NLO QCD predictions
The need for higher precision

- derive exclusion bounds from a given measurement
  - non-observation of a given hypothesis, e.g. heavy Higgs, BSM model
  - crucial to know the corresponding production cross sections precisely
  - derive limits on particle masses, model parameters
- counting experiments to extract certain parameters, couplings
  - at hadron colliders $\alpha_S$ corrections indispensable, need one-loop QCD

Stop mass bounds from Tevatron

[from the early days, T. Plehn private communication]

Higgs coupling extraction

- $\sigma(gg \to h \to \gamma\gamma) \sim \frac{\Gamma_g \Gamma_\gamma}{\Gamma}$
- $\sigma(gg \to h \to WW^*) \sim \frac{\Gamma_g \Gamma_W}{\Gamma}$
- $\sigma(qq \to qqh, h \to \tau\tau) \sim \frac{\Gamma_W \Gamma_\tau}{\Gamma}$
  - ...  
  - $\Gamma_i \sim g_{ih}, \Gamma_g \sim g_{tt}$
  - theoretical uncertainties dominate
  - systematics reduced for $\sigma$ ratios
Hard Processes at Next-to-Leading Order QCD

Anatomy of NLO QCD calculations

\[ |\mathcal{M}|^2 = |\mathcal{M}_B|^2 + \alpha_S \left( |\mathcal{M}_R|^2 + 2\Re(\mathcal{M}_B\mathcal{M}_V^\dagger) \right) \]

\[ \gamma^{\ast/2\vartheta} \quad \gamma^{\ast/2\vartheta} \quad \gamma^{\ast/2\vartheta} \]
\[ |\mathcal{M}|^2 = |\mathcal{M}_B|^2 + \alpha_S \left( |\mathcal{M}_R|^2 + 2\Re(\mathcal{M}_B \mathcal{M}_V^\dagger) \right) \]

\[ \sigma_{2\rightarrow n}^{NLO} = \int_n d^{(4)}\sigma^B + \int_{n+1}^{n} d^{(d)}\sigma^R + \int_n d^{(d)}\sigma^V \]

\[ \alpha_S \text{ real & virtual corrections} \]
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\[ \sigma_{2\to n}^{NLO} = \int_n d^{(4)}\sigma_B + \int_{n+1} d^{(d)}\sigma_R + \int_n d^{(d)}\sigma_V \]

- real-emission \( \sigma^R \)
  - \( \sim \) IR divergence for soft/collinear emission
- (UV renormalized) virtual-corrections \( \sigma^V \)
  - \( \sim \) IR divergent when propagator goes on-shell
- divergences manifest in dimensional regularisation, \( d = 4 - 2\epsilon \)
  - \( \sim \) occurrence of single & double poles, i.e. \( 1/\epsilon, 1/\epsilon^2 \)
- for IR safe observables poles cancel, the sum is finite [Kinoshita '62; Lee, Nauenberg '64]
Infrared & Collinear safe observables

- interested in observable-independent formulation

\[ \mathcal{O}^B = \int |M_B^{(n)}|^2 \, d\Phi_n \Theta^{(n)}(\mathcal{O}, \{p_n\}) \]

\[ \mathcal{O}^R = \int |M_R^{(n+1)}|^2 \, d\Phi_{n+1} \Theta^{(n+1)}(\mathcal{O}, \{p_{n+1}\}) \]

\[ \mathcal{O}^V = \int 2\Re(M_B M_V^\dagger) \, d\Phi_n \Theta^{(n)}(\mathcal{O}, \{p_n\}) \]

with \( d\Phi_{n/n+1} \) the \( n/n + 1 \) particle phase space, \( \Theta^{(n/n+1)} \) obs. measure function
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\[
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\]

\[
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\]

\[
O^V = \int 2\Re(M^B V^\dagger) \, d\Phi_n \Theta^{(n)}(O, \{p_n\})
\]

with \(d\Phi_{n/n+1}\) the \(n/n+1\) particle phase space, \(\Theta^{(n/n+1)}\) obs. measure function

