

Neutrino masses and Gravitational Waves

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In collaboration with Graham White

Overview

$m_\nu + \text{GW}$

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Models

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potential and
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GW

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Neutrino-
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Standard Model

$m_\nu + GW$

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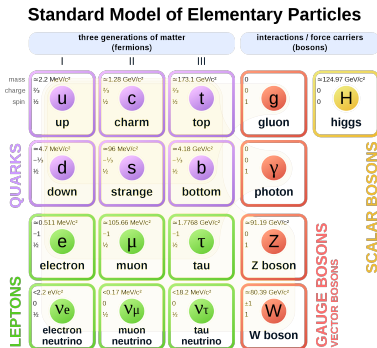
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Gauge symmetry group $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$

$$\mathcal{L}^Y = \bar{u} Y_u Q H + \bar{Q} Y_d d H + \text{h.c.}$$

$$\langle h^0 \rangle = v \rightarrow m_f = Y_f v$$

Introduction/Motivation

$m_\nu + \text{GW}$

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- Neutrino masses (Dirac)
- Non-Abelian Dark Sector (self-interacting dark matter)
- Gravitational waves from early universe
- Neutrino genesis
- Connect all of the above

Introduction

$m_\nu + \text{GW}$

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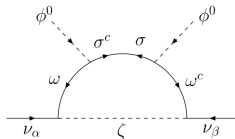
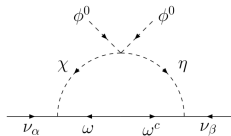
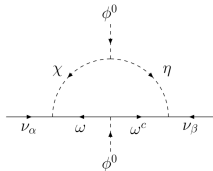
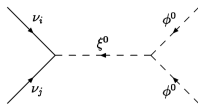
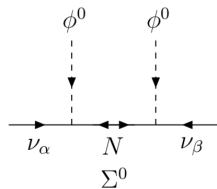
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Model*	Field	$SU(2)_L$	$U(1)_Y$	$SU(N)$	Flavor
1,2	ν_R	1	0	\square	3
1a	$N_{R,L}$	1	0	1	3
	ϕ	1	0	\square	1
1b	$\Sigma_{R,L}$	3	0	1	3
1b	ϕ	3	0	\square	1
1c	$E_{R,L}$	2	$-\frac{1}{2}$	\square	3
1c	ϕ	1	0	\square	1
2	η	2	$\frac{1}{2}$	\square	1
2	ϕ	1	0	\square	1

*10.1016/j.physletb.2016.11.027

Models' Lagrangians

m_ν + GW

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$$-\mathcal{L}_a = N_R^\dagger Y_L L H + \phi \nu_R^\dagger Y_R N_L + N_R^\dagger M_N N_L + \text{h.c.}$$

$$-\mathcal{L}_b = H \Sigma_R^\dagger Y_L L + \nu_R^\dagger Y_R \text{Tr}[\Sigma_L \phi] + \text{Tr}[\Sigma_R^\dagger M_\Sigma \Sigma_L] + \text{h.c.}$$

$$-\mathcal{L}_c = E_R^\dagger Y_L L \phi + \nu_R^\dagger Y_R E_L H + E_R^\dagger M_E E_L + \text{h.c.}$$

$$-\mathcal{L}_2 = \nu_R^\dagger Y_\nu L \eta + \text{h.c.}$$

Models' Potentials

m_ν + GW

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$$V_1 = -\mu_H^2 H^\dagger H + \frac{\lambda_H}{2} (H^\dagger H)^2 - m_\phi^2 \phi^\dagger \phi + \frac{\lambda_\phi}{2} (\phi^\dagger \phi)^2 \\ + \lambda_{H\phi} (H^\dagger H) (\phi^\dagger \phi) + \delta_{\text{SU}(2)} \lambda'_{H\phi} H^\dagger \phi \phi^\dagger H$$

