# What is *not* the hierarchy problem (of the SM Higgs)

Matěj Hudec

Výjezdní seminář ÚČJF

Malá Skála, 12 Apr 2019, lunchtime

# Our ecological footprint

Fun with the Abelian Higgs model

M. Malinský

*EPJC 73, 2415 (2013)*

*arXiv:1212.4660*

*12 pgs.*

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MASTER THESIS

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Hierarchy and decoupling

M. Hudec,
M. Malinský

*(hopefully EPJC)*

*arXiv:1902.04470*

*17 pgs.*

# Hierarchy problem – first thoughts

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- This article **needs attention from an expert on the subject**. *(July 2017)*
- This article **needs additional citations for verification**. *(January 2017)*

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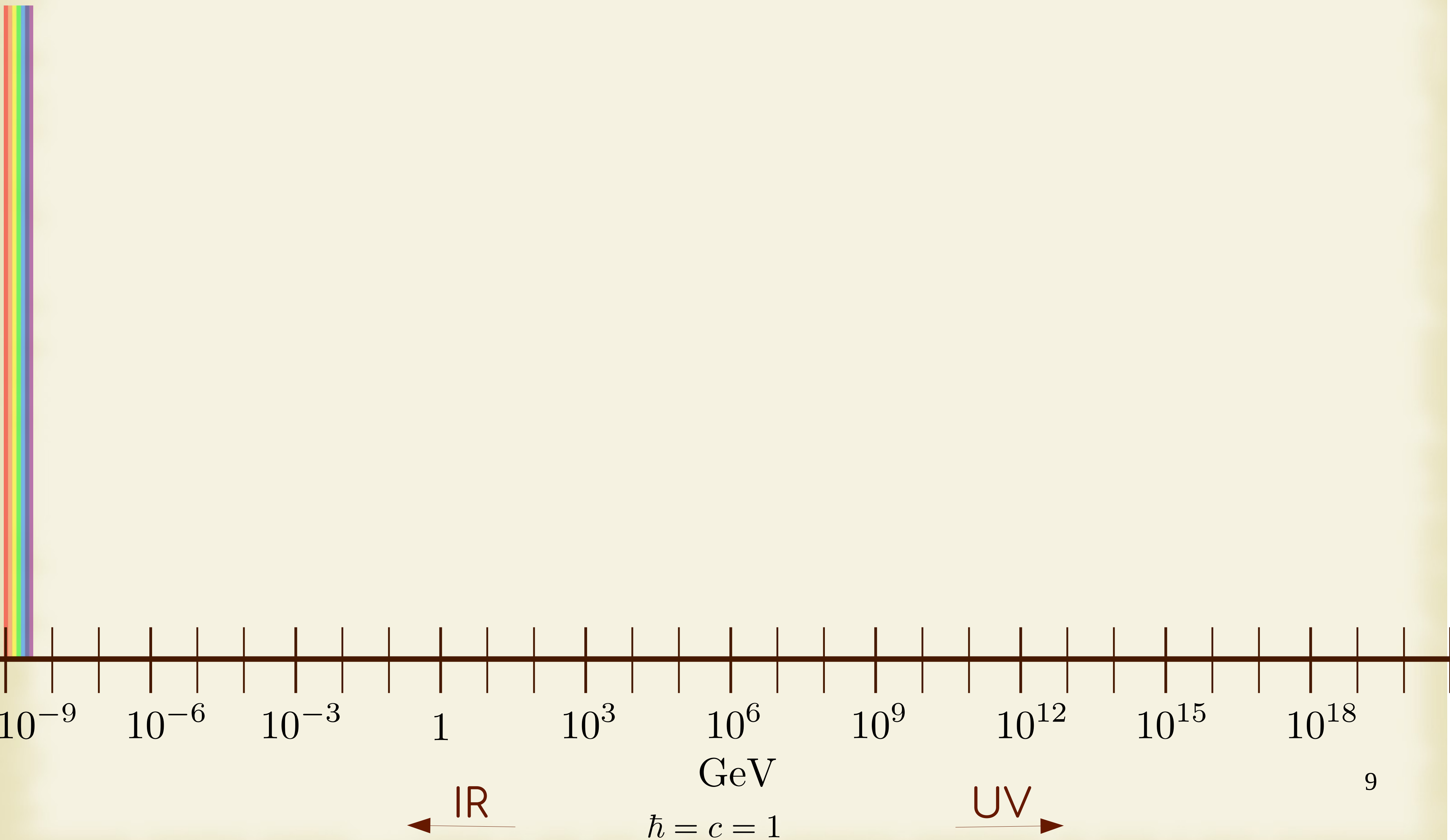
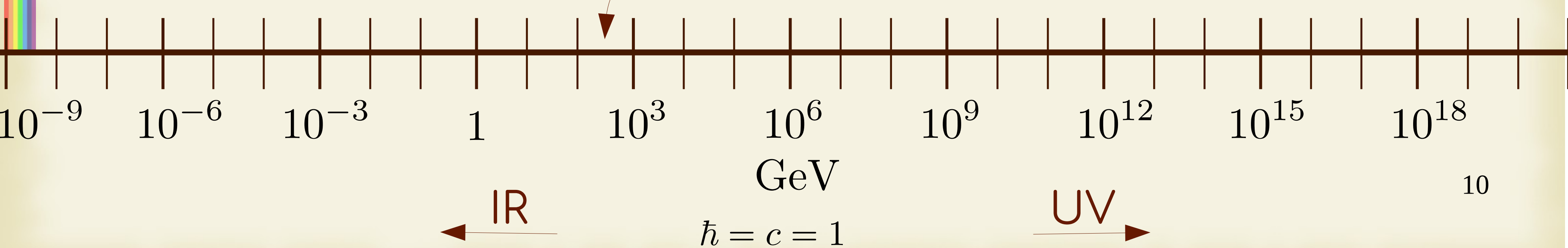
# Energy scales in High Energy Physics



# Energy scales in High Energy Physics

Electroweak scale

$$v = \left(\sqrt{2}G_F\right)^{-\frac{1}{2}}$$
$$= 256 \text{ GeV}$$



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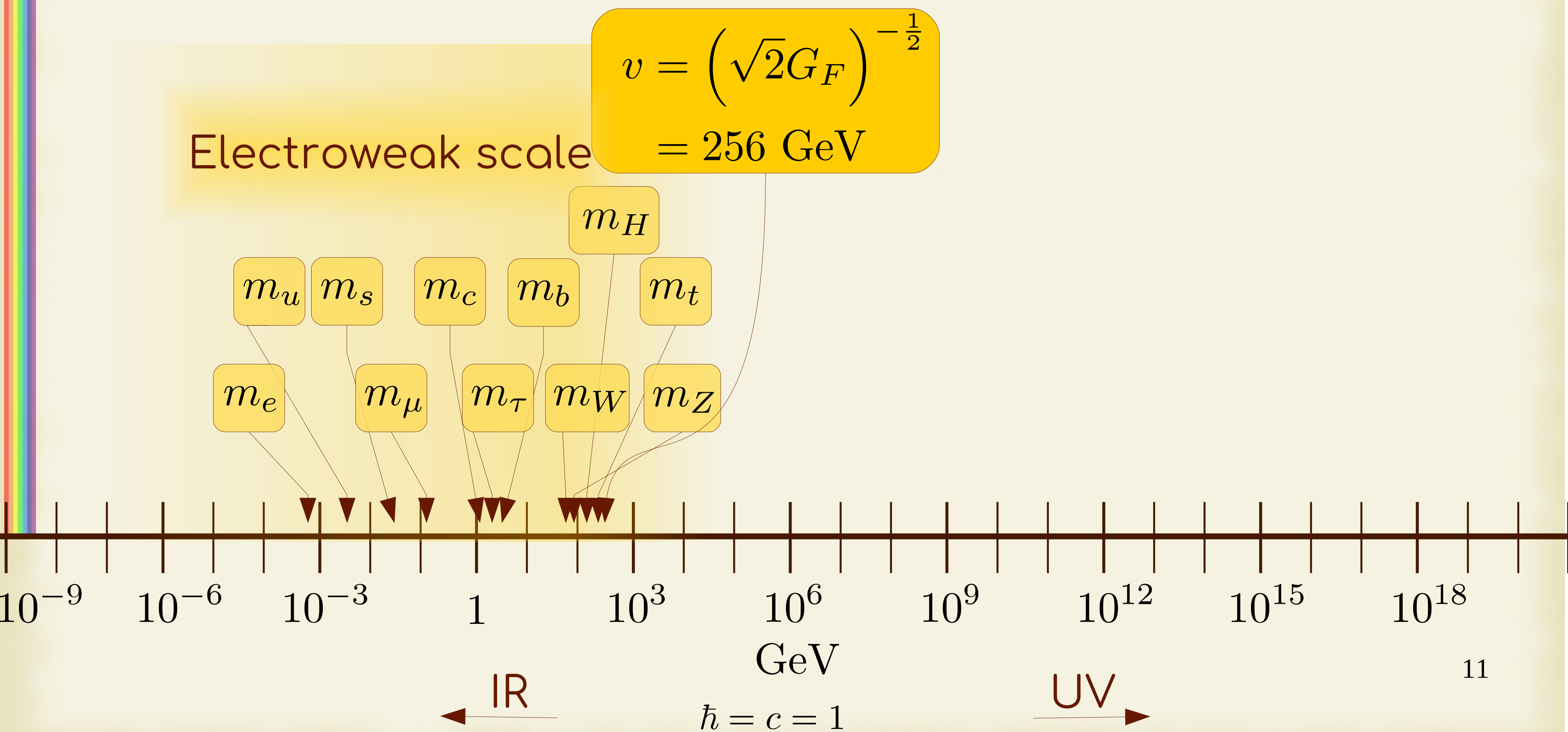

