Introduction to Quantum Chromodynamics

Lecture 4

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Quy Nhon – MCnet school 2019

18/09/19

GEFÖRDERT VOM

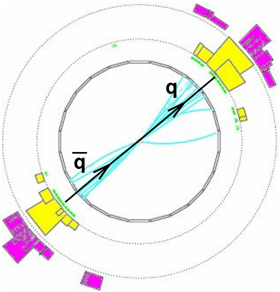
Bundesministerium
für Bildung
und Forschung

QCD jets & PDFs

The emergent picture: final-state jets

Jet definition (prel.): jets are collimated sprays of hadronic particles

- hard partons undergo soft and collinear showering
- hadrons closely correlated with the hard partons' directions



Counting jets

⤳ near perfect two-jet event

⤳ almost all energy contained in two cones

The emergent picture: final-state jets

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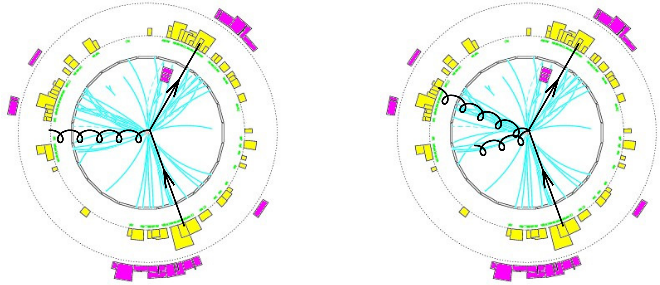


Counting jets

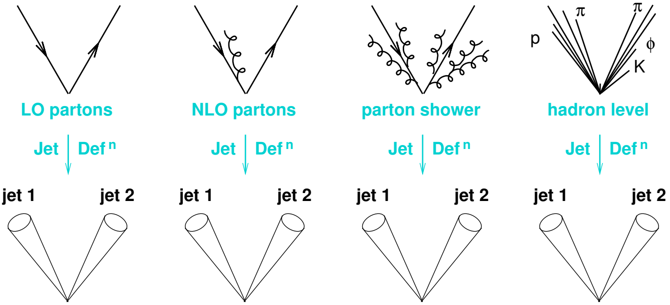
⇝ hard emissions can induce more jets

⇝ jet counting not obvious, is this a three- or four-jet event?

Defining jets

Jet definition (addendum): jet number shouldn't depend upon just a soft/collinear emission

$\leadsto$ **Infrared & collinear safety**



Infrared & Collinear safe jet definitions

crucial for comparing theory with experimental results

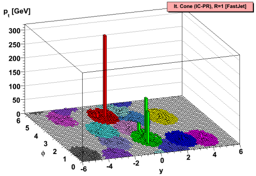
Jet algorithms

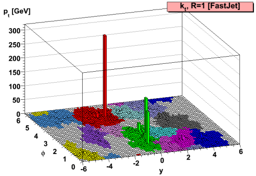
Jet definition

- group together particles into a common jets [jet algorithm]
- typical parameter is R, distance in $y - \phi$ space, determines angular reach
- combine momenta of jet constituents to yield jet momentum [recombination scheme]

two generic types of jet algorithms are commonly used:

- cone algorithms
  - widely used in the past at the Tevatron
  - jets have regular/circular shapes
  - some suffer from IR or collinear unsafety
- **sequential recombination algorithms**
  - widely used at LEP [Durham k_T algorithm]
  - jet can have irregular shapes
  - default at the LHC experiments [anti-k_T algorithm]





