"Zer∅ing" the minimal type-I seesaw

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Motivation

TEXTURE ZEROS IN YUKAWA/MASS MATRICES

↓

SYMMETRY

[Abelian, Non-abelian, Discrete, continuous,...]

Patgiri, Kumar'19
Ahuja, Gupta'18
Borgohain, Das'18
Liu, Yue, Zhao'18
Kitabayashi'18
Alcaide, Salvado, Santamaria'18
Kobayashi, Nomura'18
Barreiros, Felipe, FRJ'18
Achelashvili, Tavartkiladze'17
Varzielas, Ross, Talbert'17
Fukujita, Kaneta, Shimizu, Tanimoto, Yanagida'16
Lamprea, Peinado'16
Kitabayashi, Yasuè'16
Nath, Gosh, Gupta'16
Chen, Ding, Canales, Valle'16
Liao, Marfatia, Whisnant'15
Gautam, Singh, Gupta'15
Zhang,15
Felipe, Serodio'14
Cannonni, Ellis, Gomez'13
Fritzsch, Xing, Zhou'13
Altarelli, Feruglio, Masina, Merlo'12
Canales, Mondragon'12
...

RATIONALE:

Consider the **simplest** (canonical) **type-I seesaw** framework

Q1: Which are the maximally-restricted texture-zero structures?

Q2: Which are compatible with data?

Q3: Predictions for $m_{\beta\beta}$ and leptogenesis?

Q4: What kind of restrictions if a CP symmetry is imposed?

Seesaw and notation

Minimal pure type-I seesaw: $(n_L,n_R)=(3,2)$

Minkowski'77
Gell-Mann, Ramond, Slanskiy'79
Yanagida'79
Schechter & Valle'80
Glashow'80
Mohapatra, Senjanovic'80

King'99
Frampton, Glashow, Yanagida'02

Neutrinos: $\mathcal{L}_\nu = -\overline{\ell_L}\mathbf{Y}_\nu^*\tilde{\Phi}\nu_R - \frac{1}{2}\overline{(\nu_R)^c}\mathbf{M}_R^*\nu_R + \text{H.c.}$

Charged-leptons: $\mathcal{L}_\ell = -\overline{\ell_L}\mathbf{Y}_\ell\Phi\, e_R + \text{H.c.}$

Effective neutrino mass matrix:

$$\mathbf{M}_\nu = -v^2\mathbf{Y}_\nu\mathbf{M}_R^{-1}\mathbf{Y}_\nu^T$$

Parameter	Best Fit $\pm1\sigma$	3σ range
θ_{12} $(^\circ)$	$34.5^{+1.1}_{-1.0}$	$31.5 \to 38.0$
θ_{23} $(^\circ)$ [NO]	41.0 ± 1.1	$38.3 \to 52.8$
θ_{23} $(^\circ)$ [IO]	50.5 ± 1.0	$38.5 \to 53.0$
θ_{13} $(^\circ)$ [NO]	$8.44^{+0.18}_{-0.15}$	$7.9 \to 8.9$
θ_{13} $(^\circ)$ [IO]	$8.41^{+0.16}_{-0.17}$	$7.9 \to 8.9$
δ $(^\circ)$ [NO]	252^{+56}_{-36}	$0 \to 360$
δ $(^\circ)$ [IO]	259^{+41}_{-47}	$0 \to 31$ $142 \to 360$
Δm^2_{21} $(\times10^{-5}\ \text{eV}^2)$	7.56 ± 0.19	$7.05 \to 8.14$
$\lvert\Delta m^2_{31}\rvert$ $(\times10^{-3}\ \text{eV}^2)$ [NO]	2.55 ± 0.04	$2.43 \to 2.67$
$\lvert\Delta m^2_{31}\rvert$ $(\times10^{-3}\ \text{eV}^2)$ [IO]	2.49 ± 0.04	$2.37 \to 2.61$

[Salas et. al ' 17]

Lepton mixing: $\mathbf{U} = \mathbf{U}_\ell^\dagger\mathbf{U}_\nu$

$$\mathbf{U}_\nu^T\mathbf{M}_\nu\mathbf{U}_\nu = \text{diag}(m_1,m_2,m_3) \equiv \mathbf{d}_m$$

$$\mathbf{U}_\ell^\dagger\mathbf{M}_\ell\mathbf{U}_e = \text{diag}(m_e,m_\mu,m_\tau)$$

Parametrization:

$$\mathbf{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix}$$

Q1: Which are the maximally-restricted texture-zero structures?

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Charged leptons (#▯ = 6)

$$L_1 : \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}, \quad L_2 : \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}, \quad L_3 : \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & 0 \end{pmatrix},$$

$$L_4 : \begin{pmatrix} \times & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix}, \quad L_5 : \begin{pmatrix} 0 & 0 & \times \\ \times & 0 & 0 \\ 0 & \times & 0 \end{pmatrix}, \quad L_6 : \begin{pmatrix} 0 & \times & 0 \\ 0 & 0 & \times \\ \times & 0 & 0 \end{pmatrix}$$

Dirac Yukawa couplings (#▯ = 2) :

$$T_1 : \begin{pmatrix} 0 & \times \\ \times & 0 \\ \times & \times \end{pmatrix}, \quad T_2 : \begin{pmatrix} 0 & \times \\ \times & \times \\ \times & 0 \end{pmatrix}, \quad T_3 : \begin{pmatrix} \times & \times \\ 0 & \times \\ \times & 0 \end{pmatrix},$$

$$T_4 : \begin{pmatrix} \times & 0 \\ 0 & \times \\ \times & \times \end{pmatrix}, \quad T_5 : \begin{pmatrix} \times & 0 \\ \times & \times \\ 0 & \times \end{pmatrix}, \quad T_6 : \begin{pmatrix} \times & \times \\ \times & 0 \\ 0 & \times \end{pmatrix},$$

RH-neutrino mass matrix (#▯ = 1,2) :

$$R_1 : \begin{pmatrix} \times & 0 \\ \cdot & \times \end{pmatrix}, \quad R_2 : \begin{pmatrix} 0 & \times \\ \cdot & \times \end{pmatrix}, \quad R_3 : \begin{pmatrix} \times & \times \\ \cdot & 0 \end{pmatrix}, \quad R_4 : \begin{pmatrix} 0 & \times \\ \cdot & 0 \end{pmatrix}$$

$\mathbf{Y}_\nu$	$\mathbf{M}_R$	SEESAW	$\mathbf{M}_\nu$
T_1, T_2	R_2		A: $\begin{pmatrix} 0 & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_4, T_5	R_3		
T_1, T_4	R_1		B: $\begin{pmatrix} \times & 0 & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_2, T_5	R_1		C: $\begin{pmatrix} \times & \times & 0 \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_3, T_4	R_2		D: $\begin{pmatrix} \times & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_1, T_6	R_3		
T_3, T_6	R_1		E: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & 0 \\ \cdot & \cdot & \times \end{pmatrix}$
T_5, T_6	R_2		F: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & 0 \end{pmatrix}$
T_2, T_3	R_3		

Q2: Which are compatible with data?

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$$\text{NH }(m_1=0): m_2^2=\Delta m_{21}^2\ ,\ m_3^2=\Delta m_{31}^2$$

$$\text{IH }(m_3=0): m_1^2=|\Delta m_{31}^2|\ ,\ m_2^2=|\Delta m_{31}^2|+\Delta m_{21}^2$$

Low-energy relations due to the presence of TZs

$$\text{NH}: \quad \frac{m_2}{m_3}=-\frac{\mathbf{U}_{\alpha 3}^*\mathbf{U}_{\beta 3}^*}{\mathbf{U}_{\alpha 2}^*\mathbf{U}_{\beta 2}^*} \longrightarrow r_\nu=\left|\frac{\mathbf{U}_{\alpha 3}^*\mathbf{U}_{\beta 3}^*}{\mathbf{U}_{\alpha 2}^*\mathbf{U}_{\beta 2}^*}\right|^2$$

$$r_\nu\equiv\frac{\Delta m_{21}^2}{|\Delta m_{31}^2|}$$

$$\text{IH}: \quad \frac{m_1}{m_2}=-\frac{\mathbf{U}_{\alpha 2}^*\mathbf{U}_{\beta 2}^*}{\mathbf{U}_{\alpha 1}^*\mathbf{U}_{\beta 1}^*} \longrightarrow \frac{1}{1+r_\nu}=\left|\frac{\mathbf{U}_{\alpha 2}^*\mathbf{U}_{\beta 2}^*}{\mathbf{U}_{\alpha 1}^*\mathbf{U}_{\beta 1}^*}\right|^2$$

$$\text{NH}: \quad \text{Im}\left(\frac{m_2}{m_3}\right)=-\text{Im}\left(\frac{\mathbf{U}_{\alpha 3}^*\mathbf{U}_{\beta 3}^*}{\mathbf{U}_{\alpha 2}^*\mathbf{U}_{\beta 2}^*}\right)=0$$

$$\text{IH}: \quad \text{Im}\left(\frac{m_1}{m_2}\right)=-\text{Im}\left(\frac{\mathbf{U}_{\alpha 2}^*\mathbf{U}_{\beta 2}^*}{\mathbf{U}_{\alpha 1}^*\mathbf{U}_{\beta 1}^*}\right)=0$$

$$\cos\delta=f_1(\theta_{ij},r_\nu)$$

$$\cos\alpha=f_2(\theta_{ij},r_\nu)$$

Q2: Which are compatible with data?

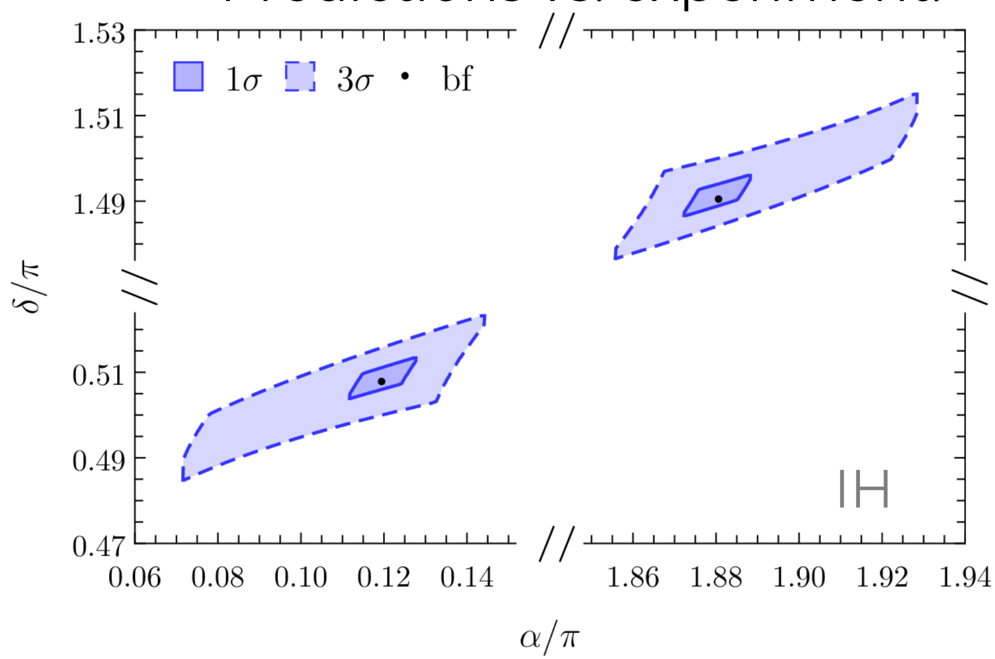
$\mathbf{Y}_\nu$	$\mathbf{M}_R$	$\mathbf{M}_\nu$
T_1, T_2	R_2	A: $\begin{pmatrix} 0 & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_4, T_5	R_3	
T_1, T_4	R_1	B: $\begin{pmatrix} \times & 0 & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_2, T_5	R_1	C: $\begin{pmatrix} \times & \times & 0 \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_3, T_4	R_2	D: $\begin{pmatrix} \times & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_1, T_6	R_3	
T_3, T_6	R_1	E: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & 0 \\ \cdot & \cdot & \times \end{pmatrix}$
T_5, T_6	R_2	F: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & 0 \end{pmatrix}$
T_2, T_3	R_3	

$$T_1: \begin{pmatrix} 0 & \times \\ \times & 0 \\ \times & \times \end{pmatrix} \quad T_2: \begin{pmatrix} 0 & \times \\ \times & \times \\ \times & 0 \end{pmatrix}, \quad T_3: \begin{pmatrix} \times & \times \\ 0 & \times \\ \times & 0 \end{pmatrix},$$

$$T_4: \begin{pmatrix} \times & 0 \\ 0 & \times \\ \times & \times \end{pmatrix} \quad T_5: \begin{pmatrix} \times & 0 \\ \times & \times \\ 0 & \times \end{pmatrix}, \quad T_6: \begin{pmatrix} \times & \times \\ \times & 0 \\ 0 & \times \end{pmatrix},$$

$$R_1: \begin{pmatrix} \times & 0 \\ \cdot & \times \end{pmatrix} \quad R_2: \begin{pmatrix} 0 & \times \\ \cdot & \times \end{pmatrix} \quad R_3: \begin{pmatrix} \times & \times \\ \cdot & 0 \end{pmatrix}$$

Predictions vs. experiment:



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$$c_\delta = 2\frac{[s_{12}^4(1+r_\nu) - c_{12}^4]s_{23}^2 s_{13}^2 + r_\nu c_{23}^2 s_{12}^2 c_{12}^2}{[s_{12}^2(1+r_\nu) + c_{12}^2]\sin(2\theta_{12})\sin(2\theta_{23})s_{13}}$$

$$c_\alpha = \frac{(2+r_\nu)c_{23}^2 s_{12}^2 c_{12}^2 - [s_{12}^4(1+r_\nu) + c_{12}^4]s_{23}^2 s_{13}^2}{2\sqrt{1+r_\nu}(c_{23}^2 + s_{23}^2 s_{13}^2)s_{12}^2 c_{12}^2}$$

Q2: Which are compatible with data?

$\mathbf{Y}_\nu$	$\mathbf{M}_R$	$\mathbf{M}_\nu$	
T_1, T_2	R_2	A: $\begin{pmatrix} 0 & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$	✗
T_4, T_5	R_3		
T_1, T_4	R_1	B: $\begin{pmatrix} \times & 0 & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$	✓
T_2, T_5	R_1	C: $\begin{pmatrix} \times & \times & 0 \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$	✓
T_3, T_4	R_2	D: $\begin{pmatrix} \times & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & \times \end{pmatrix}$	
T_1, T_6	R_3		
T_3, T_6	R_1	E: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & 0 \\ \cdot & \cdot & \times \end{pmatrix}$	
T_5, T_6	R_2	F: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & 0 \end{pmatrix}$	
T_2, T_3	R_3		

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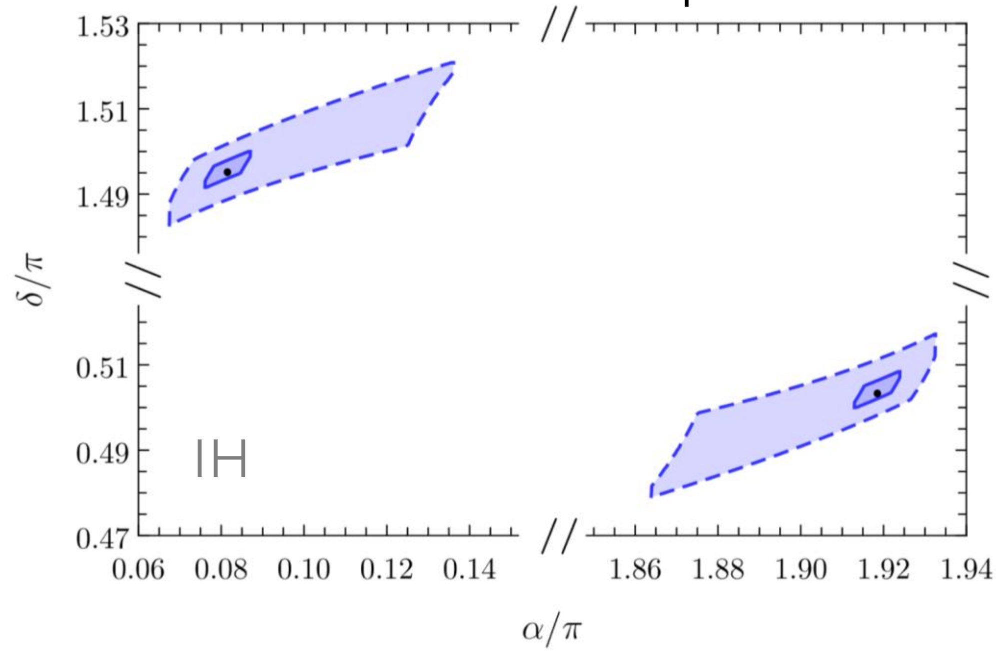
$$T_1 : \begin{pmatrix} 0 & \times \\ \times & 0 \\ \times & \times \end{pmatrix}, \quad T_2 : \begin{pmatrix} 0 & \times \\ \times & \times \\ \times & 0 \end{pmatrix}, \quad T_3 : \begin{pmatrix} \times & \times \\ 0 & \times \\ \times & 0 \end{pmatrix},$$

$$T_4 : \begin{pmatrix} \times & 0 \\ 0 & \times \\ \times & \times \end{pmatrix}, \quad T_5 : \begin{pmatrix} \times & 0 \\ \times & \times \\ 0 & \times \end{pmatrix}, \quad T_6 : \begin{pmatrix} \times & \times \\ \times & 0 \\ 0 & \times \end{pmatrix},$$

$$R_1 : \begin{pmatrix} \times & 0 \\ \cdot & \times \end{pmatrix} \quad R_2 : \begin{pmatrix} 0 & \times \\ \cdot & \times \end{pmatrix} \quad R_3 : \begin{pmatrix} \times & \times \\ \cdot & 0 \end{pmatrix}$$

Predictions vs. experiment:



$$c_\delta = -2\frac{[s_{12}^4(1+r_\nu) - c_{12}^4]c_{23}^2 s_{13}^2 + r_\nu s_{23}^2 s_{12}^2 c_{12}^2}{[s_{12}^2(1+r_\nu)+c_{12}^2]\sin(2\theta_{12})\sin(2\theta_{23})s_{13}} \qquad c_\alpha = \frac{(2+r_\nu)s_{23}^2 s_{12}^2 c_{12}^2 - [s_{12}^4(1+r_\nu)+c_{12}^4]c_{23}^2 s_{13}^2}{2\sqrt{1+r_\nu}(s_{23}^2 + c_{23}^2 s_{13}^2)s_{12}^2 c_{12}^2}$$

Q2: Which are compatible with data?

