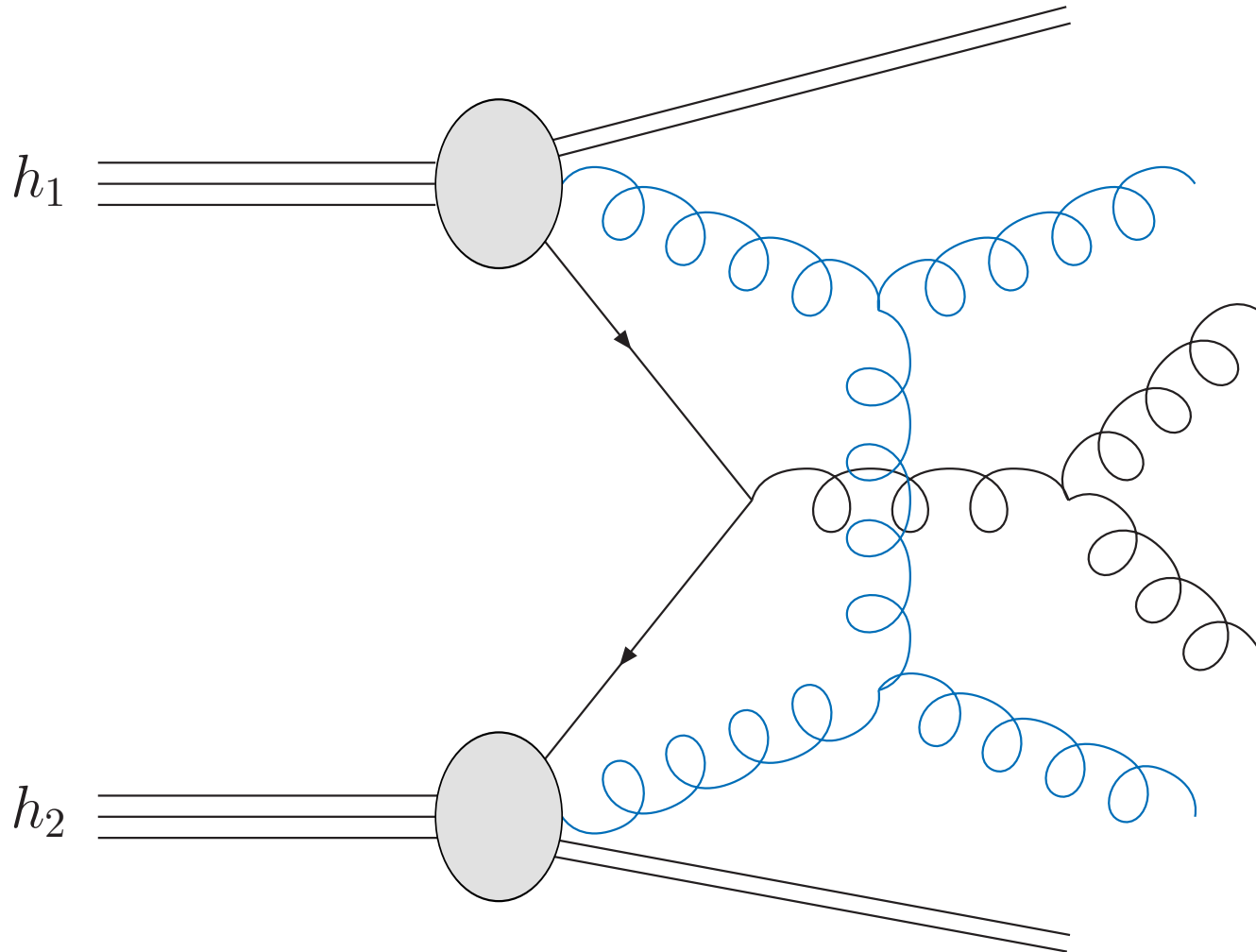


# Modelling MPI (in Herwig)

# Eikonal model basics

## Multiple hard interactions



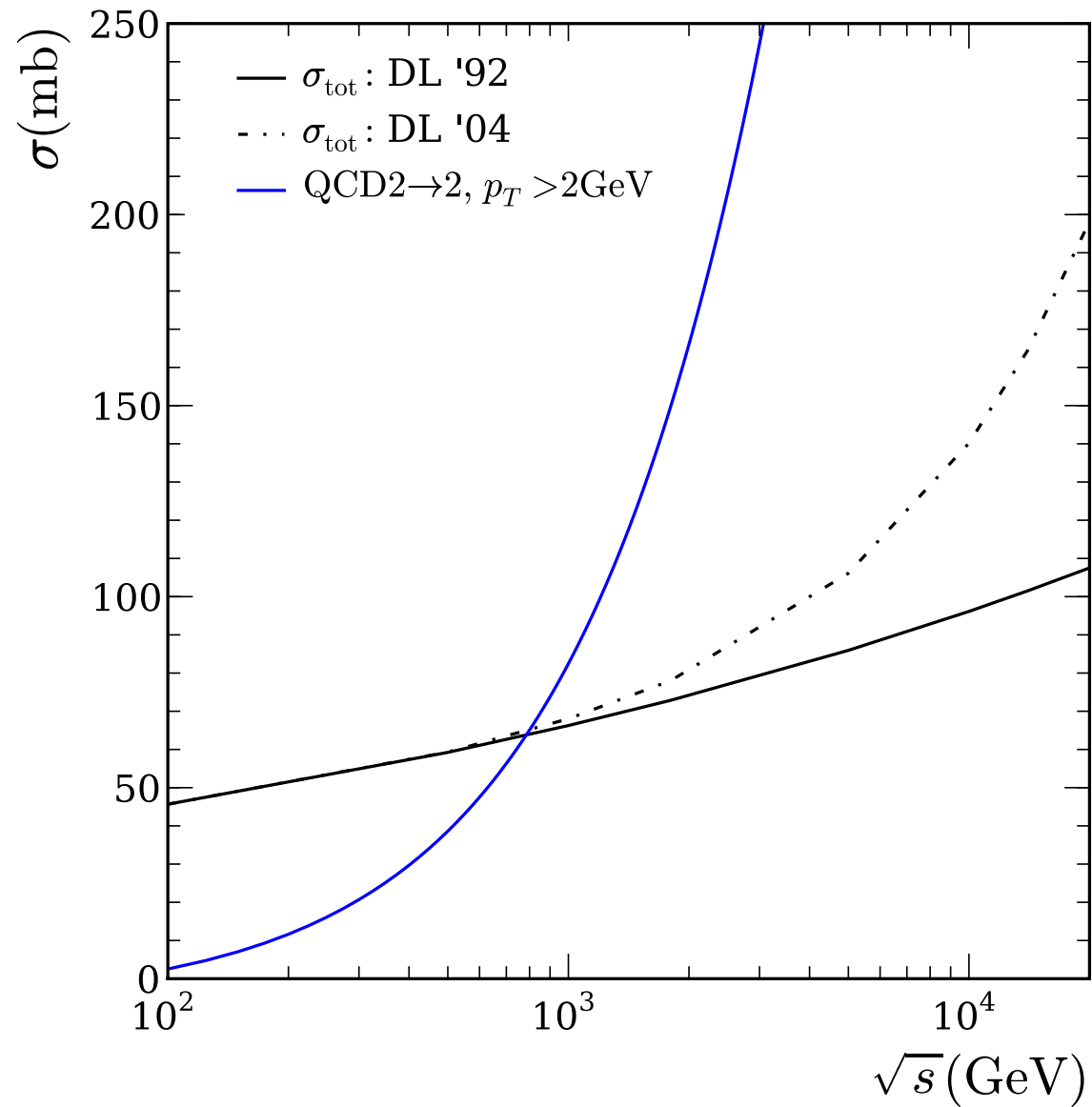
# Eikonal model basics

Starting point: hard inclusive jet cross section.

$$\sigma^{\text{inc}}(s; p_t^{\text{min}}) = \sum_{i,j} \int_{p_t^{\text{min}2}} dp_t^2 f_{i/h_1}(x_1, \mu^2) \otimes \frac{d\hat{\sigma}_{i,j}}{dp_t^2} \otimes f_{j/h_2}(x_2, \mu^2),$$

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$\sigma^{\text{inc}} > \sigma_{\text{tot}}$  eventually (for moderately small  $p_t^{\text{min}}$ ).