Sequential recombination algorithms

A generic jet finding algorithm

1. compute a distance measure y_{ij} for each pair of final-state particles
2. determine all distance measures wrt the beam y_{iB}
3. determine the minimum of all y_{ij}'s and y_{iB}'s
   1. if y_{ij} is smallest, **combine** particles ij, sum four-momenta
   2. if y_{iB} is smallest, **remove** particle i, call it a jet
4. go back to step one, until all particles are clustered into jets

in analyses one typically uses

- jets with inter-jet distances $y_{ij} > y_{\mathrm{cut}}$ [exclusive mode]
- jets with inter-jet distances $y_{ij} > y_{\mathrm{cut}}$ & $E > E_{\mathrm{cut}}$ [inclusive mode]

different algorithms use different measures: y_{ij} & y_{iB}

Sequential recombination algorithms: the k_T algorithm

recall the soft and collinear limit of the gluon-emission probability for $a \to ij$

$$dS \simeq \frac{2\alpha_s C_{A/F}}{\pi} \frac{dE_i}{\min(E_i, E_j)} \frac{d\theta_{ij}}{\theta_{ij}},$$

using $\min(E_i, E_j)$ we can avoid specifying which of i and j is soft

The k_T-algorithm distance measure

$$y_{ij} = \frac{2 \min(E_i^2, E_j^2)(1 - \cos\theta_{ij})}{Q^2}$$

- in the collinear limit: $y_{ij} \simeq \min(E_i^2 E_j^2)\theta_{ij}^2 / Q^2$
- relative transverse momentum, normalized to total energy
- soft/collinear particles get clustered first

Sequential recombination algorithms: the anti-k_T algorithm

recall the soft and collinear limit of the gluon-emission probability for $a \to ij$

$$dS \simeq \frac{2\alpha_s C_i}{\pi} \frac{dE_i}{\min(E_i, E_j)} \frac{d\theta_{ij}}{\theta_{ij}},$$

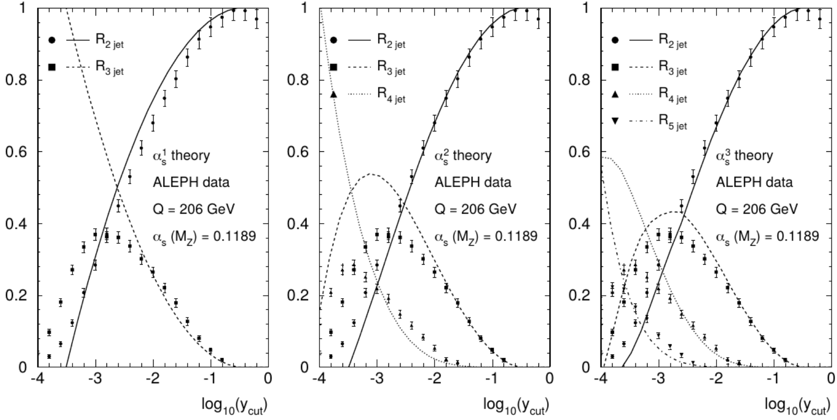
using $\min(E_i, E_j)$ we can avoid specifying which of i and j is soft

The anti-k_T-algorithm distance measure

$$y_{ij} = 2Q^2 \min(E_i^{-2}, E_j^{-2})(1 - \cos\theta_{ij})$$

- $\rightsquigarrow$ jet-finding starts out with hard objects
- $\rightsquigarrow$ softer particles get clustered into hard jets later on
- $\rightsquigarrow$ produces nicely regular shaped jets
- $\rightsquigarrow$ default in current LHC physics analyses

Jet algorithms at work: k_T jets at LEP



[Gehrmann-De Ridder et al. Phys. Rev. Lett. **100** (2008) 172001]

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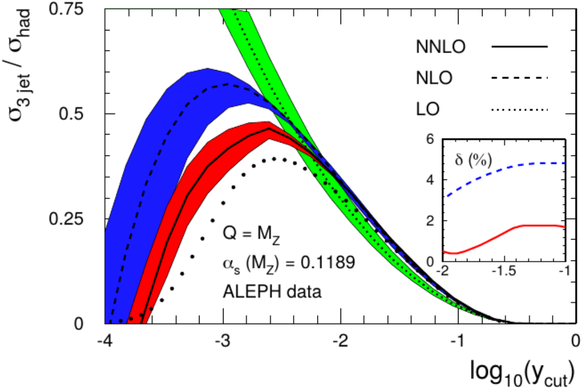