$$V_2 = -\mu_H^2 H^\dagger H + \frac{\lambda_H}{2} (H^\dagger H)^2 - \mu_\phi^2 \phi^\dagger \phi + \frac{\lambda_\phi}{2} (\phi^\dagger \phi)^2 \\ + \mu_\eta^2 \eta^\dagger \eta + \frac{\lambda_\eta}{2} (\eta^\dagger \eta)^2 + \frac{\lambda'_\eta}{2} (\eta^{\dagger, i\alpha} \eta_{j\alpha}) (\eta^{\dagger, j\beta} \eta_{i\beta}) \\ + \lambda_{H\phi} (H^\dagger H) (\phi^\dagger \phi) + \lambda_{H\eta} (H^\dagger H) (\eta^\dagger \eta) \\ + \lambda'_{H\eta} (H^\dagger \eta) (\eta^\dagger H) + \lambda_{\eta\phi} (\eta^\dagger \eta) (\phi^\dagger \phi) + \lambda'_{\eta\phi} (\phi^\dagger \eta) (\eta^\dagger \phi) \\ + \sqrt{2} \mu \eta^\dagger H \phi + \text{h.c.}$$

Models

$m_\nu + \text{GW}$

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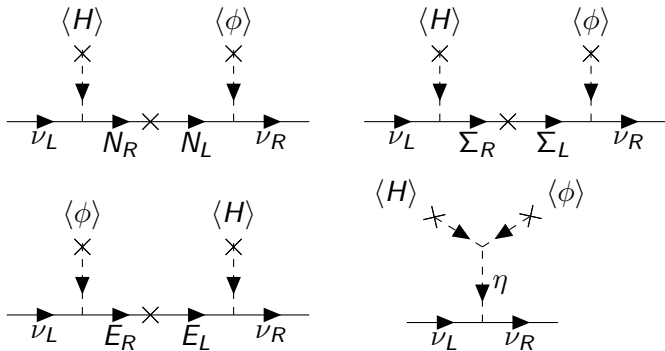
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$$m_\nu^{(a)} \cong -Y_R M_N^{-1} Y_L \frac{v v \phi}{2}$$

$$m_\nu^{(b)} \cong -Y_R M_\Sigma^{-1} Y_L \frac{v v \phi}{2}$$

$$m_\nu^{(c)} \cong -Y_R M_E^{-1} Y_L \frac{v v \phi}{2}$$

$$m_\nu \cong -Y_\nu \frac{v_\eta}{\sqrt{2}} \cong \frac{Y_\nu \mu v v \phi}{\sqrt{24} m_\eta^2}$$

Neutrino Masses

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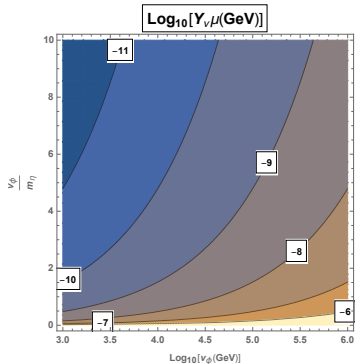
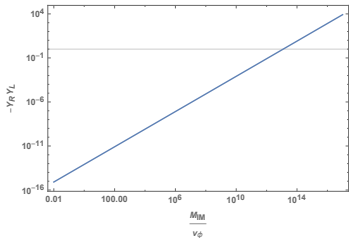
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$$\begin{aligned}
 \mathbf{Y}_L &= -i\mathcal{D}_{\sqrt{N,E,\Sigma}} R \mathcal{D}_{\sqrt{\kappa}} \mathcal{U}_{PMNS}^\dagger \\
 \mathbf{Y}_R^\dagger &= i\mathcal{D}_{\sqrt{N,E,\Sigma}}^* R \mathcal{D}_{\sqrt{\kappa}}^* \mathcal{U}_R^\dagger \\
 \mathbf{Y}_\nu &= \frac{\sqrt{24}m_\eta^2}{\mu v v_\phi} \mathcal{U}_R \text{Diag}[m_1, m_2, m_3] \mathcal{U}_{PMNS}^\dagger \\
 \mathcal{D}_\kappa &= \frac{1}{v v_\phi} \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix}
 \end{aligned}$$