Interpretation:  $\sigma^{\text{inc}}$  counts *all* partonic scatters that happen during a single  $pp$  collision  $\Rightarrow$  more than a single interaction.

$$\sigma^{\text{inc}} = \bar{n} \sigma_{\text{inel}}.$$

# Eikonal model basics

Use eikonal approximation (= independent scatters). Leads to Poisson distribution of number  $m$  of additional scatters,

$$P_m(\vec{b}, s) = \frac{\bar{n}(\vec{b}, s)^m}{m!} e^{-\bar{n}(\vec{b}, s)} .$$

Then we get  $\sigma_{\text{inel}}$ :

$$\sigma_{\text{inel}} = \int d^2\vec{b} \sum_{m=1}^{\infty} P_m(\vec{b}, s) = \int d^2\vec{b} \left( 1 - e^{-\bar{n}(\vec{b}, s)} \right) .$$

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Cf.  $\sigma_{\text{inel}}$  from scattering theory in eikonal approx. with scattering amplitude  $a(\vec{b}, s) = \frac{1}{2i} (e^{-\chi(\vec{b}, s)} - 1)$

$$\sigma_{\text{inel}} = \int d^2\vec{b} \left( 1 - e^{-2\chi(\vec{b}, s)} \right) \quad \Rightarrow \quad \chi(\vec{b}, s) = \frac{1}{2} \bar{n}(\vec{b}, s) .$$

$\chi(\vec{b}, s)$  is called *eikonal* function.

# Eikonal model basics

Calculation of  $\bar{n}(\vec{b}, s)$  from parton model assumptions:

$$\begin{aligned}\bar{n}(\vec{b}, s) &= L_{\text{partons}}(x_1, x_2, \vec{b}) \otimes \sum_{ij} \int dp_t^2 \frac{d\hat{\sigma}_{ij}}{dp_t^2} \\ &= \sum_{ij} \frac{1}{1 + \delta_{ij}} \int dx_1 dx_2 \int d^2\vec{b}' \int dp_t^2 \frac{d\hat{\sigma}_{ij}}{dp_t^2} \\ &\quad \times D_{i/A}(x_1, p_t^2, |\vec{b}'|) D_{j/B}(x_2, p_t^2, |\vec{b} - \vec{b}'|)\end{aligned}$$



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 &= A(\vec{b}) \sigma^{\text{inc}}(s; p_t^{\text{min}}) .
 \end{aligned}$$

$$\Rightarrow \chi(\vec{b}, s) = \frac{1}{2} \bar{n}(\vec{b}, s) = \frac{1}{2} A(\vec{b}) \sigma^{\text{inc}}(s; p_t^{\text{min}}) .$$

# Overlap function

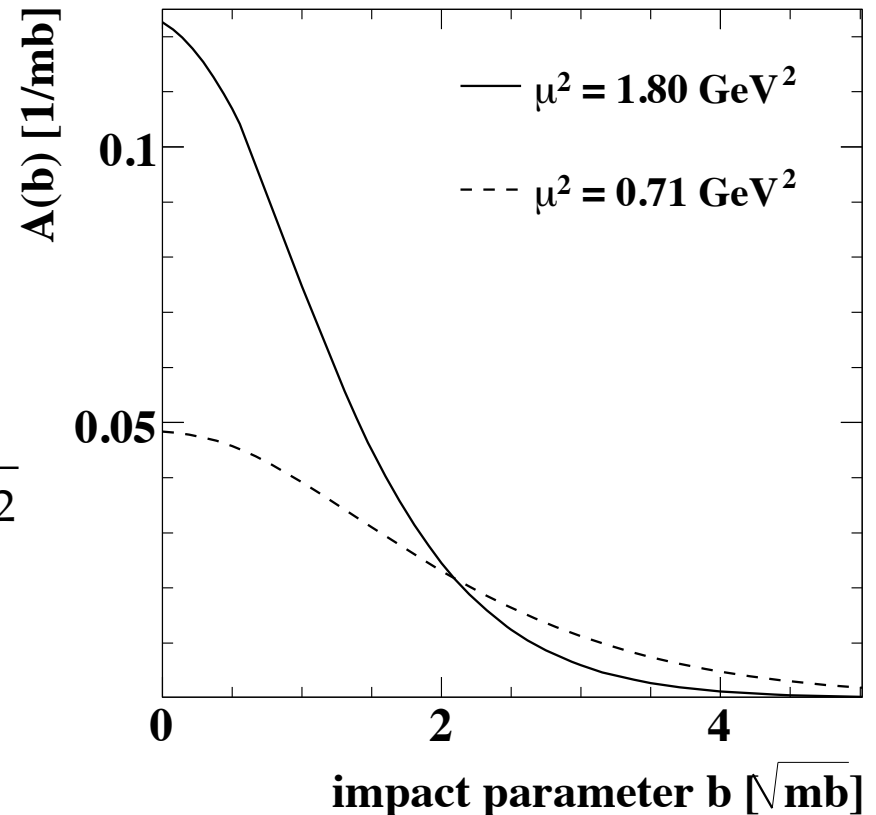
$$A(b) = \int d^2\vec{b}' G_A(|\vec{b}'|) G_B(|\vec{b} - \vec{b}'|)$$

$G(\vec{b})$  from electromagnetic FF:

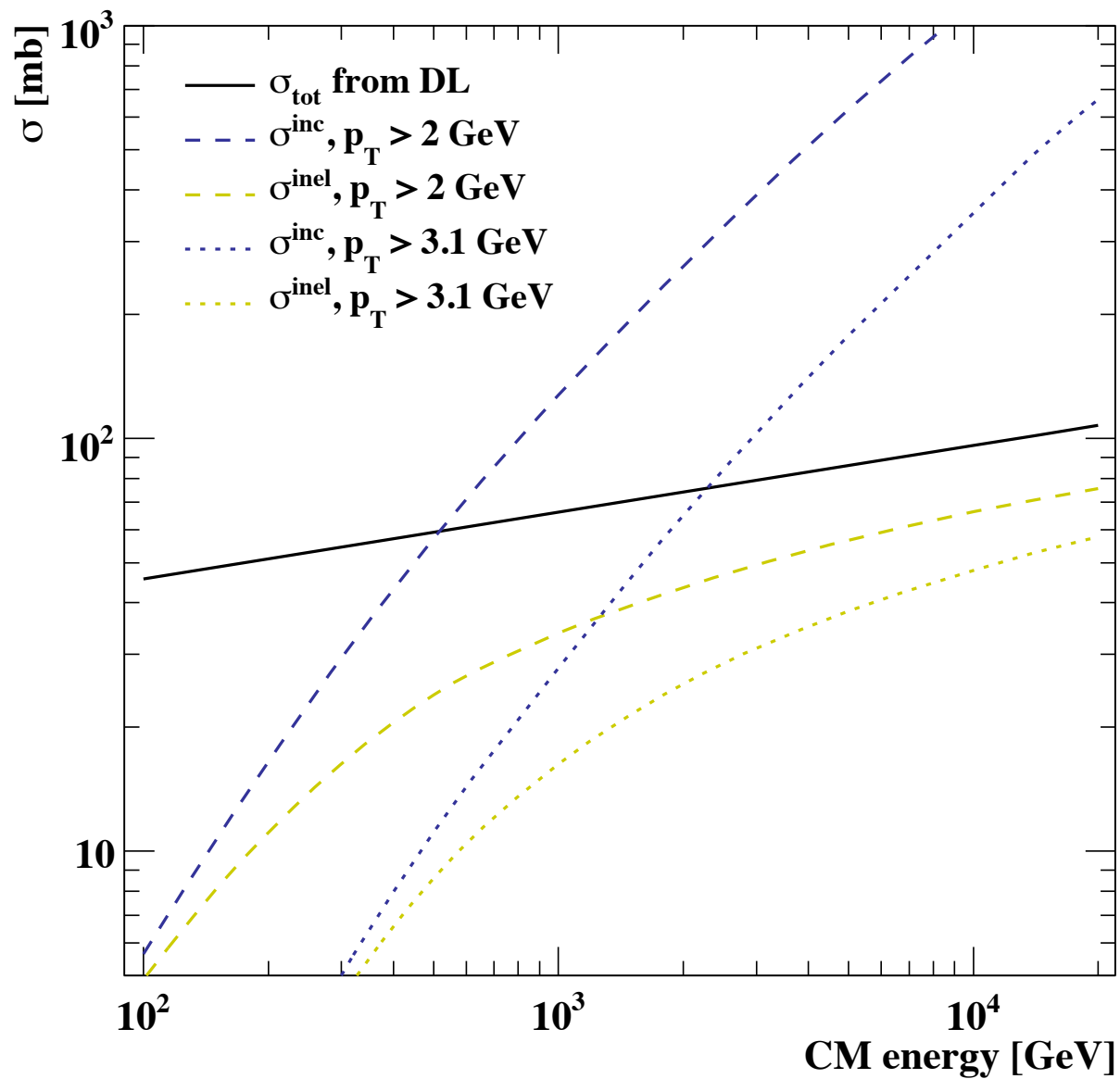
$$G_p(\vec{b}) = G_{\bar{p}}(\vec{b}) = \int \frac{d^2\vec{k}}{(2\pi)^2} \frac{e^{i\vec{k}\cdot\vec{b}}}{(1 + \vec{k}^2/\mu^2)^2}$$

But  $\mu^2$  *not fixed* to the  
electromagnetic  $0.71 \text{ GeV}^2$ .  
Free for colour charges.

$\Rightarrow$  Two main parameters:  $\mu^2, p_t^{\text{min}}$ .

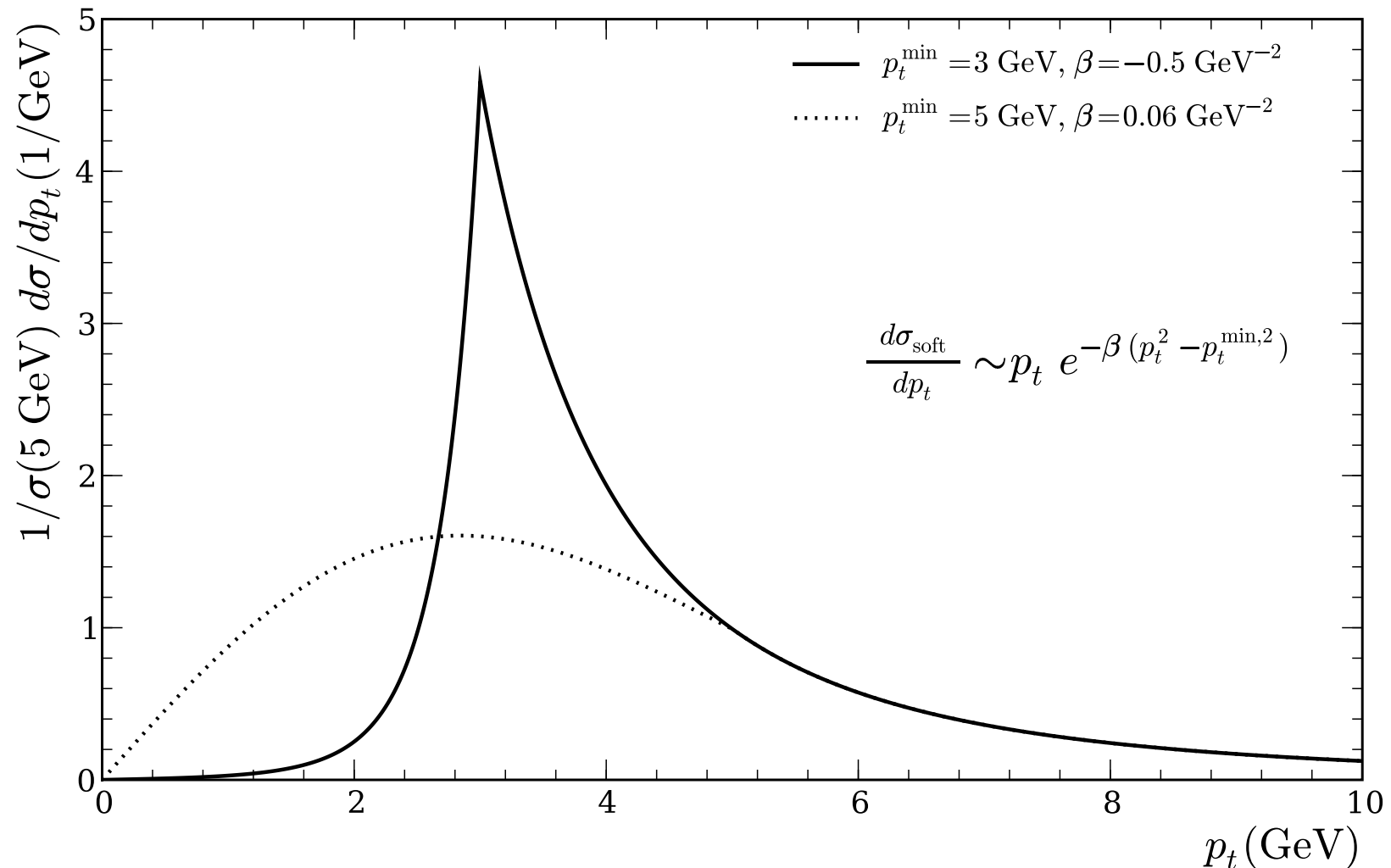


# Unitarized cross sections



# Extending into the soft region

Continuation of the differential cross section into the soft region  $p_t < p_t^{\min}$  (here:  $p_t$  integral kept fixed)



# Hot Spot model

Fix the two parameters  $\mu_{\text{soft}}$  and  $\sigma_{\text{soft}}^{\text{inc}}$  in

$$\chi_{\text{tot}}(\vec{b}, s) = \frac{1}{2} \left( A(\vec{b}; \mu) \sigma^{\text{inc}}_{\text{hard}}(s; p_t^{\text{min}}) + A(\vec{b}; \mu_{\text{soft}}) \sigma_{\text{soft}}^{\text{inc}} \right)$$

from two constraints. Require simultaneous description of  $\sigma_{\text{tot}}$  and  $b_{\text{el}}$  (measured/well predicted),

$$\sigma_{\text{tot}}(s) \stackrel{!}{=} 2 \int d^2\vec{b} \left( 1 - e^{-\chi_{\text{tot}}(\vec{b}, s)} \right) ,$$
$$b_{\text{el}}(s) \stackrel{!}{=} \int d^2\vec{b} \frac{b^2}{\sigma_{\text{tot}}} \left( 1 - e^{-\chi_{\text{tot}}(\vec{b}, s)} \right) .$$

# Diffraction final states

Strictly low mass diffraction only. Allow  $M^2$  large nonetheless.

