\,, \qquad \begin{cases} X_\psi^T m_\psi X_\psi = m_\psi^*, & \text{Majorana} \\ X_\psi^\dagger m_\psi^\dagger m_\psi X_\psi = (m_\psi^\dagger m_\psi)^*, & \text{Dirac} \end{cases}$$

with the GCP matrix X_ψ a 3×3 unitary symmetric matrix.
Remnant GCP :

$$\begin{cases} X_\nu = U_\nu \,\mathrm{diag}(\pm 1, \pm 1, \pm 1)\, U_\nu^T\,, \\ X_l = U_l \,\mathrm{diag}(e^{i\beta_e}, e^{i\beta_\mu}, e^{i\beta_\tau})\, U_l^T\,, \end{cases}$$

The mass diagonalise matrix U_ψ can be deduced from X_ψ using the takagi factorization method:

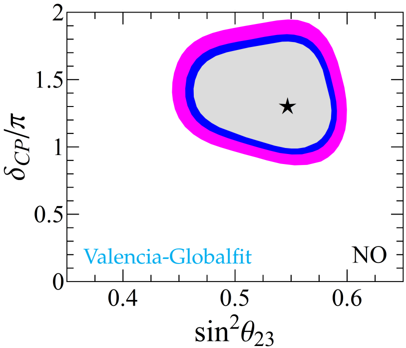
$$X_\psi = \Sigma_\psi \Sigma_\psi^T\,, \quad \Rightarrow \quad U_\psi = \Sigma_\psi O_{3\times 3} P\,,$$

with P diagonal phase matrix and $O_{3\times 3}$ real orthogonal matrix.

μ-τ reflection

Neutrino μ-τ reflection ($CP_{\mu\tau}$): [Harrison, Scott, 2002;Grimus, Lavoura, 2004]

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} {\nu_{eL}}^c \\ {\nu_{\mu L}}^c \\ {\nu_{\tau L}}^c \end{pmatrix}, \quad \Rightarrow \quad \begin{aligned} \theta_{23} &= \frac{\pi}{4}, \\ \delta_{CP} &= \pm\frac{\pi}{2}. \end{aligned}$$



Generalised μ-τ reflection:

$$X_{\mu\tau} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\Theta & i\sin\Theta \\ 0 & i\sin\Theta & \cos\Theta \end{pmatrix},$$

$$\Downarrow$$

$$\sin^2\delta_{CP}\sin^2 2\theta_{23} = \sin^2\Theta,$$

$$\sin\alpha_{21} = \sin\alpha'_{31} = 0,$$

[Chen, Ding, Gonzalez-Canales, Valle, 2015]

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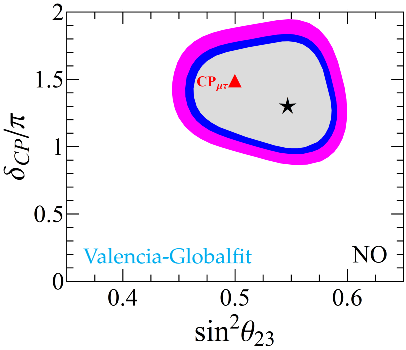

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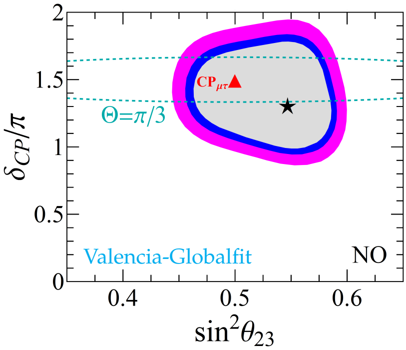

