Neutrino Oscillations

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Neutrino **flavor eigenstates** $|\nu_f\rangle \quad f = e, \mu, \tau$

produced in weak interactions are different from

mass eigenstates $|\nu_i\rangle, \ i = 1,2,3$

They are related by the unitary
mixing matrix: $U_{fi} \equiv \langle \nu_f | \nu_i \rangle$

$$|\nu_f\rangle = \left(|\nu_1\rangle\langle\nu_1| + |\nu_2\rangle\langle\nu_2| + |\nu_3\rangle\langle\nu_3| \right) |\nu_f\rangle$$

$$|\nu_f\rangle = U^*_{f1}|\nu_1\rangle + U^*_{f2}|\nu_2\rangle + U^*_{f3}|\nu_3\rangle$$

$$\left|\nu_f(L)\right\rangle = \left(\sum_i e^{-i\frac{m_1^2}{2\hbar c}\frac{L}{E}}\left|\nu_i\right\rangle\left\langle\nu_i\right|\right)\left|\nu_f\right\rangle = U_{f1}^* e^{-i\frac{m_1^2}{2\hbar c}\frac{L}{E}}\left|\nu_1\right\rangle + U_{f2}^* e^{-i\frac{m_2^2}{2\hbar c}\frac{L}{E}}\left|\nu_2\right\rangle + U_{f3}^* e^{-i\frac{m_3^2}{2\hbar c}\frac{L}{E}}\left|\nu_3\right\rangle$$

$$A_{\nu_f\to\nu_f}(L) = \left\langle\nu_f \middle| \nu_f(L)\right\rangle = \left|U_{f1}\right|^2 e^{-i\frac{m_1^2}{2\hbar c}\frac{L}{E}} + \left|U_{f2}\right|^2 e^{-i\frac{m_2^2}{2\hbar c}\frac{L}{E}} + \left|U_{f3}\right|^2 e^{-i\frac{m_3^2}{2\hbar c}\frac{L}{E}}$$

$$P_{\nu_f\to\nu_f}(L) = \left|U_{f1}\right|^4 + \left|U_{f2}\right|^4 + \left|U_{f3}\right|^4$$

$$+2\left|U_{f1}\right|^2\left|U_{f2}\right|^2\cos\left(\frac{m_2^2-m_1^2}{2\hbar c}\frac{L}{E}\right) + 2\left|U_{f1}\right|^2\left|U_{f3}\right|^2\cos\left(\frac{m_3^2-m_1^2}{2\hbar c}\frac{L}{E}\right) + 2\left|U_{f2}\right|^2\left|U_{f3}\right|^2\cos\left(\frac{m_3^2-m_2^2}{2\hbar c}\frac{L}{E}\right)$$

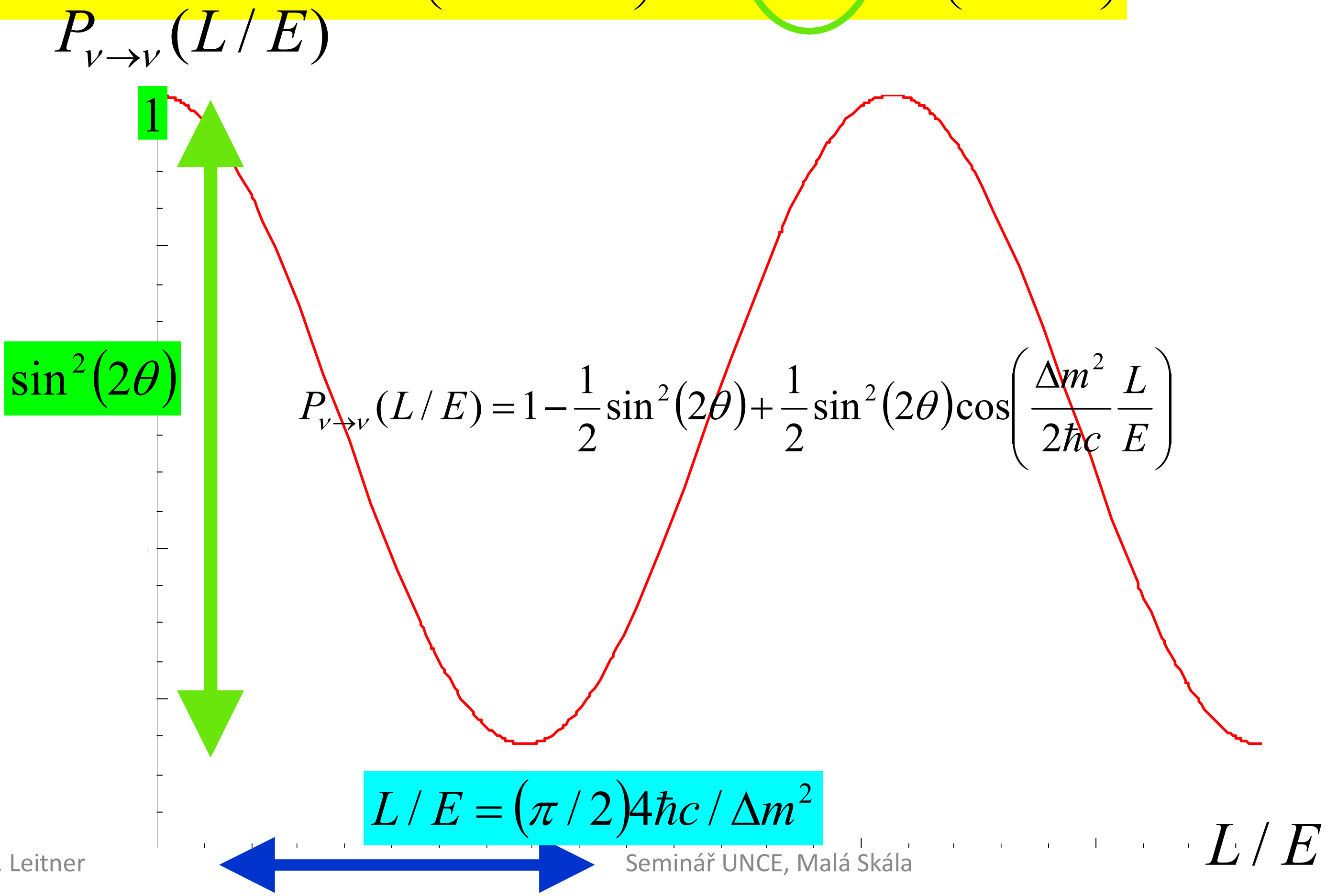
$$=1-4\left|U_{f1}\right|^2\left|U_{f2}\right|^2\sin^2\left(\frac{m_2^2-m_1^2}{4\hbar c}\frac{L}{E}\right) - 4\left|U_{f1}\right|^2\left|U_{f3}\right|^2\sin^2\left(\frac{m_3^2-m_1^2}{4\hbar c}\frac{L}{E}\right) - 4\left|U_{f2}\right|^2\left|U_{f3}\right|^2\sin^2\left(\frac{m_3^2-m_2^2}{4\hbar c}\frac{L}{E}\right)$$

$$e^{-\frac{i}{\hbar c}(Ect-PL)} \to e^{-\frac{i}{\hbar c}(E-P)L} \to e^{-\frac{i}{\hbar c}\frac{m_i^2}{2E}L}$$

Do neutrinos mass eiegenstates have the same momenta but different energies or same energies and different momena, …? See later.

