

# Addressing flavor symmetries of neutrinos at DUNE

**Newton Nath**



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In collaboration/Based on,

1805.05823/hep-ph, NN

1811.07040/hep-ph, NN, R. Srivastava & J.W.F. Valle

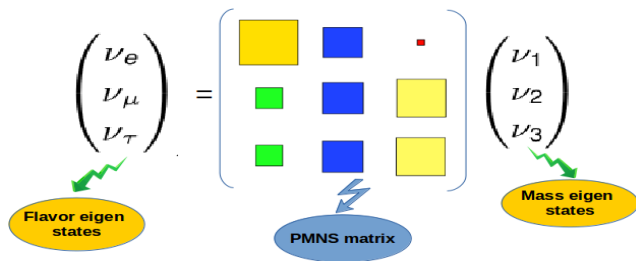
1904.05632/hep-ph, G.-J. Ding, NN, R. Srivastava & J.W.F. Valle

**FLASY-2019 @ TDLi (Shanghai) & USTC (Hefei)**

July 22 - 27

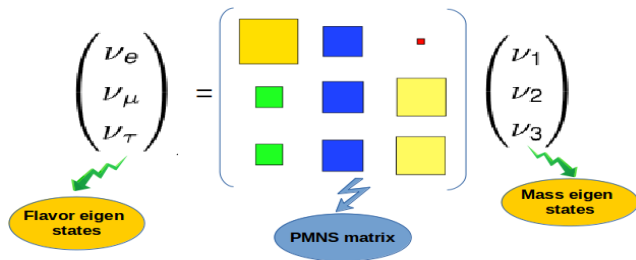
# Neutrino oscillation:

- ▶ Standard 3-flavour  $\nu$ -oscillation framework:



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PMNS matrix :

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{23} & \sin \theta_{23} \\ 0 & -\sin \theta_{23} & \cos \theta_{23} \end{pmatrix} \begin{pmatrix} \cos \theta_{13} & 0 & e^{-i\delta} \sin \theta_{13} \\ 0 & 1 & 0 \\ -e^{i\delta} \sin \theta_{13} & 0 & \cos \theta_{13} \end{pmatrix} \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 \\ -\sin \theta_{12} & \cos \theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Atmospheric,  
K2K, MINOS, T2K, etc.

Reactor  
Accelerator

Solar  
KamLAND

- ▶ 3-mixing angles, 1 CP-phase.

# Neutrino oscillation in 3 generation

- ▶ The transition probability  $\nu_\alpha \rightarrow \nu_\beta$ :

$$P_{\alpha\beta} = |\langle \nu_\beta | \nu_\alpha(t) \rangle|^2$$

where  $\alpha, \beta$  are e,  $\mu$  or  $\tau$

Full 3-flavour vacuum probability formula:

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{i < j} \text{Re}[U_{\alpha i}^* U_{\beta j}^* U_{\beta i} U_{\alpha j}] \sin^2 \frac{\Delta_{ij} L}{4E} + 2 \sum_{i < j} \text{Im}[U_{\alpha i}^* U_{\beta j}^* U_{\beta i} U_{\alpha j}] \sin 2 \frac{\Delta_{ij} L}{4E}$$

$$\Delta_{ij} = m_j^2 - m_i^2$$

Parameters of neutrino oscillation:

- ▶ Elements of U: **3-mixing**  $\angle$ 's ( $\theta_{12}$ ,  $\theta_{23}$ ,  $\theta_{13}$ ) and **1-Dirac phase** ( $\delta_{CP}$ )
- ▶ **2-independent** (mass)<sup>2</sup> differences,  $\Delta_{21} = m_2^2 - m_1^2$ ,  $\Delta_{31} = |m_3^2 - m_1^2|$

# Current Status:

de Salas, Forero, Ternes, Valle, arXiv:1708.01186, PLB782 (2018)