- formal requirements on the measurement function:

\[
\Theta^{(n+1)}(p_1, \ldots, p_i = \lambda q, \ldots, p_{n+1}) \rightarrow \Theta^{(n)}(p_1, \ldots, p_{n+1})
\]

for \(\lambda \rightarrow 0\)  
soft limit

\[
\Theta^{(n+1)}(p_1, \ldots, p_i, \ldots, p_j, \ldots, p_{n+1}) \rightarrow \Theta^{(n)}(p_1, \ldots, p, \ldots, p_{n+1})
\]

for \(p_i \rightarrow zp, \; p_j = (1 - z)p\)  
collinear limit

simply remove (soft) parton \(i\), or replace collinear pair \(\{p_i, p_j\}\) by \(p = p_i + p_j\)

\(\leadsto\) definition of infrared & collinear safe observables
Anatomy of NLO QCD calculations cont’d

\[ \sigma_{2 \to n}^{NLO} = \int_n d^4 \sigma^B + \int_{n+1} d^d \sigma^R + \int_n d^d \sigma^V \]

- finite corrections for IR safe observables
- analytical integration with cuts in \( d \)-dim infeasible for high-multi final states
- we are interested in fully-differential answer, use of MC integration methods
- however, both terms \( \sigma^R \) & \( \sigma^V \) exhibit divergences, i.e. are unbound
- \( \sigma^R \) in \((n + 1)\)-particle phase space, \( \sigma^V \) in \(n\)-particle phase space
Anatomy of NLO QCD calculations cont’d

\[ \sigma_{2 \rightarrow n}^{NLO} = \int_n d^{(4)}\sigma^B + \int_{n+1} d^{(d)}\sigma^R + \int_n d^{(d)}\sigma^V \]

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- we are interested in fully-differential answer, use of MC integration methods
- however, both terms $\sigma^R$ & $\sigma^V$ exhibit divergences, i.e. are unbound
  - $\sigma^R$ in $(n+1)$-particle phase space, $\sigma^V$ in $n$-particle phase space

Subtraction methods

- attempt to devise local *subtraction term* such that both integrals are separately finite and can be performed as ordinary multi-dimensional phase-space integrals

  - based on universality of QCD IR divergences
  - aim for IR regularisation at integrand level
Subtraction methods cont’d

- subtraction terms provide local approximation for the real-emission process

\[ \int_{n+1} d^{(d)} \sigma^A = \int_n \int_1 d^{(d)} \sigma^A \]

\[ \rightarrow \text{captures soft & collinear limits of amplitudes [1/\epsilon and 1/\epsilon^2 poles]} \]

\[ \rightarrow \text{analytically integrable (d-dim) over one-parton emission phase space} \]
Subtraction methods cont’d

- Subtraction terms provide local approximation for the real-emission process

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\[ \rightarrow \text{ analytically integrable } (d\text{-dim}) \text{ over one-parton emission phase space} \]

- Add & subtract from NLO differential cross section

\[ \sigma_{2\rightarrow n}^{NLO} = \int_{n+1} \left[ d^{(4)} \sigma^R - d^{(4)} \sigma^A \right] + \int_n \left[ d^{(4)} \sigma^B + \int_{\text{loop}} d^{(d)} \sigma^V + \int_1 d^{(d)} \sigma^A \right]_{\epsilon=0} \]
Subtraction methods cont’d

- Subtraction terms provide local approximation for the real-emission process

\[ \int_{n+1} d^{(d)} \sigma^A = \int_n \int_1 d^{(d)} \sigma^A \]

\[ \leftrightarrow \text{ captures soft & collinear limits of amplitudes } [1/\epsilon \text{ and } 1/\epsilon^2 \text{ poles}] \]

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- Add & subtract from NLO differential cross section

\[ \sigma_{2\rightarrow n}^{NLO} = \int_{n+1} \left[ d^{(4)} \sigma^R - d^{(4)} \sigma^A \right] + \int_n \left[ d^{(4)} \sigma^B + \int_{\text{loop}} d^{(d)} \sigma^V + \int_1 d^{(d)} \sigma^A \right] \]

\[ \epsilon = 0; \text{ separately finite phase-space integrals (4-dim)} \]

\[ \rightarrow \text{ can be performed by means of MC integration} \]

\[ \rightarrow \text{ no ambiguous or unphysical cut-offs/scales get introduced} \]

- Catani & Seymour presented general expression for $d^{(d)}\sigma^A$
  (known as the dipole factorisation formula)
- constructed from Born process using universal dipole terms

\[
\int_{n+1} d^{(d)}\sigma^A = \sum_{\text{dipoles}} \int_n d^{(d)}\sigma^B \otimes \int_1 d^{(d)} V_{\text{dipole}} = \int_m \left[ d^{(d)}\sigma^B \otimes I \right]
\]

spin- & color correlations $\leftrightarrow$ \hspace{1cm} $\leftrightarrow$ universal dipole terms