2x2 Mixing Amplitude of oscillations = $\sin^2(2\theta)$, oscillation length is inversely proportional to Δm^2

$$P_{\nu_f \to \nu_f}(L) = 1 - 4|U_{f1}|^2 |U_{f2}|^2 \sin^2\left(\frac{m_2^2 - m_1^2}{4\hbar c}\frac{L}{E}\right) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m_{21}^2}{4\hbar c}\frac{L}{E}\right)$$



In total there are 18 oscillation probabilities. Only 6 of them are currently measured

Currently there are no high energy electron neutrinos available

$P_{\nu e\to\nu e}$	$P_{\nu e\to\nu\mu}$	$P_{\nu e\to\nu\tau}$
$P_{\nu\mu\to\nu e}$	$P_{\nu\mu\to\nu\mu}$	$P_{\nu\mu\to\nu\tau}$
$P_{\nu\tau\to\nu e}$	$P_{\nu\tau\to\nu\mu}$	$P_{\nu\tau\to\nu\tau}$

Few events seen in OPERA

$P_{\bar{\nu}e\to\bar{\nu}e}$	$P_{\bar{\nu}e\to\bar{\nu}\mu}$	$P_{\bar{\nu}e\to\bar{\nu}\tau}$
$P_{\bar{\nu}\mu\to\bar{\nu}e}$	$P_{\bar{\nu}\mu\to\bar{\nu}\mu}$	$P_{\bar{\nu}\mu\to\bar{\nu}\tau}$
$P_{\bar{\nu}\tau\to\bar{\nu}e}$	$P_{\bar{\nu}\tau\to\bar{\nu}\mu}$	$P_{\bar{\nu}\tau\to\bar{\nu}\tau}$

No tau neutrino sources

Oscillation experiments with electron (anti)neutrinos

$\bar{\nu}_e \qquad {}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^{-} + \bar{\nu}_e$

Antineutrino source (reactor)

On the way from the source to the detector electron antineutrinos oscillate to other flavors

$$\bar{\nu}_e \rightarrow \bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau$$

Currently there are no sources of electron antineutrinos with energies above 100 MeV. We can only measure disappearance and appearance of electron neutrinos. **DISAPPEARANCE EXPERIMENT**

At the detector we could measure the probability of different flavors

$E_\nu > 1.8 MeV \qquad \bar{\nu}_e + p \rightarrow n + e^{+}$

$E_\nu > \approx 100 MeV \qquad \bar{\nu}_\mu + p \rightarrow n + \mu^{+}$

$E_\nu > 3500 MeV \qquad \bar{\nu}_\tau + p \rightarrow n + \tau^{+}$

Oscillation experiments with muon (anti)neutrinos

$\nu_\mu \quad \pi^+ \rightarrow \mu^+ + \nu_\mu$

Muon neutrinos (accelerators, cosmic rays)

On the way from the source to the detector muon (anti)neutrinos oscillate to other flavors

$$\nu_\mu \rightarrow \nu_\mu, \nu_e, \nu_\tau$$

We can measure also DISAPPEARANCE EXPERIMENT

And also appearance of electron neutrinos and for very high energies also tau neutrinos.
APPEARANCE EXPERIMENT

At the detector we could measure the probability of different flavors

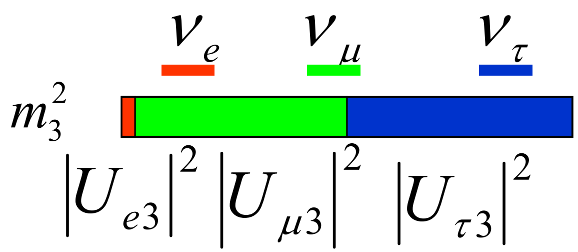
$$E_\nu >\approx 100MeV \qquad \nu_\mu + n \rightarrow p + \mu^-$$

$$E_\nu > 0MeV \qquad \nu_e + n \rightarrow p + e^-$$

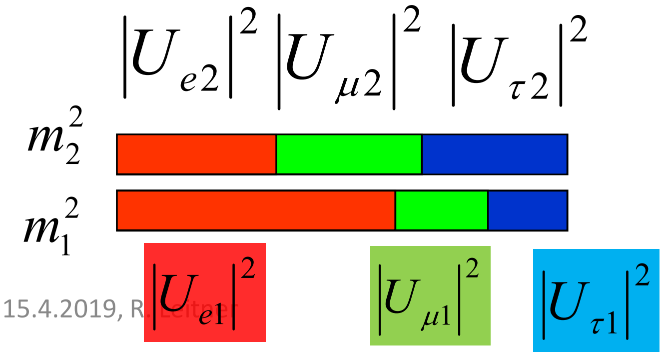
$$E_\nu > 3500MeV \qquad \nu_\tau + n \rightarrow p + \tau^-$$

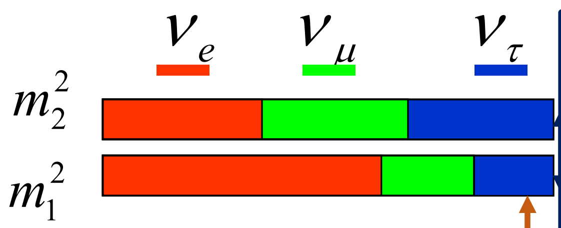
OSCILLATION PARAMETERS

Two mass splits differ by a factor of app 30



NORMAL MASS HIERARCHY (NH)



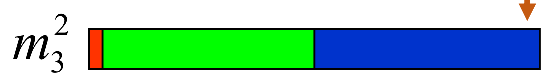


$\Delta m_{21}^2 \cong 7.54\times10^{-5}\, eV^2 \cong (8.7\, meV)^2$

INVERSE MASS HIERARCHY (IH)

$$\begin{pmatrix} \bar{\nu}_e \\ \bar{\nu}_\mu \\ \bar{\nu}_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

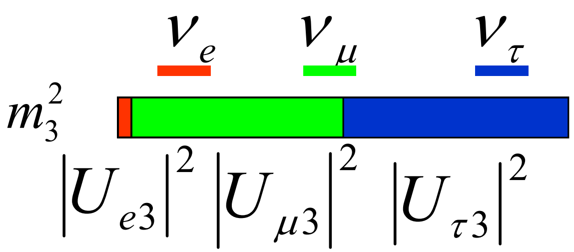
$|\Delta m_{31}^2| \cong 2.43\times10^{-3}\, eV^2 \cong (49\, meV)^2$



$\theta_{23} \simeq 45^o$

Half of both muon and tauon neutrinos in m3

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix}1 & 0 & 0\\ 0 & \cos(\theta_{23}) & \sin(\theta_{23})\\ 0 & -\sin(\theta_{23}) & \cos(\theta_{23})\end{pmatrix}\cdot$$



$\theta_{13} \simeq 8.5^o$

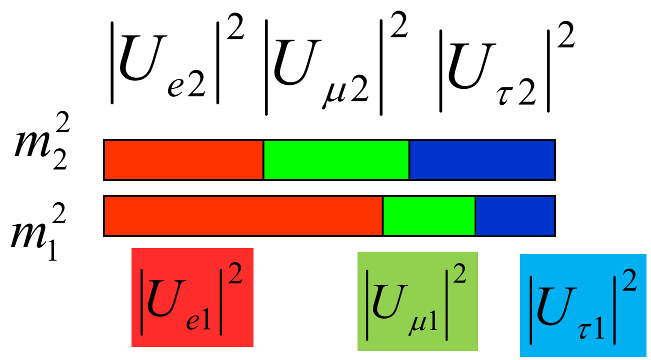
Very small fraction of electron neutrinos in m3

$$\begin{pmatrix}\cos(\theta_{13}) & 0 & \sin(\theta_{13})\cdot e^{-i\delta}\\ 0 & 1 & 0\\ -\sin(\theta_{13})\cdot e^{i\delta} & 0 & \cos(\theta_{13})\end{pmatrix}\cdot$$

