# A mini review of neutrino mu-tau reflection symmetry

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# Mu-tau interchange symmetry



- Motived by near maximality of $\theta_{23}$ and smallness of $\theta_{13}$
- For diagonal $M_l$, $M_\nu$ keeps invariant under $\nu_\mu \leftrightarrow \nu_\tau$ interchange

$$M_l = \mathrm{Diag}(m_e, m_\mu, m_\tau) \qquad XM_\nu X = M_\nu \quad \text{with} \quad X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \Longrightarrow \begin{cases} M_{e\mu} = M_{e\tau} \\ M_{\mu\mu} = M_{\tau\tau} \end{cases}$$

$$U = \begin{pmatrix} e^{i\phi_e} & & \\ & e^{i\phi_\mu} & \\ & & e^{i\phi_\tau} \end{pmatrix} \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \begin{pmatrix} e^{i\rho} & & \\ & e^{i\sigma} & \\ & & 1 \end{pmatrix}$$

$\phi_\mu = \phi_\tau + \pi$ $\qquad$ $\theta_{23} = 45^\circ$ $\qquad$ $s_{13}e^{i\delta} = 0$

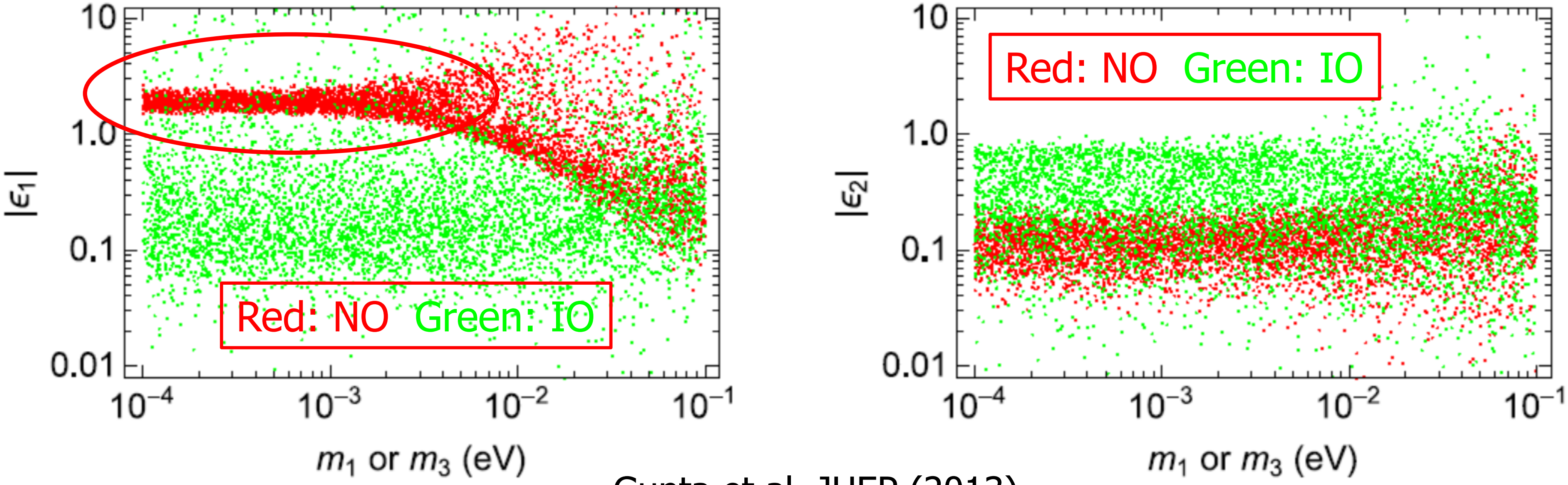
A review: Xing and **ZHZ**, RPP (2016)