- sum over dipoles contains all real-emission soft/collinear divergences
- suited for any process with massless partons (0, 1, 2 initial-state hadrons)
Dipole subtraction method cont’d

- Subtraction method for arbitrary observables (IRC safe)

\[ O^B = \int |M_B^{(n)}|^2 \, d\Phi_n \Theta^{(n)}(O, \{p_n\}) \]

\[ O^{RS} = \int d\Phi_{n+1} \left[ |M_R^{(n+1)}(\{p_n\})|^2 \Theta^{(n+1)}(O, \{p_{n+1}\}) \right. \]

\[ \left. - \sum_{i \neq j \neq k} D_{ij,k}(\{p_n\}) \Theta^{(n)}(O, p_1, .., \tilde{p}_{ij}, \tilde{p}_k, .., p_{n+1}) \right] \]

\[ O^{VI} = \int d\Phi_n \left[ 2\Re(M_B M_V^\dagger) + \int_1 [d\tilde{p}] D_{ij,k} \right] \Theta^{(n)}(O, \{p_n\}) \]

- \[ [d\tilde{p}] \] phase-space element of one-parton

- Such, differential & integrated subtraction terms cancel exactly
  \[ \rightarrow \text{requires mapping of } (n+1) \rightarrow n \text{ configuration} \]

- Need to distinguish different dipole types
Dipole subtraction method cont’d

- Dipoles involve three partons, splitting particles $i, j$ and spectator $k$
- Distinguish initial- & final-state splitter/spectator
- All contributions known, suitable for automated evaluation

**FF**

$$V_{ij,k}$$

- $p_i$, $i$
- $p_j$, $j$
- $p_k$, $k$

**FI**

$$V_{ij}^a$$

- $p_i$, $i$
- $p_j$, $j$
- $p_a$, $a$

**IF**

- $a$, $p_a$
- $p_i$, $i$
- $p_k$, $k$
- $V_{ai}^k$

**II**

- $a$, $p_a$
- $p_i$, $i$
- $p_k$, $k$
- $V_{ai,b}^{ai}$
- $b$, $p_b$
Example: $e^+ e^- \rightarrow q\bar{q}$ @ NLO QCD

- most simple scattering process
- sensitive to QCD colour charges & strong coupling $\alpha_s$
- first non-trivial contribution to jet sub-structure
  $\sim$ stringent test of QCD dynamics
Example: $e^+ e^- \rightarrow q \bar{q} @ NLO$ QCD

- most simple scattering process
- sensitive to QCD colour charges & strong coupling $\alpha_S$
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  $\sim$ stringent test of QCD dynamics

**Born-level contribution:** $(e^+ e^- \rightarrow) \gamma^* \rightarrow q \bar{q}$

$$M_{q\bar{q}} = \frac{ie \gamma_\mu}{p_1} \frac{ie \gamma_\mu}{p_2}$$

$$= \bar{u}_a(p_1) i e_q \gamma_\mu \delta_{ab} v_b(p_2)$$

$$\sigma^B_{q\bar{q}} = \sigma^{LO}_{q\bar{q}} \propto |M_{q\bar{q}}|^2 = \frac{4\pi \alpha^2}{3Q^2} e_q^2 N_c$$

where $Q^2 = (p_1 + p_2)^2$
real-emission correction, i.e. \( (e^+ e^- \rightarrow \gamma^* \rightarrow q\bar{q}g) \)

\[
M_{q\bar{q}g} = i e \gamma_\mu \frac{k,\epsilon}{p_1} + i e \gamma_\mu \frac{k,\epsilon}{p_2}
\]

\[
= -\bar{u}_a(p_1) i g_s \gamma^A t_{ab} \frac{i(\not{p_1} + \not{k})}{(p_1 + k)^2} i e_q \gamma_\mu \not{v}_b(p_2) + \bar{u}_a(p_1) i e_q \gamma_\mu \frac{i(\not{p_2} + \not{k})}{(p_2 + k)^2} i g_s \gamma^A t_{ab} \not{v}_b(p_2)
\]
Example: $e^+ e^- \rightarrow q \bar{q} \oplus$ NLO QCD