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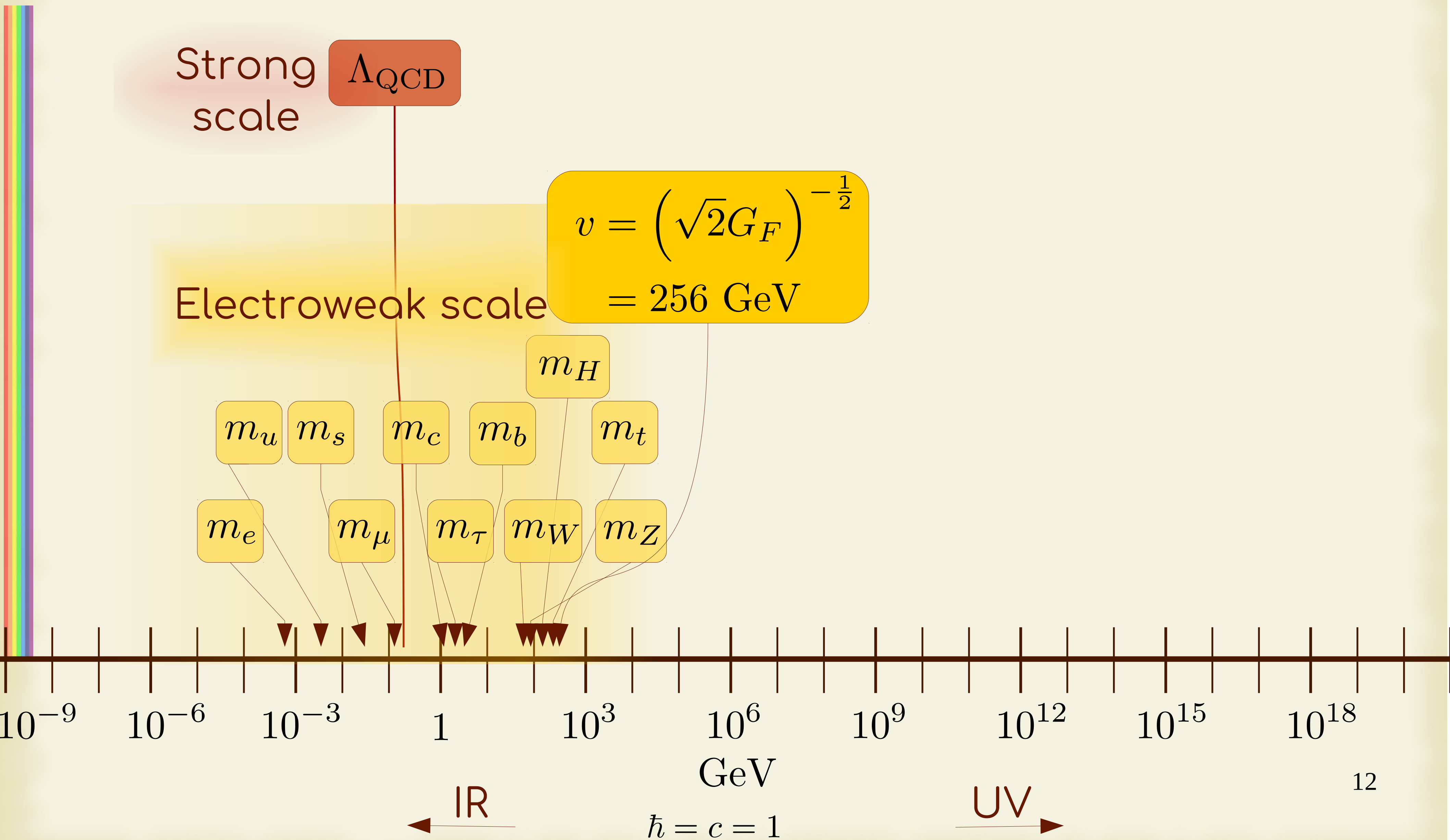

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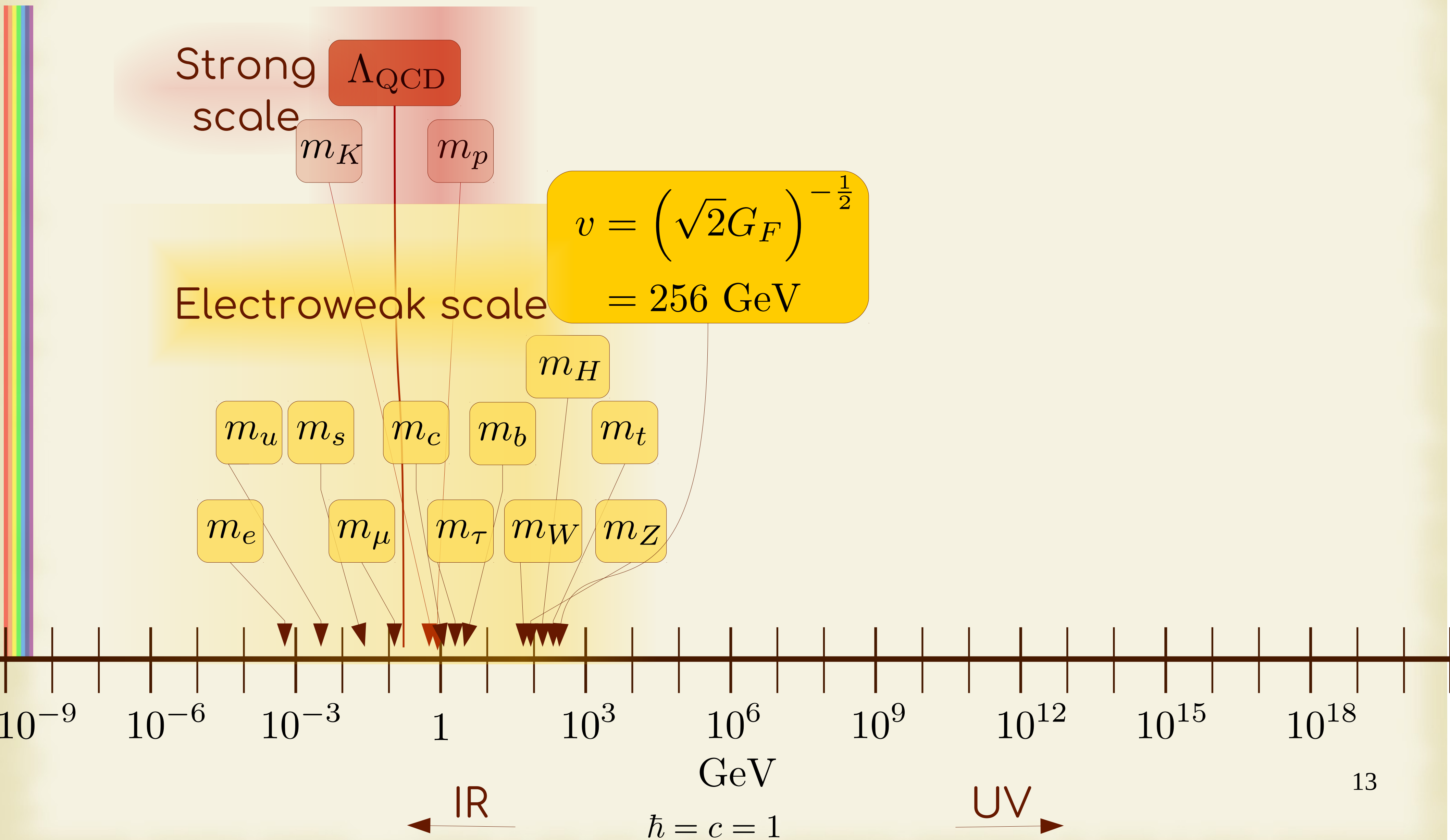

