

Scalar doublets

charged under μ - τ -philic Z_n flavor symmetry

Koji Tsumura (Kyoto U.)

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Based JHEP 06 (2019) 142 with Y. Abe (Kyoto U.), T. Toma (McGill U.)

“A μ - τ -philic scalar doublet under Z_n flavor symmetry”

Contents

- Muon $g-2$
- Flavor Symmetry
 - $U(1)_{\mu-\tau}$
 - Z_n
- Minimal Model
 - $g-2$
 - LHC, EW precision data, Lepton Universality, ...
 - Signature
- Summary

Magnetic Moment of Muon

- **g-factor** : Int. btw Spin-B (Magnetic field)

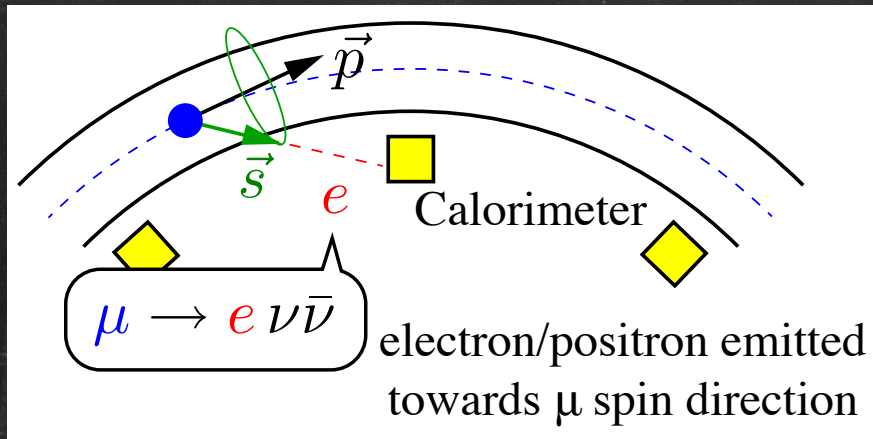
- ✓ Classical (Dirac Eq.) $\rightarrow g=2$ $\mathcal{H} = -\vec{\mu} \cdot \vec{B}$ $\left(\vec{\mu} = g \frac{e}{2m} \vec{S} \right)$
- ✓ Quantum $\rightarrow g=2(1+a_\mu)$ [$a_\mu=(g-2)/2$]

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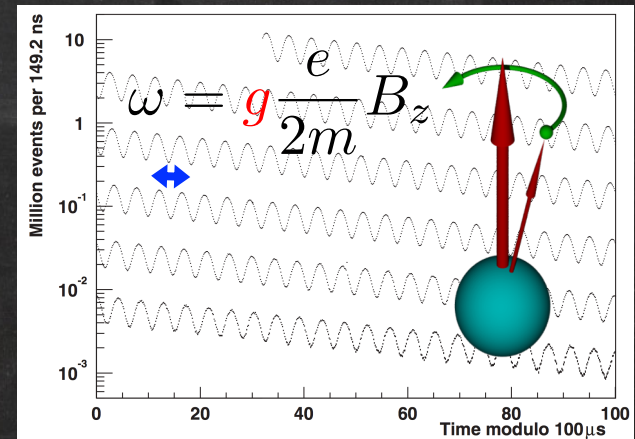
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- Experiment : **BNL E821**



Larmor Precession

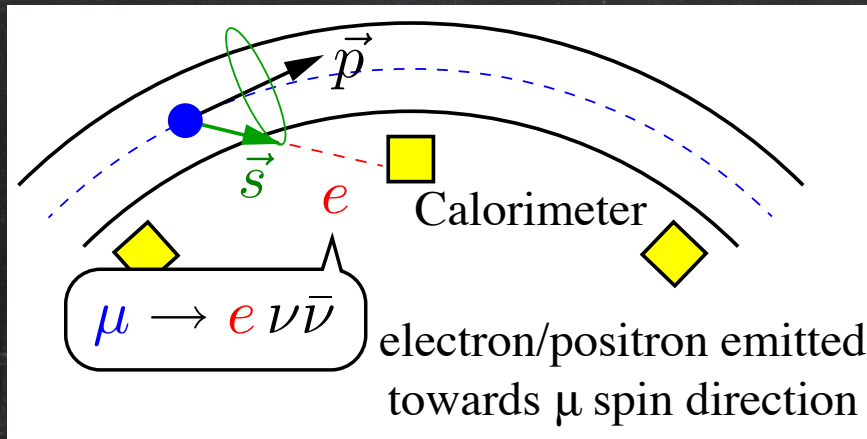


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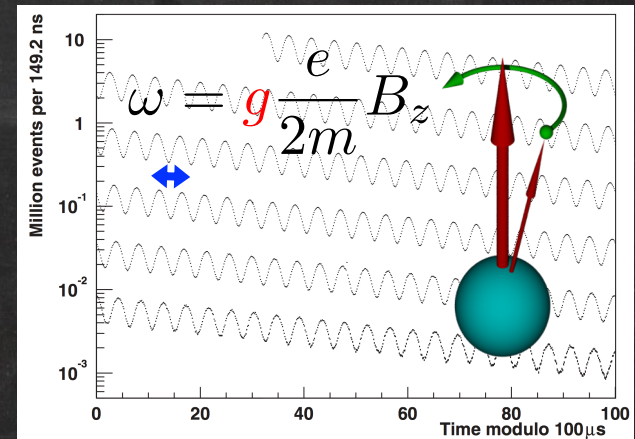
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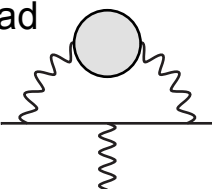
Larmor Precession



$$a_\mu^{\text{exp}} = 116\,592\,091(54)_{\text{stat}}(33)_{\text{sys}} \times 10^{-11}$$

KNT18 a_μ^{SM} update [KNT18: arXiv:1802.02995, PRD (in press)]

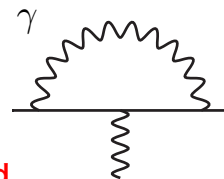
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| QED | 11658471.81 (0.02) | → | 11658471.90 (0.01) | [arXiv:1712.06060]  |
| EW | 15.40 (0.20) | → | 15.36 (0.10) | [Phys. Rev. D 88 (2013) 053005] |
| LO HLbL | 10.50 (2.60) | → | 9.80 (2.60) | [EPJ Web Conf. 118 (2016) 01016] |
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| | <u>HLMNT11</u> | → | <u>KNT18</u> | |
|-----------------|----------------|---|---------------|---|
| LO HVP | 694.91 (4.27) | → | 693.27 (2.46) | this work  |
| NLO HVP | -9.84 (0.07) | → | -9.82 (0.04) | this work |
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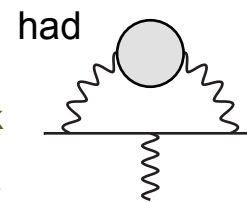
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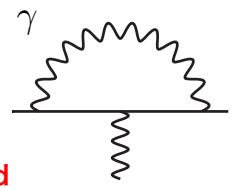
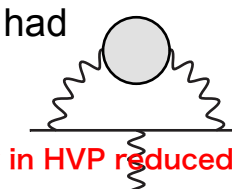


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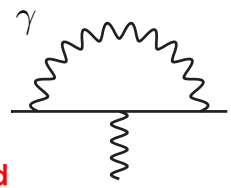
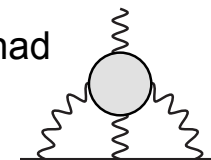
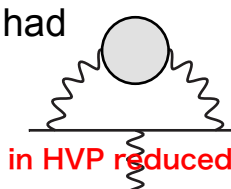


