QCD jets & PDFs
The emergent picture: final-state jets

Jet definition (prel.): jets are collimated sprays of hadronic particles
- hard partons undergo soft and collinear showering
- hadrons closely correlated with the hard partons’ directions

Counting jets
- near perfect two-jet event
- almost all energy contained in two cones
The emergent picture: final-state jets

Jet definition (prel.): jets are collimated sprays of particles
- hard partons undergo soft and collinear showering
- hadrons closely correlated with the hard partons’ directions

Counting jets
- hard emissions can induce more jets
- jet counting not obvious, is this a three- or four-jet event?
Defining jets

Jet definition (addendum): jet number shouldn’t depend upon just a soft/collinear emission

\(\sim\) Infrared & collinear safety

Infrared & Collinear safe jet definitions

crucial for comparing theory with experimental results
Jet algorithms

Jet definition

- group together particles into a common jets [jet algorithm]
- typical parameter is $R$, distance in $y - \phi$ space, determines angular reach
- combine momenta of jet constituents to yield jet momentum [recombination scheme]

Two generic types of jet algorithms are commonly used:

- cone algorithms
  - widely used in the past at the Tevatron
  - jets have regular/circular shapes
  - some suffer from IR or collinear unsafety

- sequential recombination algorithms
  - widely used at LEP [Durham $k_T$ algorithm]
  - jet can have irregular shapes
  - default at the LHC experiments [anti-$k_T$ algorithm]
Sequential recombination algorithms

A generic jet finding algorithm

1. compute a distance measure $y_{ij}$ for each pair of final-state particles
2. determine all distance measures wrt the beam $y_{iB}$
3. determine the minimum of all $y_{ij}$'s and $y_{iB}$'s
   - if $y_{ij}$ is smallest, combine particles $ij$, sum four-momenta
   - if $y_{iB}$ is smallest, remove particle $i$, call it a jet
4. go back to step one, until all particles are clustered into jets

in analyses one typically uses

- jets with inter-jet distances $y_{ij} > y_{cut}$ [exclusive mode]
- jets with inter-jet distances $y_{ij} > y_{cut} \& E > E_{cut}$ [inclusive mode]

different algorithms use different measures: $y_{ij}$ & $y_{iB}$
Sequential recombination algorithms: the $k_T$ algorithm

recall the soft and collinear limit of the gluon-emission probability for $a \rightarrow ij$

$$dS \sim \frac{2\alpha_s C_A}{\pi F} \frac{dE_i}{\min(E_i, E_j)} \frac{d\theta_{ij}}{\theta_{ij}},$$

using $\min(E_i, E_j)$ we can avoid specifying which of $i$ and $j$ is soft

---

The $k_T$-algorithm distance measure

$$y_{ij} = \frac{2 \min(E_i^2, E_j^2)(1 - \cos \theta_{ij})}{Q^2}$$

$\sim$ in the collinear limit: $y_{ij} \approx \min(E_i^2, E_j^2)\theta_{ij}^2 / Q^2$

$\sim$ relative transverse momentum, normalized to total energy

$\sim$ soft/collinear particles get clustered first
Sequential recombination algorithms: the anti-$k_T$ algorithm

recall the soft and collinear limit of the gluon-emission probability for $a \rightarrow ij$

\[
dS \approx \frac{2\alpha_s C_i}{\pi} \frac{dE_i}{\min(E_i, E_j)} \frac{d\theta_{ij}}{\theta_{ij}},
\]

using $\min(E_i, E_j)$ we can avoid specifying which of $i$ and $j$ is soft

The anti-$k_T$-algorithm distance measure

\[
y_{ij} = 2Q^2 \min(E_i^{-2}, E_j^{-2})(1 - \cos \theta_{ij})
\]

$\Rightarrow$ jet-finding starts out with hard objects
$\Rightarrow$ softer particles get clustered into hard jets later on
$\Rightarrow$ produces nicely regular shaped jets
$\Rightarrow$ default in current LHC physics analyses
Jet algorithms at work: $k_T$ jets at LEP