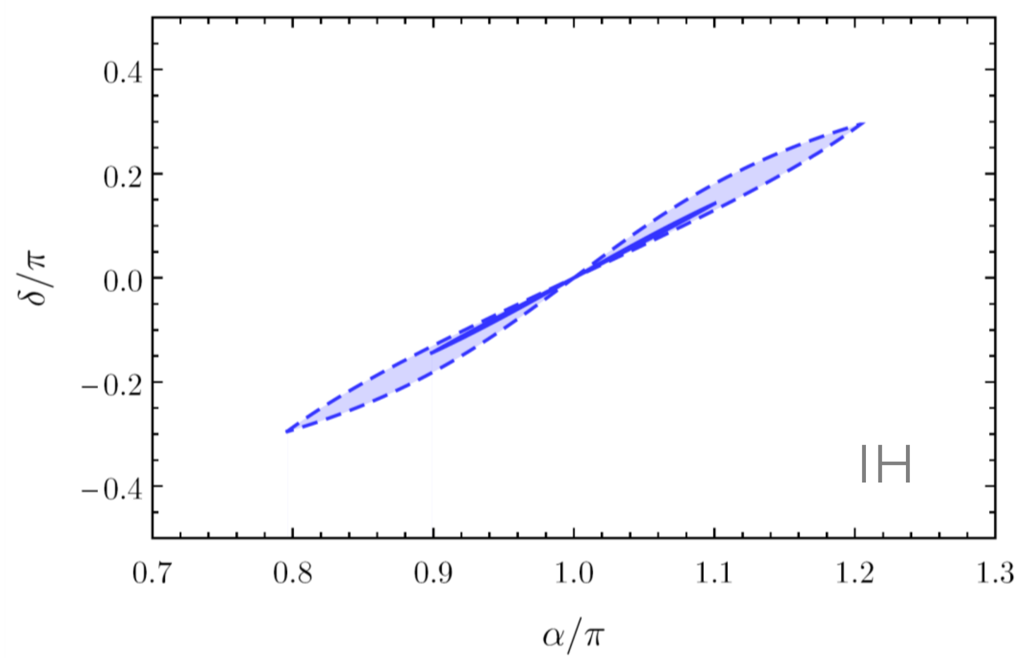
$\mathbf{Y}_\nu$	$\mathbf{M}_R$	$\mathbf{M}_\nu$
T_1, T_2	R_2	A: $\begin{pmatrix} 0 & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_4, T_5	R_3	
T_1, T_4	R_1	B: $\begin{pmatrix} \times & 0 & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_2, T_5	R_1	C: $\begin{pmatrix} \times & \times & 0 \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_3, T_4	R_2	D: $\begin{pmatrix} \times & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_1, T_6	R_3	
T_3, T_6	R_1	E: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & 0 \\ \cdot & \cdot & \times \end{pmatrix}$
T_5, T_6	R_2	F: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & 0 \end{pmatrix}$
T_2, T_3	R_3	

$$T_1: \begin{pmatrix} 0 & \times \\ \times & 0 \\ \times & \times \end{pmatrix},\quad T_2: \begin{pmatrix} 0 & \times \\ \times & \times \\ \times & 0 \end{pmatrix},\quad T_3: \begin{pmatrix} \times & \times \\ 0 & \times \\ \times & 0 \end{pmatrix},$$

$$T_4: \begin{pmatrix} \times & 0 \\ 0 & \times \\ \times & \times \end{pmatrix},\quad T_5: \begin{pmatrix} \times & 0 \\ \times & \times \\ 0 & \times \end{pmatrix},\quad T_6: \begin{pmatrix} \times & \times \\ \times & 0 \\ 0 & \times \end{pmatrix},$$

$$R_1: \begin{pmatrix} \times & 0 \\ \cdot & \times \end{pmatrix}\quad R_2: \begin{pmatrix} 0 & \times \\ \cdot & \times \end{pmatrix}\quad R_3: \begin{pmatrix} \times & \times \\ \cdot & 0 \end{pmatrix}$$

Predictions vs. experiment:



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$$c_\delta = 2\frac{(c_{12}^2\sqrt{1+r_\nu} - s_{12}^2)c_{23}^2 + (s_{12}^2\sqrt{1+r_\nu} - c_{12}^2)s_{23}^2 s_{13}^2}{(\sqrt{1+r_\nu}+1)\sin(2\theta_{12})\sin(2\theta_{23})s_{13}} \qquad c_\alpha \simeq -\frac{3+\cos(4\theta_{12}) - 16 s_{13}^2 t_{23}^2}{2\sin^2(2\theta_{12})}$$

Q2: Which are compatible with data?

$\mathbf{Y}_\nu$	$\mathbf{M}_R$	$\mathbf{M}_\nu$
T_1, T_2	R_2	A: $\begin{pmatrix} 0 & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_4, T_5	R_3	
T_1, T_4	R_1	B: $\begin{pmatrix} \times & 0 & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_2, T_5	R_1	C: $\begin{pmatrix} \times & \times & 0 \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_3, T_4	R_2	D: $\begin{pmatrix} \times & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_1, T_6	R_3	
T_3, T_6	R_1	E: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & 0 \\ \cdot & \cdot & \times \end{pmatrix}$
T_5, T_6	R_2	F: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & 0 \end{pmatrix}$
T_2, T_3	R_3	

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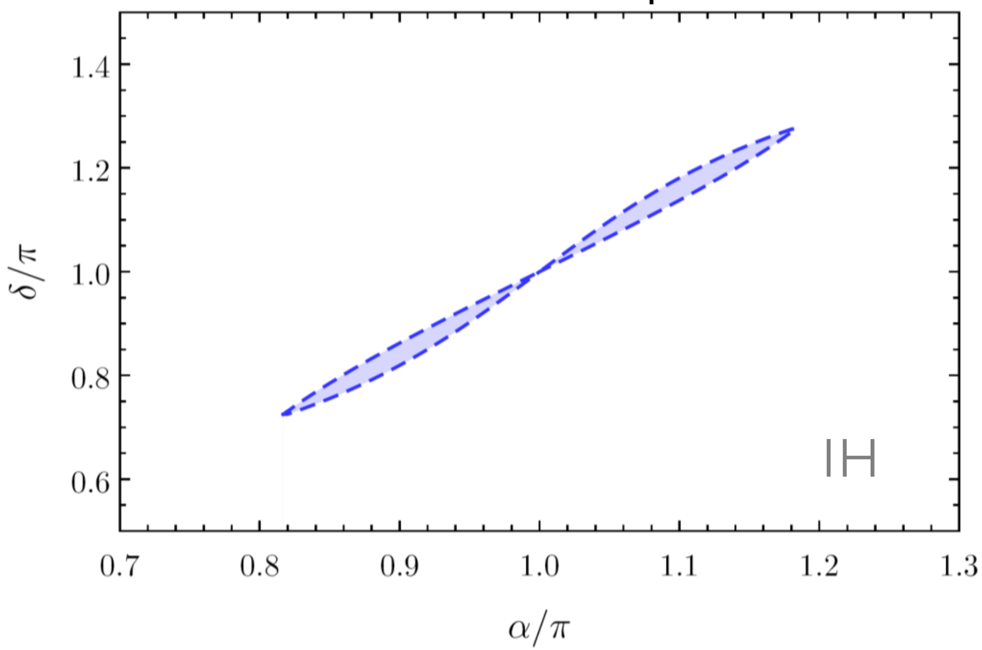
$$T_1 : \begin{pmatrix} 0 & \times \\ \times & 0 \\ \times & \times \end{pmatrix}, \quad T_2 : \begin{pmatrix} 0 & \times \\ \times & \times \\ \times & 0 \end{pmatrix}, \quad T_3 : \begin{pmatrix} \times & \times \\ 0 & \times \\ \times & 0 \end{pmatrix}$$

$$T_4 : \begin{pmatrix} \times & 0 \\ 0 & \times \\ \times & \times \end{pmatrix}, \quad T_5 : \begin{pmatrix} \times & 0 \\ \times & \times \\ 0 & \times \end{pmatrix}, \quad T_6 : \begin{pmatrix} \times & \times \\ \times & 0 \\ 0 & \times \end{pmatrix}$$

$$R_1 : \begin{pmatrix} \times & 0 \\ \cdot & \times \end{pmatrix} \quad R_2 : \begin{pmatrix} 0 & \times \\ \cdot & \times \end{pmatrix} \quad R_3 : \begin{pmatrix} \times & \times \\ \cdot & 0 \end{pmatrix}$$

Predictions vs. experiment:



$$c_\delta = 2\frac{(s_{12}^2 - c_{12}^2\sqrt{1+r_\nu})s_{23}^2 + (c_{12}^2 - s_{12}^2\sqrt{1+r_\nu})c_{23}^2 s_{13}^2}{(\sqrt{1+r_\nu}+1)\sin(2\theta_{12})\sin(2\theta_{23})s_{13}}$$

$$c_\alpha \simeq -\frac{3t_{23}^2 + t_{23}^2\cos(4\theta_{12}) + 16 s_{13}^2}{2t_{23}^2\sin^2(2\theta_{12})}$$

Q2: Which are compatible with data (the R_4 case)?

Degenerate RH neutrinos: $\mathrm{R}_4 : \begin{pmatrix} 0 & \times \\ \cdot & 0 \end{pmatrix}$

$$\mathrm{T}_1 : \begin{pmatrix} 0 & \times \\ \times & 0 \\ \times & \times \end{pmatrix}, \quad \mathrm{T}_2 : \begin{pmatrix} 0 & \times \\ \times & \times \\ \times & 0 \end{pmatrix}$$

$$\mathrm{T}_4 : \begin{pmatrix} \times & 0 \\ 0 & \times \\ \times & \times \end{pmatrix}, \quad \mathrm{T}_5 : \begin{pmatrix} \times & 0 \\ \times & \times \\ 0 & \times \end{pmatrix}$$

Specially attractive for radiative leptogenesis

Felipe, FRJ, Nobre'04
Joaquim'05
Branco, Felipe, FRJ'06

$$\mathrm{IH} : r_\nu = \frac{1}{t_{12}^4} - 1 \simeq 3.5$$

$$\mathrm{NH} : r_\nu = \frac{t_{13}^4}{s_{12}^4} \simeq 0.005$$

$\mathbf{Y}_\nu$	$\mathbf{M}_R$	$\mathbf{M}^\nu$	**Relation in** $\mathbf{M}^\nu$
$\mathrm{T}_1, \mathrm{T}_4$	R_4	$\mathrm{A}_1 : \begin{pmatrix} 0 & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & \times \end{pmatrix}$	$\frac{\mathbf{M}_{33}^\nu}{2\mathbf{M}_{23}^\nu} = \frac{\mathbf{M}_{13}^\nu}{\mathbf{M}_{12}^\nu}$
$\mathrm{T}_2, \mathrm{T}_5$		$\mathrm{A}_2 : \begin{pmatrix} 0 & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & 0 \end{pmatrix}$	$\frac{\mathbf{M}_{22}^\nu}{2\mathbf{M}_{23}^\nu} = \frac{\mathbf{M}_{12}^\nu}{\mathbf{M}_{13}^\nu}$
$\mathrm{T}_3, \mathrm{T}_6$		$\mathrm{D}_1 : \begin{pmatrix} \times & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & 0 \end{pmatrix}$	$\frac{\mathbf{M}_{11}^\nu}{2\mathbf{M}_{12}^\nu} = \frac{\mathbf{M}_{13}^\nu}{\mathbf{M}_{23}^\nu}$

Degenerate RH neutrinos: $\mathrm{R}_4 : \begin{pmatrix} 0 & \times \\ \cdot & 0 \end{pmatrix}$

$$\mathrm{T}_3 : \begin{pmatrix} \times & \times \\ 0 & \times \\ \times & 0 \end{pmatrix}$$

Specially attractive for radiative leptogenesis

Felipe, FRJ, Nobre'04
Joaquim'05
Branco, Felipe, FRJ'06

$$\mathrm{T}_6 : \begin{pmatrix} \times & \times \\ \times & 0 \\ 0 & \times \end{pmatrix}$$

$\mathbf{Y}_\nu$	$\mathbf{M}_R$	$\mathbf{M}^\nu$	Relation in $\mathbf{M}^\nu$
$\mathrm{T}_1,\ \mathrm{T}_4$	R_4	$\mathrm{A}_1: \begin{pmatrix} 0 & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & \times \end{pmatrix}$	$\dfrac{\mathbf{M}^\nu_{33}}{2\mathbf{M}^\nu_{23}} = \dfrac{\mathbf{M}^\nu_{13}}{\mathbf{M}^\nu_{12}}$
$\mathrm{T}_2,\ \mathrm{T}_5$		$\mathrm{A}_2: \begin{pmatrix} 0 & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & 0 \end{pmatrix}$	$\dfrac{\mathbf{M}^\nu_{22}}{2\mathbf{M}^\nu_{23}} = \dfrac{\mathbf{M}^\nu_{12}}{\mathbf{M}^\nu_{13}}$
$\mathrm{T}_3,\ \mathrm{T}_6$		$\mathrm{D}_1: \begin{pmatrix} \times & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & 0 \end{pmatrix}$	$\dfrac{\mathbf{M}^\nu_{11}}{2\mathbf{M}^\nu_{12}} = \dfrac{\mathbf{M}^\nu_{13}}{\mathbf{M}^\nu_{23}}$

$$c_\delta = 2\frac{(c_{12}^2\sqrt{1+r_\nu} - s_{12}^2)c_{23}^2 + (s_{12}^2\sqrt{1+r_\nu} - c_{12}^2)s_{23}^2 s_{13}^2}{(\sqrt{1+r_\nu}+1)\sin(2\theta_{12})\sin(2\theta_{23})s_{13}}$$

$$c_\delta = 2\frac{(s_{12}^2 - c_{12}^2\sqrt{1+r_\nu})s_{23}^2 + (c_{12}^2 - s_{12}^2\sqrt{1+r_\nu})c_{23}^2 s_{13}^2}{(\sqrt{1+r_\nu}+1)\sin(2\theta_{12})\sin(2\theta_{23})s_{13}}$$

Q2: Which are compatible with data (the R_4 case)?