# Approximate mu-tau interchange symmetry?



- $\theta_{13} \neq 0$ (and possibly $\theta_{23} \neq 45°$) indicates breaking of mu-tau interchange symmetry
- Symmetry breaking strength can be measured by dimensionless parameters:

$$\epsilon_1 = \frac{M_{e\mu} - M_{e\tau}}{M_{e\mu} + M_{e\tau}} \qquad \epsilon_2 = \frac{M_{\mu\mu} - M_{\tau\tau}}{M_{\mu\mu} + M_{\tau\tau}}$$




Gupta et al, JHEP (2013)

- For NO and non-degenerate neutrinos, this symmetry is not an approximate one

# Mu-tau reflection symmetry



- This symmetry serves as an alternative to mu-tau interchange symmetry
- $M_\nu$ is invariant under the combination of $\nu_\mu \leftrightarrow \nu_\tau$ interchange and charge conjugation

$$M_l = \text{Diag}(m_e, m_\mu, m_\tau)$$

$$X M_\nu X = M_\nu^* \quad \text{with} \quad X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \Longrightarrow \begin{cases} M_{e\mu} = M_{e\tau}^* \\ M_{\mu\mu} = M_{\tau\tau}^* \\ M_{ee} - \text{real} \\ M_{\mu\tau} - \text{real} \end{cases}$$

Harrison & Scott, PLB (2002)

$$U = \begin{pmatrix} e^{i\phi_e} & & \\ & e^{i\phi_\mu} & \\ & & e^{i\phi_\tau} \end{pmatrix} \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \begin{pmatrix} e^{i\rho} & & \\ & e^{i\sigma} & \\ & & 1 \end{pmatrix}$$