$M^2$  power-like,  $t$  exponential (Regge).

$$pp \rightarrow (\text{baryonic cluster}) + p .$$

Hadronic content from cluster fission/decay  $C \rightarrow hh \dots$

Cluster may be quite light. If very light, use directly

$$pp \rightarrow \Delta + p .$$

Also double diffraction implemented.

$$pp \rightarrow (\text{cluster}) + (\text{cluster}) \quad pp \rightarrow \Delta + \Delta .$$

Technically: new MEs for diffractive processes set up.

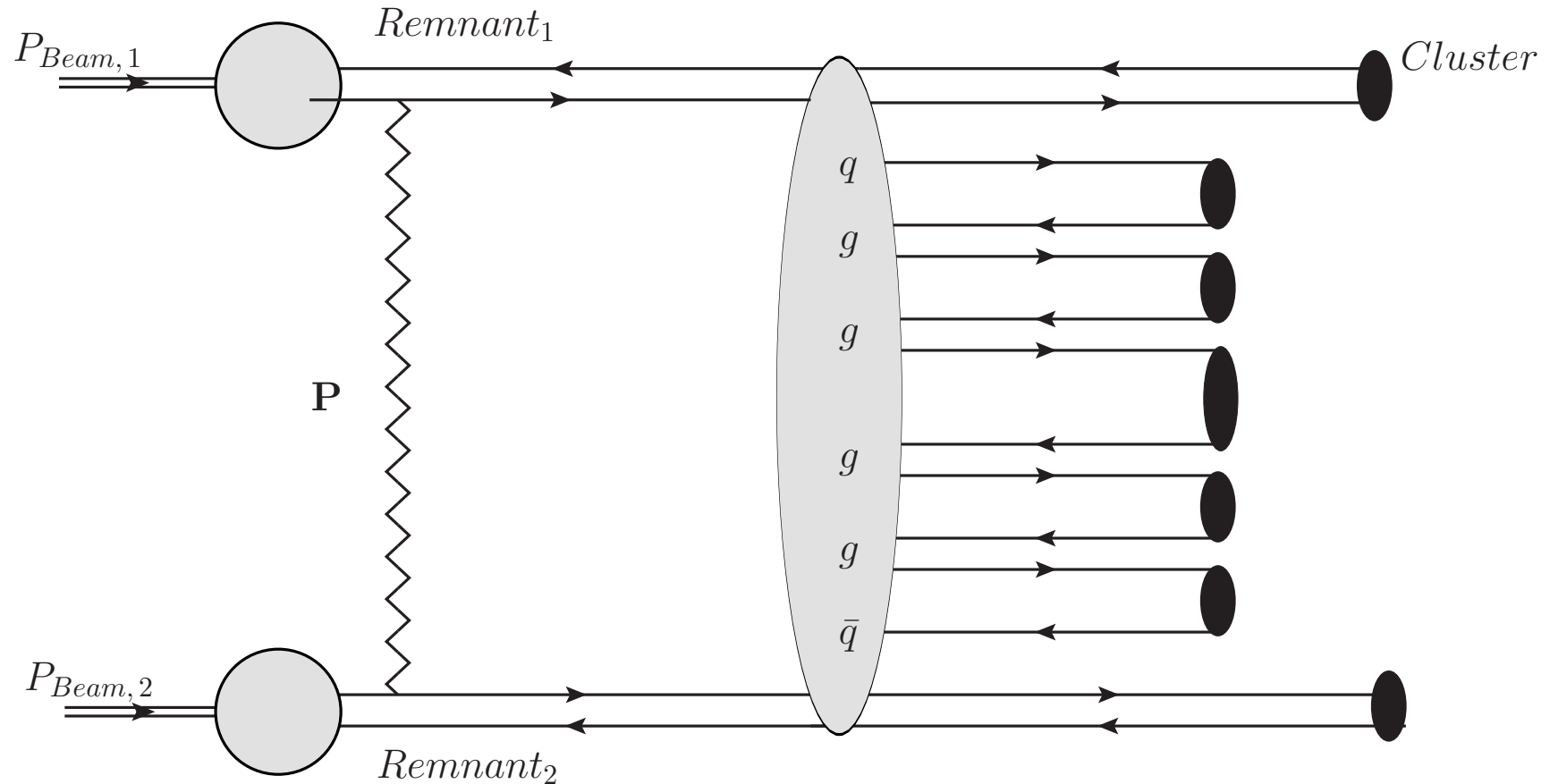
# Soft particle production model in Herwig

- #ladders =  $N_{\text{soft}}$  (MPI).
- $N$  particles from Poissonian, width  $\langle N \rangle$ .  
Model parameter  $1/\ln C \equiv n_{\text{ladder}} \rightarrow$  tuned.
- $x_i$  smeared around  $\langle x \rangle$  (calculated).
- $p_{\perp}$  from Gaussian acc to soft MPI model.
- particles are  $q, g$ , see figure.  
Symmetrically produced from both remnants.
- Colour connections between neighboured particles.



