Modelling MPI (in Herwig)
Eikonal model basics

Multiple hard interactions

$h_1$

$h_2$
Eikonal model basics

Starting point: hard inclusive jet cross section.

\[ \sigma^{inc}(s; p^\text{min}_t) = \sum_{i,j} \int_{p^\text{min}_t} dp_t f_i/h_1(x_1, \mu^2) \otimes \frac{d\hat{\sigma}_{i,j}}{dp_t^2} \otimes f_j/h_2(x_2, \mu^2), \]

\[ \sigma^{inc} > \sigma_{tot} \text{ eventually (for moderately small } p^\text{min}_t \text{).} \]
Eikonal model basics

\[ \sigma_{\text{tot}}: \text{DL '92} \]

\[ \cdots \sigma_{\text{tot}}: \text{DL '04} \]

\[ \text{QCD}2 \rightarrow 2, p_T > 2\text{GeV} \]

\[ \sqrt{s} \text{(GeV)} \]
Eikonal model basics

Starting point: hard inclusive jet cross section.

\[
\sigma^{\text{inc}}(s; p_t^{\text{min}}) = \sum_{i,j} \int_{p_t^{\text{min}}} \mathcal{d}p_t^2 f_{i/h_1}(x_1, \mu^2) \otimes \frac{\mathcal{d}\hat{\sigma}_{i,j}}{\mathcal{d}p_t^2} \otimes f_{j/h_2}(x_2, \mu^2),
\]

\[\sigma^{\text{inc}} \geq \sigma_{\text{tot}} \text{ eventually (for moderately small } p_t^{\text{min}}).\]

Interpretation: \(\sigma^{\text{inc}}\) counts all partonic scatters that happen during a single \(pp\) collision \(\Rightarrow\) more than a single interaction.

\[\sigma^{\text{inc}} = \bar{n}\sigma_{\text{inel}}.\]
Eikonal model basics

Use eikonal approximation (= independent scatters). Leads to Poisson distribution of number $m$ of additional scatters,

$$P_m(\vec{b}, s) = \frac{\bar{n}(\vec{b}, s)^m}{m!} e^{-\bar{n}(\vec{b}, s)} .$$

Then we get $\sigma_{\text{inel}}$:

$$\sigma_{\text{inel}} = \int d^2\vec{b} \sum_{m=1}^{\infty} P_m(\vec{b}, s) = \int d^2\vec{b} \left( 1 - e^{-\bar{n}(\vec{b}, s)} \right) .$$
Eikonal model basics

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Cf. $\sigma_{\text{inel}}$ from scattering theory in eikonal approx. with scattering amplitude $a(\vec{b}, s) = \frac{1}{2i}(e^{-\chi(\vec{b}, s)} - 1)$

$$\sigma_{\text{inel}} = \int d^2\vec{b} \left( 1 - e^{-2\chi(\vec{b}, s)} \right) \Rightarrow \chi(\vec{b}, s) = \frac{1}{2} \bar{n}(\vec{b}, s).$$

$\chi(\vec{b}, s)$ is called eikonal function.
Eikonal model basics

Calculation of \( \bar{n}(\vec{b}, s) \) from parton model assumptions:

\[
\bar{n}(\vec{b}, s) = L_{\text{partons}}(x_1, x_2, \vec{b}) \otimes \sum_{ij} \int dp_t^2 \frac{d\hat{\sigma}_{ij}}{dp_t^2} 
\]

\[
= \sum_{ij} \frac{1}{1 + \delta_{ij}} \int dx_1 dx_2 \int d^2\vec{b}' \int dp_t^2 \frac{d\hat{\sigma}_{ij}}{dp_t^2} 
\]

\[
\times D_{i/A}(x_1, p_t^2, |\vec{b}'|)D_{j/B}(x_2, p_t^2, |\vec{b} - \vec{b}'|)
\]
Eikonal model basics

