Special Galileon coupled to photon

Construction of Lagrangians

Filip Přeučil

Institute of Particle and Nuclear Physics

April 12, 2019

References

K. Kampf, J. Novotný
Unification of Galileon Dualities
J. J. High Energ. Phys. 2014, 6 (2014)
arXiv:1403.6813 [hep-th]

J. Novotný
Geometry of Special Galileon
Phys. Rev. D 95, 065019 (2017)
arXiv:1612.01738 [hep-th]

Special Galileon

- scalar field theory in d dimensions

$$\mathcal{L} = \sum_{n=1}^{d+1} g_n \phi \left(\varepsilon^{\mu_1 \dots \mu_d} \varepsilon^{\nu_1 \dots \nu_d} \prod_{i=1}^{n-1} \partial_{\mu_i} \partial_{\nu_i} \phi \prod_{j=n}^{d} \eta_{\mu_j \nu_j} \right)$$

for sGal: $g_{2k} = -\frac{(-1)^k}{d!\, 2k} \binom{d}{2k-1} \alpha^{2(1-k)}, \qquad g_{2k+1} = 0$

- Gal symmetry: $\phi \to \phi + a + b^\mu x_\mu$
- sGal symmetry: $\phi \to \phi - \frac{1}{2} G^{\mu\nu} \left(\alpha^2 x_\mu x_\nu + \partial_\mu \phi \partial_\nu \phi \right)$, where $G^{\mu\nu}$ is symmetric and traceless
- Gal symmetry ensures $\mathcal{O}(p^2)$ soft-behavior, sGal symmetry even $\mathcal{O}(p^3)$

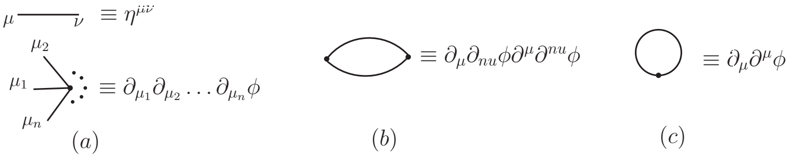
Special Galileon — geometry

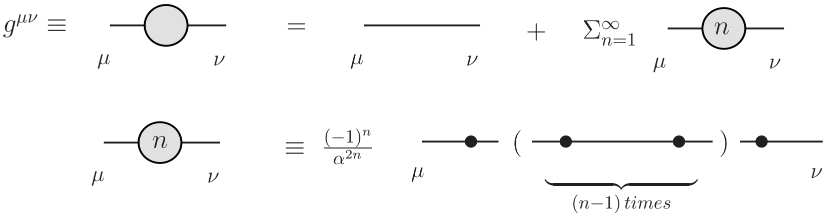
- fluctuations of a d-dimensional brane in a $2d$-dimensional flat space
- metric on the brane: $g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{\alpha^2}\partial_\mu\partial\phi \cdot \partial\partial_\nu\phi$
- inverse metric: $g^{\mu\nu} = \eta^{\mu\nu} - \frac{1}{\alpha^2}\partial^\mu\partial\phi \cdot \partial\partial^\nu\phi + \ldots$

- Christoffel symbols, Riemann tensor, etc. as usual
- external curvature: $\mathcal{K}_{\mu\nu\varrho} = -\frac{1}{\alpha}\partial_\mu\partial_\nu\partial_\varrho\phi$
- Gauss equation implies: $R_{\alpha\beta\mu\nu} = g^{\varrho\sigma}(\mathcal{K}_{\varrho\mu\alpha}\mathcal{K}_{\sigma\nu\beta} - \mathcal{K}_{\varrho\mu\beta}\mathcal{K}_{\sigma\nu\alpha})$

- another invariants: $\mathrm{d}^d Z \equiv (-1)^{d-1}\det\left(\eta + \frac{\mathrm{i}}{\alpha}\partial\partial\phi\right)\mathrm{d}^d x$
  $\mathrm{d}^d \bar{Z} \equiv (-1)^{d-1}\det\left(\eta - \frac{\mathrm{i}}{\alpha}\partial\partial\phi\right)\mathrm{d}^d x$
- it holds: $\sqrt{\mathrm{d}^d Z \mathrm{d}^d \bar{Z}} = \sqrt{g}\ \mathrm{d}^d x$

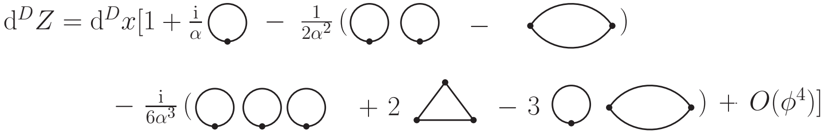
Graphical representation[1]