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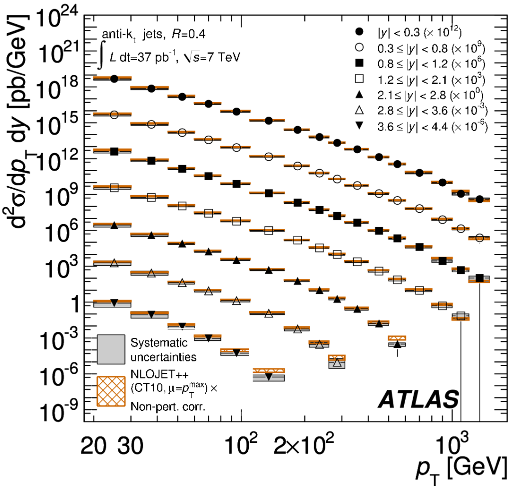
Jet algorithms at work: anti-k_T jets at LHC

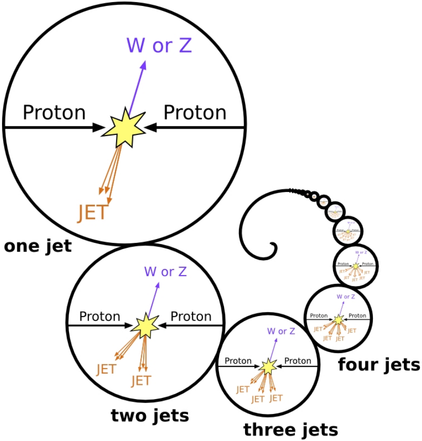
anti-k_T inclusive jets at LHC



The V+jets processes

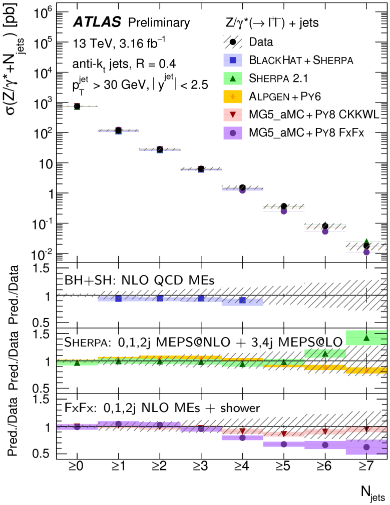
- background to many BSM searches
- wide range of kinematics

  $\hookrightarrow$ multi-scale QCD problem



$N_{\rm jets}$: jet multiplicity

[ATLAS-CONF-2016-046]



Jet algorithms at work: anti-k_T jets at LHC

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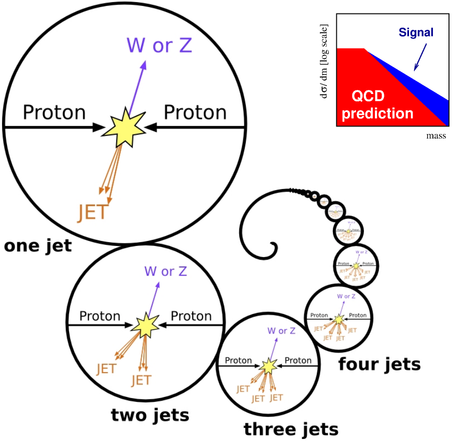
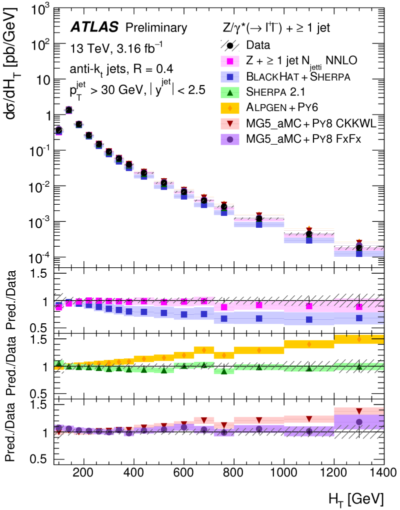


H_T: scalar sum of jet p_T's

[ATLAS-CONF-2016-046]



Processes with incoming hadrons

- so far considered processes with final-state hadrons only
- to predict cross sections for processes involving initial-state hadrons, detailed understanding of the *short distance* structure of protons is needed
- at hadron colliders all processes, even of intrinsically electroweak nature, e.g. γ, W, Z, h, are induced by quarks & gluons

