Introduction to Quantum Chromodynamics

Lecture 3

Steffen Schumann

Institute for Theoretical Physics

University of Göttingen

Quy Nhon – MCnet school 2019

17/09/19

GEFÖRDERT VOM

Bundesministerium für Bildung und Forschung

NLO QCD predictions

Hard Processes at Next-to-Leading Order QCD

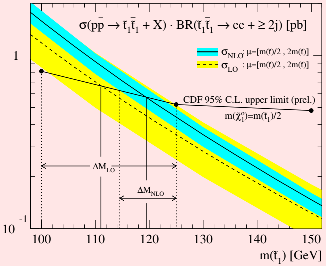
The need for higher precision

- derive exclusion bounds from a given measurement
  - non-observation of a given hypothesis, e.g. heavy Higgs, BSM model
  - crucial to know the corresponding production cross sections precisely
  - derive limits on particle masses, model parameters
- counting experiments to extract certain parameters, couplings

$\rightsquigarrow$ at hadron colliders α_S corrections indispensible, need one-loop QCD

stop mass bounds from Tevatron

[from the early days, T. Plehn private communication]



Higgs coupling extraction

- $\sigma(gg \to h \to \gamma\gamma) \sim \frac{\Gamma_g \Gamma_\gamma}{\Gamma}$
- $\sigma(gg \to h \to WW^*) \sim \frac{\Gamma_g \Gamma_W}{\Gamma}$
- $\sigma(qq \to qqh,\ h \to \tau\tau) \sim \frac{\Gamma_W \Gamma_\tau}{\Gamma}$
- ...

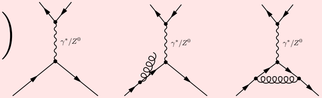
$\rightsquigarrow$ $\Gamma_i \sim g_{iih}$, $\Gamma_g \sim g_{tth}$

$\rightsquigarrow$ theoretical uncertainties dominate

$\rightsquigarrow$ systematics reduced for σ ratios

Anatomy of NLO QCD calculations

$$|\mathcal{M}|^2 = |\mathcal{M}_B|^2 + \alpha_S \left(|\mathcal{M}_R|^2 + 2\Re(\mathcal{M}_B \mathcal{M}_V^\dagger) \right)$$



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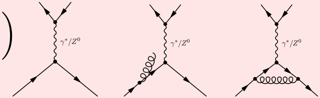

$$\sigma_{2\to n}^{NLO} = \int_n \mathrm{d}^{(4)}\sigma^B + \underbrace{\int_{n+1} \mathrm{d}^{(d)}\sigma^R + \int_n \mathrm{d}^{(d)}\sigma^V}_{\alpha_S \text{ real \& virtual corrections}}$$

Hard Processes at Next-to-Leading Order QCD

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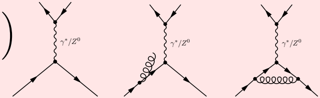

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- real-emission σ^R

  $\rightsquigarrow$ IR divergence for soft/collinear emission
- (UV renormalized) virtual-corrections σ^V

  $\rightsquigarrow$ IR divergent when propagator goes on-shell
- divergences manifest in dimensional regularisation, $d = 4 - 2\epsilon$

  $\rightsquigarrow$ occurence of single & double poles, i.e. $1/\epsilon$, $1/\epsilon^2$
- for IR safe observables poles cancel, the sum is finite [Kinoshita '62; Lee, Nauenberg '64]

Hard Processes at Next-to-Leading Order QCD

Infrared & Collinear safe observables

- interested in observable-independent formulation

$$\mathcal{O}^{\mathrm{B}} = \int |\mathcal{M}_{\mathrm{B}}^{(n)}|^2 \, d\Phi_n \Theta^{(n)}(\mathcal{O}, \{p_n\})$$

$$\mathcal{O}^{\mathrm{R}} = \int |\mathcal{M}_{\mathrm{R}}^{(n+1)}|^2 \, d\Phi_{n+1} \Theta^{(n+1)}(\mathcal{O}, \{p_{n+1}\})$$

$$\mathcal{O}^{\mathrm{V}} = \int 2\Re(\mathcal{M}_{\mathrm{B}} \mathcal{M}_{\mathrm{V}}^{\dagger}) \, d\Phi_n \Theta^{(n)}(\mathcal{O}, \{p_n\})$$

with $d\Phi_{n/n+1}$ the $n/n+1$ particle phase space, $\Theta^{(n/n+1)}$ obs. measure function

Hard Processes at Next-to-Leading Order QCD

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\end{aligned}$$

with $\mathrm{d}\Phi_{n/n+1}$ the $n/n+1$ particle phase space, $\Theta^{(n/n+1)}$ obs. measure function