Jet algorithms at work: $k_T$ jets at LEP

Jet algorithms at work: anti-$k_T$ jets at LHC
Jet algorithms at work: anti-$k_T$ jets at LHC

**The V+jets processes**
- background to many BSM searches
- wide range of kinematics
  $\rightarrow$ multi-scale QCD problem

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**$N_{\text{jets}}$: jet multiplicity**

$\sigma(Z/\gamma^* \rightarrow l^+l^-) + \text{jets}$

- ATLAS Preliminary
- 13 TeV, 3.16 fb$^{-1}$
- anti-$k_T$ jets, $R = 0.4$
- $p_T^{\text{jet}} > 30$ GeV, $|y^{\text{jet}}| < 2.5$

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BH+SH: NLO QCD MEs
SHERPA: 0,1,2j MEPS@NLO + 3,4j MEPS@LO
FxFx: 0,1,2j NLO MEs + shower

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$N_{\text{jets}}$: jet multiplicity
Jet algorithms at work: anti-$k_T$ jets at LHC

The $V+jets$ processes
- background to many BSM searches
- wide range of kinematics

$\rightarrow$ multi-scale QCD problem

$H_T$: scalar sum of jet $p_T$'s

[ATLAS-CONF-2016-046]
Processes with incoming hadrons

- so far considered processes with final-state hadrons only
- to predict cross sections for processes involving initial-state hadrons, detailed understanding of the short distance structure of protons is needed
- at hadron colliders all processes, even of intrinsically electroweak nature, e.g. $\gamma, W, Z, h$, are induced by quarks & gluons
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**Starting point: the naïve parton model**

- quarks deeply bound inside proton
- binding forces responsible for confinement due to soft gluons $\mathcal{O} \simeq \Lambda_{\text{QCD}}$
- the exchange of hard gluons would break the proton apart [recoil]
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Starting point: the naïve parton model

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$Q^2 \equiv -q^2$
Processes with incoming hadrons: factorization

**hadronic cross section in the naïve parton model**

\[
\sigma(s) = \sum_{ij} \int dx_1 f_{i/p}(x_1) \int dx_2 f_{j/p}(x_2) \hat{\sigma}_{ij} \to x(x_1x_2s)
\]

**factorized cross section**

- assume partons move collinear with the protons: \( p_i = x_i P_i \)
- partonic cms energy: \( \hat{s} = x_1x_2s \)
- \( f_{i/p} \) Parton-Distribution-Functions parametrize number densities of quarks inside protons
Parton-Distribution-Functions: sum rules

- $|p\rangle = |u \ u \ d\rangle$, the valence quark distributions

\[ \sim \int_0^1 dx \left( f_{u/p}(x) - f_{\bar{u}/p}(x) \right) = 2 \quad \& \quad \int_0^1 dx \left( f_{d/p}(x) - f_{\bar{d}/p}(x) \right) = 1 \]

- fraction of proton's momentum carried by quarks

\[ \sum_q \int_0^1 dx \ x f_{q/p}(x) \approx 0.5 \]

- well, we kind of forgot the gluons, carry $\approx 0.5$ of protons' momentum

- gluons appear in splitting processes $q \rightarrow qg$

- let's better check impact of higher-order QCD corrections
most fluctuations inside the proton happen at times $t_{\text{had}} \sim 1/\Lambda_{\text{QCD}}$
Factorization revised

most fluctuations inside the proton happen at times $t_{\text{had}} \sim 1/\Lambda_{QCD}$

