UCLouvain
Institut de recherche en mathématique et physique
Centre de Cosmologie, Physique des Particules et Phénoménologie

From Lagrangians to Events (II)

Olivier Mattelaer

Plan

Plan

Yesterday

- BSM Model
- Matrix-element and decay

Today

- Phase-Space integration at LO
- Merging at LO (MLM and CKKW(L))

Friday

- aMC@NLO versus NLO computation

Phase-Space

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Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism

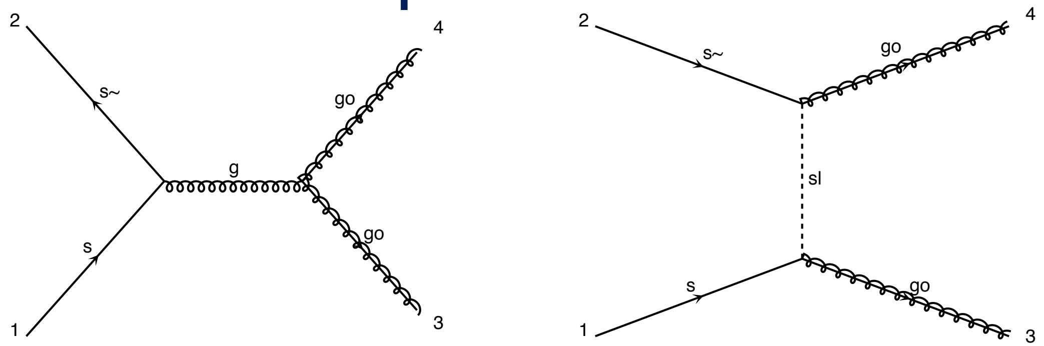

diagram 1 QCD=2, QED=0

diagram 2 QCD=2, QED=0

- Evaluate the matrix-element

$$|\mathcal{M}|^2$$ ➡Need Feynman Rules!

- Phase-Space Integration

$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi(n)$$

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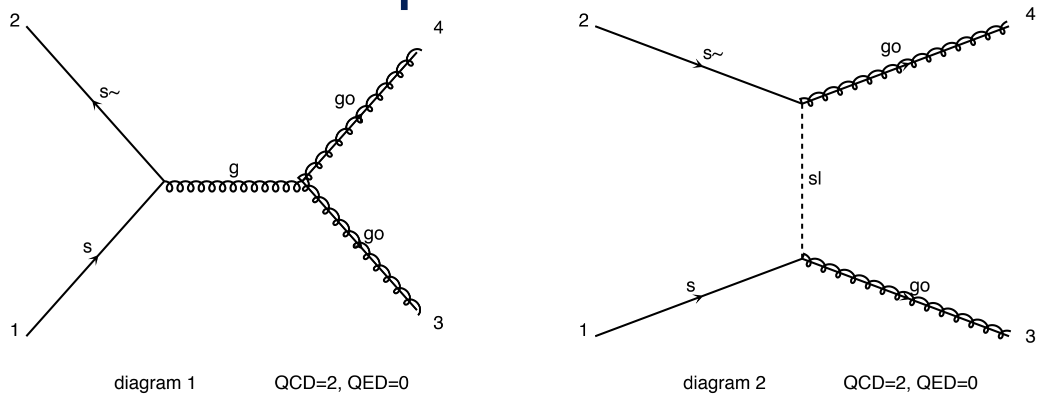

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Easy enough

Hard

Very Hard (in general)

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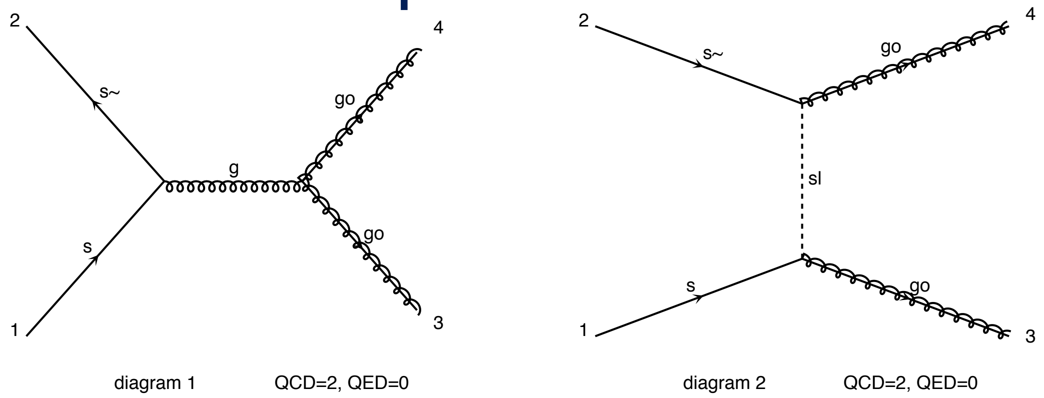

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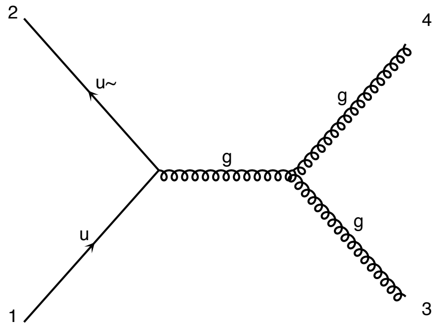
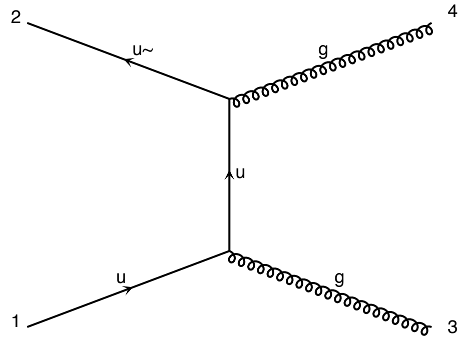
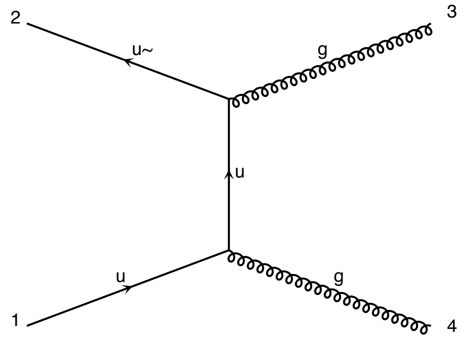
Now

Example: QCD 2 → 2







$$\propto \frac{1}{\hat{s}} = \frac{1}{(p_1 + p_2)^2} \qquad \propto \frac{1}{\hat{t}} = \frac{1}{(p_1 - p_3)^2} \qquad \propto \frac{1}{\hat{u}} = \frac{1}{(p_1 - p_4)^2}$$

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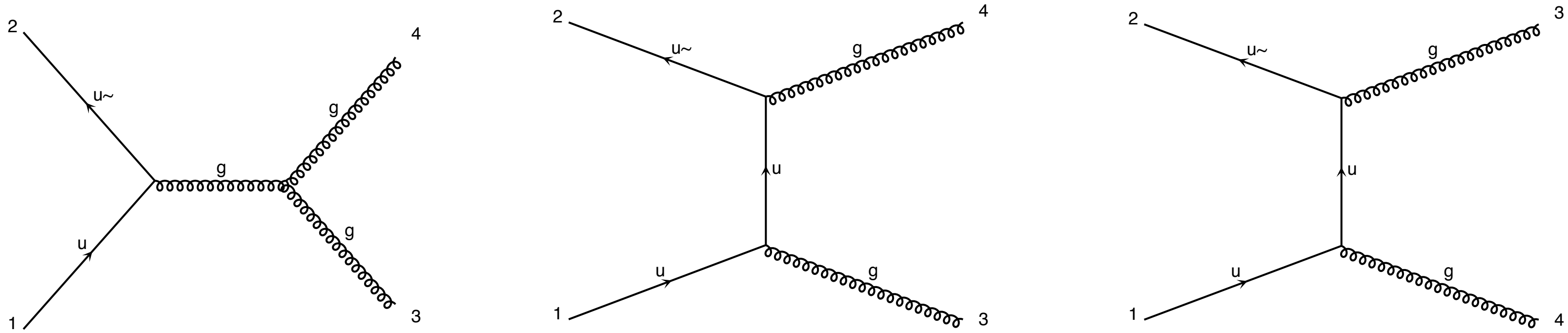

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Three very different pole structures contributing to the same matrix element.

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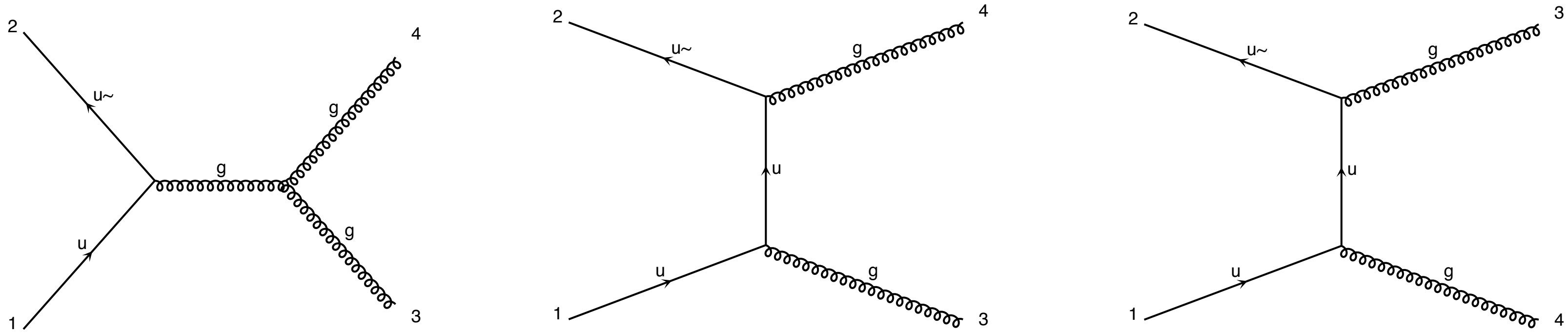

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ToolBox of yesterday: Multi-Channel

Multi-channel based on single diagrams*

*Method used in MadGraph

Trick in MadEvent: Split the complexity

$$\int |M_{tot}|^2 = \int \frac{\sum_i |M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2 = \sum_i \int \frac{|M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2$$

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$$\boxed{\frac{|M_{tot}|^2}{\sum_j |M_j|^2}} \approx 1$$

Key Idea

- Any single diagram is "easy" to integrate (pole structures/ suitable integration variables known from the propagators)
- Divide integration into pieces, based on diagrams
- All other peaks taken care of by denominator sum

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≈ 1

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N Integral

- Errors add in quadrature so no extra cost
- "Weight" functions already calculated during $|M|^2$ calculation
- Parallel in nature

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P1_qq_wpwm

s= 725.73 ± 2.07 (pb)

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G2.2	377.6	1.67	142.285	7941.0	21
G3	239	1.16	220.04	10856.0	45.5
G1	109.1	0.378	70.88	3793.0	34.8

P1_gg_wpwm

s= 20.714 ± 0.332 (pb)

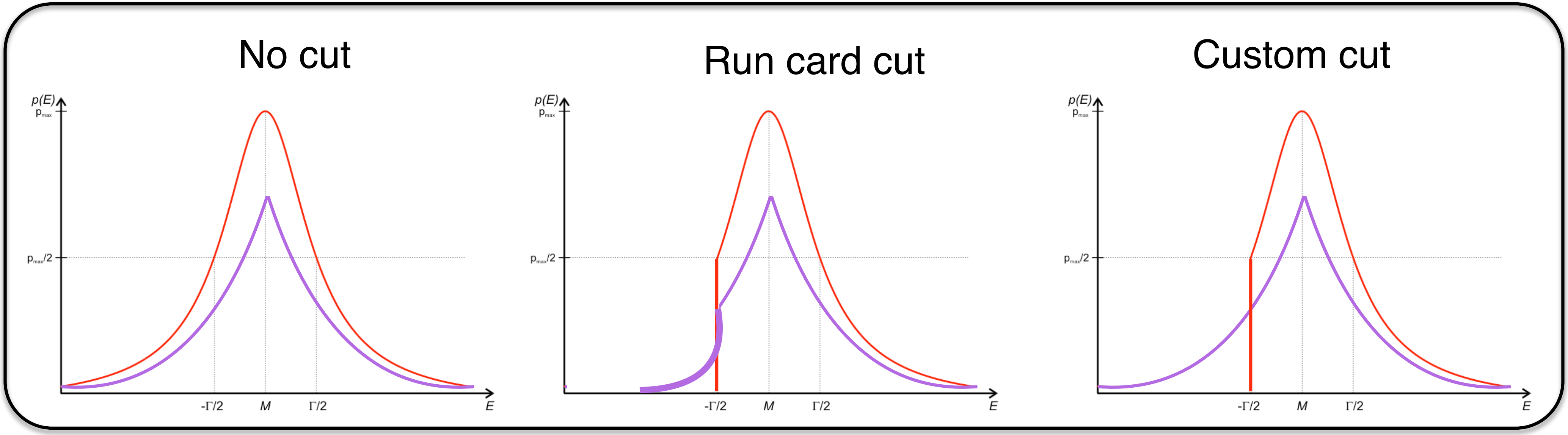
Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G1.2	20.71	0.332	7.01	373.0	18

term of the above sum.

each term might not be gauge invariant

Cut Impact

- Events are generated according to our best knowledge of the function
  - ➡Basic cut include in this "best knowledge"
  - ➡Custom cut are ignored



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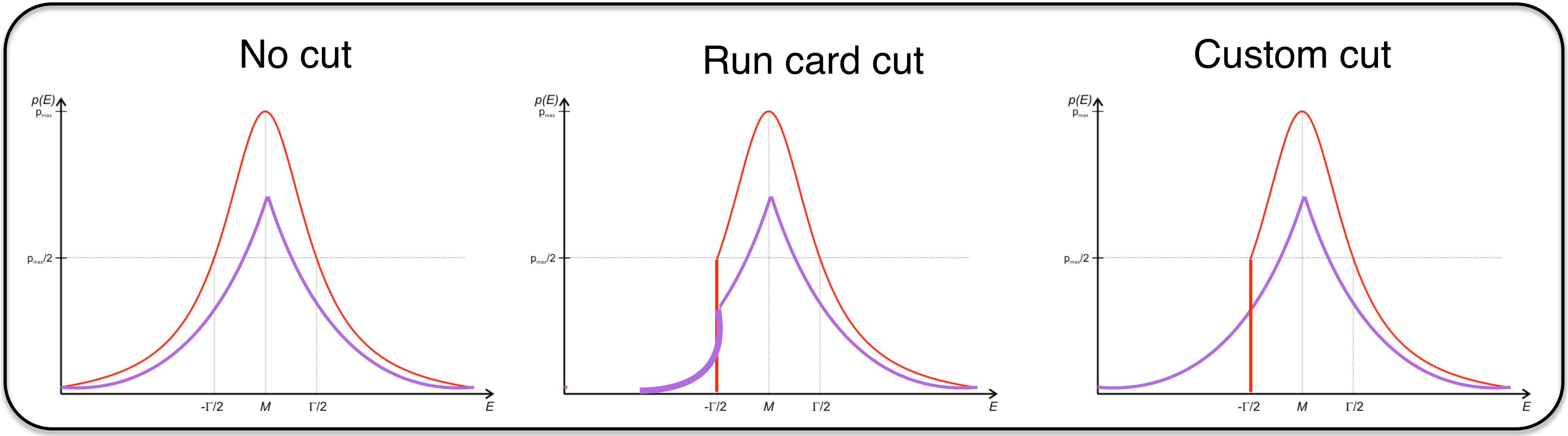

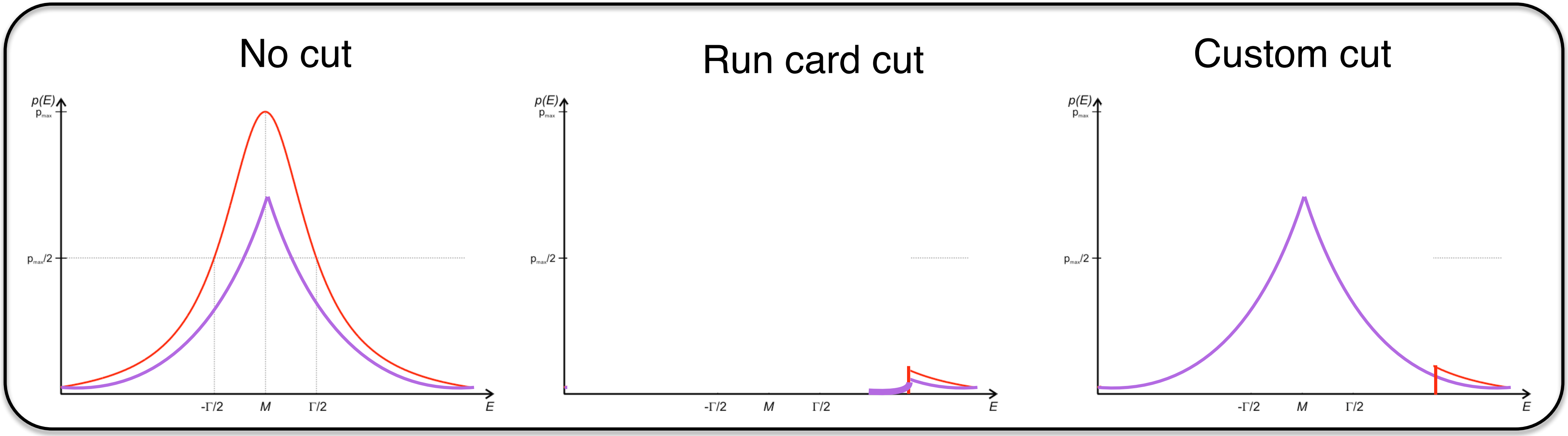

Might miss the contribution and think it is just zero.