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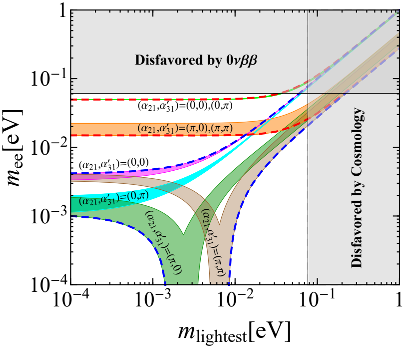
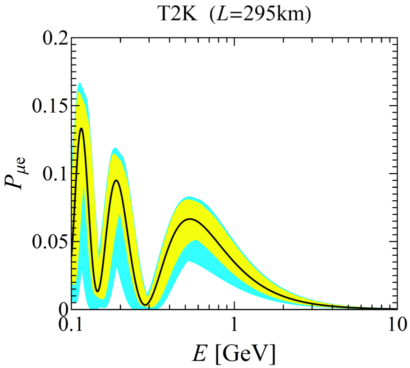
Generalised μ-τ reflection





Example: $\Theta = \frac{\pi}{3}$

$$X_{\mu\tau} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & i\sqrt{3}/2 \\ 0 & i\sqrt{3}/2 & 1/2 \end{pmatrix},$$

Numerical example: $\chi^2 = 0.015$

$$\theta_1 = 0.720\pi\,, \quad \theta_2 = 0.047\pi\,, \quad \theta_3 = 0.809\pi\,,$$
$$\sin^2\theta_{12} = 0.320\,, \quad \sin^2\theta_{13} = 0.0216\,,$$
$$\sin^2\theta_{23} = 0.547\,, \quad \delta_{CP} = 1.336\pi\,.$$

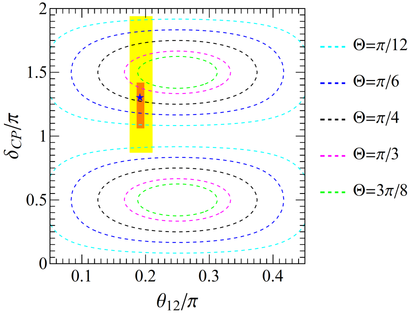
Two-Flavor GCP of Charged Leptons

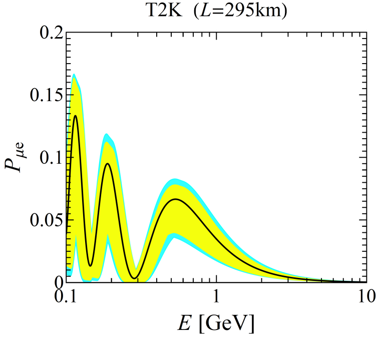
Generalised e-μ:

$$\begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} \Rightarrow \underbrace{\begin{pmatrix} \cos\Theta & i\sin\Theta & 0 \\ i\sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{X_{e\mu}} \begin{pmatrix} {e_L}^c \\ {\mu_L}^c \\ {\tau_L}^c \end{pmatrix}, \qquad \Sigma_{e\mu} = \begin{pmatrix} \cos\frac{\Theta}{2} & i\sin\frac{\Theta}{2} & 0 \\ i\sin\frac{\Theta}{2} & \cos\frac{\Theta}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$U = \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3}) O_3 \Sigma_{e\mu}^\dagger$$

$$\theta_{12} - \delta_{CP} \quad \text{correlation} : \qquad \sin^2 2\theta_{12} \sin^2\delta_{CP} = \sin^2\Theta$$

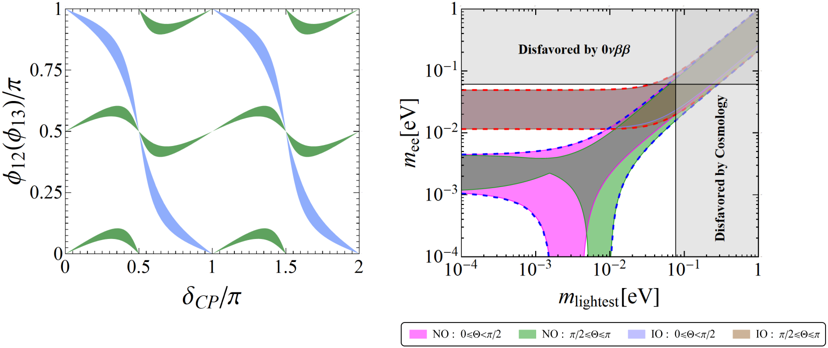




Generalised e-μ of charged leptons

Generalised e-μ reflection:

