Introduction to Quantum Chromodynamics

Lecture 1

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Quy Nhon – MCnet school 2019

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GEFÖRDERT VOM

Bundesministerium
für Bildung
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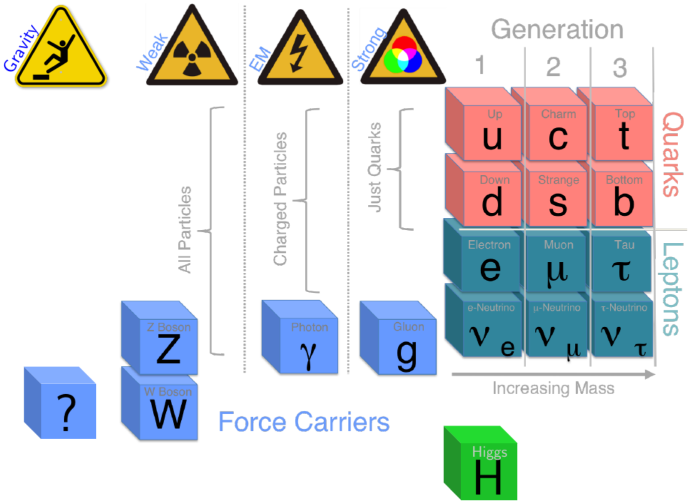
Invitation to QCD: The Standard Model



The two faces of QCD

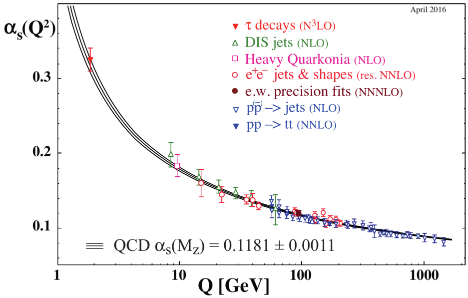
- theory of the strong interaction [non-abelian $\mathrm{SU}(3)_C$ gauge group]

$$Q^2 \frac{\mathrm{d}\alpha_s}{\mathrm{d}Q^2} = \beta(\alpha_s) = -(b_0\alpha_s^2 + b_1\alpha_s^3 + \dots)$$

- asymptotic free if $Q^2 \gg \Lambda_{\mathrm{QCD}}^2$
  $\hookrightarrow$ perturbative regime
- confinement when $Q^2 \simeq \Lambda_{\mathrm{QCD}}^2$
  $\hookrightarrow$ non-perturbative regime
  $\hookrightarrow$ parton-to-hadron transition

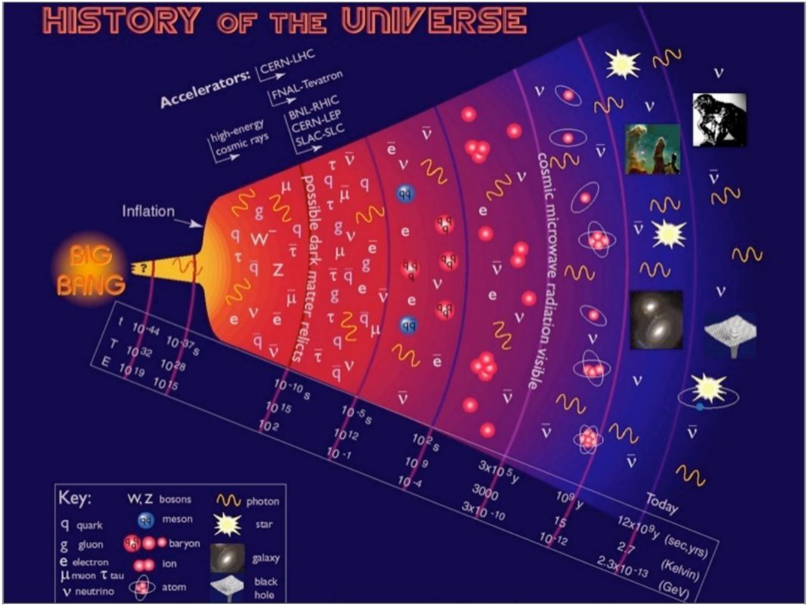
The QCD running coupling

[Patrignani *et al.* Chin. Phys. C **40** (2016) no.10, 100001]



strong-interaction phenomena pose most severe theory challenges: sizable quantum corrections, non-perturbative effects