Calculation of $\tilde{n}(\vec{b}, s)$ from parton model assumptions:

$$\tilde{n}(\vec{b}, s) = L_{\text{partons}}(x_1, x_2, \vec{b}) \otimes \sum_{ij} \int dp^2_t \frac{d\hat{\sigma}_{ij}}{dp^2_t}$$

$$= \sum_{ij} \frac{1}{1 + \delta_{ij}} \int dx_1 dx_2 \int d^2\vec{b}' \int dp^2_t \frac{d\hat{\sigma}_{ij}}{dp^2_t}$$

$$\times \left[ D_{i/A}(x_1, p^2_t, |\vec{b}'|)D_{j/B}(x_2, p^2_t, |\vec{b} - \vec{b}'|) \right]$$

$$= \sum_{ij} \frac{1}{1 + \delta_{ij}} \int dx_1 dx_2 \int d^2\vec{b}' \int dp^2_t \frac{d\hat{\sigma}_{ij}}{dp^2_t}$$

$$\times f_{i/A}(x_1, p^2_t)G_A(|\vec{b}'|)f_{j/B}(x_2, p^2_t)G_B(|\vec{b} - \vec{b}'|)$$

$$= A(\vec{b})\sigma^{\text{inc}}(s; p^2_{t_{\text{min}}}) .$$
Eikonal model basics

Calculation of $\tilde{n}(\vec{b}, s)$ from parton model assumptions:

$$
\tilde{n}(\vec{b}, s) = L_{\text{partons}}(x_1, x_2, \vec{b}) \otimes \sum_{ij} \int dp_t^2 \frac{d\sigma_{ij}}{dp_t^2} 
$$

$$
= \sum_{ij} \frac{1}{1 + \delta_{ij}} \int dx_1 dx_2 \int d^2\vec{b}' \int dp_t^2 \frac{d\sigma_{ij}}{dp_t^2} 
$$

$$
\times D_{i/A}(x_1, p_t^2, |\vec{b}'|) D_{j/B}(x_2, p_t^2, |\vec{b} - \vec{b}'|) 
$$

$$
= \sum_{ij} \frac{1}{1 + \delta_{ij}} \int dx_1 dx_2 \int d^2\vec{b}' \int dp_t^2 \frac{d\sigma_{ij}}{dp_t^2} 
$$

$$
\times f_{i/A}(x_1, p_t^2) G_A(|\vec{b}'|) f_{j/B}(x_2, p_t^2) G_B(|\vec{b} - \vec{b}'|) 
$$

$$
= A(\vec{b}) \sigma^{\text{inc}}(s; p_t^{\text{min}}). 
$$

$$
\Rightarrow \chi(\vec{b}, s) = \frac{1}{2} \tilde{n}(\vec{b}, s) = \frac{1}{2} A(\vec{b}) \sigma^{\text{inc}}(s; p_t^{\text{min}}). 
$$
Overlap function

\[ A(b) = \int d^2\vec{b}' G_A(|\vec{b}'|) G_B(|\vec{b} - \vec{b}'|) \]

\[ G(\vec{b}) \text{ from electromagnetic FF:} \]

\[ G_p(\vec{b}) = G_{\bar{p}}(\vec{b}) = \int \frac{d^2\vec{k}}{(2\pi)^2} \frac{e^{i\vec{k} \cdot \vec{b}}}{(1 + \vec{k}^2 / \mu^2)^2} \]

But \( \mu^2 \) not fixed to the electromagnetic 0.71 GeV\(^2\).
Free for colour charges.

\[ \Rightarrow \text{Two main parameters: } \mu^2, p^\text{min}_t. \]
Unitarized cross sections

\[ \sigma_{\text{tot}} \] from DL
\[ \sigma^{\text{inc}}, p_T > 2 \text{ GeV} \]
\[ \sigma^{\text{inel}}, p_T > 2 \text{ GeV} \]
\[ \sigma^{\text{inc}}, p_T > 3.1 \text{ GeV} \]
\[ \sigma^{\text{inel}}, p_T > 3.1 \text{ GeV} \]
Extending into the soft region

Continuation of the differential cross section into the soft region $p_t < p_t^{\text{min}}$ (here: $p_t$ integral kept fixed)

\[
\frac{d\sigma_{\text{soft}}}{dp_t} \sim p_t e^{-\beta (p_t^2 - p_t^{\text{min},2})}
\]