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Starting point: the naïve parton model

- quarks deeply bound inside proton
- binding forces responsible for confinement due to soft gluons $\mathcal{O} \simeq \Lambda_{\mathrm{QCD}}$
- the exchange of hard gluons would break the proton apart [recoil]

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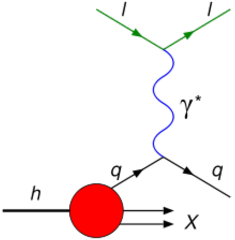
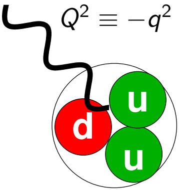
$\rightsquigarrow$ learn about the proton structure via Deep-Inelastic-Scattering (DIS)

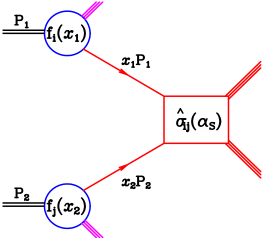




Processes with incoming hadrons: factorization

hadronic cross section in the naïve parton model

$$\sigma(s) = \sum_{ij} \int dx_1 f_{i/p}(x_1) \int dx_2 f_{j/p}(x_2)\, \hat{\sigma}_{ij\to X}(x_1 x_2 s)$$



factorized cross section

- assume partons move collinear with the protons: $p_i = x_i P_i$
- partonic cms energy: $\hat{s} = x_1 x_2 s$
- $f_{i/p}$ Parton-Distribution-Funtions parametrize number densities of quarks inside protons

Parton-Distribution-Functions: sum rules

- $|p\rangle = |\, u\ u\ d\rangle$, the valence quark distributions

$$\rightsquigarrow \int_0^1 dx\, \left(f_{u/p}(x) - f_{\bar{u}/p}(x)\right) = 2 \quad \& \quad \int_0^1 dx\, \left(f_{d/p}(x) - f_{\bar{d}/p}(x)\right) = 1$$

- fraction of proton's momentum carried by quarks

$$\sum_q \int_0^1 dx\, x f_{q/p}(x) \simeq 0.5$$

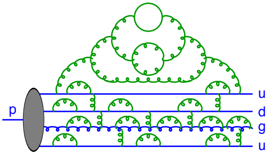
$\rightsquigarrow$ well, we kind of forgot the gluons, carry $\simeq 0.5$ of protons' momentum

$\rightsquigarrow$ gluons appear in splitting processes $q \to qg$

$\rightsquigarrow$ let's better check impact of higher-order QCD corrections

Factorization revised

most fluctuations inside the proton happen at times $t_{\rm had} \sim 1/\Lambda_{QCD}$



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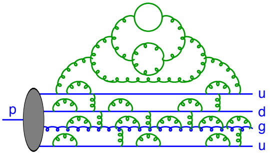
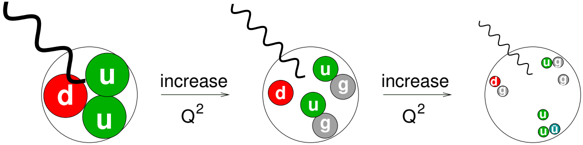
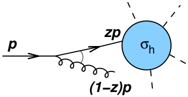


- a hard interaction (e.g. γ^* in DIS) probes much shorter times $t_{\rm hard} \sim 1/Q$
- hard probes take instantaneous snapshots of hadron structure
- PDFs are scale dependent objects: $f_{i/p}(x) \to f_{i/p}(x, Q^2)$