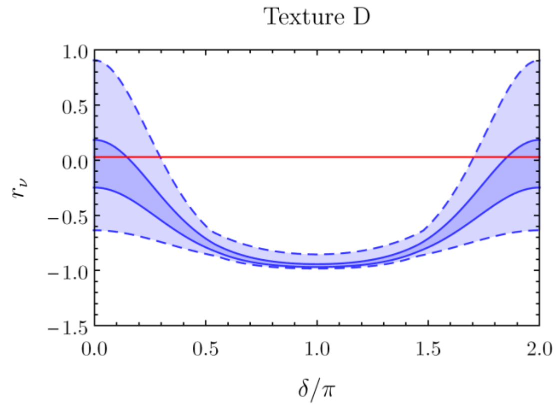
Degenerate RH neutrinos: $R_4 : \begin{pmatrix} 0 & \times \\ \cdot & 0 \end{pmatrix}$

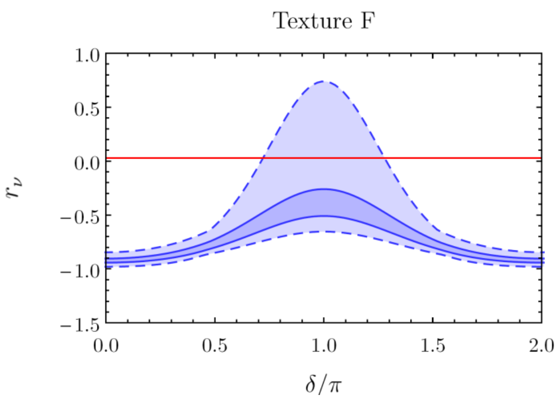
$$T_3 : \begin{pmatrix} \times & \times \\ 0 & \times \\ \times & 0 \end{pmatrix}$$

$$T_6 : \begin{pmatrix} \times & \times \\ \times & 0 \\ 0 & \times \end{pmatrix}$$

Specially attractive for radiative leptogenesis

Felipe, FRJ, Nobre'04
Joaquim'05
Branco, Felipe, FRJ'06





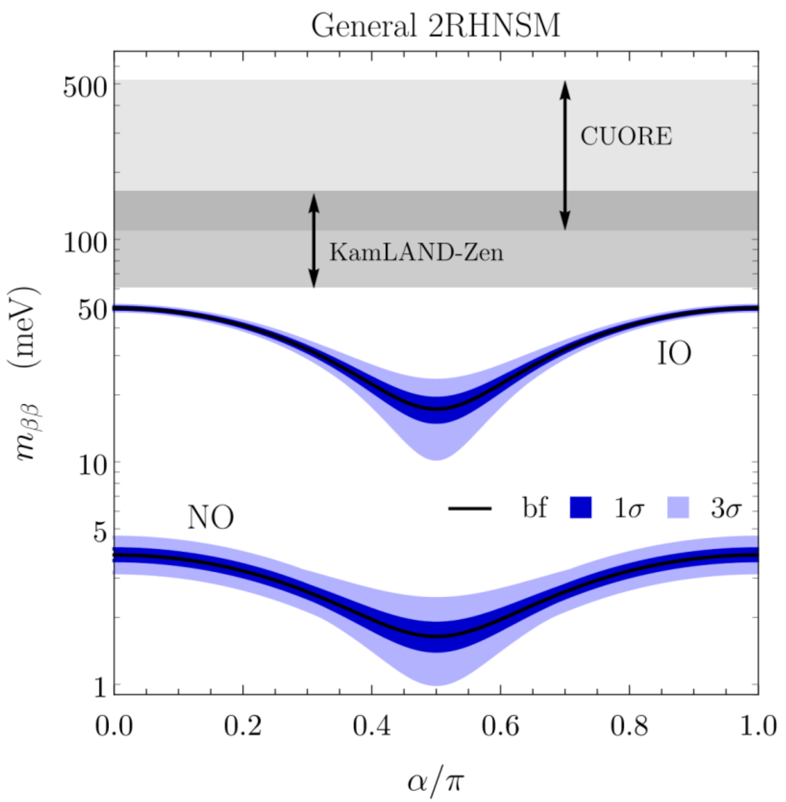
$\mathbf{Y}_\nu$	$\mathbf{M}_R$	$\mathbf{M}^\nu$	Relation in $\mathbf{M}^\nu$
$T_1,\ T_4$		$A_1: \begin{pmatrix} 0 & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & \times \end{pmatrix}$	$\frac{M^\nu_{33}}{2M^\nu_{23}} = \frac{M^\nu_{13}}{M^\nu_{12}}$
$T_2,\ T_5$	R_4	$A_2: \begin{pmatrix} 0 & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & 0 \end{pmatrix}$	$\frac{M^\nu_{22}}{2M^\nu_{23}} = \frac{M^\nu_{12}}{M^\nu_{13}}$
$T_3,\ T_6$		$D_1: \begin{pmatrix} \times & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & 0 \end{pmatrix}$	$\frac{M^\nu_{11}}{2M^\nu_{12}} = \frac{M^\nu_{13}}{M^\nu_{23}}$

Q3: Predictions for $m_{\beta\beta}$?

In the (3,2) type I seesaw (2RHNSM):

$$\text{NO: } m_{\beta\beta} = \sqrt{|\Delta m^2_{31}|}\,|\sqrt{r_\nu}c^2_{13}s^2_{12}e^{-2i\alpha} + s^2_{13}|$$

$$\text{IO: } m_{\beta\beta} = c^2_{13}\sqrt{|\Delta m^2_{31}|}\,|c^2_{12} + \sqrt{1+r_\nu}s^2_{12}e^{-2i\alpha}|$$

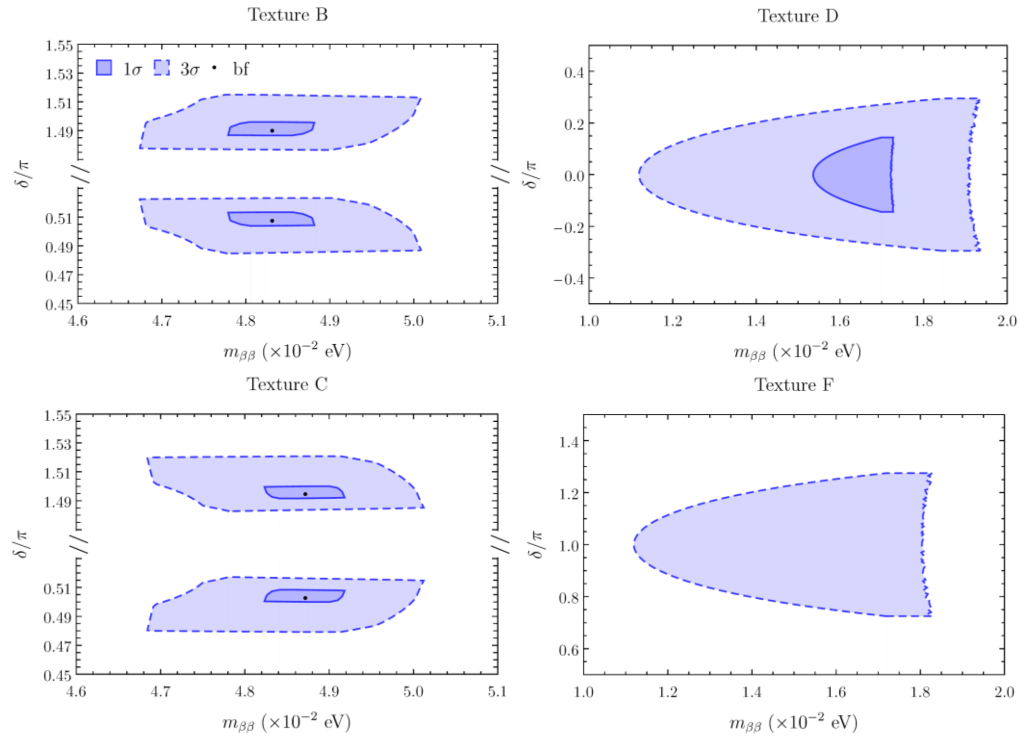


Barreiros, Felipe, FRJ'18

Barreiros, Felipe, FRJ'18

Maximally-restricted zero textures

$$B: \begin{pmatrix} \times & 0 & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix} \qquad D: \begin{pmatrix} \times & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & \times \end{pmatrix}$$

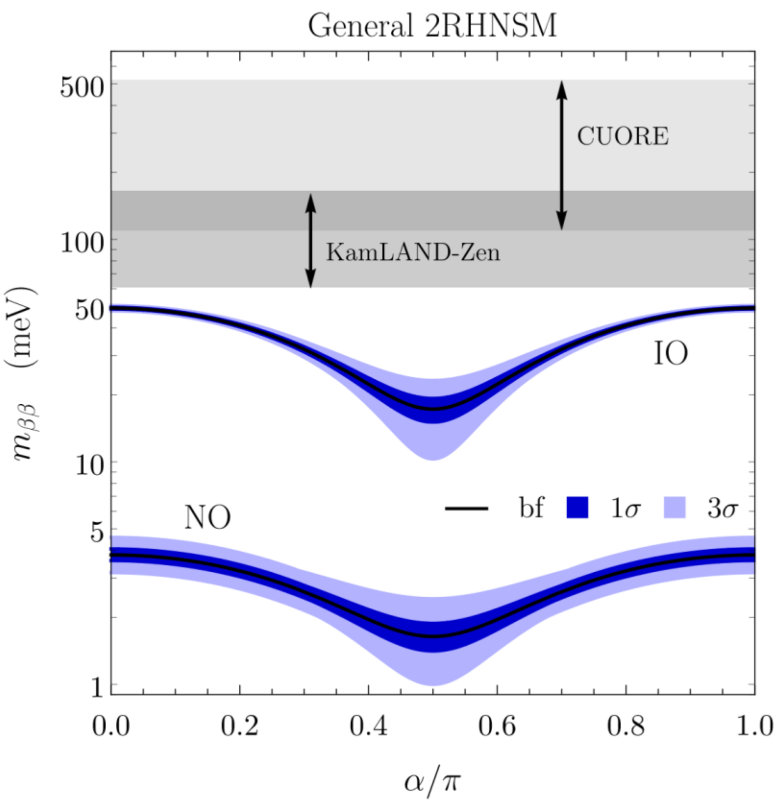


$$C: \begin{pmatrix} \times & \times & 0 \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix} \qquad F: \begin{pmatrix} \times & \times & 0 \\ \cdot & \times & \times \\ \cdot & \cdot & 0 \end{pmatrix}$$

In the (3,2) type I seesaw (2RHNSM):

$$\text{NO: } m_{\beta\beta} = \sqrt{|\Delta m^2_{31}|}|\sqrt{r_\nu} c^2_{13} s^2_{12} e^{-2i\alpha} + s^2_{13}|$$

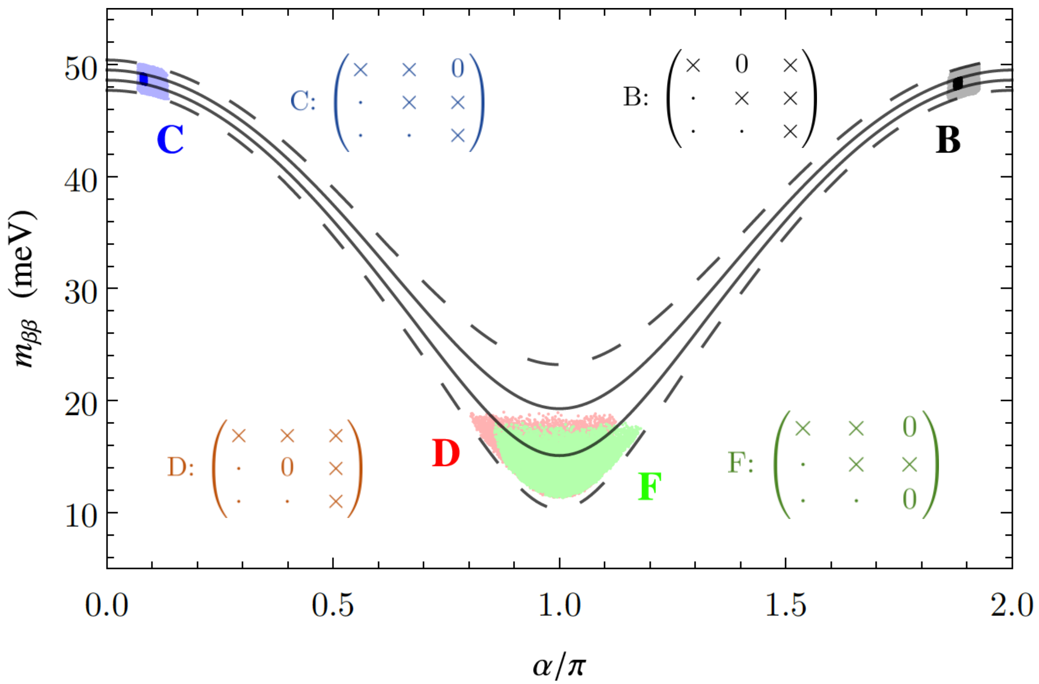
$$\text{IO: } m_{\beta\beta} = c^2_{13}\sqrt{|\Delta m^2_{31}|}|c^2_{12} + \sqrt{1+r_\nu} s^2_{12} e^{-2i\alpha}|$$



Barreiros, Felipe, FRJ'18

Maximally-restricted zero textures

Interplay $(\beta\beta)_{0\nu}$ / neutrino oscillations



- Improvement of the $m_{\beta\beta}$ limit would disfavour textures B and C
- All textures with R_1 would be excluded

- Reproduce the value of the baryon to photon ratio:

$$\eta_B^0 = (6.11 \pm 0.04) \times 10^{-10}$$

Fukugita, Yanagida' 86
Barbieri et. al.'99
Abada et. al'06
Blanchet, Di Bari'06
Giudice et. al.'11

- Casas-Ibarra parametrization: $\mathbf{Y}_\nu = v^{-1} \mathbf{U}^* \, \mathbf{d}_m^{1/2} \, \mathbf{O} \, \mathbf{d}_M^{1/2} \mathbf{U}_R^\dagger$

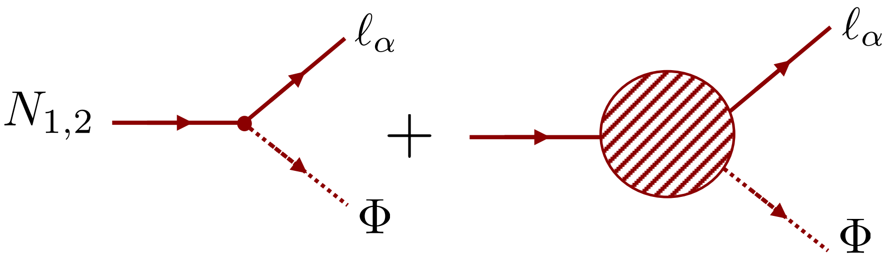
Casas & Ibarra' 01
Ibarra,Ross'04

- For the 2RHNSM: $\mathbf{O}_{\rm NH} = \begin{pmatrix} 0 & 0 \\ \cos z & -\sin z \\ \xi \sin z & \xi \cos z \end{pmatrix} \;, \; \mathbf{O}_{\rm IH} = \begin{pmatrix} \cos z & -\sin z \\ \xi \sin z & \xi \cos z \\ 0 & 0 \end{pmatrix} \;, \; \xi = \pm 1$

$\mathbf{M}_R$	$\tan z$ for $\mathbf{Y}^\nu_{\beta 1} = 0$	$\tan z$ for $\mathbf{Y}^\nu_{\beta 2} = 0$
R_1	$-\xi \sqrt{\frac{m_1}{m_2}} \frac{\mathbf{U}^*_{\beta 1}}{\mathbf{U}^*_{\beta 2}}$	$\xi \sqrt{\frac{m_2}{m_1}} \frac{\mathbf{U}^*_{\beta 2}}{\mathbf{U}^*_{\beta 1}}$
R_2	Any $\tan z$ with δ, α obeying $\mathbf{M}^\nu_{\beta\beta} = 0$	$\frac{-i \sqrt{m_1} M_1 \mathbf{U}^*_{\beta 1} + \xi \sqrt{m_2} M_2 \mathbf{U}^*_{\beta 2}}{\sqrt{m_1} M_2 \mathbf{U}^*_{\beta 1} + i \xi \sqrt{m_2} M_1 \mathbf{U}^*_{\beta 2}}$
R_3	$\frac{i \sqrt{m_1} M_1 \mathbf{U}^*_{\beta 1} + \xi \sqrt{m_2} M_2 \mathbf{U}^*_{\beta 2}}{\sqrt{m_1} M_2 \mathbf{U}^*_{\beta 1} - i \xi \sqrt{m_2} M_1 \mathbf{U}^*_{\beta 2}}$	Any $\tan z$ with δ, α obeying $\mathbf{M}^\nu_{\beta\beta} = 0$