$\theta_{12} \simeq 34^o$

2/3 of electron neutrinos in m1 and 1/3 in m2

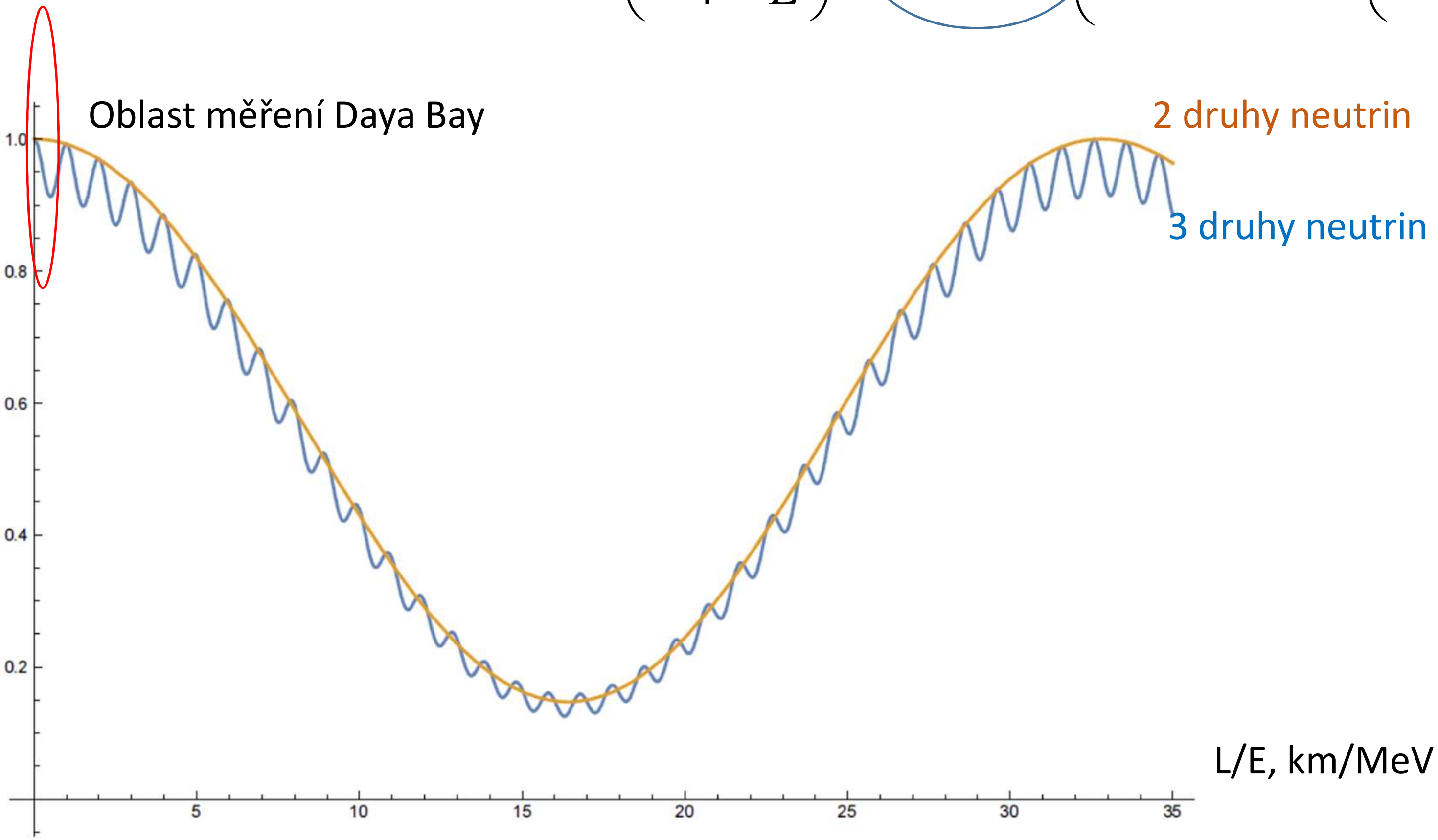
$$\begin{pmatrix}\cos(\theta_{12}) & \sin(\theta_{12}) & 0\\ -\sin(\theta_{12}) & \cos(\theta_{12}) & 0\\ 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$



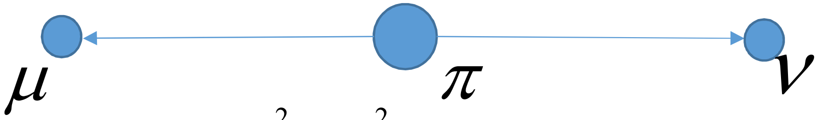
Pravděpodobnost oscilací pro elektronová (anti)neutrina pro 2 druhy a 3 druhy neutrin

$$P^{2x2}_{ve \to ve} = 1 - \sin^2(2\theta_{12})\sin^2\left(\frac{\Delta m^2_{21}}{4}\frac{L}{E}\right)$$

$$P^{3x3}_{ve \to ve} = 1 - \cos^4(\theta_{13})\sin^2(2\theta_{12})\sin^2\left(\frac{\Delta m^2_{21}}{4}\frac{L}{E}\right) - \sin^2(2\theta_{13})\left(\cos^2(\theta_{12})\sin^2\left(\frac{\Delta m^2_{31}}{4}\frac{L}{E}\right) + \sin^2(\theta_{12})\sin^2\left(\frac{\Delta m^2_{32}}{4}\frac{L}{E}\right)\right)$$



Neutrinos have both different energies and momenta. Momenta usually differ (much) more.



Neutrinos from pion decay at rest

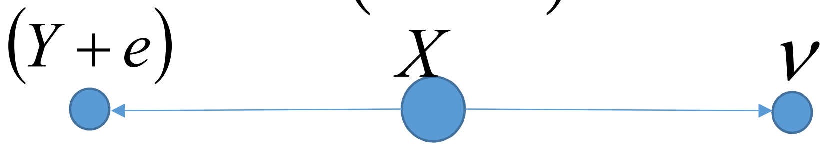
$$E_\nu = \frac{m_\pi}{2} + \frac{m_\nu^2 - m_\mu^2}{2m_\pi} = E_0 + \frac{m_\nu^2}{2m_\pi} \cong 30MeV + \frac{m_\nu^2}{280MeV}$$

$$P_\nu = \sqrt{E_\nu^2 - m_\nu^2} = \sqrt{\left(E_0 + \frac{m_\nu^2}{2m_\pi}\right)^2 - m_\nu^2} \cong E_0 + \frac{m_\nu^2}{2m_\pi} - \frac{m_\nu^2}{2E_0} = 30MeV + \frac{m_\nu^2}{280MeV} - \frac{m_\nu^2}{60MeV}$$

$$E_\nu - P_\nu = \frac{m_\nu^2}{2E_0}$$

Reactor neutrinos of 4 MeV from decays of ~100 GeV heavy nuclei

$$X \to (Y+e) + \nu$$



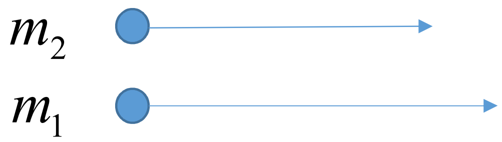
$$E_\nu = \frac{m_X}{2} + \frac{m_\nu^2 - m_{Ye}^2}{2m_X} = E_0 + \frac{m_\nu^2}{2m_X} \cong 4MeV + \frac{m_\nu^2}{100GeV}$$

$$P_\nu = \sqrt{E_\nu^2 - m_\nu^2} \cong E_0 + \frac{m_\nu^2}{2m_X} - \frac{m_\nu^2}{2E_0} = 4MeV + \frac{m_\nu^2}{100GeV} - \frac{m_\nu^2}{4MeV}$$