Matching/Merging

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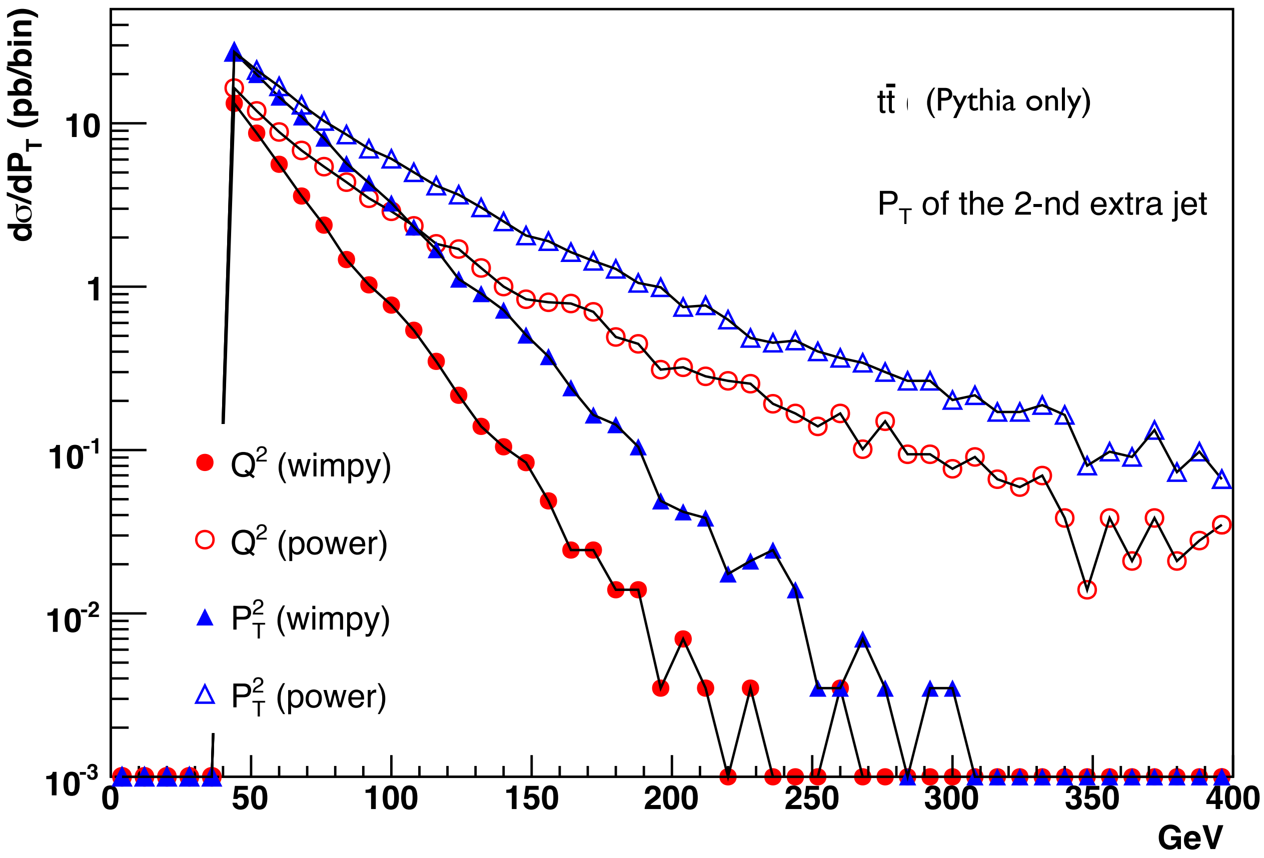
PS alone vs matched samples

Parton Shower are using collinear approximation how accurate does it works



Matrix Elements vs. Parton Showers

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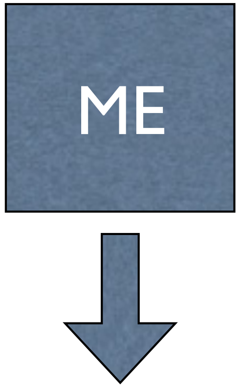

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2. Computationally expensive
3. Limited number of particles
4. Valid when partons are hard and well separated
5. Quantum interference correct
6. Needed for multi-jet description

Matrix Elements vs. Parton Showers

ME	Shower MC
1. Fixed order calculation	1. Resums logs to all orders
2. Computationally expensive	2. Computationally cheap
3. Limited number of particles	3. No limit on particle multiplicity
4. Valid when partons are hard and well separated	4. Valid when partons are collinear and/or soft
5. Quantum interference correct	5. Partial interference through angular ordering
6. Needed for multi-jet description	6. Needed for hadronization

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Approaches are complementary: merge them!

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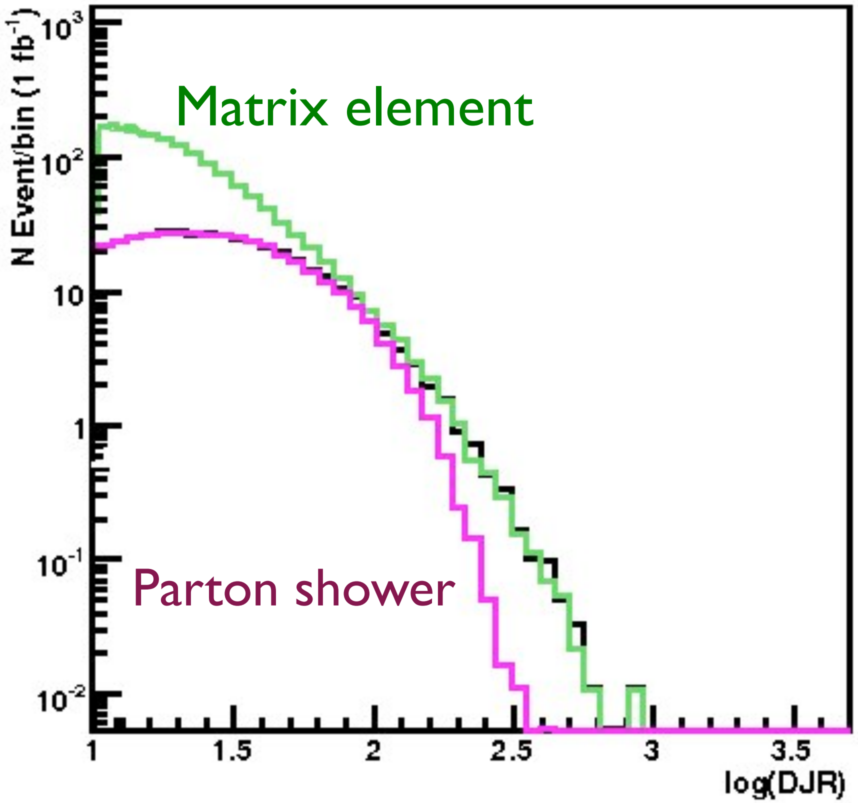
Approaches are complementary: merge them!

Difficulty: avoid double counting, ensure smooth distributions

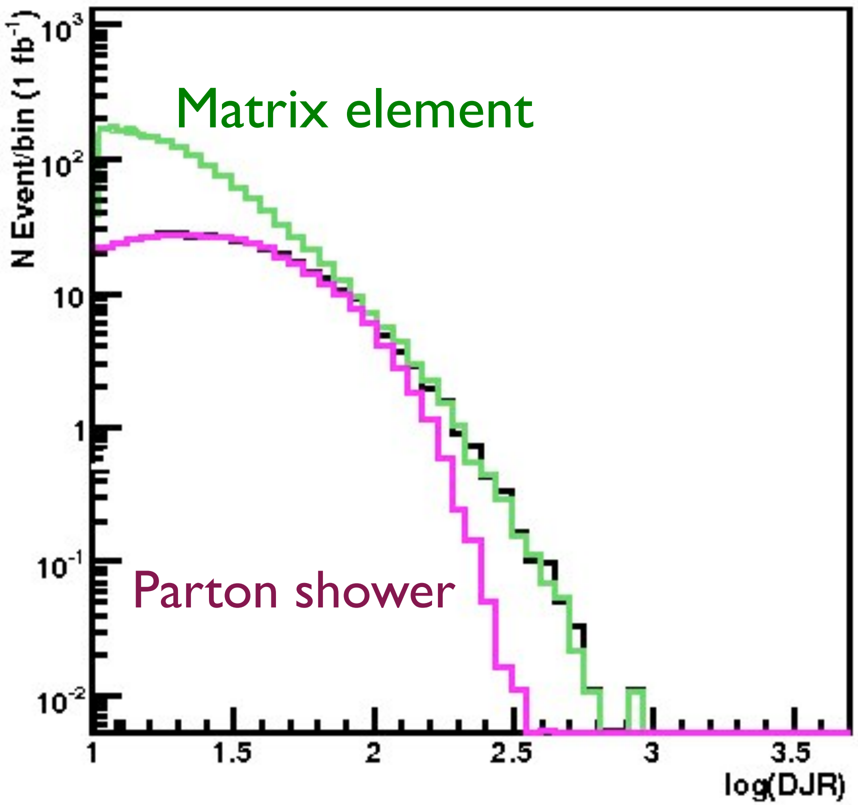
Goal for ME-PS merging/matching



2nd QCD radiation jet in top pair production at the LHC, using MadGraph + Pythia

Goal for ME-PS merging/matching

- Regularization of matrix element divergence



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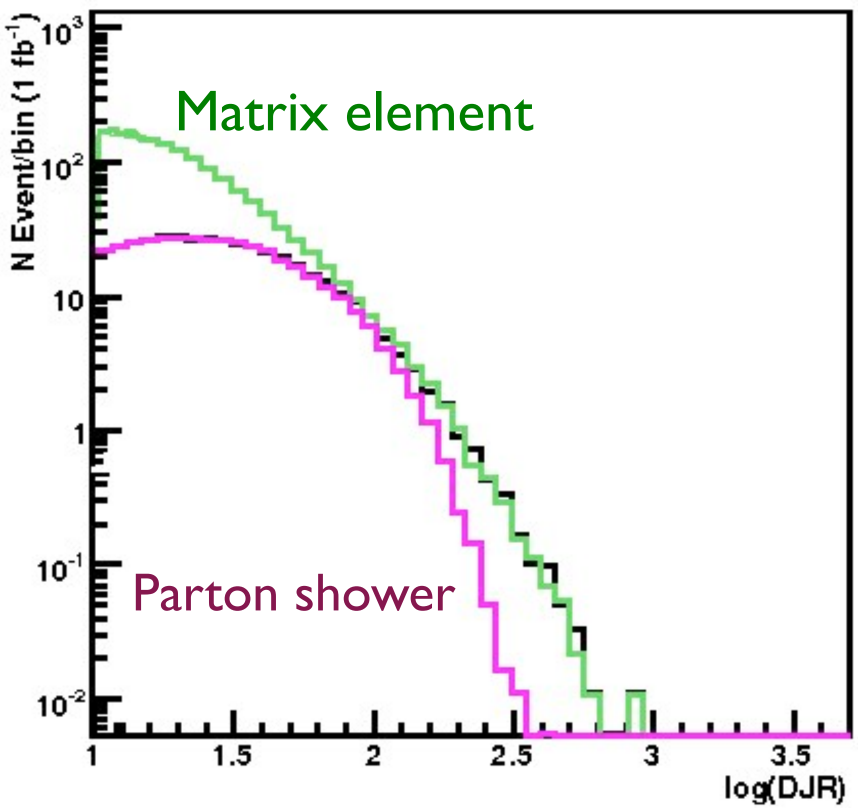

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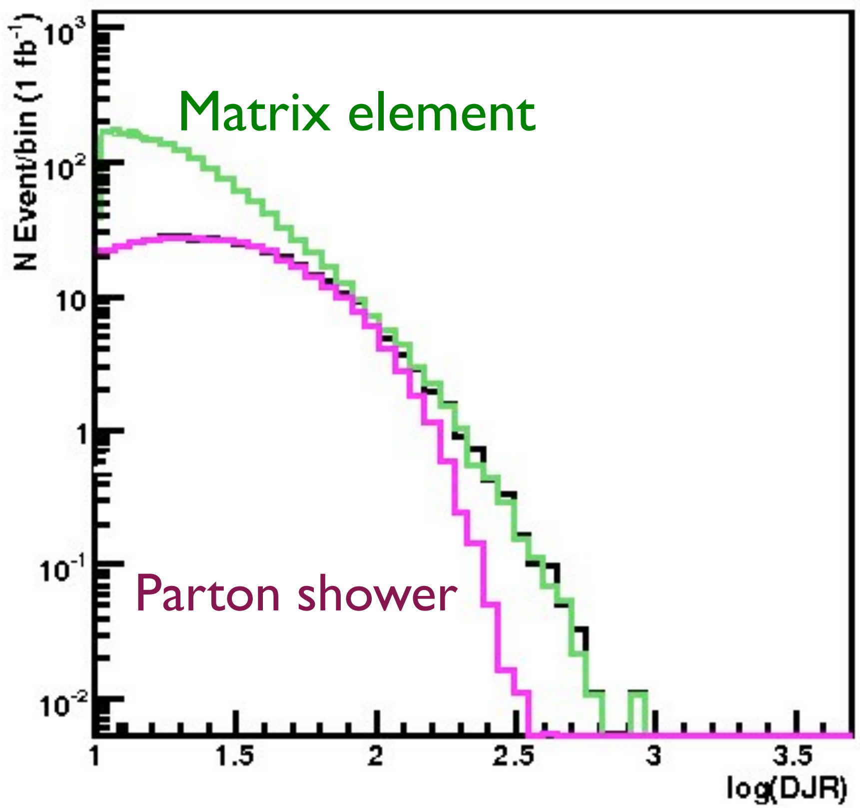

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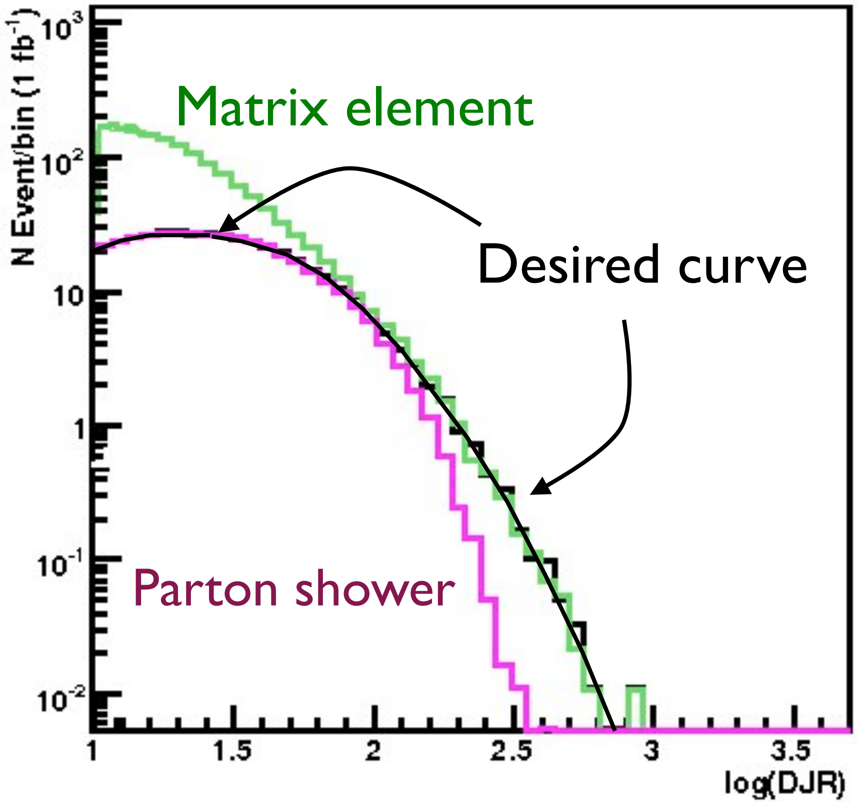

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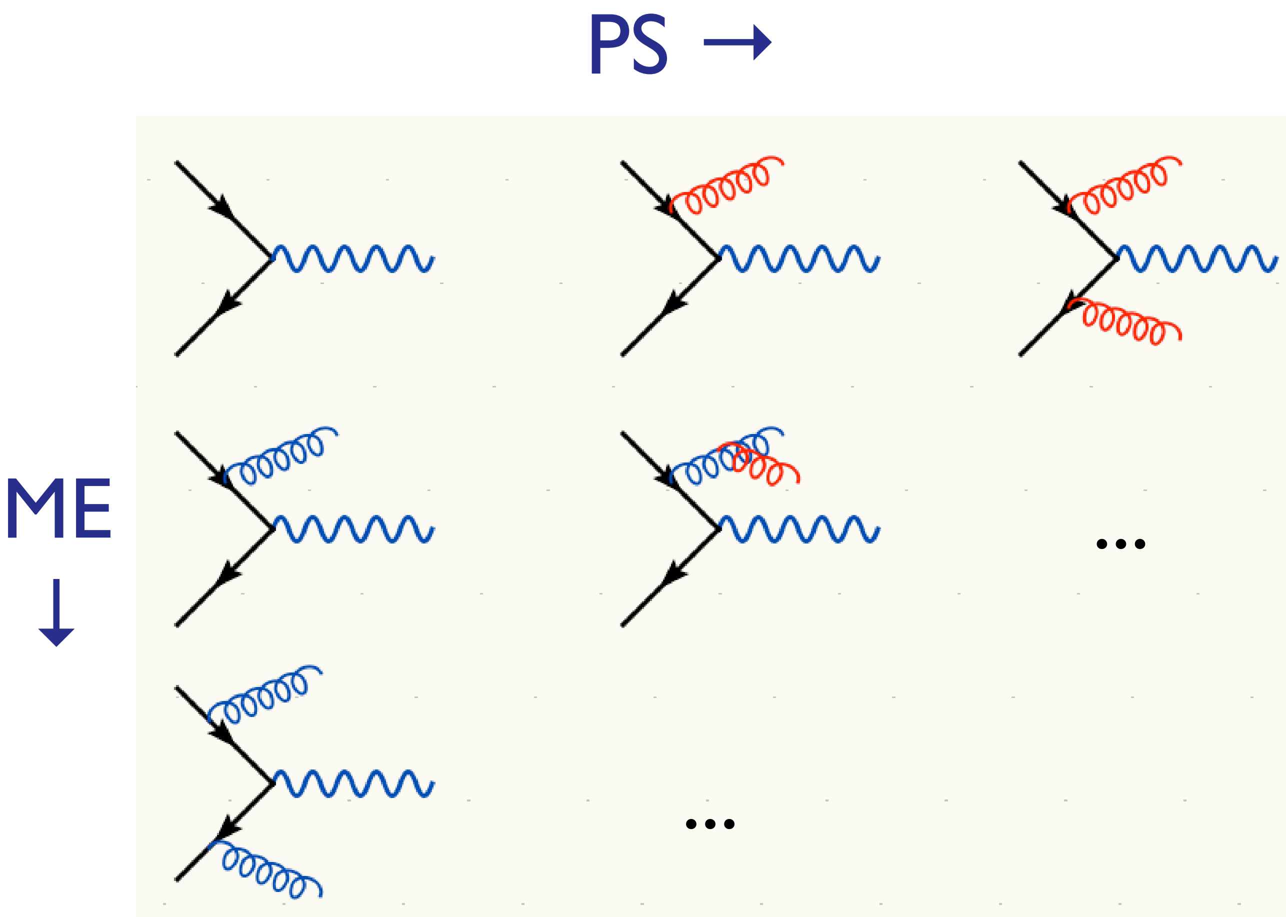
Merging ME with PS

[Mangano]
[Catani, Krauss, Kuhn, Webber]
[Lönnblad]