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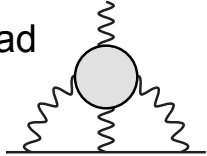
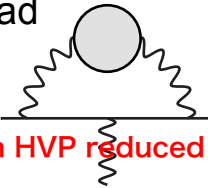
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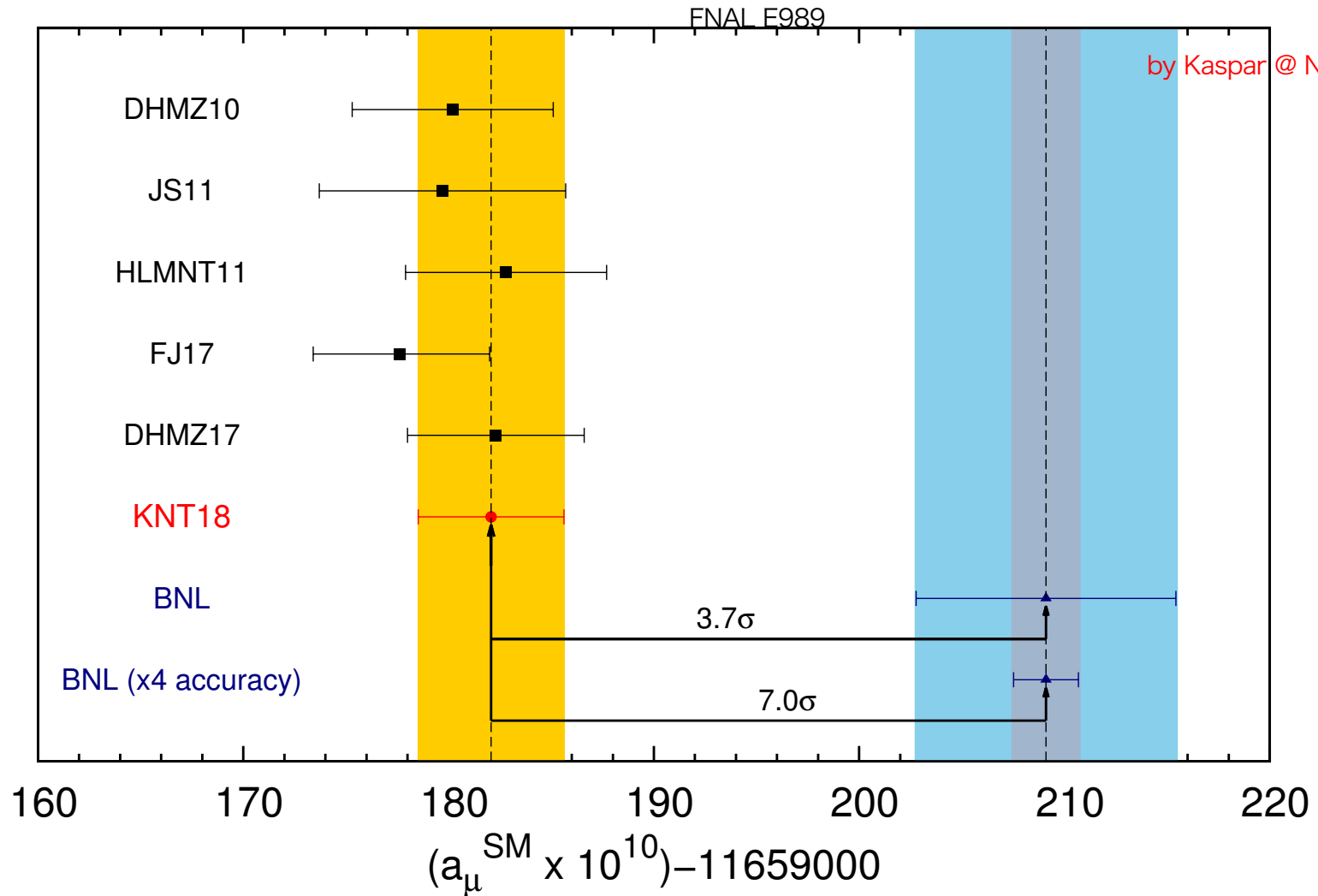
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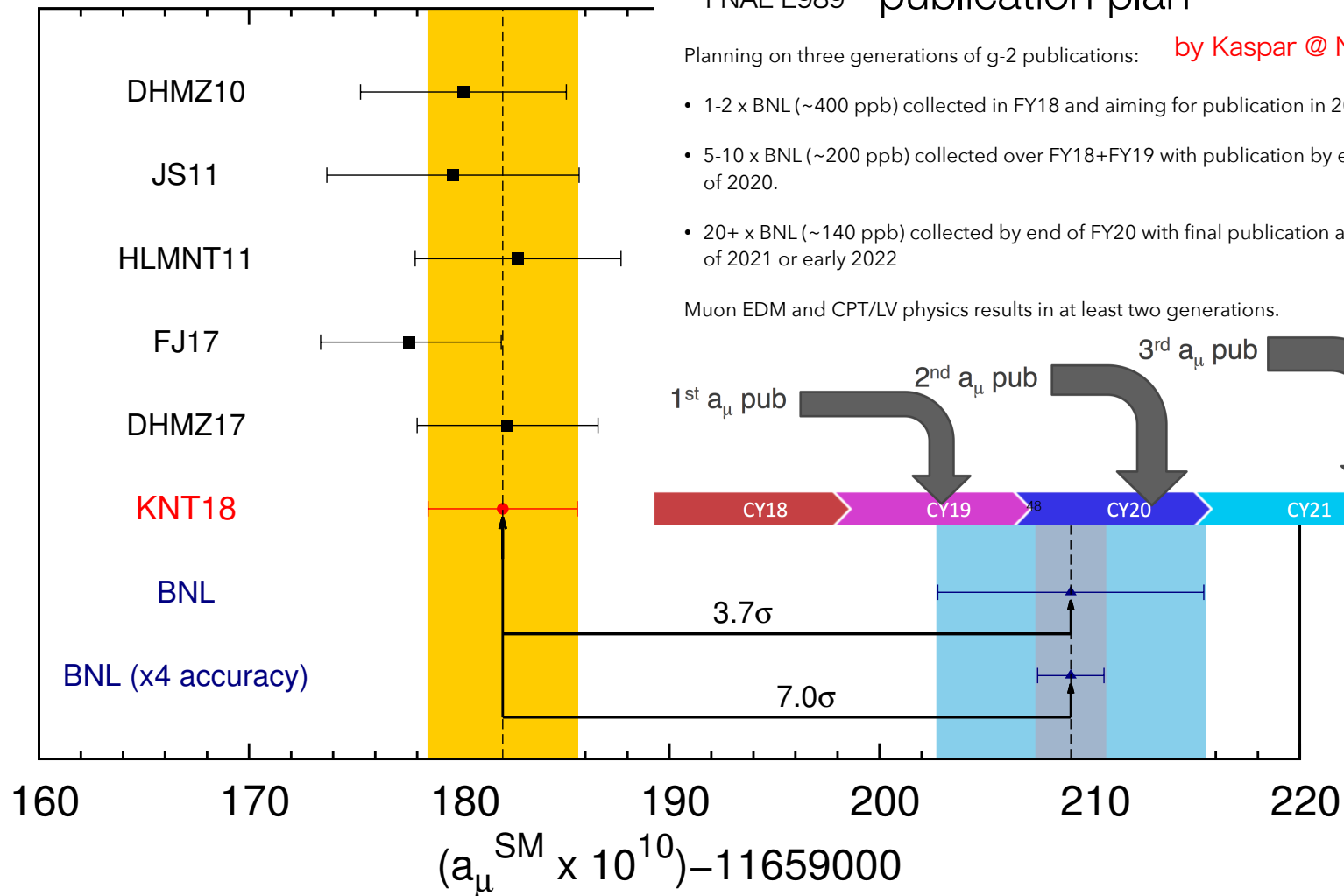
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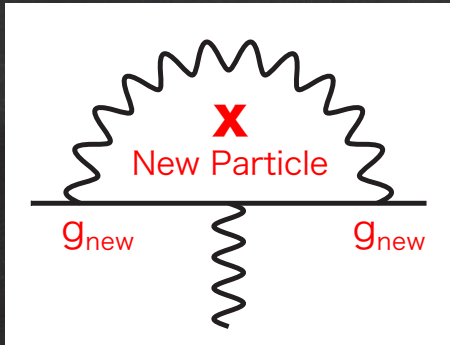
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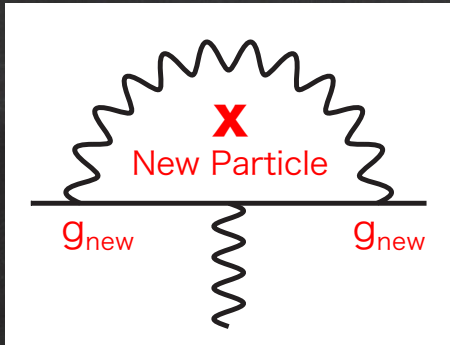


New Physics



$$\Delta a_{\mu}^{\text{new}} \sim \frac{g_{\text{new}}^2}{(4\pi)^2} \frac{M_{\mu}^2}{M_X^2}$$

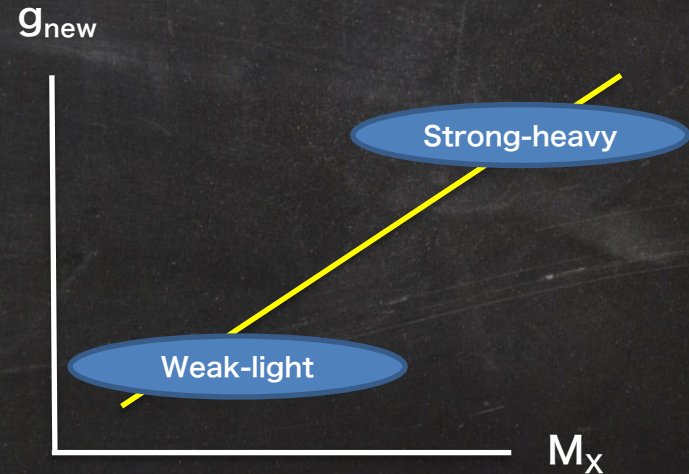
New Physics



$$\Delta a_{\mu}^{\text{new}} \sim \frac{g_{\text{new}}^2}{(4\pi)^2} \frac{M_{\mu}^2}{M_X^2}$$