$M_{1,2}$ are free

Q3: Predictions for leptogenesis (flavoured regime)?



$$\epsilon_i^\alpha = \frac{\Gamma(N_i \to \Phi \ell_\alpha) - \Gamma(N_i \to \Phi^\dagger \bar{\ell}_\alpha)}{\sum_\beta [\Gamma(N_i \to \Phi \ell_\beta) + \Gamma(N_i \to \Phi^\dagger \bar{\ell}_\beta)]}$$

$$10^9 \text{ GeV} \lesssim M_{1,2} \lesssim 10^{12} \text{ GeV}$$

$$\epsilon_1^\alpha = -\frac{M_2}{8\pi v^2} \frac{A_1^\alpha [f(x_2) + g(x_2)] + B_1^\alpha g'(x_2)}{m_1 |c_z|^2 + m_2 |s_z|^2}$$

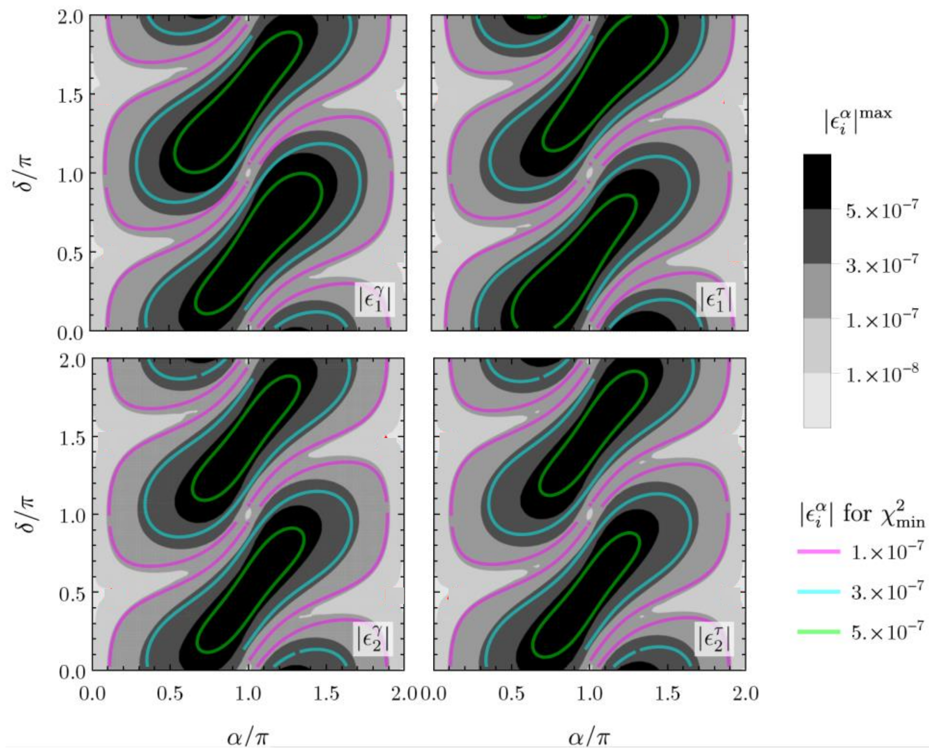
$$\epsilon_2^\alpha = -\frac{M_1}{8\pi v^2} \frac{A_2^\alpha [f(x_1) + g(x_1)] + B_2^\alpha g'(x_1)}{m_1 |s_z|^2 + m_2 |c_z|^2}$$

with: $A_i^\alpha \equiv A_i^\alpha(m_{1,2}, \mathbf{U}_{\alpha 1}, \mathbf{U}_{\alpha 2}, z, \xi)$

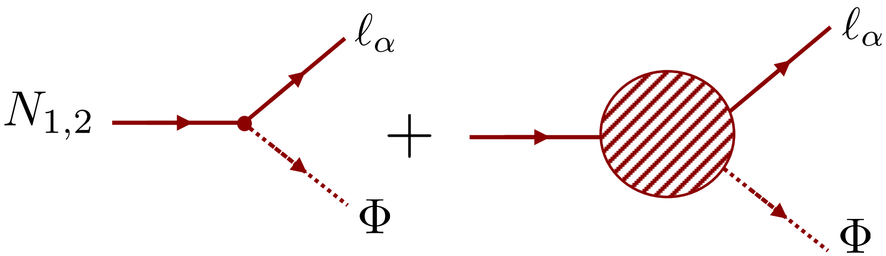
$B_i^\alpha \equiv B_i^\alpha(m_{1,2}, \mathbf{U}_{\alpha 1}, \mathbf{U}_{\alpha 2}, z, \xi)$

e and μ in equilibrium: $\epsilon_i^\gamma \equiv \epsilon_i^e + \epsilon_i^\mu$

$\mathbf{Y}^\nu_{11} = 0$, with R_1 — Barreiros, Felipe, FRJ'18



Q3: Predictions for leptogenesis (flavoured regime)?



$$\epsilon_1^\alpha = -\frac{M_2}{8\pi v^2}\frac{A_1^\alpha\left[f(x_2)+g(x_2)\right]+B_1^\alpha g'(x_2)}{m_1|c_z|^2+m_2|s_z|^2}$$

$$\epsilon_2^\alpha = -\frac{M_1}{8\pi v^2}\frac{A_2^\alpha\left[f(x_1)+g(x_1)\right]+B_2^\alpha g'(x_1)}{m_1|s_z|^2+m_2|c_z|^2}$$

with: $A_i^\alpha \equiv A_i^\alpha(m_{1,2},\mathbf{U}_{\alpha 1},\mathbf{U}_{\alpha 2},z,\xi)$

$B_i^\alpha \equiv B_i^\alpha(m_{1,2},\mathbf{U}_{\alpha 1},\mathbf{U}_{\alpha 2},z,\xi)$

e and μ in equilibrium: $\epsilon_i^\gamma \equiv \epsilon_i^e + \epsilon_i^\mu$

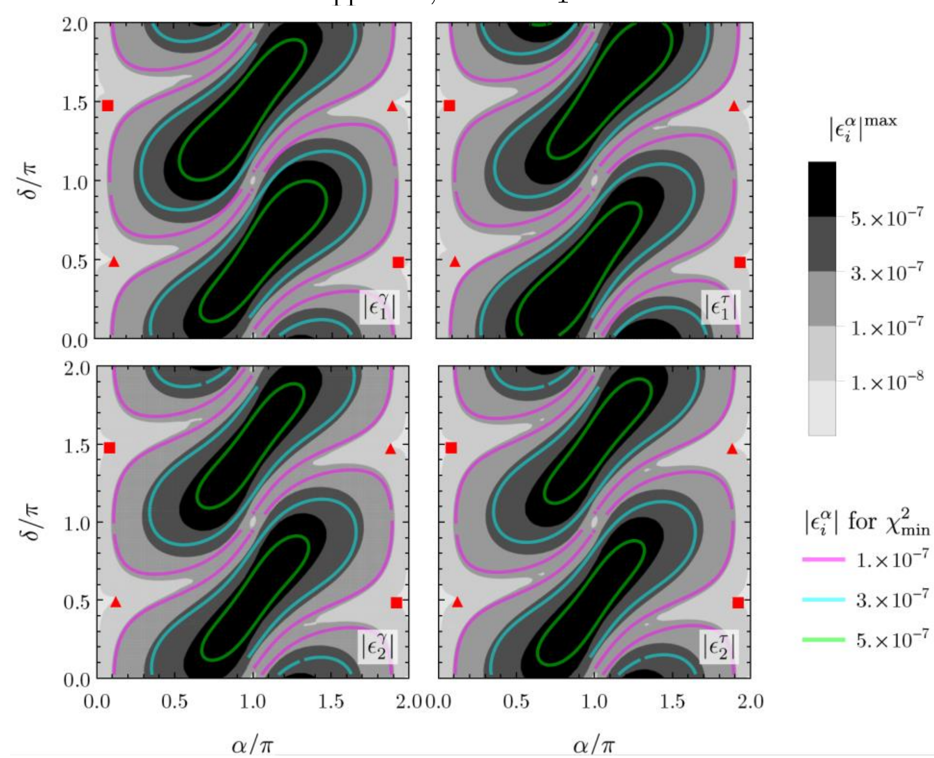
■ (T1,R1) ▲ (T2,R1)

$$\epsilon_i^\alpha = \frac{\Gamma(N_i\to\Phi\ell_\alpha)-\Gamma(N_i\to\Phi^\dagger\bar{\ell}_\alpha)}{\sum_\beta[\Gamma(N_i\to\Phi\ell_\beta)+\Gamma(N_i\to\Phi^\dagger\bar{\ell}_\beta)]}$$

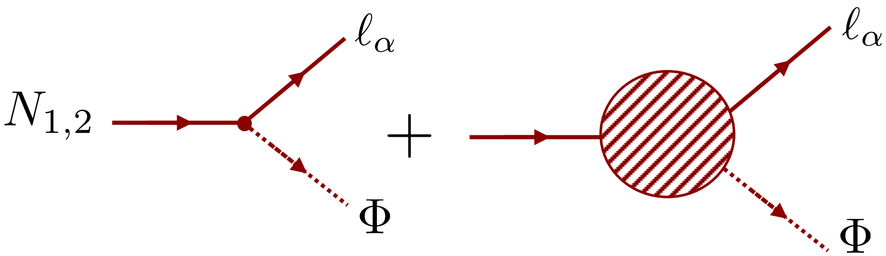
$$10^9\ \text{GeV} \lesssim M_{1,2} \lesssim 10^{12}\ \text{GeV}$$

$\mathbf{Y}^\nu_{11}=0$, with R_1

Barreiros, Felipe, FRJ'18



Q3: Predictions for leptogenesis (flavoured regime)?



$$\epsilon_i^\alpha = \frac{\Gamma(N_i \to \Phi \ell_\alpha) - \Gamma(N_i \to \Phi^\dagger \bar{\ell}_\alpha)}{\sum_\beta [\Gamma(N_i \to \Phi \ell_\beta) + \Gamma(N_i \to \Phi^\dagger \bar{\ell}_\beta)]}$$

Antusch, di Bari, Jones, King'11

$$\epsilon_1^\alpha = -\frac{M_2}{8\pi v^2} \frac{A_1^\alpha \left[f(x_2) + g(x_2)\right] + B_1^\alpha g'(x_2)}{m_1 |c_z|^2 + m_2 |s_z|^2}$$

$$\epsilon_2^\alpha = -\frac{M_1}{8\pi v^2} \frac{A_2^\alpha \left[f(x_1) + g(x_1)\right] + B_2^\alpha g'(x_1)}{m_1 |s_z|^2 + m_2 |c_z|^2}$$

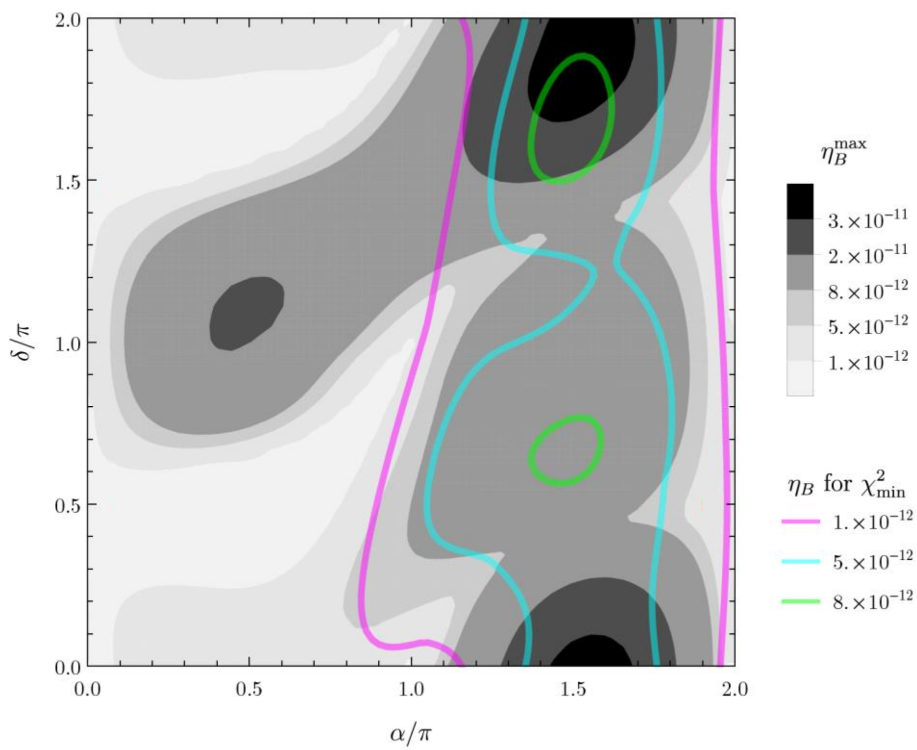
with: $A_i^\alpha \equiv A_i^\alpha(m_{1,2}, \mathbf{U}_{\alpha 1}, \mathbf{U}_{\alpha 2}, z, \xi)$

$B_i^\alpha \equiv B_i^\alpha(m_{1,2}, \mathbf{U}_{\alpha 1}, \mathbf{U}_{\alpha 2}, z, \xi)$

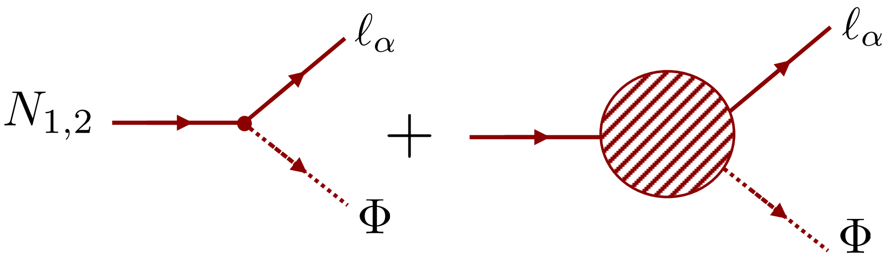
e and μ in equilibrium: $\epsilon_i^\gamma \equiv \epsilon_i^e + \epsilon_i^\mu$

$$10^9 \text{ GeV} \lesssim M_{1,2} \lesssim 10^{12} \text{ GeV}$$

$\mathbf{Y}^\nu_{11} = 0$, with R_1 Barreiros, Felipe, FRJ'18



Q3: Predictions for leptogenesis (flavoured regime)?