- formal requirements on the measurement function:

$$\begin{aligned}
\Theta^{(n+1)}(p_1, \ldots, p_i = \lambda q, \ldots, p_{n+1}) &\to \Theta^{(n)}(p_1, \ldots, p_{n+1}) \\
\text{for } \lambda \to 0 & \quad \text{soft limit} \\
\Theta^{(n+1)}(p_1, \ldots, p_i, \ldots, p_j, \ldots, p_{n+1}) &\to \Theta^{(n)}(p_1, \ldots, p, \ldots, p_{n+1}) \\
\text{for } p_i \to zp\,, \quad p_j = (1-z)p & \quad \text{collinear limit}
\end{aligned}$$

simply remove (soft) parton i, or replace collinear pair $\{p_i, p_j\}$ by $p = p_i + p_j$

$\leadsto$ definition of infrared & collinear safe observables

Anatomy of NLO QCD calculations cont'd

$$\sigma_{2\to n}^{NLO} = \int_n \mathrm{d}^{(4)}\sigma^B + \underbrace{\int_{n+1} \mathrm{d}^{(d)}\sigma^R + \int_n \mathrm{d}^{(d)}\sigma^V}_{\alpha_S \text{ real \& virtual corrections}}$$

- finite corrections for IR safe observables
- analytical integration with cuts in d-dim infeasible for high-multi final states
- we are interested in fully-differential answer, use of MC integration methods
  however, both terms σ^R & σ^V exhibit divergences, i.e. are unbound
  σ^R in $(n+1)$-particle phase space, σ^V in n-particle phase space

Hard Processes at Next-to-Leading Order QCD

Anatomy of NLO QCD calculations cont'd

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Subtraction methods

- attempt to devise local *subtraction term* such that both integrals are separately finite and can be performed as ordinary multi-dimensional phase-space integrals

  $\rightsquigarrow$ based on universality of QCD IR divergences

  $\rightsquigarrow$ aim for IR regularisation at integrand level

Subtraction methods cont'd

- subtraction terms provide local approximation for the real-emission process

$$\int_{n+1} \mathrm{d}^{(d)}\sigma^{\mathrm{A}} = \int_n \int_1 \mathrm{d}^{(d)}\sigma^{\mathrm{A}}$$

$\hookrightarrow$ captures soft & collinear limits of amplitudes [$1/\epsilon$ and $1/\epsilon^2$ poles]

$\hookrightarrow$ analytically integrable (d-dim) over one-parton emission phase space

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- add & subtract from NLO differential cross section

$$\sigma^{NLO}_{2\to n} = \int_{n+1} \left[\mathrm{d}^{(4)}\sigma^{\mathrm{R}} - \mathrm{d}^{(4)}\sigma^{\mathrm{A}}\right] + \int_n \left[\mathrm{d}^{(4)}\sigma^{\mathrm{B}} + \int_{\mathrm{loop}} \mathrm{d}^{(d)}\sigma^{\mathrm{V}} + \int_1 \mathrm{d}^{(d)}\sigma^{\mathrm{A}}\right]_{\epsilon=0}$$

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  $\rightsquigarrow$ separately finite phase-space integrals (4-dim)

  $\rightsquigarrow$ can be performed by means of MC integration

  $\rightsquigarrow$ no ambiguous or unphysical cut-offs/scales get introduced

Hard Processes at Next-to-Leading Order QCD

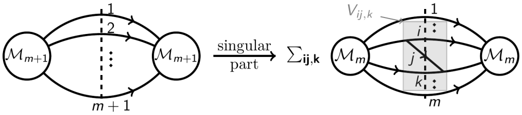
Dipole subtraction method [Catani, Seymour Nucl. Phys. B **485** (1997) 291]

- Catani & Seymour presented general expression for $\mathrm{d}^{(d)}\sigma^{\mathrm{A}}$ (known as the dipole factorisation formula)
- constructed from Born process using universal dipole terms

$$\int_{n+1} \mathrm{d}^{(d)}\sigma^{\mathrm{A}} = \sum_{\text{dipoles}} \int_n \mathrm{d}^{(d)}\sigma^{\mathrm{B}} \otimes \int_1 \mathrm{d}^{(d)} V_{\text{dipole}} = \int_m \left[\mathrm{d}^{(d)}\sigma^{\mathrm{B}} \otimes I\right]$$

spin- & color correlations ⟵ ↪ universal dipole terms

- sum over dipoles contains all real-emission soft/collinear divergences
- suited for any process with massless partons (0, 1, 2 initial-state hadrons)
- generalisation to massive partons [Catani,Dittmaier, Seymour, Trocsanyi Nucl. Phys. B **627** (2002) 189]