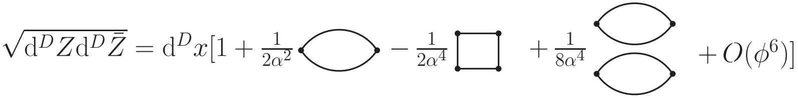




[1]Figures courtesy of J. Novotný

Graphical representation





Minimal coupling to photon

- $\mathcal{L}^{\rm 4D}_{\rm sGal} = -\frac{1}{2}\phi\Box\phi + \frac{1}{24\alpha^2}\phi\left[(\Box\phi)^3 + 2\langle\partial\partial\phi^{\cdot 3}\rangle - 3\Box\phi\partial\partial\phi : \partial\partial\phi\right]$
- the simplest possible term to add to the sGal action is schematically

$$\int F^2 \mathrm{d}\mu(a,b)$$

- ...or not so schematically

$$\int -\frac{1}{4}F_{\mu\alpha}F_{\nu\beta}g^{\mu\nu}g^{\alpha\beta}\ \mathrm{d}\mu(a,b)$$

where $\mu(a,b) \equiv a\sqrt{\mathrm{d}^d Z \mathrm{d}^d \bar{Z}} + b\mathrm{d}^d Z + b^\star \mathrm{d}^d \bar{Z}$

- we must have $a + b + b^\star = 1$ (photon kinetic term normalization)
- a must be real (action hermiticity)
- this term yields interaction vertices with 2 photons and how many galileons we want

Minimal coupling to photon

- example: expanding up to $\mathcal{O}(\phi^4)$ yields for $b = 0$:

$$\mathcal{L}_a = -\frac{1}{4}a\Bigg\{F_{\mu\alpha}F^{\mu\alpha}\left[1+\frac{1}{2\alpha^2}\partial\partial\phi:\partial\partial\phi-\frac{1}{4\alpha^2}\langle\partial\partial\phi^{\cdot 4}\rangle+\frac{1}{8\alpha^2}(\partial\partial\phi:\partial\partial\phi)^2\right]$$
$$-\frac{2}{\alpha^2}F_{\mu\alpha}F_{\nu}{}^{\alpha}\partial^\mu\partial\phi\cdot\partial\partial^\nu\phi\left[1+\frac{1}{2\alpha^2}\partial\partial\phi:\partial\partial\phi\right]$$
$$+\frac{2}{\alpha^4}F_{\mu\alpha}F_{\nu}{}^{\alpha}\partial^\mu\partial\phi\cdot\partial\partial\phi\cdot\partial\partial\phi\cdot\partial\partial^\nu\phi$$
$$+\frac{1}{\alpha^4}F_{\mu\alpha}F_{\nu\beta}\partial^\mu\partial\phi\cdot\partial\partial^\nu\phi\partial^\alpha\partial\phi\cdot\partial\partial^\beta\phi\Bigg\}$$

- similarly, we can compute the part corresponding to $a = 0$

Next-to-minimal coupling to photon

- let us try 4 photons now
- possible terms to add to the action are schematically

$$\int F^4\mathcal{K}^2\mathrm{d}\mu,\quad \int F^4(\nabla\mathcal{K})\mathrm{d}\mu,\quad \int F^3(\nabla F)\mathcal{K}\mathrm{d}\mu,\quad \int(\nabla F)^2F^2\mathrm{d}\mu$$

  where ∇ is the covariant derivative constructed using the metric g

- the easiest to analyze is the derivative-less one
- there are in fact 12 unique terms of this type (assuming parity conservation), so we can write

$$S_{4\gamma} = \sum_{i=1}^{12} c_i \int [F^4\mathcal{K}^2]_i \mathrm{d}\mu(a_i, b_i)$$