Antusch, di Bari, Jones, King'11

$$\epsilon_1^\alpha = -\frac{M_2}{8\pi v^2}\frac{A_1^\alpha\,[f(x_2)+g(x_2)]+B_1^\alpha g'(x_2)}{m_1|c_z|^2+m_2|s_z|^2}$$

$$\epsilon_2^\alpha = -\frac{M_1}{8\pi v^2}\frac{A_2^\alpha\,[f(x_1)+g(x_1)]+B_2^\alpha g'(x_1)}{m_1|s_z|^2+m_2|c_z|^2}$$

with: $A_i^\alpha \equiv A_i^\alpha(m_{1,2},\mathbf{U}_{\alpha 1},\mathbf{U}_{\alpha 2},z,\xi)$

$B_i^\alpha \equiv B_i^\alpha(m_{1,2},\mathbf{U}_{\alpha 1},\mathbf{U}_{\alpha 2},z,\xi)$

e and μ in equilibrium: $\epsilon_i^\gamma \equiv \epsilon_i^e + \epsilon_i^\mu$

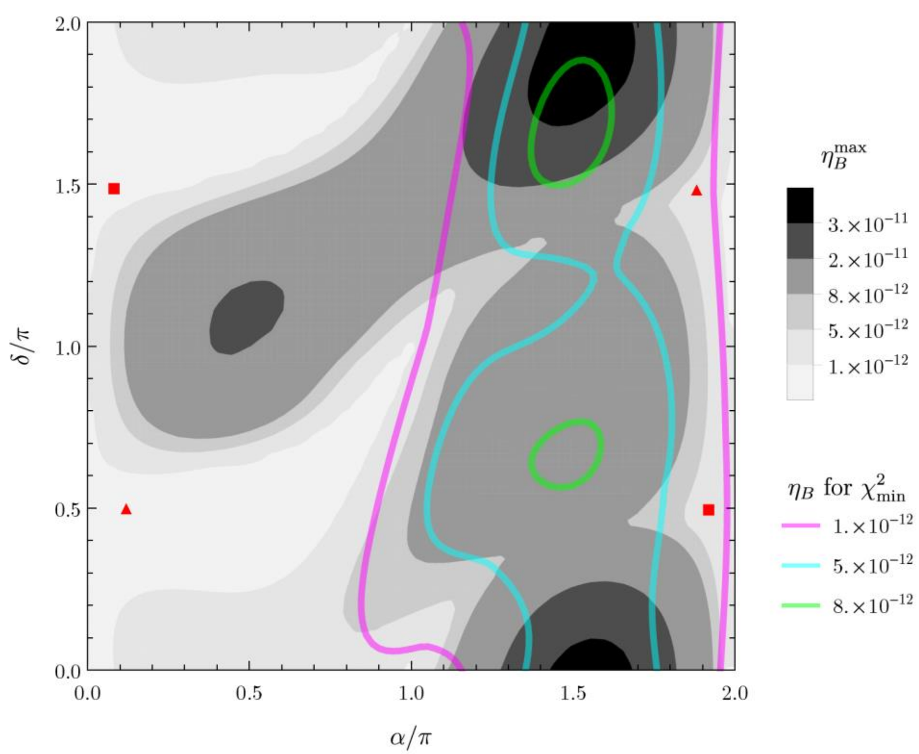
■ (T1,R1) ▲ (T2,R1)

THE BAU IS TOO SMALL!

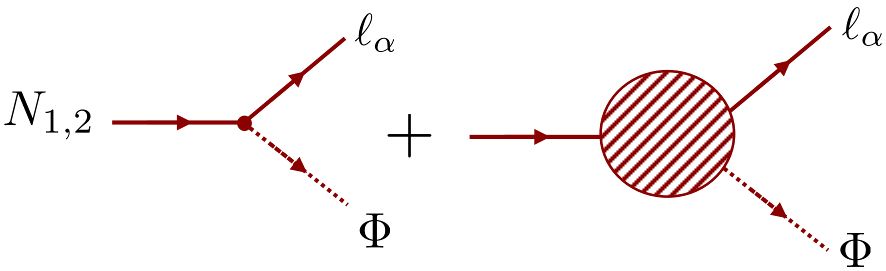
$$\epsilon_i^\alpha = \frac{\Gamma(N_i\to\Phi\ell_\alpha)-\Gamma(N_i\to\Phi^\dagger\bar{\ell}_\alpha)}{\sum_\beta[\Gamma(N_i\to\Phi\ell_\beta)+\Gamma(N_i\to\Phi^\dagger\bar{\ell}_\beta)]}$$

$$10^9\ \mathrm{GeV} \lesssim M_{1,2} \lesssim 10^{12}\ \mathrm{GeV}$$

$\mathbf{Y}^\nu_{11}=0$, with R_1 Barreiros, Felipe, FRJ'18



Q3: Predictions for leptogenesis (unflavoured regime)?



$M_1 \gtrsim 10^{12}$ GeV (unflavoured regime)

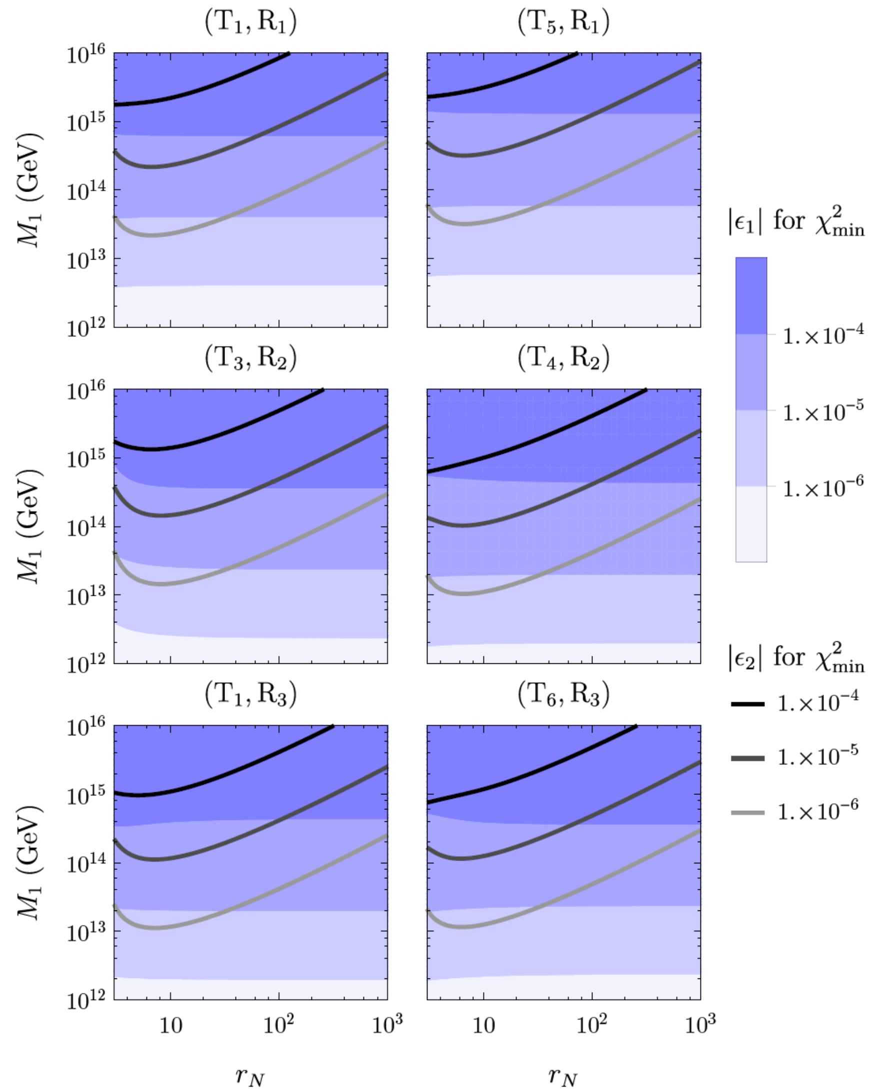
$$\epsilon_1 = -\frac{M_2}{8\pi v^2}\frac{\Delta m^2_{21}\,\mathrm{Im}[s_z^2]}{m_1\,|c_z|^2 + m_2\,|s_z|^2}\,[f(x_2)+g(x_2)]$$

$$\epsilon_2 = -\frac{M_1}{8\pi v^2}\frac{\Delta m^2_{21}\,\mathrm{Im}[c_z^2]}{m_1\,|s_z|^2 + m_2\,|c_z|^2}\,[f(x_1)+g(x_1)]$$

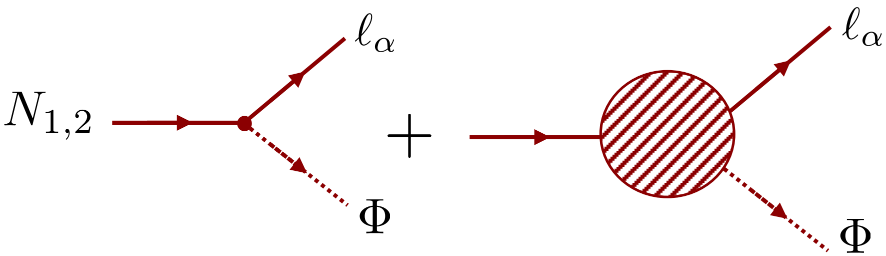
For the “best-fit” textures:

T$_1$, T$_4$	R$_1$	B: $\begin{pmatrix} \times & 0 & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$	✗	✓(1σ)
T$_2$, T$_5$	R$_1$	C: $\begin{pmatrix} \times & \times & 0 \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$	✗	✓(1σ)
T$_3$, T$_4$	R$_2$	D: $\begin{pmatrix} \times & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & \times \end{pmatrix}$	✗	✓(1σ)
T$_1$, T$_6$	R$_3$			

Barreiros, Felipe, FRJ’18



Q3: Predictions for leptogenesis (unflavoured regime)?



$$\epsilon_1 = -\frac{M_2}{8\pi v^2}\frac{\Delta m^2_{21}\,\mathrm{Im}[s_z^2]}{m_1\,|c_z|^2 + m_2\,|s_z|^2}\,[f(x_2)+g(x_2)]$$

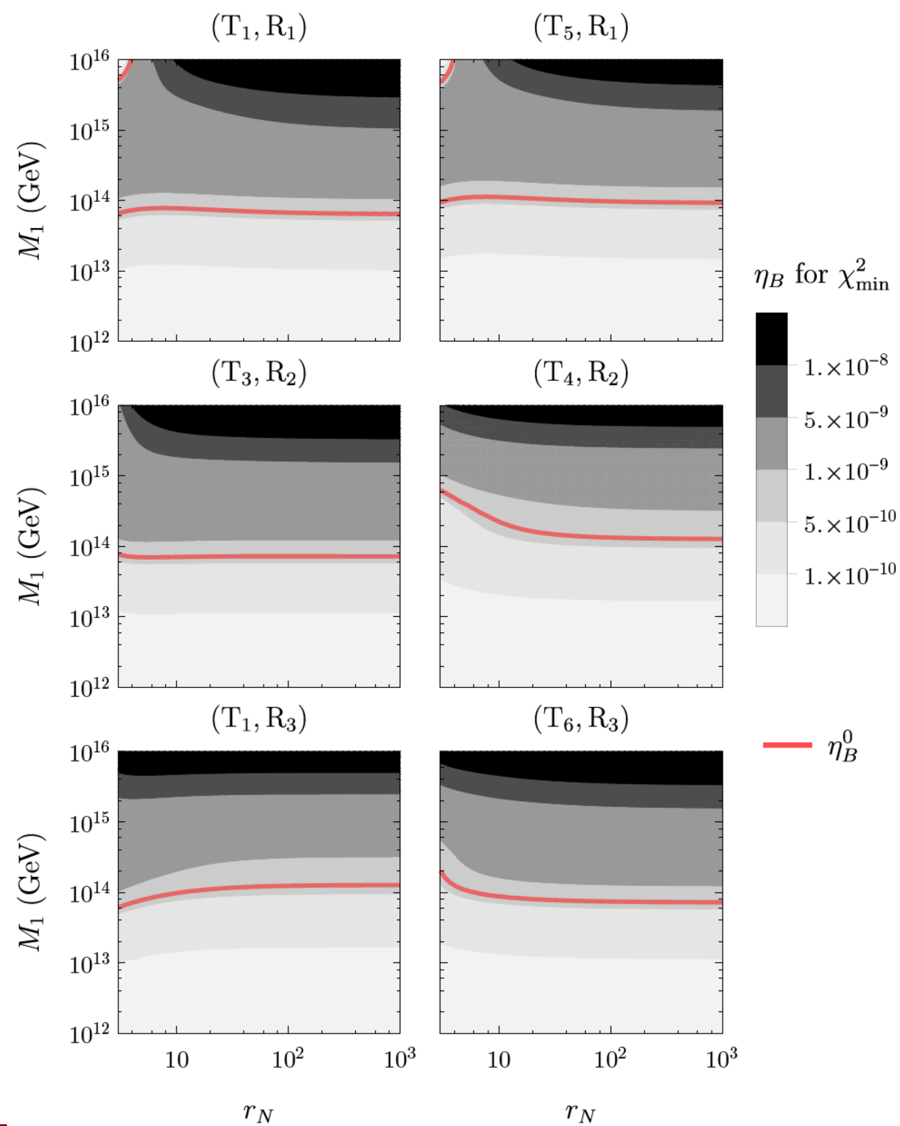
$$\epsilon_2 = -\frac{M_1}{8\pi v^2}\frac{\Delta m^2_{21}\,\mathrm{Im}[c_z^2]}{m_1\,|s_z|^2 + m_2\,|c_z|^2}\,[f(x_1)+g(x_1)]$$

For the "best-fit" textures:

T_1, T_4	R_1	B: $\begin{pmatrix} \times & 0 & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$	✗	✓(1σ)
T_2, T_5	R_1	C: $\begin{pmatrix} \times & \times & 0 \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$	✗	✓(1σ)
T_3, T_4	R_2	D: $\begin{pmatrix} \times & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & \times \end{pmatrix}$	✗	✓(1σ)
T_1, T_6	R_3			

THE BAU IS OK!

Barreiros, Felipe, FRJ'18



Q4: What kind of restrictions if a CP symmetry is imposed?

CP SYMMETRY:

$$\nu_L \to i\,\mathbf{X}_\nu\,\gamma_0\,\nu_L^c \qquad \nu_R \to i\,\mathbf{X}_R\,\gamma_0\,\nu_R^c \longrightarrow \mathcal{L}_\nu = -\overline{\ell_L}\mathbf{Y}_\nu^*\tilde{\Phi}\nu_R - \frac{1}{2}\overline{(\nu_R)^c}\mathbf{M}_R^*\nu_R + \text{H.c.}$$

Branco, Lavoura, Rebelo'86
Grimus, Rebelo'97
Feruglio, Hagedorn, Ziegler'12'14
Holthausen, Lindner, Schmidt'12
Branco, Felipe, FRJ'12
Li, Ding'14
Chen, Yao,Ding'15
King, Ding'16
Chen, Ding, King ´16
Chen, Ding, Canales, Valle'16
Chen, Chuliá, Srivastava,Valle'18
Nan, Ding'18
Penedo, Petcov, Titov'18
Samanta, Sinha, Ghosal'18
Barreiros, Felipe,FRJ'18
Novichkov, Penedo, Petcov, Titov'19

$$\mathbf{X}_\nu^\dagger\mathbf{Y}_\nu^*\mathbf{X}_R = \mathbf{Y}_\nu$$

$$\mathbf{X}_R^T\mathbf{M}_R^*\mathbf{X}_R = \mathbf{M}_R$$

Seesaw $\longrightarrow$ $\mathbf{M}_\nu = -v^2\mathbf{Y}^\nu\mathbf{M}_R^{-1}\mathbf{Y}^{\nu T}$

$$\mathbf{X}_\nu^T\mathbf{M}_\nu\mathbf{X}_\nu = \mathbf{M}_\nu^*$$

For non-degenerate light neutrinos: $\mathbf{U}_\nu^\dagger\mathbf{X}_\nu\mathbf{U}_\nu^* = \widehat{\mathbf{X}}_\nu = \text{diag}(\pm1,\pm1,\pm1)$

For one massless neutrino: $\widehat{\mathbf{X}}_\nu = \begin{cases} \text{diag}(e^{i\phi},a,b) & \text{for NO} \\ \text{diag}(a,b,e^{i\phi}) & \text{for IO} \end{cases}, \quad (a,b) = (\pm1,\pm1)$

In our case: $\mathbf{U}_\nu = \mathcal{P}_{ij}\mathbf{U} \longrightarrow \mathbf{X}_\nu = \mathbf{U}\widehat{\mathbf{X}}_\nu\mathbf{U}^T$

The lepton mixing matrix U fixes the LH neutrino CP transformation

Q4: What kind of restrictions if a CP symmetry is imposed?