Parameter	Best fit $\pm 1\sigma$	$2\sigma$ range	$3\sigma$ range	
$\Delta m_{21}^2$ [ $10^{-5} \text{eV}^2$ ]	$7.55^{+0.20}_{-0.16}$	7.20–7.94	7.05–8.14	
$ \Delta m_{31}^2 $ [ $10^{-3} \text{eV}^2$ ] (NO)	$2.50 \pm 0.03$	2.44–2.57	2.41–2.60	
$ \Delta m_{31}^2 $ [ $10^{-3} \text{eV}^2$ ] (IO)	$2.42^{+0.03}_{-0.04}$	2.34–2.47	<u>2.31–2.51</u>	
$\sin^2 \theta_{12}/10^{-1}$	$3.20^{+0.20}_{-0.16}$	2.89–3.59	2.73–3.79	
$\theta_{12}/^\circ$	$34.5^{+1.2}_{-1.0}$	32.5–36.8	31.5–38.0	
$\sin^2 \theta_{23}/10^{-1}$ (NO)	$5.47^{+0.20}_{-0.30}$	4.67–5.83	4.45–5.99	
$\theta_{23}/^\circ$	$47.7^{+1.2}_{-1.7}$	43.1–49.8	<u>41.8–50.7</u>	
$\sin^2 \theta_{23}/10^{-1}$ (IO)	$5.51^{+0.18}_{-0.30}$	4.91–5.84	4.53–5.98	
$\theta_{23}/^\circ$	$47.9^{+1.0}_{-1.7}$	44.5–48.9	42.3–50.7	
$\sin^2 \theta_{13}/10^{-2}$ (NO)	$2.160^{+0.083}_{-0.069}$	2.03–2.34	1.96–2.41	
$\theta_{13}/^\circ$	$8.45^{+0.16}_{-0.14}$	8.2–8.8	8.0–8.9	
$\sin^2 \theta_{13}/10^{-2}$ (IO)	$2.220^{+0.074}_{-0.076}$	2.07–2.36	1.99–2.44	
$\theta_{13}/^\circ$	$8.53^{+0.14}_{-0.15}$	8.3–8.8	8.1–9.0	
$\delta/\pi$ (NO)	$1.32^{+0.21}_{-0.15}$	1.01–1.75	0.87–1.94	
$\delta/^\circ$	$238^{+38}_{-27}$	182–315	<u>157–349</u>	
$\delta/\pi$ (IO)	$1.56^{+0.13}_{-0.15}$	1.27–1.82	1.12–1.94	
$\delta/^\circ$	$281^{+23}_{-27}$	229–328	202–349	

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- NO preferred over IO by  $3.4\sigma$

# Questions?

Theory behind the origin of  $\nu$  masses & mixings?

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- ▶ Seesaw mechanism  $\Rightarrow$  tiny  $\nu$ -masses.
- ▶ Symmetry based studies  $\Rightarrow$   $\nu$ -mixings.



## $\mu - \tau$ symmetry:

### First seed:

- ▶ Fukuyama, Nishiura proposed  $\mu - \tau$  symmetry in the  $M_\nu$ ,

$$M_\nu = \begin{pmatrix} 0 & A & \pm A \\ A & B & C \\ \pm A & C & B \end{pmatrix}. \quad (1)$$

where  $A, B, C \in \mathcal{R}$ .

arXiv:hep-ph/9702253 & 1701.04985, (PTEP 2017)

- ▶ Leads to  $\theta_{23} = \mp 45^\circ$  and  $\theta_{13} = 0^\circ$ .
- ▶ (1,1)-entry = 0  $\Rightarrow$  small  $\theta_{12}$ .

## Cont...

- ▶ Easy generalization,

$$M_\nu = \begin{pmatrix} D & A & \pm A \\ A & B & C \\ \pm A & C & B \end{pmatrix}. \quad (2)$$

- ▶ Leads to,

$$\tan 2\theta_{12} = \frac{2\sqrt{2}A}{B \pm C - D}. \quad (3)$$

- ▶ For  $B \pm C - D = A$ , we get,

$$U_{HPS} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/3} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/3} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}. \quad (4)$$

- $U_{HPS}$  was 1st proposed by Harrison, Perkins & Scott arXiv:hep-ph/0202074, PLB530 (2002)

- ▶ Also, famously called 'Tri-Bi-Maximal' leptonic mixing pattern.