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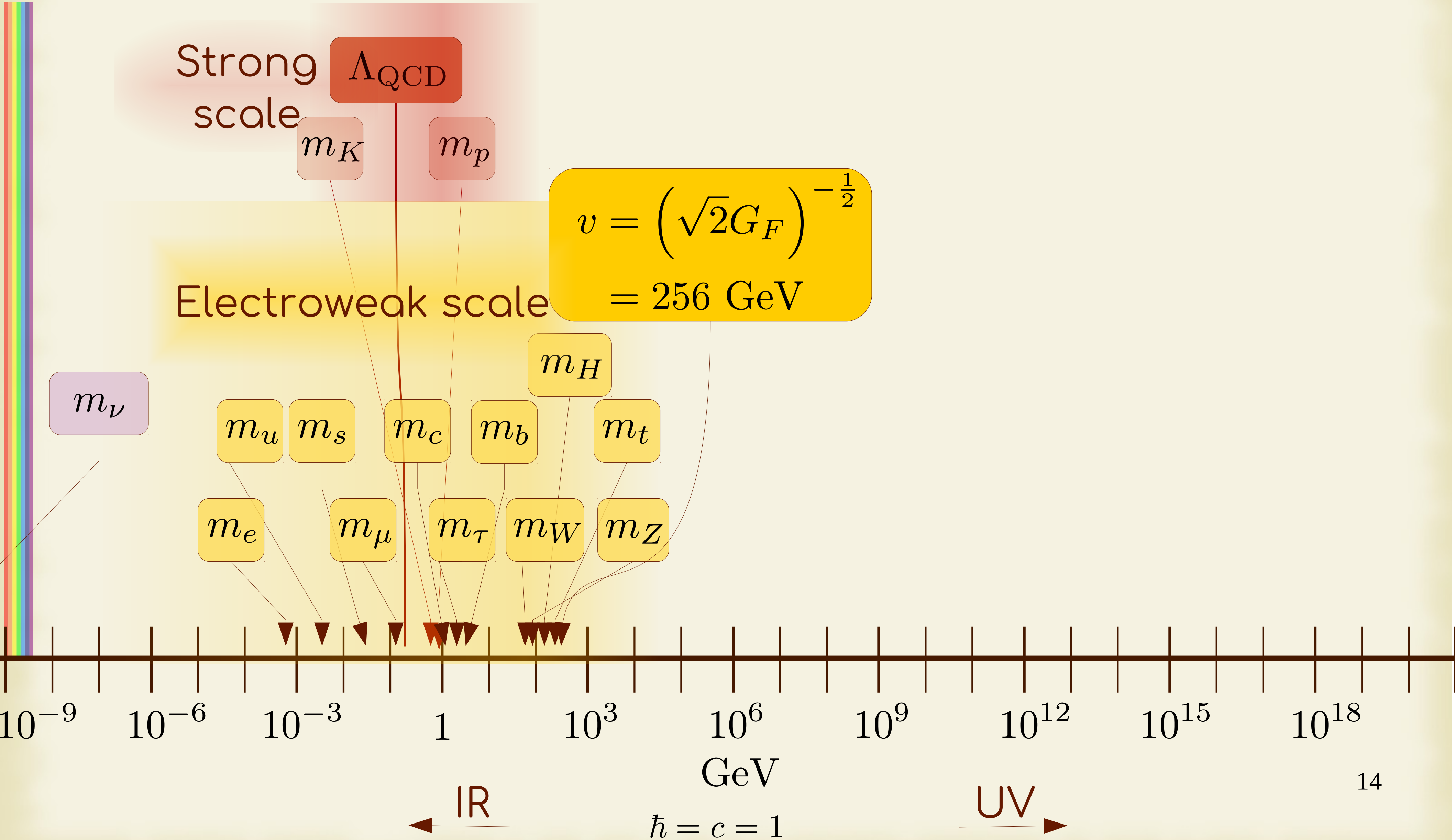

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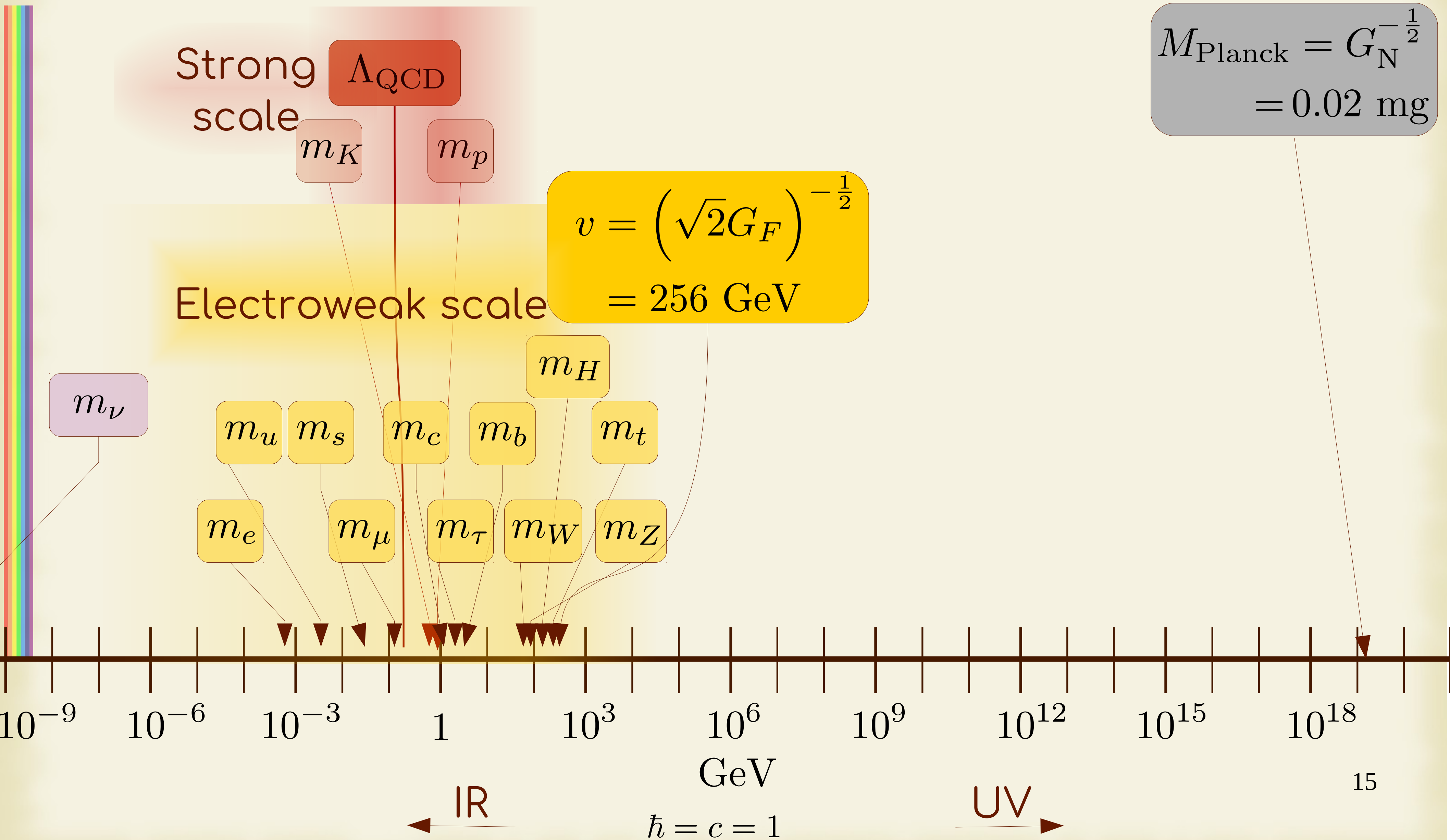