To fit the observed discrepancy

$$g_{\text{new}} \approx g_{\text{weak}} \text{ and } M_X \approx M_W$$
$$(g_{\text{new}} \approx 10^{-3} \text{ and } M_X \approx M_{\mu})$$



Flavor symmetry and a_μ

Gauged $U(1)_{\mu-\tau}$ Model

- A minimal ext. of SM
- Anomaly free

| | l_e | e_R | l_μ | μ_R | l_τ | τ_R |
|-------------------------|-------|-------|---------|---------|----------|----------|
| L_μ | 0 | 0 | +1 | +1 | 0 | 0 |
| L_τ | 0 | 0 | 0 | 0 | +1 | +1 |
| $U(1)_{L_\mu - L_\tau}$ | 0 | 0 | +1 | +1 | -1 | -1 |

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Lagrangian

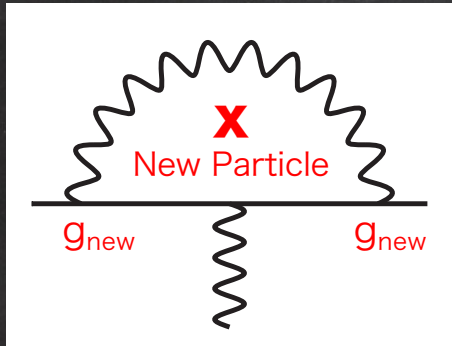
Kinetic mixing w/ SM gauge boson

$$\mathcal{L}_{\text{new}} = -\frac{1}{4} Z'_{\alpha\beta} Z'^{\alpha\beta} + \frac{M_{Z'}^2}{2} Z'_\alpha Z'^\alpha + \frac{\epsilon}{2} B_{\alpha\beta} Z'^{\alpha\beta} + g' Z'_\alpha (+\bar{\mu}\gamma^\alpha \mu + \bar{\nu}_\mu \gamma^\alpha \nu_\mu - \bar{\tau}\gamma^\alpha \tau - \bar{\nu}_\tau \gamma^\alpha \nu_\tau)$$

New gauge int. for μ and τ

Gauged U(1)_{μ-τ} and a_μ

- Z' can give a sufficient amount of $\Delta a_{\mu}^{\text{new}}$



$$\Delta a_{\mu}^{\text{new}} \sim \frac{g_{\text{new}}^2}{(4\pi)^2} \frac{M_{\mu}^2}{M_X^2}$$

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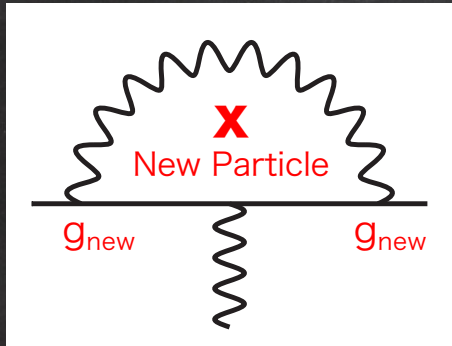
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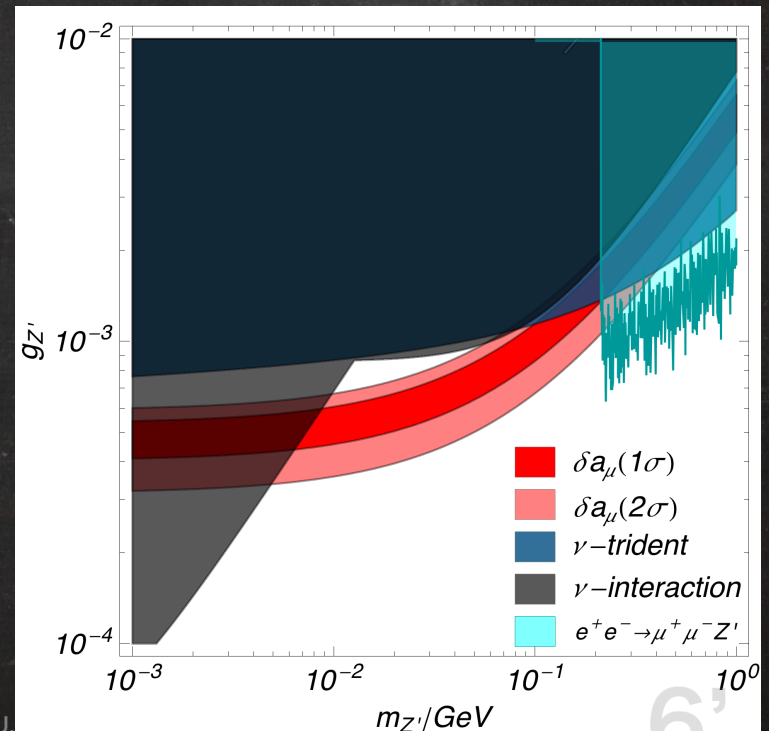
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“weakly” interacting low mass Z'

Ibe, Nakano, Suzuki (16)



Global $U(1)_{\mu-\tau}$?

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- No Z' boson
- Flavor charged scalars?

| | $U(1)_{\mu-\tau}$ |
|------------------------|-------------------|
| $[L_e, L_\mu, L_\tau]$ | $[0, +1, -1]$ |
| $[e_R, \mu_R, \tau_R]$ | $[0, +1, -1]$ |
| H | 0 |
| Φ | +2 |
| $\underline{\Phi}$ | -2 |

$$-\mathcal{L}_{\text{yukawa}} = (\overline{e_R} \quad \overline{\mu_R} \quad \overline{\tau_R}) \begin{pmatrix} y_e H^\dagger & & \\ & y_\mu H^\dagger & y_{\mu\tau} \underline{\Phi}^\dagger \\ & y_{\tau\mu} \Phi^\dagger & y_\tau H^\dagger \end{pmatrix} \begin{pmatrix} l_e \\ l_\mu \\ l_\tau \end{pmatrix} + \text{H.c.}$$

A pair of scalar doublets can have Yukawa int.

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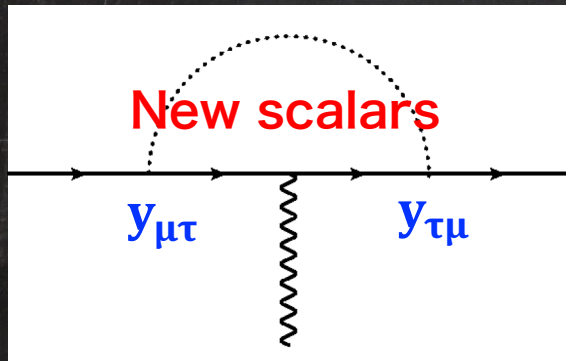
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$$\Delta a_\mu^{\text{new}} \simeq \frac{y_{\mu\tau} y_{\tau\mu}}{(4\pi)^2} \frac{M_\mu^2}{M_\Phi^2} \left(\frac{M_\tau}{M_\mu} \right)$$

Chirality enhancement!!

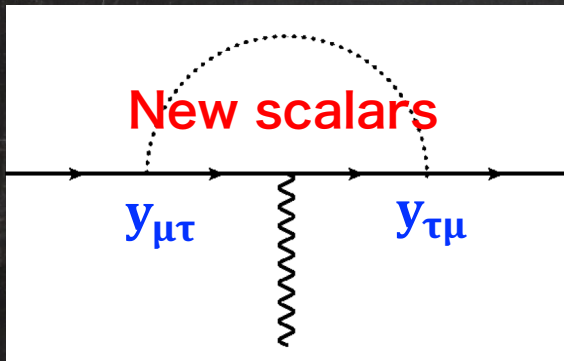
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Chirality enhancement!!

$U(1)$ breaking is NOT necessary!!

Global U(1) _{μ - τ} and M _{ν}

- Large neutrino mixing

Choubey, Rodejohann (05)
Ota, Rodejohann (06)

$$M_\nu \propto \begin{pmatrix} m_{11} & & \\ & & m_{23} \\ & m_{23} & \end{pmatrix}$$

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- Symmetry breaking by S

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- ✓ Fit with ν oscillation data
- ✓ Massless NGB

Global $U(1)_{\mu-\tau}$ and M_ν

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- ✓ Fit with ν oscillation data
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⇒ Explicit $U(1)_{\mu-\tau}$ breaking : Z_n

Z_n flavor symmetries

Z_n subgroup of $U(1)_{\mu-\tau}$

| | $U(1)_{\mu-\tau}$ | Z_2 | Z_3 | Z_4 | Z_n |
|--------------------------|-------------------|-------------|-------------------------|---------------|-----------------------------------|
| $[L_e, L_\mu, L_\tau]$ | $[0, +1, -1]$ | $[+, -, -]$ | $[1, \omega, \omega^2]$ | $[1, +i, -i]$ | $[1, \omega, \underline{\omega}]$ |
| $[e_R, \mu_R, \tau_R]$ | $[0, +1, -1]$ | $[+, -, -]$ | $[1, \omega, \omega^2]$ | $[1, +i, -i]$ | $[1, \omega, \underline{\omega}]$ |
| H | 0 | + | 1 | 1 | 1 |
| Φ | +2 | + | ω^2 | -1 | ω^2 |
| <u>Φ</u> | -2 | + | ω | -1 | <u>ω^2</u> |

For $n \geq 5$, global $U(1)_{\mu-\tau}$ is recovered

$$V \sim +\lambda_5 (H^\dagger \Phi) (H^\dagger \underline{\Phi})$$

Z_n flavor symmetries

Z_n subgroup of U(1)_{μ-τ}

| | U(1) _{μ-τ} | Z ₂ | Z ₃ | Z ₄ | Z _n |
|---|---------------------|----------------|-------------------------|----------------|-----------------------|
| [L _e , L _μ , L _τ] | [0, +1, -1] | [+, -, -] | [1, ω, ω ²] | [1, +i, -i] | [1, ω, ω] |
| [e _R , μ _R , τ _R] | [0, +1, -1] | [+, -, -] | [1, ω, ω ²] | [1, +i, -i] | [1, ω, ω] |
| H | 0 | + | 1 | 1 | 1 |
| Φ | +2 | + | ω ² | -1 | ω ² |
| <u>Φ</u> | -2 | + | ω | -1 | <u>ω</u> ² |

Φ and Φ are identical

$$V \sim +\lambda_5 (H^\dagger \Phi)^2$$

Candidates for the Minimal Model

Z_n conserving Yukawa int.