Dipole subtraction method cont'd

- subtraction method for arbitrary observables (IRC safe)

$$
\begin{aligned}
\mathcal{O}^{\mathrm{B}} &= \int |\mathcal{M}_{\mathrm{B}}^{(n)}|^2 \, \mathrm{d}\Phi_n \Theta^{(n)}(\mathcal{O}, \{p_n\}) \\
\mathcal{O}^{\mathrm{RS}} &= \int \mathrm{d}\Phi_{n+1} \Big[|\mathcal{M}_{\mathrm{R}}^{(n+1)}(\{p_n\})|^2 \, \Theta^{(n+1)}(\mathcal{O}, \{p_{n+1}\}) \\
&\qquad - \sum_{i \neq j \neq k} \mathcal{D}_{ij,k}(\{p_n\}) \;\; \Theta^{(n)}(\mathcal{O}, p_1, .., \tilde{p}_{ij}, \tilde{p}_k, .., p_{n+1}\}) \Big] \\
\mathcal{O}^{\mathrm{VI}} &= \int \mathrm{d}\Phi_n \left[2\Re(\mathcal{M}_{\mathrm{B}} \mathcal{M}_{\mathrm{V}}^{\dagger}) + \int_1 [\mathrm{d}\tilde{p}] \mathcal{D}_{ij,k} \right] \Theta^{(n)}(\mathcal{O}, \{p_n\})
\end{aligned}
$$

- $[\mathrm{d}\tilde{p}]$ phase-space element of one-parton
- such, differential & integrated subtraction terms cancel exactly

  $\hookrightarrow$ requires mapping of $(n+1) \to n$ configuration

  $\hookrightarrow$ need to distinguish different dipole types

Dipole subtraction method cont'd

- dipoles involve three partons, splitting particles i, j and spectator k
- distinguish initial- & final-state splitter/spectator
- all contributions known, suitable for automated evaluation

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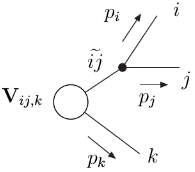

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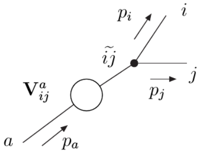

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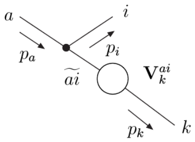

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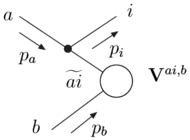

Example: $e^+e^- \to q\bar{q}$ @ NLO QCD

- most simple scattering process
- sensitive to QCD colour charges & strong coupling α_S
- first non-trivial contribution to jet sub-structure

$\rightsquigarrow$ stringent test of QCD dynamics

Example: $e^+e^- \to q\bar{q}$ @ NLO QCD

- most simple scattering process
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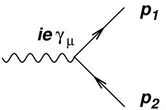
  $\rightsquigarrow$ stringent test of QCD dynamics

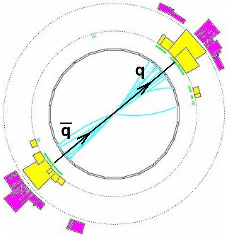
Born-level contribution: $(e^+e^- \to)\gamma^* \to q\bar{q}$

$\mathcal{M}_{q\bar{q}} =$ 

$$= \bar{u}_a(p_1) ie_q \gamma_\mu \delta_{ab} v_b(p_2)$$

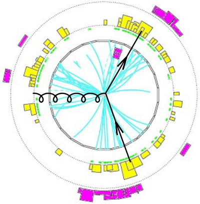
$$\sigma^{\mathrm{B}}_{q\bar{q}} = \sigma^{\mathrm{LO}}_{q\bar{q}} \propto |\mathcal{M}_{q\bar{q}}|^2 = \frac{4\pi\alpha^2}{3Q^2} e_q^2 N_c$$

where $Q^2 = (p_1 + p_2)^2$

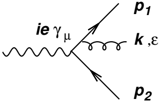


Example: $e^+e^- \to q\bar{q}$ @ NLO QCD

real-emission correction, i.e. $(e^+e^- \to)\gamma^* \to q\bar{q}g$



$\mathcal{M}_{q\bar{q}g} =$



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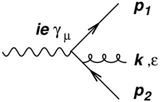

$$= -\bar{u}_a(p_1) i g_s \not{\epsilon} t^A_{ab} \frac{i(\not{p}_1 + \not{k})}{(p_1+k)^2} i e_q \gamma_\mu v_b(p_2) + \bar{u}_a(p_1) i e_q \gamma_\mu \frac{i(\not{p}_2 + \not{k})}{(p_2+k)^2} i g_s \not{\epsilon} t^A_{ab} v_b(p_2)$$