Invitation to QCD: The cosmological perspective

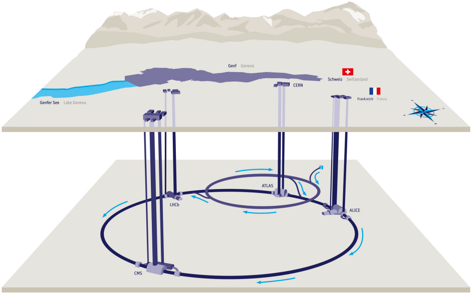


Invitation to QCD: The LHC

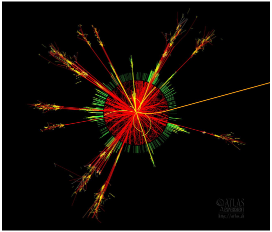
LHC experimental conditions

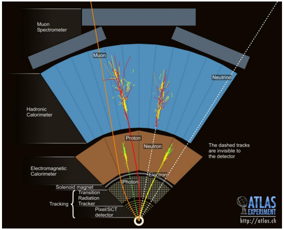
- 27 km circumference accelerator
- colliding protons of 6.5 TeV energy
- bunch crossing every 25 ns
- four interaction points

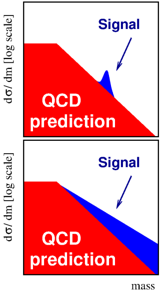
  ↪ ATLAS, CMS, LHCb, ALICE



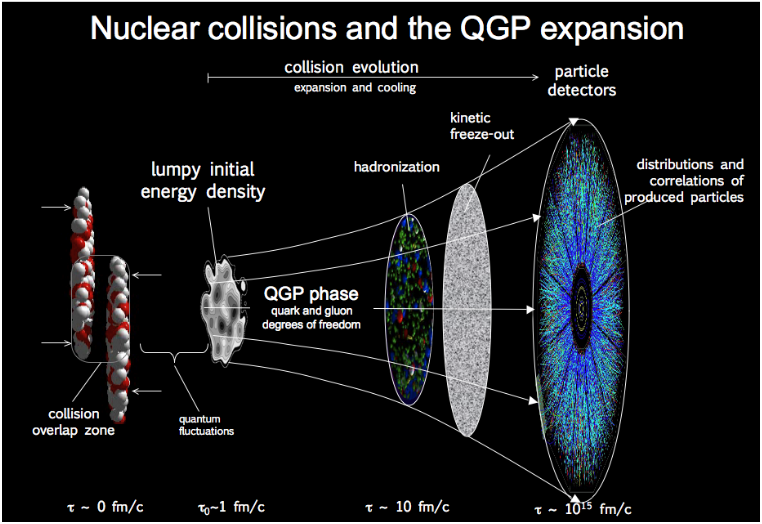
need to identify & measure final states of individual pp clashes







Invitation to QCD: heavy ion collisions

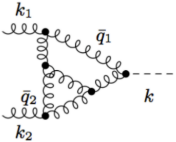


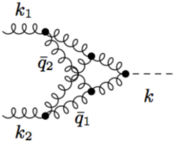
Predictive Methods: Perturbation Theory

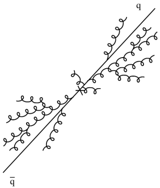
- QCD is an interacting QFT
- can employ order-by-order expansions in coupling α_s

$$\mathcal{O} \approx C_0 + C_1\alpha_s + C_2 \underbrace{\alpha_s^2}_{\text{small}} + C_3 \underbrace{\alpha_s^3}_{\text{smaller}} + \underbrace{\ldots}_{\text{negligible?}}$$

- calculational complexity grows extremely fast with powers of α_s
  proliferation of contributions: real-emission & loop diagrams



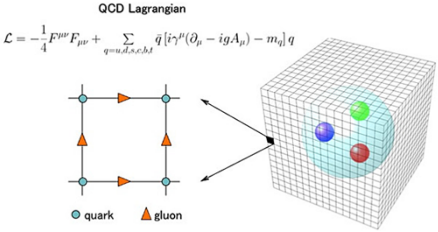


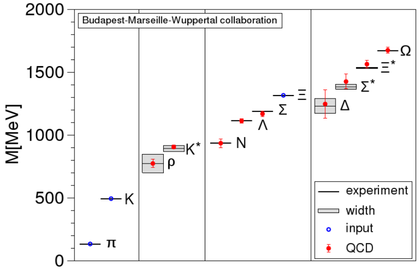


- truncate expansion at fixed order: LO, NLO, NNLO, ... prediction
- all-orders resummation of logarithmically enhanced terms: LL, NLL, ...