Structure of Yukawa int.

$$-\mathcal{L}_{Z_2}^{\text{yukawa}} = \overline{\ell}_R \begin{pmatrix} y_e H^\dagger + y_{ee} \Phi^\dagger & & \\ & y_\mu H^\dagger + y_{\mu\mu} \Phi^\dagger & g_{\mu\tau} H^\dagger + y_{\mu\tau} \Phi^\dagger \\ & g_{\tau\mu} H^\dagger + y_{\tau\mu} \Phi^\dagger & y_\tau H + y_{\tau\tau} \Phi^\dagger \end{pmatrix} L + \text{H.c.}$$

$$-\mathcal{L}_{Z_3}^{\text{yukawa}} = \overline{\ell}_R \begin{pmatrix} y_e H^\dagger & \underline{y_{e\mu}} \Phi^\dagger & y_{e\tau} \Phi^\dagger \\ y_{\mu e} \Phi^\dagger & y_\mu H^\dagger & \underline{y_{\mu\tau}} \Phi^\dagger \\ \underline{y_{\tau e}} \Phi^\dagger & y_{\tau\mu} \Phi^\dagger & y_\tau H^\dagger \end{pmatrix} L + \text{H.c.}$$

$$-\mathcal{L}_{Z_4}^{\text{yukawa}} = \overline{\ell}_R \begin{pmatrix} y_e H^\dagger & & \\ & y_\mu H^\dagger & y_{\mu\tau} \Phi^\dagger \\ & y_{\tau\mu} \Phi^\dagger & y_\tau H^\dagger \end{pmatrix} L + \text{H.c.}$$

$$-\mathcal{L}_{Z_{n \geq 5}}^{\text{yukawa}} = \overline{\ell}_R \begin{pmatrix} y_e H^\dagger & & \\ & y_\mu H^\dagger & \underline{y_{\mu\tau}} \Phi^\dagger \\ & y_{\tau\mu} \Phi^\dagger & y_\tau H^\dagger \end{pmatrix} L + \text{H.c.}$$

Z_n conserving Yukawa int.

Structure of Yukawa int.

No distinction btw H and Φ
(Φ tends to have VEV)

$$-\mathcal{L}_{Z_2}^{\text{yukawa}} = \overline{\ell}_R \begin{pmatrix} y_e H^\dagger + y_{ee} \Phi^\dagger & & & \\ & y_\mu H^\dagger + y_{\mu\mu} \Phi^\dagger & g_{\mu\tau} H^\dagger + y_{\mu\tau} \Phi^\dagger & \\ & g_{\tau\mu} H^\dagger + y_{\tau\mu} \Phi^\dagger & & y_\tau H + y_{\tau\tau} \Phi^\dagger \end{pmatrix} L + \text{H.c.}$$

$$-\mathcal{L}_{Z_3}^{\text{yukawa}} = \overline{\ell}_R \begin{pmatrix} y_e H^\dagger & \underline{y_{e\mu}} \Phi^\dagger & y_{e\tau} \Phi^\dagger \\ y_{\mu e} \Phi^\dagger & y_\mu H^\dagger & \underline{y_{\mu\tau}} \Phi^\dagger \\ \underline{y_{\tau e}} \Phi^\dagger & y_{\tau\mu} \Phi^\dagger & y_\tau H^\dagger \end{pmatrix} L + \text{H.c.}$$

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$$-\mathcal{L}_{Z_4}^{\text{yukawa}} = \overline{\ell}_R \begin{pmatrix} y_e H^\dagger & & \\ & y_\mu H^\dagger & y_{\mu\tau} \Phi^\dagger \\ & y_{\tau\mu} \Phi^\dagger & y_\tau H^\dagger \end{pmatrix} L + \text{H.c.}$$

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Z_n conserving Yukawa int.

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Source of LFV

$$-\mathcal{L}_{Z_4}^{\text{yukawa}} = \overline{\ell}_R \begin{pmatrix} y_e H^\dagger & & \\ & y_\mu H^\dagger & y_{\mu\tau} \Phi^\dagger \\ & y_{\tau\mu} \Phi^\dagger & y_\tau H^\dagger \end{pmatrix} L + \text{H.c.}$$

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**Candidates
for the Minimal Model**

Minimal Model

Z₄ model

- Z₂ symmetric 2HDM

$$V(H, \Phi) = -\mu_H^2 H^\dagger H + m_\Phi^2 \Phi^\dagger \Phi + \lambda_1 (H^\dagger H)^2 + \lambda_2 (\Phi^\dagger \Phi)^2 \\ + \lambda_3 (H^\dagger H)(\Phi^\dagger \Phi) + \lambda_4 (H^\dagger \Phi)(\Phi^\dagger H) + \left[+ \frac{\lambda_5}{2} (H^\dagger \Phi)^2 + \text{H.c.} \right]$$

Z₄ model

- **Z₂ symmetric 2HDM**

$$V(H, \Phi) = -\mu_H^2 H^\dagger H + m_\Phi^2 \Phi^\dagger \Phi + \lambda_1 (H^\dagger H)^2 + \lambda_2 (\Phi^\dagger \Phi)^2 \\ + \lambda_3 (H^\dagger H)(\Phi^\dagger \Phi) + \lambda_4 (H^\dagger \Phi)(\Phi^\dagger H) + \left[+ \frac{\lambda_5}{2} (H^\dagger \Phi)^2 + \text{H.c.} \right]$$

- **Positive mass square for Φ**

$$H = \begin{pmatrix} 0 \\ (v + h_{125})/\sqrt{2} \end{pmatrix} \quad \Phi = \begin{pmatrix} \phi^+ \\ (\rho + i\eta)/\sqrt{2} \end{pmatrix}$$

Z₄ model

- Z₂ symmetric 2HDM

$$M_\rho^2 - M_\eta^2 = \lambda_5 v^2$$

$$V(H, \Phi) = -\mu_H^2 H^\dagger H + m_\Phi^2 \Phi^\dagger \Phi + \lambda_1 (H^\dagger H)^2 + \lambda_2 (\Phi^\dagger \Phi)^2 \\ + \lambda_3 (H^\dagger H)(\Phi^\dagger \Phi) + \lambda_4 (H^\dagger \Phi)(\Phi^\dagger H) + \left[+ \frac{\lambda_5}{2} (H^\dagger \Phi)^2 + \text{H.c.} \right]$$

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Z₄ model

- **Z₂ symmetric 2HDM**

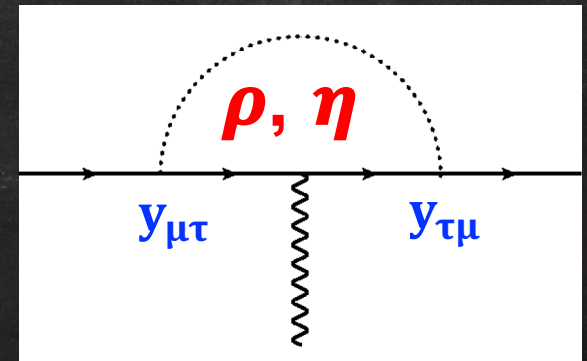
$$M_\rho^2 - M_\eta^2 = \lambda_5 v^2$$

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$$-\mathcal{L}_{Z_4}^{\text{yukawa}} = \overline{\ell}_R \begin{pmatrix} y_e H^\dagger & & \\ & y_\mu H^\dagger & y_{\mu\tau} \Phi^\dagger \\ & y_{\tau\mu} \Phi^\dagger & y_\tau H^\dagger \end{pmatrix} L + \text{H.c.}$$