real-emission correction, i.e. $(e^+ e^- \rightarrow \gamma^*) \rightarrow q \bar{q}g$

$$M_{q \bar{q}g} = -\bar{u}_a(p_1)ig_s \epsilon^{\mu} t^A_{ab} \left(\frac{i(p_1 + k)}{(p_1 + k)^2}\right) ie_q \gamma_\mu \nu_b(p_2) + \bar{u}_a(p_1)ie_q \gamma_\mu \left(\frac{i(p_2 + k)}{(p_2 + k)^2}\right) ig_s \epsilon^{\mu} \nu_{Ab}(p_2)$$

defining $Q^2 = (p_1 + p_2 + k)^2 = s$ and $x_i = 2p_i \cdot Q / Q^2$, we find

$$|M_{q \bar{q}g}|^2 = C_F \frac{8\pi \alpha_s}{Q^2} \frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)} |M_{q \bar{q}}|^2 \sim \text{singular when } x_{1/2} \rightarrow 1$$

associated phase-space element

$$d\Phi^{(3)} = \frac{Q^2}{16\pi^2} dx_1 dx_2 \ \Theta(1 - x_1)\Theta(1 - x_2)\Theta(x_1 + x_2 - 1)$$
Example: $e^+ e^- \rightarrow q\bar{q} \oplus$ NLO QCD
eal-emission subtraction term there are two FF dipoles contributing

$$D_{13,2}(p_1, p_2, k) = \frac{1}{2p_1 k} V_{qg,\bar{q}} |M_{q\bar{q}}|^2$$

$$D_{23,1}(p_1, p_2, k) = \frac{1}{2p_2 k} V_{\bar{q}g,q} |M_{q\bar{q}}|^2$$

corresponding subtracted real-emission cross section ($d = 4$)

$$\sigma_{q\bar{q}g}^{RS} = \int_3 \left[ d\sigma_{\epsilon=0}^R - d\sigma_{\epsilon=0}^A \right] \left( \tilde{p}_{ij} + \tilde{p}_k = Q, \tilde{p}_k = \frac{1}{x_k} p_k, \tilde{p}_{ij} = Q - \frac{1}{x_k} p_k \right)$$

$$= |M_{q\bar{q}}|^2 \frac{C_F \alpha_s}{2\pi} \int_0^1 dx_1 \, dx_2 \, \Theta(x_1 + x_2 - 1) \left\{ \right.$$

$$\left. \left[ \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \right] \Theta^{(3)}(p_1, p_2, p_3) \right.$$\vspace{-4ex}

$$\left. - \left[ \frac{1}{1-x_2} \left( \frac{2}{2-x_1-x_2} - (1+x_1) \right) + \frac{1-x_1}{x_2} \right] \Theta^{(2)}(\tilde{p}_{13}, \tilde{p}_2) \right.$$\vspace{-4ex}

$$\left. - \left[ \frac{1}{1-x_1} \left( \frac{2}{2-x_1-x_2} - (1+x_2) \right) + \frac{1-x_2}{x_1} \right] \Theta^{(2)}(\tilde{p}_{23}, \tilde{p}_1) \right\}$$

\(\sim\) finite as $x_{1/2} \rightarrow 1$ (implying $\Theta^{(3)} \rightarrow \Theta^{(2)}$)
Example: $e^+e^- \rightarrow q\bar{q} \oplus$ NLO QCD

virtual correction & integrated dipole contribution ($d = 4 - 2\epsilon$)

\[
V \supset \begin{array}{c}
\begin{array}{c}
-\text{i}e\gamma_{\mu}
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\text{x}
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\text{i}e\gamma_{\mu}
\end{array}
\end{array}
\]

\[
|\mathcal{M}^{(d)}_{q\bar{q}}|_{\text{(1-loop)}}^{2 \over \overline{\text{MS}}} = |\mathcal{M}^{(d)}_{q\bar{q}}|^{2} \frac{C_F \alpha_s}{2\pi} \frac{1}{\Gamma(1 - \epsilon)} \left( \frac{4\pi\mu^2}{Q^2} \right)^{\epsilon} \left\{ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + \mathcal{O}(\epsilon) \right\}
\]
Example: \( e^+ e^- \rightarrow q\bar{q} \) @ NLO QCD

