The data

$\underline{SU(3)}$ $a = 1$ to 8 $\qquad \underline{SU(2)}$ $k = 1$ to 3 $\qquad \underline{U(1)}$

$[T^a, T^b] = i f^{abc} T^c$, $[T^j, T^k] = i\epsilon_{jkl} T^l$

$D_\mu = \partial_\mu + i g_s T^a A^a_\mu + i g T^k W^k_\mu + i g' Y B_\mu$

$[D_\mu, D_\nu] = i g_s T^a F^a_{\mu\nu} + i g T^k W^k_{\mu\nu} + i g' Y B_{\mu\nu}$

$$\mathcal{L}_{SM} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \frac{1}{4} W^k_{\mu\nu} W^{k\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \quad \leftarrow \mathrm{Tr}([D_\mu, D_\nu][D^\mu, D^\nu])$$

$$+ \sum_{i=1}^{3} \Big\{ Q_i^\dagger i D_\mu \sigma_-^\mu Q_i + \underbrace{u_{Ri}^\dagger i D_\mu \sigma_+^\mu u_{Ri}}_{= u^{c\dagger}_{Ri} i D_\mu \sigma_-^\mu u^c_R} + \underbrace{d_{Ri}^\dagger i D_\mu \sigma_+^\mu d_{Ri}}_{= d^{c\dagger}_{Ri} i D_\mu \sigma_-^\mu d_{Ri}} + L_i^\dagger i D_\mu \sigma_-^\mu L_i + \underbrace{l_{Ri}^\dagger i D_\mu \sigma_+^\mu l_{Ri}}_{+ \nu_{Ri}^\dagger i D_\mu \sigma_+^\mu \nu_{Ri}} \Big\}$$

$$+ (D_\mu \phi)^\dagger (D^\mu \phi) - \frac{\lambda}{4} (\phi^\dagger \phi)^2 - \mu^2 \phi^\dagger \phi$$

$$+ \sum_{i,j=1}^{3} \Big\{ y^u_{ij} \overset{-\frac{1}{6}+\frac{2}{3}-\frac{1}{2}}{Q_i^\dagger u_{Rj} \phi} + y^d_{ij} Q_i^\dagger d_{Rj} \phi^c + y^l_{ij} L_i^\dagger l_{Rj} \phi^c + \text{h.c.}$$

after 1998
$$\begin{cases} + \dfrac{y^\nu_{ij}}{2\Lambda} (\phi^{c\dagger} L_i)^T (i\sigma^2) (\phi^{c\dagger} L_j) + \text{h.c.} \\ + y^\nu_{ij} L_i^\dagger \nu_{Rj} \phi + \text{h.c.} \end{cases}$$

$$\begin{cases} F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu \\ W^k_{\mu\nu} = \partial_\mu W^k_\nu - \partial_\nu W^k_\mu - g \epsilon^{klm} W^l_\mu W^m_\nu \\ B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \end{cases}$$

	$SU(3)$	$SU(2)$	$U(1)$	Y for ψ_L
$Q_i = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}$	3	2	$\frac{1}{6}$	$\frac{1}{6} \times 6 = +1$
u^c_{Ri}: u_{Ri}	3^* 3	1	$+2/3$	$-2/3 \times 3 = -2$
d^c_{Ri}: d_{Ri}	3^* 3	1	$-1/3$	$+1/3 \times 3 = 1$
$L_i = \begin{pmatrix} \nu_{Li} \\ l_{Li} \end{pmatrix}$	1	2	$-\frac{1}{2}$	$-\frac{1}{2} \times 2 = -1$
l^c_{Ri}: l_{Ri}	1	1	-1	$+1 \times 1 = +1$
ν_{Ri}	1	1	0	
ϕ	1	2	$-\frac{1}{2}$	
ϕ^c	1	2	$+\frac{1}{2}$	

$\to 10$; $\Big\} 5^*$; sum $= 0$

1975
Georgi-Glashow $SU(5)$

$\phi \to \phi' = U(x) \phi$

$\underline{D_\mu \to D'_\mu = U D_\mu U^{-1}}$

$$\begin{cases} \dfrac{g_s^2}{4\pi} \sim \dfrac{1}{8.5} \\ \dfrac{g^2}{4\pi} \sim \dfrac{1}{30} \\ \dfrac{g'^2}{4\pi} \sim \dfrac{1}{98} \end{cases}$$

$J^3 = \frac{\sigma^3}{2}$

$J^3 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Lorentz transformation

$x^\mu \to x'^\mu = L^\mu{}_\nu x^\nu$

$A^\mu \to A'^\mu = L^\mu{}_\nu A^\nu$

θ_k, η_k

$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$ $\qquad \gamma^\mu = \begin{pmatrix} 0 & \sigma_-^\mu \\ \sigma_+^\mu & 0 \end{pmatrix}$

$\bar\Psi = \Psi^\dagger \gamma^0$ $\qquad \sigma_\pm^\mu = (1, \pm\vec\sigma)$

$S_L(\theta_k, \eta_k) = e^{\sum_{k=1}^{3} \frac{\sigma^k}{2}(-i\theta_k - \eta_k)}$ $\qquad (\psi_L^\dagger, \psi_R^\dagger) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = (\psi_R^\dagger, \psi_L^\dagger)$

$S_R(\theta_k, \eta_k) = e^{\sum_{k=1}^{3} \frac{\sigma^k}{2}(-i\theta_k + \eta_k)}$

$$\bar\Psi \Psi = (\psi_R^\dagger, \psi_L^\dagger) \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = \psi_R^\dagger \psi_L + \psi_L^\dagger \psi_R$$

$$\bar\Psi \gamma^\mu \Psi = (\psi_R^\dagger, \psi_L^\dagger) \begin{pmatrix} 0 & \sigma_+^\mu \\ \sigma_-^\mu & 0 \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = \psi_R^\dagger \sigma_+^\mu \psi_R + \psi_L^\dagger \sigma_-^\mu \psi_L$$

$\psi_L \to \psi'_L = S_L \psi_L$

$\psi_R \to \psi'_R = S_R \psi_R$

$$\begin{cases} \underline{S_L^\dagger S_R = S_R^\dagger S_L = 1} \\ \underline{S_L^\dagger \sigma_-^\mu S_L = L^\mu{}_\nu \sigma_-^\nu}, \quad \underline{S_R^\dagger \sigma_+^\mu S_R = L^\mu{}_\nu \sigma_+^\nu} \end{cases}$$