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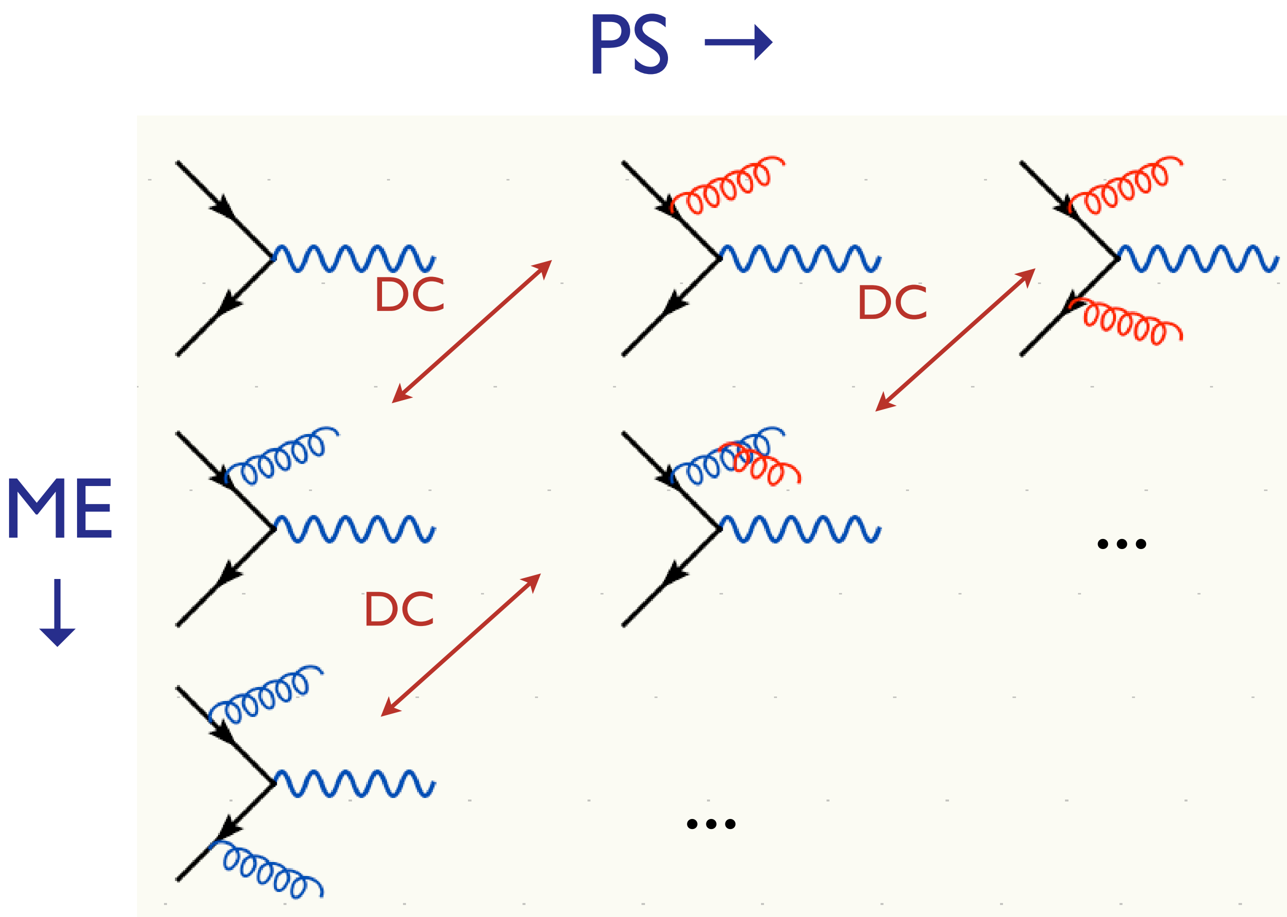

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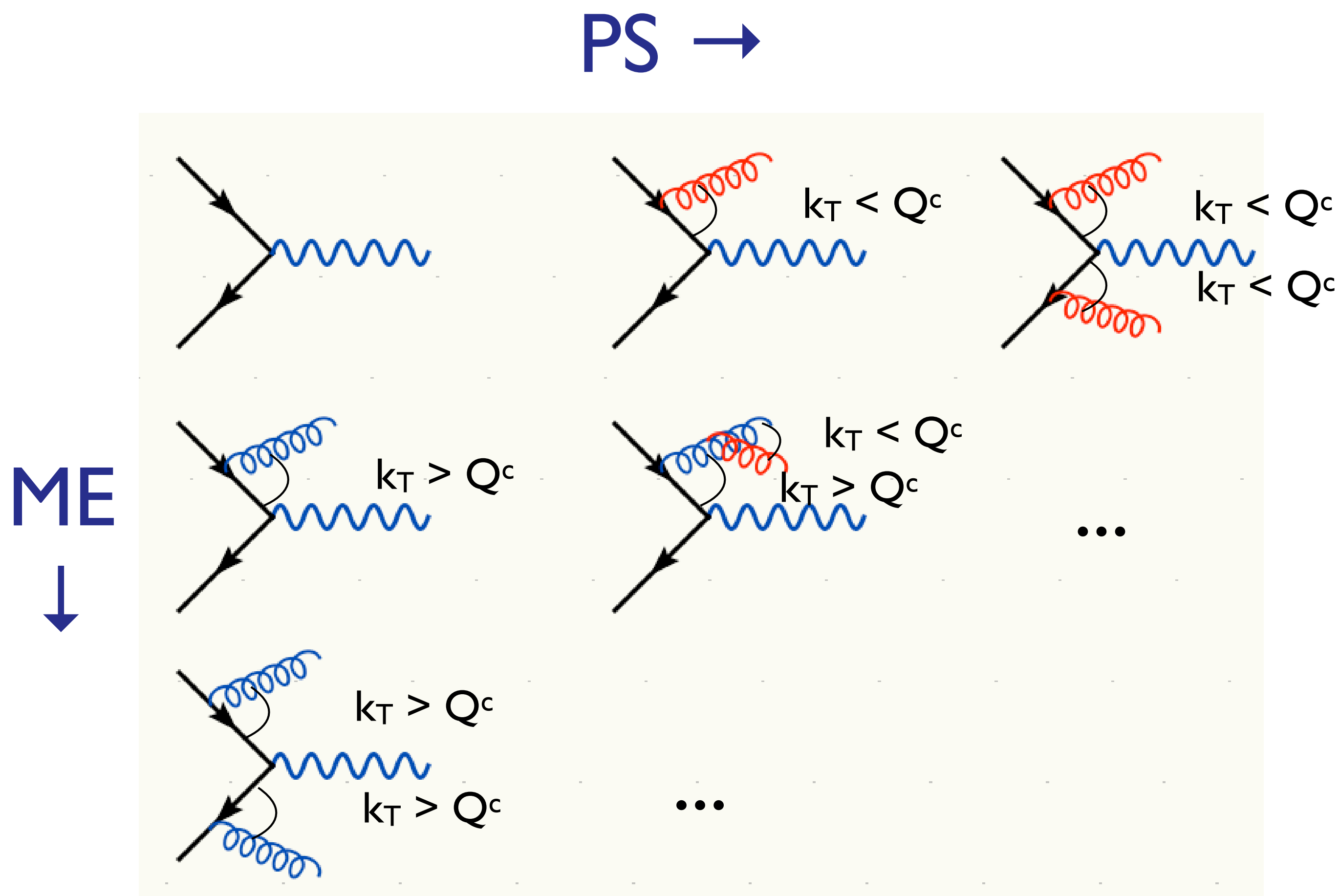

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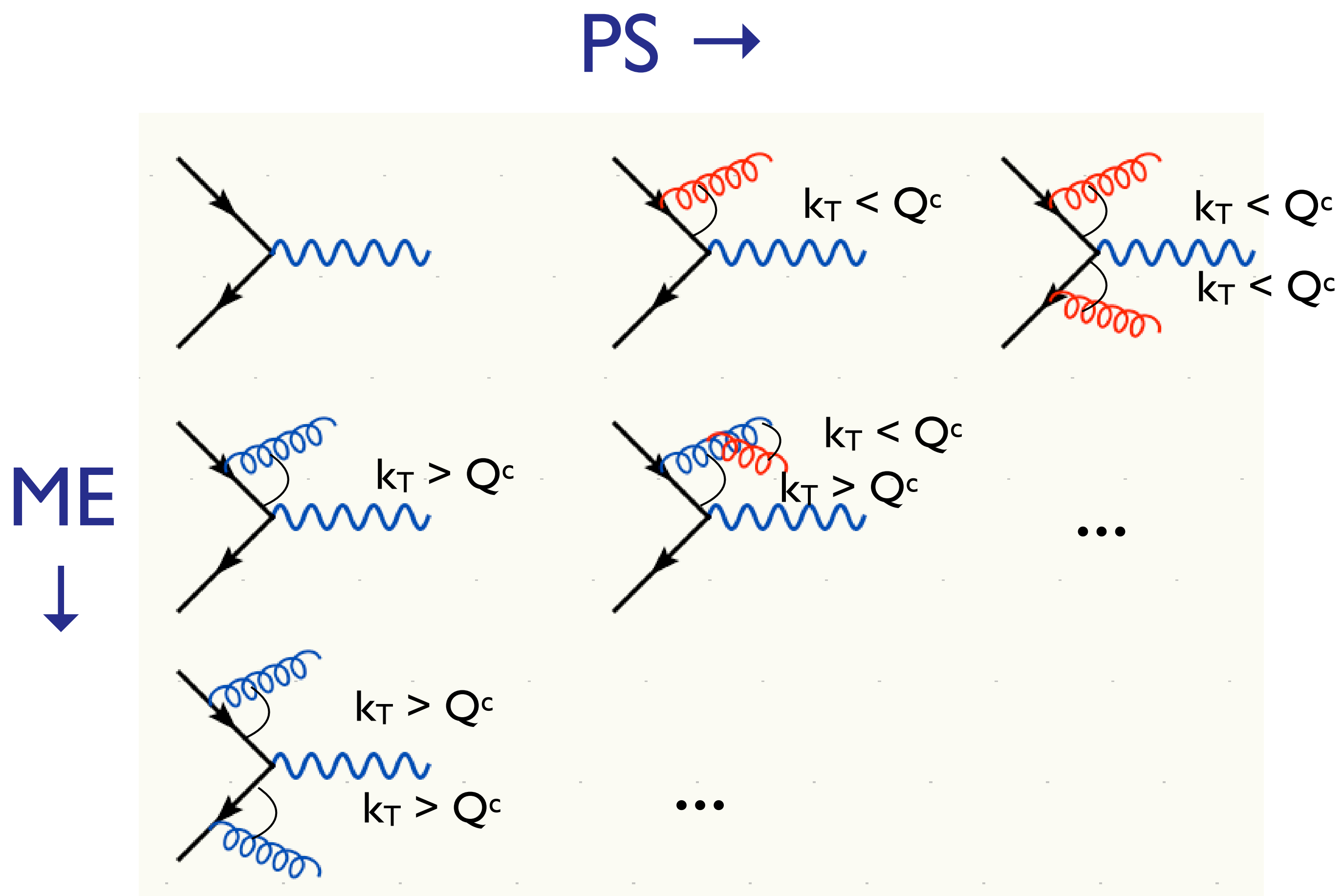
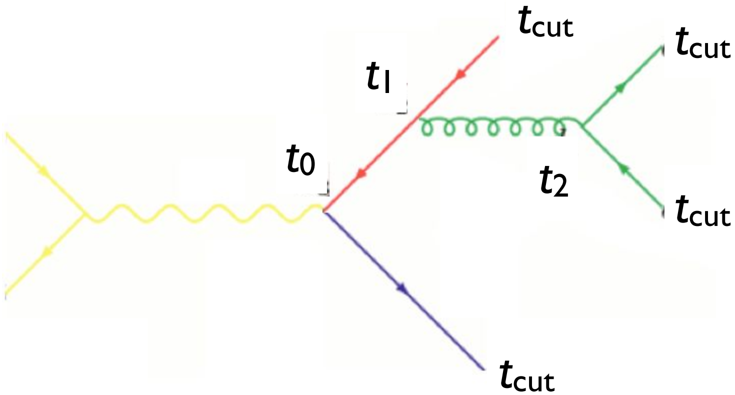


Double counting between ME and PS easily avoided using phase space cut between the two: PS below cutoff, ME above cutoff.

Merging ME with PS

- So double counting problem easily solved, but what about getting smooth distributions that are independent of the precise value of Q^c?
- Below cutoff, distribution is given by PS
  - need to make ME look like PS near cutoff
- Let's take another look at the PS!

Merging ME with PS



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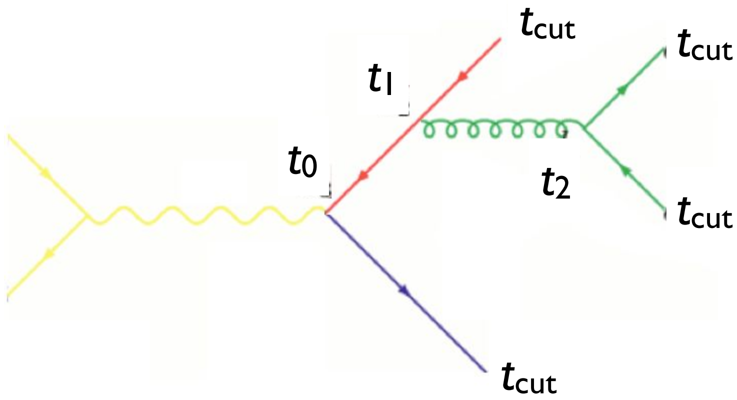
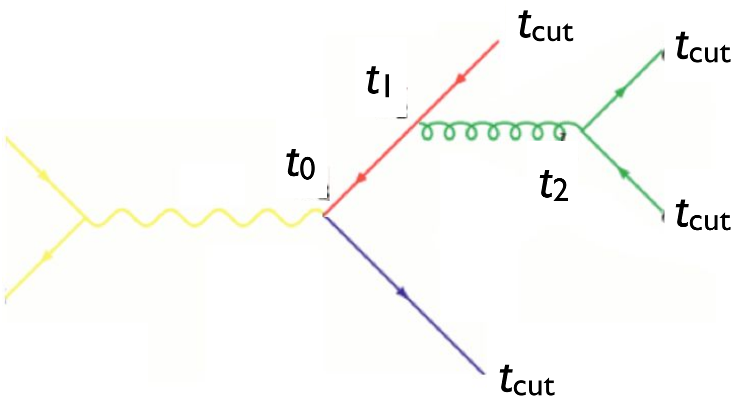


- How does the PS generate the configuration above?

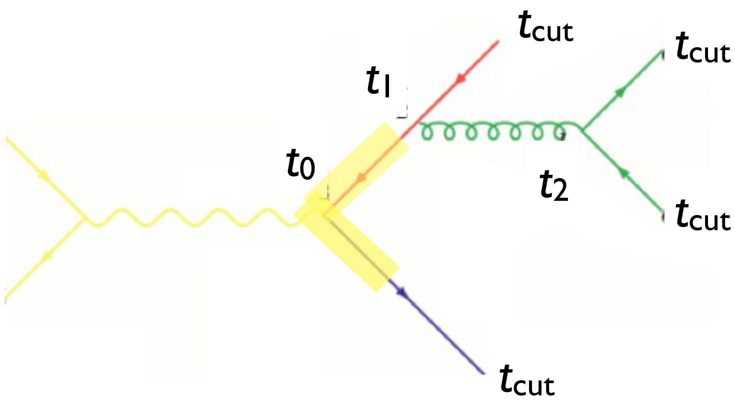
Merging ME with PS



- How does the PS generate the configuration above?
- Probability for the splitting at t_1 is given by

$$(\Delta_q(t_1, t_0))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z)$$

Merging ME with PS



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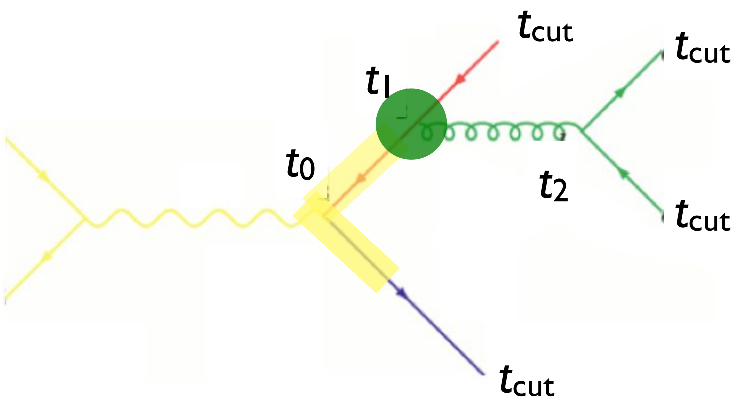

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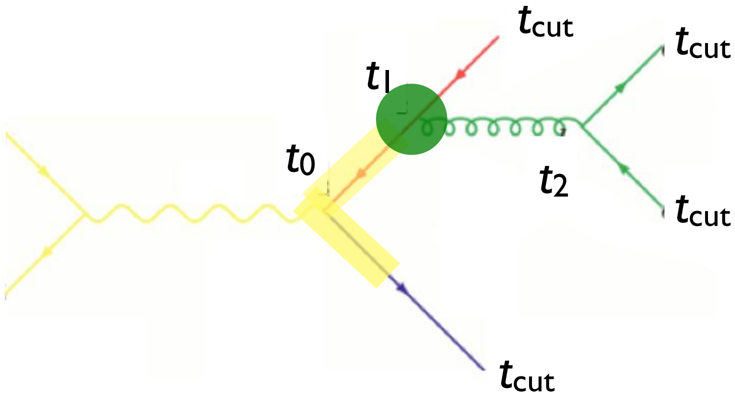

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and for the whole tree

$$(\Delta_q(t_{cut}, t_0))^2 \Delta_g(t_2, t_1) (\Delta_q(t_{cut}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$

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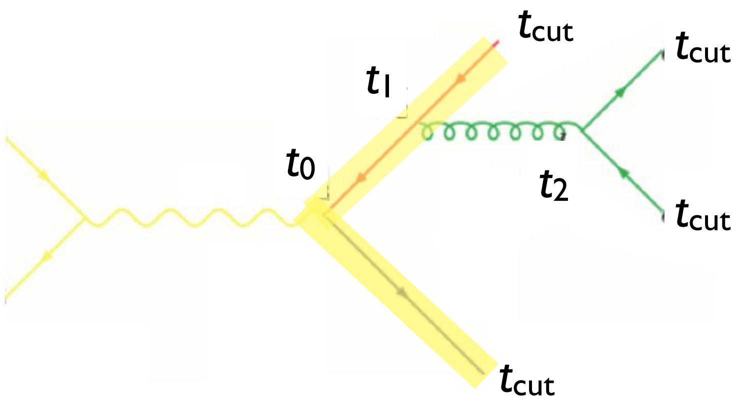