- ▶  $U_{HPS} \Rightarrow \sin \theta_{12} = \frac{1}{\sqrt{3}} \Rightarrow$  'trimaximal mixing',  
 $\sin \theta_{23} = \frac{1}{\sqrt{2}} \Rightarrow$  'bimaximal mixing'  
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and  $\theta_{13} = 0^\circ$ .

- ▶  $M_\nu$  is unchanged under:  $\nu_e \leftrightarrow \nu_\mu, \nu_\mu \leftrightarrow \pm \nu_\tau$  ( $\mu - \tau$  permutation symmetry).

## $\mu - \tau$ reflection symmetry

Originally proposed by Harrison & Scott, PLB547 (2002)

- ▶ At present  $\theta_{13} = 0^\circ$  is excluded at more than  $5\sigma$ .
- ▶ **Many discrete flavor groups are rule out now.**

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- ▶ **Many discrete flavor groups are rule out now.**
- ▶ Extend the ' $\mu - \tau$  permutation symmetric' matrix to a complex matrix as

$$M_\nu = \begin{pmatrix} D & A & \pm A^* \\ A & B & C \\ \pm A^* & C & B^* \end{pmatrix}; \quad C, D \in \mathbb{R} \text{ \& } A, B \in \mathbb{C}. \quad (5)$$

- ▶ One finds,  $X^T M_\nu X = M_\nu^*$  with

$$X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \quad (6)$$

- ▶  $M_\nu$  can be diagonalized by the unitary matrix

$$U = \begin{pmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ \pm v_1^* & \pm v_2^* & \pm v_3^* \end{pmatrix}. \quad (7)$$

- ▶ One finds two well known predictions:  $\theta_{23} = 45^\circ, s_{13} \cos \delta = 0$ .
- ▶ Allows  $\theta_{13} \neq 0$  and  $\delta = \pm 90^\circ$  (...wow).
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- ▶ Excellent agreement with the latest data.

# Implications

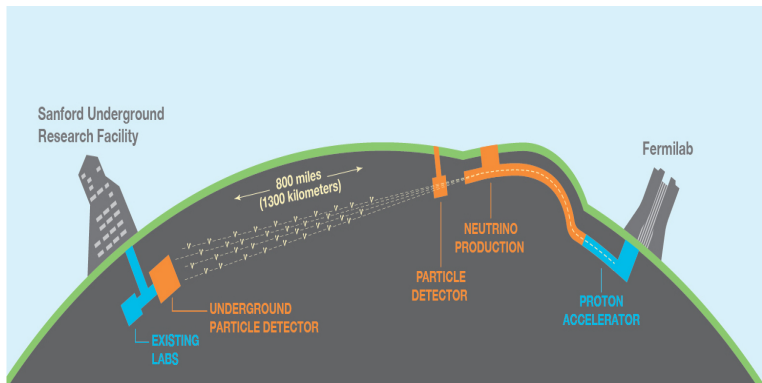
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# Implications

- ▶ Interesting to look for the consequences of different flavor symmetries in long baseline neutrino oscillation experiments.
- ▶ DUNE (Deep Underground Neutrino Experiment), a proposed long baseline experiment at Fermilab, USA.
- ▶ DUNE will improve the precision of  $\theta_{23}$  and play a key role to probe  $\delta$ . [Acciarri et al.(DUNE), arXiv:1512.06148].



# DUNE



- ▶ DUNE : Neutrinos travel from Fermilab to Sanford Underground Research Facility (SURF), 1300 km, 2.3 GeV, 1.07 MW,  $4 \times 10$  kt-LArTPC detector.  
[\[Alio et al.\(DUNE\), arXiv:1601.09550\].](#)
- ▶ Their first 2-modules are expected to be completed in 2024, with the beam operational in 2026.
- ▶ To simulate the data we use GLoBES package.