Energies of different mass eigenstates are almost the same, momenta differs much more

$$E_\nu = E_0 + \delta_\nu\left(=\frac{m_\nu^2}{2m_X}\right)$$

Let us take two neutrinos and boost them to the rest frame of the heavier one.



$$P_\nu \cong E_0 + \delta_\nu\left(=\frac{m_\nu^2}{2m_X}\right) - \Delta_\nu\left(=\frac{m_\nu^2}{2E_0}\right)$$

Let us go to the rest frame of heavier neutrino (m2)

$$\beta = \frac{P_2}{E_2} = \frac{E_0+\delta_2-\Delta_2}{E_0+\delta_2} = 1-\frac{\Delta_2}{E_0+\delta_2} \cong 1-\frac{\Delta_2}{E_0} = 1-\frac{m_2^2}{2E_0^2} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{\frac{m_2^2}{2E_0^2}\left(2-\frac{m_2^2}{2E_0^2}\right)}} \cong \frac{E_0}{m_2} \qquad P^* = \gamma(P-\beta E) \quad E^* = \gamma(E-\beta P)$$

$$P_2^* = \frac{E_0}{m_2}\left(E_0+\delta_2-\Delta_2-\left(1-\frac{m_2^2}{2E_0^2}\right)(E_0+\delta_2)\right) = \frac{E_0}{m_2}\left(-\Delta_2+\frac{m_2^2}{2E_0^2}E_0\right) = 0$$

$$E_2^* = \frac{E_0}{m_2}\left(E_0+\delta_2-\left(1-\frac{m_2^2}{2E_0^2}\right)(E_0+\delta_2-\Delta_2)\right) = \frac{E_0}{m_2}\left(\frac{m_2^2}{2E_0}+\Delta_2\right) = m_2$$

$$P_1^* = \frac{E_0}{m_2}\left(E_0+\delta_1-\Delta_1-\left(1-\frac{m_2^2}{2E_0^2}\right)(E_0+\delta_1)\right) = \frac{E_0}{m_2}\left(-\Delta_1+\frac{m_2^2}{2E_0^2}E_0\right) = \frac{m_2^2-m_1^2}{m_2} = \frac{\Delta m_{21}^2/2}{m_2}$$

$$E_1^* = \frac{E_0}{m_2}\left(E_0+\delta_1-\left(1-\frac{m_2^2}{2E_0^2}\right)(E_0+\delta_1-\Delta_2)\right) = \frac{E_0}{m_2}\left(\frac{m_2^2}{2E_0}+\Delta_1\right) = \frac{(m_2^2+m_1^2)/2}{m_2} = \frac{\langle m^2\rangle}{m_2}$$

The rest frame of heavier neutrino 2:

$$P_2^* = 0 \qquad E_2^* = m_2$$

$$P_1^* = \frac{\Delta m_{21}^2 / 2}{m_2} \qquad E_1^* = \frac{\langle m^2 \rangle}{m_2} \qquad \langle m^2 \rangle = \frac{m_1^2 + m_2^2}{2}$$

m_2^2

m_1^2

$$\Delta m_{21}^2 \cong 7.54 \times 10^{-5}\, eV^2 \cong (8.7\ meV)^2$$

$$\Rightarrow \beta_1^* = \frac{\Delta m_{21}^2 / 2}{\langle m^2 \rangle}$$

$$m_1 = 0 \Rightarrow \beta_1^* = 1 \ldots P_1^* = E_1^* = \frac{(9\ meV)^2 / 2}{(9\ meV)} \cong 4{,}5 meV$$

$$m_1 = 0{,}1 eV \Rightarrow \beta_1^* = \frac{(9 meV)^2 / 2}{(100 meV)^2} = 0{,}004$$

The difference of phases in m2 rest frame and lab frame:

$$P_2^* = 0 \qquad E_2^* = m_2 \qquad e^{-i\phi_2} = e^{-i\frac{1}{\hbar c}\left(E_2^* ct^* - P_2^* x^*\right)} = e^{-i\frac{1}{\hbar c} m_2^* ct^*}$$

$$P_1^* = \frac{\Delta m_{21}^2 / 2}{m_2} \qquad E_1^* = \frac{\langle m^2 \rangle}{m_2} \qquad e^{-i\phi_1} = e^{-i\frac{1}{\hbar c}\left(E_1^* ct^* - P_1^* x^*\right)} = e^{-i\frac{1}{\hbar c}\left(\frac{\langle m^2 \rangle}{m_2} ct^* - \frac{\Delta m_{21}^2 / 2}{m_2} x^*\right)}$$

$$\phi_2 - \phi_1 = \frac{1}{\hbar c}\left(m_2^* ct^* - \frac{\langle m^2 \rangle}{m_2} ct^* + \frac{\Delta m_{21}^2 / 2}{m_2} x^*\right) = \frac{1}{\hbar c}\frac{\Delta m_{21}^2}{2m_2}\left(ct^* + x^*\right) \xrightarrow[\gamma = E_0 / m_2]{} \frac{1}{\hbar c}\frac{\Delta m_{21}^2}{2E_0} L$$

$$\phi_2 - \phi_1 = \frac{1}{\hbar c}\frac{\Delta m_{21}^2}{2m_2}\left(ct^* + x^*\right) \xrightarrow[\gamma = E_0 / m_2]{} \frac{1}{\hbar c}\frac{\Delta m_{21}^2}{2E_0} L$$

Is the energy conserved during the oscillations?

$$E_e(L=0) = \langle \nu_e | H | \nu_e \rangle = \langle \nu_1 \cos\vartheta + \nu_2 \sin\vartheta | H | \nu_1 \cos\vartheta + \nu_2 \sin\vartheta \rangle$$

$$= \langle \nu_1 | H | \nu_1 \rangle \cos^2\vartheta + \langle \nu_2 | H | \nu_2 \rangle \sin^2\vartheta = E_1 \cos^2\vartheta + E_2 \sin^2\vartheta$$

$$E_e(L) = \langle \nu_e(L) | H | \nu_e(L) \rangle = \left\langle \nu_1 \left(e^{-i\frac{M_1^2 L}{2E}} \right)^* \cos\vartheta + \nu_2 \left(e^{-i\frac{M_2^2 L}{2E}} \right)^* \sin\vartheta \middle| H \middle| \nu_1 e^{-i\frac{M_1^2 L}{2E}} \cos\vartheta + \nu_2 e^{-i\frac{{}_2^2 L}{2E}} \sin\vartheta \right\rangle$$

$$= E_1 \cos^2\vartheta + E_2 \sin^2\vartheta$$

Is it true even if electron neutrino completely changes to muon one?