Correlations :
$$\begin{cases} \sin 2\phi_{12} = -\dfrac{\cos 2\theta_{12} \sin 2\delta_{CP}}{1-\sin^2 2\theta_{12}\sin^2\delta_{CP}}\,, \\ \sin^2 2\phi_{13} = \dfrac{\sin^2 2\delta_{CP}\sin^4\theta_{12}}{1-\sin^2 2\theta_{12}\sin^2\delta_{CP}}\,, \end{cases}$$



Generalised e-τ of charged leptons

Generalised e-τ reflection:

$$X_{e\tau} = \begin{pmatrix} \cos\Theta & 0 & i\sin\Theta \\ 0 & 1 & 0 \\ i\sin\Theta & 0 & \cos\Theta \end{pmatrix},$$

Mixing parameters:

$$\begin{aligned}
\sin^2\theta_{13} &= \cos^2\theta_2\cos^2\theta_3\sin^2\frac{\Theta}{2} + \sin^2\theta_2\cos^2\frac{\Theta}{2}\,, \\
\sin^2\theta_{12} &= \frac{\sin^2\theta_3\cos^2\theta_2}{1-\cos^2\theta_2\cos^2\theta_3\sin^2\frac{\Theta}{2} - \sin^2\theta_2\cos^2\frac{\Theta}{2}}\,, \\
\sin^2\theta_{23} &= \frac{\sin^2\frac{\Theta}{2}\left(\sin\theta_1\sin\theta_2\cos\theta_3 + \sin\theta_3\cos\theta_1\right)^2 + \sin^2\theta_1\cos^2\theta_2\cos^2\frac{\Theta}{2}}{1-\cos^2\theta_2\cos^2\theta_3\sin^2\frac{\Theta}{2} - \sin^2\theta_2\cos^2\frac{\Theta}{2}}\,, \\
J_{CP} &= \frac{1}{4}\sin\theta_3\cos\theta_2\sin\Theta\left(\sin2\theta_1\left(\cos^2\theta_3 - \sin^2\theta_2\sin^2\theta_3\right) + \sin\theta_2\sin2\theta_3\cos2\theta_1\right), \\
I_1 &= \sin^2\theta_3\cos^2\theta_2\sin\theta_2\cos\theta_2\cos\theta_3\sin\Theta\,, \\
I_2 &= \frac{1}{4}\left[4\sin\theta_2\cos^3\theta_2\cos^3\theta_3\sin\Theta + 4\sin^3\theta_2\cos\theta_2\cos\theta_3\sin\Theta\right].
\end{aligned}$$

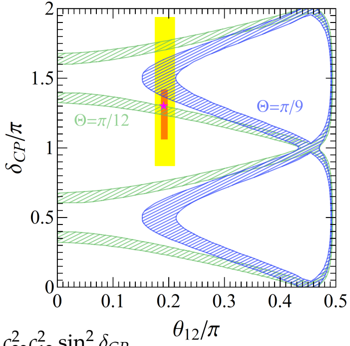
correlations ?

Generalised e-τ of charged leptons

e-τ correlations ?

Notice that Generalised e-μ and Generalised e-τ are related by

$$X_{e\tau} = P_{23} X_{e\mu} P_{23}\,,$$



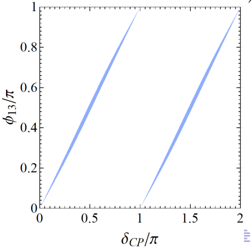
as a result, from the e-μ correlation: $\sin^2 2\theta_{12} \sin^2 \delta_{CP} = \sin^2 \Theta$ we can deduce the following relation:

$$\sin^2\Theta = \frac{4 s_{13}^2 c_{13}^2 s_{23}^2 c_{23}^2 c_{12}^2 \sin^2\delta_{CP}}{s_{23}^2 c_{23}^2 (c_{12}^2 + s_{12}^2 s_{13}^2)^2 \sin^2\delta_{CP} + \left(s_{23} c_{23} (c_{12}^2 - s_{12}^2 s_{13}^2) \cos\delta_{CP} + s_{13} s_{12} c_{12} \cos 2\theta_{23}\right)^2}\,,$$