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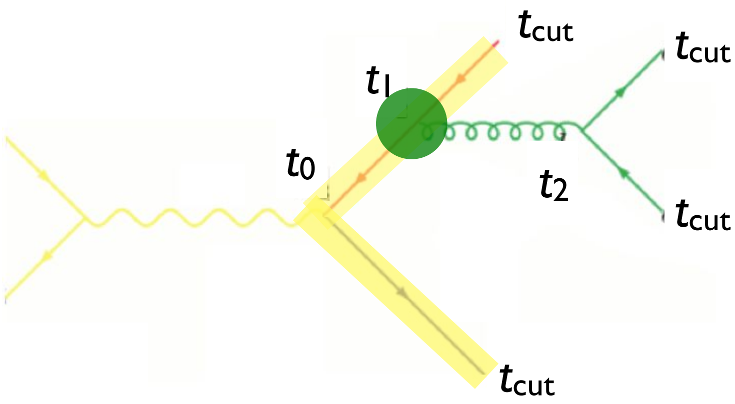

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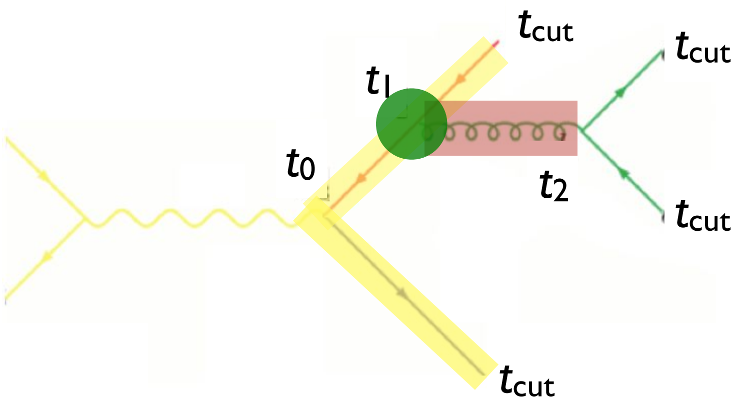

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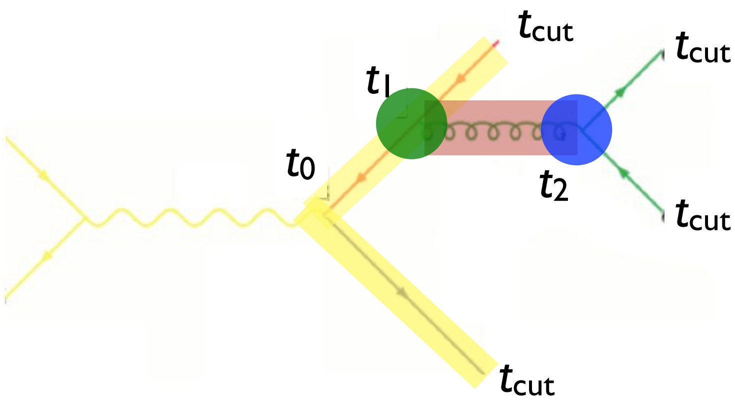

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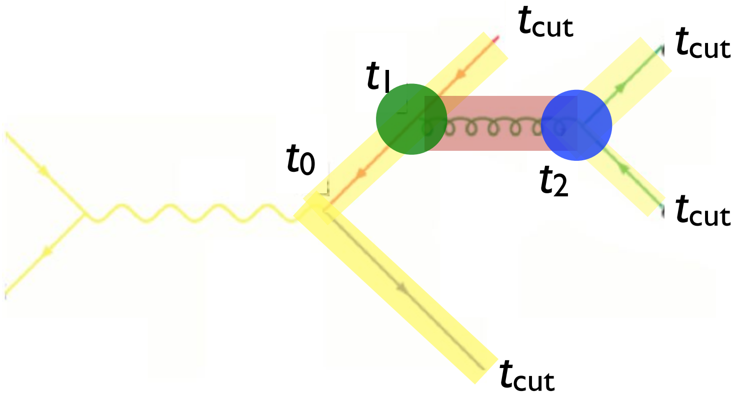

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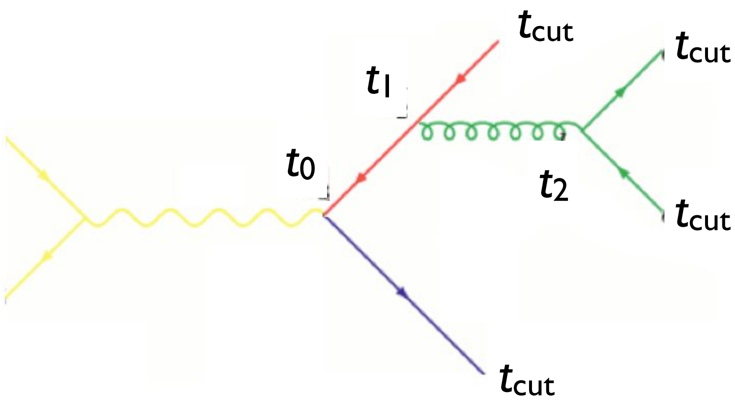

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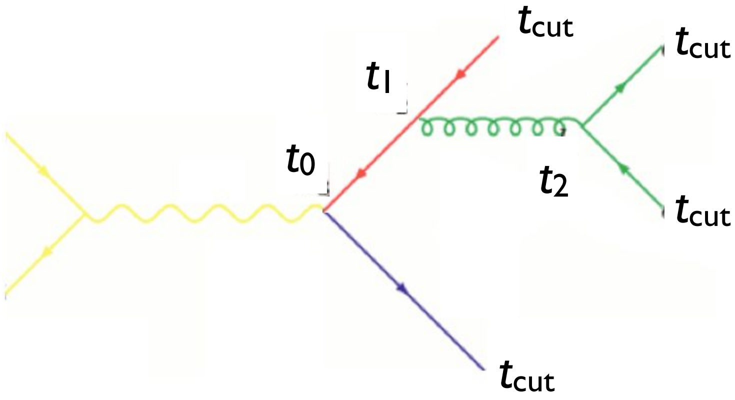

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Corresponds to the matrix element
BUT with α_s evaluated at the scale of each splitting

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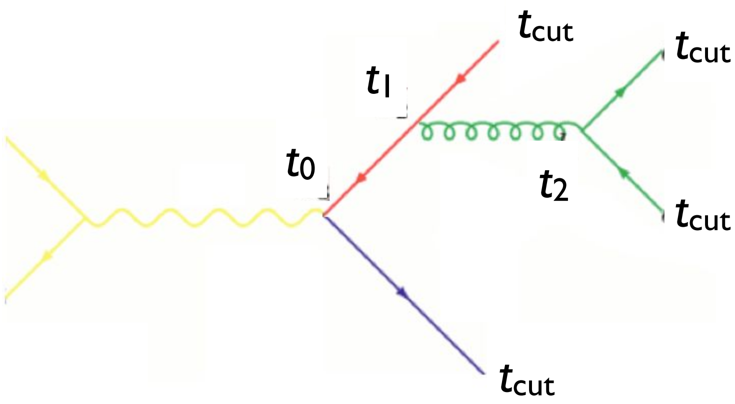

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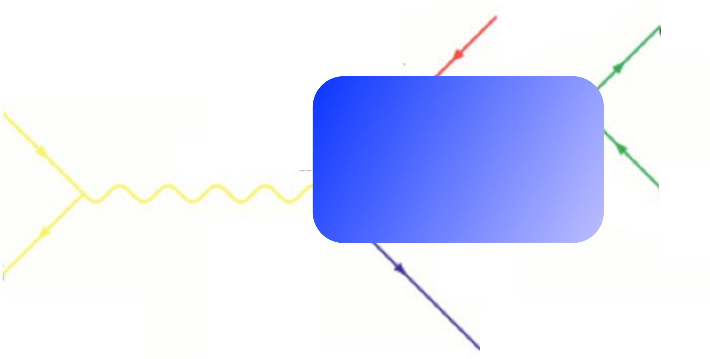
Sudakov suppression due to not allowing additional radiation above the scale $t_{\rm cut}$

Merging ME with PS



$$|\mathcal{M}|^2(\hat{s}, p_3, p_4, ...)$$

t

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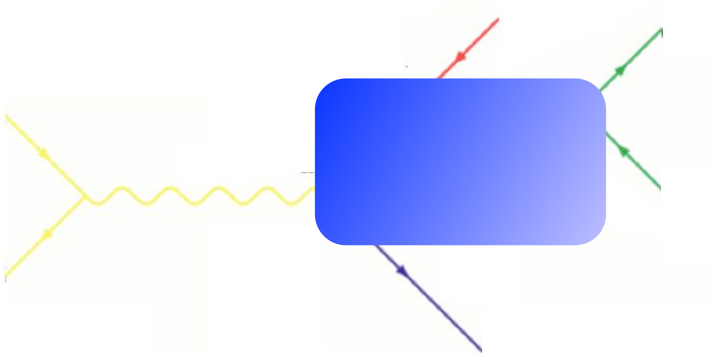

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- To get an equivalent treatment of the corresponding matrix element, do as follows:

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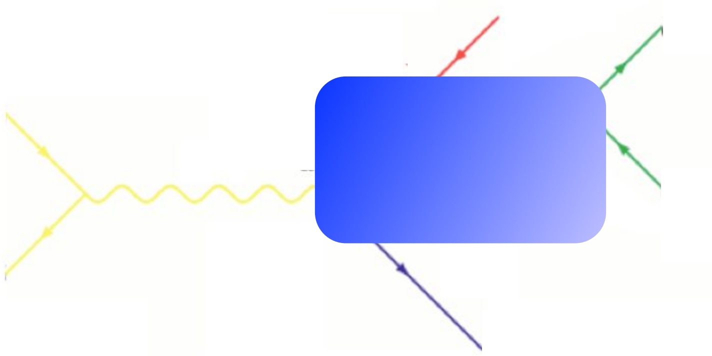

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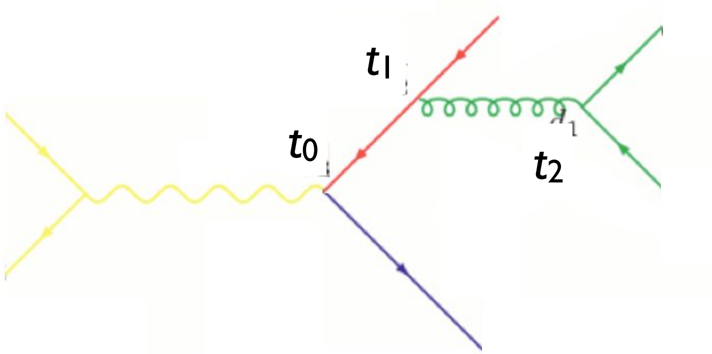

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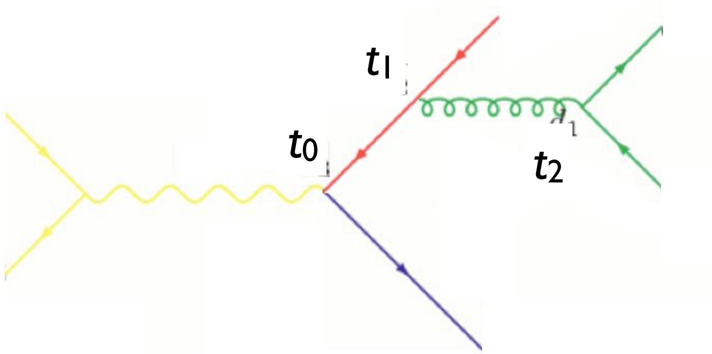

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1. Cluster the event using some clustering algorithm
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2. Reweight α_s in each clustering vertex with the clustering scale

$$|\mathcal{M}|^2 \to |\mathcal{M}|^2 \frac{\alpha_s(t_1)}{\alpha_s(t_0)} \frac{\alpha_s(t_2)}{\alpha_s(t_0)}$$

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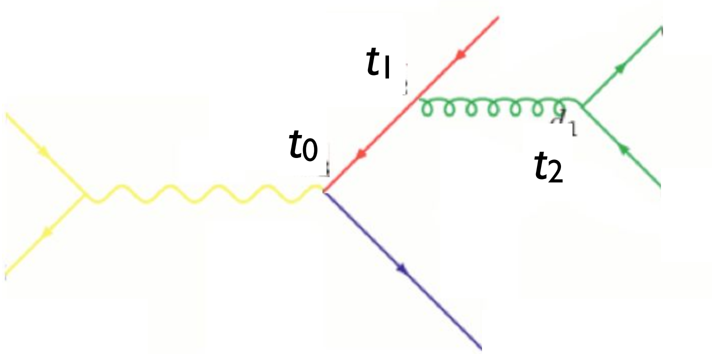

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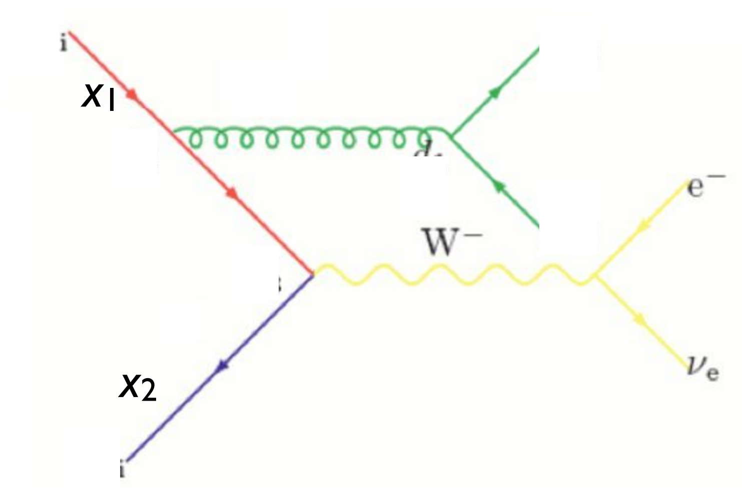
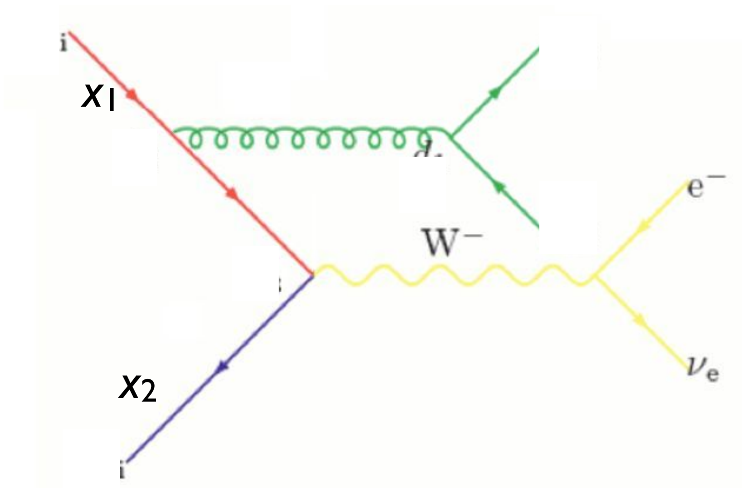
3. Use some algorithm to apply the equivalent Sudakov suppression $(\Delta_q(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\rm cut}, t_2))^2$

Matching for initial state radiation



Matching for initial state radiation

- We are of course not interested in e^+e^- but p-p(bar)
  - what happens for initial state radiation?



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- Let's do the same exercise as before:

$$\mathcal{P} = (\Delta_{Iq}(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\rm cut}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi}\frac{P_{gq}(z)}{z}\frac{f_q(x_1, t_1)}{f_q(x_1', t_1)}\frac{\alpha_s(t_2)}{2\pi}P_{qg}(z')$$
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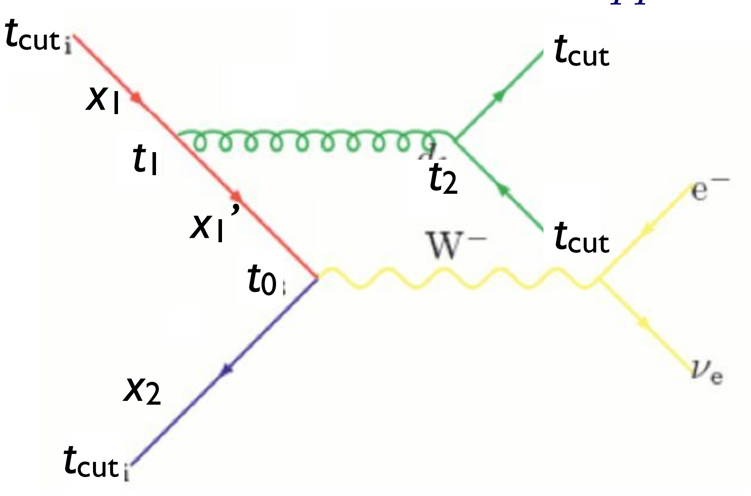

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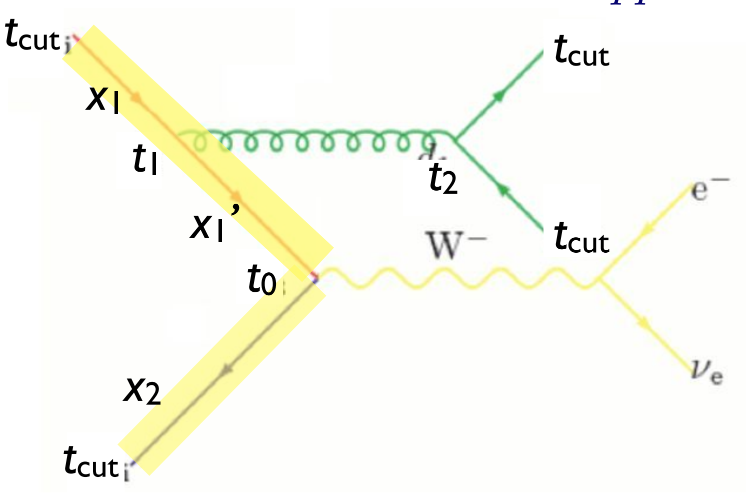

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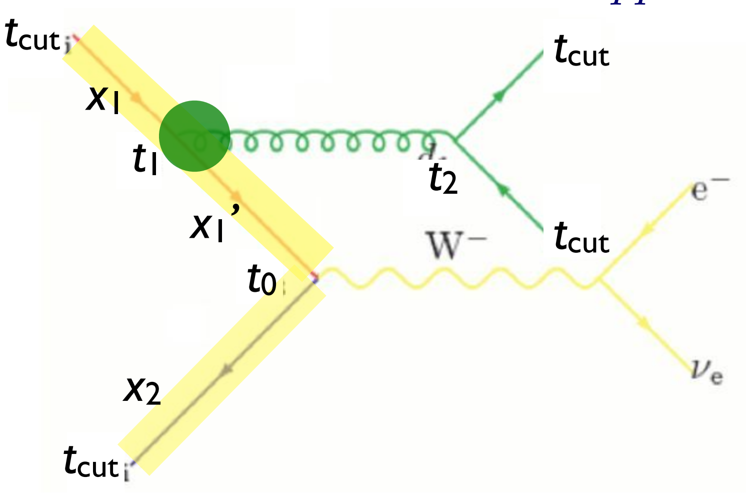