**virtual correction & integrated dipole contribution \((d = 4 - 2\epsilon)\)**

\[
V \supset \begin{array}{c}
-\text{i}e \gamma_\mu \\
\end{array} \begin{array}{c}
x \\
\end{array} \begin{array}{c}
\text{i}e \gamma_\mu \\
\end{array}
\]

\[
|\mathcal{M}_{q\bar{q}}^{(d)}|_{(1-\text{loop})}^{2} \overset{\overline{\text{MS}}}{=} |\mathcal{M}_{q\bar{q}}^{(d)}|^{2} \cdot \frac{C_F \alpha_s}{2\pi} \cdot \frac{1}{\Gamma(1 - \epsilon)} \cdot \left( \frac{4\pi \mu^2}{Q^2} \right)^{\epsilon} \left\{ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + O(\epsilon) \right\}
\]

integrated dipole kernels

\[
I(\epsilon) = \frac{C_F \alpha_s}{2\pi} \cdot \frac{1}{\Gamma(1 - \epsilon)} \cdot \left( \frac{4\pi \mu^2}{Q^2} \right)^{\epsilon} \left\{ \frac{2}{\epsilon^2} + \frac{3}{\epsilon} + 10 - \pi^2 + O(\epsilon) \right\}
\]
Example: $e^+ e^- \to q \bar{q} \ @ \text{NLO QCD}$

**Virtual correction & integrated dipole contribution** ($d = 4 - 2\epsilon$)

\[
V \supset \begin{array}{c}
-\frac{ie\gamma_\mu}{x} \\
\frac{ie\gamma_\mu}{x}
\end{array}
\]

\[
|\mathcal{M}_{q\bar{q}}^{(d)}|_{1\text{-loop}}^2 \equiv |\mathcal{M}_{q\bar{q}}^{(d)}|^2 = \frac{C_F \alpha_s}{2\pi} \frac{1}{\Gamma(1-\epsilon)} \left(\frac{4\pi \mu^2}{Q^2}\right)^\epsilon \left\{ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + O(\epsilon) \right\}
\]

**Integrated dipole kernels**

\[
I(\epsilon) = \frac{C_F \alpha_s}{2\pi} \frac{1}{\Gamma(1-\epsilon)} \left(\frac{4\pi \mu^2}{Q^2}\right)^\epsilon \left\{ \frac{2}{\epsilon^2} + \frac{3}{\epsilon} + 10 - \pi^2 + O(\epsilon) \right\}
\]

Combining both contributions yields

\[
\sigma_{q\bar{q}}^{VI} = \int_2 \left[ d\sigma_{q\bar{q}}^{V} + \int_1 d\sigma_{q\bar{q}}^{A} \right]_{\epsilon=0} = |\mathcal{M}_{q\bar{q}}|^2 \frac{C_F \alpha_s}{\pi} \int dy_{12} \delta(1 - y_{12}) \Theta^{(2)}(p_1, p_2)
\]

where $y_{12} = 2p_1 \cdot p_2 / Q^2$
Example: $e^+ e^- \rightarrow q \bar{q} \oplus$ NLO QCD

**total scattering cross section**

measurement functions given by $\Theta^{(3)} = \Theta^{(2)} = 1$

$$\sigma_{q\bar{q}}^{NLO} = \int_3 d\sigma_{q\bar{q}g}^{RS} + \int_2 [d\sigma_{q\bar{q}}^{B} + d\sigma_{q\bar{q}}^{VI}] = \sigma_{q\bar{q}}^{LO} \left(1 + \frac{3}{4} C_F \frac{\alpha_s}{\pi}\right) + \mathcal{O}(\alpha_s^2)$$

**Discussion:**

- fully inclusive total cross section [IR safe]
- $\sim$ real & virtual IR singularities properly cancelled
- yields modest, finite correction
- dependent on strong-coupling parameter and its running
The emerging picture

- corrections to $\sigma_{\text{tot}}$ dominated by hard, large-angle gluons
- soft gluons play no role for $\sigma_{\text{tot}}$
  - collision characterised by $t_{\text{hard}} \sim 1/Q$
  - soft-gluons emitted on long time scales $t_{\text{soft}} \sim 1/(E\theta^2)$
    - cannot influence cross section
  - transition to hadrons occurs on long time scales $t_{\text{had}} \sim 1/\Lambda_{\text{QCD}}$
    - can thus be ignored

- with proper choice for scale of $\alpha_s$, $\mu^2 = Q^2$, perturbation theory works well