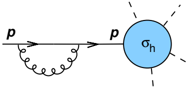
Factorization revised: the factorization scale

consider soft & collinear emissions from an inital-state quark



$$\sigma_{g+h}(p) \simeq \sigma_h(zp)\frac{\alpha_s C_F}{\pi}\frac{dz}{1-z}\frac{dk_t^2}{k_t^2}$$

where we assume σ_h involves momentum transfer $Q \gg k_t$



$$\sigma_{V+h}(p) \simeq -\sigma_h(p)\frac{\alpha_s C_F}{\pi}\frac{dz}{1-z}\frac{dk_t^2}{k_t^2}$$

total cross section receives contributions from both

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi}\underbrace{\int_0^{Q^2}\frac{dk_t^2}{k_t^2}}_{\text{infinite}}\underbrace{\int_0^1\frac{dz}{1-z}[\sigma_h(zp)-\sigma_h(p)]}_{\text{finite}}$$

regulate the singularity in the k_t integral by μ_F, the factorization scale
absorb the singularity into redefined, scale dependent, PDFs

Factorization revised: the factorization scale

define renormalised distribution function, e.g. $f_q(x, \mu_F^2)$

$$
\begin{aligned}
f_q(x,\mu_F^2) \quad = \quad & f_q^0(x) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{\mathrm{d}z}{z} f_q^0(x/z)\left[P_{q\leftarrow q}(z)\ln\left(\frac{\mu_F^2}{\mu_0^2}\right) + C_q(z) + \dots\right] \\
& + \frac{\alpha_s}{2\pi}\int_x^1 \frac{\mathrm{d}z}{z} f_g^0(x/z)\left[P_{q\leftarrow g}(z)\ln\left(\frac{\mu_F^2}{\mu_0^2}\right) + C_g(z) + \dots\right]
\end{aligned}
$$

- "bare" PDFs $f^0_{q/g}(x)$ defined at input scale μ_0^2
- emissions with $k_t \lesssim \mu_F$ implicitely included in PDFs
- emissions with $k_t \gtrsim \mu_F$ described by the hard process
- typically we identify μ_F^2 with the inherent process scale Q^2

Factorization into hard and soft component (resummed in PDFs)

$$
\sigma_{pp\to X_{\mathrm{part}}}(s;\mu_R^2,\mu_F^2) \equiv \sum_{ij}\int \mathrm{d}x_1 \mathrm{d}x_2\; f_{i/p}(x_1,\mu_F^2) f_{j/p}(x_2,\mu_F^2)\; \mathrm{d}\hat{\sigma}_{ij\to X_{\mathrm{part}}}(\hat{s};\{p_X\},\mu_R^2,\mu_F^2)
$$

Scale evolution of the PDFs

- change of PDFs wrt μ_F^2 covered by perturbative QCD
  $\rightsquigarrow$ DGLAP evolution equations

[Dokshitzer Sov. Phys. JETP **46** (1977) 641][Gribov, Lipatov Sov. J. Nucl. Phys. **15** (1972) 438][Altarelli, Parisi Nucl. Phys. B **126** (1977) 298]

$$\mu_F^2 \frac{\partial f_i(x,\mu_F^2)}{\partial \mu_F^2} = \sum_j \int_x^1 \frac{\mathrm{d}z}{z} \frac{\alpha_s}{2\pi} P_{i\leftarrow j}(z) f_j(x/z,\mu_F^2)$$

- based on *regularised* splitting functions, finite in the limit $z \to 1$
  employ plus-prescription

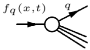
$$[g(z)]_+ \equiv g(z) - \delta(1-z) \int_0^1 dy\, g(y)$$

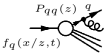
$$\int_0^1 \mathrm{d}z\ [g(z)]_+ f(z) = \int_0^1 \mathrm{d}z\, g(z)\,(f(z) - f(1))$$

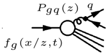
Scale evolution of the PDFs

coupled system of integro-differential equations for the parton content

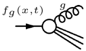
$$t\frac{\partial f_q(x,t)}{\partial t} = \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{\mathrm{d}z}{z}\left[P_{q\leftarrow q}(z) f_q(x/z,t) + P_{q\leftarrow g}(z) f_g(x/z,t)\right]$$

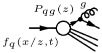


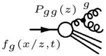




$$t\frac{\partial f_g(x,t)}{\partial t} = \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{\mathrm{d}z}{z}\left[P_{g\leftarrow q}(z) f_q(x/z,t) + P_{g\leftarrow g}(z) f_g(x/z,t)\right]$$