Spontaneous Symmetry Breaking

m_ν + GW

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$$SM : SU(3)_c \times SU(2)_L \times U(1)_Y \xrightarrow{H} SU(3)_c \times U(1)_{em}$$

$$DS : SU(N) \xrightarrow{\phi} SU(N-1)$$

$$m_{W_D^\pm} = \frac{1}{2} g_D^2 (v_\phi) v_\phi^2 \times (N-1) \text{ complex}$$

$$m_{Z_D} = g_D^2 (v_\phi) v_\phi^2 \frac{N-1}{2N}$$

Thermal potential

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$$V_0 = -\mu_H^2 H^\dagger H + \frac{\lambda_H}{2} (H^\dagger H)^2 - m_\phi^2 \phi^\dagger \phi + \frac{\lambda_\phi}{2} (\phi^\dagger \phi)^2 \\ + \lambda_{H\phi} (H^\dagger H) (\phi^\dagger \phi) + \delta_{\text{SU}(2)} \lambda'_{H\phi} H^\dagger \phi \phi^\dagger H$$

$$V_{\text{CW}} = n_{\text{GB}} \frac{m_{\text{GB}}^4}{64\pi^2} \left(\log \left[\frac{m_{\text{GB}}^2}{\mu^2} \right] - \frac{5}{6} \right) + \sum_{i \neq \text{GB}} n_i \frac{m_i^4}{64\pi^2} \left(\log \left[\frac{m_i^2}{\mu^2} \right] - \frac{3}{2} \right)$$

$$V_{T \neq 0} = \sum_i \frac{T^4}{2\pi^2} J_B \left(\frac{m_i^2 + \Pi_i}{T^2} \right)$$

$$V(\phi, T) = V_0(\phi) + V_{\text{CW}}(\phi, \mu) + V_{T \neq 0}(\phi, T)$$

Thermal Potential

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$$m_{W_D}^2(\phi) = \frac{1}{2} g_D^2 \phi^2 \times (N-1) \text{complex}$$

$$m_{Z_D}^2(\phi) = \frac{1}{2} g_D^2 \phi^2 \frac{N-1}{N}$$

$$m_{\text{Re}[\phi]}^2(\phi) = -\mu^2 + 3 \frac{\lambda \phi}{2} \phi^2$$

$$m_H^2(\phi) = -\mu_H^2 + \lambda_H \phi \phi^2 / 2$$

$$m_{\text{Im}[\phi]}^2(\phi) = -\mu^2 + \frac{\lambda \phi}{2} \phi^2$$

$$m_{\phi_{N-1}}^2(\phi) = -\mu^2 + \frac{\lambda \phi}{2} \phi^2$$

$$\Pi_{W_D} = \frac{11}{3} g_D^2 T^2$$

$$\Pi_{Z_D} = \frac{20 + N}{6N} g_D^2 T^2$$

$$\Pi_{\text{Re}[\phi]} = \frac{T^2}{12} \left(2N \frac{\lambda \phi}{2} + 4\lambda_H \phi + 3g_D^2 \left(2(N-1) + \frac{N-1}{N} \right) \right)$$

$$\Pi_H = \frac{T^2}{12} (4\lambda_H + N\lambda_H \phi)$$

$$\Pi_{\text{Im}[\phi]} = \frac{T^2}{12} \left(N \frac{\lambda \phi}{2} + 4\lambda_H \phi + 3g_D^2 \left(2(N-1) + \frac{N-1}{N} \right) \right)$$

$$\Pi_{\phi_{N-1}} = \frac{T^2}{12} \left(N \frac{\lambda \phi}{2} + 4\lambda_H \phi + 3g_D^2 \left(2(N-1) + \frac{1}{(N-1)N} \right) \right)$$