[arXiv:hep-ph/0407333, 0701187.](#)

# Framework:

- ▶ We consider,

$$M_D = \begin{pmatrix} ae^{i\phi_a} & ae^{-i\phi_a} \\ be^{i\phi_b} & ce^{i\phi_c} \\ ce^{-i\phi_c} & be^{-i\phi_b} \end{pmatrix}, M_R = \text{diag}(M_1, M_1). \quad (9)$$

- ▶ Within type-I seesaw:

$$\begin{aligned} -M_\nu &= M_D M_R^{-1} M_D^T, \\ &= \frac{1}{M_1} \begin{pmatrix} 2a^2 \cos 2\phi_a & abe^{i(\phi_a+\phi_b)} + ace^{-i(\phi_a-\phi_c)} & abe^{-i(\phi_a+\phi_b)} + ace^{i(\phi_a-\phi_c)} \\ - & b^2 e^{2i\phi_b} + c^2 e^{2i\phi_c} & 2bc \cos(\phi_b - \phi_c) \\ - & - & b^2 e^{-2i\phi_b} + c^2 e^{-2i\phi_c} \end{pmatrix}. \end{aligned}$$

$$\bullet M_{ee} = M_{ee}^*, \quad M_{\mu\tau} = M_{\mu\tau}^*, \quad M_{e\mu} = M_{e\tau}^*, \quad M_{\mu\mu} = M_{\tau\tau}^*$$

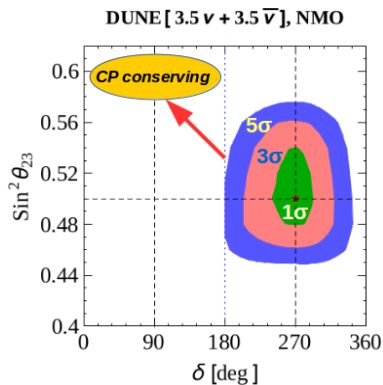
- ▶ Predicts non-zero  $\theta_{13}$  with,

$$\theta_{23} = 45^\circ, \quad \delta = \pm 90^\circ. \quad (10)$$

NN, arXiv: 1805.05823, PRD98 (2018)

# Cont...

- ▶ DUNE's Potential:

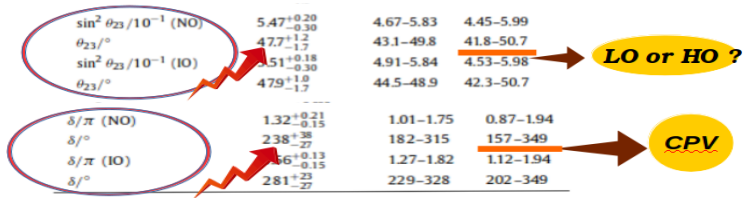


- ▶ CP-conservation (CPC) hypothesis can be ruled out at  $> 3\sigma$ .

NN, arXiv: 1805.05823, PRD98 (2018)

Cont...

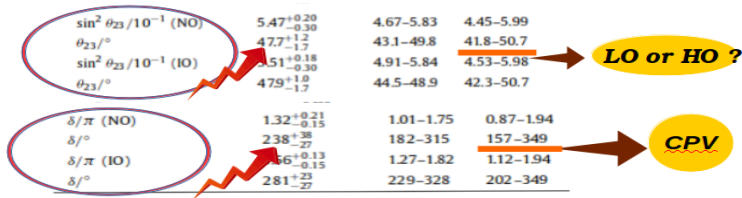
Reminder: Best-fit preferences  $\theta_{23}, \delta \Rightarrow$



Looking for more realistic model

Cont...

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## Looking for more realistic model

- ▶ Break  $\mu - \tau$  reflection symmetry.
- ▶ Generalized CP symmetry.
- ▶ Bi-large ansatz.