$$\phi_e = \pi/2, \quad \phi_\mu = -\phi_\tau$$

$$\theta_{23} = 45^\circ$$

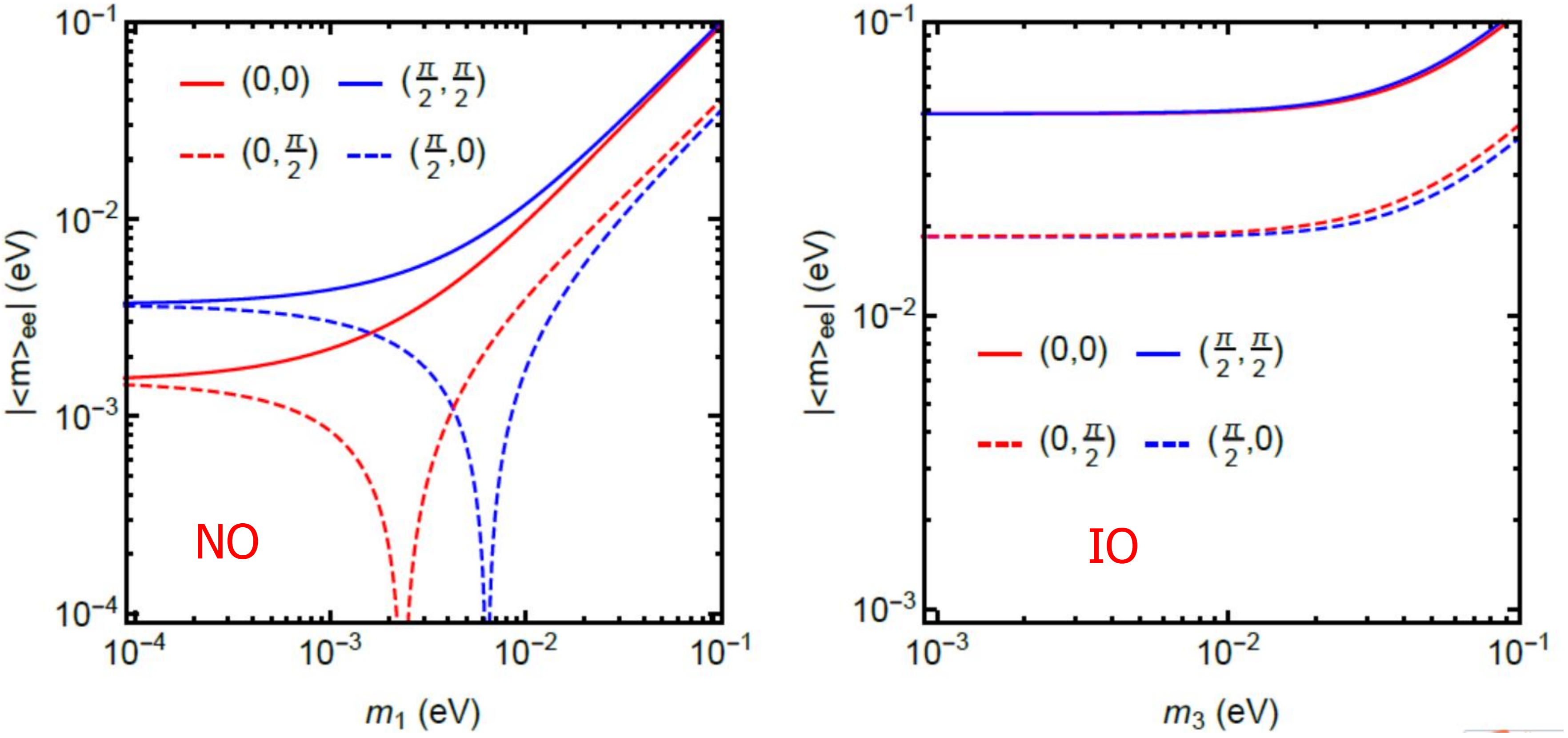
$$\delta = 270^\circ$$

$$\rho = 0 \text{ or } \pi/2, \quad \sigma = 0 \text{ or } \pi/2$$

# Implications for neutrino-less double beta decay

$$|\langle m\rangle_{ee}| = \left|m_1 e^{2i\rho} c_{12}^2 c_{13}^2 + m_2 e^{2i\sigma} s_{12}^2 c_{13}^2 + m_3 s_{13}^2 e^{-2i\delta}\right|$$

- For δ=270° and (ρ, σ)=(0, 0), (0, π/2), (π/2, 0) and (π/2, π/2), one has




- CP parities can be distinguished if mass ordering and absolute mass scale known

# Implications for leptogenesis

- Seesaw mechanism can generate matter-antimatter asymmetry via leptogenesis

$$M_l = \mathrm{Diag}(m_e, m_\mu, m_\tau)$$
$$M_\mathrm{N} = \mathrm{Diag}(M_1, M_2, M_3)$$

$$Y_\nu = \begin{pmatrix} y_{e1} & y_{e2} & y_{e3} \\ y_{\mu 1} & y_{\mu 2} & y_{\mu 3} \\ y_{\tau 1} & y_{\tau 2} & y_{\tau 3} \end{pmatrix} \quad \text{with} \quad \begin{cases} y_{ei} \text{ being real} \\ y_{\mu i} = y^*_{\tau i} \end{cases}$$

- CP asymmetry between decays of $N_1$ and CP-conjugate processes vanishes

$$Y_\nu^\dagger Y_\nu \text{ being real} \quad \Longrightarrow \quad \epsilon_1 = -\frac{3}{16\pi (Y_\nu^\dagger Y_\nu)_{11}} \sum_{i \neq 1} \mathrm{Im}\left[(Y_\nu^\dagger Y_\nu)^2_{1i}\right] \frac{M_1}{M_i} = 0$$

- To make leptogenesis viable, one can

Ahn et al, 0811.1458; Liu, Yue & **ZHZ**, JHEP (2017)

✓ break mu-tau reflection symmetry

✓ have the flavor effects take effect

Mohapatra & Nishi, JHEP (2015)

# When flavor effects are relevant



- Under mu-tau reflection symmetry, flavored CP asymmetries: $\epsilon_e=0$ and $\epsilon_\mu=-\epsilon_\tau$
- When $M_1 < 10^{12}$ GeV, tau leptons enter in thermal equilibrium

$$\frac{n_B - \bar{n}_B}{s} \sim (\epsilon_e + \epsilon_\mu)\,\kappa_f \left(\frac{417}{589}\tilde{m}_e + \boxed{\frac{417}{589}}\tilde{m}_\mu\right) + \epsilon_\tau \kappa_f \left(\boxed{\frac{390}{589}}\tilde{m}_\tau\right)$$