$$E_\mu = \langle \nu_\mu | H | \nu_\mu \rangle = \langle -\nu_1 \sin\vartheta + \nu_2 \cos\vartheta | H | -\nu_1 \sin\vartheta + \nu_2 \cos\vartheta \rangle = E_1 \sin^2\vartheta + E_2 \cos^2\vartheta$$

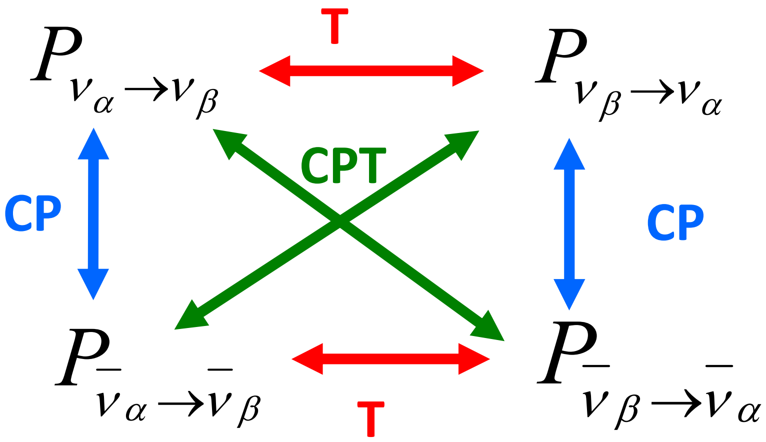
$E_\mu \neq E_e$ **The energies looks different**

But full change to another neutrino flavor is possible only for maximal mixing angle

$$\vartheta = 45^o \Rightarrow \sin^2\vartheta = \cos^2\vartheta \Rightarrow E_1 \sin^2\vartheta + E_2 \cos^2\vartheta = E_1 \cos^2\vartheta + E_2 \sin^2\vartheta = (E_1 + E_2)/2$$

and energy is also conserved

CP and T violation in neutrino oscillations



Current and future experiments could test CP Invariance in:

$$P_{\nu_\mu \to \nu_e} \stackrel{?}{=} P_{\bar{\nu}_\mu \to \bar{\nu}_e}$$

Few remarks on Unitarity triangles.
Matrix of squares of PMNS elements.

Four elements are needed to determine the rest. One of the two following examples is with CP phase =0 and one with maximal CP phase 90 degrees.

If there is no CP phase =0, three out of four values should be enough.

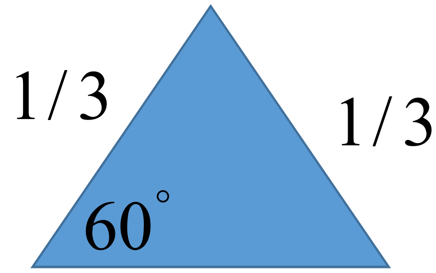
Try to find a solution.

$$|U1|^2 = \begin{pmatrix} 0{,}652 & 0{,}326 & ? \\ 0{,}174 & 0{,}330 & ? \\ ? & ? & ? \end{pmatrix} \qquad |U2|^2 = \begin{pmatrix} 0{,}652 & 0{,}326 & ? \\ 0{,}245 & 0{,}266 & ? \\ ? & ? & ? \end{pmatrix}$$

Let us consider 1/3 probability of mixing all neutrino states.

$$|U|^2 = \begin{pmatrix} 1/3 & 1/3 & ? \\ 1/3 & 1/3 & ? \\ ? & ? & ? \end{pmatrix} \Rightarrow |U| = \begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix}$$

$$|U| = \begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix}$$



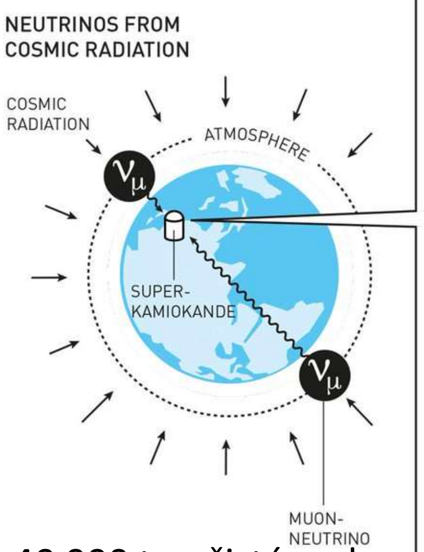
$$1/\sqrt{3} \times 1/\sqrt{3} = 1/3$$

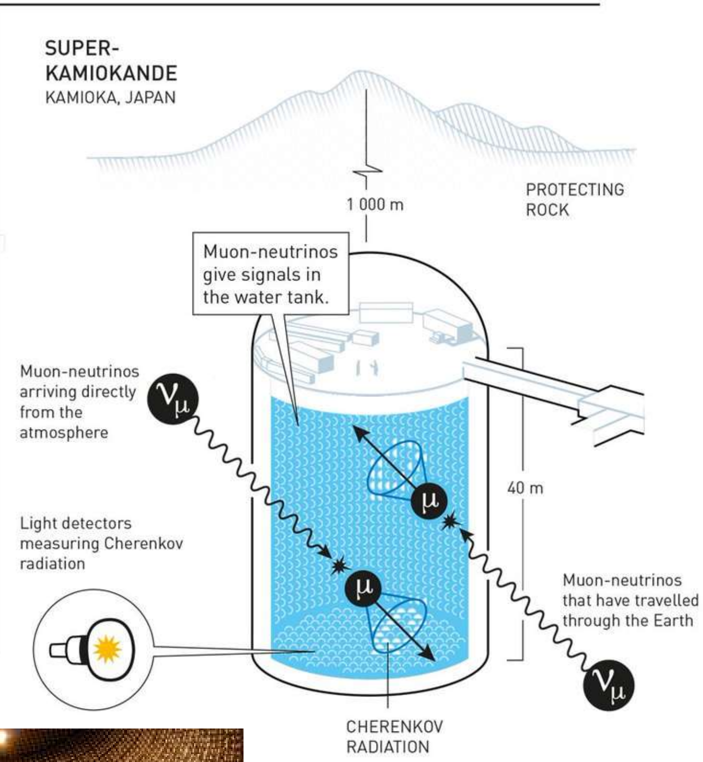
Two different rows or columns must be perpendicular each to other
There are three vectors, te lentgh of each is 1/3.
They form a triangle with the angle of 60 degrees. It is the value of CP violatng phase

Neutrinos from extended source
Poor resolution in the measurement of E/L
Decoherence

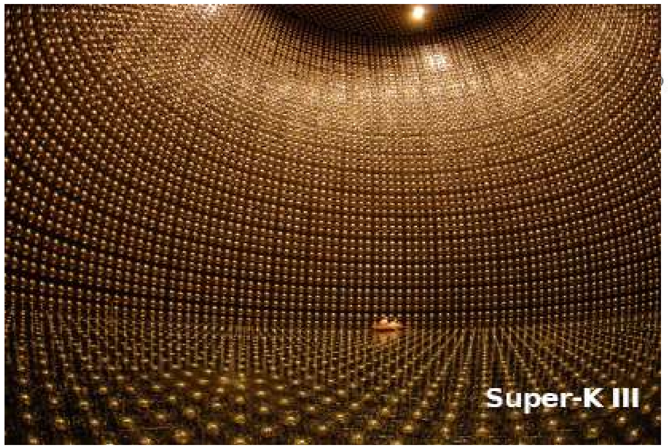
Due to all these effects we measure distorted oscillation curve, ultimately only the mean value between maximum and minimum of oscillations