RH NEUTRINO CP transformation: $\mathbf{U}_R^T \mathbf{X}_R \mathbf{U}_R = \widehat{\mathbf{X}}_R$

- Impact on the Casas-Ibarra orthogonal Matrix O

Invariance:

$$\mathbf{X}_\nu^\dagger \mathbf{Y}_\nu^* \mathbf{X}_R = \mathbf{Y}_\nu$$

+

Casas-Ibarra:

$$\mathbf{Y}^\nu = v^{-1} \mathbf{U}^* \, \mathbf{d}_m^{1/2} \, \mathbf{O} \, \mathbf{d}_M^{1/2} \mathbf{U}_R^\dagger$$

$$\mathbf{X}_R = \mathbf{U}_R^* \widehat{\mathbf{X}}_R \mathbf{U}_R^\dagger$$

$$\mathbf{X}_\nu = \mathbf{U} \widehat{\mathbf{X}}_\nu \mathbf{U}^T$$

Constraint on O imposed by the CPt

$$\widehat{\mathbf{X}}_\nu^\dagger \mathbf{O}^* \widehat{\mathbf{X}}_R = \mathbf{O}$$

- Non-degenerate RH neutrinos $M_1 \neq M_2 \neq ... \neq M_n$:

$$\widehat{\mathbf{X}}_\nu = \begin{cases} \text{diag}(e^{i\phi}, \pm 1, \pm 1) & \text{for NO} \\ \text{diag}(\pm 1, \pm 1, e^{i\phi}) & \text{for IO} \end{cases}$$

$$\widehat{\mathbf{X}}_R = \text{diag}(\pm 1, \pm 1, \pm 1, ...)$$

$$\Longrightarrow \quad \mathbf{O}_{ij} = \pm \mathbf{O}_{ij}^*$$

C-C Li, G-J Ding'14

<u>Degeneracies:</u>

$$M_1 = M_2 = ... = M_n$$

$$\widehat{\mathbf{X}}_R = \begin{pmatrix} \mathbf{O}_n & 0 \\ 0 & \mathbf{D}_{N-n} \end{pmatrix}$$

$\mathbf{O}_n$ - $n \times n$ othogonal matrix

In the non-degenerate subspace:

$$\mathbf{D}_{N-n} = \text{diag}(\pm 1, \pm 1, \cdots, \pm 1)$$

Fully-degenerate spectrum:

$$(n = N): \quad \widehat{\mathbf{X}}_R \equiv \mathbf{O}_N = \mathbf{O}^\dagger \widehat{\mathbf{X}}_\nu \mathbf{O}$$

$\mathbf{O}_N$ must be real since $\widehat{\mathbf{X}}_R$ is unitary

Barreiros, Felipe, FRJ '19

Q4: What kind of restrictions if a CP symmetry is imposed?

IN THE 2 RH NEUTRINO CASE:

$$\mathbf{O}_{\rm NH} = \begin{pmatrix} 0 & 0 \\ \cos z & -\sin z \\ \xi \sin z & \xi \cos z \end{pmatrix}, \quad \mathbf{O}_{\rm IH} = \begin{pmatrix} \cos z & -\sin z \\ \xi \sin z & \xi \cos z \\ 0 & 0 \end{pmatrix}$$

Invariance under CPt

$$\widehat{\mathbf{X}}_\nu^\dagger \mathbf{O}^* \widehat{\mathbf{X}}_R = \mathbf{O}$$

$$M_1 \neq M_2 \qquad \mathbf{O}_{ij} = \pm \mathbf{O}_{ij}^*$$

$\widehat{\mathbf{X}}_R$	(a,b)	$\mathbf{O}$ (NO)	$\mathbf{O}$ (IO)	Label
diag(1,1)	$(1,1)$	$\begin{pmatrix} 0 & 0 \\ \cos z & -\sin z \\ \xi\sin z & \xi\cos z \end{pmatrix}$	$\begin{pmatrix} \cos z & -\sin z \\ \xi\sin z & \xi\cos z \\ 0 & 0 \end{pmatrix}$	$\mathbf{O}_{\rm I}$
	$(1,-1)$	✗	✗	✗
	$(-1,1)$	✗	✗	✗
	$(-1,-1)$	✗	✗	✗
diag(1,−1)	$(1,1)$	✗	✗	✗
	$(1,-1)$	$\pm\begin{pmatrix} 0 & 0 \\ \cosh z & -i\sinh z \\ i\xi\sinh z & \xi\cosh z \end{pmatrix}$	$\pm\begin{pmatrix} \cosh z & -i\sinh z \\ i\xi\sinh z & \xi\cosh z \\ 0 & 0 \end{pmatrix}$	$\mathbf{O}_{\rm II}$
	$(-1,1)$	$\pm\begin{pmatrix} 0 & 0 \\ i\sinh z & -\cosh z \\ \xi\cosh z & i\xi\sinh z \end{pmatrix}$	$\pm\begin{pmatrix} i\sinh z & -\cosh z \\ \xi\cosh z & i\xi\sinh z \\ 0 & 0 \end{pmatrix}$	$\mathbf{O}_{\rm III}$
	$(-1,-1)$	✗	✗	✗

...with z real.

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Q4: What happens when a CP symmetry is imposed?

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$$\mathrm{R}_1: \begin{pmatrix} \times & 0 \\ \cdot & \times \end{pmatrix}, \quad \mathrm{R}_2: \begin{pmatrix} 0 & \times \\ \cdot & \times \end{pmatrix},$$

$$\mathrm{R}_3: \begin{pmatrix} \times & \times \\ \cdot & 0 \end{pmatrix}, \quad \mathrm{R}_4: \begin{pmatrix} 0 & \times \\ \cdot & 0 \end{pmatrix},$$

$$\mathrm{T}_1: \begin{pmatrix} 0 & \times \\ \times & 0 \\ \times & \times \end{pmatrix}, \mathrm{T}_2: \begin{pmatrix} 0 & \times \\ \times & \times \\ \times & 0 \end{pmatrix}, \mathrm{T}_3: \begin{pmatrix} \times & \times \\ 0 & \times \\ \times & 0 \end{pmatrix},$$

$$\mathrm{T}_4: \begin{pmatrix} \times & 0 \\ 0 & \times \\ \times & \times \end{pmatrix}, \mathrm{T}_5: \begin{pmatrix} \times & 0 \\ \times & \times \\ 0 & \times \end{pmatrix}, \mathrm{T}_6: \begin{pmatrix} \times & \times \\ \times & 0 \\ 0 & \times \end{pmatrix}.$$

Degenerate RH neutrinos ($M_1 = M_2$)

$$\widehat{\mathbf{X}}_R \equiv \mathbf{O}_2 = \mathbf{O}^\dagger \widehat{\mathbf{X}}_\nu \mathbf{O}$$

$(a,b) = \pm(1,1)$: $\mathbf{O}_2$ must be real and so $\mathbf{O} \equiv \mathbf{O}_{\rm I}$

$(a,b) = \pm(1,-1)$: $\mathbf{O}_2$ is automatically real

$$\mathrm{T}_{1,2,4,5} + \mathrm{R}_1 \ (M_1 = M_2) + (a,b) = \pm(1,-1)$$

The results do not change

Q4: What kind of restrictions if a CP symmetry is imposed?

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$\mathrm{R}_1 \ (M_1 = M_2) + (a,b) = \pm(1,1)$

$\mathrm{R}_{1,2,3} \ (M_1 \neq M_2)$

$\mathbf{O}$ is parameterized by a single real parameter z

- Remember from the previous analysis:

$\mathbf{M}_R$	$\tan z$ for $\mathbf{Y}^{\nu}_{\alpha 1} = 0$	$\tan z$ for $\mathbf{Y}^{\nu}_{\alpha 2} = 0$
R_1	$-\xi\sqrt{\frac{m_1}{m_2}}\frac{\mathbf{U}^*_{\alpha 1}}{\mathbf{U}^*_{\alpha 2}}$	$\xi\sqrt{\frac{m_2}{m_1}}\frac{\mathbf{U}^*_{\alpha 2}}{\mathbf{U}^*_{\alpha 1}}$
R_2	Any $\tan z$ with δ, α obeying $\mathbf{M}^{\nu}_{\alpha\alpha} = 0$ →	$\frac{-i\sqrt{m_1}M_1\mathbf{U}^*_{\alpha 1} + \xi\sqrt{m_2}M_2\mathbf{U}^*_{\alpha 2}}{\sqrt{m_1}M_2\mathbf{U}^*_{\alpha 1} + i\,\xi\sqrt{m_2}M_1\mathbf{U}^*_{\alpha 2}}$
R_3	$\frac{i\sqrt{m_1}M_1\mathbf{U}^*_{\alpha 1} + \xi\sqrt{m_2}M_2\mathbf{U}^*_{\alpha 2}}{\sqrt{m_1}M_2\mathbf{U}^*_{\alpha 1} - i\,\xi\sqrt{m_2}M_1\mathbf{U}^*_{\alpha 2}}$	← Any $\tan z$ with δ, α obeying $\mathbf{M}^{\nu}_{\alpha\alpha} = 0$

$M_{1,2}$ are free

The CPt will constrain the heavy neutrino spectrum

$$r_N \equiv M_1/M_2$$

Q4: What kind of restrictions if a CP symmetry is imposed?

$\mathbf{Y}_\nu$	$\mathbf{M}_R$	$\mathbf{M}_\nu$
$T_1,\ T_4$	R_1	B: $\begin{pmatrix} \times & 0 & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
$T_2,\ T_5$	R_1	C: $\begin{pmatrix} \times & \times & 0 \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
$T_3,\ T_4$	R_2	D: $\begin{pmatrix} \times & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & \times \end{pmatrix}$
$T_1,\ T_6$	R_3	
$T_3,\ T_6$	R_1	E: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & 0 \\ \cdot & \cdot & \times \end{pmatrix}$
$T_5,\ T_6$	R_2	F: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & 0 \end{pmatrix}$
$T_2,\ T_3$	R_3	

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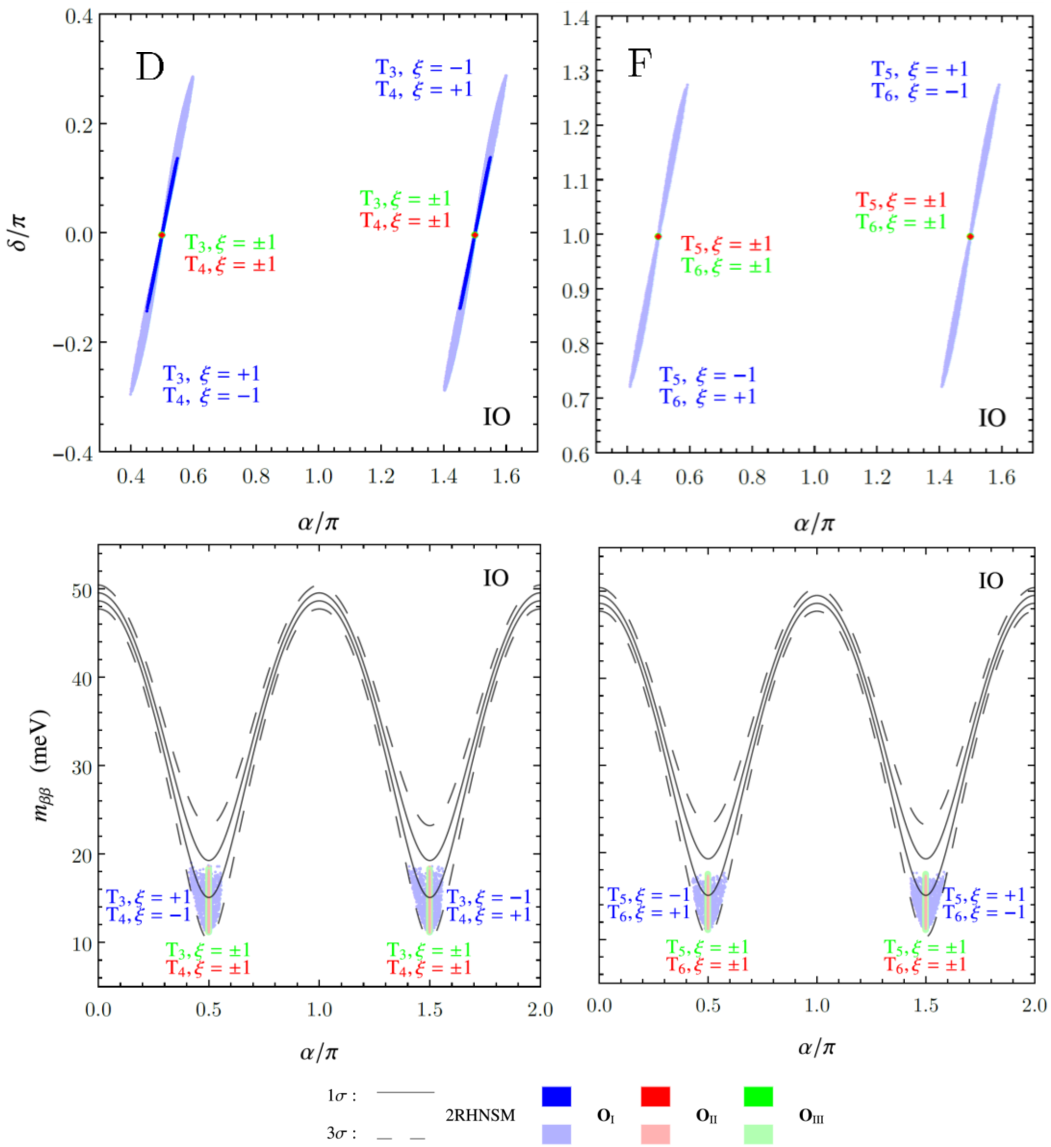

Q4: What kind of restrictions if a CP symmetry is imposed?

$\mathbf{Y}_\nu$	$\mathbf{M}_R$	$\mathbf{M}_\nu$
T_1, T_4	R_1	B: $\begin{pmatrix} \times & 0 & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_2, T_5	R_1	C: $\begin{pmatrix} \times & \times & 0 \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_3, T_4	R_2	D: $\begin{pmatrix} \times & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_1, T_6	R_3	
T_3, T_6	R_1	E: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & 0 \\ \cdot & \cdot & \times \end{pmatrix}$
T_5, T_6	R_2	F: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & 0 \end{pmatrix}$
T_2, T_3	R_3	

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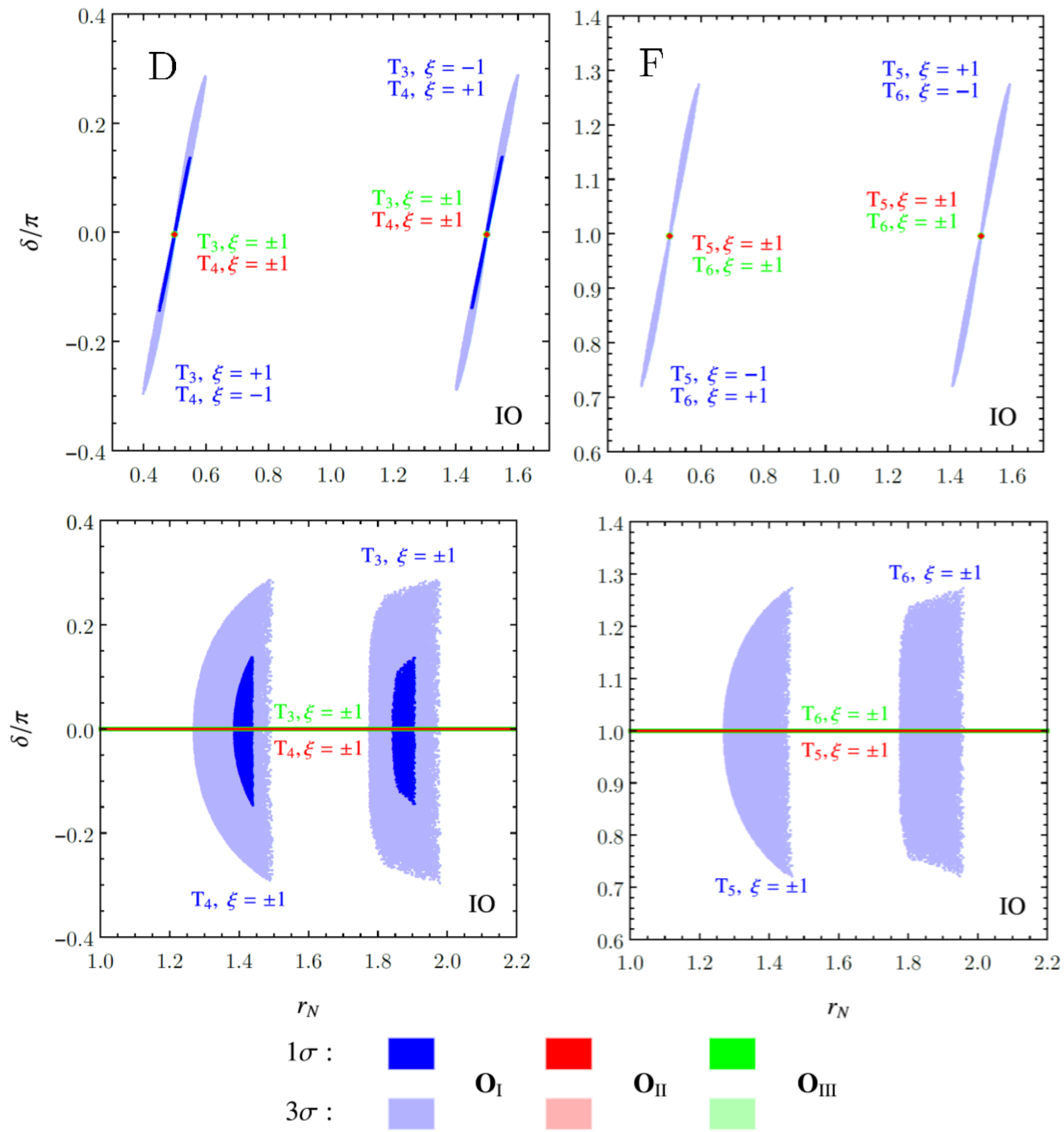


Q4: What kind of restrictions if a CP symmetry is imposed?