Contributions from mu and tau flavors do not cancel completely

- When $M_1 < 10^{9}$ GeV, mu leptons enter in thermal equilibrium

$$\frac{n_B - \bar{n}_B}{s} \sim \epsilon_e \kappa_f \left(\frac{151}{179}\tilde{m}_e\right) + \epsilon_\mu \kappa_f \left(\boxed{\frac{344}{537}}\tilde{m}_\mu\right) + \epsilon_\tau \kappa_f \left(\boxed{\frac{344}{537}}\tilde{m}_\tau\right)$$

Contributions from mu and tau flavors would cancel completely

Mohapatra & Nishi, JHEP (2015)

# RG induced symmetry breaking

- There are experimental trends for $\theta_{23} \neq 45°$ and $\delta \neq 270°$
- Because of $m_\mu \neq m_\tau$, RG running effect can induce symmetry breaking
- Significant deviations is possible in MSSM in some parameter space

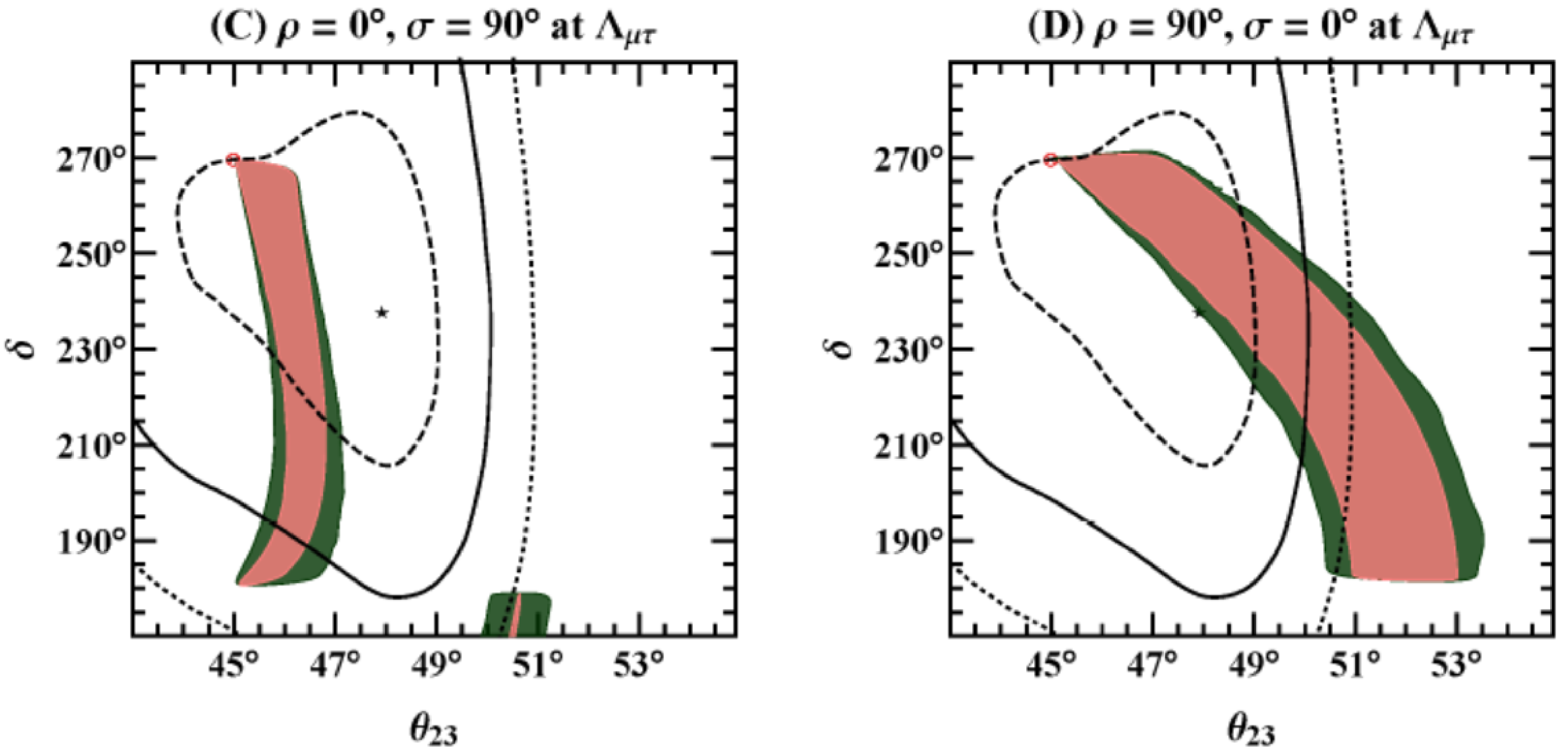
Zhou, 1409.8600

**ZHZ**, JHEP (2017)

Example:

$\theta_{23}=48°$

$\delta=238°$

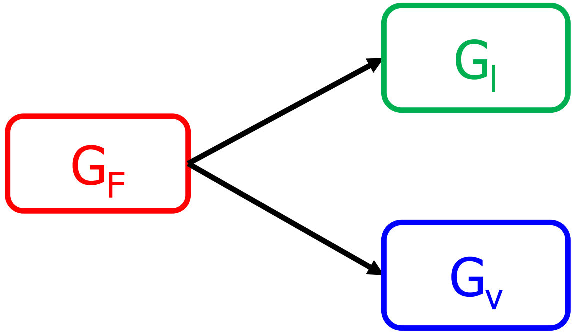



Huang et al, CPC (2018)

# Model realization

- Residual symmetry idea: mu-tau reflection symmetry and $G_l$ which ensures diagonal $M_l$ close into a whole group $G_F$

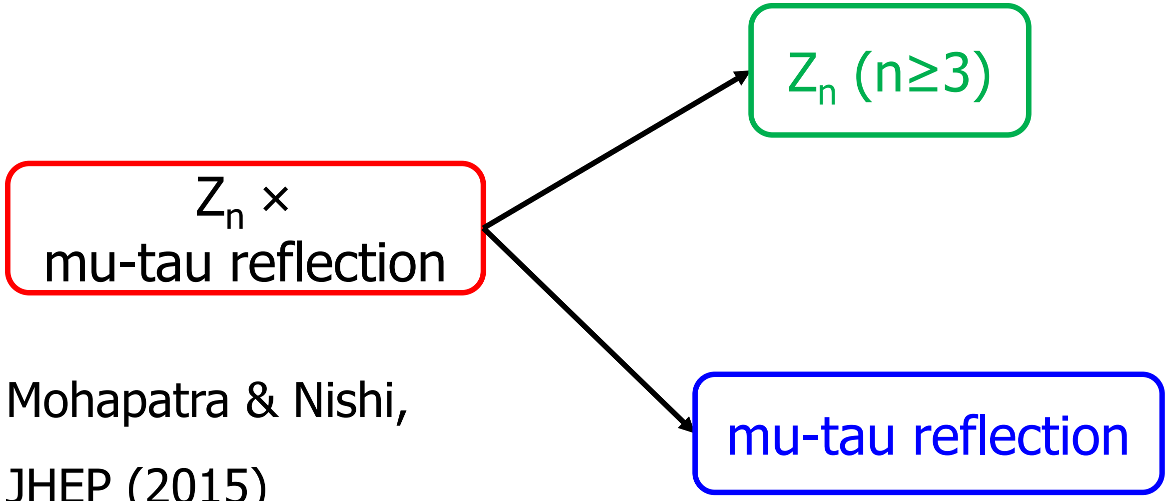
  A review: King & Luhn, RPP (2013)