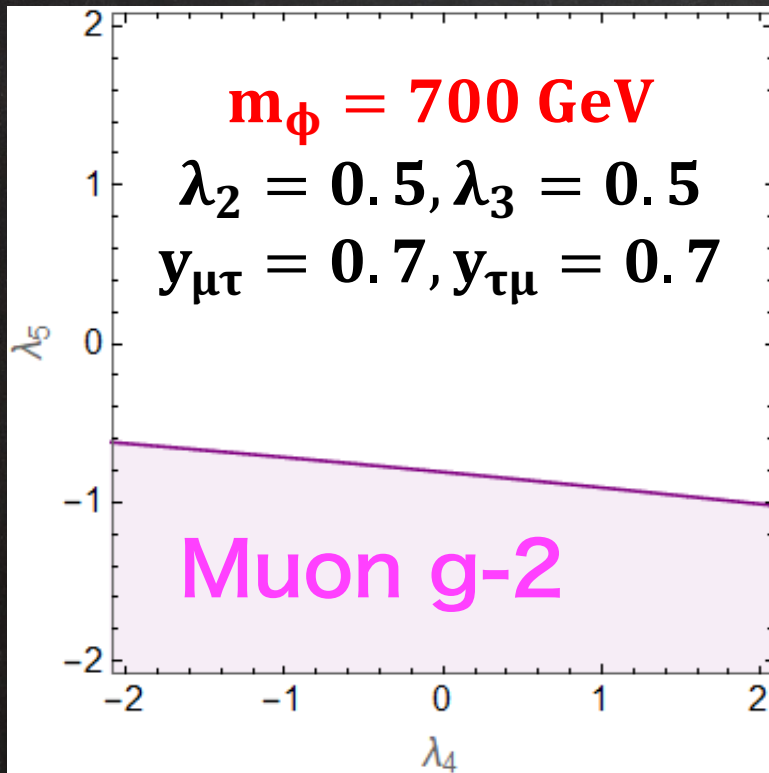


Z₄ model and a_μ

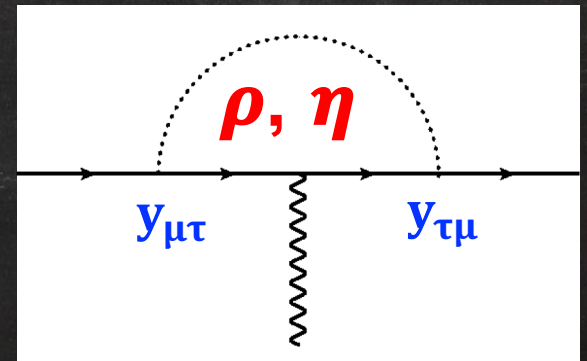
● Muon g-2

$$M_\rho^2 - M_\eta^2 = \lambda_5 v^2$$

$$\Delta a_\mu^{\text{new}} \simeq \frac{\text{Re}(y_{\mu\tau} y_{\tau\mu})}{(4\pi)^2} \frac{M_\mu^2}{M_\phi^2} \left(\frac{M_\tau}{M_\mu}\right) \left(\frac{\lambda_5 v^2}{M_\phi^2}\right) \left(\frac{5}{2} + \ln \frac{M_\tau^2}{M_\phi^2}\right)$$

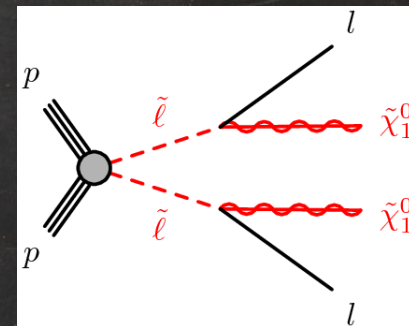


- ✓ Chirality Enhancement
- ✓ Mass splitting btw ρ and η
- ✓ Large Yukawa (w/ heavy scalar)

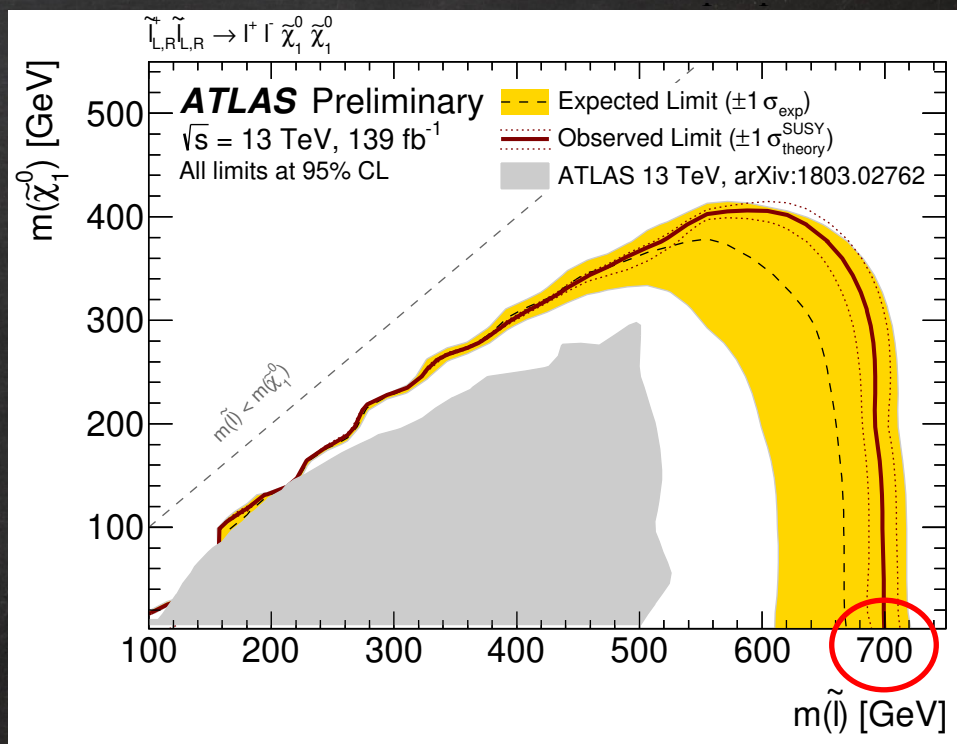
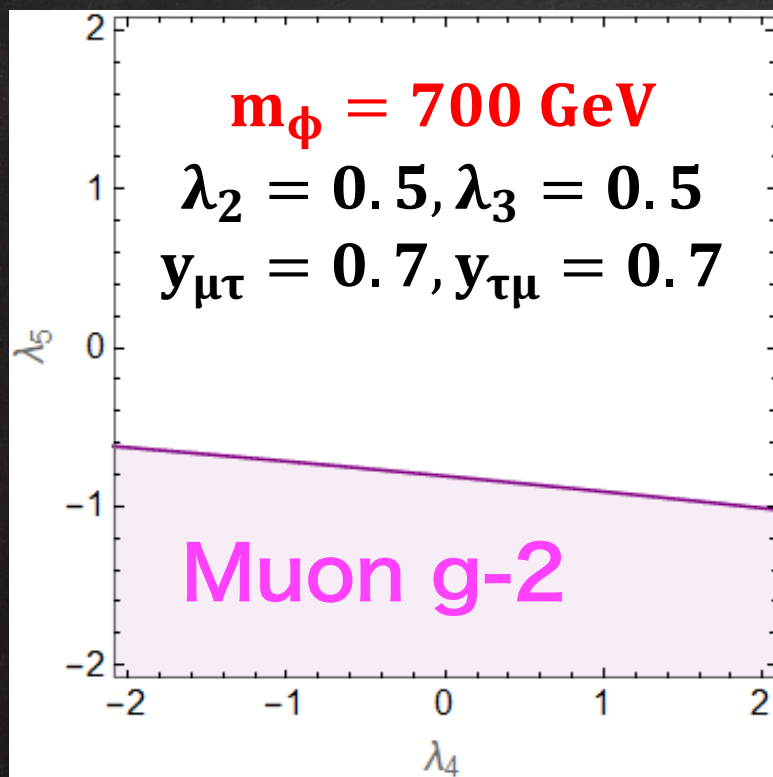


Z₄ model @ LHC

- EW pair production and $\phi^\pm \rightarrow \ell^\pm \nu$
(Slepton mass bound w/ massless neutralino)