$\sigma_{\text{tot}} = \sigma_{q\bar{q}} \left( \begin{array}{c}
1 \\
\text{LO} \quad +1.045 \frac{\alpha_s(Q^2)}{\pi} \\
\text{NLO} \quad +0.94 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^2 \\
\text{NNLO} \quad -15 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^3 \\
\text{NNNLO} \quad + \cdots
\end{array} \right)$

[coefficients given for $Q^2 = M_Z^2$, including EW corrections]

**Total cross sections are inclusive quantities, inclusive in the number of additional QCD partons!**
The scale ambiguity

- fixed-order prediction depends on (arbitrary) renormalisation scale $\mu$
- estimate theory uncertainty through scale variations, e.g. $\frac{Q}{2} < \mu < 2Q$
  $\sim$ provides estimate of uncalculated higher-order terms

$$
\sigma_{tot}^{\text{NLO}}(\mu^2) = \sigma_{q\bar{q}} \left( 1 + C_1 \alpha_s(\mu^2) \right) \\
\text{with } \alpha_s(\mu^2) = \alpha_s(Q^2) - 2b_0 \alpha_s^2(Q^2) \ln \left( \frac{\mu}{Q} \right) + \mathcal{O}(\alpha_s^3) \\
= \sigma_{q\bar{q}} \left( 1 + C_1 \alpha_s(Q^2) - 2C_1 b_0 \alpha_s^2(Q^2) \ln \left( \frac{\mu}{Q} \right) + \mathcal{O}(\alpha_s^3) \right)
$$

compare to

$$
\sigma_{tot}^{\text{NNLO}}(\mu^2) = \sigma_{q\bar{q}} \left( 1 + C_1 \alpha_s(\mu^2) + C_2(\mu^2) \alpha_s^2(\mu^2) \right) \\
\text{with } C_2(\mu^2) = C_2(Q^2) + 2C_1 b_0 \alpha_s^2(Q^2) \ln \left( \frac{\mu}{Q} \right)
$$
The scale ambiguity

\[ \frac{\sigma_{ee \rightarrow \text{hadrons}}}{\sigma_{ee \rightarrow \text{qq}}} = \frac{\mu_R}{Q} \]

- Use scale variations to estimate theory uncertainty
- Probe residual dependence on (uncalculated) higher orders
NLO QCD: brief summary

Motivations

- accurate cross-section estimates
- real-emission kinematics corrections
- reduced systematic uncertainties [scale dependences]
NLO QCD: brief summary

**Motivations**
- accurate cross-section estimates
- real-emission kinematics corrections
- reduced systematic uncertainties

**Anatomy**

\[ \hat{\sigma}_{ij \to X} = \int d\Phi_{ij \to X} \left[ \sum \text{lowest order term} + \sum 2 \text{Re} \{ \text{quantum corrections} \} \right] \]

\[ + \sum_{k \in \{q,g\}} \int d\Phi_{i,j \to X+k} \sum \text{radiative corrections} \]

\[ \leftrightarrow \text{subtraction of infrared singularities in real- & virtual corrections} \]


\[ \leftrightarrow \text{dedicated one-loop amplitude codes: BLACKHAT, OPENLOOPS/COLLIER, RECOLA, ...} \]
Interlude: QCD jets & PDFs
The emergent picture: final-state jets

Jet definition (prel.): jets are collimated sprays of hadronic particles

- hard partons undergo soft and collinear showering
- hadrons closely correlated with the hard partons’ directions

Counting jets

- near perfect two-jet event
- almost all energy contained in two cones
The emergent picture: final-state jets

Jet definition (prel.): jets are collimated sprays of particles

- hard partons undergo soft and collinear showering
- hadrons closely correlated with the hard partons’ directions

Counting jets

→ hard emissions can induce more jets
→ jet counting not obvious, is this a three- or four-jet event?
Defining jets

Jet definition (addendum): jet number shouldn’t depend upon just a soft/collinear emission

→ Infrared & collinear safety

Infrared & Collinear safe jet definitions
crucial for comparing theory with experimental results
Jet algorithms

Jet definition

- group together particles into a common jets [jet algorithm]
- typical parameter is $R$, distance in $y-\phi$ space, determines angular reach
- combine momenta of jet constituents to yield jet momentum [recombination scheme]

Two generic types of jet algorithms are commonly used:

- cone algorithms
  - widely used in the past at the Tevatron
  - jets have regular/circular shapes
  - some suffer from IR or collinear unsafety