- a hard interaction (e.g. $\gamma^*$ in DIS) probes much shorter times $t_{\text{hard}} \sim 1/Q$
- hard probes take instantaneous snapshots of hadron structure
- PDFs are scale dependent objects: $f_{i/p}(x) \rightarrow f_{i/p}(x, Q^2)$
Factorization revised: the factorization scale

consider soft & collinear emissions from an initial-state quark

\[ \sigma_{g+h}(p) \simeq \sigma_h(zp) \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2} \]

where we assume \( \sigma_h \) involves momentum transfer \( Q \gg k_t \)

\[ \sigma_{V+h}(p) \simeq -\sigma_h(p) \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2} \]

total cross section receives contributions from both

\[ \sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi} \int_0^{Q^2} \frac{dk_t^2}{k_t^2} \int_0^1 \frac{dz}{1-z} [\sigma_h(zp) - \sigma_h(p)] \]

regulate the singularity in the \( k_t \) integral by \( \mu_F \), the factorization scale
absorb the singularity into redefined, scale dependent, PDFs
Factorization revised: the factorization scale

define renormalised distribution function, e.g. $f_q(x, \mu_F^2)$

$$f_q(x, \mu_F^2) = f^0_q(x) + \frac{\alpha_s}{2\pi} \int_0^1 \frac{dz}{z} f^0_q(x/z) \left[ P_{q\leftarrow q}(z) \ln \left( \frac{\mu_F^2}{\mu_0^2} \right) + C_q(z) + \ldots \right]$$

$$+ \frac{\alpha_s}{2\pi} \int_0^1 \frac{dz}{z} f^0_g(x/z) \left[ P_{q\leftarrow g}(z) \ln \left( \frac{\mu_F^2}{\mu_0^2} \right) + C_g(z) + \ldots \right]$$

- “bare” PDFs $f^0_{q/g}(x)$ defined at input scale $\mu_0^2$
- emissions with $k_t \lesssim \mu_F$ implicitly included in PDFs
- emissions with $k_t \gtrsim \mu_F$ described by the hard process
- typically we identify $\mu_F^2$ with the inherent process scale $Q^2$

Factorization into hard and soft component (resummed in PDFs)

$$\sigma_{pp\rightarrow X_{\text{part}}}(s; \mu_R^2, \mu_F^2) \equiv \sum_{ij} \int d^4x_1 d^4x_2 \ f_{i/p}(x_1, \mu_F^2) f_{j/p}(x_2, \mu_F^2) \ d\hat{s}_{ij} \cdot x_{\text{part}}(\hat{s}; \{ pX \}, \mu_R^2, \mu_F^2)$$
Scale evolution of the PDFs

- change of PDFs wrt $\mu_F^2$ covered by perturbative QCD
  $\sim$ DGLAP evolution equations


$$\mu_F^2 \frac{\partial f_i(x, \mu_F^2)}{\partial \mu_F^2} = \sum_j \int_x^1 \frac{dz}{z} P_{i \leftarrow j}(z) f_j(x/z, \mu_F^2)$$

- based on \textit{regularised} splitting functions, finite in the limit $z \to 1$
  employ plus-prescription

$$\left[ g(z) \right]_+ \equiv g(z) - \delta(1 - z) \int_0^1 dy \ g(y)$$

$$\int_0^1 dz \ \left[ g(z) \right]_+ f(z) = \int_0^1 dz \ g(z) (f(z) - f(1))$$
Scale evolution of the PDFs

coupled system of integro-differential equations for the parton content

\[
\begin{align*}
\frac{t}{\alpha_s(t)} \frac{\partial f_q(x, t)}{\partial t} &= \frac{\alpha_s(t)}{2\pi} \int_x^1 \frac{dz}{z} [P_{q\leftarrow q}(z)f_q(x/z, t) + P_{q\leftarrow g}(z)f_g(x/z, t)] \\
\frac{t}{\alpha_s(t)} \frac{\partial f_g(x, t)}{\partial t} &= \frac{\alpha_s(t)}{2\pi} \int_x^1 \frac{dz}{z} [P_{g\leftarrow q}(z)f_q(x/z, t) + P_{g\leftarrow g}(z)f_g(x/z, t)]
\end{align*}
\]
PDFs for the LHC

$Q = 10 \text{ GeV}$

$Q = 1 \text{ TeV}$

- current PDF sets extracted from DIS, $p\bar{p}$, $pp$ & fixed target data
- more and more LHC data gets included in fits
- much, much more to come over the next years
NLO QCD: state of the art
NLO QCD: state of the art