- Mu-tau reflection symmetry is essentially a generalize CP symmetry

$$XT^*X^\dagger = T' \in G_l$$

Feruglio et al, JHEP (2013); Holthausen et al, JHEP (2013)




Mohapatra & Nishi, JHEP (2015)

In charged lepton sector, one can break $G_\nu$ by using flavon fields with $G_\nu$ charge but invariant under $G_l$ to preserve it

In neutrino sector, one can break $G_l$ by using flavon fields with $G_l$ charge but invariant under $G_\nu$ to preserve it

# Combined with texture zero

- Zero textures may arise as a result of Abelian symmetry — Grimus et al, EPJC (2004)

$$T = \begin{pmatrix} \omega_{15}^{3} & & \\ & \omega_{15} & \\ & & \omega_{15}^{11} \end{pmatrix}$$
with $\omega_{15} = \exp(i2\pi/15)$

$$\longrightarrow \quad \overline{\nu_\alpha^c}\nu_\beta \sim \begin{pmatrix} \omega_{15}^{6} & \omega_{15}^{4} & \omega_{15}^{14} \\ & \omega_{15}^{2} & \omega_{15}^{12} \\ & & \omega_{15}^{7} \end{pmatrix}$$

Absence of flavon fields with charge $\omega_{15}^{9}$ would render $M_{ee}=0$

- Two possible one-zero textures: $M_{ee}=0$ or $M_{\mu\tau}=0$ — Nishi & Sánchez-Vega, JHEP (2017)

| Case | $(M_\nu)_{\alpha\beta}=0$ | ordering | CP parities | $m_0$ | $m_{\beta\beta}$ | $\sum m_\nu$ |
|---|---|---|---|---|---|---|
| I | $(ee)$ | NO | $(-++)$ | $4.4-9.0$ | 0 | $63-74$ |
| II | $(ee)$ | NO | $(+-+)$ | $1.1-3.9$ | 0 | $59-65$ |
| III | $(\mu\tau)$ | NO | $(++-)$ | $151-185$ | $142-178$ | $460-561$ |
| IV | $(\mu\tau)$ | IO | $(+-+)$ | $15-30$ | $14.3-29.3$ | $116-148$ |

# Implemented in minimal seesaw



$$M_l = \mathrm{Diag}(m_e, m_\mu, m_\tau) \qquad M_N = \mathrm{Diag}(M_1, M_2)$$

$$Y_\nu = \begin{pmatrix} y_{e1} & y_{e2} \\ y_{\mu 1} & y_{\mu 2} \\ y_{\tau 1} & y_{\tau 2} \end{pmatrix} \quad \text{with} \quad \begin{cases} y_{ei} \text{ being real} \\ y_{\mu i} = y_{\tau i}^* \end{cases}$$

Liu, Yue & **ZHZ,** JHEP (2017); Nath et al, EPJC (2018)

- Neutrino masses are fixed by $m_1(m_3)=0$ with measured mass-squared differences
- Mixing parameters are fixed by predictions with measured $\theta_{12}$ and $\theta_{13}$
- These predictions are stable against renormalization group (RG) running
- A successful leptogenesis is possible when only considering flavor effects

Littlest seesaw

$$\omega = e^{i2\pi/3}$$

$$M_\nu = m_s \begin{pmatrix} 1 & 3 & 1 \\ 3 & 9+11\omega & 3+11\omega \\ 1 & 3+11\omega & 1+11\omega \end{pmatrix}$$

$$\Longrightarrow \begin{cases} \theta_{13} \simeq 7.8^\circ \\ \theta_{12} \simeq 34.5^\circ \\ \theta_{23} = 45^\circ \\ \delta = 270^\circ \end{cases} + \begin{cases} m_1 = 0 \\ m_2 \simeq 3.2 m_s \\ m_3 \simeq 20.5 m_s \end{cases}$$

King & Nishi, PLB (2018); King & Zhou, JHEP (2019)