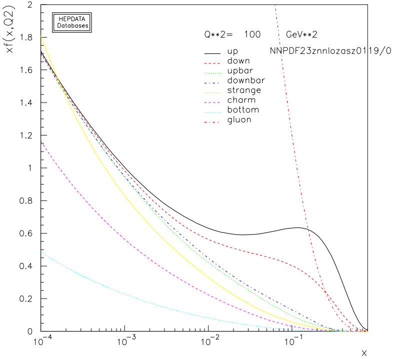




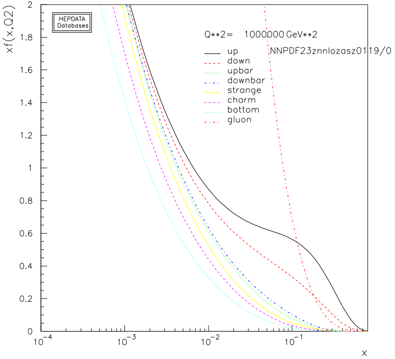


PDFs for the LHC

$Q = 10$ GeV



$Q = 1$ TeV



- current PDF sets extracted from DIS, $p\bar{p}$, pp & fixed target data
- more and more LHC data gets included in fits
- much, much more to come over the next years

NLO QCD: state of the art

NLO QCD: state of the art

Fully automated differential NLO calculations

$$\sigma_{2\to n}^{NLO} = \int_{n+1}\left[\mathrm{d}^{(4)}\sigma^{\mathrm{R}} - \mathrm{d}^{(4)}\sigma^{\mathrm{A}}\right] + \int_{n}\left[\mathrm{d}^{(4)}\sigma^{B} + \int_{\mathrm{loop}}\mathrm{d}^{(d)}\sigma^{\mathrm{V}} + \int_{1}\mathrm{d}^{(d)}\sigma^{\mathrm{A}}\right]_{\epsilon=0}$$

Monte-Carlo codes

- all the tree-level bits
- subtraction of singularities
- efficient phase-space integration

One-Loop codes

- Loop amplitudes, *i.e.* $2\Re(\mathcal{M}_{\mathrm{B}}\mathcal{M}_{\mathrm{V}}^{\dagger})$
- Loop integration

$\rightsquigarrow$ $1/\epsilon$, $1/\epsilon^2$ coefficients & finite terms

some recent NLO calculations:

2009 $W + 3$jets, $t\bar{t} + 1$jet

2010 $W + 4$jets, $Z + 3$jets

2011 $Z + 4$jets, $t\bar{t} + 2$jets, 4jets

2012 $\gamma + 3$jets

2013 $W + 5$jets, 5jets

2014 $\gamma\gamma + 3$jets

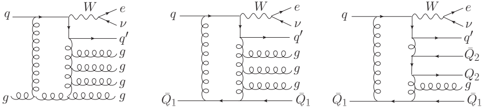
OL tools & names:

- BlackHat: Bern et al.
- HelacNLO: Bevilacqua et al.
- OpenLoops: Pozzorini et al.
- GoSam: Cullen et al.
- NJET: Biedermann et al.
- Recola: Denner et al.

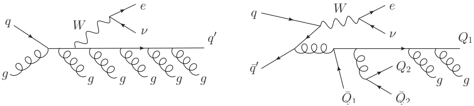
NLO QCD: state of the art – $W + 5$jets

$W + 5$j @ NLO: The challenge

- one-loop corrections



- real emission corrections



first evaluation

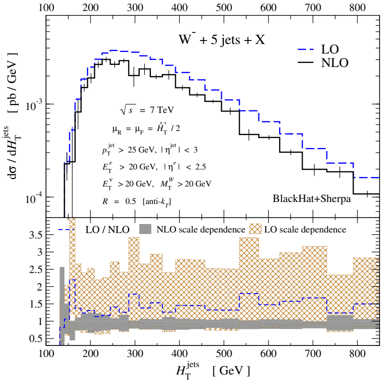
- BlackHat+Sherpa: Bern et al. [Phys. Rev. D **88** (2013) 1, 014025]
  - BlackHat: on-shell methods for one-loop amplitudes [arXiv:0808.0941]
  - Sherpa: dipole subtraction, real-emission, phase space, steering
  - ↪ fully differential partonic event generator with NLO accuracy