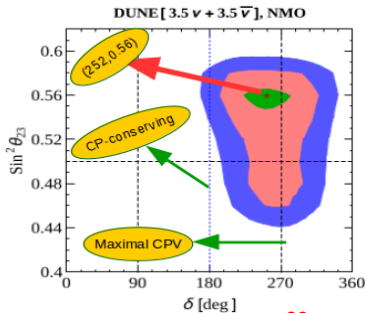
Recent topics: Modular symmetries, tri-direct CP approaches etc...

# Cont...

Break  $M_D$ :

$$\widehat{M}_D = \begin{pmatrix} ae^{i\phi_a} & ae^{-i\phi_a} \\ be^{i\phi_b} & ce^{i\phi_c} \\ ce^{-i\phi_c} & b(1+\epsilon)e^{-i\phi_b} \end{pmatrix}.$$

$$\widehat{M}_\nu \simeq M_\nu - \epsilon \frac{be^{-i\phi_b}}{M_1} \begin{pmatrix} 0 & 0 & ae^{-i\phi_a} \\ 0 & 0 & ce^{i\phi_c} \\ be^{-i\phi_a} & ce^{i\phi_c} & 2be^{-2i\phi_b} \end{pmatrix} + \mathcal{O}(\epsilon^2).$$



- **Best-fit:**  $238^{+38}_{-27}, 0.547^{+0.02}_{-0.03}$

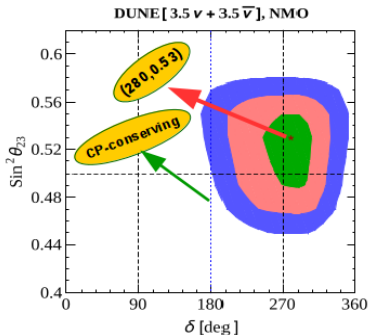


# Cont...

Break  $M_R$ :

$$\widehat{M}_R = \begin{pmatrix} M_1 & 0 \\ 0 & M_1(1 + \epsilon) \end{pmatrix}.$$

$$\widehat{M}_\nu \simeq M_\nu - \frac{\epsilon}{M_1} \begin{pmatrix} a^2 e^{-2i\phi_a} & ace^{-i(\phi_a - \phi_c)} & abe^{-i(\phi_a + \phi_b)} \\ - & b^2 e^{-2i\phi_b} & bce^{-i(\phi_b - \phi_c)} \\ - & - & c^2 e^{-2i\phi_c} \end{pmatrix} + \mathcal{O}(\epsilon^2).$$



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# Generalized CP (gCP) symmetry

► **Reminder:**

$$\psi \rightarrow X\psi ; \mu - \tau \text{ permutation symmetry ,}$$

$$\psi \rightarrow X\psi^c ; \mu - \tau \text{ reflection symmetry ,}$$

where

$$X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} . \quad (11)$$

► In gCP, one assumes

$$\psi \xrightarrow{CP} iX_\psi \gamma^0 \psi^c \quad (12)$$

$X_\psi$  are the generalized CP transformation matrices

[ Feruglio, Hagedorn, Ziegler, arXiv:1211.5560, Chen, Li, Ding, arXiv:1412.8352, Chen, Ding, Gonzalez-Canales, Valle, arXiv:1512.01551 ]

with

$$X_\psi^T m_\psi X_\psi = m_\psi^* , \quad (\text{Majorana fields})$$

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$$X_\psi ?$$

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$$\boxed{X_\psi ?} \quad \boxed{U_{PMNS} ?}$$

**Recent studies:** Chen, Ding, Gonzalez-Canales, Valle, arXiv:1604.03510, Chen, Chulia, Ding,

Srivastava, Valle, arXiv:1802.04275, Joshipura, Patel , arXiv : 1805.02002, Lu, Ding, arXiv:

1806.02301, Barreiros, Felipe, Joaquim, arXiv:1810.05454

# Cont...

- **Steps to find  $U_{PMNS}$ :** [Chen, Chulia, Ding, Srivastava, Valle, arXiv:1802.04275]

$$\begin{aligned} U_{\psi}^T m_{\psi} U_{\psi} &= \text{diag}(m_1, m_2, m_3), \quad (\text{Majorana fields}) \\ U_{\psi}^{\dagger} M_{\psi}^2 U_{\psi} &= \text{diag}(m_1^2, m_2^2, m_3^2), \quad (\text{Dirac fields}). \end{aligned} \quad (14)$$

- $U_{\psi}$  satisfies the following constraint

$$U_{\psi}^{\dagger} X_{\psi} U_{\psi}^* \equiv P = \begin{cases} \text{diag}(\pm 1, \pm 1, \pm 1), & \text{for Majorana fields,} \\ \text{diag}(e^{i\delta_1}, e^{i\delta_2}, e^{i\delta_3}), & \text{for Dirac fields,} \end{cases} \quad (15)$$

- **Unitary-symmetric matrix  $X_{\psi}$  can be decomposed as  $X_{\psi} = \Sigma \cdot \Sigma^T$ .**
- Subsequently,  $P^{-\frac{1}{2}} U_{\psi}^{\dagger} \Sigma \equiv O_3, \Rightarrow U_{\psi} = \Sigma O_3^T P^{-\frac{1}{2}}$ ,  
where  $O_3$ ,

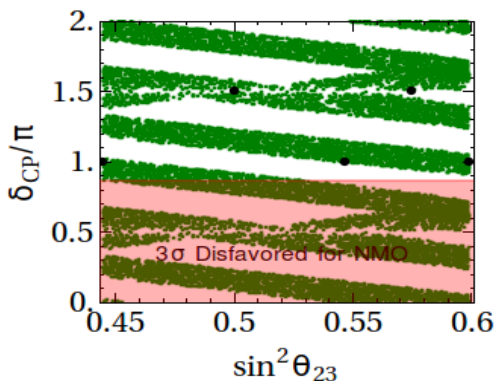
$$O_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{\theta_1} & s_{\theta_1} \\ 0 & -s_{\theta_1} & c_{\theta_1} \end{pmatrix} \begin{pmatrix} c_{\theta_2} & 0 & s_{\theta_2} \\ 0 & 1 & 0 \\ -s_{\theta_2} & 0 & c_{\theta_2} \end{pmatrix} \begin{pmatrix} c_{\theta_3} & s_{\theta_3} & 0 \\ -s_{\theta_3} & c_{\theta_3} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

- **Maximum possible zeros in  $X_{\psi}$ : 4** ; [Chen, Ding, Gonzalez-Canales, Valle, arXiv:1604.03510]
- $X_{\psi}$  : 11 possibilities, 8 are compatible with latest data.

## Cont...

▶ No-Zeros in  $X$  :  $\frac{1}{\sqrt{3}} \begin{pmatrix} e^{i\alpha} & e^{i(\frac{\alpha+\beta}{2} + \frac{2\pi}{3})} & e^{i(\frac{\alpha+\gamma}{2} + \frac{2\pi}{3})} \\ e^{i(\frac{\alpha+\beta}{2} + \frac{2\pi}{3})} & e^{i\beta} & e^{i(\frac{\beta+\gamma}{2} + \frac{2\pi}{3})} \\ e^{i(\frac{\alpha+\gamma}{2} + \frac{2\pi}{3})} & e^{i(\frac{\beta+\gamma}{2} + \frac{2\pi}{3})} & e^{i\gamma} \end{pmatrix}$

- ▶ This leads,

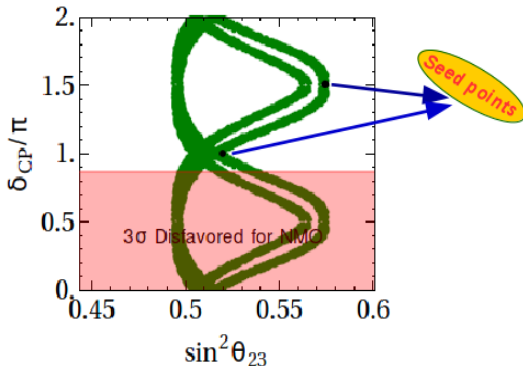


NN, Srivastava, Valle, arXiv:1811.07040, PRD99 (2019)