Thermal parameters

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$$\frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r} \frac{\partial \phi}{\partial r} = \frac{\partial V(\phi, T)}{\partial \phi} \quad \phi'(0) = 0 \quad \phi(\infty) = 0$$

$$S_E = 4\pi \int_0^\infty r^2 dr \left[\left(\frac{\partial \phi}{\partial r} \right)^2 + V(\phi, T) \right]$$

$$p(t_N) t_N^4 \approx 1 \quad p(T) \approx T^4 e^{-S_E(S_b; T)/T} \quad T^2 t = \sqrt{\frac{45}{16\pi^3}} \frac{M_p}{\sqrt{g_\star}}$$

$$\alpha = \frac{\Delta \left(V - T \frac{\partial V}{\partial T} \right)}{\pi^2 g_\star T^4 / 30} \Bigg|_{T_N}$$

$$\frac{\beta}{H} = T \left(\frac{d(S_E/T)}{dT} \right) \Bigg|_{T_N}$$

Thermal parameters

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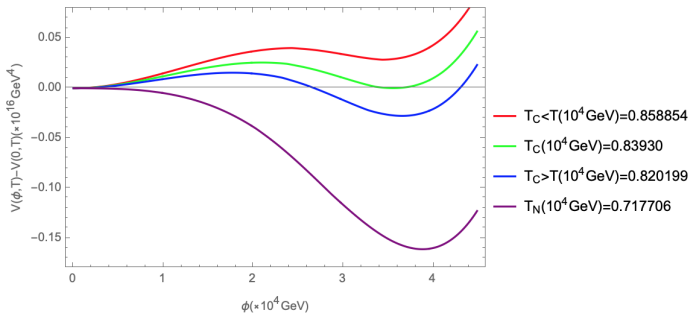
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$$\lambda_\phi = 0.0009, \lambda_{h\phi} = 0.05, g_D = 0.5, N = 5, \frac{\phi_C}{T_C} = 4.268, \alpha = 0.278, \beta/H = 1942$$

$$V(0, T_C) = V(\phi_C, T_C) \quad \frac{\phi_C}{T_C} > 1$$

Gravitational waves

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$$h^2 \Omega_{sw}(f) = 8.5 \times 10^{-6} \left(\frac{100}{g_\star} \right)^{1/3} \Gamma^2 \bar{U}_f^4 \left(\frac{H_\star}{\beta} \right) v_w S_{sw}(f)^\dagger$$

$$\Gamma = 4/3, \quad \bar{U}_f^2 = \frac{3}{4} \kappa_f \alpha_{T_\star}, \quad S_{sw}(f) = \left(\frac{f}{f_{sw}} \right)^3 \left(\frac{7}{4 + 3(f/f_{sw})^2} \right)^{7/2}$$

$$f_{sw} = 8.9 \mu\text{Hz} \frac{1}{v_w} \left(\frac{\beta}{H_\star} \right) \left(\frac{z_p}{10} \right) \left(\frac{T_\star}{100\text{GeV}} \right) \left(\frac{g_\star}{100} \right)^{1/6}$$

$$h^2 \Omega_{turb}(f) = 3.35 \times 10^{-4} \left(\frac{H_\star}{\beta} \right) \left(\frac{\kappa_{turb}(\alpha_{T_\star}) \alpha_{T_\star}}{1 + \alpha_{T_\star}} \right)^{3/2} \left(\frac{100}{g_\star} \right)^{1/3} v_w S_{turb}(f)$$

$$S_{turb}(f) = \frac{(f/f_{turb})^3}{[1 + (f/f_{turb})]^{11/3} (1 + 8\pi f/h_\star)} \quad h_\star = 16.5 \mu\text{Hz} \left(\frac{T_\star}{100\text{GeV}} \right) \left(\frac{g_\star}{100} \right)^{1/6}$$

$$f_{turb} = 27 \mu\text{Hz} \frac{1}{v_w} \left(\frac{\beta}{H_\star} \right) \left(\frac{T_\star}{100\text{GeV}} \right) \left(\frac{g_\star}{100} \right)^{1/6}$$