Two texture-zero scenarios + CP are very restrictive(*) (and difficult to implement in minimal SM extensions e.g. 2HDM).

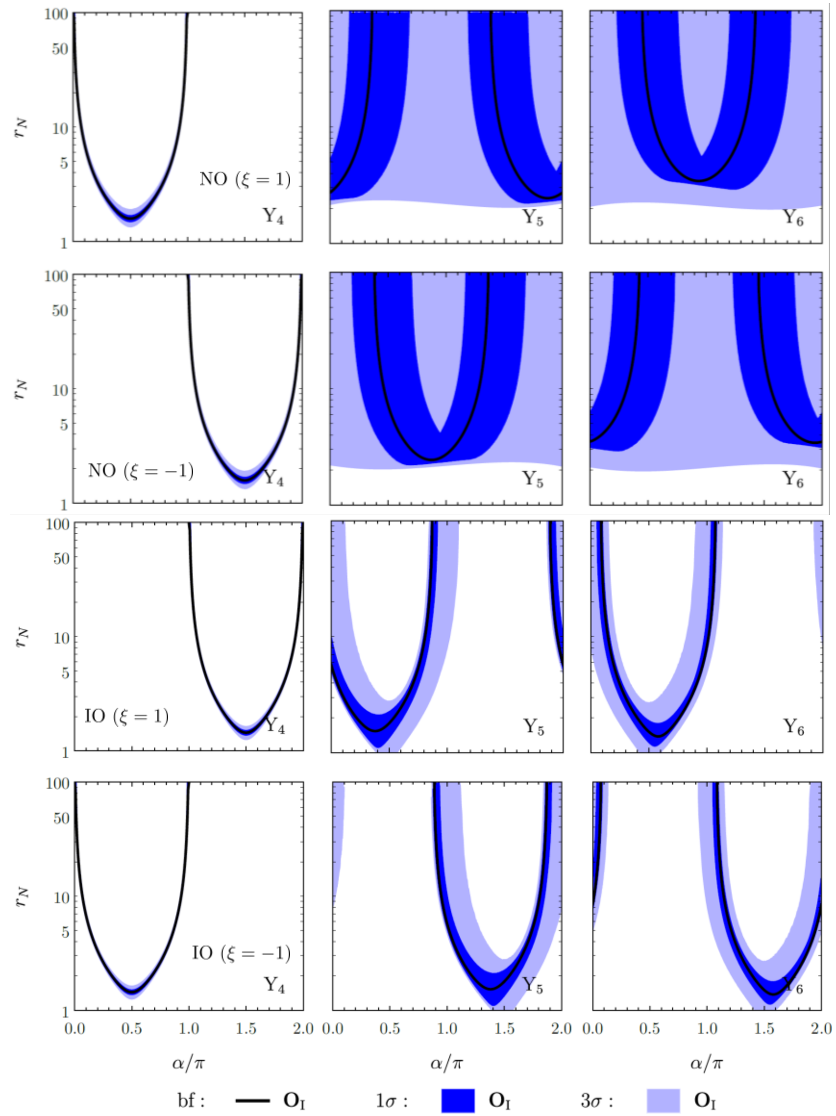
- ONE TZ SCENARIOS

Example: $R_2 : \begin{pmatrix} 0 & \times \\ \cdot & \times \end{pmatrix}$,

$$Y_4 : \begin{pmatrix} \times & 0 \\ \times & \times \\ \times & \times \end{pmatrix}, Y_5 : \begin{pmatrix} \times & \times \\ \times & 0 \\ \times & \times \end{pmatrix}, Y_6 : \begin{pmatrix} \times & \times \\ \times & \times \\ \times & 0 \end{pmatrix}$$

- Interesting relations among CP phases
- Contraints on r_N
- Correlations between r_N and α

MANY OTHER INTERESTING FEATURES
(Perhaps) more straightforward implementation.



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(*) And only compatible with data for a IH neutrino mass spectrum

CONCLUSIONS

- Maximally-restricted texture zero patterns in the 2RHN seesaw: **IH only**
- **Compatible with neutrino data** at 1σ with $\delta \sim 3\pi/2$ (in the charged-lepton mass basis. Consider permutations.)

$\mathbf{Y}^\nu$	$\mathbf{M}_R$	$\mathbf{M}^\nu$	NH	IH
T_1, T_4	R_1	B: $\begin{pmatrix} \times & 0 & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$	✗	✓(1σ)
T_2, T_5	R_1	C: $\begin{pmatrix} \times & \times & 0 \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$	✗	✓(1σ)

$$T_1 : \begin{pmatrix} 0 & \times \\ \times & 0 \\ \times & \times \end{pmatrix}, \quad T_2 : \begin{pmatrix} 0 & \times \\ \times & \times \\ \times & 0 \end{pmatrix}$$

$$T_4 : \begin{pmatrix} \times & 0 \\ 0 & \times \\ \times & \times \end{pmatrix}, \quad T_5 : \begin{pmatrix} \times & 0 \\ \times & \times \\ 0 & \times \end{pmatrix}$$

$$R_1 : \begin{pmatrix} \times & 0 \\ \cdot & \times \end{pmatrix}$$

- **Sharp predictions** for neutrinoless double beta decay

- **Leptogenesis works** (in the unflavoured regime) for $M_1 \gtrsim 10^{14}\,\mathrm{GeV}$
- CP transformations: further constraints on CP phases and heavy neutrino mass spectrum (leptogenesis still to be analysed)

谢谢！

谢谢！

THANK YOU!

SUMMARY TABLE

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$\mathbf{Y}_\nu$	$\mathbf{M}_R$	$\mathbf{M}_\nu$	NH	IH
T_1, T_2	R_2	A: $\begin{pmatrix} 0 & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$	✗	✗
T_4, T_5	R_3			
T_1, T_4	R_1	B: $\begin{pmatrix} \times & 0 & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$	✗	✓(1σ)
T_2, T_5	R_1	C: $\begin{pmatrix} \times & \times & 0 \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$	✗	✓(1σ)
T_3, T_4	R_2	D: $\begin{pmatrix} \times & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & \times \end{pmatrix}$	✗	✓(1σ)
T_1, T_6	R_3			
T_3, T_6	R_1	E: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & 0 \\ \cdot & \cdot & \times \end{pmatrix}$	✗	✗
T_5, T_6	R_2	F: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & 0 \end{pmatrix}$	✗	✓(3σ)
T_2, T_3	R_3			

δ is in the 1σ interval

Parameter	Best Fit $\pm1\sigma$	3σ range
θ_{12} (°)	$34.5^{+1.1}_{-1.0}$	$31.5 \to 38.0$
θ_{23} (°) [NO]	41.0 ± 1.1	$38.3 \to 52.8$
θ_{23} (°) [IO]	50.5 ± 1.0	$38.5 \to 53.0$
θ_{13} (°) [NO]	$8.44^{+0.18}_{-0.15}$	$7.9 \to 8.9$
θ_{13} (°) [IO]	$8.41^{+0.16}_{-0.17}$	$7.9 \to 8.9$
δ (°) [NO]	252^{+56}_{-36}	$0 \to 360$
δ (°) [IO]	259^{+41}_{+47}	$0 \to 31$ $142 \to 360$
Δm^2_{21} ($\times 10^{-5}$ eV^2)	7.56 ± 0.19	$7.05 \to 8.14$
$\|\Delta m^2_{31}\|$ ($\times 10^{-3}$ eV^2) [NO]	2.55 ± 0.04	$2.43 \to 2.67$
$\|\Delta m^2_{31}\|$ ($\times 10^{-3}$ eV^2) [IO]	2.49 ± 0.04	$2.37 \to 2.61$

Examples of symmetry implementations

[Kobayashi, Nomura, Okada'18]

Abelian symmetry realization – U(1)

Fields	L_{L_e}	L_{L_μ}	L_{L_τ}	e_R	μ_R	τ_R	N_{R_1}	N_{R_2}	H_{SM}	H_1	H_2	H_3	H_4	φ_1	φ_2	φ_3
$SU(2)_L$	**2**	**2**	**2**	**1**	**1**	**1**	**1**	**1**	**2**	**2**	**2**	**2**	**2**	**1**	**1**	**1**
$U(1)_Y$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	-1	-1	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0
$U(1)_{\mu-\tau}$	0	1	-1	0	1	-1	n_1	n_2	0	n_1	n_2-1	n_1+1	n_2+1	-2n_1	-2n_2	-n_1-n_2

Non-abelian symmetry realization – D_4

Fields	L_{L_ℓ}	L_{L_τ}	ℓ_R	τ_R	N_{R_i}	N_{R_τ}	H	H_2	η_1	$\eta_{1'}$	η_D	φ_8	φ'_8	φ_{10}	ζ	φ_2
$SU(2)_L$	**2**	**2**	**1**	**1**	**1**	**1**	**2**	**2**	**2**	**2**	**2**	**1**	**1**	**1**	**2**	**1**
$U(1)_Y$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	-1	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	0
$U(1)_{B-L}$	-1	-1	-1	-1	-4	5	0	0	-3	-3	-3	8	8	10	-6	2
D_4	2	1	2	1	2	1	1	$1'$	1	$1'$	2	1	$1'$	2	1	1

Compatibility with data (Texture A)

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$\mathbf{Y}_\nu$	$\mathbf{M}_R$	$\mathbf{M}_\nu$
T_1, T_2	R_2	A: $\begin{pmatrix} 0 & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_4, T_5	R_3	
T_1, T_4	R_1	B: $\begin{pmatrix} \times & 0 & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_2, T_5	R_1	C: $\begin{pmatrix} \times & \times & 0 \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_3, T_4	R_2	D: $\begin{pmatrix} \times & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_1, T_6	R_3	
T_3, T_6	R_1	E: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & 0 \\ \cdot & \cdot & \times \end{pmatrix}$
T_5, T_6	R_2	F: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & 0 \end{pmatrix}$
T_2, T_3	R_3	

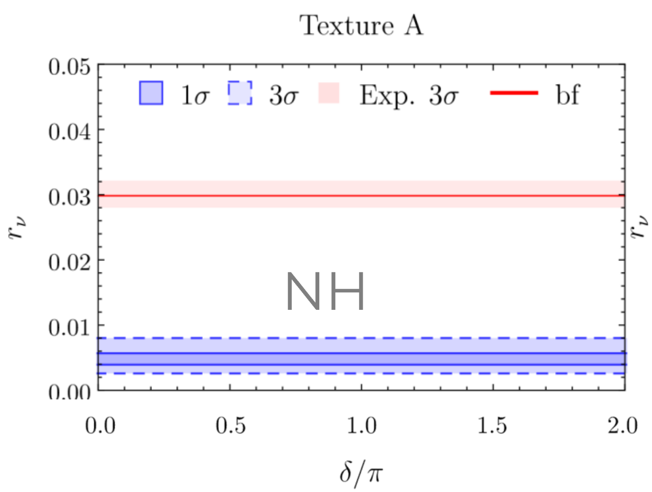
$$T_1 : \begin{pmatrix} 0 & \times \\ \times & 0 \\ \times & \times \end{pmatrix}, \quad T_2 : \begin{pmatrix} 0 & \times \\ \times & \times \\ \times & 0 \end{pmatrix}, \quad T_3 : \begin{pmatrix} \times & \times \\ 0 & \times \\ \times & 0 \end{pmatrix},$$

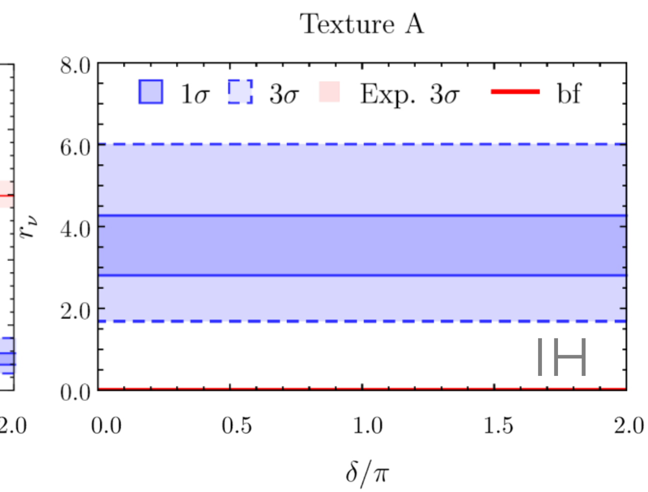
$$T_4 : \begin{pmatrix} \times & 0 \\ 0 & \times \\ \times & \times \end{pmatrix}, \quad T_5 : \begin{pmatrix} \times & 0 \\ \times & \times \\ 0 & \times \end{pmatrix}, \quad T_6 : \begin{pmatrix} \times & \times \\ \times & 0 \\ 0 & \times \end{pmatrix},$$

$$R_1 : \begin{pmatrix} \times & 0 \\ \cdot & \times \end{pmatrix} \qquad R_2 : \begin{pmatrix} 0 & \times \\ \cdot & \times \end{pmatrix} \qquad R_3 : \begin{pmatrix} \times & \times \\ \cdot & 0 \end{pmatrix}$$

Predictions vs. experiment:

$$\text{NH} : r_\nu = \frac{t_{13}^4}{s_{12}^4} \simeq 0.005 \qquad \text{IH} : r_\nu = \frac{1}{t_{12}^4} - 1 \simeq 3.5$$





Incompatible for both NH and IH

Q2: Which are compatible with data?

$\mathbf{Y}_\nu$	$\mathbf{M}_R$	$\mathbf{M}_\nu$
T_1, T_2	R_2	A: $\begin{pmatrix} 0 & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_4, T_5	R_3	
T_1, T_4	R_1	B: $\begin{pmatrix} \times & 0 & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_2, T_5	R_1	C: $\begin{pmatrix} \times & \times & 0 \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_3, T_4	R_2	D: $\begin{pmatrix} \times & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_1, T_6	R_3	
T_3, T_6	R_1	E: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & 0 \\ \cdot & \cdot & \times \end{pmatrix}$
T_5, T_6	R_2	F: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & 0 \end{pmatrix}$
T_2, T_3	R_3	

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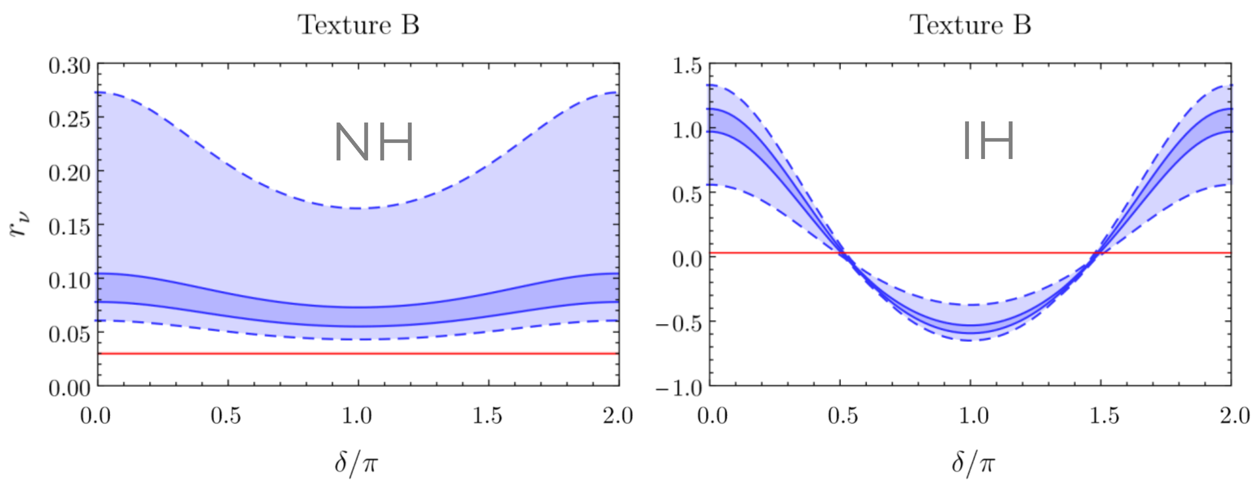
$$T_1 : \begin{pmatrix} 0 & \times \\ \times & 0 \\ \times & \times \end{pmatrix}, \quad T_2 : \begin{pmatrix} 0 & \times \\ \times & \times \\ \times & 0 \end{pmatrix}, \quad T_3 : \begin{pmatrix} \times & \times \\ 0 & \times \\ \times & 0 \end{pmatrix},$$

$$T_4 : \begin{pmatrix} \times & 0 \\ 0 & \times \\ \times & \times \end{pmatrix}, \quad T_5 : \begin{pmatrix} \times & 0 \\ \times & \times \\ 0 & \times \end{pmatrix}, \quad T_6 : \begin{pmatrix} \times & \times \\ \times & 0 \\ 0 & \times \end{pmatrix},$$

$$R_1 : \begin{pmatrix} \times & 0 \\ \cdot & \times \end{pmatrix} \quad R_2 : \begin{pmatrix} 0 & \times \\ \cdot & \times \end{pmatrix} \quad R_3 : \begin{pmatrix} \times & \times \\ \cdot & 0 \end{pmatrix}$$

Predictions vs. experiment:



(In)compatible for IH (NH)

Q2: Which are compatible with data?