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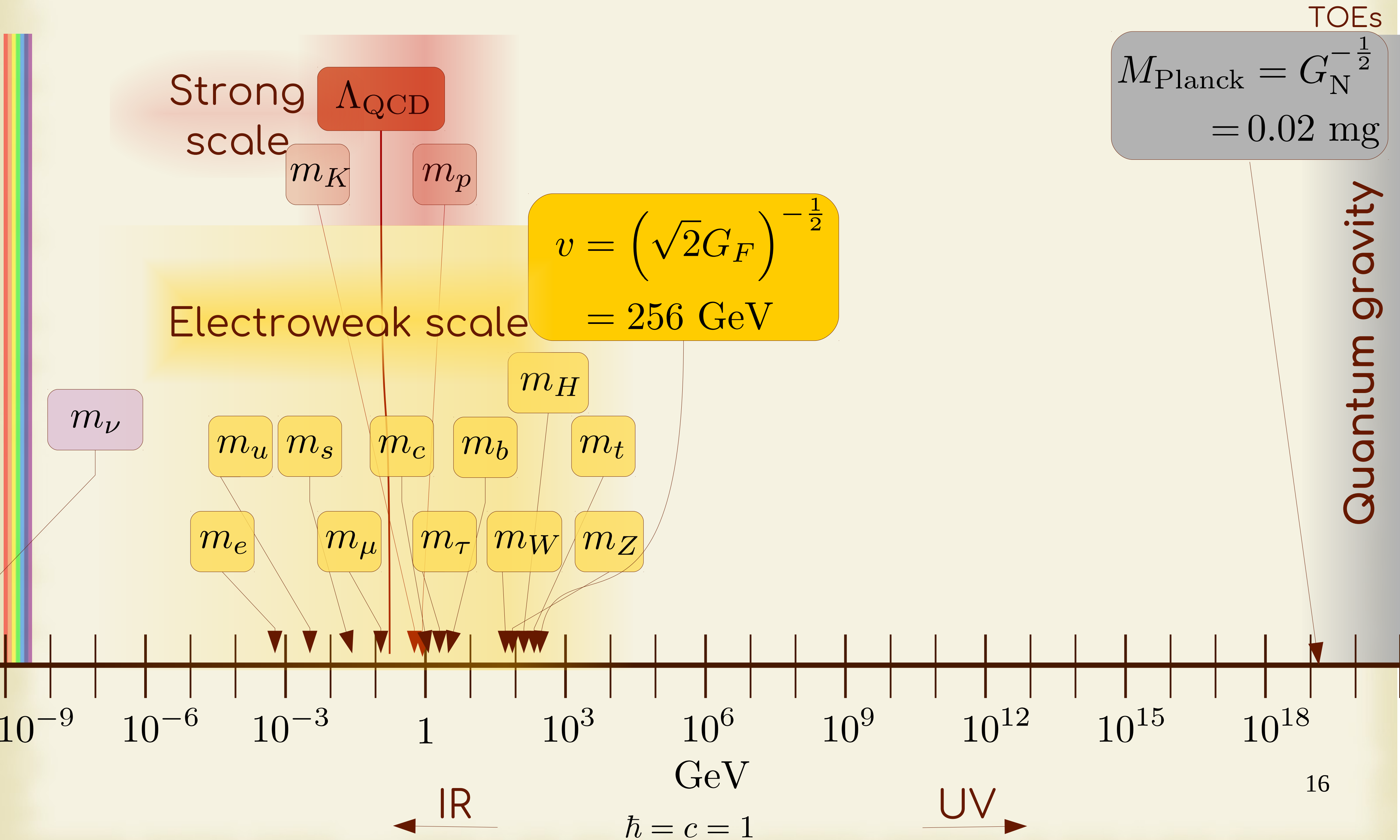

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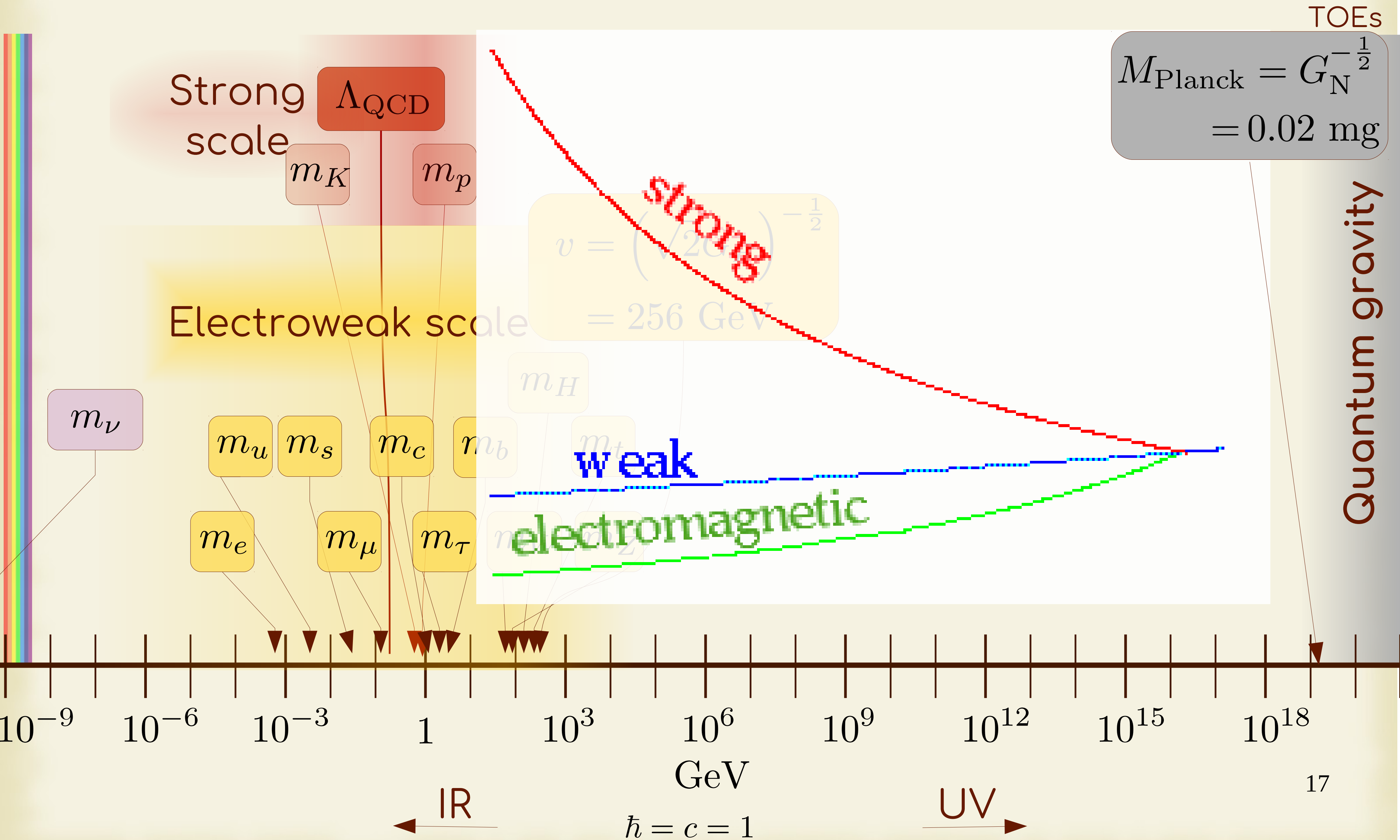

