Amplitudové metody pro efektivní polní teorie

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Outline:

- Role model: gluon amplitudes
- Effective Field Theories
- From NLSM to Periodic Table of Scalar Theories
- Further avenues
- Summary

Introduction: amplitudes

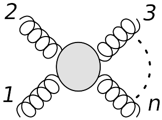
Objective of amplitude community:

Study a priori known objects from different perspective

Example in mind: gluon amplitudes

- 1986: Parke and Taylor calculated 6-point gluon-scattering
- simplification: tree-level, no-fermions
- final result: extremely simple
- other way of calculation?

Example: gluon amplitudes



At tree level:

- colour ordering $\to$ stripped amplitude

$$M^{a_1 \ldots a_n}(p_1, \ldots p_n) = \sum_{\sigma/Z_n} \mathrm{Tr}(t^{a_{\sigma(1)}} \ldots t^{a_{\sigma(n)}}) M_\sigma(p_1, \ldots, p_n)$$

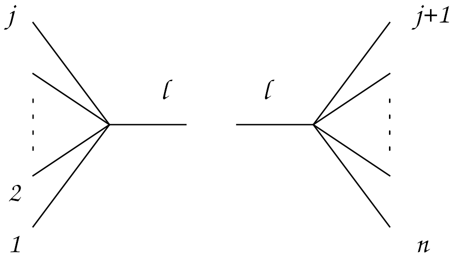
- $M_\sigma(p_{\sigma(1)}, \ldots, p_{\sigma(n)}) = M(p_1, \ldots, p_n) \equiv M(1, 2, \ldots n)$
- propagators $\Rightarrow$ the only poles of M_σ
- thanks to ordering the only possible poles are:

$$P_{ij}^2 = (p_i + p_{i+1} + \ldots + p_{j-1} + p_j)^2$$

Pole structure

Weinberg's theorem (one particle unitarity): on the factorization channel

$$\lim_{P_{1j}^2 \to 0} M(1,2,\dots n) = \sum_{h_l} M_L(1,2\dots j,l) \times \frac{1}{P_{1j}^2} \times M_R(l,j+1,\dots n)$$



BCFW relations, preliminaries

[Britto, Cachazo, Feng, Witten '05]

Reconstruct the amplitude from its poles (in complex plane)

- shift in two external momenta

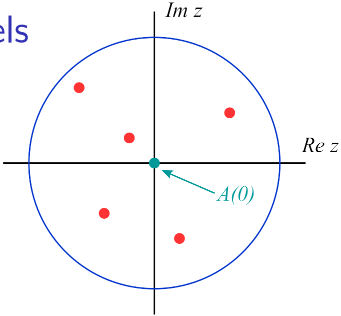
$$p_i \to p_i + zq, \qquad p_j \to p_j - zq$$

- keep p_i and p_j on-shell, i.e.

$$q^2 = q \cdot p_i = q \cdot p_j = 0$$

- amplitude becomes a meromorphic function $A(z)$
- only simple poles coming from propagators $P_{ab}(z)$
- original function is $A(0)$

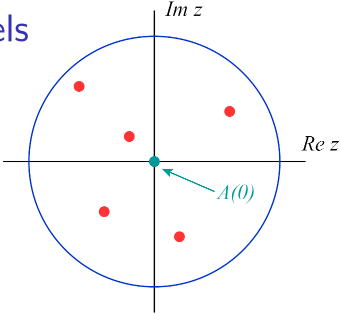
BCFW relations: factorization channels



Cauchy's theorem

$$\frac{1}{2\pi \mathrm{i}} \int \frac{dz}{z} A(z) = A(0) + \sum_k \frac{\mathrm{Res}\,(A, z_k)}{z_k}$$

BCFW relations: factorization channels



Cauchy's theorem

$$0 = \frac{1}{2\pi \mathrm{i}} \int \frac{dz}{z} A(z) = A(0) + \sum_k \frac{\mathrm{Res}\,(A, z_k)}{z_k}$$

If $A(z)$ vanishes for $z \to \infty$

$$A = A(0) = -\sum_k \frac{\mathrm{Res}\,(A, z_k)}{z_k}$$

BCFW relations

$P_{ab}^2(z) = 0$ if one and only one i (or j) in $(a, a+1, \ldots, b)$.