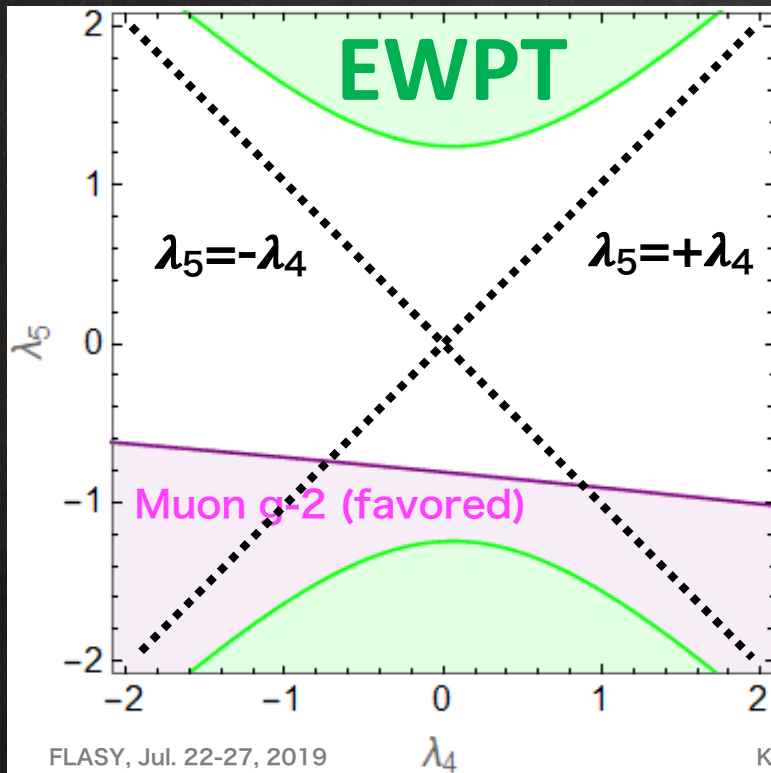
$M_\phi \gtrsim 700 \text{ GeV}$



Z₄ model and EW precision Test

- Peskin-Takeuchi's T parameter

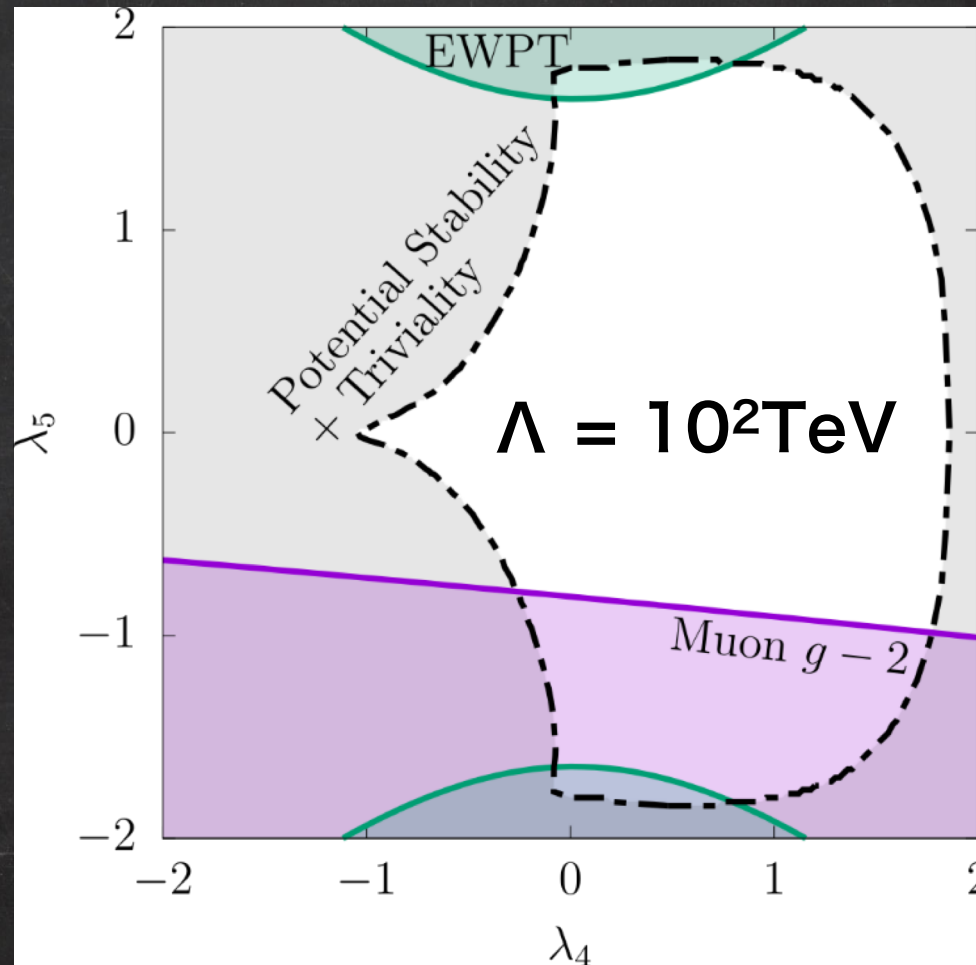
$$\alpha_{\text{EM}} T \simeq \frac{\lambda_4^2 - \lambda_5^2}{12(4\pi)^2} \frac{v^2}{M_\phi^2} \quad (T = 0.09 \pm 0.13)$$



✓ EWPT favors mass degeneracy

$$\begin{cases} M_\rho^2 &= M_\phi^2 + (\lambda_4 + \lambda_5) \frac{v^2}{2} \\ M_\eta^2 &= M_\phi^2 + (\lambda_4 - \lambda_5) \frac{v^2}{2} \end{cases}$$

Z₄ model and Theory constraints



Triviality

$$|\lambda_i(\Lambda)| < 4\pi$$

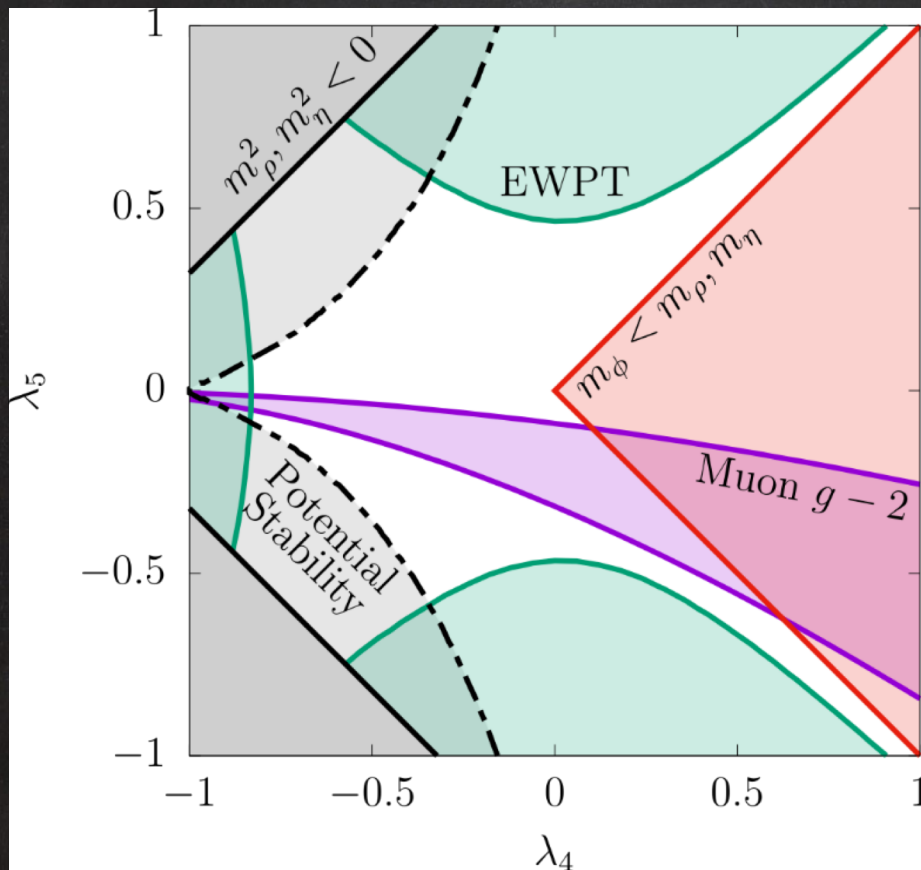
Stability

$$2\sqrt{\lambda_1(\Lambda)\lambda_2(\Lambda)} + \lambda_3(\Lambda) + \lambda_4(\Lambda) \pm |\lambda_5(\Lambda)| > 0$$

LHC bound revisited

- Cascade decay $\phi^+ \rightarrow W^+ \rho, W^+ \eta$

Slepton mass bound cannot be applied



Low mass solution possible

$$m_\phi = 200 \text{ GeV}$$

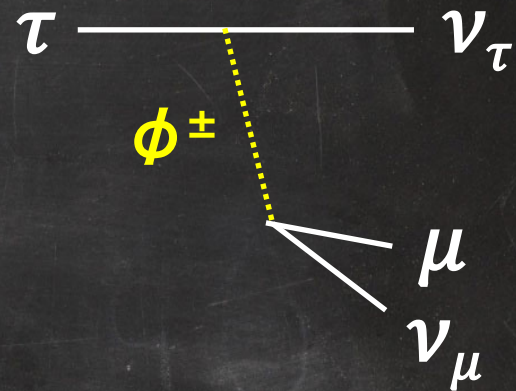
$$\lambda_2 = 0.5, \lambda_3 = 0.5$$

$$y_{\mu\tau} = 0.2, y_{\tau\mu} = 0.2$$

Lepton Universality Violation

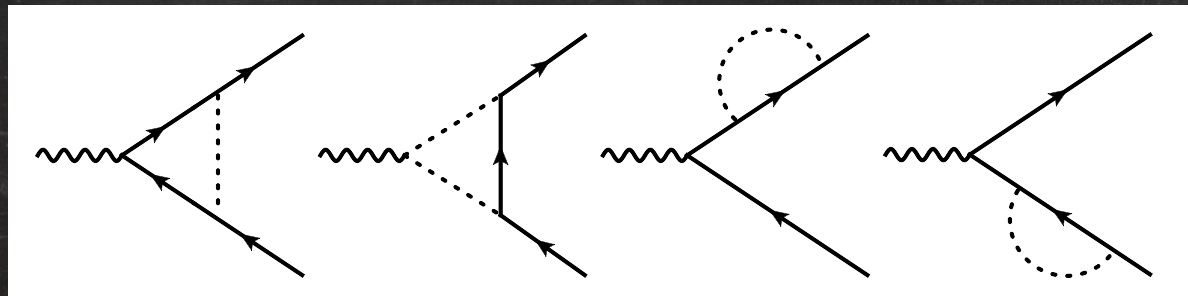
- τ Michel decay @ Tree

$$\Gamma \simeq \Gamma^{\text{SM}} \left(1 + \frac{|y_{\mu\tau}|^2 |y_{\tau\mu}|^2}{32G_F^2 M_\phi^2} \right)$$