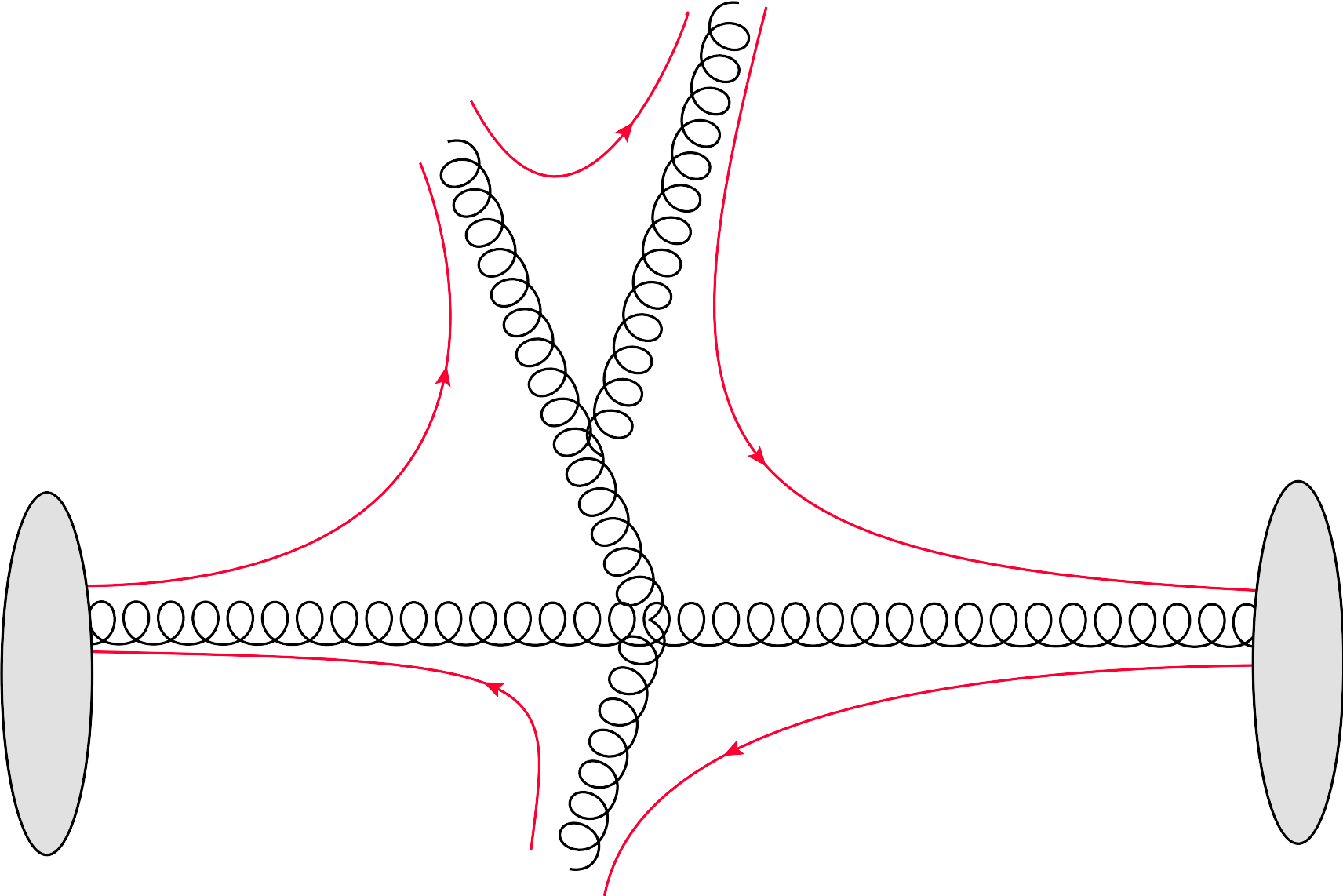
# Soft particle production model in Herwig

Single soft ladder with MinBias initiating process.

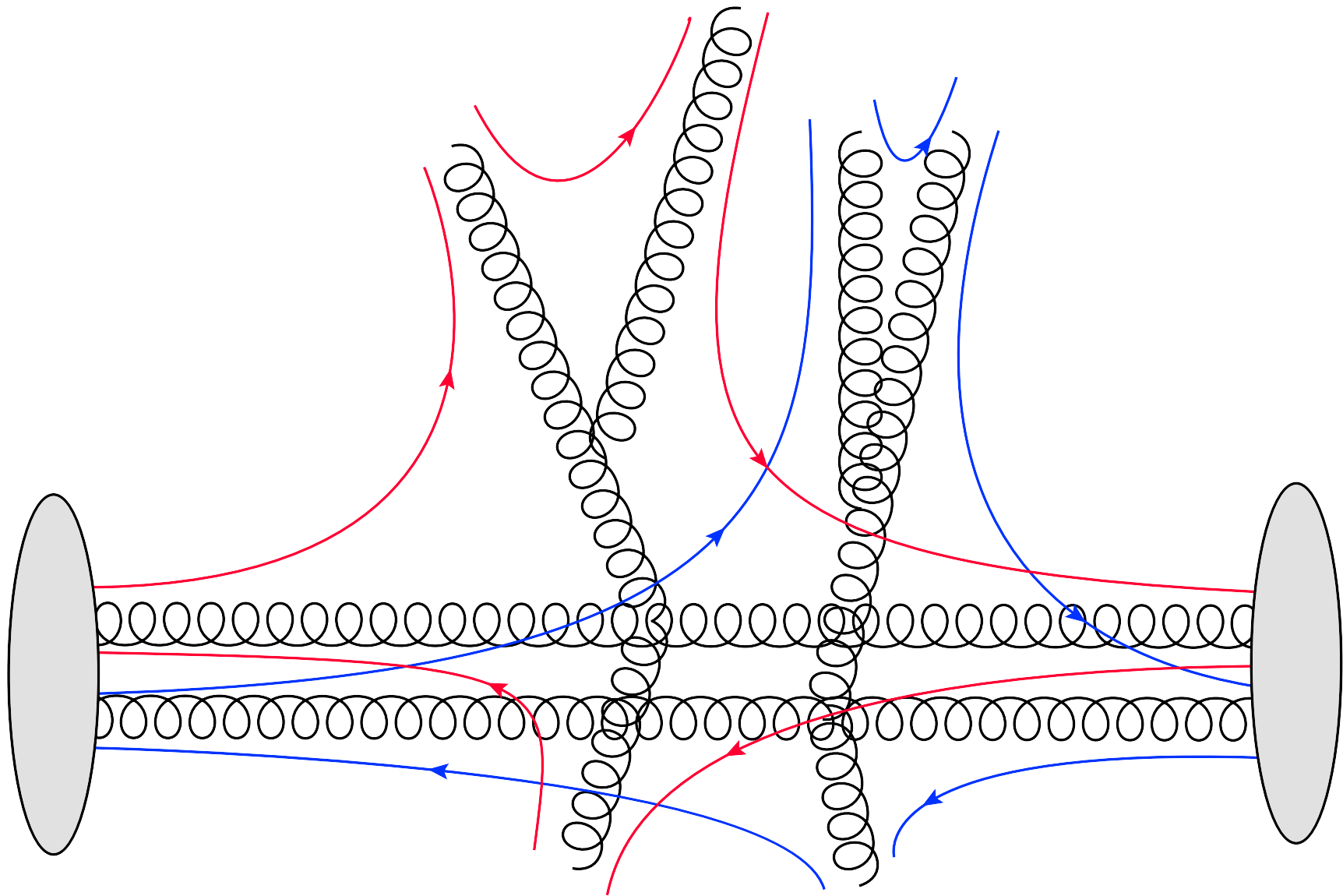


Further hard/soft MPI scatters possible.