SuperKamiokande – muon neutrinos deficit

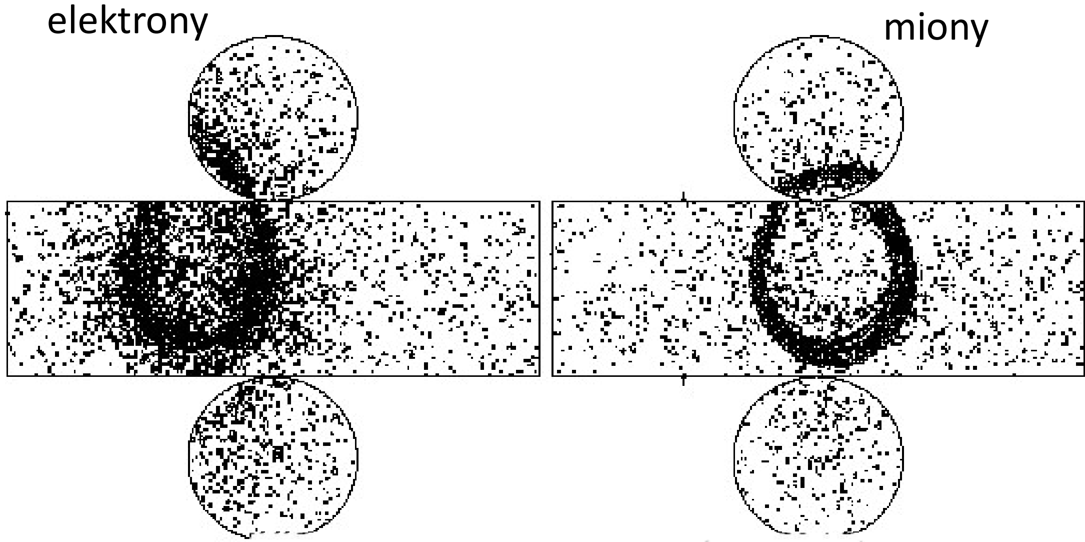




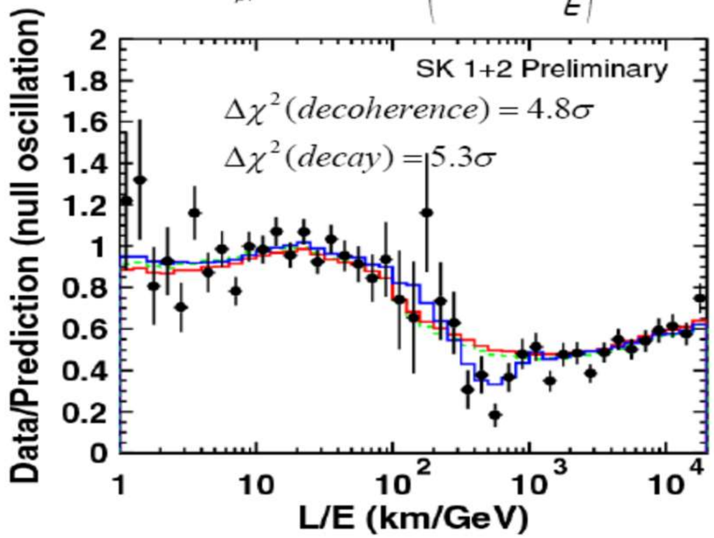
40 000 tun čisté vody a
11 000 20 palcových
fotonásobičů



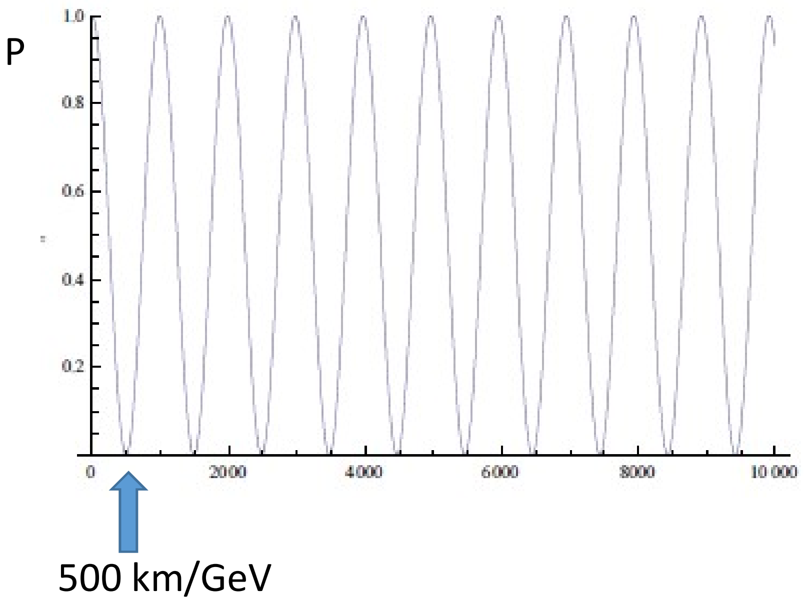
T. Kajita



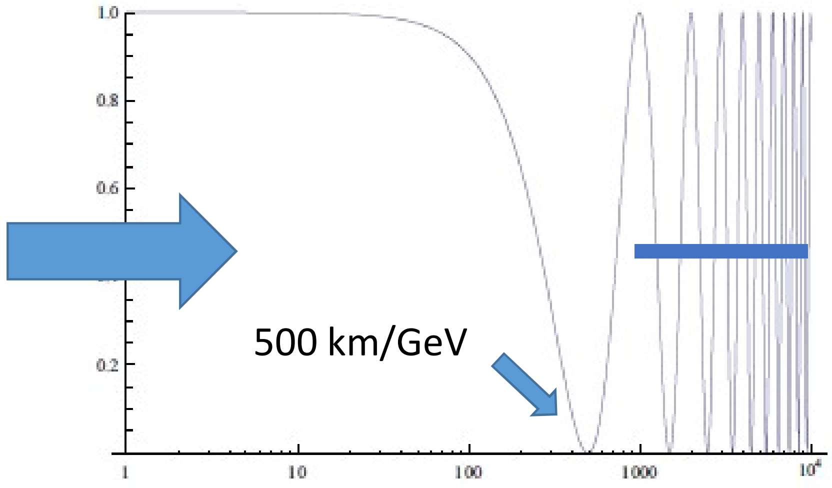
$$P_{\mu\tau} = \sin^2 2\theta \sin^2\left(1.27 \Delta m^2 \frac{L}{E}\right)$$



Experiment SuperKamiokande byl původně určen pro hledání rozpadu protonu. Očekávalo se několik rozpadů za rok a interakce neutrin byly hlavním pozadím.