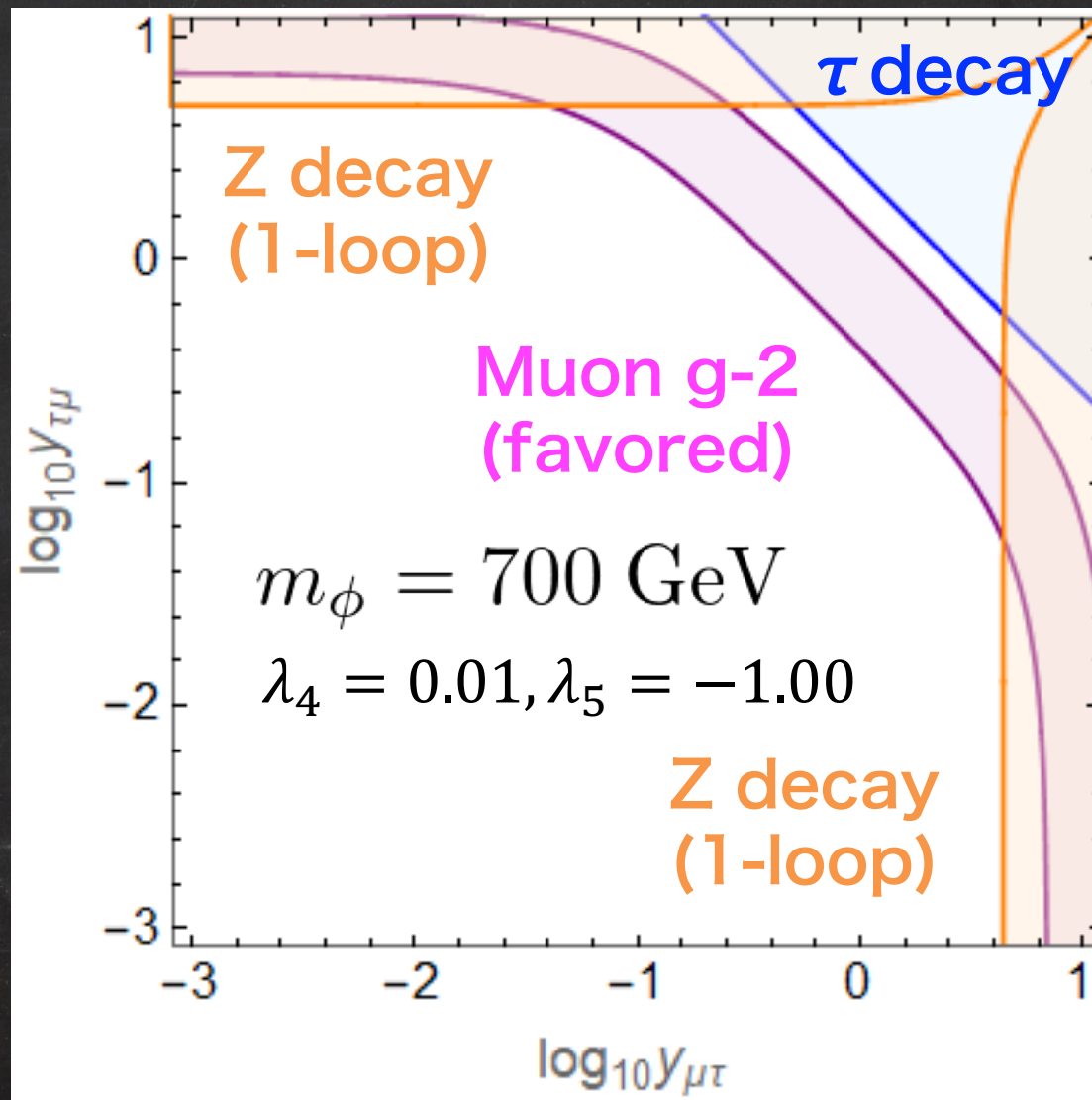


- Z decay @ 1-loop

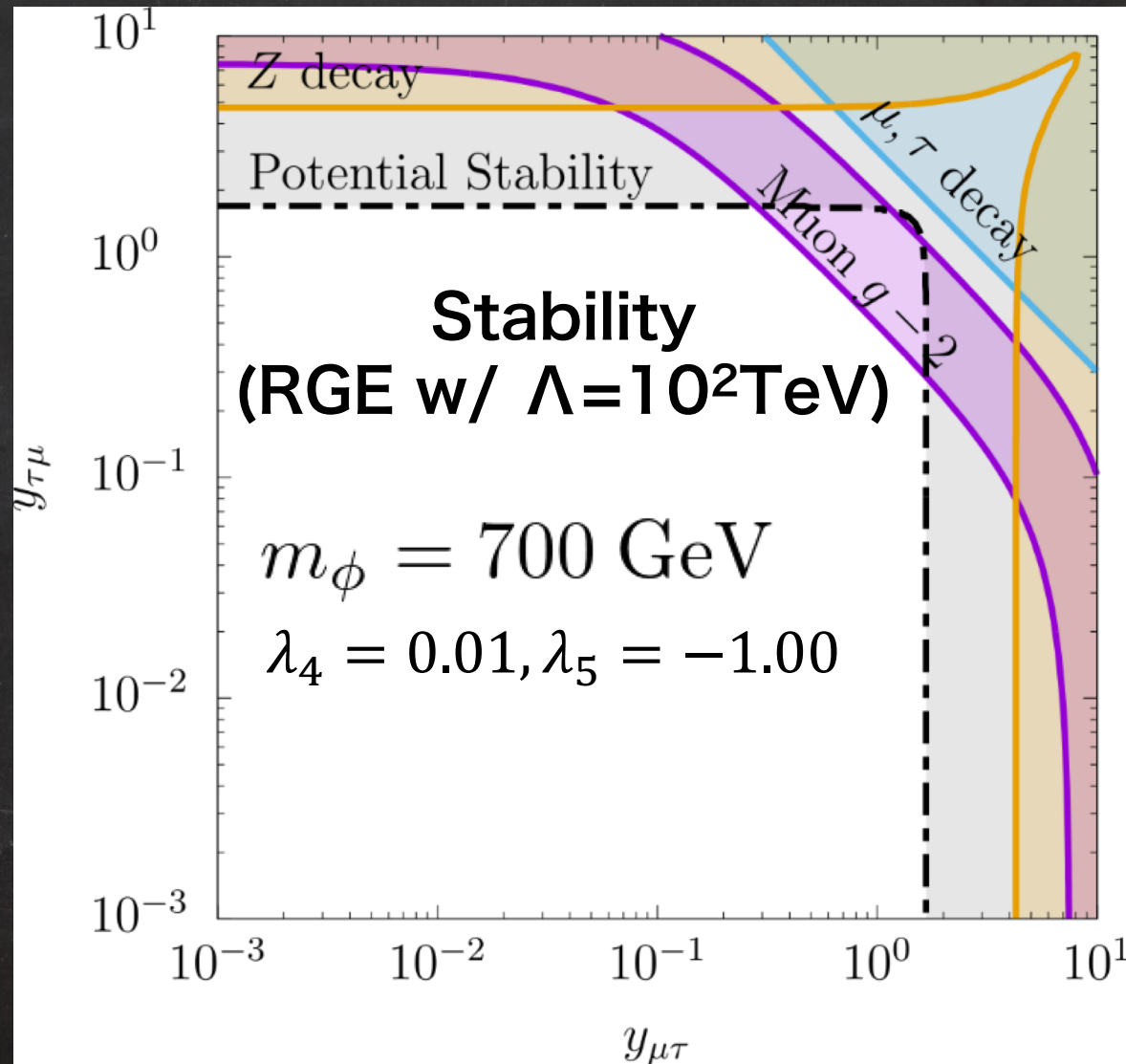
$$\delta g_{Z\mu\bar{\mu}}^L, \delta g_{Z\tau\bar{\tau}}^R \propto |y_{\tau\mu}|^2, \quad \delta g_{Z\mu\bar{\mu}}^R, \delta g_{Z\tau\bar{\tau}}^L \propto |y_{\mu\tau}|^2$$



Constraints on Yukawa plane



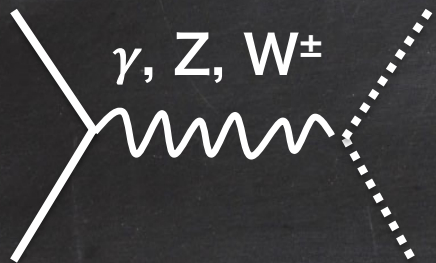
Constraints on Yukawa plane



Signals ?