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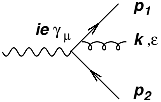
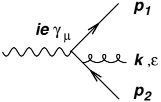
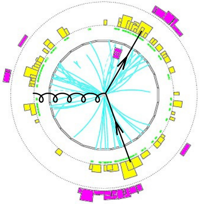
$\mathcal{M}_{q\bar{q}g} =$ 

$+$ 



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defining $Q^2 = (p_1 + p_2 + k)^2 = s$ and $x_i = 2p_i \cdot Q/Q^2$, we find

$$|\mathcal{M}_{q\bar{q}g}|^2 = C_F \frac{8\pi\alpha_S}{Q^2} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} |\mathcal{M}_{q\bar{q}}|^2$$ $\rightsquigarrow$ **singular when $x_{1/2} \to 1$**

associated phase-space element

$$\mathrm{d}\Phi^{(3)} = \frac{Q^2}{16\pi^2} dx_1 dx_2 \;\; \Theta(1-x_1)\Theta(1-x_2)\Theta(x_1+x_2-1)$$

real-emission subtraction term there are two FF dipoles contributing

$$\mathcal{D}_{13,2}(p_1,p_2,k) = \frac{1}{2p_1 k} V_{qg,\bar{q}} \, |\mathcal{M}_{q\bar{q}}|^2$$
$$\mathcal{D}_{23,1}(p_1,p_2,k) = \frac{1}{2p_2 k} V_{\bar{q}g,q} \, |\mathcal{M}_{q\bar{q}}|^2$$

corresponding subtracted real-emission cross section ($d = 4$)

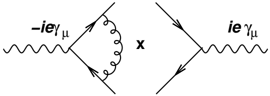
$$\sigma^{\mathrm{RS}}_{q\bar{q}g} = \int_3 \left[d\sigma^R_{\epsilon=0} - d\sigma^A_{\epsilon=0} \right] \qquad \left(\widetilde{p}_{ij} + \widetilde{p}_k = Q \,, \widetilde{p}_k = \frac{1}{x_k} p_k \,, \widetilde{p}_{ij} = Q - \frac{1}{x_k} p_k \right)$$

$$= |\mathcal{M}_{q\bar{q}}|^2 \frac{C_F \alpha_s}{2\pi} \int_0^1 dx_1 \; dx_2 \; \Theta(x_1 + x_2 - 1) \Bigg\{ \left[\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \right] \Theta^{(3)}(p_1,p_2,p_3)$$
$$- \left[\frac{1}{1-x_2} \left(\frac{2}{2-x_1-x_2} - (1+x_1) \right) + \frac{1-x_1}{x_2} \right] \Theta^{(2)}(\widetilde{p}_{13}, \widetilde{p}_2)$$
$$- \left[\frac{1}{1-x_1} \left(\frac{2}{2-x_1-x_2} - (1+x_2) \right) + \frac{1-x_2}{x_1} \right] \Theta^{(2)}(\widetilde{p}_{23}, \widetilde{p}_1) \Bigg\}$$

$\rightsquigarrow$ **finite as $x_{1/2} \to 1$ (implying $\Theta^{(3)} \to \Theta^{(2)}$)**

Example: $e^+e^- \to q\bar{q}$ @ NLO QCD

virtual correction & integrated dipole contribution ($d = 4 - 2\epsilon$)

$V \supset$ 

$$|\mathcal{M}^{(d)}_{q\bar{q}}|^2_{(1-loop)} \stackrel{\overline{\text{MS}}}{=} |\mathcal{M}^{(d)}_{q\bar{q}}|^2 \, \frac{C_F \alpha_s}{2\pi} \frac{1}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon} \left\{ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + \mathcal{O}(\epsilon) \right\}$$

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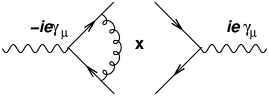
$V \supset$ 

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integrated dipole kernels

$$I(\epsilon) = \frac{C_F \alpha_s}{2\pi} \frac{1}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon \left\{ \frac{2}{\epsilon^2} + \frac{3}{\epsilon} + 10 - \pi^2 + \mathcal{O}(\epsilon) \right\}$$

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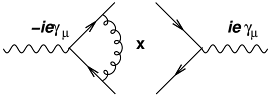
$V \supset$ 