$$h^2 \Omega(f) = h^2 \Omega_{sw}(f) + h^2 \Omega_{turb}(f)$$

†10.1098/rsta.2017.0126

†10.1098/rsta.2017.0126

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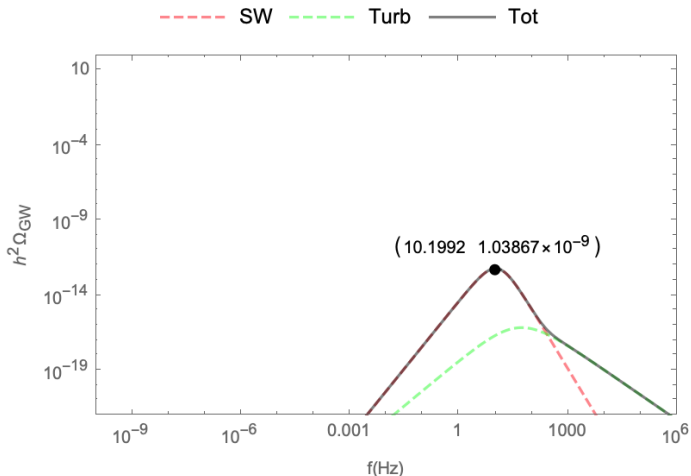
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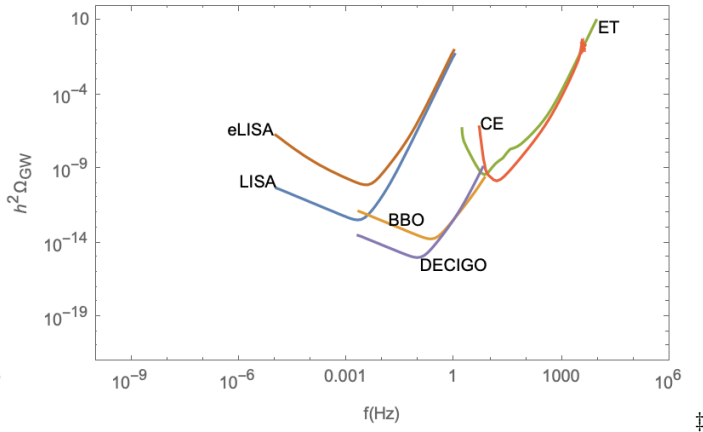
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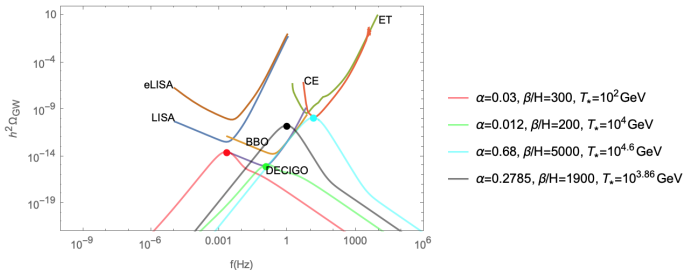


[†]gwplotter

Gravitational waves

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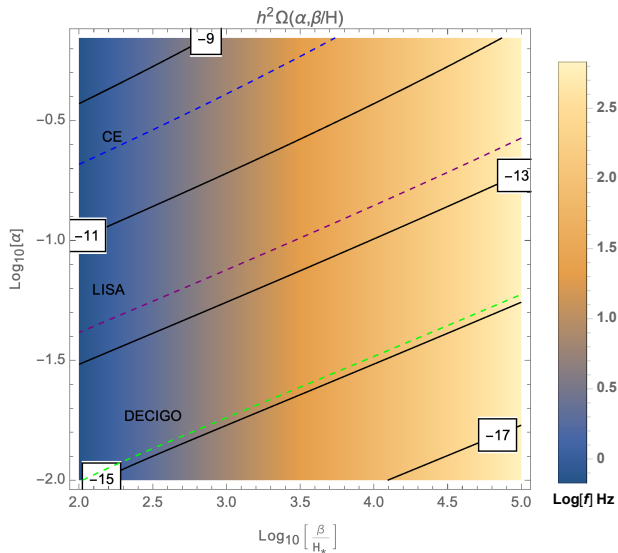