- this introduces $12 \times 3 = 36$ new free parameters to the model

Next-to-minimal coupling to photon

- the list of the terms is the following

$$
\begin{aligned}
I_1 &= \mathcal{K}_{\mu\nu\alpha} g^{\mu\varrho} g^{\nu\sigma} g^{\alpha\beta} \mathcal{K}_{\varrho\sigma\beta} \times \mathcal{F} \\
I_2 &= \mathcal{K}_{\mu\nu\alpha} g^{\mu\varrho} g^{\nu\sigma} g^{\alpha\beta} \mathcal{K}_{\varrho\sigma\beta} \times \mathcal{G} \\
I_3 &= g^{\mu\nu} \mathcal{K}_{\mu\nu\alpha} g^{\alpha\beta} \mathcal{K}_{\beta\varrho\sigma} g^{\varrho\sigma} \times \mathcal{F} \\
I_4 &= g^{\mu\nu} \mathcal{K}_{\mu\nu\alpha} g^{\alpha\beta} \mathcal{K}_{\beta\varrho\sigma} g^{\varrho\sigma} \times \mathcal{G} \\
I_5 &= \mathcal{K}_{\mu\nu\alpha} g^{\mu\varrho} g^{\nu\sigma} \mathcal{K}_{\varrho\sigma\beta} [g^{\alpha\kappa} F_{\kappa\gamma} g^{\gamma\delta} F_{\delta\lambda} g^{\lambda\beta}] \times \mathcal{F} \\
I_6 &= \mathcal{K}_{\mu\nu\alpha} g^{\mu\varrho} \mathcal{K}_{\varrho\sigma\beta} [g^{\alpha\kappa} F_{\kappa\lambda} g^{\gamma\beta}][g^{\nu\delta} F_{\delta\lambda} g^{\lambda\sigma}] \times \mathcal{F} \\
I_7 &= g^{\mu\nu} \mathcal{K}_{\mu\nu\alpha} g^{\alpha\beta} \mathcal{K}_{\beta\varrho\sigma} [g^{\varrho\kappa} F_{\kappa\gamma} g^{\lambda\delta} F_{\delta\lambda} g^{\lambda\sigma}] \times \mathcal{F} \\
I_8 &= g^{\mu\nu} \mathcal{K}_{\mu\nu\alpha} [g^{\alpha\kappa} F_{\kappa\gamma} g^{\gamma\delta} F_{\delta\lambda} g^{\lambda\beta}] \mathcal{K}_{\beta\varrho\sigma} g^{\varrho\sigma} \times \mathcal{F}
\end{aligned}
$$

Next-to-minimal coupling to photon

$$
\begin{aligned}
I_9 &= \mathcal{K}_{\mu\nu\alpha} g^{\mu\varrho} g^{\nu\sigma} \mathcal{K}_{\varrho\sigma\beta} [g^{\alpha\kappa} F_{\kappa\gamma} g^{\gamma\delta} F_{\delta\lambda} g^{\lambda\varepsilon} F_{\varepsilon\iota} g^{\iota\zeta} F_{\zeta\xi} g^{\xi\beta}] \\
I_{10} &= \mathcal{K}_{\mu\nu\alpha} g^{\mu\varrho} \mathcal{K}_{\varrho\sigma\beta} [g^{\nu\delta} F_{\delta\lambda} g^{\lambda\sigma}][g^{\alpha\kappa} F_{\kappa\gamma} g^{\gamma\delta} F_{\delta\lambda} g^{\lambda\varepsilon} F_{\varepsilon\iota} g^{\iota\beta}] \\
I_{11} &= \mathcal{K}_{\mu\nu\alpha} g^{\mu\varrho} \mathcal{K}_{\varrho\sigma\beta} [g^{\nu\delta} F_{\delta\lambda} g^{\lambda\varepsilon} F_{\varepsilon\iota} g^{\iota\sigma}][g^{\alpha\kappa} F_{\kappa\gamma} g^{\gamma\delta} F_{\delta\lambda} g^{\lambda\beta}] \\
I_{12} &= \mathcal{K}_{\mu\nu\alpha} \mathcal{K}_{\varrho\sigma\beta} [g^{\mu\varphi} F_{\varphi\psi} g^{\psi\varrho}][g^{\nu\delta} F_{\delta\lambda} g^{\lambda\varepsilon} F_{\varepsilon\iota} g^{\iota\sigma}][g^{\alpha\kappa} F_{\kappa\lambda} g^{\gamma\beta}]
\end{aligned}
$$

- the terms might be linearly dependent, not known yet

Next-to-minimal coupling to photon — derivative terms

- currently work in progress
- example:

$$\nabla_\mu K_{\alpha\beta\gamma} = -\frac{1}{\alpha}\partial_\mu\partial_\alpha\partial_\beta\partial_\gamma\phi + K_{\varrho\beta\gamma}g^{\varrho\sigma}\Gamma_{\sigma\mu\alpha} + K_{\alpha\varrho\gamma}g^{\varrho\sigma}\Gamma_{\sigma\mu\beta} + K_{\alpha\beta\varrho}g^{\varrho\sigma}\Gamma_{\sigma\mu\gamma}$$

Thank you for your attention