NLO QCD: state of the art – $W + 5$jets

BlackHat+Sherpa: 7 TeV LHC predictions [Phys. Rev. D **88** (2013) 1, 014025]

- consider anti-k_t jets with $p_T^{\text{jet}} > 25$ GeV & R=0.5

process	W^- – LO	W^- – NLO	W^+ – LO	W^+ – NLO
xsec [pb]	$1.076(0.003)^{+0.985}_{-0.480}$	$0.77(0.02)^{+0.07}_{-0.19}$	$2.005(0.006)^{+1.815}_{-0.888}$	$1.45(0.04)^{+0.12}_{-0.34}$



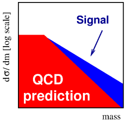
central scale

$$\mu_R = \mu_F = \mu = \hat{H}'_T/2$$

$$\hat{H}'_T \equiv \sum_i p_{T,i}^{\text{jet}} + \sqrt{M_W^2 + p_{T,W}^2}$$

scale variations

$$\mu/2, \mu/\sqrt{2}, \mu, \sqrt{2}\mu, 2\mu$$

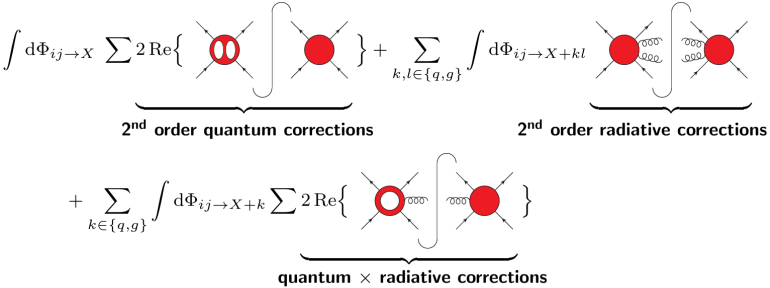


Beyond NLO QCD

NNLO QCD precision

The anatomy of NNLO QCD calculations

- terms contributing at NNLO



$\hookrightarrow$ two-loop corrections known for more and more processes now
$pp \to V, VV, VH, t\bar{t}, Hj, Zj, jj, \ldots$

$\hookrightarrow$ fully general local infrared subtraction schemes in the making
[Antennas: Gehrmann-De Ridder *et al.* JHEP **0509** (2005) 056, q_T-subt: Catani, Grazzini Phys. Rev. Lett. **98** (2007) 222002, N-jettiness: Gaunt *et al.* JHEP **1509** (2015) 058, CoLoRFulNNLO: Phys. Rev. D **94** (2016) no.7, 074019, ...]

NNLO QCD precision

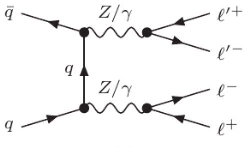
Fully differential $ZZ \to 4l$ production at NNLO QCD

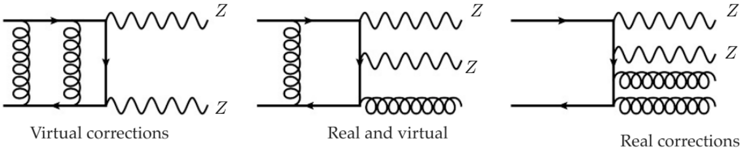
[Kallweit, Wiesemann: Phys. Lett. B **786** (2018) 382]

- based on q_T subtraction

  [Catani, Grazzini Phys. Rev. Lett. **98** (2007) 222002]
- implementation in MATRIX code

  [Grazzini et al. Eur. Phys. J. C **78** (2018) no.7, 537]