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combining both contributions yields

$$\sigma^{VI}_{q\bar{q}} = \int_2 \left[d\sigma^V_{q\bar{q}} + \int_1 d\sigma^A_{q\bar{q}}\right]_{\epsilon=0} = |\mathcal{M}_{q\bar{q}}|^2 \frac{C_F \alpha_s}{\pi} \int dy_{12} \, \delta(1 - y_{12}) \, \Theta^{(2)}(p_1, p_2)$$

where $y_{12} = 2p_1 \cdot p_2 / Q^2$

Example: $e^+e^- \to q\bar{q}$ @ NLO QCD

total scattering cross section

measurement functions given by $\Theta^{(3)} = \Theta^{(2)} = 1$

$$\sigma_{q\bar{q}}^{\mathrm{NLO}} = \int_3 \mathrm{d}\sigma_{q\bar{q}g}^{\mathrm{RS}} + \int_2 \left[\mathrm{d}\sigma_{q\bar{q}}^{B} + \mathrm{d}\sigma_{q\bar{q}}^{\mathrm{VI}}\right] = \sigma_{q\bar{q}}^{\mathrm{LO}} \left(1 + \frac{3}{4} C_F \frac{\alpha_{\mathrm{s}}}{\pi}\right) + \mathcal{O}(\alpha_{\mathrm{s}}^2)$$

Discussion:

- fully inclusive total cross section [IR safe]

  $\rightsquigarrow$ real & virtual IR singularities properly cancelled
- yields modest, finite correction
- dependent on strong-coupling parameter and its running

Example: $e^+e^- \to q\bar{q}$ @ NLO QCD

The emerging picture

- corrections to σ_{tot} dominated by hard, large-angle gluons
- soft gluons play no role for σ_{tot}
  - collision characterised by $t_{\text{hard}} \sim 1/Q$
  - soft-gluons emitted on long time scales $t_{\text{soft}} \sim 1/(E\theta^2)$
    $\rightsquigarrow$ cannot influence cross section
  - transition to hadrons occurs on long time scales $t_{\text{had}} \sim 1/\Lambda_{\text{QCD}}$
    $\rightsquigarrow$ can thus be ignored
- with proper choice for scale of α_s, $\mu^2 = Q^2$, perturbation theory works well

$$\sigma_{tot} = \sigma_{q\bar{q}} \left(\underbrace{1}_{\text{LO}} \underbrace{+1.045 \frac{\alpha_s(Q^2)}{\pi}}_{\text{NLO}} \underbrace{+0.94 \left(\frac{\alpha_s(Q^2)}{\pi}\right)^2}_{\text{NNLO}} \underbrace{-15 \left(\frac{\alpha_s(Q^2)}{\pi}\right)^3}_{\text{NNNLO}} + \cdots \right)$$

[coefficients given for $Q^2 = M_Z^2$, including EW corrections]

Total cross sections are inclusive quantities,

inclusive in the number of additional QCD partons!

Scale Uncertainties of Fixed-Order Predictions

The scale ambiguity

- fixed-order prediction depends on (arbitrary) renormalisation scale μ
- estimate theory uncertainty through scale variations, e.g. $\frac{Q}{2} < \mu < 2Q$

  $\rightsquigarrow$ provides estimate of uncalculated higher-order terms

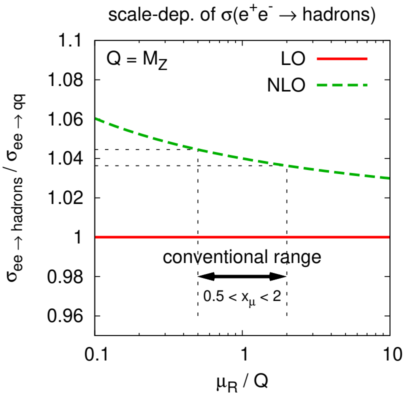
$$\begin{aligned} \sigma_{tot}^{\text{NLO}}(\mu^2) &= \sigma_{q\bar{q}}\left(1 + C_1\alpha_s(\mu^2)\right) \\ &\quad \text{with } \alpha_s(\mu^2) = \alpha_s(Q^2) - 2b_0\alpha_s^2(Q^2)\ln\left(\frac{\mu}{Q}\right) + \mathcal{O}(\alpha_s^3) \\ &= \sigma_{q\bar{q}}\left(1 + C_1\alpha_s(Q^2) - 2C_1b_0\alpha_s^2(Q^2)\ln\left(\frac{\mu}{Q}\right) + \mathcal{O}(\alpha_s^3)\right) \end{aligned}$$