# Combination with TM symmetry

- When TM1 and mu-tau reflection symmetries are imposed simultaneously

$$M_\nu = \begin{pmatrix} a+b+c & b & b \\ b & c & a \\ b & a & c \end{pmatrix} + \mathrm{i}d \begin{pmatrix} 0 & 1 & -1 \\ 1 & -2 & 0 \\ -1 & 0 & 2 \end{pmatrix} \Longrightarrow s_{12}^2 = \frac{1}{3}\frac{1}{1-s_{13}^2} = 0.341$$

$\theta_{12} \approx 36°$

Rodejohann & Xu, PRD (2017)

- Friedberg-Lee symmetry: $M_\nu$ is invariant under translation of neutrino fields

$$\nu_e \to \nu_e - 2\xi$$
$$\nu_{\mu,\tau} \to \nu_{\mu,\tau} + \xi$$

ξ: space-time independent element of Grassmann algebra

$$M_\nu = \begin{pmatrix} b+b^* & 2b & 2b^* \\ 2b & a+4b & -a \\ 2b^* & -a & a+4b^* \end{pmatrix} \Longrightarrow \begin{cases} m_1 = 0 \\ s_{12}^2 = \frac{1}{3}\frac{1-3s_{13}^2}{1-s_{13}^2} = 0.318 \end{cases}$$

$\theta_{12} \approx 34°$

**ZZH,** PRD (2015)

# Implemented in 3+1 neutrino framework



- Short baseline oscillation anomaly, reactor anomaly and Gallium anomaly indicate existence of eV sterile neutrino mixed with active neutrinos

A review: Giunti & Lasserre, 1901.08330

$$M_\nu = \begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} & M_{es} \\ & M_{\mu\mu} & M_{\mu\tau} & M_{\mu s} \\ & & M_{\tau\tau} & M_{\tau s} \\ & & & M_{ss} \end{pmatrix} \Longrightarrow \begin{cases} M_{e\mu} = M_{e\tau}^* \\ M_{\mu\mu} = M_{\tau\tau}^* \\ M_{ee} - \text{real} \\ M_{\mu\tau} - \text{real} \end{cases} + \begin{cases} M_{\mu s} = M_{\tau s}^* \\ M_{es} - \text{real} \\ M_{ss} - \text{real} \end{cases}$$

Chakraborty et al, 1904.10184

- 7 predictions for 12 physical mixing parameters result from mu-tau reflection symmetry

With measured $\theta_{12}$ and $\theta_{13}$, only 3 physical mixing parameters are left unconstrained

$$\rho = \sigma = -\phi_e = -\delta_{13} = \gamma - \delta_{14} \quad \phi_s = -\gamma \quad t_{24} = s_{34} \quad 2\gamma - \delta_{24} + \phi_\mu + \phi_\tau = 0$$

$$2\gamma - \delta_{24} - \rho = \pi/2 \quad c_{24}c_{34}t_{23}\sin(\gamma - \delta_{24}) + \sin\gamma = 0 \quad t_{23}\cos(\gamma - \delta_{24}) - c_{24}c_{34}\cos\gamma = 0$$