$\mu_\phi (10^4 \text{ GeV})$	$\mu_H (10^4 \text{ GeV})$	λ_ϕ	λ_H	$\lambda_{h\phi}$	g_D	N	g_*
0.212168	1.58143	0.0009	3.04	0.05	0.5	5	184.25
$\frac{\phi_C}{T_C}$	$T_C (10^4 \text{ GeV})$	$T_N (10^4 \text{ GeV})$	α	$\frac{\beta}{H_*}$			
4.268	0.8393	0.717706	0.278	1942			

Gravitational waves

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Connection between SSB and confinement

m_ν + GW

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$$\alpha_d^{-1}(m_2) = \alpha_d^{-1}(m_1) + \frac{-b_d}{2\pi} \ln \left(\frac{m_2}{m_1} \right)$$

$$\beta_d(g_d) = \frac{-g_d^3}{(4\pi)^2} \left[\frac{11}{3} C_2(G) - \frac{4}{3} \kappa_f S_2(R_F) - \frac{1}{3} \kappa_s S_2(R_S) + \frac{2\kappa_f}{(4\pi)^2} Y_4(R_F) \right]$$

$$b_d = -\frac{11}{3}(N-1) + \frac{1}{3}N_f + \frac{1}{6}N_s$$

Connection between SSB and confinement

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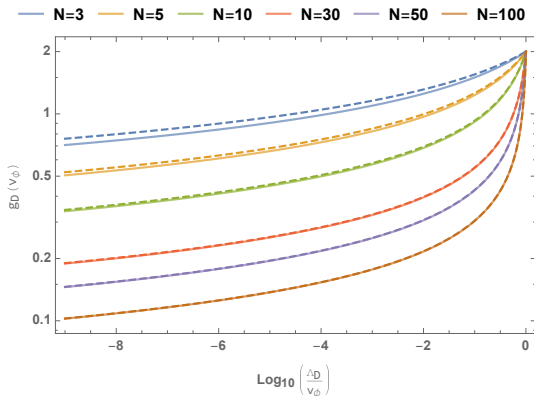
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Correlation between $g_d(v_\phi)$ and $\frac{\Lambda_D}{v_\phi}$ for different values of N where for solid(dashed) curves $N_f = 2(5)$. If $\Lambda_D > 0.2$ MeV (BBN) and $v_\phi \sim O(10^{4-6} \text{ GeV})$ then $\frac{\Lambda_D}{v_\phi}$ can be as low as $O(10^{-8} - 10^{-10})$.

Confinement of $SU(N - 1)$

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Gravitational waves from confinement:

- $N_C = 2$ and $N_F \geq 2$ ¶
- $N_C \geq 3$ and $N_F \geq 3$ ||
- $N_F = 0$ **
- $\mu_{B,D} \gtrsim \Lambda_{\text{conf}}$ ††

¶ 10.1103/PhysRevD.62.045012

|| 10.1103/PhysRevD.29.338

** 10.1088/1126-6708/2005/02/033;10.1103/PhysRevD.82.114505;10.1103/PhysRevD.92.055034

†† 10.1142/S0217751X05027965

Neutrino mass models and gravitational waves

m_ν + GW

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- $h^2\Omega \propto \frac{v_\phi}{m_\phi}$
- $h^2\Omega \propto g_D$
- $h^2\Omega \propto n_{f_i}$ where $m_{f_i} \propto v_\phi$
- $h^2\Omega \propto N$
- $h^2\Omega \propto g_\star^{-1}$
- Some models: Pati-Salam, LR, Dark Sector, GUT

Neutrino genesis

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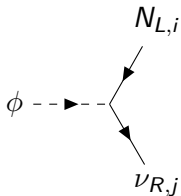
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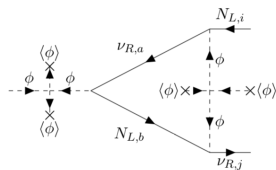
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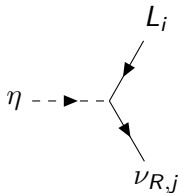
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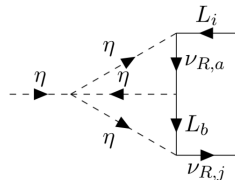
$$\sim Y_R^{ij}$$



$$\sim \frac{1}{16\pi^2} Y_R^{ia} Y_R^{*ab} Y_R^{bj} \lambda_\phi^2 \langle \phi \rangle^4$$



$$\sim Y_\nu^{ij}$$



$$\sim \frac{1}{(16\pi^2)^2} Y_\nu^{ia} Y_\nu^{*ab} Y_\nu^{bj} \lambda_\eta^{\text{eff}}$$