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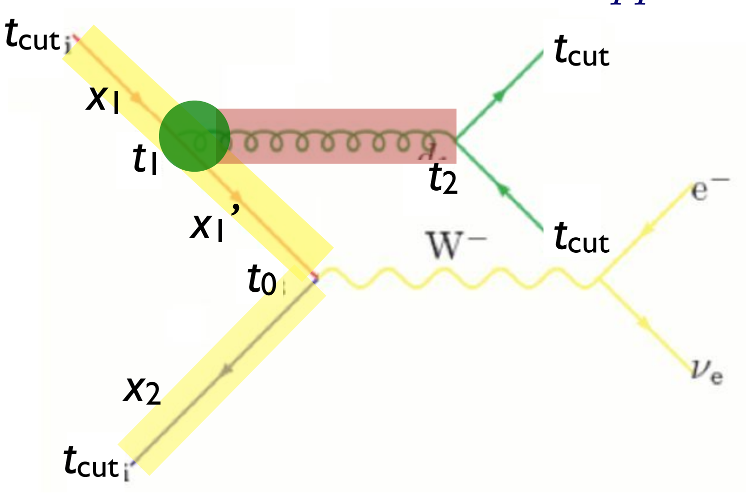

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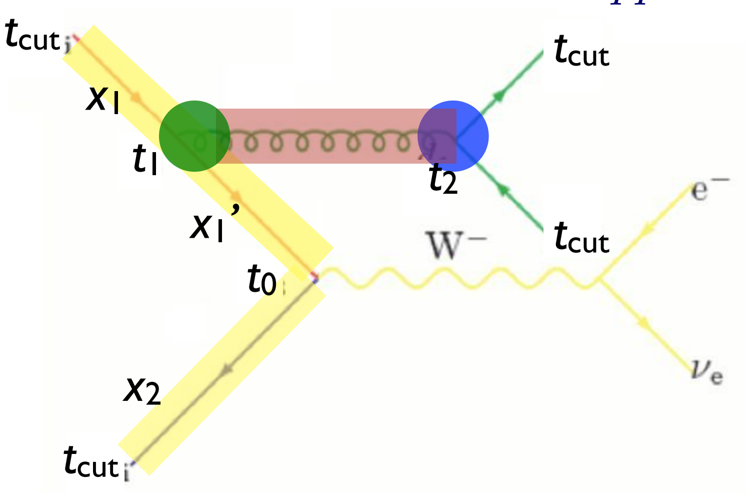

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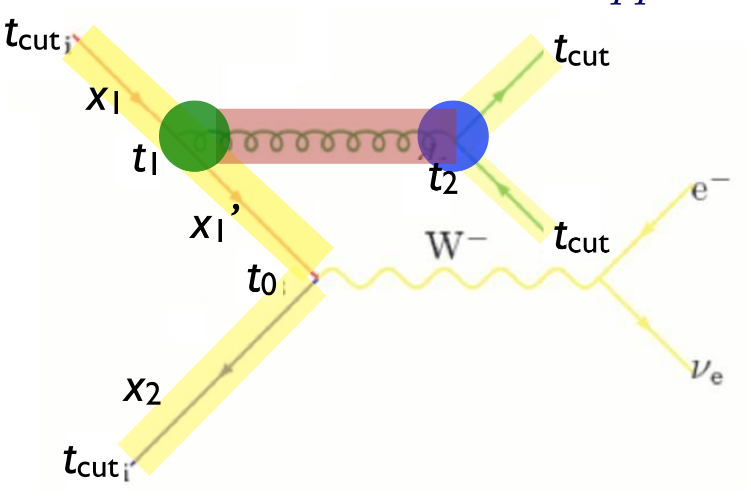

Matching for initial state radiation

$$(\Delta_{Iq}(t_{\rm cut},t_0))^2\Delta_g(t_2,t_1)(\Delta_q(t_{\rm cut},t_2))^2\frac{\alpha_s(t_1)}{2\pi}\frac{P_{gq}(z)}{z}\frac{f_q(x_1,t_1)}{f_q(x_1',t_1)}\frac{\alpha_s(t_2)}{2\pi}P_{qg}(z')$$
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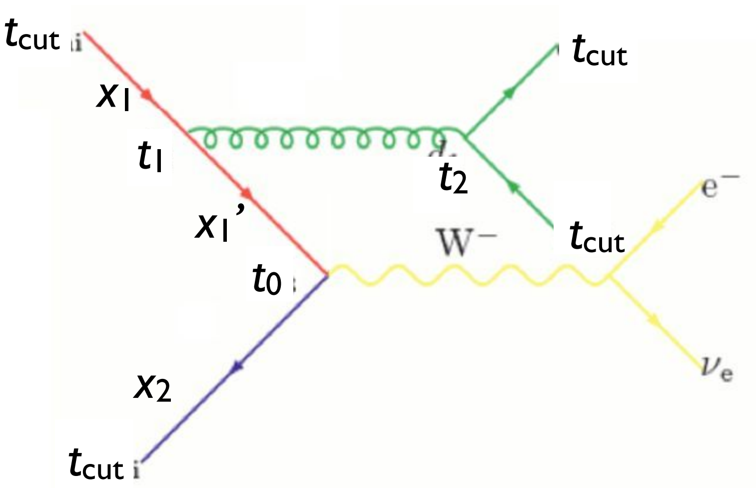

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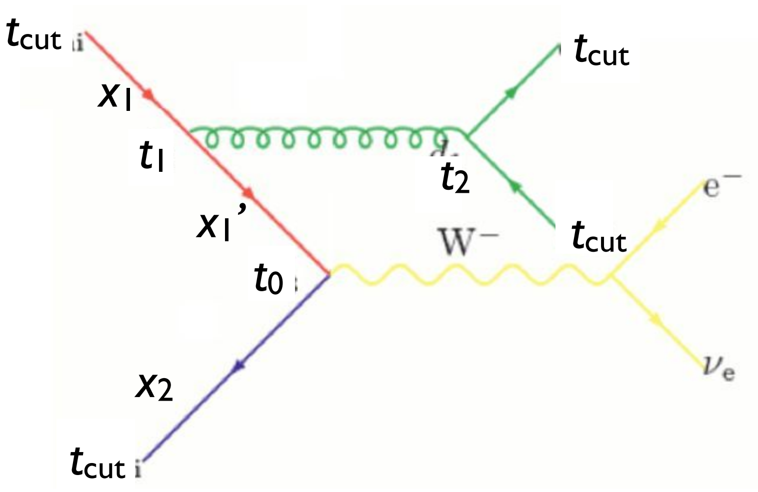
ME (convoluted with PDF) with α_s evaluated at the scale of each splitting



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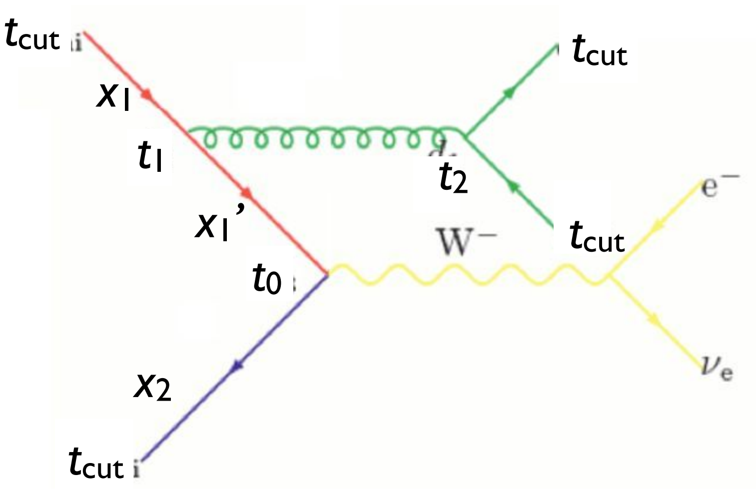
PDF reweighting



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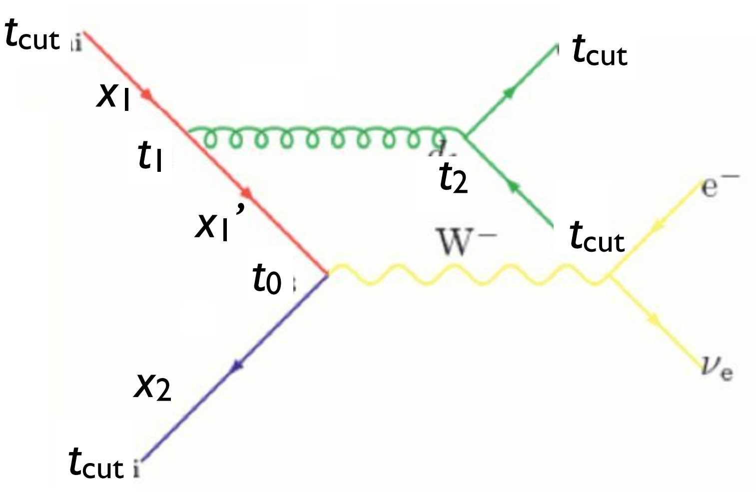
PDF reweighting

Sudakov suppression due to non-branching above scale $t_{\rm cut}$



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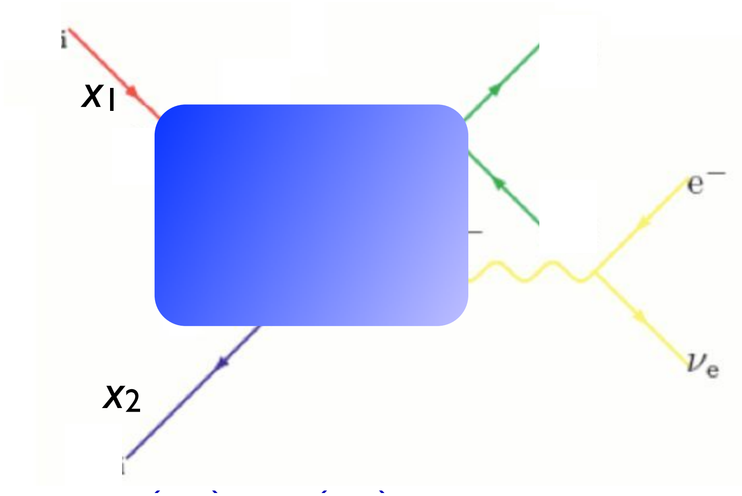
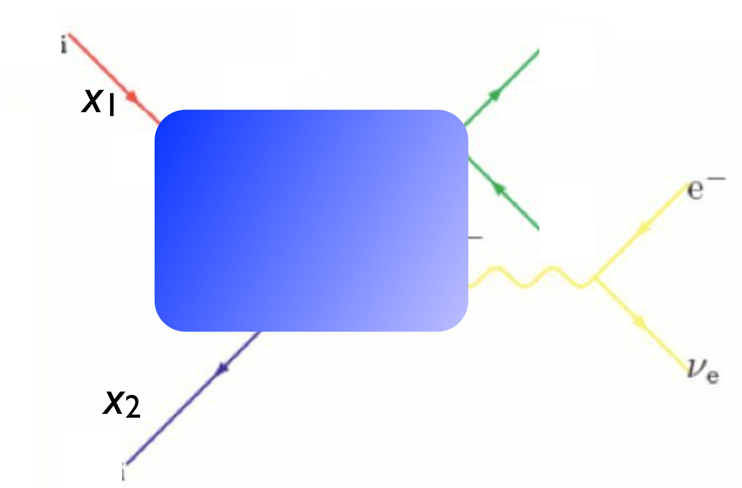


Matching for initial state radiation

- Again, use a clustering scheme to get a parton shower history



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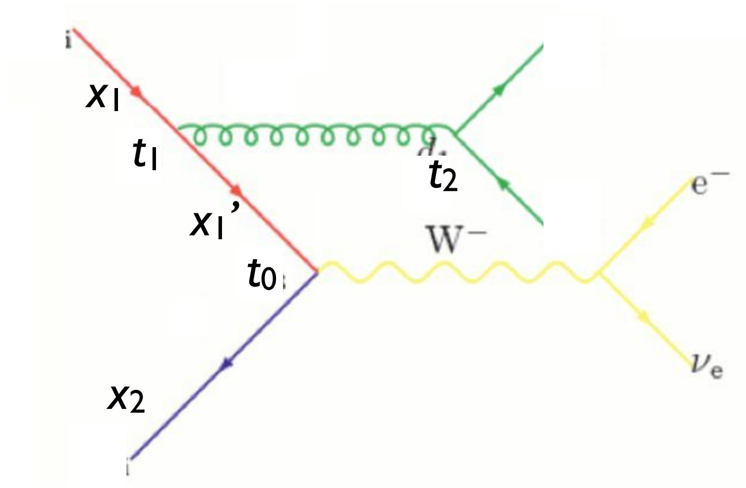
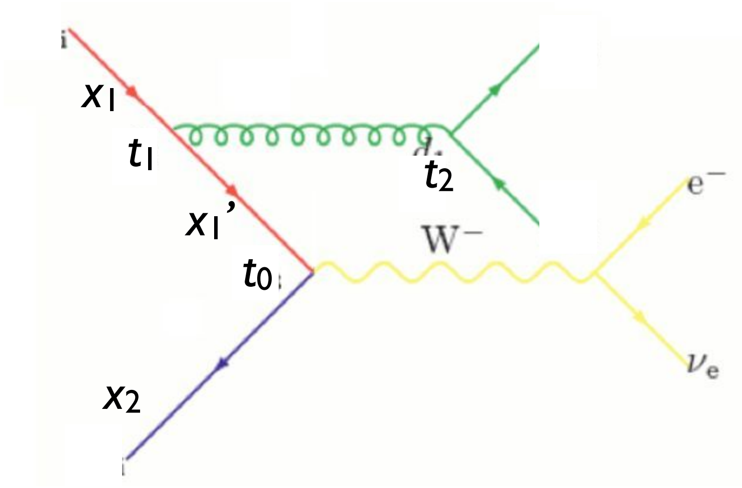


Matching for initial state radiation

- Again, use a clustering scheme to get a parton shower history
- Now, reweight both due to α_s and PDF

$$|\mathcal{M}|^2 \to |\mathcal{M}|^2 \frac{\alpha_s(t_1)}{\alpha_s(t_0)} \frac{\alpha_s(t_2)}{\alpha_s(t_0)} \frac{f_q(x_1', t_0)}{f_q(x_1', t_1)}$$



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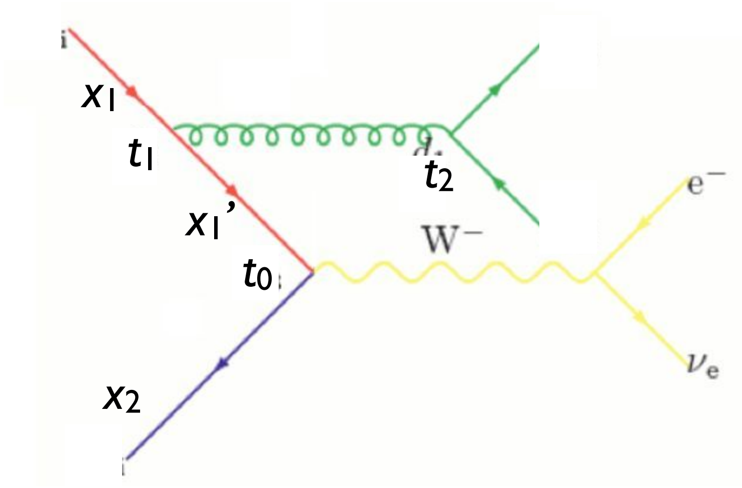
- Use some algorithm to apply the equivalent Sudakov suppression



Matching schemes

- We still haven't specified how to apply the Sudakov reweighting to the matrix element
- Three general schemes available in the literature:
    - ➡ CKKW scheme [Catani,Krauss,Kuhn,Webber 2001; Krauss 2002]
    - ➡ Lönnblad scheme (or CKKW-L) [Lönnblad 2002]
    - ➡ MLM scheme [Mangano *unpublished* 2002; Mangano et al. 2007]

CKKW matching

[Catani, Krauss, Kuhn, Webber 2001]
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CKKW matching

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- Apply the required Sudakov suppression

$$(\Delta_{Iq}(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\rm cut}, t_2))^2$$

analytically, using the best available (NLL) Sudakovs.

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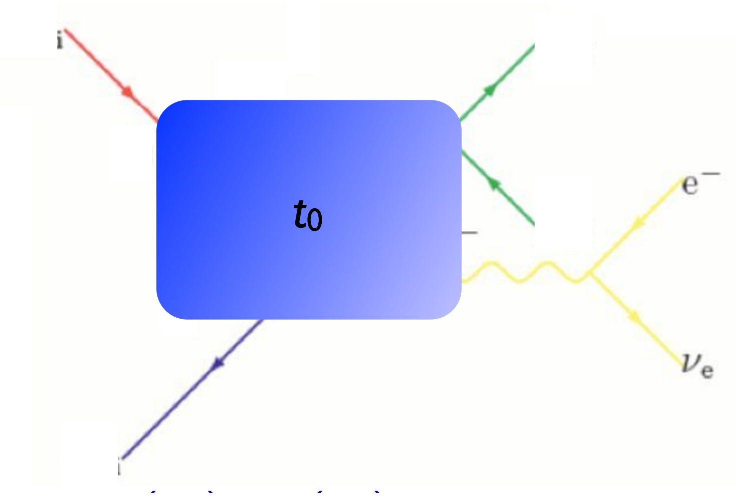
- Perform "truncated showering": Run the parton shower starting at t_0, but forbid any showers above the cutoff scale $t_{\rm cut}$.