Predictive Methods: Lattice computations

- use discretized version of the QCD action on finite lattice
- numerically evaluate field configuations





[Durr et al. Science **322** (2008) 1224]

address the strong coupling regime: parton condensation, hadron spectra, phase transitions

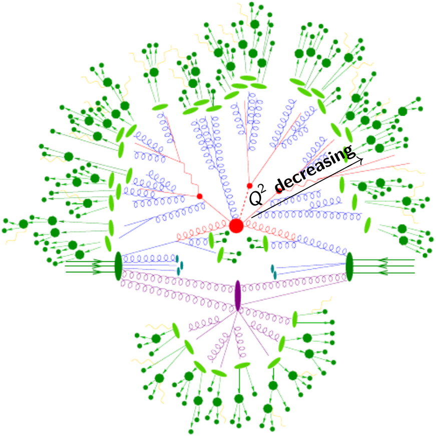
Predictive Methods: Monte Carlo simulations

Stochastic simulation of fully exclusive collision events

[Buckley *et al.* Phys. Rept. **504** (2011) 145]

$\hookrightarrow$ factorize short- & long range physics

- perturbative phases
  - **Hard interaction**
    exact matrix elements $|\mathcal{M}|^2$
    LO,NLO,NNLO – QCD, NLO – EW
  - **Radiativ corrections**
    parton showers in the initial and final state
    resummation of soft-collinear logs: LL, NLL
- non-perturbative phases
  - **Hadronization**
    parton–hadron transition
  - **Hadron Decays**
    phase space or effective theories
  - **Underlying Event**
    beyond factorization: modelling



MC generators, i.e. PYTHIA, HERWIG, SHERPA, provide pseudo data for direct comparison to experimental data

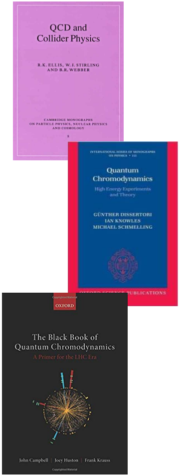
Outline of the lecture

1. **General properties of QCD**
   - non-abelian gauge symmetries
   - the QCD Lagrangian
   - the QCD running coupling
2. **Infrared singularities**
   - IR singularities
   - Exponentiation and Resummation
   - NLO QCD corrections
3. **Hadronic initial- and final states**
   - Parton fragmentation & QCD jets
   - Parton Density Functions

Recommended literature

QCD textbooks

- R. K. Ellis, W. J. Stirling and B. R. Webber
  QCD and collider physics
  Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. **8** (1996) 1
- G. Dissertori, I. Knowles and M. Schmelling
  Quantum Chromodynamics: High Energy Experiments and Theory
  International Series of Monogr (2009)
- J. Campbell, J. Huston and F. Krauss
  The Black Book of Quantum Chromodynamics
  Oxford University Press (2018)
- J. Collins
  Foundations of Perturbative QCD
  Cambridge University Press (2011)



QCD lecture notes

- G. Salam *Elements of QCD for hadron colliders*
  arXiv:1011.5131 [hep-ph]
- P. Skands *Introduction to QCD* arXiv:1207.2389 [hep-ph]

Generalities: Basics of QCD

Generalities: the naïve parton model

1st tenet: hadronic matter made of quarks

- the fermionic quarks carry fractional charges
  $Q_{u,c,t} = +\frac{2}{3}$, $Q_{d,s,b} = -\frac{1}{3}$
- three-quark states form baryons: $|B\rangle = |\, q_1 \;\, q_2 \;\, q_3\rangle$

  $\rightsquigarrow$ baryons are fermions, follow Fermi-Dirac statistics

  $\rightsquigarrow$ wave functions must be totally anti-symmetric
- mesons thought of as bound states of quark & anti-quark: $|M\rangle = |\, q_1 \;\, \bar{q}_2\rangle$

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obstacle at the time: how to account for spin-3/2 baryons?