Fully automated differential NLO calculations

\[ \sigma_{2 \rightarrow n}^{NLO} = \int_{n+1} \left[ d^{(4)}\sigma^R - d^{(4)}\sigma^A \right] + \int_n \left[ d^{(4)}\sigma^B + \int_{\text{loop}} d^{(d)}\sigma^V + \int_1 d^{(d)}\sigma^A \right] \epsilon=0 \]

Monte-Carlo codes

- all the tree-level bits
- subtraction of singularities
- efficient phase-space integration

One-Loop codes

- Loop amplitudes, i.e. \( 2\Re(M_B M_V^\dagger) \)
- Loop integration
- \( \sim 1/\epsilon, 1/\epsilon^2 \) coefficients & finite terms

some recent NLO calculations:

2009 \( W + 3\text{jets}, t\bar{t} + 1\text{jet} \)
2010 \( W + 4\text{jets}, Z + 3\text{jets} \)
2011 \( Z + 4\text{jets}, t\bar{t} + 2\text{jets}, 4\text{jets} \)
2012 \( \gamma + 3\text{jets} \)
2013 \( W + 5\text{jets}, 5\text{jets} \)
2014 \( \gamma\gamma + 3\text{jets} \)

OL tools & names:

- BlackHat: Bern et al.
- HelacNLO: Bevilacqua et al.
- OpenLoops: Pozzorini et al.
- GoSam: Cullen et al.
- NJET: Biedermann et al.
- Recola: Denner et al.
NLO QCD: state of the art – $W + 5j$ets

$W + 5j$ @ NLO: The challenge

- one-loop corrections

- real emission corrections

first evaluation

- **BlackHat**+**Sherpa**: Bern et al. [Phys. Rev. D 88 (2013) 1, 014025]
  - **BlackHat**: on-shell methods for one-loop amplitudes [arXiv:0808.0941]
  - **Sherpa**: dipole subtraction, real-emission, phase space, steering
  ↔ fully differential partonic event generator with NLO accuracy
consider anti-\(k_t\) jets with \(p_T^{\text{jet}} > 25\) GeV \& \(R=0.5\)

<table>
<thead>
<tr>
<th>process</th>
<th>(W^- - \text{LO})</th>
<th>(W^- - \text{NLO})</th>
<th>(W^+ - \text{LO})</th>
<th>(W^+ - \text{NLO})</th>
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</thead>
<tbody>
<tr>
<td>xsec [pb]</td>
<td>(1.076(0.003)^{+0.985}_{-0.480})</td>
<td>(0.77(0.02)^{+0.07}_{-0.19})</td>
<td>(2.005(0.006)^{+1.815}_{-0.888})</td>
<td>(1.45(0.04)^{+0.12}_{-0.34})</td>
</tr>
</tbody>
</table>

central scale
\[ \mu_R = \mu_F = \mu = \hat{H}_T'/2 \]

scale variations
\[ \mu/2, \mu/\sqrt{2}, \mu, \sqrt{2}\mu, 2\mu \]
Beyond NLO QCD
NNLO QCD precision

The anatomy of NNLO QCD calculations

- terms contributing at NNLO

\[
\int \! d\Phi_{ij \to X} \sum \text{2 Re}\left\{ \begin{array}{c}
\includegraphics[width=1cm]{diagram1}
\end{array} \right\} + \sum_{k, l \in \{q, g\}} \int \! d\Phi_{ij \to X + kl} \begin{array}{c}
\includegraphics[width=1cm]{diagram2}
\end{array}
\]

\[2^{\text{nd}} \text{ order quantum corrections}\]

\[2^{\text{nd}} \text{ order radiative corrections}\]

\[+ \sum_{k \in \{q, g\}} \int \! d\Phi_{ij \to X + k} \sum \text{2 Re}\left\{ \begin{array}{c}
\includegraphics[width=1cm]{diagram3}
\end{array} \right\} \]

\[\text{quantum } \times \text{ radiative corrections}\]

\[\leftrightarrow \text{ two-loop corrections known for more and more processes now}
pp \to V, VV, VH, t\bar{t}, Hj, Zj, jj, \ldots\]

\[\leftrightarrow \text{ fully general local infrared subtraction schemes in the making}\]