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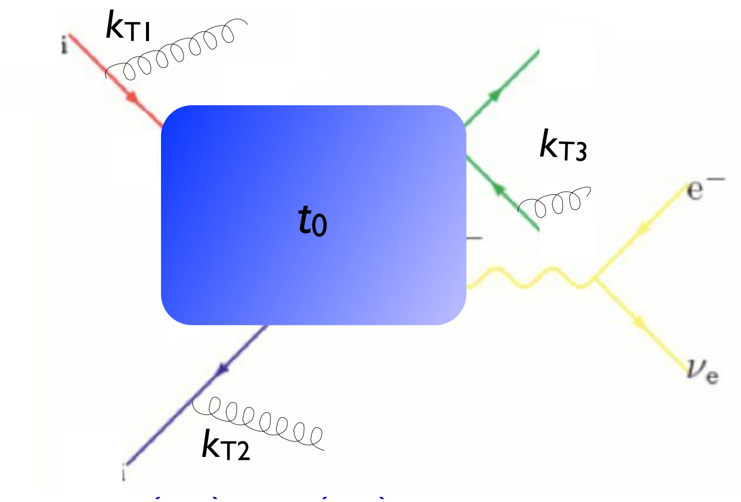

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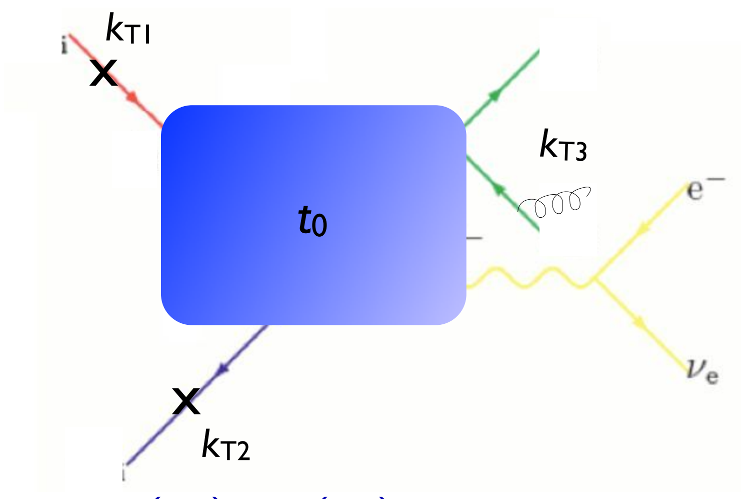

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- Apply the required Sudakov suppression

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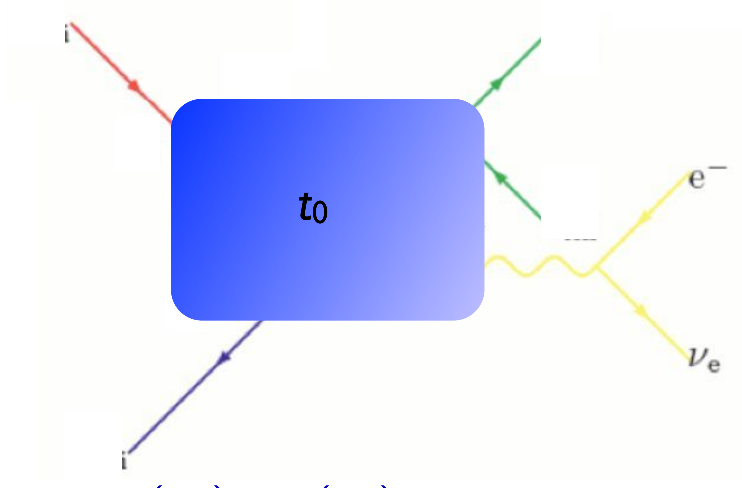
  analytically, using the best available (NLL) Sudakovs.
- Perform "truncated showering": Run the parton shower starting at t_0, but forbid any showers above the cutoff scale t_{cut}.

  ✓ Best theoretical treatment of matrix element
  - Requires dedicated PS implementation
  - Mismatch between analytical Sudakov and (non-NLL) shower
  - Implemented in Sherpa (v. 1.1)

CKKW-L matching

[Lönnblad 2002]
[Hoeche et al. 2009]



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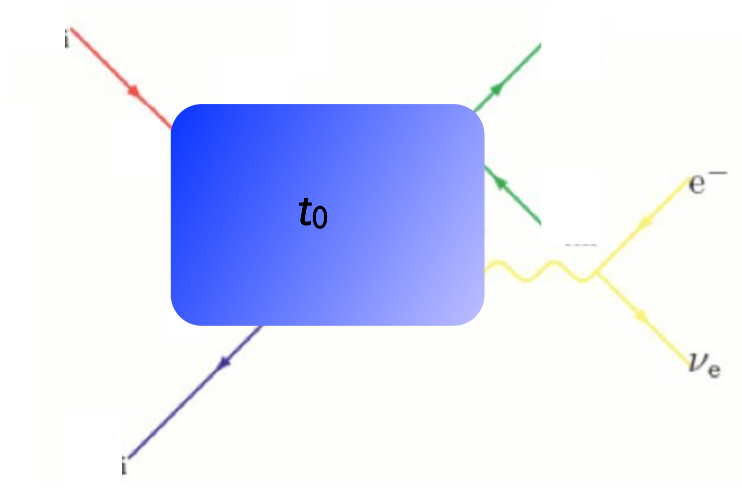

- Cluster back to "parton shower history"

CKKW-L matching

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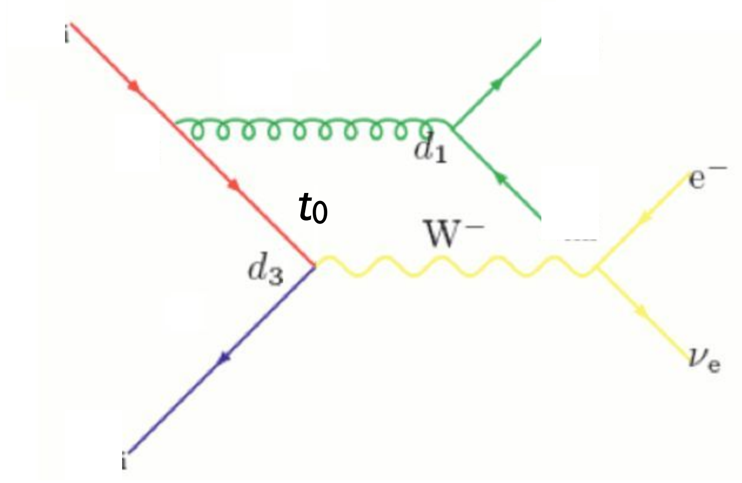

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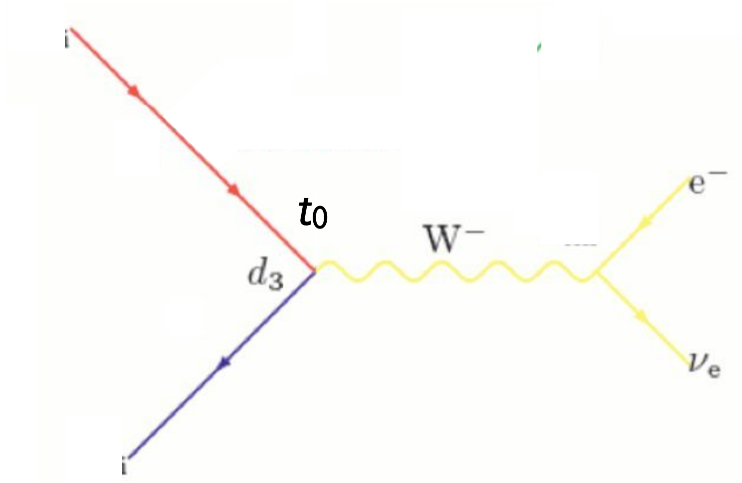

- Cluster back to "parton shower history"
- Perform showering step-by-step for each step in the parton shower history, starting from the clustering scale for that step

CKKW-L matching

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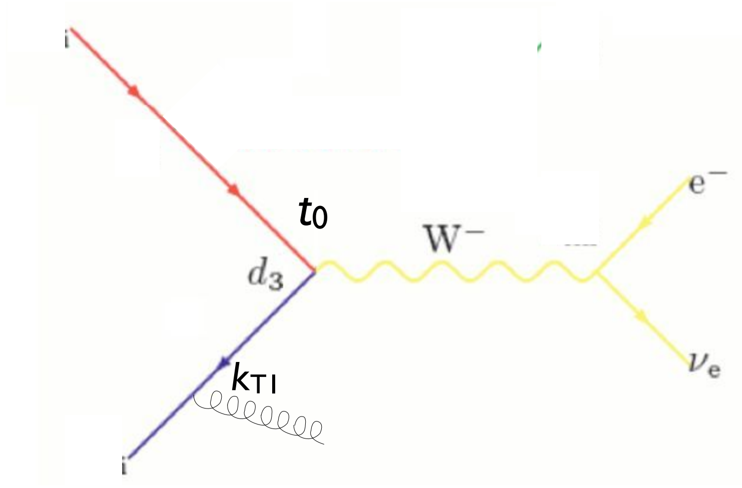

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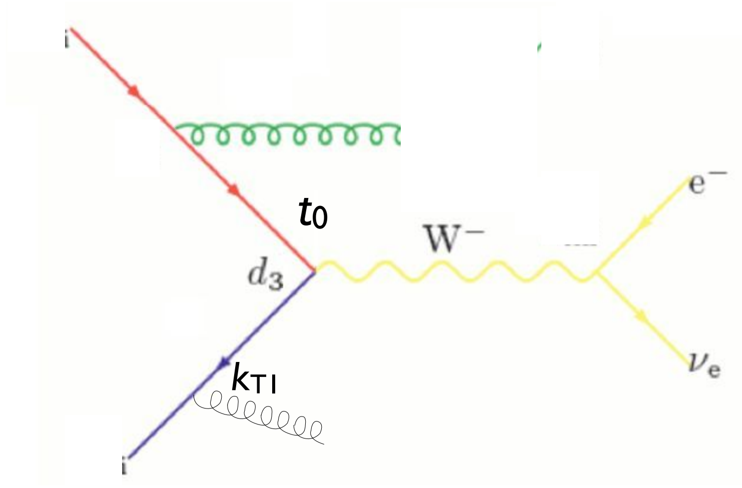

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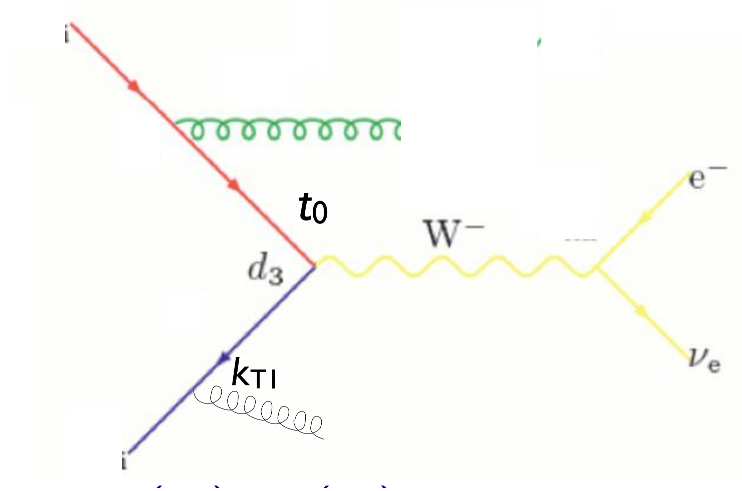

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CKKW-L matching

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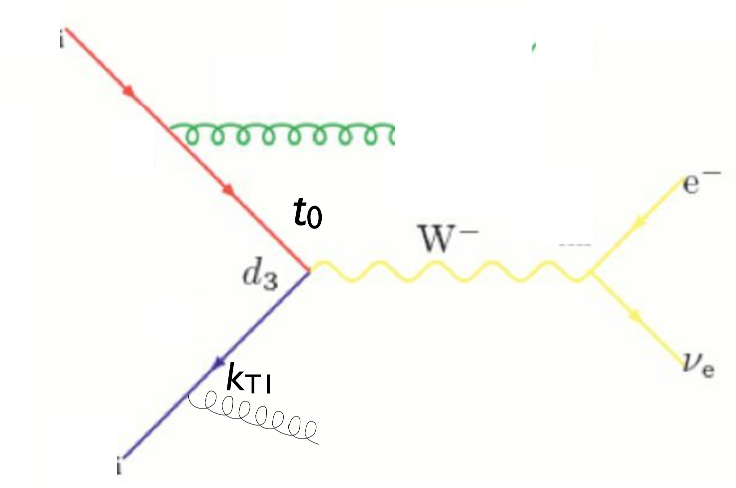

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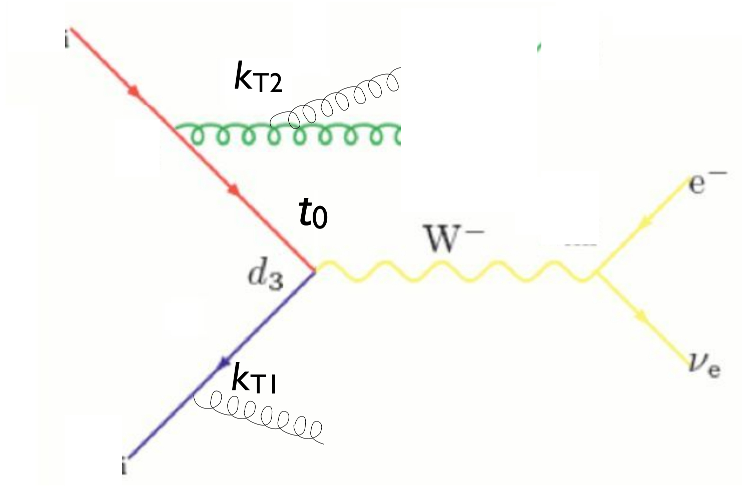
- Perform showering step-by-step for each step in the parton shower history, starting from the clustering scale for that step
- Veto the event if any shower is harder than the clustering scale for the next step (or t_{cut}, if last step)
- Keep any shower emissions that are softer than the clustering scale for the next step

CKKW-L matching

[Lönnblad 2002]
[Hoeche et al. 2009]



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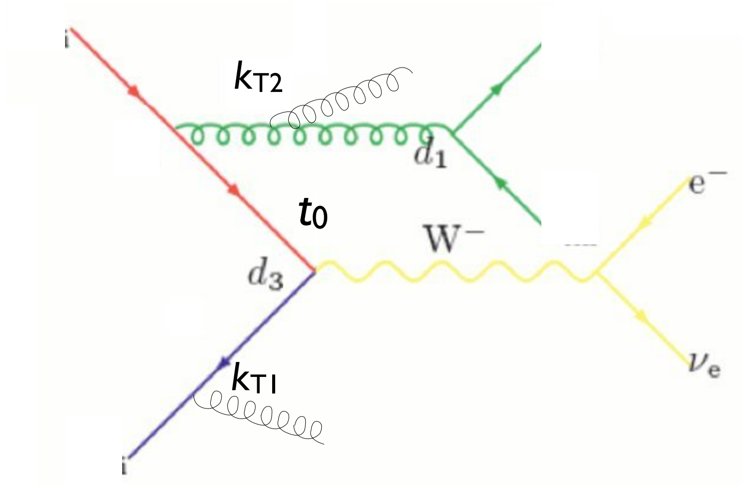

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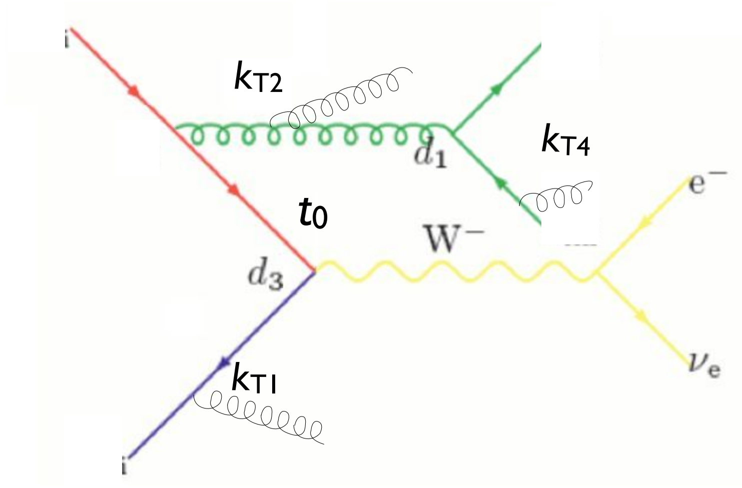

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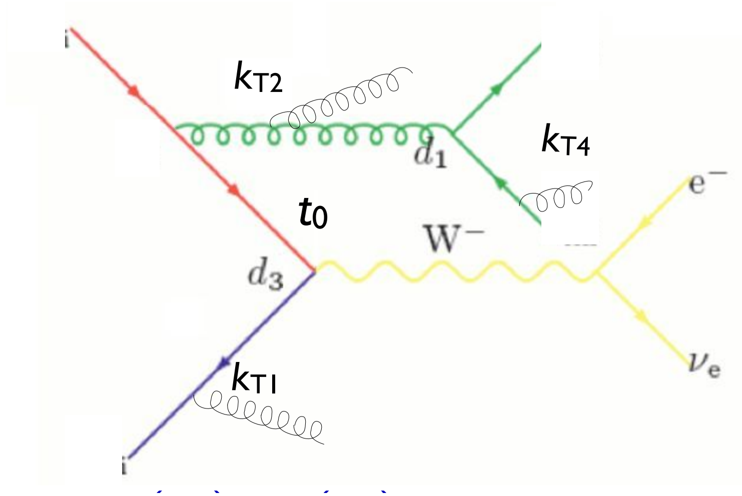
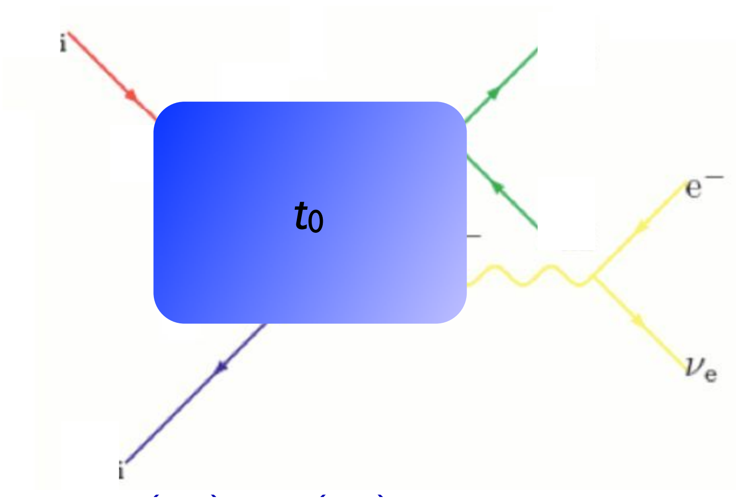


- ✓ Automatic agreement between Sudakov and shower
- Requires dedicated PS implementation
  - ➡ Need multiple implementations to compare between showers
- Implemented in Adriane, Sherpa (v. 1.2), and Pythia 8

MLM matching

[M.L. Mangano, ~2002, 2007]
[J.A. et al 2007, 2008]



MLM matching

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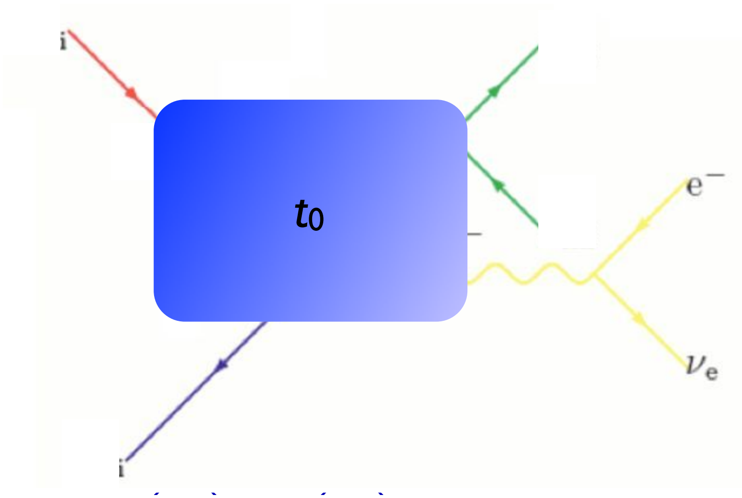
- The simplest way to do the Sudakov suppression is to run the shower on the event, starting from t_0!