- consider the resonance $|\Delta^{++}\rangle = |\, u_\uparrow \; u_\uparrow \; u_\uparrow\rangle$
  $\rightsquigarrow$ symmetrical state in space, spin & flavour
- introduce new degree of freedom: colour index $a \in \{1, 2, 3\}$
  $\rightsquigarrow$ $|\Delta^{++}\rangle = \epsilon_{abc}\,|\, u_{a\uparrow} \; u_{b\uparrow} \; u_{c\uparrow}\rangle$
  $\rightsquigarrow$ baryon wave-function totally anti-symmetric in that index

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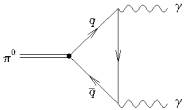
2nd tenet: hadronic matter must be colour-singlet states

Generalities: colour degree of freedom

- consider the decay $\pi^0 \to \gamma\gamma$ $\left[\left|\pi^0\right\rangle = \frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle - \left|d\bar{d}\right\rangle\right)\right]$



$$\Gamma^{\text{theo}}(\pi^0 \to \gamma\gamma) = \xi^2 \left(\frac{\alpha}{\pi}\right)^2 \frac{1}{64\pi} \frac{m_\pi^3}{f_\pi^2} = 7.6\,\xi^2\,\text{eV}$$

electric charge and colour factor ξ given by

$$\xi = N_c \left[\left(\frac{2}{3}\right)^2 - \left(\frac{1}{3}\right)^2\right] = \frac{N_c}{3}$$

experimental value is $\Gamma^{\text{exp}} = 7.74 \pm 0.55\,\text{eV}$ [PDG] $\rightsquigarrow \xi = 1 \rightsquigarrow N_c = 3$

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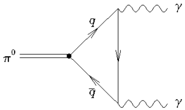

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- consider the ratio $R = \sum_q \sigma_{\rm tot}(e^+e^- \to q\bar{q})/\sigma_{\rm tot}(e^+e^- \to \mu^+\mu^-)$

u, d, s only $R = N_c \left[\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2\right] = N_c\,\frac{2}{3}$

u, d, s, c, b only $R = N_c \left[2 \times \left(\frac{2}{3}\right)^2 + 3 \times \left(-\frac{1}{3}\right)^2\right] = N_c\,\frac{11}{9}$

$\rightsquigarrow$ data consistent with $N_c = 3$

the group of unitary 3×3 matrices U with $\det(U) = +1$

$\rightsquigarrow$ $SU(3)$ generators, hermitian & traceless Gell-Mann matrices

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda^8 = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{-2}{\sqrt{3}} \end{pmatrix}$$

by convention we define $t^A_{ab} \equiv \frac{1}{2}\lambda^A_{ab} \quad \rightsquigarrow \quad U = \exp\{i\alpha_A t^A\}$

$$[t^A, t^B] = i f_{ABC} t^C \text{ [non-abelian!]}$$

with f_{ABC} the $SU(3)$ structure constants (anti-symmetric in all indices)

$\rightsquigarrow$ **(Anti-)Quark fields in (anti-)triplet representation**

$[3 \otimes \bar{3} = 8 \oplus 1,\ 3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1]$

$\rightsquigarrow$ **Gluon gauge field in octet representation**

construction principles

- renormalizable, local QFT in 4D
- invariance of $\mathcal{L}_{\text{QCD}}$ under local $SU(3)_c$ gauge transformations

$\hookrightarrow$ quark fields in fundamental representation

$$\Psi_q(x) \quad \to \quad U(x)\Psi_q(x) = \exp\left(i\alpha_A(x)t^A\right)\Psi_q(x)$$

$\hookrightarrow$ gluon gauge field in adjoint representation

$$A_\mu^A(x)t^A \quad \to \quad U(x)A_\mu^A(x)t^A U^{-1}(x) + \frac{i}{g_s}\left(\partial_\mu U(x)\right)U^{-1}(x)$$

the free quark part

denote quark fields by ψ_q^a, where a denotes a colour index $a \in \{1, 2, 3\}$

$$\mathcal{L}_{\text{free Dirac}} = \sum_q \bar{\psi}_q^a \, i\delta_{ab}\gamma^\mu \partial_\mu \, \psi_q^b - m_q \bar{\psi}_q^a \psi_q^a \quad \text{with} \quad q \in \{u, d, s, c, b, t\}$$

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the pure gluon part

denote the gluon fields by A_μ^A, with $A \in \{1, .., 8\}$

$$\mathcal{L}_{\text{pure Gluon}} = -\frac{1}{4} F_{\mu\nu}^A F^{A\mu\nu} \quad \text{with} \quad F_{\mu\nu}^A = \partial_\mu A_\nu^A - \partial_\nu A_\mu^A - g_s f_{ABC} A_\mu^B A_\nu^C$$