NNLO QCD precision

**Fully differential $ZZ \rightarrow 4l$ production at NNLO QCD**


- based on $q_T$ subtraction
- implementation in **Matrix** code
NNLO QCD precision

Fully differential $ZZ \rightarrow 4l$ production at NNLO QCD


<table>
<thead>
<tr>
<th>channel</th>
<th>$\sigma_{\text{LO}}$ [fb]</th>
<th>$\sigma_{\text{NLO}}$ [fb]</th>
<th>$\sigma_{\text{NNLO}}$ [fb]</th>
<th>$\sigma_{\text{ATLAS}}$ [fb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^-\mu^+\mu^-$</td>
<td>8.188(1)$^{+2.4%}_{-3.2%}$</td>
<td>11.30(0)$^{+2.5%}_{-2.0%}$</td>
<td>12.92(1)$^{+2.8%}_{-2.2%}$</td>
<td>12.4$^{+0.7\text{(stat)}}^{-0.5\text{(syst)}}^{+0.3\text{(lumi)}}$</td>
</tr>
<tr>
<td>$e^+e^-e^+e^-$</td>
<td>4.654(0)$^{+2.3%}_{-3.1%}$</td>
<td>6.410(2)$^{+2.5%}_{-2.0%}$</td>
<td>7.310(8)$^{+2.7%}_{-2.1%}$</td>
<td>5.9$^{+0.8\text{(stat)}}^{-0.4\text{(syst)}}^{\pm0.1\text{(lumi)}}$</td>
</tr>
<tr>
<td>$\mu^+\mu^-\mu^+\mu^-$</td>
<td>3.565(0)$^{+2.6%}_{-3.5%}$</td>
<td>4.969(5)$^{+2.5%}_{-2.0%}$</td>
<td>5.688(6)$^{+2.9%}_{-2.2%}$</td>
<td>4.9$^{+0.6\text{(stat)}}^{+0.3\text{(syst)}}^{\pm0.1\text{(lumi)}}$</td>
</tr>
<tr>
<td>$e^+e^-\nu\nu$</td>
<td>5.558(0)$^{+0.1%}_{-0.5%}$</td>
<td>4.806(1)$^{+3.5%}_{-3.9%}$</td>
<td>5.083(8)$^{+1.9%}_{-0.6%}$</td>
<td>5.0$^{+0.8\text{(stat)}}^{-0.4\text{(syst)}}^{\pm0.1\text{(lumi)}}$</td>
</tr>
<tr>
<td>$\mu^+\mu^-\nu\nu$</td>
<td>5.558(0)$^{+0.1%}_{-0.5%}$</td>
<td>4.770(4)$^{+3.6%}_{-4.0%}$</td>
<td>5.035(9)$^{+1.8%}_{-0.5%}$</td>
<td>4.7$^{+0.7\text{(stat)}}^{+0.5\text{(syst)}}^{\pm0.1\text{(lumi)}}$</td>
</tr>
<tr>
<td>total rate</td>
<td>4982(0)$^{+1.9%}_{-2.7%}$</td>
<td>6754(2)$^{+2.4%}_{-2.0%}$</td>
<td>7690(5)$^{+2.7%}_{-2.1%}$</td>
<td>7300$^{+400\text{(stat)}}^{-300\text{(syst)}}^{+200\text{(lumi)}}$</td>
</tr>
</tbody>
</table>

[ATLAS data 8 TeV: Aaboud et al. JHEP 1701 (2017) 099]
Fully differential $ZZ \rightarrow 4\ell$ production at NNLO QCD


[ATLAS data 8 TeV: Aaboud et al. JHEP 1701 (2017) 099]