Liu, Nath & **ZHZ**, 1908.XXXXX

# Generalized mu-tau reflection symmetry

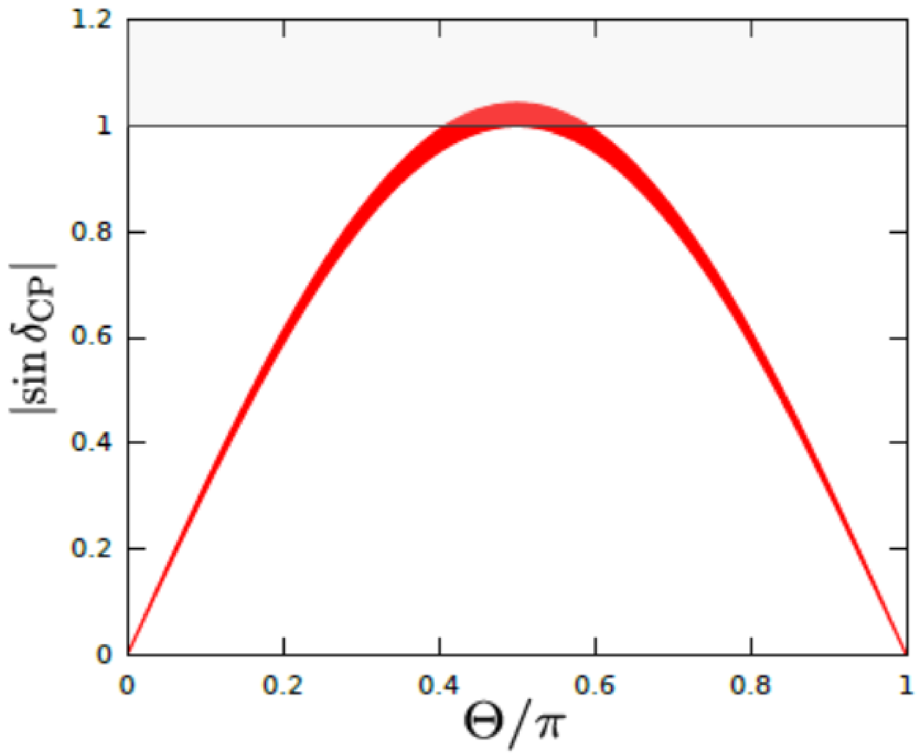
- $M_\nu$ is invariant under generalized mu-tau reflection transformation

$$X M_\nu X = M_\nu^* \quad \text{with} \quad X = \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & e^{i\beta}\cos\Theta & ie^{i\frac{\beta+\gamma}{2}}\sin\Theta \\ 0 & ie^{i\frac{\beta+\gamma}{2}}\sin\Theta & e^{i\gamma}\cos\Theta \end{pmatrix}$$

Chen et al, PLB (2016)

- Majorana phases lie at CP-conserving values
- $\theta_{23}$ and $\delta$ can deviate from maximal values and are correlated according to

$$\sin^2\delta \sin^2 2\theta_{23} = \sin^2\Theta$$



# Uniqueness of mu-tau reflection symmetry

- Is there other special form of $M_\nu$ that can give maximal $\theta_{23}$ and $\delta$

  Kitabayashi & Yasue, PLB (2005); Xing & Zhou, PLB (2010)

- $M_\nu$ should be diagonalized by $U(\theta_{23}=\pi/4, \delta=\pm\pi/2)$ to give 3 real mass eigenvalues

$$U^T M_\nu U^* = \begin{pmatrix} m_1 & 0 & 0 \\ & m_2 & 0 \\ & & m_3 \end{pmatrix} \Longrightarrow \left\{ \begin{array}{ll} \mathrm{Re}\left[(U^T M_\nu U^*)_{12}\right] = 0 & \mathrm{Im}\left[(U^T M_\nu U^*)_{12}\right] = 0 \\ \mathrm{Re}\left[(U^T M_\nu U^*)_{13}\right] = 0 & \mathrm{Im}\left[(U^T M_\nu U^*)_{13}\right] = 0 \\ \mathrm{Re}\left[(U^T M_\nu U^*)_{23}\right] = 0 & \mathrm{Im}\left[(U^T M_\nu U^*)_{23}\right] = 0 \\ \mathrm{Im}\left[(U^T M_\nu U^*)_{33}\right] = 0 & \end{array} \right.$$

- Simplicity of $M_\nu$ is characterized by that some conditions hold spontaneously

  When hold spontaneously, mu-tau reflection symmetry is reproduced

- Other compact form of $M_\nu$ like that in mu-tau reflection symmetry not found

  Liu, Yue & **ZHZ**, JHEP (2018); PRD (2019)

# Summary and outlook

- Mu-tau reflection symmetry has interesting consequences for neutrino mixing parameters, neutrino-less double beta decay, and leptogenesis

- Its various combinations with other approaches for addressing neutrino mixing and masses yield fruitful consequences

- Most importantly, we need more precise experimental results to test these ideas

Thanks for your attention!