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coupling the quarks to gluons

minimal coupling of quarks with gluons, consistent with local gauge invariance

$$\mathcal{L}_{\text{interaction}} = \sum_q g_s \bar{\psi}_q^a \, \gamma^\mu \, t_{ab}^A A_\mu^A \, \psi_q^b \quad \text{with} \quad g_s^2 = 4\pi\alpha_s$$

the classical QCD Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\mathrm{QCD}} &= \mathcal{L}_{\text{free Dirac}} + \mathcal{L}_{\text{interaction}} + \mathcal{L}_{\text{pure Gluon}} \\
&= \sum_q \bar{\psi}_q^a \left(i\delta_{ab}\gamma^\mu \partial_\mu + g_s \gamma^\mu t^A_{ab} A^A_\mu \right) \psi_q^b - m_q \bar{\psi}_q^a \psi_q^a - \frac{1}{4} F^A_{\mu\nu} F^{A\mu\nu}
\end{aligned}
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&= \sum_q \bar{\psi}_q^a \left(i\gamma^\mu (D_\mu)_{ab} - \delta_{ab} m_q\right) \psi_q^b - \frac{1}{4} F_{\mu\nu}^A F^{A\mu\nu}
\end{aligned}
$$

with $(D_\mu)_{ab} = \delta_{ab}\partial_\mu - i g_s t_{ab}^A A_\mu^A$ the $SU(3)_c$ covariant derivative

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with $(D_\mu)_{ab} = \delta_{ab}\partial_\mu - ig_s t^A_{ab}A^A_\mu$ the $SU(3)_c$ covariant derivative

QCD e.o.m.

$$\begin{aligned}(i\not{D} - m_q\mathbb{1})\,\psi_q &= 0 \\ D^\mu F^A_{\mu\nu} &= g_3\sum_q \bar{\psi}_q t^A\gamma_\nu\psi_q\end{aligned}$$

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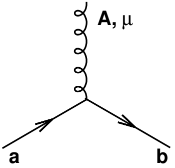
D^\mu F^A_{\mu\nu} &= g_3 \sum_q \bar{\psi}_q t^A \gamma_\nu \psi_q
\end{aligned}$$

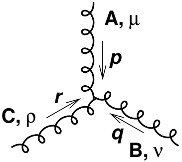
$\rightsquigarrow$ interacting field theory (decoupling when $g_3 \to 0$)
$\rightsquigarrow$ non-abelian gauge field theory [to be quantized]
$\rightsquigarrow$ quark & gluon propagators
$\rightsquigarrow$ quark–gluon & gluon self interactions

Generalities: Basics of QCD – The Feynman rules

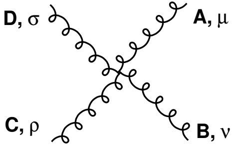
$$\mathcal{L}_{\mathrm{QCD}} \supset \bar{\psi}^a_q \left(-ig_s \gamma^\mu t^A_{ab} A^A_\mu\right) \psi^b_q - g_s f^{ABC} (\partial_\mu A^A_\nu) A^{B\mu} A^{C\nu} - \frac{1}{4} g_s^2 f^{XAB} f^{XCD} A^{A\mu} A^{B\nu} A^C_\mu A^D_\nu$$



$-ig_s t^A_{ba} \gamma^\mu$

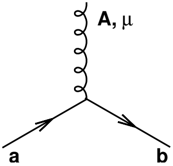


$-g_s f^{ABC}[(p-q)^\rho g^{\mu\nu}$
$+(q-r)^\mu g^{\nu\rho}$
$+(r-p)^\nu g^{\rho\mu}]$



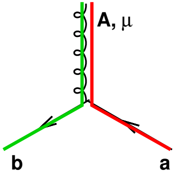
$-ig_s^2 f^{XAC} f^{XBD}[g^{\mu\nu} g^{\rho\sigma} -$
$g^{\mu\sigma} g^{\nu\gamma}] + (C,\gamma) \leftrightarrow$
$(D,\rho) + (B,\nu) \leftrightarrow (C,\gamma)$

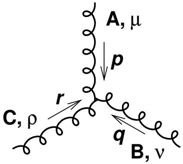
Generalities: Basics of QCD – The Feynman rules