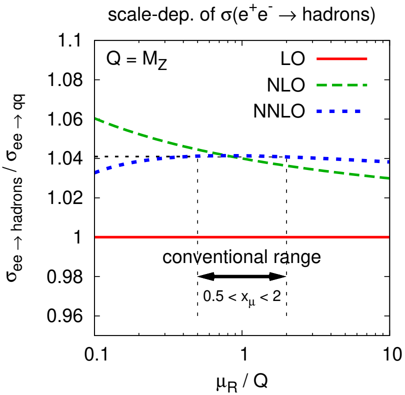
compare to

$$\begin{aligned} \sigma_{tot}^{\text{NNLO}}(\mu^2) &= \sigma_{q\bar{q}}\left(1 + C_1\alpha_s(\mu^2) + C_2(\mu^2)\alpha_s^2(\mu^2)\right) \\ &\quad \text{with } C_2(\mu^2) = C_2(Q^2) + 2C_1b_0\alpha_s^2(Q^2)\ln\left(\frac{\mu}{Q}\right) \end{aligned}$$

Scale Uncertainties of Fixed-Order Predictions

The scale ambiguity





⇝ use scale variations to estimate theory uncertainty

⇝ probe residual dependence on (uncalculated) higher orders

NLO QCD: brief summary

Motivations

- accurate cross-section estimates
- real-emission kinematics corrections
- reduced systematic uncertainties [scale dependences]

NLO QCD: brief summary

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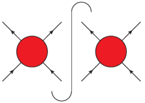
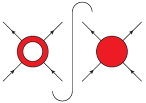
Anatomy

$$\hat{\sigma}_{ij\to X} = \int \mathrm{d}\Phi_{ij\to X} \Big[$$



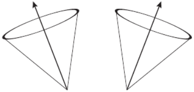
lowest order term

$$+ \sum 2\,\mathrm{Re}\Big\{$$

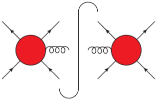


$$\Big\} \Big]$$

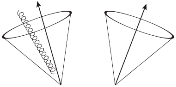
quantum corrections



$$+ \sum_{k\in\{q,g\}} \int \mathrm{d}\Phi_{ij\to X+k}$$



radiative corrections



↪ subtraction of infrared singularities in real- & virtual corrections

[Catani, Seymour Nucl. Phys. B **485** (1997) 291, Frixione *et al.* Nucl. Phys. B **467** (1996) 399]

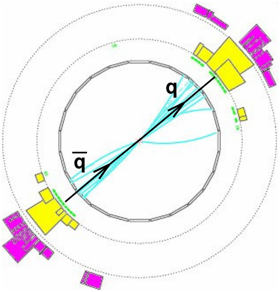
↪ dedicated one-loop amplitude codes: BlackHat, OpenLoops/Collier, Recola, ...

Interlude: QCD jets & PDFs

The emergent picture: final-state jets

Jet definition (prel.): jets are collimated sprays of hadronic particles

- hard partons undergo soft and collinear showering
- hadrons closely correlated with the hard partons' directions



Counting jets

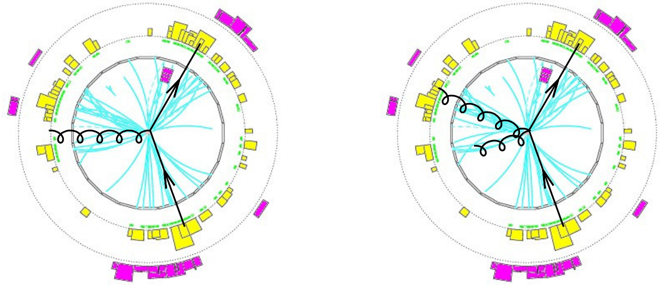
$\rightsquigarrow$ near perfect two-jet event

$\rightsquigarrow$ almost all energy contained in two cones

The emergent picture: final-state jets

Jet definition (prel.): jets are collimated sprays of particles

- hard partons undergo soft and collinear showering
- hadrons closely correlated with the hard partons' directions



Counting jets

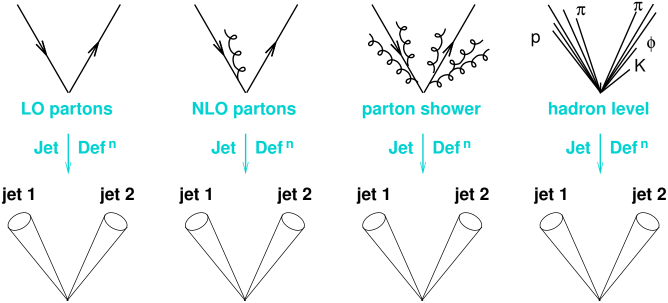
⇝ hard emissions can induce more jets

⇝ jet counting not obvious, is this a three- or four-jet event?