$\mathbf{Y}_\nu$	$\mathbf{M}_R$	$\mathbf{M}_\nu$
T_1, T_2	R_2	A: $\begin{pmatrix} 0 & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_4, T_5	R_3	
T_1, T_4	R_1	B: $\begin{pmatrix} \times & 0 & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_2, T_5	R_1	C: $\begin{pmatrix} \times & \times & 0 \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_3, T_4	R_2	D: $\begin{pmatrix} \times & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_1, T_6	R_3	
T_3, T_6	R_1	E: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & 0 \\ \cdot & \cdot & \times \end{pmatrix}$
T_5, T_6	R_2	F: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & 0 \end{pmatrix}$
T_2, T_3	R_3	

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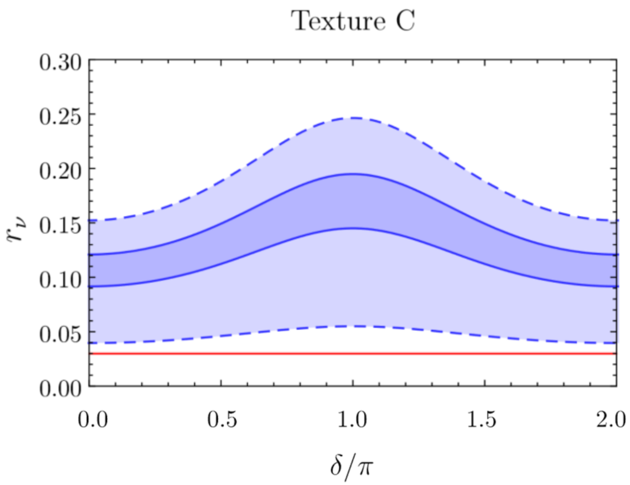
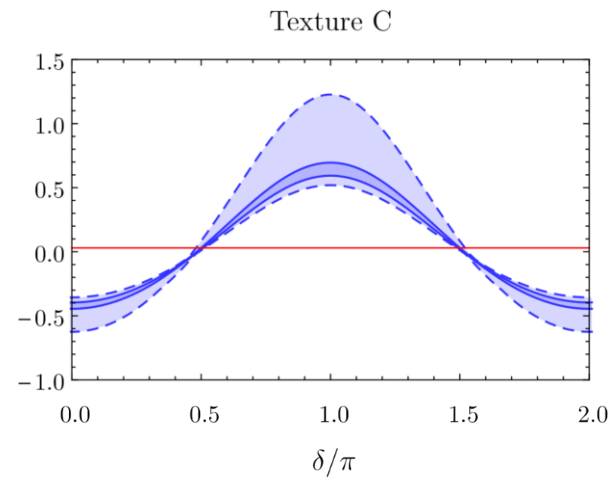
$$T_1: \begin{pmatrix} 0 & \times \\ \times & 0 \\ \times & \times \end{pmatrix}, \quad T_2: \begin{pmatrix} 0 & \times \\ \times & \times \\ \times & 0 \end{pmatrix}, \quad T_3: \begin{pmatrix} \times & \times \\ 0 & \times \\ \times & 0 \end{pmatrix},$$

$$T_4: \begin{pmatrix} \times & 0 \\ 0 & \times \\ \times & \times \end{pmatrix}, \quad T_5: \begin{pmatrix} \times & 0 \\ \times & \times \\ 0 & \times \end{pmatrix}, \quad T_6: \begin{pmatrix} \times & \times \\ \times & 0 \\ 0 & \times \end{pmatrix},$$

$$R_1: \begin{pmatrix} \times & 0 \\ \cdot & \times \end{pmatrix} \quad R_2: \begin{pmatrix} 0 & \times \\ \cdot & \times \end{pmatrix} \quad R_3: \begin{pmatrix} \times & \times \\ \cdot & 0 \end{pmatrix}$$

Predictions vs. experiment:





(In)compatible for IH (NH)

Q2: Which are compatible with data?

$\mathbf{Y}_\nu$	$\mathbf{M}_R$	$\mathbf{M}_\nu$
T_1, T_2	R_2	A: $\begin{pmatrix} 0 & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_4, T_5	R_3	
T_1, T_4	R_1	B: $\begin{pmatrix} \times & 0 & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_2, T_5	R_1	C: $\begin{pmatrix} \times & \times & 0 \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_3, T_4	R_2	D: $\begin{pmatrix} \times & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_1, T_6	R_3	
T_3, T_6	R_1	E: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & 0 \\ \cdot & \cdot & \times \end{pmatrix}$
T_5, T_6	R_2	F: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & 0 \end{pmatrix}$
T_2, T_3	R_3	

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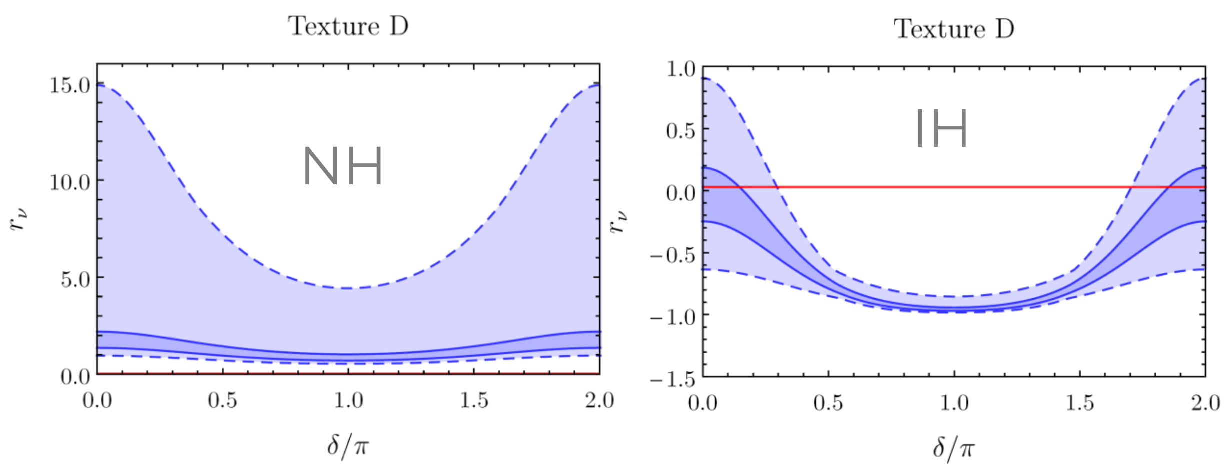
$$T_1 : \begin{pmatrix} 0 & \times \\ \times & 0 \\ \times & \times \end{pmatrix}, \quad T_2 : \begin{pmatrix} 0 & \times \\ \times & \times \\ \times & 0 \end{pmatrix}, \quad T_3 : \begin{pmatrix} \times & \times \\ 0 & \times \\ \times & 0 \end{pmatrix},$$

$$T_4 : \begin{pmatrix} \times & 0 \\ 0 & \times \\ \times & \times \end{pmatrix}, \quad T_5 : \begin{pmatrix} \times & 0 \\ \times & \times \\ 0 & \times \end{pmatrix}, \quad T_6 : \begin{pmatrix} \times & \times \\ \times & 0 \\ 0 & \times \end{pmatrix},$$

$$R_1 : \begin{pmatrix} \times & 0 \\ \cdot & \times \end{pmatrix} \quad R_2 : \begin{pmatrix} 0 & \times \\ \cdot & \times \end{pmatrix} \quad R_3 : \begin{pmatrix} \times & \times \\ \cdot & 0 \end{pmatrix}$$

Predictions vs. experiment:



(In)compatible for IH (NH)

Q2: Which are compatible with data?

$\mathbf{Y}_\nu$	$\mathbf{M}_R$	$\mathbf{M}_\nu$
T_1, T_2	R_2	A: $\begin{pmatrix} 0 & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_4, T_5	R_3	
T_1, T_4	R_1	B: $\begin{pmatrix} \times & 0 & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_2, T_5	R_1	C: $\begin{pmatrix} \times & \times & 0 \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_3, T_4	R_2	D: $\begin{pmatrix} \times & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_1, T_6	R_3	
T_3, T_6	R_1	E: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & 0 \\ \cdot & \cdot & \times \end{pmatrix}$
T_5, T_6	R_2	F: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & 0 \end{pmatrix}$
T_2, T_3	R_3	

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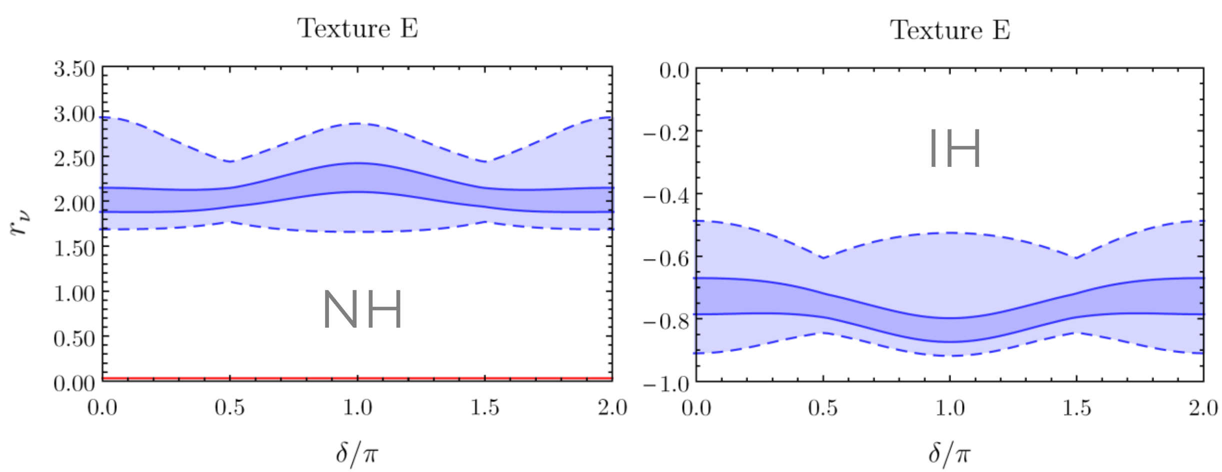
$$T_1 : \begin{pmatrix} 0 & \times \\ \times & 0 \\ \times & \times \end{pmatrix}, \quad T_2 : \begin{pmatrix} 0 & \times \\ \times & \times \\ \times & 0 \end{pmatrix}, \quad T_3 : \begin{pmatrix} \times & \times \\ 0 & \times \\ \times & 0 \end{pmatrix},$$

$$T_4 : \begin{pmatrix} \times & 0 \\ 0 & \times \\ \times & \times \end{pmatrix}, \quad T_5 : \begin{pmatrix} \times & 0 \\ \times & \times \\ 0 & \times \end{pmatrix}, \quad T_6 : \begin{pmatrix} \times & \times \\ \times & 0 \\ 0 & \times \end{pmatrix},$$

$$R_1 : \begin{pmatrix} \times & 0 \\ \cdot & \times \end{pmatrix} \quad R_2 : \begin{pmatrix} 0 & \times \\ \cdot & \times \end{pmatrix} \quad R_3 : \begin{pmatrix} \times & \times \\ \cdot & 0 \end{pmatrix}$$

Predictions vs. experiment:



Not compatible!

Q2: Which are compatible with data?

$\mathbf{Y}_\nu$	$\mathbf{M}_R$	$\mathbf{M}_\nu$
T_1, T_2	R_2	A: $\begin{pmatrix} 0 & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_4, T_5	R_3	
T_1, T_4	R_1	B: $\begin{pmatrix} \times & 0 & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_2, T_5	R_1	C: $\begin{pmatrix} \times & \times & 0 \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_3, T_4	R_2	D: $\begin{pmatrix} \times & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & \times \end{pmatrix}$
T_1, T_6	R_3	
T_3, T_6	R_1	E: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & 0 \\ \cdot & \cdot & \times \end{pmatrix}$
T_5, T_6	R_2	F: $\begin{pmatrix} \times & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & 0 \end{pmatrix}$
T_2, T_3	R_3	

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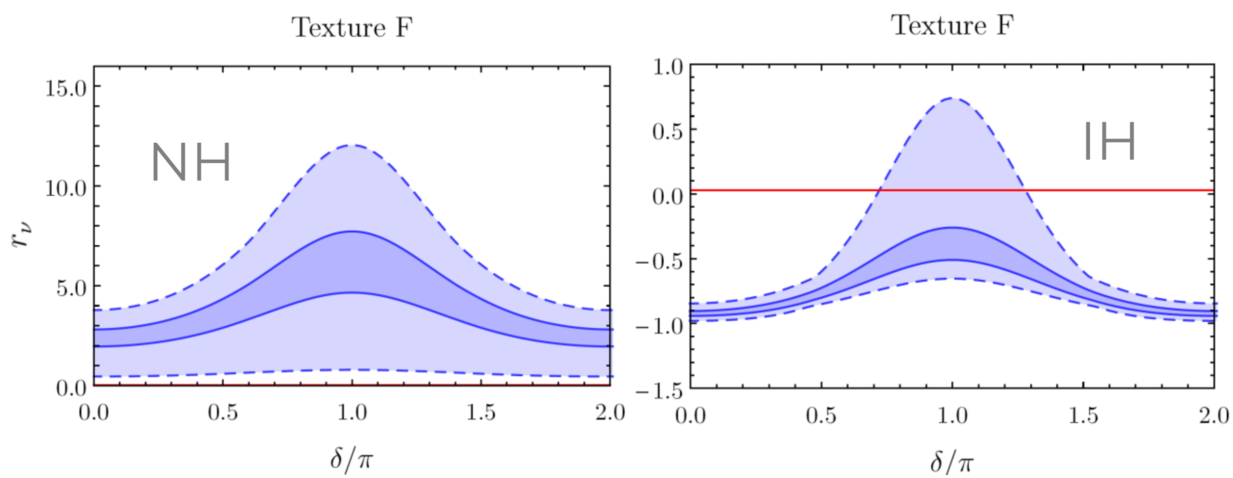
$$T_1 : \begin{pmatrix} 0 & \times \\ \times & 0 \\ \times & \times \end{pmatrix}, \quad T_2 : \begin{pmatrix} 0 & \times \\ \times & \times \\ \times & 0 \end{pmatrix}, \quad T_3 : \begin{pmatrix} \times & \times \\ 0 & \times \\ \times & 0 \end{pmatrix},$$

$$T_4 : \begin{pmatrix} \times & 0 \\ 0 & \times \\ \times & \times \end{pmatrix}, \quad T_5 : \begin{pmatrix} \times & 0 \\ \times & \times \\ 0 & \times \end{pmatrix}, \quad T_6 : \begin{pmatrix} \times & \times \\ \times & 0 \\ 0 & \times \end{pmatrix},$$

$$R_1 : \begin{pmatrix} \times & 0 \\ \cdot & \times \end{pmatrix} \quad R_2 : \begin{pmatrix} 0 & \times \\ \cdot & \times \end{pmatrix} \quad R_3 : \begin{pmatrix} \times & \times \\ \cdot & 0 \end{pmatrix}$$

Predictions vs. experiment:



(In)compatible for IH (NH)