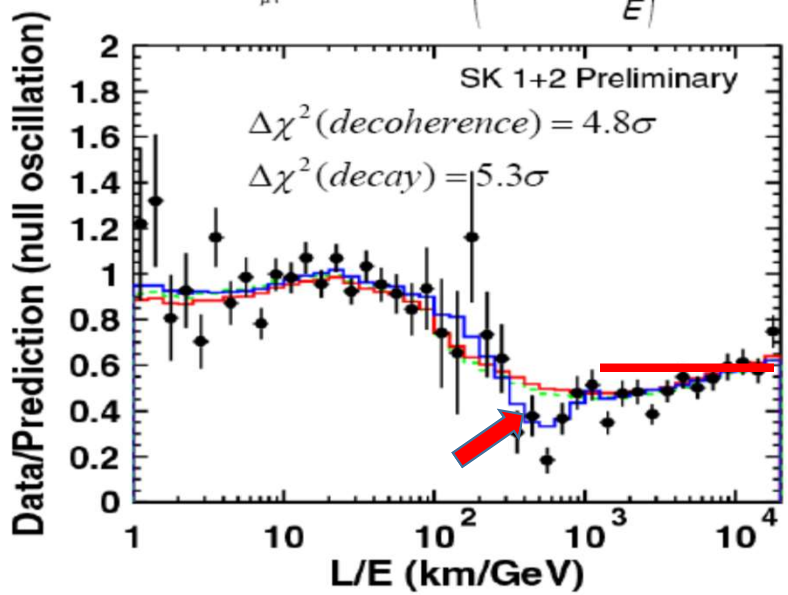


L/E [km/GeV]



Due to uncertainties in measurement of L/E only a mean value is measured

$$P_{\mu\tau} = \sin^2 2\theta \sin^2\left(1.27\,\Delta m^2 \frac{L}{E}\right)$$



Position of the minimum at 500 km/GeV:

$$1{,}27 \cdot \Delta M^2[eV^2] \cdot 500 km/GeV = \pi/2$$

$$\Delta M^2[eV^2] = 2{,}5 \cdot 10^{-3} eV^2 = (50\, meV)^2$$

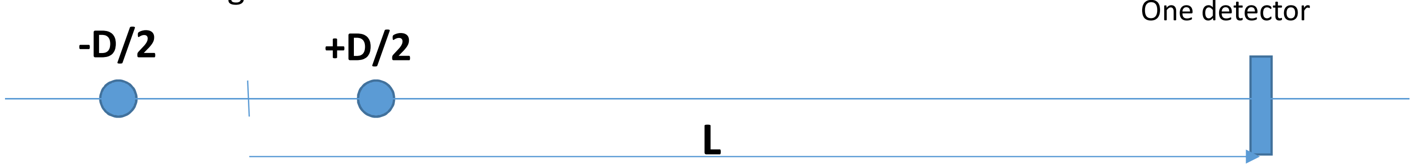
Mean value of the deficit is 0.5 means that oscillations are between 1 and 0

$$\theta \cong 45°$$

The oscillation curves in real experiment are often modified because of

- multiple sources or extended neutrino sources or detectors
- the E and L are measured with limited precision

Simplest case – two equal sources
with flux weighted distance L to the detector

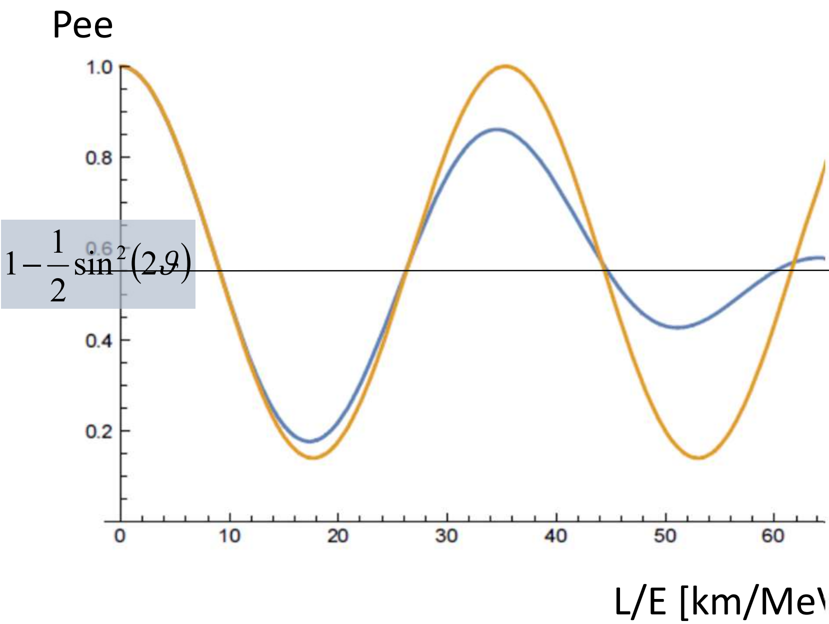


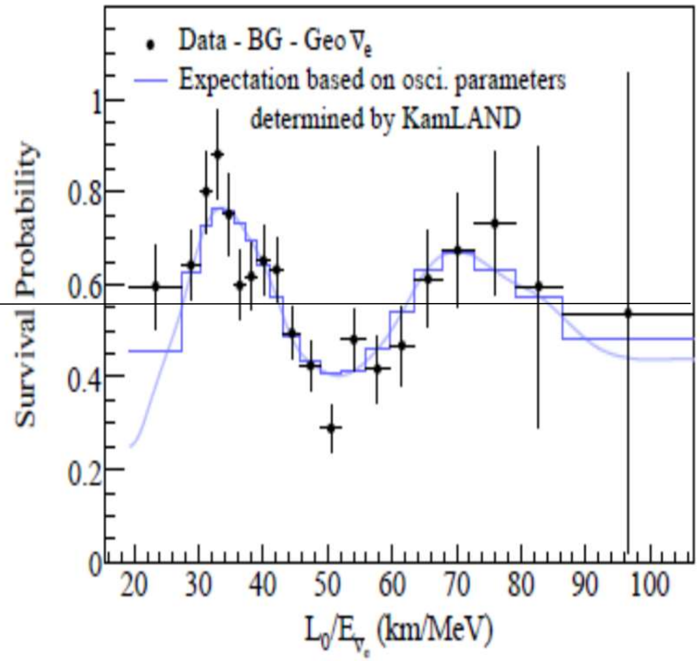
$$P_2ee\left(\frac{L}{E}\right)=\frac{1}{2}\left(Pee\left(\frac{L+D/2}{E}\right)+Pee\left(\frac{L-D/2}{E}\right)\right)=1-\frac{1}{2}\sin^2(2\vartheta)+\frac{1}{2}\sin^2(2\vartheta)\frac{\cos\left(2\frac{\Delta m^2}{4\hbar c}\frac{L+D/2}{E}\right)+\cos\left(2\frac{\Delta m^2}{4\hbar c}\frac{L-D/2}{E}\right)}{2}=$$

$$P_2ee\left(\frac{L}{E}\right)=1-\frac{1}{2}\sin^2(2\vartheta)+\frac{1}{2}\sin^2(2\vartheta)\cos\left(\frac{\Delta m^2}{2\hbar c}\frac{L}{E}\right)\cos\left(\frac{\Delta m^2}{4\hbar c}\frac{D}{L}\frac{L}{E}\right)\neq 1-\frac{1}{2}\sin^2(2\vartheta)+\frac{1}{2}\sin^2(2\vartheta)\cos\left(\frac{\Delta m^2}{2\hbar c}\frac{L}{E}\right)=P_1ee\left(\frac{L}{E}\right)$$

$$Pee\left(\frac{L}{E},\frac{D}{L}\right)=1-\frac{1}{2}\sin^2(2\vartheta)+\frac{1}{2}\sin^2(2\vartheta)\cos\left(\frac{\Delta m^2}{2\hbar c}\frac{L}{E}\right)\cos\left(\frac{\Delta m^2}{4\hbar c}\frac{D}{L}\frac{L}{E}\right)$$