$p_t^{\text{min}} = 3$ GeV, $\beta = -0.5$ GeV$^{-2}$

$p_t^{\text{min}} = 5$ GeV, $\beta = 0.06$ GeV$^{-2}$
Fix the two parameters $\mu_{\text{soft}}$ and $\sigma_{\text{soft}}^{\text{inc}}$ in

$$
\chi_{\text{tot}}(\vec{b}, s) = \frac{1}{2} \left( A(\vec{b}; \mu)\sigma_{\text{hard}}^{\text{inc}}(s; p_t^{\text{min}}) + A(\vec{b}; \mu_{\text{soft}})\sigma_{\text{soft}}^{\text{inc}} \right)
$$

from two constraints. Require simultaneous description of $\sigma_{\text{tot}}$ and $b_{\text{el}}$ (measured/well predicted),

$$
\sigma_{\text{tot}}(s) \doteq 2 \int d^2\vec{b} \left( 1 - e^{-\chi_{\text{tot}}(\vec{b}, s)} \right),
$$

$$
b_{\text{el}}(s) \doteq \int d^2\vec{b} \frac{b^2}{\sigma_{\text{tot}}} \left( 1 - e^{-\chi_{\text{tot}}(\vec{b}, s)} \right).
$$
Diffractive final states

Strictly low mass diffraction only. Allow $M^2$ large nonetheless. $M^2$ power-like, $t$ exponential (Regge).

$$pp \rightarrow \text{(baryonic cluster)} + p.$$ 

Hadronic content from cluster fission/decay $C \rightarrow hh\ldots$

Cluster may be quite light. If very light, use directly

$$pp \rightarrow \Delta + p.$$ 

Also double diffraction implemented.

$$pp \rightarrow \text{(cluster)} + \text{(cluster)} \quad pp \rightarrow \Delta + \Delta.$$ 

Technically: new MEs for diffractive processes set up.
# Soft particle production model in Herwig

- \#ladders = \( N_{\text{soft}} \) (MPI).
- \( N \) particles from Poissonian, width \( \langle N \rangle \).
  Model parameter \( 1/\ln C \equiv n_{\text{ladder}} \to \text{tuned} \).
- \( x_i \) smeared around \( \langle x \rangle \) (calculated).
- \( p_\perp \) from Gaussian acc to soft MPI model.
- particles are \( q, g \), see figure.
  Symmetrically produced from both remnants.
- Colour connections between neighboured particles.
Soft particle production model in Herwig

Single soft ladder with MinBias initiating process.

Further hard/soft MPI scatters possible.
Colour correlations in hadronic collisions
Colour correlations in hadronic collisions

![Diagram of colour correlations in hadronic collisions](image)
Colour correlations in hadronic collisions
Colour reconnection (CR) in Herwig

Extend cluster hadronization:

- QCD parton showers provide pre-confinement $\Rightarrow$ colour-anticolour pairs

Plain CR, iterate cluster pairs in "random order":

- Allow CR if the cluster mass decreases, $M_{il} + M_{kj} < M_{ij} + M_{kl}$
- Accept alternative clustering with probability $p_{reco}$ (model parameter)
  this allows to switch on CR smoothly

Alternative Statistical CR (Metropolis)
Colour reconnection (CR) in Herwig

Extend cluster hadronization:

- QCD parton showers provide *pre-confinement* ⇒ colour-anticolour pairs
- \( \rightarrow \) clusters
Colour reconnection (CR) in Herwig

Extend cluster hadronization:

- QCD parton showers provide *pre-confinement* ⇒ colour-anticolour pairs
- → *clusters*
- CR in the cluster hadronization model: allow *reformation* of clusters, e.g. \((il) + (jk)\)
Colour reconnection (CR) in Herwig

Extend cluster hadronization:

- QCD parton showers provide *pre-confinement* ⇒ colour-anticolour pairs
- → clusters
- CR in the cluster hadronization model: allow *reformation* of clusters, e.g. \((il) + (jk)\)

**Plain CR**, iterate cluster pairs in “random order”:

- Allow CR if the cluster mass decreases,
  \[ M_{il} + M_{kj} < M_{ij} + M_{kl}, \]
- Accept alternative clustering with probability \(p_{\text{reco}}\) (model parameter) ⇒ this allows to switch on CR smoothly
- Alternative **Statistical CR** (Metropolis)

[SG, C. Röhr, A. Siodmok, EPJ C72 (2012) 2225]
Colour reconnections

- Sensitivity to CR already known since UA1.
- (From Sjöstrand / van Zijl)
MPI Summary

- MPI (with colour reconnections) currently model of choice.
- Describes averages and fluctuations.
- Not always universal, but all models tunable.
- Soft component needed for MB modelling.
- Constraints from inclusive cross sections.
- Different emphasis on hard/soft modelling between generators.
- Many details still only models.
Brief graphical summary
Brief graphical summary
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for details go to: www.montecarlonet.org
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www.montecarlonet.org

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