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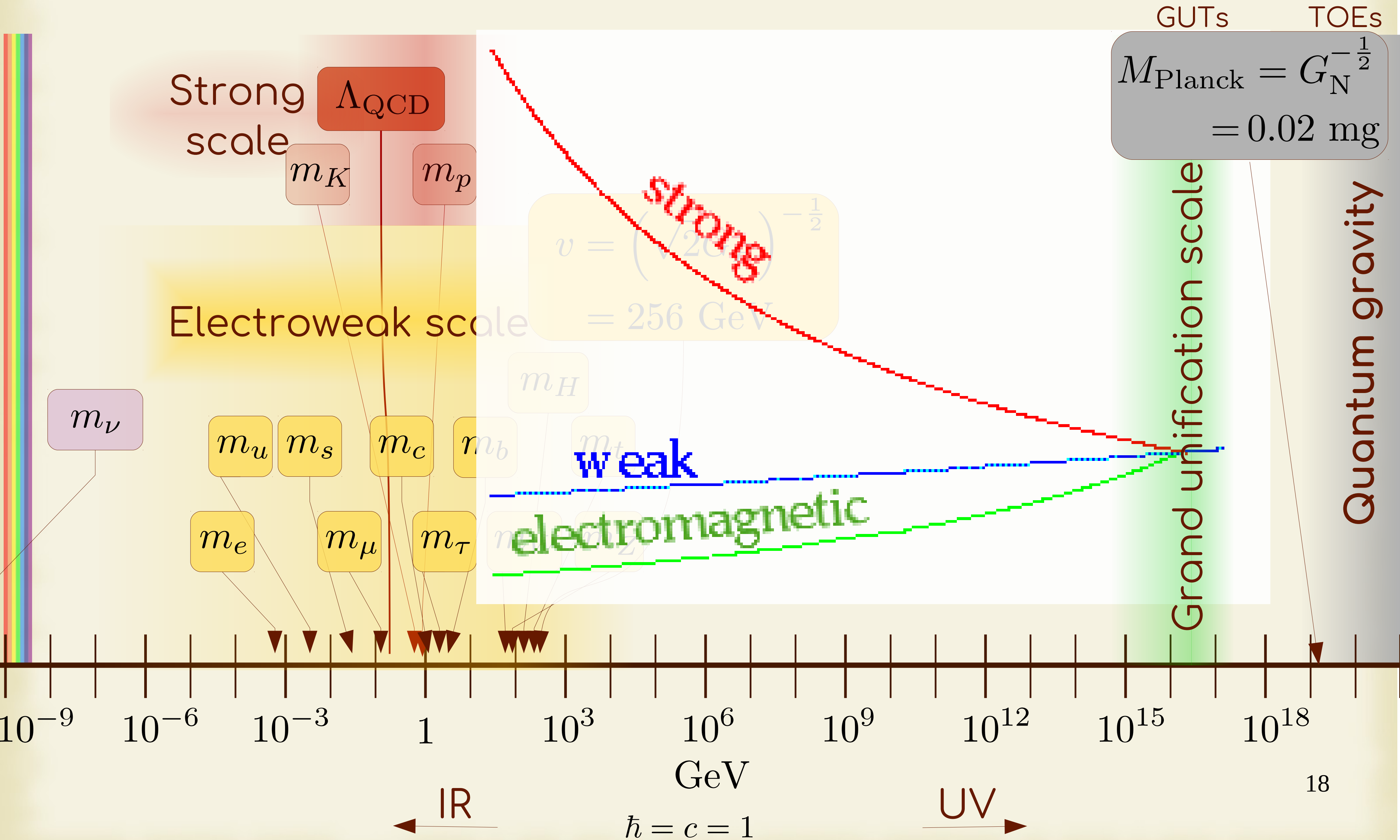

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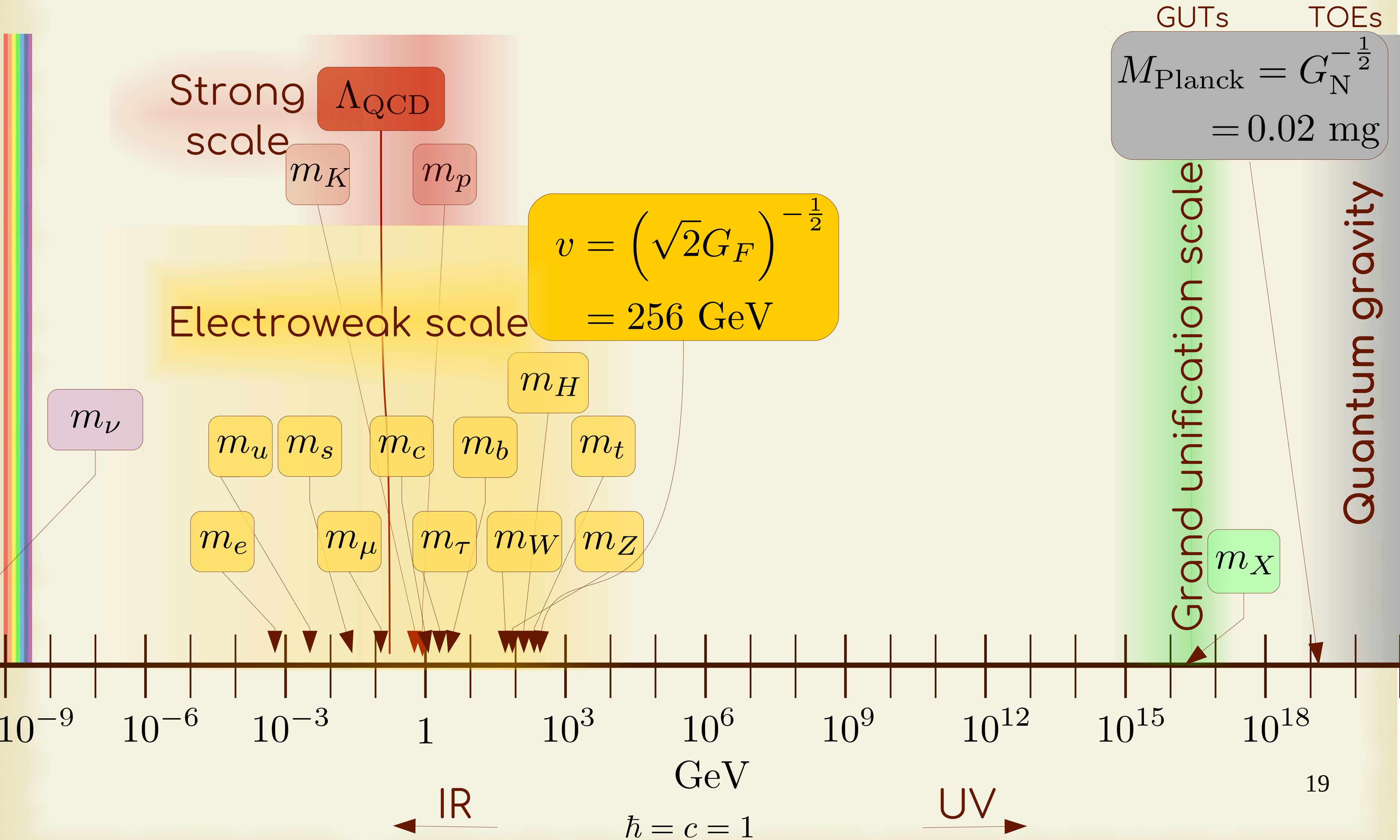