In a similar way:

$$\sin^2 2\phi_{12} \simeq \frac{2\sin^4\theta_{13} \sin 2\delta_{CP} \sin\delta_{CP}}{\cos^4\theta_{12}} \Big[\cos\delta_{CP} - 4\sin\theta_{13} \tan\theta_{12} \cot 2\theta_{23} \cos 2\delta_{CP}\Big]\,,$$

$$\sin 2\phi_{13} \simeq \sin 2\delta_{CP} - 4\sin\theta_{13} \tan\theta_{12} \cot 2\theta_{23} \sin\delta_{CP} \cos 2\delta_{CP}$$



Generalised μ-τ of charged leptons

Generalised μ-τ reflection:

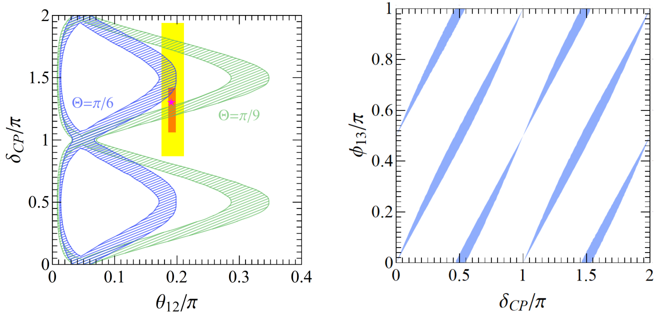
$$X_{\mu\tau} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\Theta & i\sin\Theta \\ 0 & i\sin\Theta & \cos\Theta \end{pmatrix},$$

correlations:

$$\sin^2\Theta \simeq \frac{4\sin^2\theta_{13}\sin^2\delta_{CP}}{\sin^2\theta_{12}}\left[1 + 4\sin\theta_{13}\cot\theta_{12}\cot 2\theta_{23}\cos\delta_{CP}\right],$$

$$\sin^2 2\phi_{12} \simeq \frac{2\sin^4\theta_{13}\sin 2\delta_{CP}\sin\delta_{CP}}{\sin^4\theta_{12}}\left[\cos\delta_{CP} + 4\sin\theta_{13}\cot\theta_{12}\cot 2\theta_{23}\cos 2\delta_{CP}\right],$$

$$\sin 2(\phi_{13} - \phi_{12}) \simeq \sin 2\delta_{CP} + 4\sin\theta_{13}\cot\theta_{12}\cot 2\theta_{23}\sin\delta_{CP}\cos 2\delta_{CP}\,.$$