Suppose $i \in (a, \ldots, b) \not\ni j$

$$P_{ab}^2(z) = (p_a + \ldots + p_{i-1} + p_i + zq + p_{i+1} + \ldots + p_b)^2 =$$
$$= P_{ab}^2 + 2q \cdot P_{ab} z = 0$$

solution

$$z_{ab} = -\frac{P_{ab}^2}{2(q \cdot P_{ab})} \qquad \Rightarrow \qquad P_{ab}^2(z) = -\frac{P_{ab}^2}{z_{ab}}(z - z_{ab})$$

Thus

$$\text{Res}(A, z_{ab}) = \sum_s A_L^{-s}(z_{ab}) \times \frac{-z_{ab}}{P_{ab}^2} \times A_R^s(z_{ab})$$

and for allowed helicities it factorizes into two subamplitudes

BCFW relations

Using Cauchy's formula, we have finally as a result

$$A = \sum_{k,s} A_L^{-s_k}(z_k)\frac{1}{P_k^2}A_R^{s_k}(z_k)$$

- based on two-line shift (convenient choice: adjacent i,j)
- recursive formula (down to 3-pt amplitudes)
- number of terms small = number of factorization channels
- all ingredients are on shell

BCFW Example: gluon amplitudes

od diagrams for n-body gluon scatterings at tree level

n	3	4	5	6	7	8
# diagrams (inc.crossing)	1	4	25	220	2485	34300
# diagrams (col.ordered)	1	3	10	38	154	654
# BCFW terms	–	1	2	3	6	20

[C.Cheung: TASI Lectures '17]
[KK, Novotny, Trnka '13]

BCFW recursion relations: problems

We have assumed that

$$A(z) \to 0, \qquad \text{for} \qquad z \to \infty$$

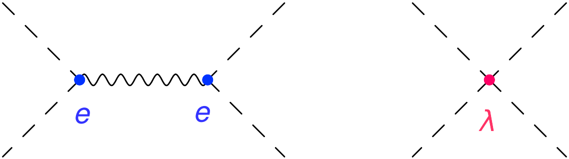
More generally we have to include a boundary term in Cauchy's theorem.

This is intuitively clear: we can formally use the derived BCFW recursion relations to obtain any higher n amplitude starting with the leading interaction. But this does not have to be the correct answer.

BCFW recursion relations: problems

example: scalar-QED

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |D\phi|^2 - \frac{1}{4}\lambda|\phi|^4$$



Due to the power-counting the boundary term is proportional to

$$B \sim 2e^2 - \lambda$$

In order to eliminate the boundary term we have to set $\lambda = 2e^2$, then the original BCFW works.

I.e. we needed some further information (e.g. supersymmetry) to determine the λ piece.

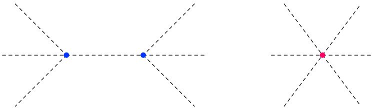
Effective field theories

Effective field theories: general picture

Now we have infinitely many unfixed "λ" terms. Schematically

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \lambda_4(\partial^{m_4}\phi)^4 + \lambda_6(\partial^{m_6}\phi)^6 + \dots$$

Example: 6pt scattering, Feynman diagrams



Corresponding amplitude:

$$\mathcal{M}_6 = \sum_{I=poles} \lambda_4^2 \frac{\cdots}{P_I} + \lambda_6(\dots)$$

λ_6 part: not fixed by the pole behaviour.

Task: to find a condition in order to link these two terms

Effective field theories: introduction

Usual steps:

Symmetry $\rightarrow$ Lagrangian $\rightarrow$ Amplitudes $\rightarrow$ physical quantities
(cross-section, masses, decay constants, . . .)

In our work – opposite direction:

Amplitudes $\rightarrow$ Lagrangian $\rightarrow$ Symmetry

Our aim: classification of interesting EFTs

works done in collaborations with Clifford Cheung, Jiri Novotny, Chia-Hsien Shen, Jaroslav Trnka and Congkao Wen

Effective field theories: scalar theories

As simple as possible: a spin-0 massless degree of freedom with a three-point interaction.