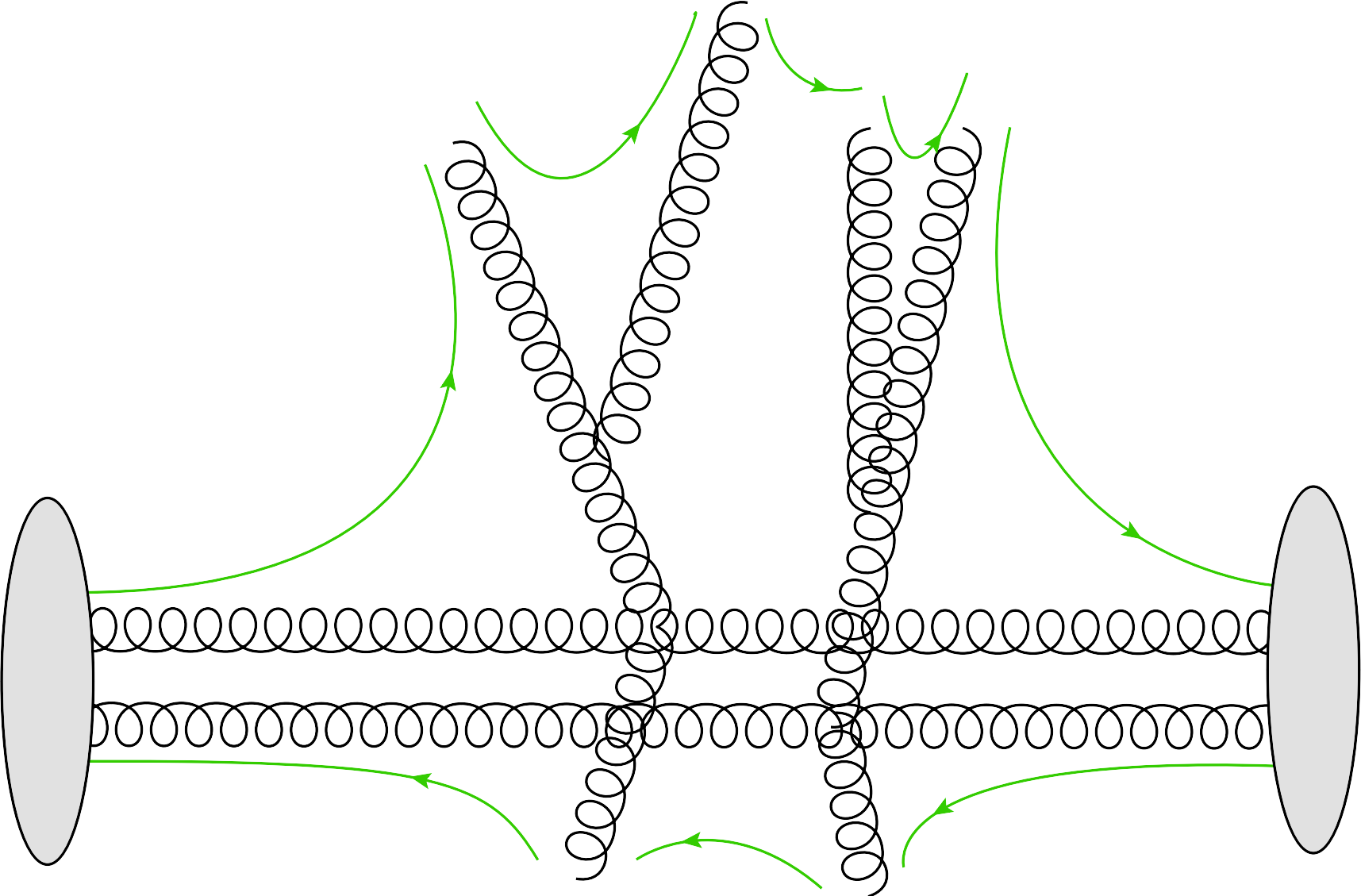
# Colour correlations in hadronic collisions



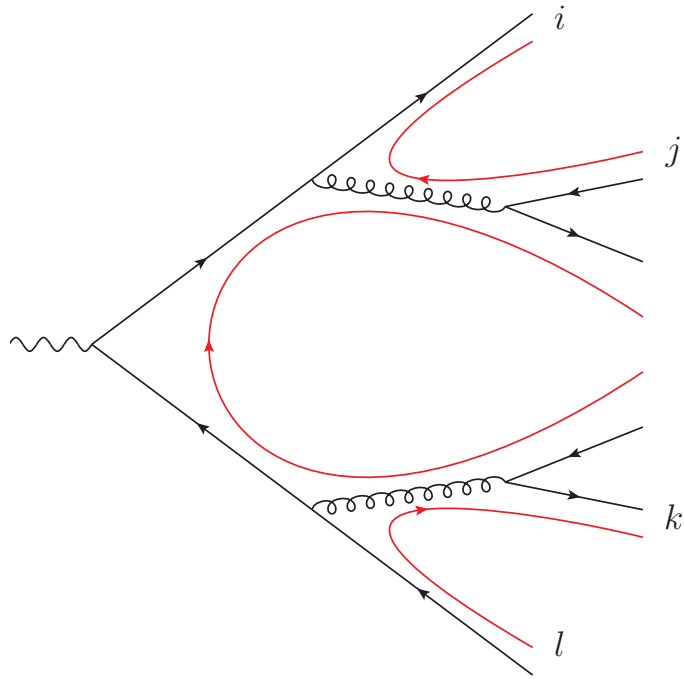
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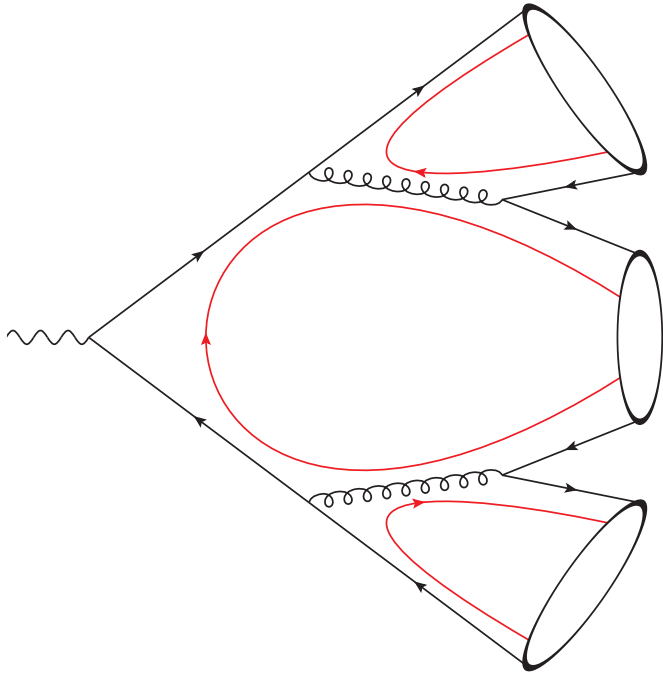
# Colour reconnection (CR) in Herwig



Extend cluster hadronization:

- QCD parton showers provide *pre-confinement*  $\Rightarrow$  colour-anticolour pairs