NNLO QCD precision

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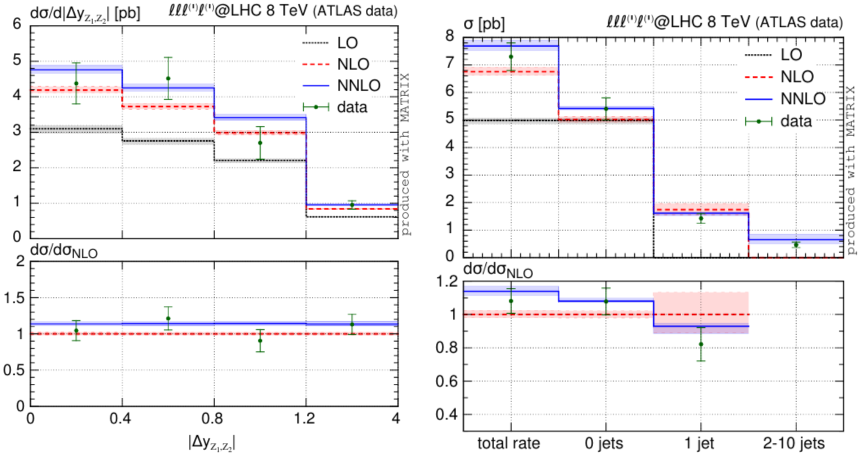
channel	$\sigma_{\rm LO}$ [fb]	$\sigma_{\rm NLO}$ [fb]	$\sigma_{\rm NNLO}$ [fb]	$\sigma_{\rm ATLAS}$ [fb]
$e^+e^-\mu^+\mu^-$	$8.188(1)^{+2.4\%}_{-3.2\%}$	$11.30(0)^{+2.5\%}_{-2.0\%}$	$12.92(1)^{+2.8\%}_{-2.2\%}$	$12.4\ ^{+1.0}_{-1.0}(\text{stat})\ ^{+0.6}_{-0.5}(\text{syst})\ ^{+0.3}_{-0.2}(\text{lumi})$
$e^+e^-e^+e^-$	$4.654(0)^{+2.3\%}_{-3.1\%}$	$6.410(2)^{+2.5\%}_{-2.0\%}$	$7.310(8)^{+2.7\%}_{-2.1\%}$	$5.9\ ^{+0.8}_{-0.8}(\text{stat})\ ^{+0.4}_{-0.4}(\text{syst}) \pm 0.1(\text{lumi})$
$\mu^+\mu^-\mu^+\mu^-$	$3.565(0)^{+2.6\%}_{-3.5\%}$	$4.969(5)^{+2.5\%}_{-2.0\%}$	$5.688(6)^{+2.9\%}_{-2.2\%}$	$4.9\ ^{+0.6}_{-0.5}(\text{stat})\ ^{+0.3}_{-0.2}(\text{syst}) \pm 0.1(\text{lumi})$
$e^+e^-\nu\nu$	$5.558(0)^{+0.1\%}_{-0.5\%}$	$4.806(1)^{+3.5\%}_{-3.9\%}$	$5.083(8)^{+1.9\%}_{-0.6\%}$	$5.0\ ^{+0.8}_{-0.7}(\text{stat})\ ^{+0.5}_{-0.4}(\text{syst}) \pm 0.1(\text{lumi})$
$\mu^+\mu^-\nu\nu$	$5.558(0)^{+0.1\%}_{-0.5\%}$	$4.770(4)^{+3.6\%}_{-4.0\%}$	$5.035(9)^{+1.8\%}_{-0.5\%}$	$4.7\ ^{+0.7}_{-0.7}(\text{stat})\ ^{+0.5}_{-0.4}(\text{syst}) \pm 0.1(\text{lumi})$
total rate	$4982(0)^{+1.9\%}_{-2.7\%}$	$6754(2)^{+2.4\%}_{-2.0\%}$	$7690(5)^{+2.7\%}_{-2.1\%}$	$7300\ ^{+400}_{-400}(\text{stat})\ ^{+300}_{-300}(\text{syst})\ ^{+200}_{-100}(\text{lumi})$

[ATLAS data 8 TeV: Aaboud et al. JHEP **1701** (2017) 099]

Fully differential $ZZ \to 4l$ production at NNLO QCD

[Kallweit, Wiesemann: Phys. Lett. B **786** (2018) 382]



[ATLAS data 8 TeV: Aaboud et al. JHEP **1701** (2017) 099]