Defining jets

Jet definition (addendum): jet number shouldn't depend upon just a soft/collinear emission

$\rightsquigarrow$ **Infrared & collinear safety**



Infrared & Collinear safe jet definitions

crucial for comparing theory with experimental results

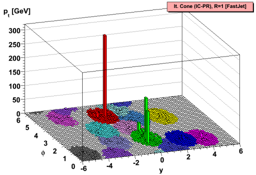
Jet algorithms

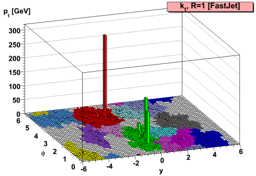
Jet definition

- group together particles into a common jets [jet algorithm]
- typical parameter is R, distance in $y - \phi$ space, determines angular reach
- combine momenta of jet constituents to yield jet momentum [recombination scheme]

two generic types of jet algorithms are commonly used:

- cone algorithms
  - widely used in the past at the Tevatron
  - jets have regular/circular shapes
  - some suffer from IR or collinear unsafety
- **sequential recombination algorithms**
  - widely used at LEP [Durham k_T algorithm]
  - jet can have irregular shapes
  - default at the LHC experiments [anti-k_T algorithm]





A generic jet finding algorithm

1. compute a distance measure y_{ij} for each pair of final-state particles
2. determine all distance measures wrt the beam y_{iB}
3. determine the minimum of all y_{ij}'s and y_{iB}'s
    1. if y_{ij} is smallest, **combine** particles ij, sum four-momenta
    2. if y_{iB} is smallest, **remove** particle i, call it a jet
4. go back to step one, until all particles are clustered into jets

in analyses one typically uses

- jets with inter-jet distances $y_{ij} > y_{\mathrm{cut}}$ [exclusive mode]
- jets with inter-jet distances $y_{ij} > y_{\mathrm{cut}}$ & $E > E_{\mathrm{cut}}$ [inclusive mode]

different algorithms use different measures: y_{ij} & y_{iB}

Sequential recombination algorithms: the k_T algorithm

recall the soft and collinear limit of the gluon-emission probability for $a \to ij$

$$dS \simeq \frac{2\alpha_s C_{A/F}}{\pi} \frac{dE_i}{\min(E_i, E_j)} \frac{d\theta_{ij}}{\theta_{ij}},$$

using $\min(E_i, E_j)$ we can avoid specifying which of i and j is soft

The k_T-algorithm distance measure

$$y_{ij} = \frac{2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij})}{Q^2}$$

- in the collinear limit: $y_{ij} \simeq \min(E_i^2 E_j^2)\theta_{ij}^2/Q^2$
- relative transverse momentum, normalized to total energy
- soft/collinear particles get clustered first

Sequential recombination algorithms: the anti-k_T algorithm

recall the soft and collinear limit of the gluon-emission probability for $a \to ij$

$$dS \simeq \frac{2\alpha_s C_i}{\pi} \frac{dE_i}{\min(E_i, E_j)} \frac{d\theta_{ij}}{\theta_{ij}} ,$$

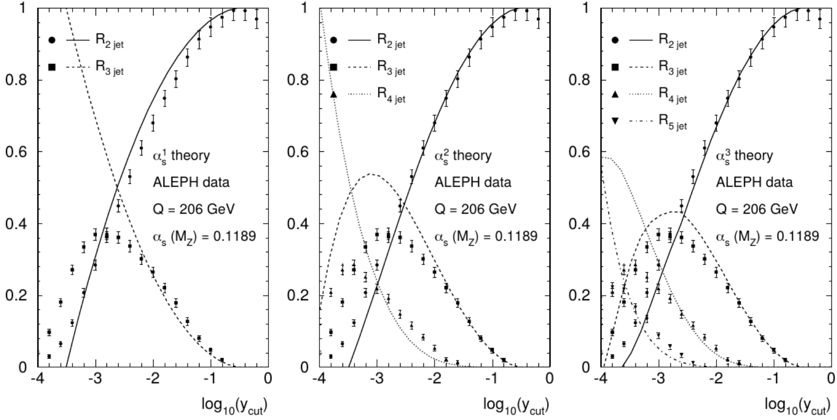
using $\min(E_i, E_j)$ we can avoid specifying which of i and j is soft

The anti-k_T-algorithm distance measure

$$y_{ij} = 2Q^2 \min(E_i^{-2}, E_j^{-2})(1 - \cos\theta_{ij})$$

- $\rightsquigarrow$ jet-finding starts out with hard objects
- $\rightsquigarrow$ softer particles get clustered into hard jets later on
- $\rightsquigarrow$ produces nicely regular shaped jets
- $\rightsquigarrow$ default in current LHC physics analyses