$\propto g_s t^A_{ba}$

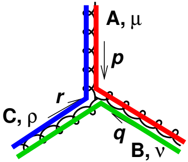
$\Longleftrightarrow$





$\propto g_s f^{ABC}$

$\Longleftrightarrow$



- gluon emission repaints the quark colour
- gluon carries colour and anti-colour
- gluon emission repaints the gluon colours

subtleties

- for a valid gluon propagator, the gauge needs to be fixed, e.g. R_ξ gauges:

$$\mathcal{L}_{\text{kin}} \to \mathcal{L}_{\text{kin}} - \frac{1}{2\xi}\left(\partial_\mu A^\mu\right)^2 \ \rightsquigarrow\ D^{\mu\nu}(p) = \frac{-\eta^{\mu\nu} + (1-\xi)p^\mu p^\nu/p^2}{p^2 + i\epsilon}$$

[physical results gauge independent, different choices ease calculations]

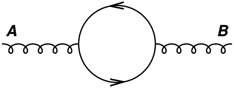
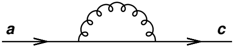
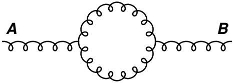
- for consistent quantization need to add so-called Faddeev–Popov ghosts

$$\mathcal{L}_{\text{ghost}} = \partial_\mu \bar{c}_A \partial^\mu c^A + g_s f_{ABE} A^{\mu,A}\left(\partial_\mu \bar{c}_B\right) c_E$$

[adjoint scalars, Grassmann-type, cancel unphysical gluon polarizations]

some useful $SU(N_c)$ colour algebra relations

$\hookrightarrow$ appear when summing squared amplitudes colours

trace relation	corresponding diagram
$Tr\{t^A t^B\} = T_R\,\delta^{AB}\,, \quad T_R = \frac{1}{2}$	
$\sum_A t^A_{ab} t^A_{bc} = C_F\,\delta_{ac}\,, \quad C_F = \frac{N_c^2 - 1}{2N_c}$	
$\sum_{C,D} f^{ACD} f^{BCD} = C_A\,\delta^{AB}\,, \quad C_A = N_c$	

QCD: $N_c \equiv$ number of colours $= 3 \quad \rightsquigarrow \quad C_A = 3 \quad \& \quad C_F = \frac{4}{3}$

Generalities: The QCD running coupling

as other couplings/parameters α_s is scale dependent [momentum scale μ^2]

$$\frac{d\alpha_s(\mu^2)}{d\ln\mu^2} = \beta(\alpha_s(\mu^2))\,, \qquad \beta(\alpha_s) = -\alpha_s^2(b_0 + b_1\alpha_s + b_2\alpha_s^2 + \ldots)$$

where

$$b_0 = \frac{11C_A - 2n_f}{12\pi}\,, \qquad b_1 = \frac{17C_A^2 - 5C_A n_f - 3C_F n_f}{24\pi^2} = \frac{153 - 19n_f}{24\pi^2}$$

Note sign of β: **Asymptotic freedom**, due to gluon self interaction

[Nobel prize in 2004 to Gross, Politzer & Wilczek]

- $\rightsquigarrow$ at high scales μ^2 coupling becomes small, quarks & gluons are almost free interactions weak, perturbation theory works
- $\rightsquigarrow$ at low scales coupling becomes strong, quarks & gluons interact strongly perturbation theory fails

Generalities: The QCD running coupling

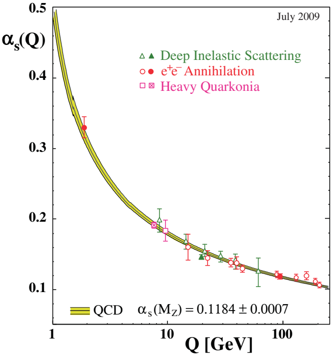
as other couplings/parameters α_s is scale dependent [momentum scale μ^2]

$\rightsquigarrow$ ignoring all terms other than $b_0 = (33 - 2n_f)/12\pi$, we get for $\alpha_s(\mu^2)$

$$\frac{d\alpha_s(\mu^2)}{d\ln\mu^2} = -b_0\alpha_s^2 \quad \rightsquigarrow \quad \alpha_s(\mu^2) = \frac{\alpha_s(\mu_0^2)}{1 + b_0\alpha_s(\mu_0^2)\ln\frac{\mu^2}{\mu_0^2}} = \frac{1}{b_0 \ln\frac{\mu^2}{\Lambda^2_{\rm QCD}}}$$

result expressed in terms of

- reference scale μ_0^2, e.g. M_Z^2
- non-perturbative constant $\Lambda_{\rm QCD} \simeq 0.2$ GeV
  - fundamental scale of QCD
  - sets scale for hadron masses



- perturbation theory valid for $\mu \gg \Lambda_{\rm QCD}$
- non-perturbative description $\mu \simeq \Lambda_{\rm QCD}$