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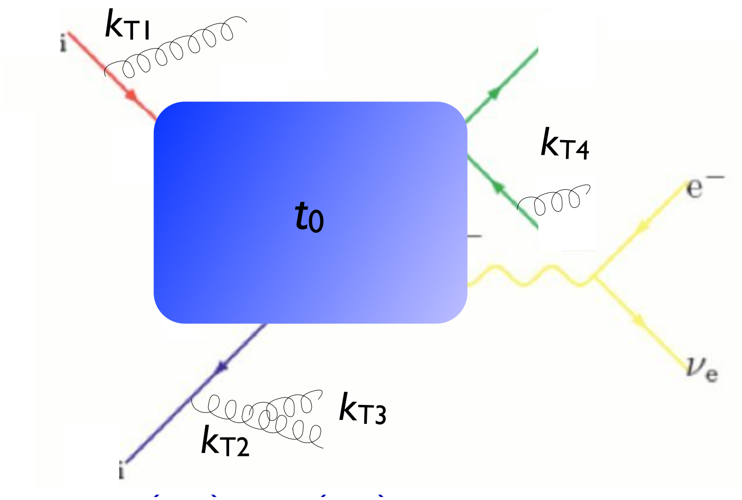

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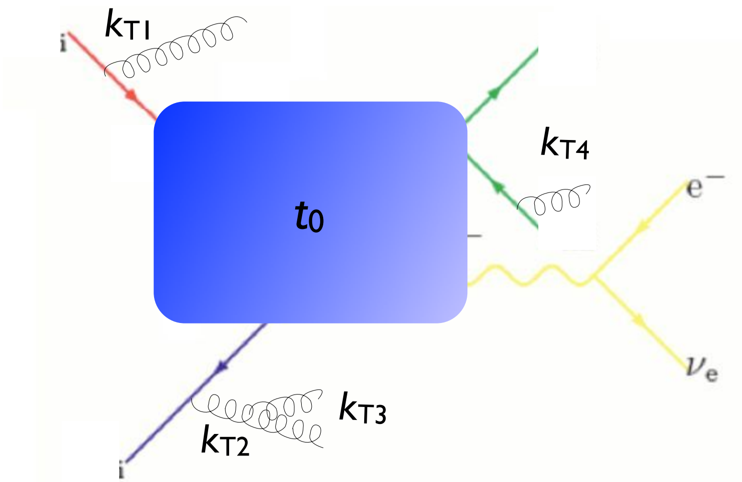

- Perform jet clustering after PS - if hardest jet $k_{T1} > t_{cut}$ or there are jets not matched to partons, reject the event

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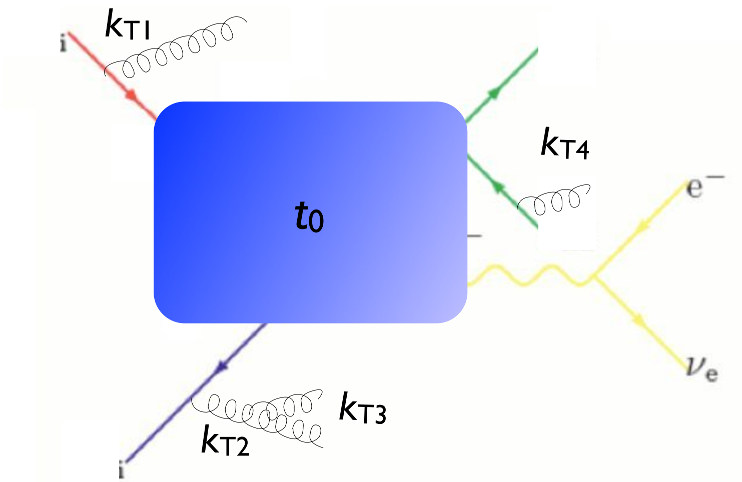

- Perform jet clustering after PS - if hardest jet $k_{T1} > t_{cut}$ or there are jets not matched to partons, reject the event
- The resulting Sudakov suppression from the procedure is

$$(\Delta_{Iq}(t_{\rm cut}, t_0))^2 (\Delta_q(t_{\rm cut}, t_0))^2$$

which turns out to be a good enough approximation of the correct expression $$(\Delta_{Iq}(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\rm cut}, t_2))^2$$

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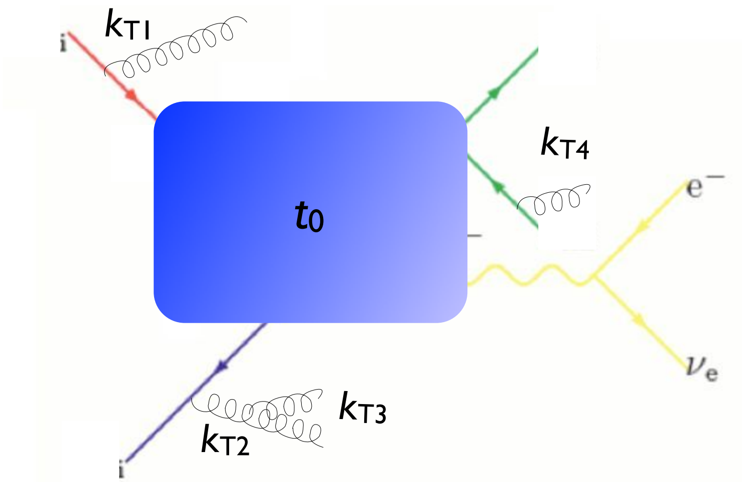


✓ Simplest available scheme

✓ Allows matching with any shower, without modification

➡ Sudakov suppression not exact, minor mismatch with shower

- Implemented in AlpGen, HELAC, MadGraph+Pythia 6

Highest multiplicity sample

- In the previous, assumed we can simulate all parton multiplicities by the ME
- In practice, we can only do limited number of final-state partons with matrix element (up to 4-5 or so)
- For the highest jet multiplicity that we generate with the matrix element, we need to allow additional jets above the matching scale t_{cut}, since we will otherwise not get a jet-inclusive description – but still can't allow PS radiation harder than the ME partons

➡ Need to replace t_{cut} by the clustering scale for the softest ME parton for the highest multiplicity

matching schemes

- We have a number of choices to make in the above procedure. The most important are:

1. The clustering scheme used to determine the parton shower history of the ME event
2. What to use for the scale *of hard emission*
3. How to divide the phase space between parton showers and matrix elements

Cluster schemes

1. The clustering scheme used inside MadGraph and Sherpa to determine the parton shower history is the Durham k_T scheme. For e^+e^-:

$$k_{Tij}^2 = 2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})$$

   and for hadron collisions, the minimum of:

$$k_{Ti\mathrm{beam}} = m_i^2 + p_{Ti}^2 = (E_i + p_{zi})(E_i - p_{zi})$$

   and

$$k_{Tij}^2 = \min(p_{Ti}^2, p_{Tj}^2)R_{ij}$$

   with $R_{ij} = 2[\cosh(y_i - y_j) - \cos(\phi_i - \phi_j)] \simeq (\Delta y)^2 + (\Delta\phi)^2$

   Find the smallest k_{Tij} (or $k_{Ti\mathrm{beam}}$), combine partons *i* and *j* (or *i* and the beam), and continue until you reach a 2 → 2 (or 2 → 1) scattering.

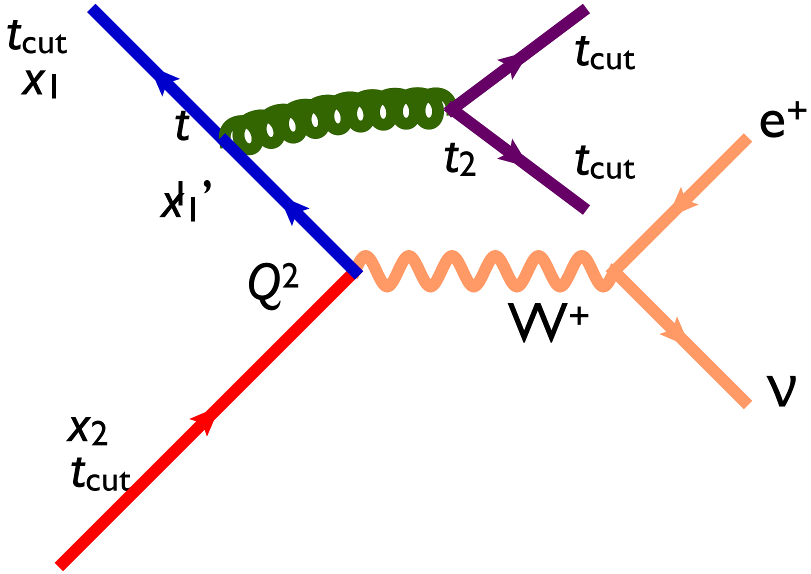
2. In AlpGen a more naive cone algorithm is used.

MLM in MadGraph

- Cannot use the standard k_T clustering:
  - MadGraph and Sherpa only allow clustering according to valid diagrams in the process. This means that, e.g., two quarks or quark-antiquark of different flavor are never clustered, and the clustering always gives a physically allowed parton shower history.
  - If there is an on-shell propagator in the diagram (e.g. a top quark), only clustering according to diagrams with this propagator is allowed.

Hard scale

2. The clustering provides a convenient choice for factorization scale Q^2:



Cluster back to the 2 → 2 (here qq → W-g) system, and use the W boson transverse mass in that system.

Phase-space division

3. How to divide the phase space between PS and ME:
   This is where the schemes really differ:

   AlpGen: MLM Cone
   MadGraph: MLM Cone, k_T or shower-k_T
   Sherpa: CKKW

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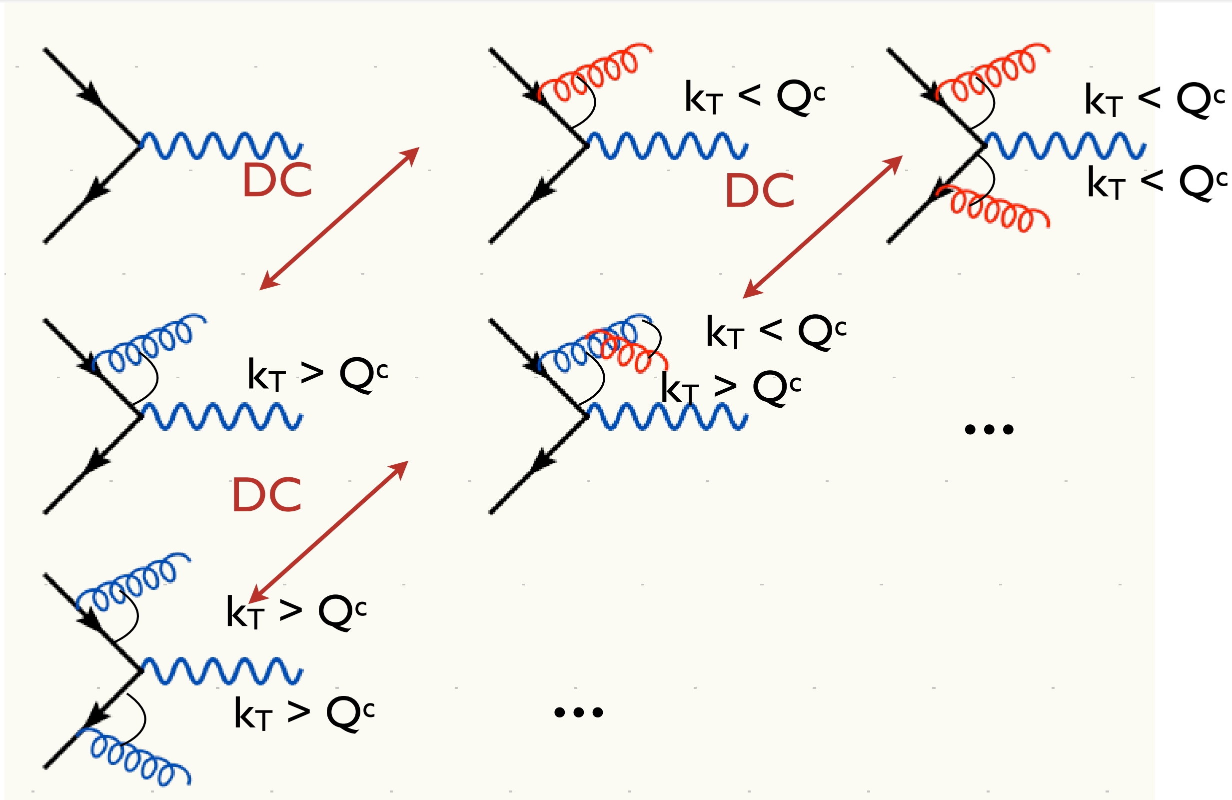
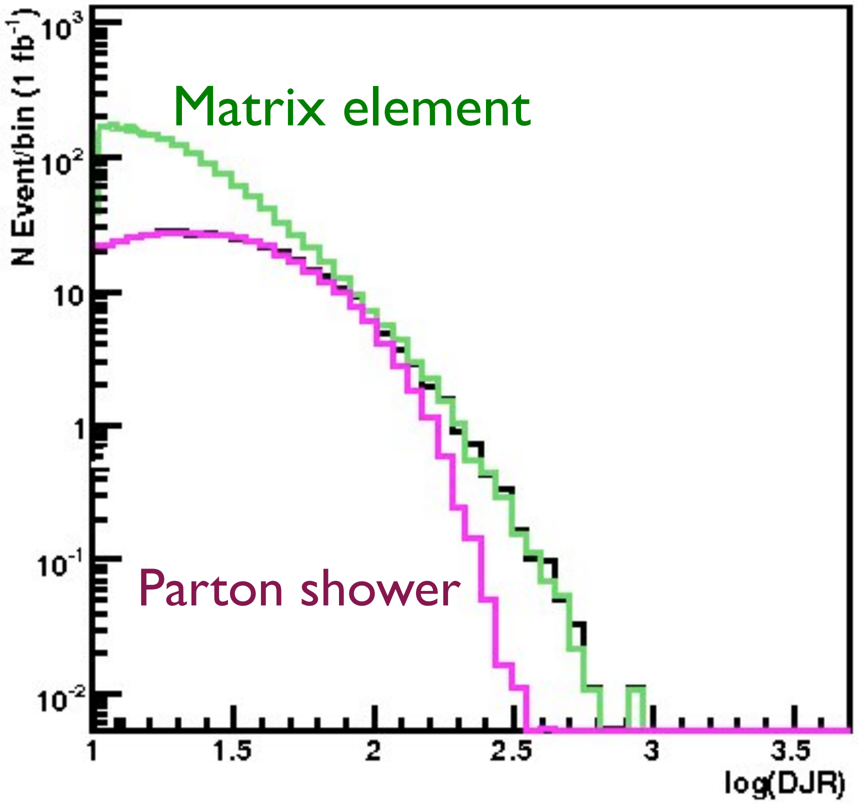


Back to the “matching goal”

- Regularization of matrix element divergence
- Correction of the parton shower for large momenta
- Smooth jet distributions



2nd QCD radiation jet in top pair production at the LHC, using MadGraph + Pythia

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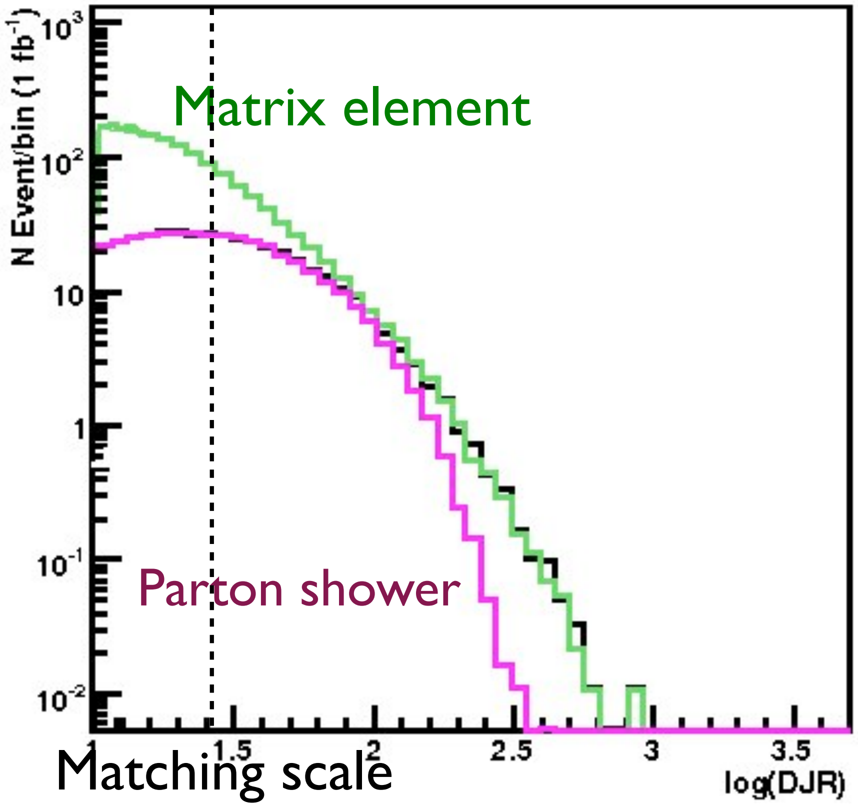