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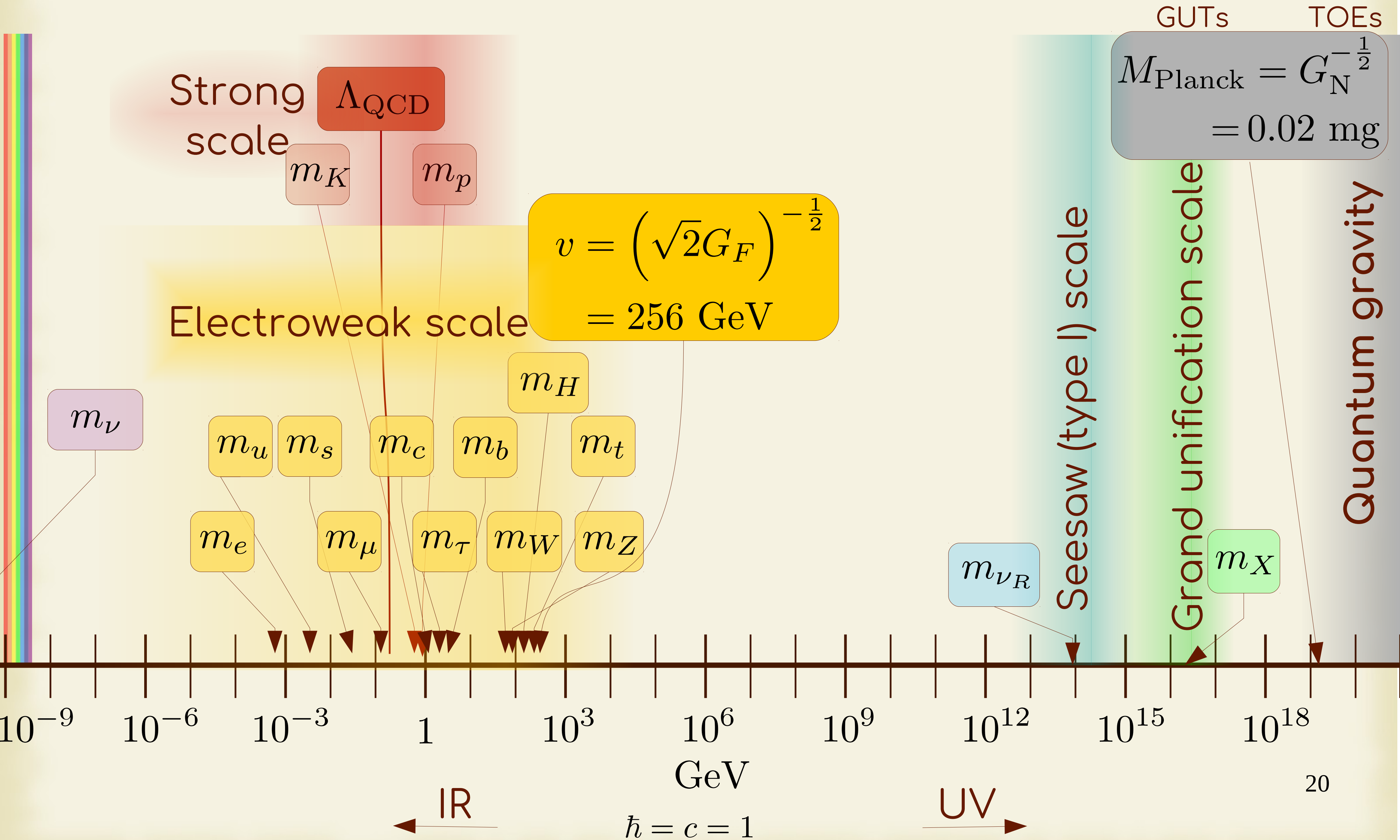

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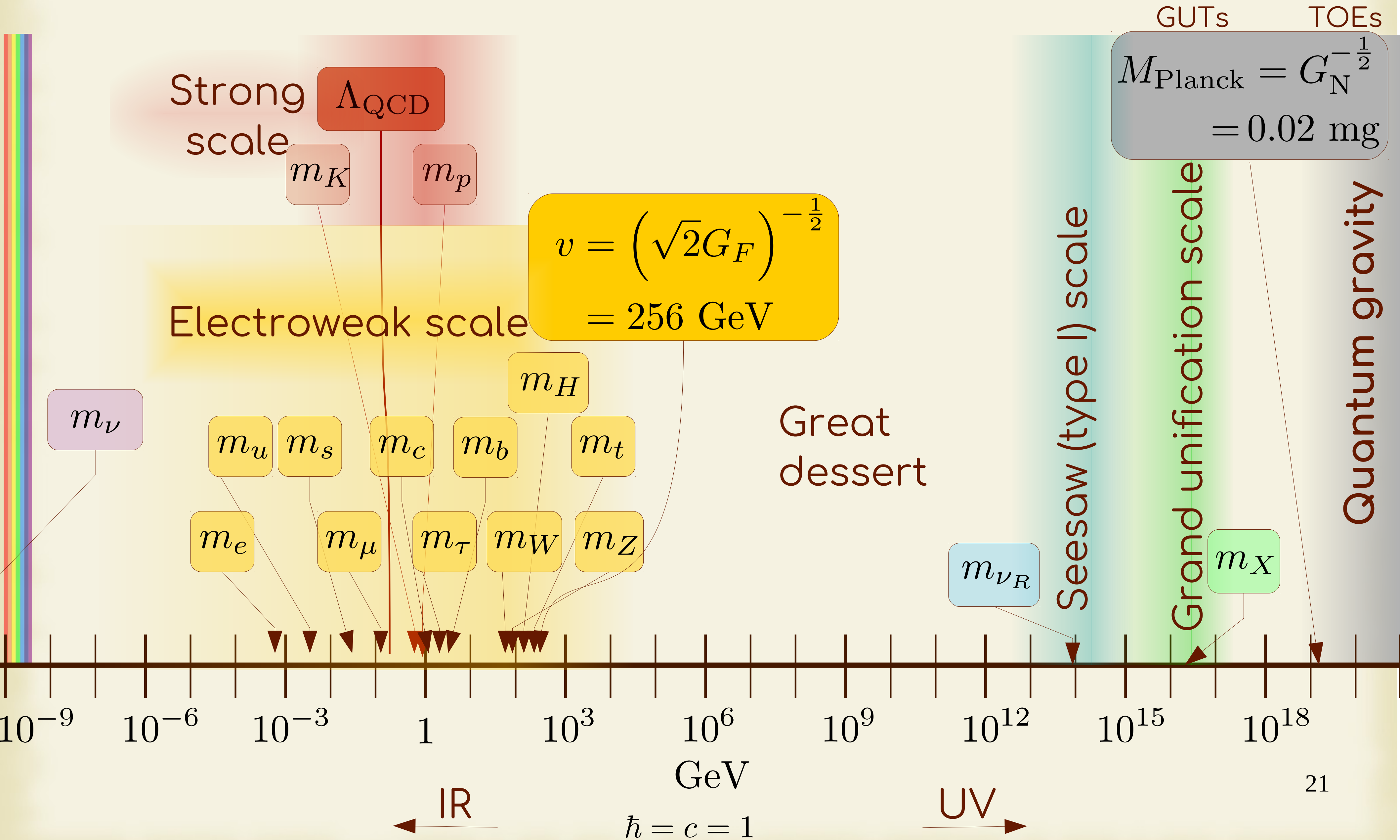

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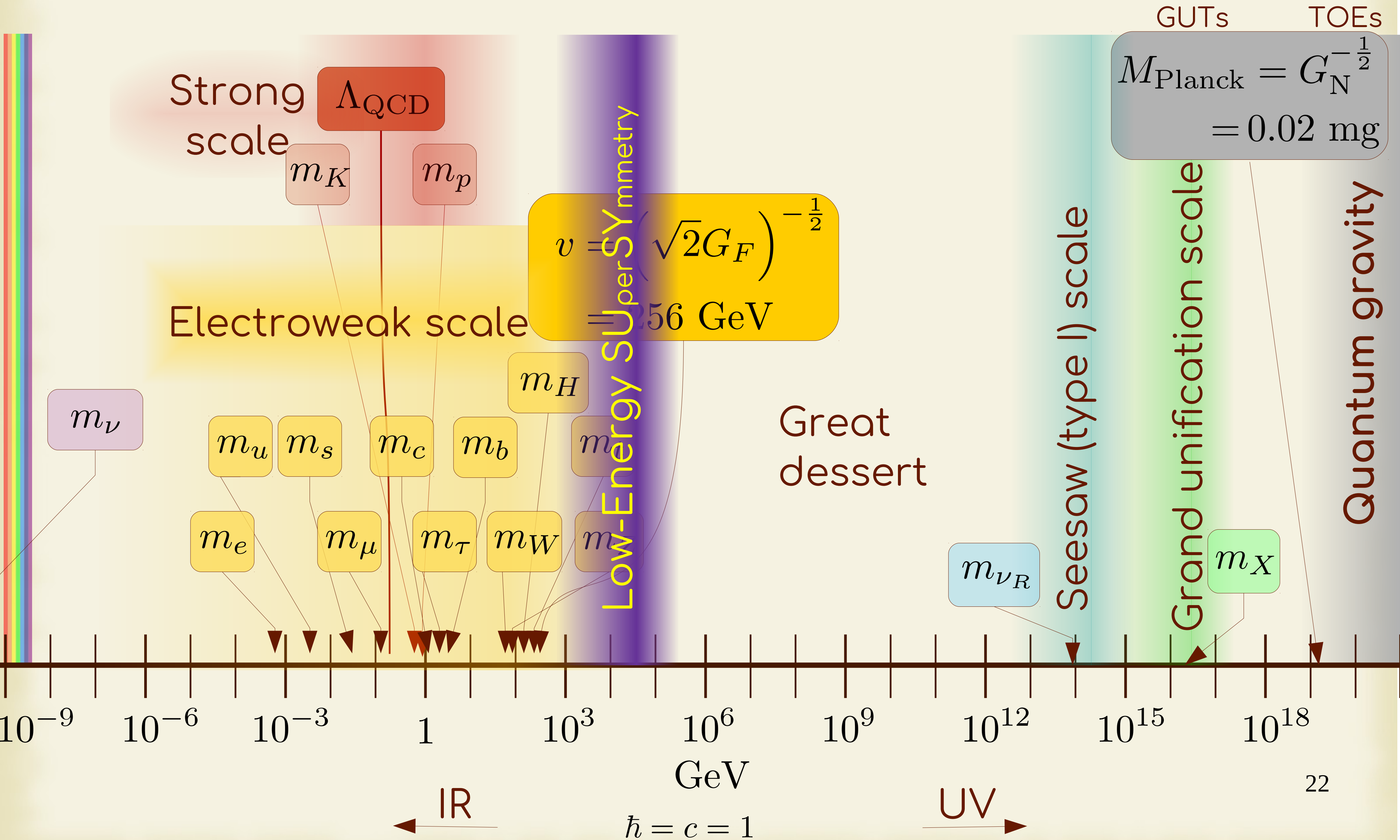