Neutrino genesis

$m_\nu + \text{GW}$

$$n_B = n_L = O(1)n_{\nu_R} = O(1)\varepsilon_A n_{\phi, \eta}^{\ddagger\dagger} \rightarrow \varepsilon_A \sim O(10^{-8})$$

$$\begin{aligned} \varepsilon_A &= \frac{\Gamma(\phi \rightarrow N_L^\dagger \nu_R) - \Gamma(\phi^\dagger \rightarrow N_L \nu_R^\dagger)}{\Gamma(\phi \rightarrow N_L^\dagger \nu_R) + \Gamma(\phi^\dagger \rightarrow N_L \nu_R^\dagger)} \\ &= \frac{\text{Im}[Z]\text{Im}[\mathcal{F}]}{16\pi^2} \left[\frac{|X^{ij}|^2 + |X^{i\alpha} X^{*\alpha\beta} X^{\beta j}|^2 |\mathcal{F}|^2 (16\pi^2)^{-2}}{2} + \frac{\text{Re}[Z]\text{Re}[\mathcal{F}]}{16\pi^2} \right]^{-1} \end{aligned}$$

$$Z = X^{ij} \left(X^{i\alpha} X^{*\alpha\beta} X^{\beta j} \right)^* \quad X^{ij} = R_{N\alpha}^i \frac{Y_R^{\alpha\beta}}{\sqrt{2}} U_{R\beta}^j$$

$$\begin{aligned} \mathcal{F} = F(x_i, x_R, x_I, x_N) &= (1 - x_i)^{-1} \left[(1 - x_I) \log \left(\frac{x_I}{x_I - 1} \right) - (1 - x_R) \log \left(\frac{x_R}{x_R - 1} \right) - \log \left(\frac{x_I}{x_R} \right) \right. \\ &\quad \left. + (x_N - x_i x_I) C_0(0, m_{N_f}^2, m_{s_i}^2; m_N, m_{s_I}, 0) - (x_N - x_i x_R) C_0(0, m_{N_f}^2, m_{s_i}^2; m_N, m_{s_R}, 0) \right] \end{aligned}$$

$$x_i = \left(\frac{m_{s_i}}{m_{N_f}} \right)^2 \quad x_{R,I} = \left(\frac{m_{s_{R,I}}}{m_{N_f}} \right)^2 \quad x_N = \left(\frac{m_N}{m_{N_f}} \right)^2 \quad m_{s_I} > m_{s_R} > m_{N_f}, m_N$$

$$R \neq \mathbb{1}$$

$\ddagger\dagger$ 10.1103/PhysRevLett.84.4039

Parameters and Constraints

m_ν +GW

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Parameter	Constrained by
N, N_F	GW, DM
g_D	DM, GW, DS
λ_H, ν	SM
v_ϕ	GW, m_ν , Direct searches
μ_H, μ_ϕ	V minimization
$\lambda_\phi, \lambda_{H\phi}$	GW, m_ϕ
$Y_{R,L,\nu}$	m_ν , Dirac leptogenesis, PMNS
M_{IM}	m_ν , Dirac leptogenesis, DM

Future tasks

$m_\nu + \text{GW}$

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- Correlation between GW signals and DM dynamics
- GUT completions (3 possible GW signals)
- Radiative neutrino models

Conclusion

$m_\nu + \text{GW}$

Introduction

Models

Thermal
potential and
parameters

GW

Confinement

Neutrino-
genesis

Conclusion

- Minimal Dirac neutrino models
- Correlated GW signals
- Confined Dark Sector
- Baryon asymmetry via leptogenesis
- Probing neutrino models via GW

Thanks for your attention!