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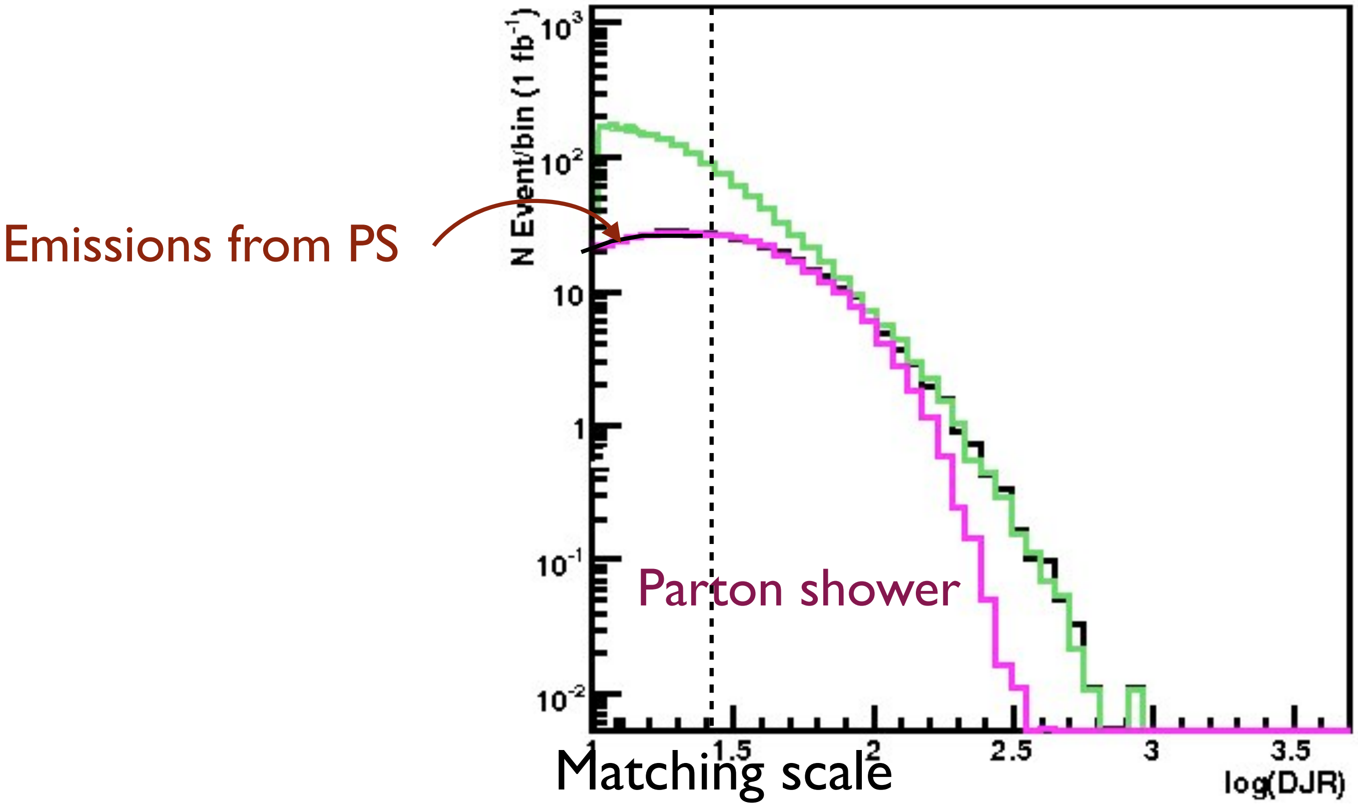

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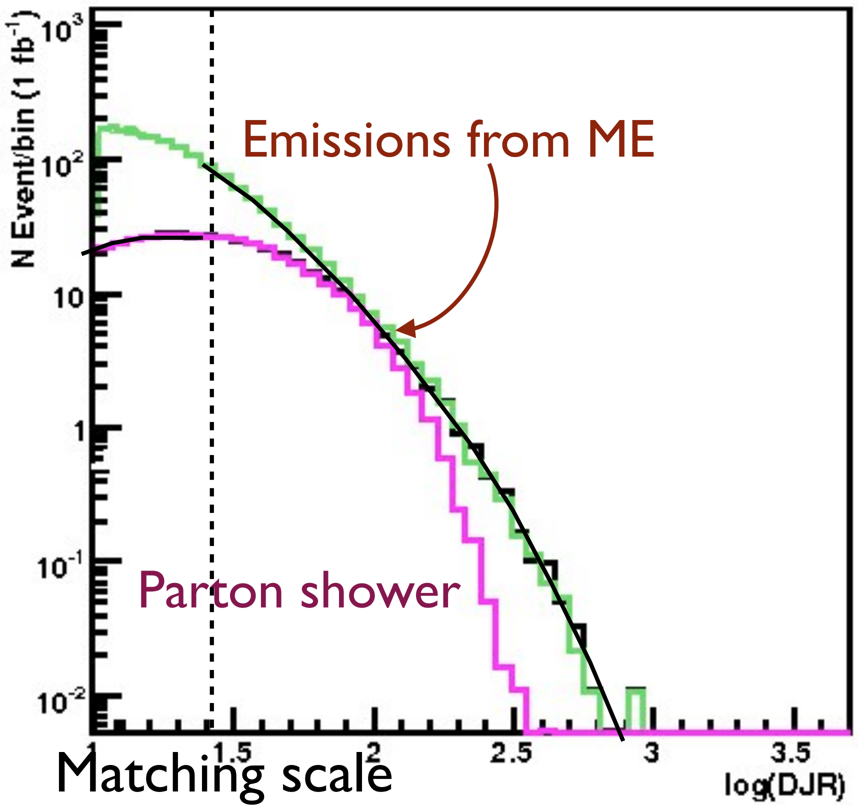

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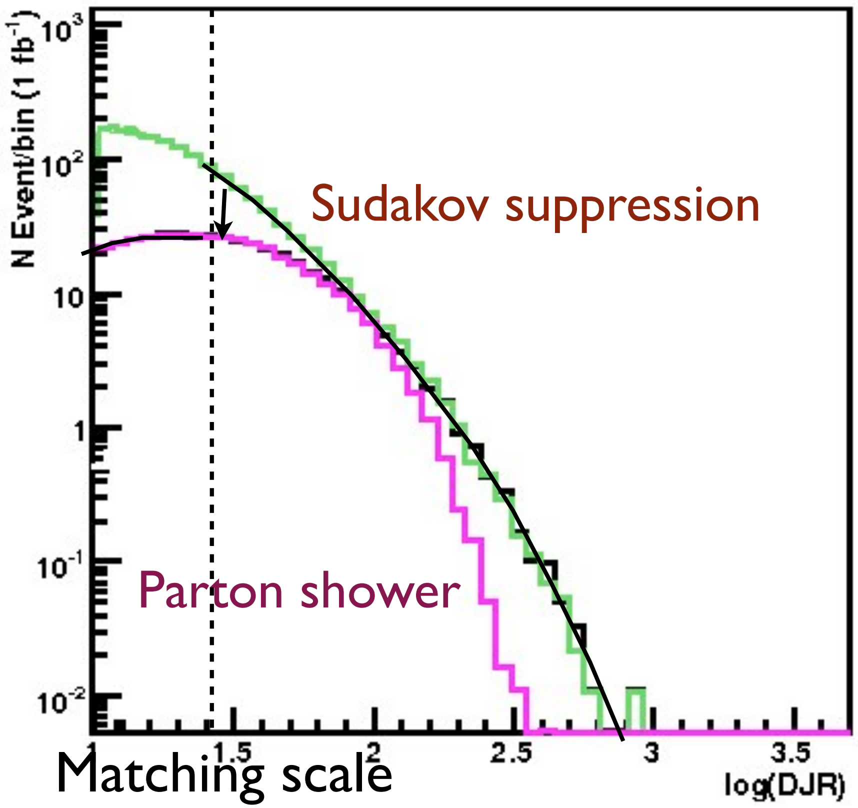

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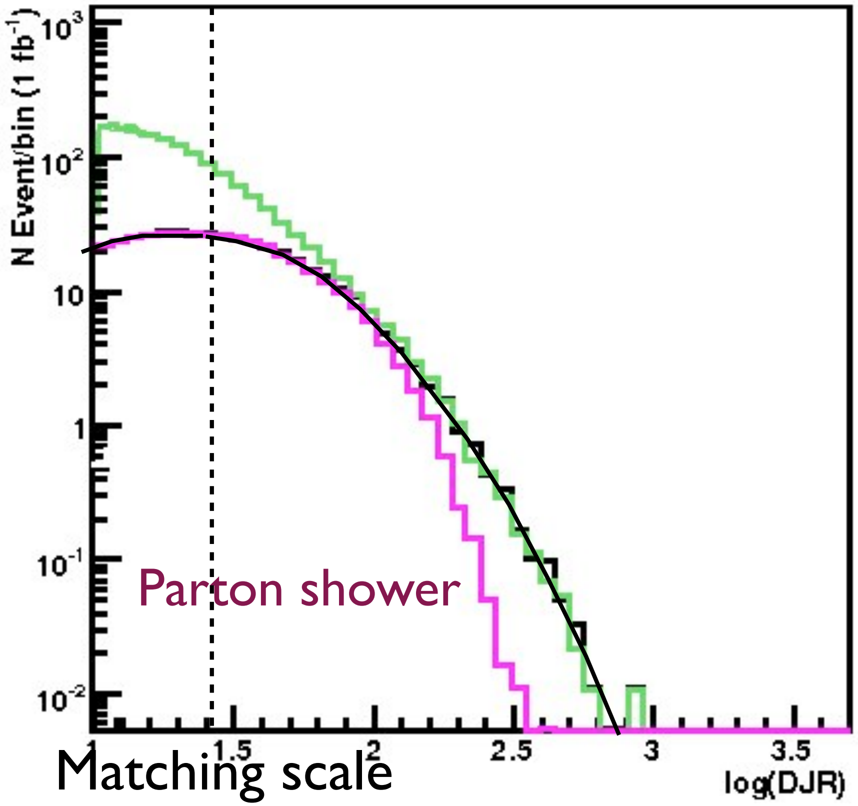

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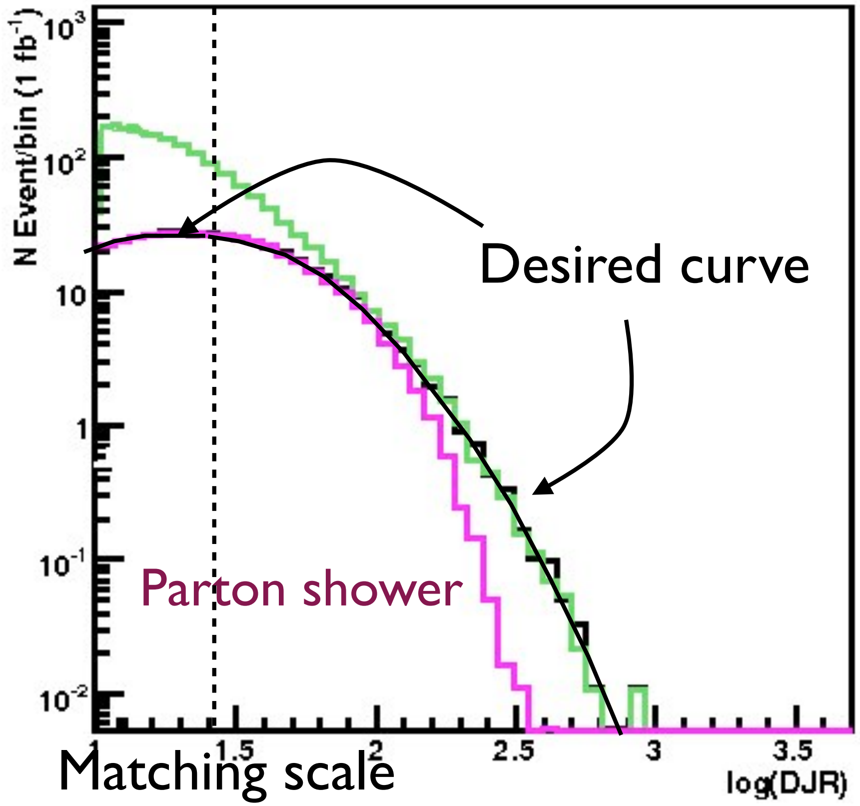

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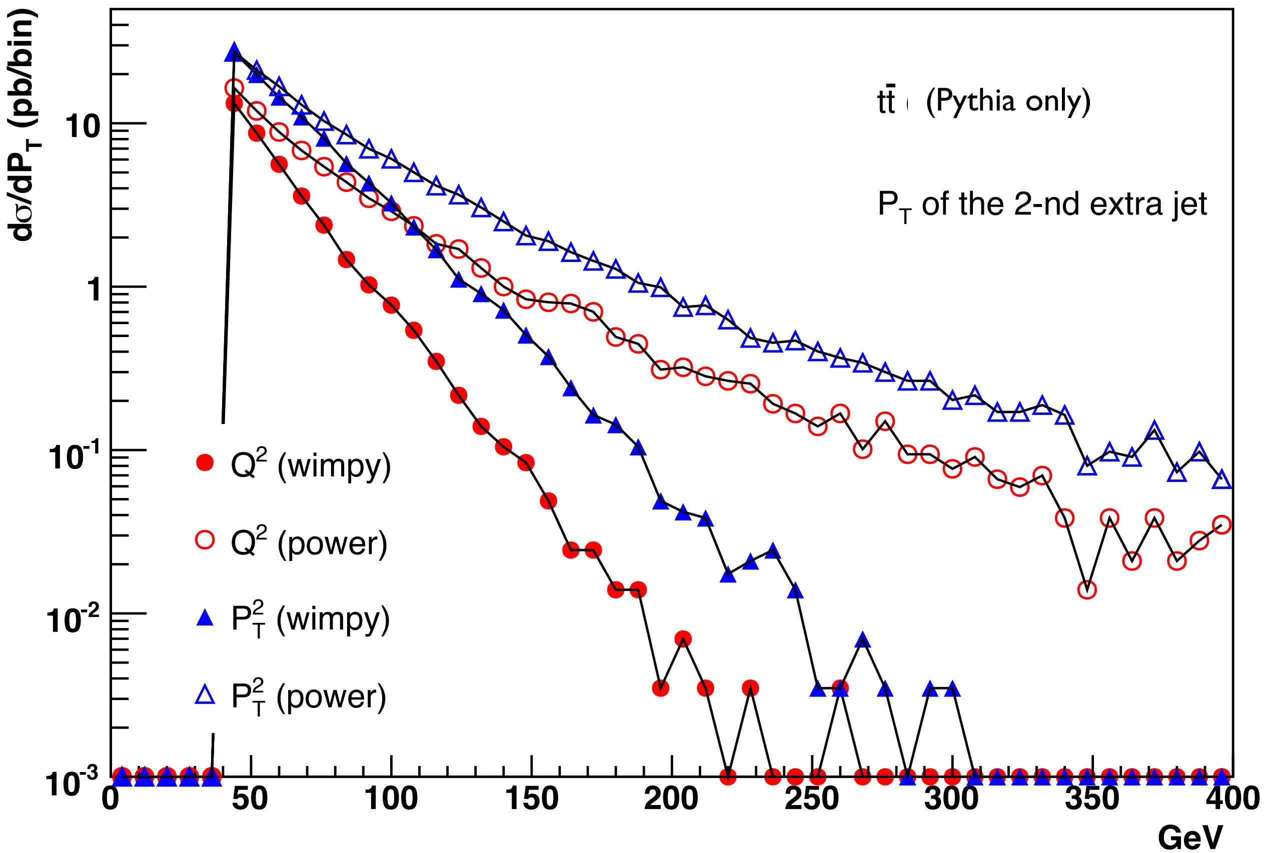
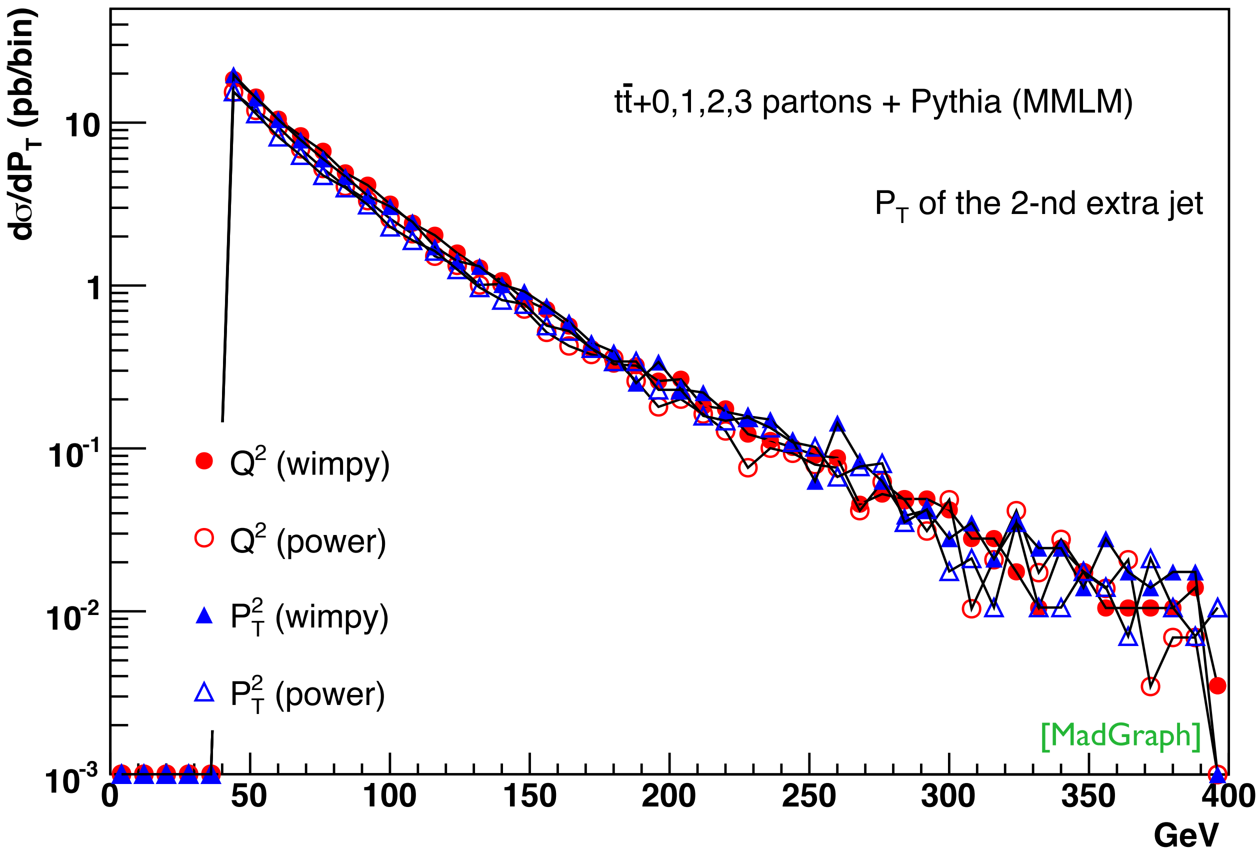
PS alone vs matched samples

In the soft-collinear approximation of Parton Shower MCs, parameters are used to tune the result ⇒ Large variation in results (small prediction power)



PS alone vs ME matching

In a matched sample these differences are irrelevant since the behavior at high pt is dominated by the matrix element.



Summary of Matching Procedure

1. Generate ME events (with different parton multiplicities) using parton-level cuts ($p_T^{ME}/\Delta R$ or k_T^{ME})
2. Cluster each event and reweight α_s and PDFs based on the scales in the clustering vertices
3. Apply Sudakov factors to account for the required non-radiation above clustering cutoff scale and generate parton shower emissions below clustering cutoff:
    a. (CKKW) Analytical Sudakovs + truncated showers
    b. (CKKW-L) Sudakovs from truncated showers
    c. (MLM) Sudakovs from reclustered shower emissions