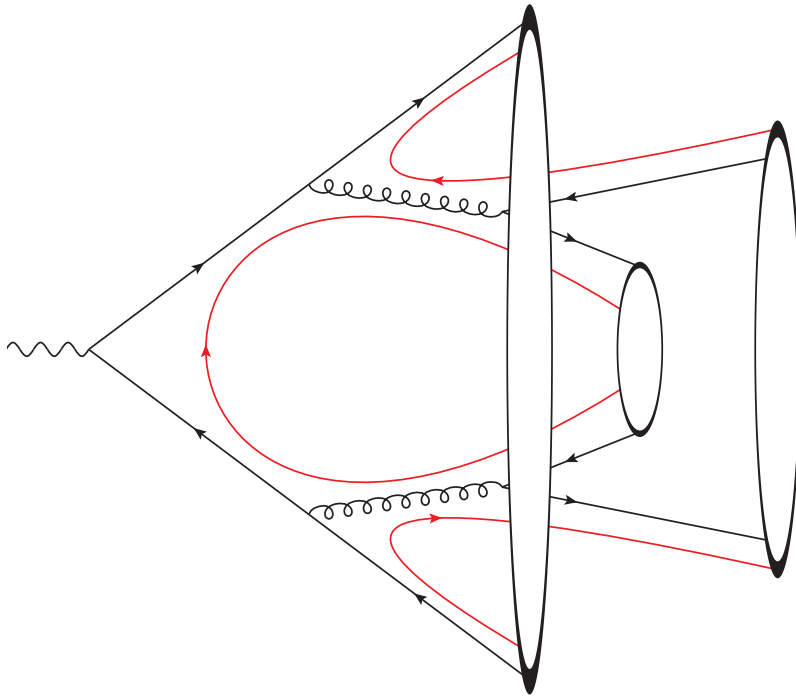
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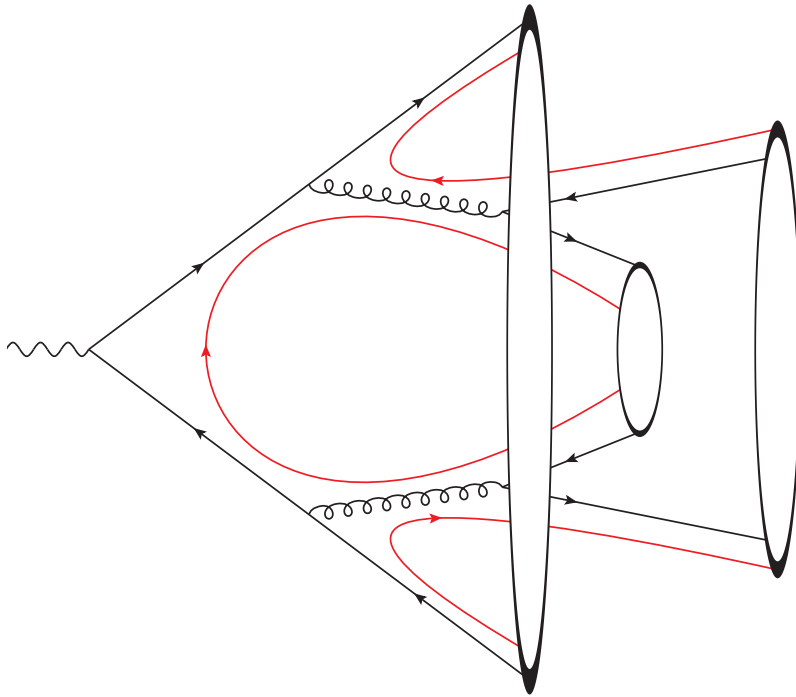
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**Plain CR**, iterate cluster pairs in “random order”:

- Allow CR if the cluster mass decreases,

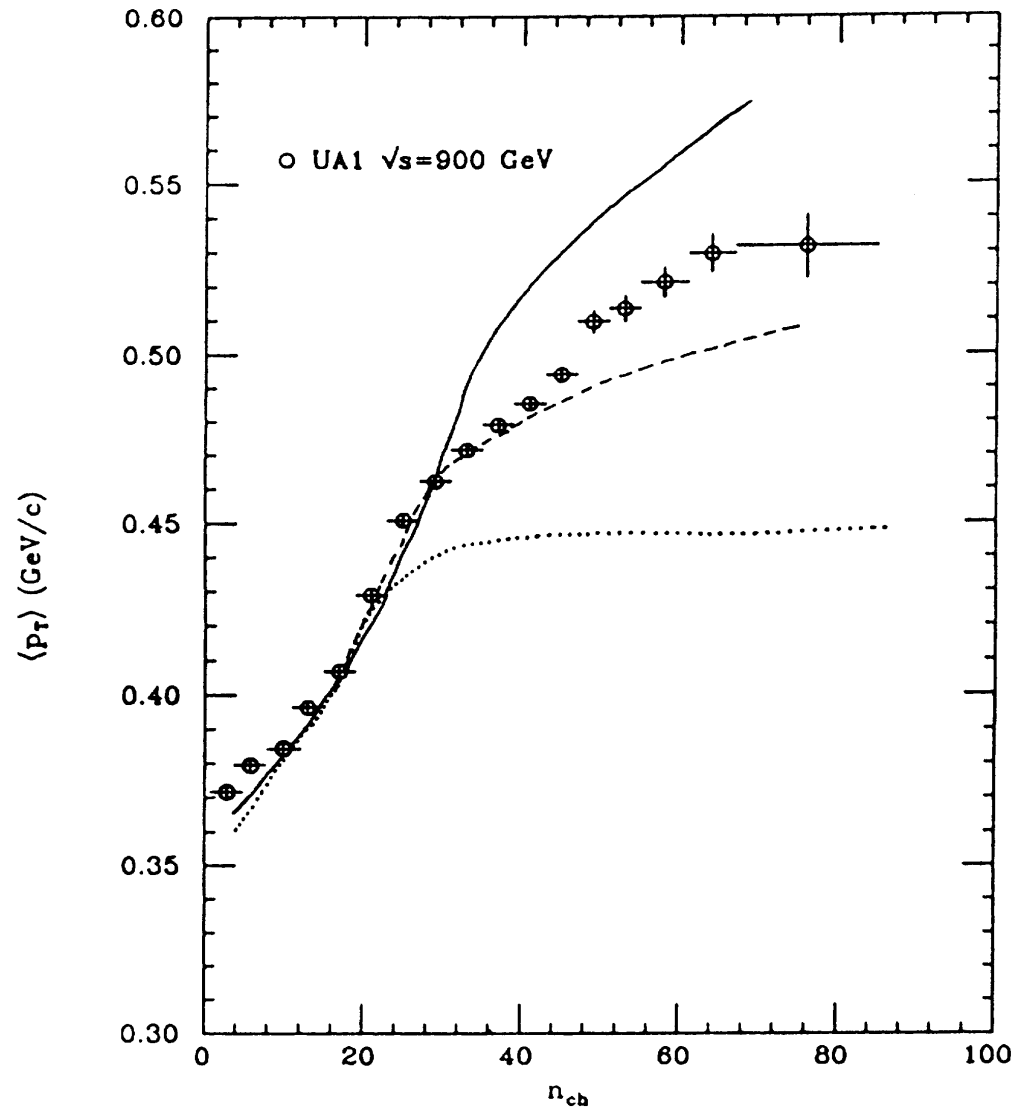
$$M_{il} + M_{kj} < M_{ij} + M_{kl},$$

- Accept alternative clustering with probability  $p_{\text{reco}}$  (model parameter)  $\Rightarrow$  this allows to switch on CR smoothly
- Alternative **Statistical CR** (Metropolis)

[SG, C. Röhr, A. Siodmok, EPJ C72 (2012) 2225]