General formula for three-particle amplitude

$$A(1^{h_1}2^{h_2}3^{h_3}) = \begin{cases} \langle 12\rangle^{h_3-h_1-h_2}\langle 23\rangle^{h_1-h_2-h_3}\langle 31\rangle^{h_2-h_3-h_1}, & \Sigma h_i \leq 0 \\ [12]^{h_1+h_2-h_3}[23]^{h_2+h_3-h_1}[31]^{h_3+h_1-h_2}, & \Sigma h_i \geq 0 \end{cases}$$

Used a spinor-helicity notation, e.g. $p_i \cdot p_j \sim \langle ij\rangle[ij]$

For scalars ($h_i = 0$) this is a constant - corresponding to $\mathcal{L}_{int} = \lambda\phi^3$. All derivatives can be removed by equations of motions (boxes)

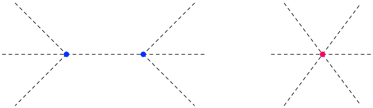
$$\mathcal{L}_{int} = (\partial_\alpha \ldots \partial_\omega \phi)(\partial^\alpha \ldots \partial^\omega \phi)\phi \quad \rightarrow \quad \mathcal{L}_{int} = (\Box\phi)(\ldots)$$

Effective field theories: scalar theories

We start with (m counts number of derivatives)

$$\mathcal{L}_{int} = \lambda_4 \partial^m \phi^4$$

n.b. we want to connect this four-point vertex with the 6-point contact terms



This rules out again no-derivative terms, as the powercounting dictates:

$$\partial^m \times \frac{1}{\partial^2} \times \partial^m \quad \rightarrow \quad \partial^{2m-2}\phi^6$$

Simplest example: two derivatives, single scalar

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \lambda_4\partial^2\phi^4 + \lambda_6\partial^2\phi^6 + \ldots$$

How to connect λ_4 and λ_6?
Well Lagrangian, an infinite series, looks complicated, but it is not the case. It represents a free theory:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi\underbrace{(1 + \lambda_4\phi^2 + \ldots)}_{F(\phi)}$$

$F(\phi)$ can be removed by a field redefinition

Non-trivial simplest example:

- more derivatives
- more flavours ($\phi \to \phi_1, \phi_2, \ldots$)

More flavours

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi^i\partial^\mu\phi^i + \lambda_{ijkl}\partial_\mu\phi^i\partial^\mu\phi^j\phi^k\phi^l + \lambda_{i_1\dots i_6}\partial_\mu\phi^{i_1}\partial^\mu\phi^{i_2}\phi^{i_3}\dots\phi^{i_6} + \dots$$

- Can be used for systematic studies of two species, three species, etc.
- Very complicated generally
- Assume some simplification, organize using a group structure

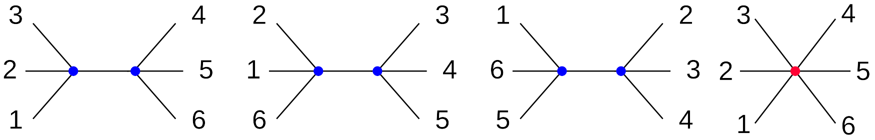
$$\phi = \phi^a T^a$$

- motivated by the ‘gluon case’: flavour ordering [KK,Novotny,Trnka ’13]

$$A^{a_1\dots a_n} = \sum_{perm} \mathrm{Tr}(T^{a_1}\dots T^{a_n})A(p_1,\dots p_n)$$

More flavours: stripped amplitude

first non-trivial case 6pt scattering:



power-counting is ok:

$$\lambda_4^2 \; p^2 \frac{1}{p^2} p^2 + \lambda_6 \; p^2$$

in order to combine the pole and contact term we need to consider some limit. The most natural candidate: We will demand soft limit, i.e.

$$A \to 0, \qquad \text{for} \quad p \to 0$$

$\Rightarrow \lambda_4^2 \sim \lambda_6$

Standard direction(s)

Assuming the shift symmetry

$$\phi^a \to \phi^a + \epsilon^a$$

$\Rightarrow$ Noether current

$$A_\mu^a = \frac{\delta \mathcal{L}}{\delta \partial^\mu \phi^a}$$

$\Rightarrow$ Ward identity $\Rightarrow$ LSZ

$$\langle 0 | A_\mu^a(x) | \phi^b(p) \rangle = i F \delta^{ab} p_\mu \mathrm{e}^{-ipx}$$