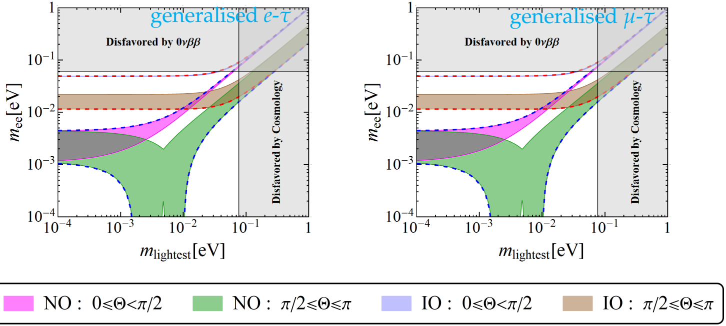
Generalised μ-τ of charged leptons

The relation between $X_{\mu\tau}$ and $X_{e\tau}$

$$X_{\mu\tau} = P_{12} X_{e\tau} P_{12}\,,$$

$$P_{12} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \begin{array}{c} \theta_{13}^{\mu\tau} = \theta_{13}^{e\tau}\,, \quad \theta_{23}^{\mu\tau} = \theta_{23}^{e\tau}\,, \quad \theta_{12}^{\mu\tau} = \pi/2 - \theta_{12}^{e\tau}\,, \\ \delta_{CP}^{\mu\tau} = \delta_{CP}^{e\tau} + \pi\,, \quad \phi_{12}^{\mu\tau} = -\phi_{12}^{e\tau}\,, \quad \phi_{13}^{\mu\tau} = \phi_{13}^{e\tau} - \phi_{12}^{e\tau}\,. \end{array}$$

As a result: $|m_{ee}^{\mu\tau}| = |m_{ee}^{e\tau}|(m_1 \leftrightarrow m_2) \quad \Rightarrow \quad |m_{ee}^{\mu\tau}| \approx |m_{ee}^{e\tau}|$.



$\Delta(6n^2)$ as an origin of two flavor GCP

$3_{1,1}$ representation of $\Delta(6n^2)$ generators:

$$a = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad b = -\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad c = \begin{pmatrix} \eta & 0 & 0 \\ 0 & \eta^{-1} & 0 \\ 1 & 0 & 1 \end{pmatrix}, \quad d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \eta & 0 \\ 1 & 0 & \eta^{-1} \end{pmatrix},$$

with $\eta = e^{\frac{2\pi i}{n}}$. Consider the case $\Delta(6n^2)$ flavor symmetry is broken down to K_4 in the neutrino sector.

- $X_l = c^{\xi} d^{\psi}$, $G_\nu = K_4(c^{\frac{n}{2}}, abc^y)$, $X_{\nu_r} = \{c^\gamma d^{2y+2\gamma}, abc^{y+\gamma} d^{2y+2\gamma}\}$

In the neutrino diagonal basis: [Ding, King, Neder, 2014]

$$X_{\text{lepton}} = \begin{pmatrix} e^{-i\vartheta}\cos\Theta & ie^{-i\vartheta}\sin\Theta & 0 \\ ie^{-i\vartheta}\sin\Theta & e^{-i\vartheta}\cos\Theta & 0 \\ 0 & 0 & e^{2i\vartheta} \end{pmatrix}.$$

with $\Theta = \frac{2(y+\xi)-\psi}{n}\pi$ and $\vartheta = \frac{2(y+\gamma)-\psi}{n}\pi$.

Revamping TBM According to GCP

Complex TBM Mixing

$$U_{cTBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{e^{-i\rho}}{\sqrt{3}} & 0 \\ -\frac{e^{i\rho}}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{e^{-i\sigma}}{\sqrt{2}} \\ \frac{e^{i(\rho+\sigma)}}{\sqrt{6}} & -\frac{e^{i\sigma}}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix},$$

Generalised CP

$$X_i = U_{cTBM}\, \hat{d}_i\, U^T_{cTBM},$$

$$i = 1,2,3,4,$$

$\hat{d}_1 = \text{diag}\,(1,\text{-}1,\text{-}1)$, $\hat{d}_2 = \text{diag}\,(\text{-}1,1,\text{-}1)$,

$\hat{d}_3 = \text{diag}\,(\text{-}1,\text{-}1,1)$, $\hat{d}_4 = \text{diag}\,(1,1,1)$,

Variants of the TBM mixing pattern can be obtained when the neutrino mass matrix respects only a partial set of the above CP symmetries X_i. As an example we consider X_2 and X_3 are partially preserved, which gives the following generalised version of the TBM ansatz (gTBM):

$$U_{gTBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{e^{-i\rho}\cos\theta}{\sqrt{3}} & -\frac{ie^{-i\rho}\sin\theta}{\sqrt{3}} \\ -\frac{e^{i\rho}}{\sqrt{6}} & \frac{\cos\theta}{\sqrt{3}} - \frac{ie^{-i\sigma}\sin\theta}{\sqrt{2}} & \frac{e^{-i\sigma}\cos\theta}{\sqrt{2}} - \frac{i\sin\theta}{\sqrt{3}} \\ \frac{e^{i(\rho+\sigma)}}{\sqrt{6}} & -\frac{e^{i\sigma}\cos\theta}{\sqrt{3}} - \frac{i\sin\theta}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} + \frac{ie^{i\sigma}\sin\theta}{\sqrt{3}} \end{pmatrix}.$$