Jet algorithms at work: k_T jets at LEP



[Gehrmann-De Ridder et al. Phys. Rev. Lett. **100** (2008) 172001]

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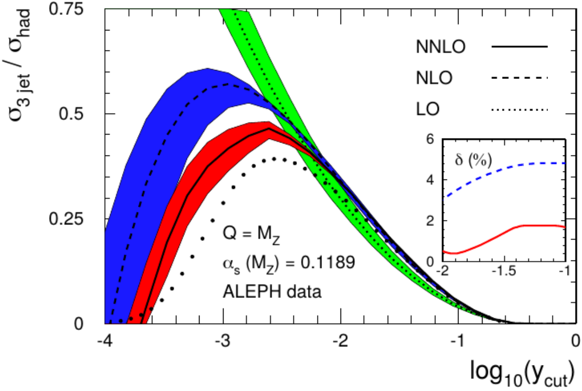
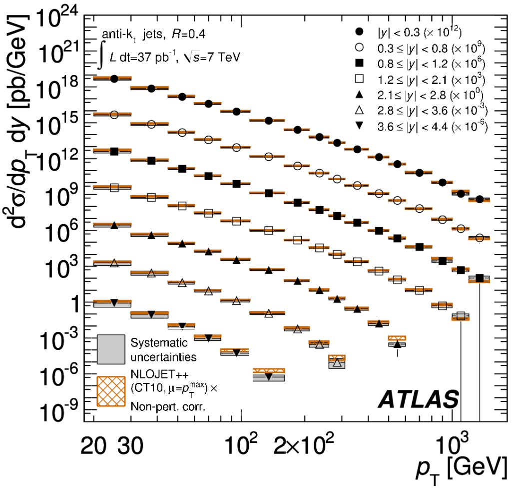


[Gehrmann-De Ridder et al. Phys. Rev. Lett. **100** (2008) 172001]

Jet algorithms at work: anti-k_T jets at LHC

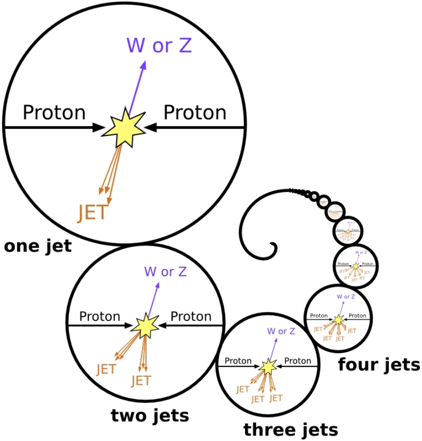
anti-k_T inclusive jets at LHC



Jet algorithms at work: anti-k_T jets at LHC

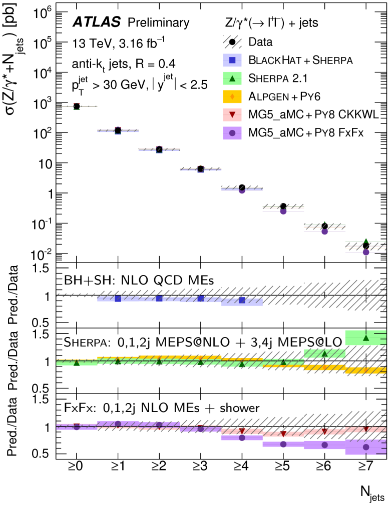
The V+jets processes

- background to many BSM searches
- wide range of kinematics
  ↪ multi-scale QCD problem



$N_{\rm jets}$: jet multiplicity

[ATLAS-CONF-2016-046]



Jet algorithms at work: anti-k_T jets at LHC

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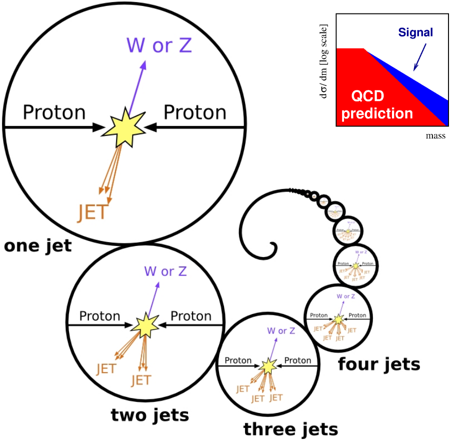
- wide range of kinematics

$\hookrightarrow$ multi-scale QCD problem



H_T: scalar sum of jet p_T's

[ATLAS-CONF-2016-046]