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Example: SU(5) Grand Unified Theory

$$m_{\rm H}^2 = -2\left(\mu_5^2 + \frac{b}{g_5^2} m_X^2\right)$$

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*"Doublet – triplet splitting"*

$m_H = 125\ \mathrm{GeV}$

$m_\Delta \sim 10^{16}\ \mathrm{GeV}$

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Everything was TREE LEVEL!

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# More scales – why people care (2)

Example 2: Loop corrections in simple models

$$\mathcal{L} = \ldots + m_\psi \bar{\psi}\psi + m_\phi^2 \phi^\dagger \phi + \text{heavy part}\,(\Lambda; \phi, \psi)$$

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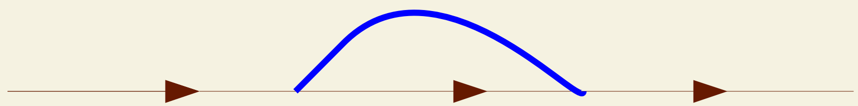
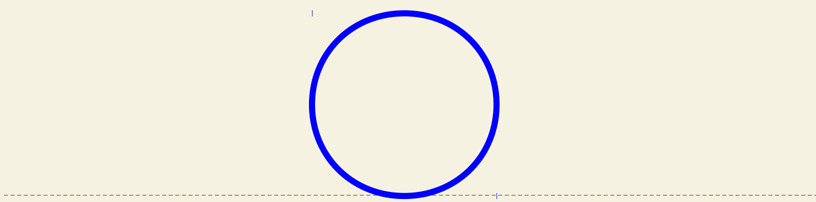


$$m_{\psi\ \text{phys}} = m_\psi(\mu) + \frac{m_\psi(\mu)}{16\pi^2} \log \frac{\Lambda^2}{\mu^2}$$



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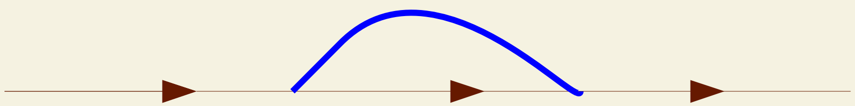

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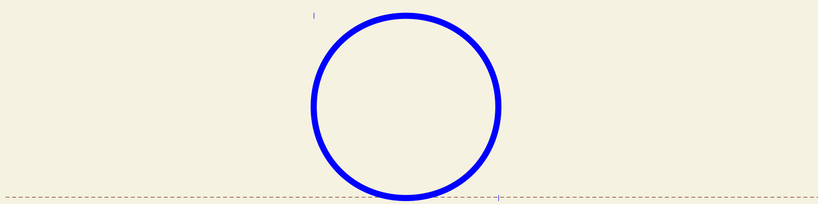

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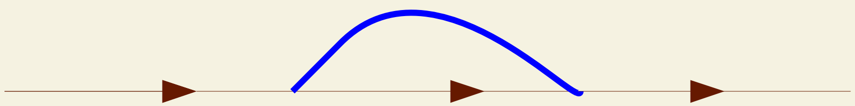

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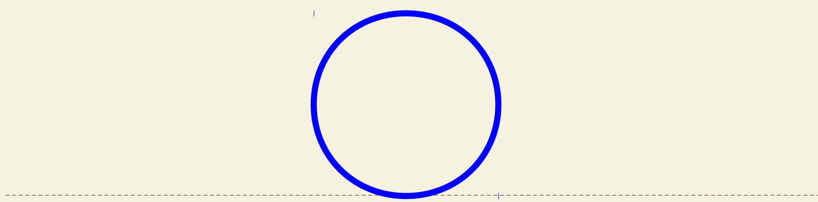

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*Common wisdom:* scalars are more sensitive on higher-energy scales than fermions.

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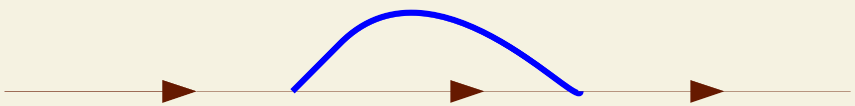

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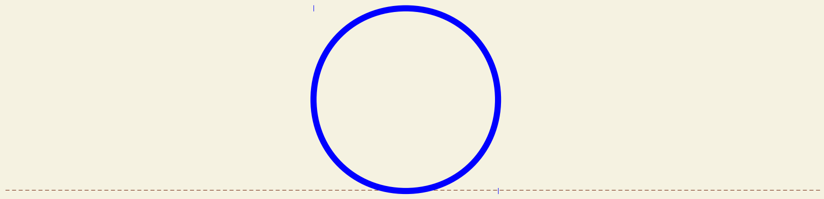

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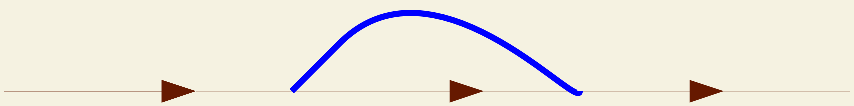

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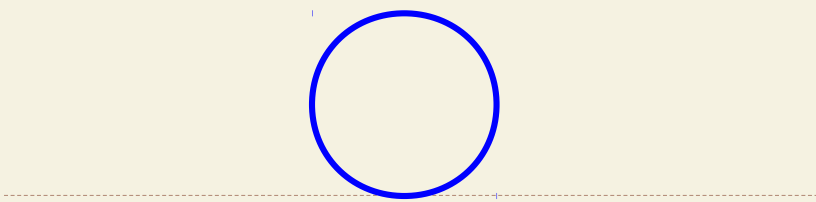

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This can be solved by SUSY, unlike Example 1.

# Dark side of the internet

"If the Standard Model is used to calculate quantum corrections to Fermi's constant, it appears … surprisingly large, closer to a Newton's constant."

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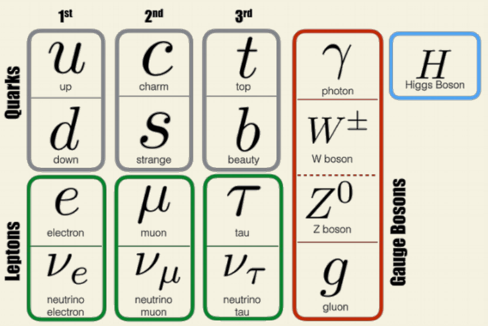
"If the Standard Model is used to calculate quantum corrections to Fermi's constant, it appears … surprisingly large, closer to a Newton's constant."