$\Rightarrow$ Adler zero

$$\lim_{p \to 0} \langle f | i + \phi^a(p) \rangle = 0$$

$\Rightarrow$ CCWZ: non-linear sigma model

$$\mathcal{L} = \frac{F^2}{2} \mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U), \qquad U = \mathrm{e}^{\frac{i}{F} \phi^a T^a}$$

[Weinber'66], [Ian Low '14-'15]

Natural classification: σ and ρ

Soft limit of one external leg of the tree-level amplitude

$$A(tp_1, p_2, \ldots, p_n) = \mathcal{O}(t^{\sigma}), \qquad \text{as} \quad tp_1 \to 0$$

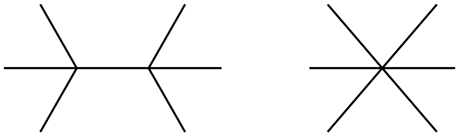
Interaction term

$$\mathcal{L} = \partial^m \phi^n$$

Then another natural parameter is (counts the homogeneity)

$$\rho = \frac{m-2}{n-2} \qquad \text{"averaging number of derivatives"}$$

e.g. $\mathcal{L} = \partial^m \phi^4 + \partial^{\widetilde{m}} \phi^6$



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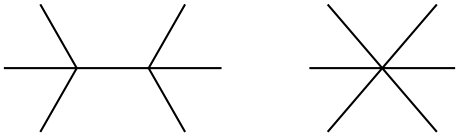

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$2m - 2 - 2 = \tilde{m} - 2 \Rightarrow \frac{2m-4}{4} = \frac{\tilde{m}-2}{4} \Rightarrow$

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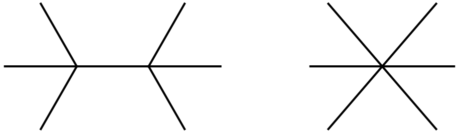


so these two diagrams can mix: $p^{2m-2} \sim p^{\widetilde{m}}$

$2m - 2 - 2 = \tilde{m} - 2 \Rightarrow \frac{2m-4}{4} = \frac{\tilde{m}-2}{4} \Rightarrow \rho = \widetilde{\rho}$

rho is same if they talk to each other

Non-trivial cases

$$\text{for: } \mathcal{L} = \partial^m \phi^n : \qquad m < \sigma n$$

or

$$\sigma > \frac{(n-2)\rho + 2}{n}$$

i.e.

ρ	σ at least
0	1
1	2
2	2
3	3

i.e. non-trivial regime for $\rho \leq \sigma$

First case: $\rho = 0$ (i.e. two derivatives)

Schematically for single scalar case

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \sum_i \lambda_4^i(\partial^2\phi^4) + \sum_i \lambda_6^i(\partial^2\phi^6) + \dots$$

similarly for multi-flavour (ϕ_i: $\phi_1, \phi_2, \dots$).
non-trivial case

$$\sigma = 1$$

Outcome:

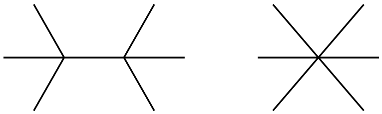
- single scalar: free theory
- multiple scalars (with flavour-ordering): non-linear sigma model

n.b. it represents a generalization of [Susskind, Frye '70], [Ellis, Renner '70]

Second case: $\rho = 1$, $\sigma = 2$ (double soft limit)

1. focus on the lowest combination and fix the form:

$$\mathcal{L}_{int} = c_2(\partial\phi \cdot \partial\phi)^2 + c_3(\partial\phi \cdot \partial\phi)^3 \qquad \text{condition: } c_3 = 4c_2^4$$



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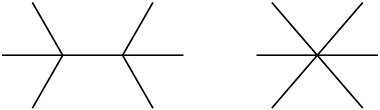

2. find the symmetry

$$\phi \to \phi - b_\rho x^\rho + b_\rho \partial^\rho \phi \, \phi \quad \text{(again up to 6pt so far)}$$

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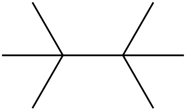