## Cont...

▶ One-Zero in  $X$  : 
$$\begin{pmatrix} e^{i\alpha} c_\Theta^2 & e^{i\gamma} c_\Theta s_\Theta & e^{i\beta} s_\Theta \\ e^{i\gamma} c_\Theta s_\Theta & e^{i(-\alpha+2\gamma)} s_\Theta^2 & -e^{i\alpha_1} c_\Theta \\ e^{i\beta} s_\Theta & -e^{i\alpha_1} c_\Theta & 0 \end{pmatrix}$$

▶ This leads,



▶ **Seed points: (0.575, 1.5 $\pi$ ) & (0.52, 1.0 $\pi$ )**

## Cont...

- ▶ Two-Zeros in  $X$  :

$$\begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & e^{i\beta} c_{\Theta} & ie^{i(\beta+\gamma)/2} s_{\Theta} \\ 0 & ie^{i(\beta+\gamma)/2} s_{\Theta} & e^{i\gamma} c_{\Theta} \end{pmatrix} \quad (16)$$

- ▶  $\Rightarrow \sin^2 \delta \sin^2 2\theta_{23} = \sin^2 \Theta$  , ( $\Theta$  is the model parameter.)

'Generalized  $\mu - \tau$  reflection symmetry'

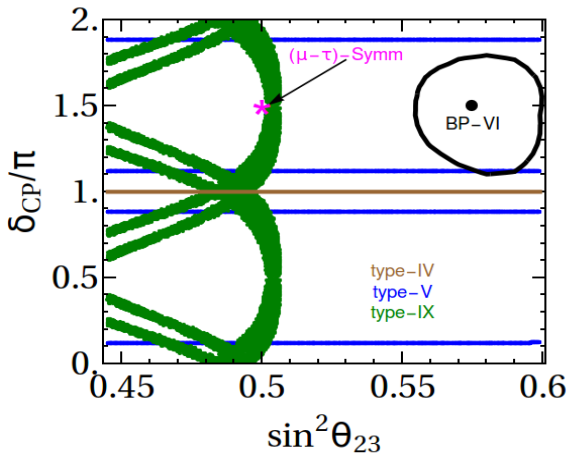
- ▶  $\Theta = \pm\pi/2 \Rightarrow$  'exact  $\mu - \tau$  reflection symmetry',

i.e.  $\theta_{23} = 45^\circ, \delta = \pm 90^\circ$

- ▶  $\Theta = 0 \Rightarrow$  **CP-conservation.**
- ▶  $\Theta \neq 0 \Rightarrow$  **deviations from  $\mu - \tau$  reflection symmetry.**

## Cont...

- ▶ DUNE's capability:



- ▶ Precise measurement of  $\theta_{23}, \delta$  by DUNE can rule out various models.



# Bi-large ansatze

## Motivation:

- ▶ Smallest leptonic-mixing angle  $\simeq$  largest of the quark-mixing angle.
- ▶ Cabibbo angle ( $\lambda$ ) may act as the universal seed for quark and lepton mixings.
- ▶ Bi-large patterns arise from the simplest GUTs model.

Boucenna, Morisi, Tortala, Valle: 1206.2555, Roy, Morisi, Singh, Valle: 1410.3658

- ▶  $\nu$ -mixing angles are related with  $\lambda$ ,

$$\sin \theta_{23} = 1 - \lambda, \quad \sin \theta_{12} = 2\lambda, \quad \sin \theta_{13} = \lambda \quad \text{with } \lambda = 0.22453. \quad (17)$$