Revamping TBM according to GCP

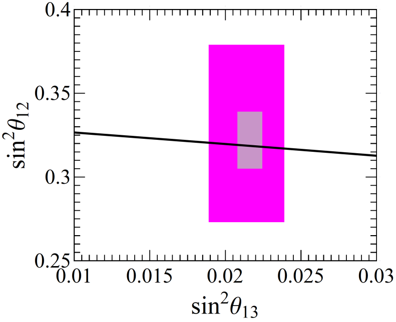
gTBM mixing parameters:

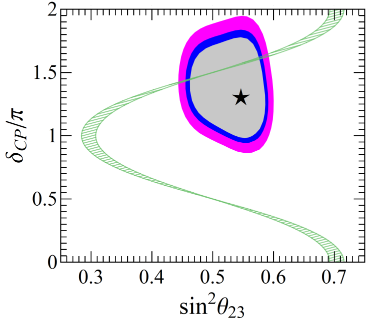
$$\sin^2\theta_{12} = \frac{\cos^2\theta}{\cos^2\theta+2}\,, \qquad \sin^2\theta_{23} = \frac{1}{2} + \frac{\sqrt{6}\sin 2\theta\sin\sigma}{2\cos^2\theta+4}\,, \qquad \sin^2\theta_{13} = \frac{\sin^2\theta}{3}\,,$$

$$\tan\delta_{CP} = \frac{(\cos^2\theta+2)\cot\sigma}{(5\cos^2\theta-2)}\,, \qquad \phi_{12} = \rho\,, \qquad \phi_{13} = \rho + \frac{\pi}{2}\,.$$

correlations:

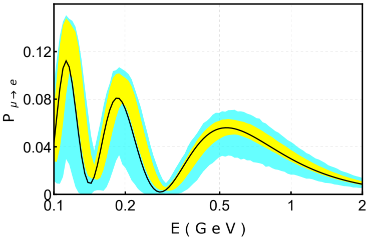
$$\cos^2\theta_{12}\,\cos^2\theta_{13} = \frac{2}{3}\,, \qquad \tan 2\theta_{23}\cos\delta_{CP} = \frac{5\sin^2\theta_{13}-1}{4\tan\theta_{12}\sin\theta_{13}}\,.$$





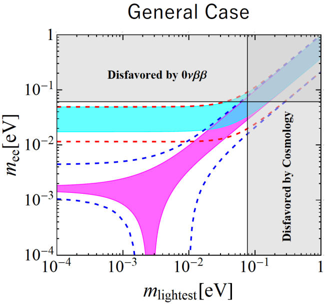
Revamping TBM according to GCP

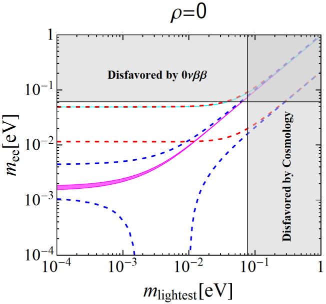
Neutrino Osscillasion:



$0\nu\beta\beta$ Decay:

$$|m_{ee}| = \frac{1}{3}\left|2e^{2i\rho}m_1 + m_2\cos^2\theta - m_3\sin^2\theta\right| ,$$





Summary

- We have considered in detail the two flavor generalised CP of charged leptons, such as generalised e-μ, e-τ and μ-τ symmetries. In each case we have obtained strong correlations involving the mixing angles and CP phases.
- Using the generalised CP symmetry, we proposed a realistic generalization of the TBM ansatz which accounts for non-zero measured value of θ_{13} and makes testable predictions for the other parameters of lepton mixing.

thank you