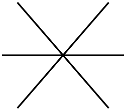

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3. ansatz of the form

$$c_n(\partial\phi \cdot \partial\phi)^n + c_{n+1}(\partial\phi \cdot \partial\phi)^n \partial\phi \cdot \partial\phi$$

4. in order to cancel: $2(n+1)c_{n+1} = (2n-1)c_n$
   i.e. $c_1 = \frac{1}{2} \Rightarrow c_2 = \frac{1}{8}, c_3 = \frac{1}{16}, c_4 = \frac{5}{128}, \ldots$

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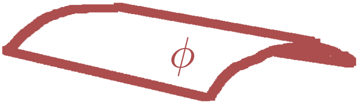
solution:

$$\mathcal{L} = -\sqrt{1 - (\partial\phi \cdot \partial\phi)}$$

This theory known as a scalar part of the Dirac-Born-Infeld [1934] – DBI action
Scalar field can be seen as a fluctuation of a 4-dim brane in five-dim Minkowski space



Remark: soft limit and symmetry are "equivalent"

Third case: $\rho = 2$, $\sigma = 2$ (double soft limit)

Similarly to previous case we will arrive to a unique solution: the Galileon Lagrangian

$$\mathcal{L} = \sum_{n=1}^{d+1} d_n \phi \mathcal{L}^{\mathrm{der}}_{n-1}$$

$$\mathcal{L}^{\mathrm{der}}_n = \varepsilon^{\mu_1 \ldots \mu_d} \varepsilon^{\nu_1 \ldots \nu_d} \prod_{i=1}^{n} \partial_{\mu_i} \partial_{\nu_i} \phi \prod_{j=n+1}^{d} \eta_{\mu_j \nu_j} = -(d-n)! \det \left\{ \partial^{\nu_i} \partial_{\nu_j} \phi \right\} .$$

It possesses the Galilean shift symmetry

$$\phi \to \phi + a + b_\mu x^\mu$$

(leads to EoM of second-order in field derivatives)

Surprise: $\rho = 2$, $\sigma = 3$ (enhanced soft limit)

- general galileon: three parameters (in 4D)
- only two relevant (due to dualities [de Rham, Keltner, Tolley '14] [KK, Novotny '14])

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- symmetry explanation: hidden symmetry [K. Hinterbichler and A. Joyce 1501.07600]

$$\phi \to \phi + s_{\mu\nu} x^\mu x^\nu - 12\lambda_4 s^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

New recursion for effective theories

[Cheung, KK, Novotny, Shen, Trnka 2015]

The high energy behaviour forbids a naive Cauchy formula

$$A(z) \neq 0 \qquad \text{for} \quad z \to \infty$$

Can we instead use the soft limit directly?

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Can we instead use the soft limit directly? yes!
The standard BCFW not applicable, we propose a special shift:

$$p_i \to p_i(1 - za_i) \qquad \text{on all external legs}$$

This leads to a modified Cauchy formula

$$\oint \frac{dz}{z} \frac{A(z)}{\Pi_i(1 - a_i z)^\sigma} = 0$$

note there are no poles at $z = 1/a_i$ (by construction).
Now we can continue in analogy with BCFW

Vector EFTs

[Cheung, KK, Novotny, Shen, Trnka, Wen '18]

Further avenues

- similarly for vector EFT:

$$\mathcal{L}_{\mathrm{BI}} = 1 - \sqrt{(-1)^{D-1}\mathrm{det}(\eta_{\mu\nu} + F_{\mu\nu})},$$

  (see [Cheung, KK, Novotny, Shen, Trnka, Wen '18])
- so far avoided the fermionic degrees of freedom (see e.g. Elvang et al.'18)
- multiple flavours – especially without flavour ordering
- only two-flavour case fully classified
- preliminary study of the mixed scalar-vector case (Galileon-BI): more promising than the pure Galileon-like BI
- spin-2: similar to Galileon-like studies – no exceptional candidate
- non-abelian Born-Infeld
- non-zero masses (technically possible)
- more generally: breaking the shift symmetries
- loop corrections – focused on the exceptional theories

Summary

- We have offered a new tool for effective field theories
- motivated by the amplitude methods employed for renormalizable theories
- used for classification of scalar theories
- one new theory discovered: special galileon
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- our natural EFTs appear also in CHY [Cachazo, He, Yuan] formalism

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Thank you!