# Colour reconnections

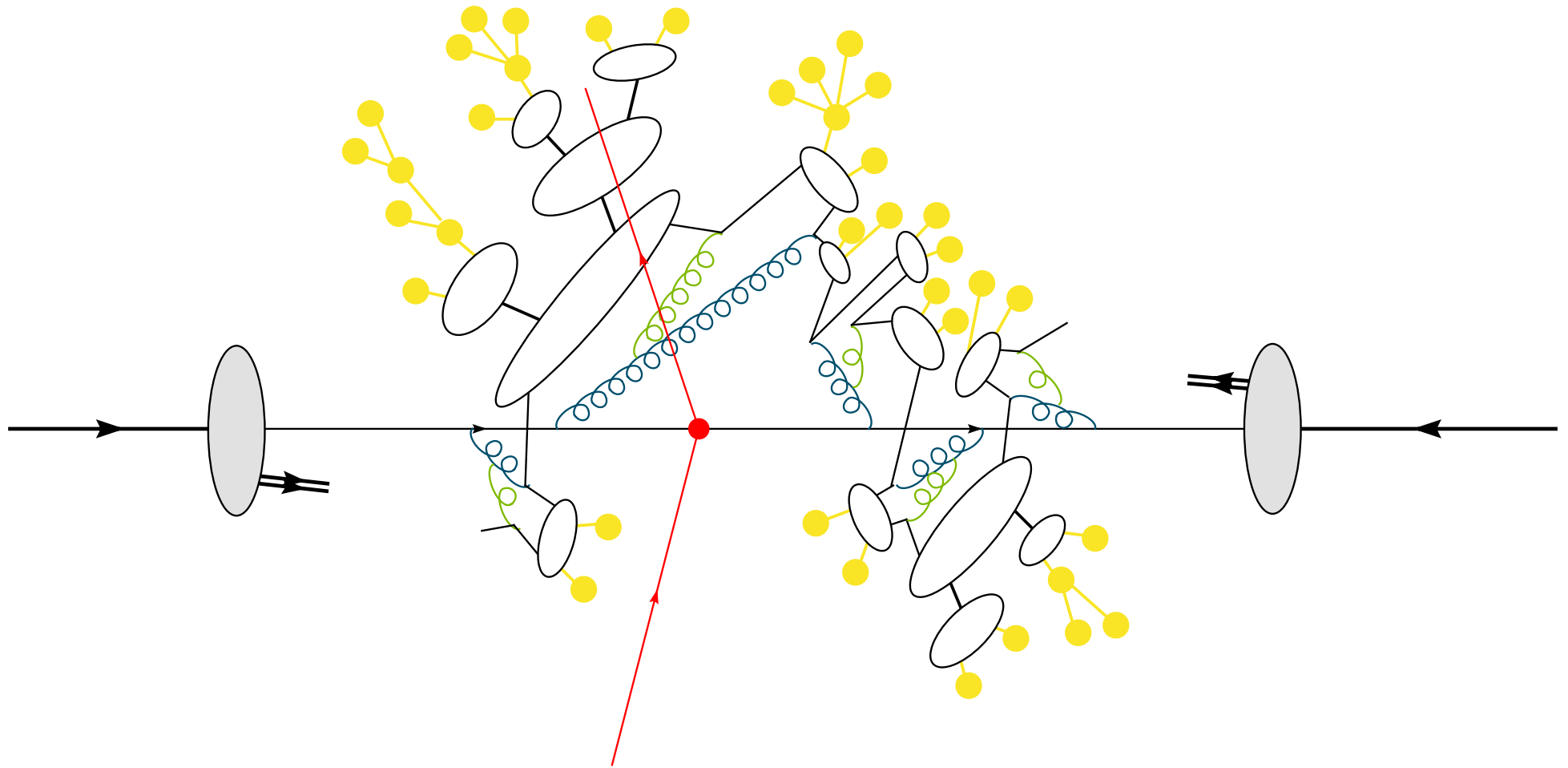


- Sensitivity to CR already known since UA1.
- (From Sjöstrand/van Zijl)

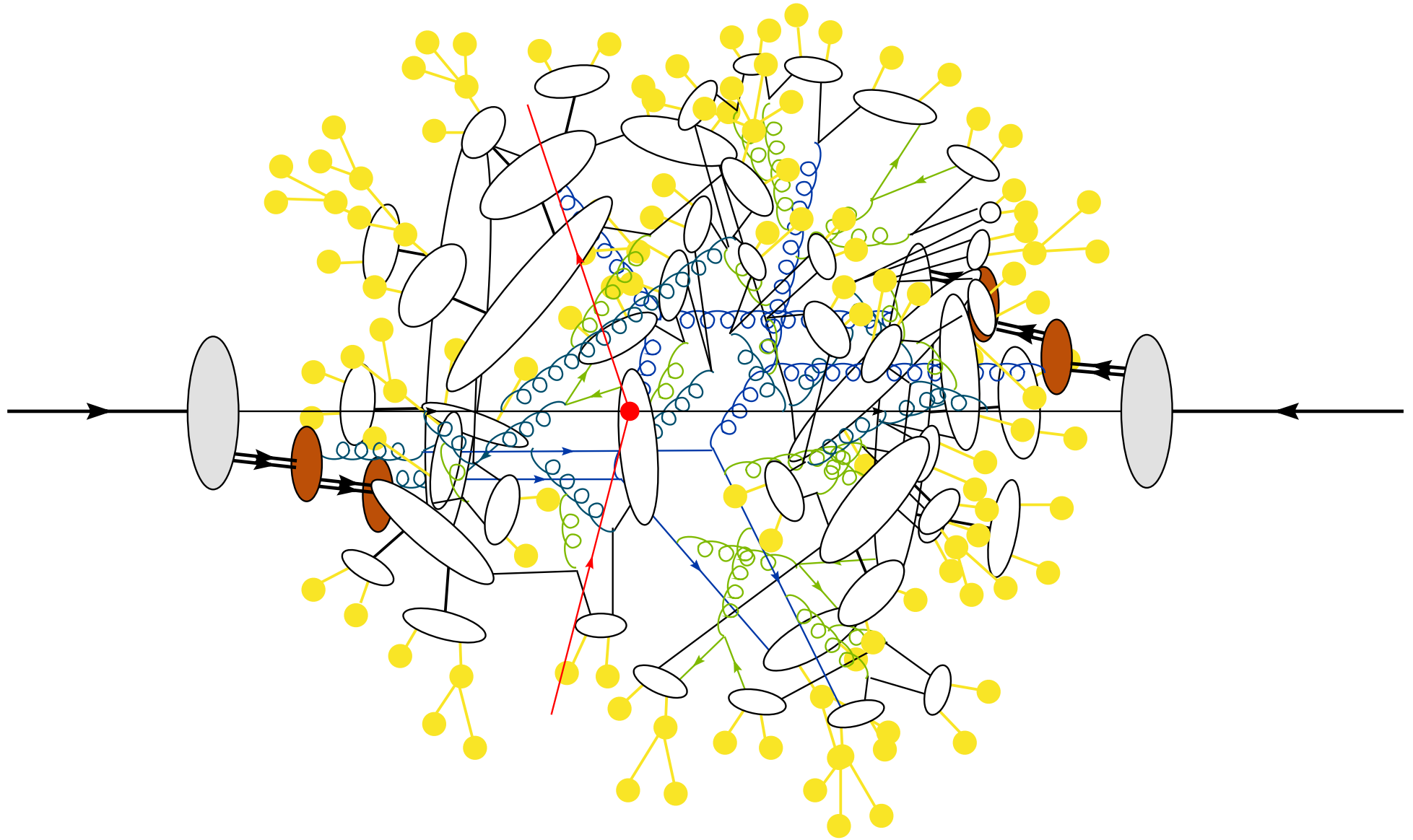
# MPI Summary

- MPI (with colour reconnections) currently model of choice.
- Describes averages *and* fluctuations.
- Not always universal, but all models tunable.
- soft component needed for MB modelling.
- Constraints from inclusive cross sections.
- Different emphasis on hard/soft modelling between generators.
- Many details still only models.

# Brief graphical summary



# Brief graphical summary



# Monte Carlo

## training studentships



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MCnet funded short-term studentships,  
PhD students spend 3-6 months at one node  
on a project related to their thesis work.