Collider signature

- μ - τ specific scalar from EW pair production



ρ, η, ϕ^\pm

$$\mathcal{B}(\rho, \eta \rightarrow \mu\tau) \simeq 100\%$$

⇒ Clear $M_{\tau\mu}$ signal with small LFV BG

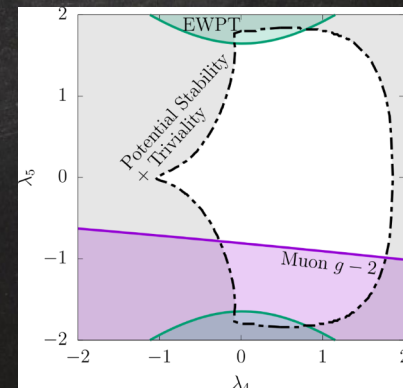
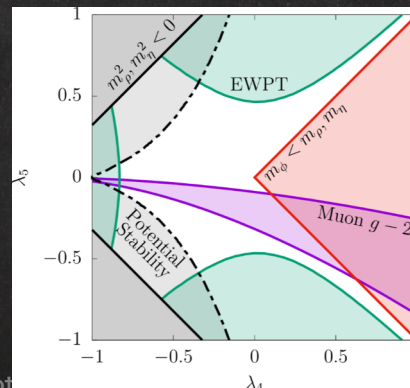
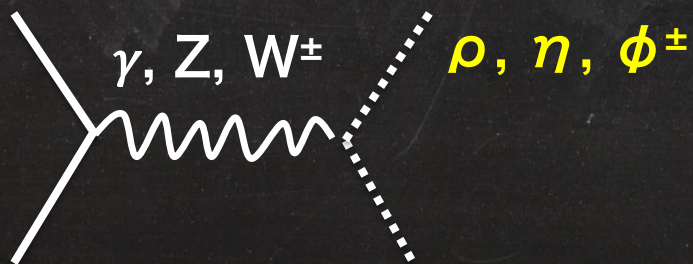
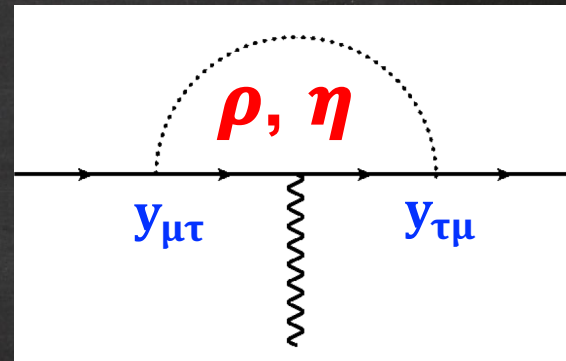
Summary

Contents

- Muon $g-2$
- Flavor Symmetry
 - $U(1)_{\mu-\tau}$
 - Z_n
- Minimal Model
 - $g-2$
 - LHC, EW precision data, Lepton Universality, ...
 - Signature
- Summary

Summary

- Muon $g-2 \rightarrow 3.7\sigma$ anomaly
- Flavor Symmetry
 - $U(1)_{\mu-\tau}$
 - Z_n
- Minimal Model $\rightarrow Z_4$ symmetry w/ 1 more scalar doublet
 - $g-2$ (Chirality enhancement)
 - LHC, EW precision data, Lepton Universality, ...
 - Signature



Acknowledgment

Thank you

- The work of KT is supported by the MEXT Grant-in-Aid for Scientific Research on Innovation Areas (Grant No. 16H00868)
- This visit is supported by The Kyoto University Foundation

Backup

Z₄ model and M_ν

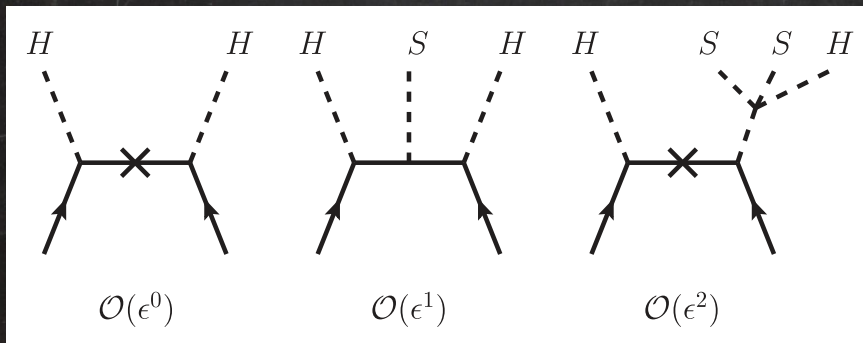
- Large neutrino mixing

Choubey, Rodejohann (05)
Ota, Rodejohann (06)

$$M_\nu \propto \begin{pmatrix} m_{11} & & \\ & & m_{23} \\ & m_{23} & \end{pmatrix}$$

- Symmetry breaking by S

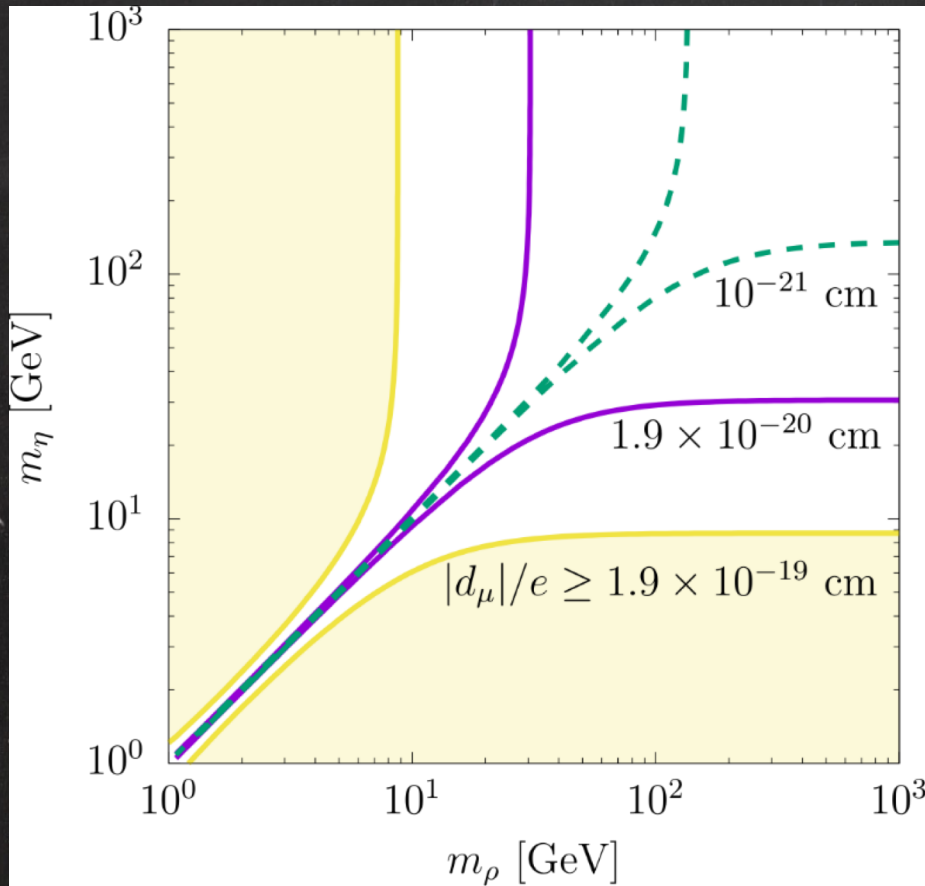
$$M_\nu \propto \begin{pmatrix} m_{11} & \epsilon_{12} \langle S^* \rangle & \epsilon_{13} \langle S \rangle \\ \epsilon_{12} \langle S^* \rangle & \mathcal{O}(\kappa \epsilon^2) & m_{23} \\ \epsilon_{13} \langle S \rangle & m_{23} & \mathcal{O}(\kappa \epsilon^2) \end{pmatrix}$$



Induced VEV for Φ

$$+ \kappa S^2 H^\dagger \Phi$$

Muon EDM



$$\frac{d_\mu}{e} \simeq \frac{\text{Im}(y_{\mu\tau}y_{\tau\mu})}{2(4\pi)^2} \left[-\frac{M_\tau \lambda_5 v^2}{6M_\phi^4} \right]$$

$$\text{Im}(y_{\mu\tau}y_{\tau\mu}) = 1.00$$

(Depend of Imaginary Part, Not guaranteed)