FALSE

# Standard Model



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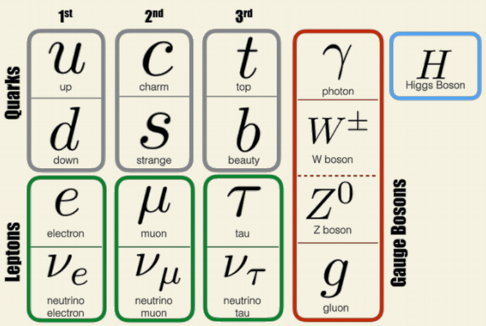
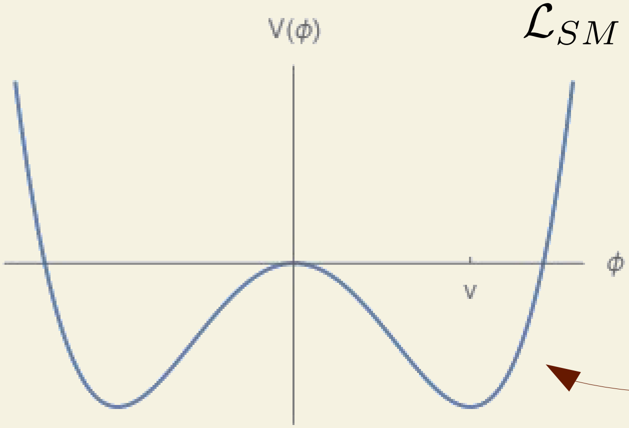



$$
\begin{aligned}
\mathcal{L}_{SM} = \quad & -\tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} - \tfrac{1}{2}A^{i}_{j\mu\nu}A^{j\mu\nu}_{i} - \tfrac{1}{2}G^{\alpha}_{\mu\nu\beta}G^{\beta\mu\nu}_{\alpha} \\
& + \overline{L}_i i \not{D}^{i}_{j} L^j + \overline{e} i \not{D} e + \overline{Q}_{i\alpha} i \not{D}^{i\alpha}_{j\beta} Q^{j\beta} + \overline{u}_\alpha i \not{D}^{\alpha}_{\beta} u^\beta + \overline{d}_\alpha i \not{D}^{\alpha}_{\beta} d^\beta \\
& + (D_\mu \Phi)^{\dagger}_{i} (D^\mu \Phi)^i \\
& + \left( \overline{Q}_{i\alpha} Y_{(d)} d^\alpha \varepsilon^{ij} \Phi^{\dagger}_{j} + \overline{Q}_{i\alpha} Y_{(u)} u^\alpha \Phi^i + \overline{L}_i Y_{(e)} e \, \Phi^i \right) + \text{h.c.} \\
& - V(\Phi)
\end{aligned}
$$

# Standard Model

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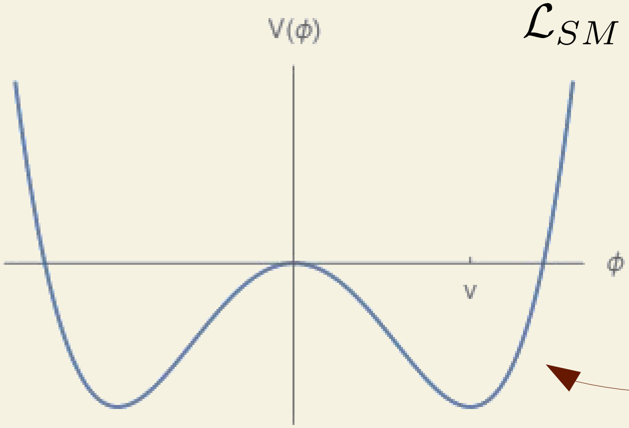
Higgs potential: $V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2$

# Standard Model

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Single dimensionful parameter in the Lagrangian!

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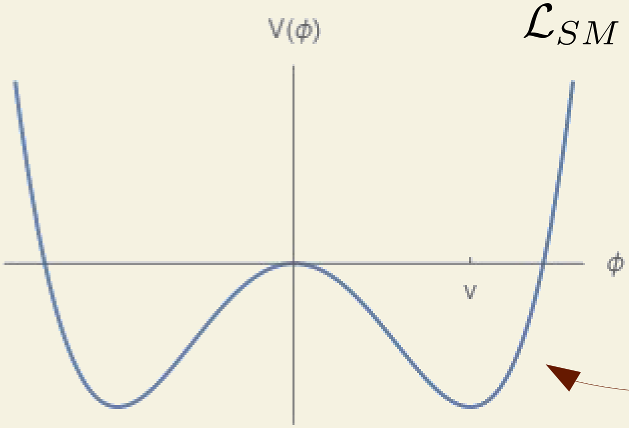


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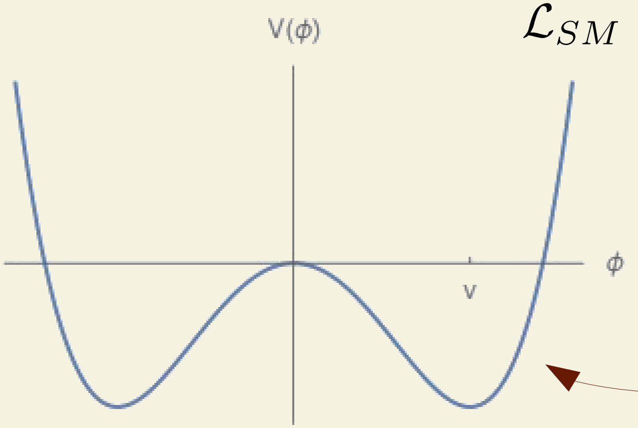
Tree level: $\mu/\sqrt{\lambda} = v = \left(G_F\sqrt{2}\right)^{-\frac{1}{2}}$

Vaccum Expectation Value of the Higgs field = “the VEV”

# Standard Model




$$\begin{aligned}\mathcal{L}_{SM} = \; & -\tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} - \tfrac{1}{2}A^{i}_{j\mu\nu}A^{j\mu\nu}_{i} - \tfrac{1}{2}G^{\alpha}_{\mu\nu\beta}G^{\beta\mu\nu}_{\alpha} \\ & +\overline{L}_i i\not{D}^{i}_{j}L^{j} + \overline{e}i\not{D}e + \overline{Q}_{i\alpha}i\not{D}^{i\alpha}_{j\beta}Q^{j\beta} + \overline{u}_{\alpha}i\not{D}^{\alpha}_{\beta}u^{\beta} + \overline{d}_{\alpha}i\not{D}^{\alpha}_{\beta}d^{\beta} \\ & + (D_\mu\Phi)^{\dagger}_{i}(D^{\mu}\Phi)^{i} \\ & + \left(\overline{Q}_{i\alpha}Y_{(d)}d^{\alpha}\varepsilon^{ij}\Phi^{\dagger}_{j} + \overline{Q}_{i\alpha}Y_{(u)}u^{\alpha}\Phi^{i} + \overline{L}_i Y_{(e)}e\,\Phi^{i}\right) + \text{h.c.} \\ & - V(\Phi)\end{aligned}$$

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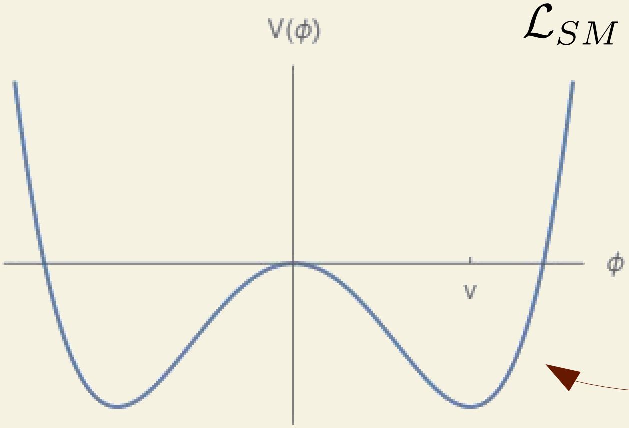


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Quantum (loop) level: only more complicated prefactors

# Effective potential - crash-course

Analogue to scalar potential (e.g. Higgs) but with quantum loops incorporated.

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Vaccuum Expectation Value of the scalar field

$$v = \langle \phi \rangle$$

is determined by minimum condition.

$$\left\langle \frac{\partial}{\partial \phi} V_{\text{eff}} \right\rangle = 0$$

Mass of the corresponding particle given by curvature in the minimum

$$m^2 = \left\langle \frac{\partial^2}{[\partial \phi]^2} V_{\text{eff}} \right\rangle$$

# What if $V_{eff} = V_{eff}(\phi^2)$

*Examples*:

- SM
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- SM + heavy neutrino

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|---|---|---|
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**Higgs boson mass always proportional to electroweak VEV!**

**Even for BSM people $m_H$ = 125 GeV should have been no surprise!**

# Hierarchy problem is ***not***

- a problem of the SM alone

  *as it is a single scale theory*

- caused solely by loop corrections

  *tree-level  fine-tuning issues are also common*

- resolved fully in SUSY GUTs,

  *tree level fine tuning still there*

- a problem of smallness of $m_H$= 125 GeV

  *but rather of the whole electroweak scale*

- a problem at all for
  "totally unBayessed" people.

# LUNCH !!!