- ▶  $\nu$ -part of mixing matrix  $U_{BL1}$  is given by

$$U_{BL1} \approx \begin{bmatrix} 1 - \frac{5\lambda^2}{2} & 2\lambda & -\lambda \\ \lambda - 2\sqrt{2}\lambda^{3/2} & \sqrt{2}\lambda - \frac{\lambda^{3/2}}{2\sqrt{2}} & 1 - \lambda - \frac{\lambda^2}{2} \\ 2\lambda + \sqrt{2}\lambda^{3/2} & -1 + \lambda & \sqrt{2}\lambda - \frac{\lambda^{3/2}}{2\sqrt{2}} \end{bmatrix} + h.o.$$

Ding, NN, Srivastava, Valle, arXiv:1904.05632, PLB796 (2019)

## Cont...

- ▶ The  $SO(10)$  GUT-motivated, CKM-type charged-lepton corrections,

$$U_{l_1} = R_{23}(\theta_{23}^{CKM})\Phi R_{12}(\theta_{12}^{CKM})\Phi^\dagger \simeq \begin{bmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda e^{-i\phi} & 0 \\ -\lambda e^{i\phi} & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3 e^{i\phi} & -A\lambda^2 & 1 \end{bmatrix},$$

with  $\sin \theta_{12}^{CKM} = \lambda$  and  $\sin \theta_{23}^{CKM} = A\lambda^2$ , where  $\lambda, A$  are the Wolfenstein parameters.

- ▶ The lepton mixing matrix is simply given by  $U = U_{l_1}^\dagger U_{BL1} \Rightarrow$

$$\sin^2 \theta_{13} \simeq 4\lambda^2(1 - \lambda) \cos^2 \frac{\phi}{2},$$

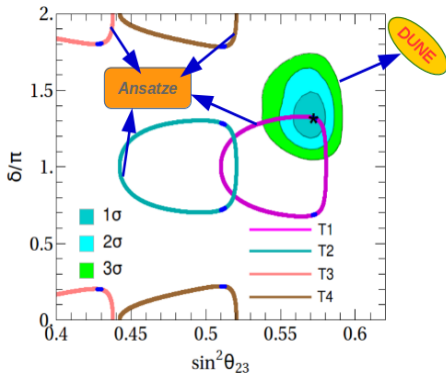
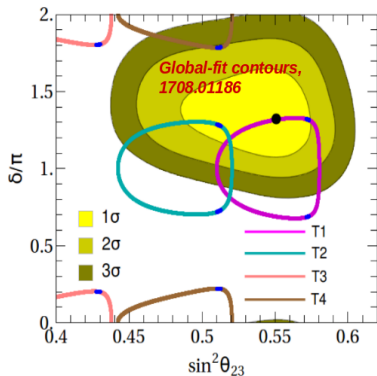
$$\sin^2 \theta_{12} \simeq 2\lambda^2(2 - 2\sqrt{2}\lambda \cos \phi + \lambda),$$

$$\sin^2 \theta_{23} \simeq (1 - \lambda)^2 - 2\sqrt{2}A\lambda^{5/2} - 2\lambda^3(1 + 2 \cos \phi),$$

$$J_{CP} \simeq -2(\sqrt{2} + \sqrt{\lambda})\lambda^{5/2} \sin \phi.$$

# Cont...

- ▶ Left panel  $\Rightarrow$  **Global-fit**, right panel  $\Rightarrow$  **DUNE analysis**.



Ding, NN, Srivastava, Valle, arXiv:1904.05632, PLB796 (2019)

- ▶ Types - 1, 2, 4 are compatible with latest data.
- ▶ Blue marks are obtained by requiring 1-3 mixing angle to lie within its current 3 $\sigma$  range.

# Wrap-up

- ▶ Focus was to introduce the current status of different flavor symmetries to explain realistic leptonic mixing patterns.
- ▶  $\mu - \tau$  reflection symmetry, generalized CP symmetry, bi-large ansatz have been discussed.
- ▶ Impact of these symmetries on DUNE have been examined.
- ▶ DUNE with its high statistics and ability to measure  $(\theta_{23}, \delta)$  with high precision, will serve as an excellent experiment to test these different mixing patterns.

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**thank you**