- sequential recombination algorithms
  - widely used at LEP [Durham $k_T$ algorithm]
  - jet can have irregular shapes
  - default at the LHC experiments [anti-$k_T$ algorithm]
Sequential recombination algorithms

A generic jet finding algorithm

1. Compute a distance measure $y_{ij}$ for each pair of final-state particles.
2. Determine all distance measures wrt the beam $y_{iB}$.
3. Determine the minimum of all $y_{ij}$'s and $y_{iB}$'s:
   - If $y_{ij}$ is smallest, **combine** particles $ij$, sum four-momenta.
   - If $y_{iB}$ is smallest, **remove** particle $i$, call it a jet.
4. Go back to step one, until all particles are clustered into jets.

In analyses one typically uses

- Jets with inter-jet distances $y_{ij} > y_{\text{cut}}$ [exclusive mode].
- Jets with inter-jet distances $y_{ij} > y_{\text{cut}}$ & $E > E_{\text{cut}}$ [inclusive mode].

**Different algorithms use different measures:** $y_{ij}$ & $y_{iB}$.
Sequential recombination algorithms: the $k_T$ algorithm

recall the soft and collinear limit of the gluon-emission probability for $a \rightarrow ij$

$$dS \sim \frac{2\alpha_s C_A/F}{\pi} \frac{dE_i}{\min(E_i, E_j)} \frac{d\theta_{ij}}{\theta_{ij}},$$

using $\min(E_i, E_j)$ we can avoid specifying which of $i$ and $j$ is soft

The $k_T$-algorithm distance measure

$$y_{ij} = \frac{2 \min(E_i^2, E_j^2)(1 - \cos \theta_{ij})}{Q^2}$$

$\sim$ in the collinear limit: $y_{ij} \sim \min(E_i^2 E_j^2) \theta_{ij}^2 / Q^2$

$\sim$ relative transverse momentum, normalized to total energy

$\sim$ soft/collinear particles get clustered first
Sequential recombination algorithms: the anti-$k_T$ algorithm

recall the soft and collinear limit of the gluon-emission probability for $a \rightarrow ij$

$$dS \sim \frac{2\alpha_s C_i}{\pi} \frac{dE_i}{\min(E_i, E_j)} \frac{d\theta_{ij}}{\theta_{ij}},$$

using $\min(E_i, E_j)$ we can avoid specifying which of $i$ and $j$ is soft

The anti-$k_T$-algorithm distance measure

$$y_{ij} = 2Q^2 \min(E_i^{-2}, E_j^{-2})(1 - \cos \theta_{ij})$$

$\sim$ jet-finding starts out with hard objects
$\sim$ softer particles get clustered into hard jets later on
$\sim$ produces nicely regular shaped jets
$\sim$ default in current LHC physics analyses
Jet algorithms at work: $k_T$ jets at LEP

Jet algorithms at work: $k_T$ jets at LEP

Jet algorithms at work: anti-\(k_T\) jets at LHC

anti-\(k_T\) inclusive jets at LHC

\[
\int d\sigma/dy \ dy \left[ \text{pb}/\text{GeV} \right]
\]

- \(\text{anti-}k_t\) jets, \(R=0.4\)
- \(|y| < 0.3 \times 10^1\)
- \(0.3 \leq |y| < 0.8 \times 10^3\)
- \(0.8 \leq |y| < 1.2 \times 10^4\)
- \(1.2 \leq |y| < 2.1 \times 10^5\)
- \(2.1 \leq |y| < 2.8 \times 10^5\)
- \(2.8 \leq |y| < 3.6 \times 10^5\)
- \(3.6 \leq |y| < 4.4 \times 10^5\)

\[\sigma = \text{NLOJET} \times \text{ATLAS}\]

Systematic uncertainties

Non-pert. corr.
Jet algorithms at work: anti-$k_T$ jets at LHC

The $V$+jets processes
- background to many BSM searches
- wide range of kinematics
- multi-scale QCD problem

$N_{\text{jets}}$: jet multiplicity

[ATLAS-CONF-2016-046]
Jet algorithms at work: anti-$k_T$ jets at LHC

The $V+\text{jets}$ processes
- background to many BSM searches
- wide range of kinematics
  $\rightarrow$ multi-scale QCD problem

$H_T$: scalar sum of jet $p_T$'s

[ATLAS-CONF-2016-046]