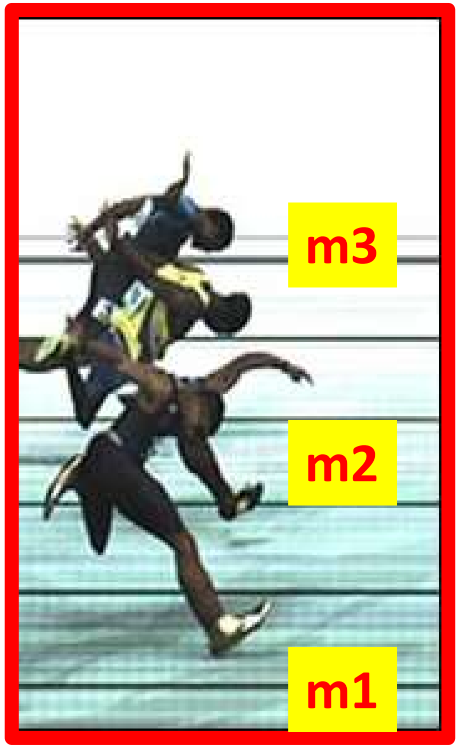
Two reactors 170 and 190 km from the detector





Problem Calculate the disappearance electron neutrino probability Pee(L/E) for following cases. **A)** The source that extends from –D/2 to D/2 and has constant linear power density 1/D; **B)** One source, the variables L and E in are measured with a Gaussian resolutions ←L, ←E.

COHERENCE



DECOHERENCE

Problem A. Calculate differences in arrival times (ct) of ν1 ν2, ν1 ν3 for 4 MeV electron neutrinos at distances of 2 km, 150 mil. km, 150 k light-years.

B. Evaluate the disappearance Pee and appearance Pemu and Petau probabilities in the case of full decoherence. Check that the sum of probabilities is equal to 1.

$$\frac{\Delta x}{x} = \Delta\beta = \frac{\Delta m^2}{2E^2}$$

4 MeV neutrinos

1 GeV neutrinos

$$\Delta\beta_{31} = \frac{2.5\cdot 10^{-3}\,eV^2}{2\left(4\cdot 10^6\,eV\right)^2} \cong 0.78\cdot 10^{-16} \qquad \Delta\beta_{31} = \frac{2.5\cdot 10^{-3}\,eV^2}{2\left(10^9\,eV\right)^2} \cong 1.25\cdot 10^{-21}$$

$$\Delta\beta_{21} = \frac{7.5\cdot 10^{-5}\,eV^2}{2\left(4\cdot 10^6\,eV\right)^2} \cong 0.23\cdot 10^{-17} \qquad \Delta\beta_{31} = \frac{7.5\cdot 10^{-5}\,eV^2}{2\left(10^9\,eV\right)^2} \cong 3.75\cdot 10^{-23}$$

$$x = 2km \Rightarrow \Delta x = 0.78\cdot 10^{-16}\cdot 2\cdot 10^{18}\,fm = 160\,fm\ \left(\Delta\beta_{31}\right)$$ 2 km from reactor

$$x = 150milkm \Rightarrow \Delta x = 0.23\cdot 10^{-17}\cdot 1.5\cdot 10^{26}\,fm = 0.3\cdot 10^{9}\,fm = 300nm$$ **Sun**

$$x = 150000ly \Rightarrow \Delta x = 0.78\cdot 10^{-16}\cdot 1.5\cdot 10^{5}\cdot 3\cdot 10^{7}\cdot 3\cdot 10^{5}\,km = 105km$$ Supernova

$$x = 800km \Rightarrow \Delta x = 3.75\cdot 10^{-23}\cdot 800\cdot 10^{18}\,fm = 0.03\,fm$$ Accelerator 1GeV at 800 km

Mass ordering

$$P_{\nu\to\nu}(L/E) = 1 - \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2}{4\hbar c}\frac{L}{E}\right)$$

Oscillation curve is not sensitive to the sign of Dm2

How did we learned that m2>m1?

How we could determine mass ordering

Normal: m3>m1,m2

Inverted: m3<m1,m2

Rotate mass eigenstates back to the flavor states at the detector

Transport the mass eigenstates to the detector

Rotate to the mass eigenstates at the source

Initial flavor state at the source

$$\begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{+i\Delta m_{21}^2 x/4\hbar cE} & 0 \\ 0 & e^{-i\Delta m_{21}^2 x/4\hbar cE} \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_e(0) \\ \nu_{\mu\tau}(0) \end{pmatrix}$$

$$\begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{+i\Delta m_{21}^2 x/4\hbar cE} & 0 \\ 0 & e^{-i\Delta m_{21}^2 x/4\hbar cE} \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\left(\Delta m_{21}^2 x/4\hbar cE\right) + i\sin\left(\Delta m_{21}^2 x/4\hbar cE\right)\cos 2\theta \\ -i\sin\left(\Delta m_{21}^2 x/4\hbar cE\right)\sin 2\theta \end{pmatrix}$$

SOLUTIONS IN VACUUM OR MATTER WITH A CONSTANT DENSITY

Variable matter density

$$V(x) = (\hbar c)^3 \sqrt{2} G_F N_e(x)$$ electron density

$$\Delta M_{21}^2(x) = \Delta m_{21}^2 \sqrt{\sin^2 2\theta + \left(\cos 2\theta - \frac{4E\hbar c}{\Delta m_{21}^2} \frac{V(x)}{2} \right)^2}$$

$$\cos 2\Theta(x) = \frac{\cos 2\theta - \frac{4E\hbar c}{\Delta m_{21}^2} \frac{V(x)}{2}}{\sqrt{\sin^2 2\theta + \left(\cos 2\theta - \frac{4E\hbar c}{\Delta m_{21}^2} \frac{V(x)}{2} \right)^2}}$$

Parameters are sensitive to the sign of dm12

Rotate mass eigenstates back to the flavor states at the detector

Transport the mass eigenstates to the detector

Rotate to the mass eigenstates at the source

Initial flavor state at the source

$$\begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix} = \begin{pmatrix} \cos\Theta(x) & \sin\Theta(x) \\ -\sin\Theta(x) & \cos\Theta(x) \end{pmatrix} \begin{pmatrix} e^{+i\int_0^x \Delta M_{21}^2(y)dy/4\hbar cE} & 0 \\ 0 & e^{-i\int_0^x \Delta M_{21}^2(y)dy/4\hbar cE} \end{pmatrix} \begin{pmatrix} \cos\Theta(0) & -\sin\Theta(0) \\ \sin\Theta(0) & \cos\Theta(0) \end{pmatrix} \begin{pmatrix} \nu_e(0) \\ \nu_{\mu\tau}(0) \end{pmatrix}$$

$$\cos 2\Theta(0) = \frac{\cos 2\theta - \frac{4E\hbar c}{\Delta m_{21}^2}\frac{V(x)}{2}}{\sqrt{\sin^2 2\theta + \left(\cos 2\theta - \frac{4E\hbar c}{\Delta m_{21}^2}\frac{V(x)}{2}\right)^2}} \xrightarrow{\frac{4E\hbar c}{\Delta m_{21}^2}\frac{V(x)}{2} >> 1} -1$$

$$\cos 2\Theta(L) = \cos 2\theta$$

Solution for neutrinos with high energy ~>5 MeV created in very dense core of the Sun and detected at the Earth can distinguish mass ordering.

$$\begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{+i\int_0^x \Delta M_{21}^2(y)dy/4\hbar cE} & 0 \\ 0 & e^{-i\int_0^x \Delta M_{21}^2(y)dy/4\hbar cE} \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

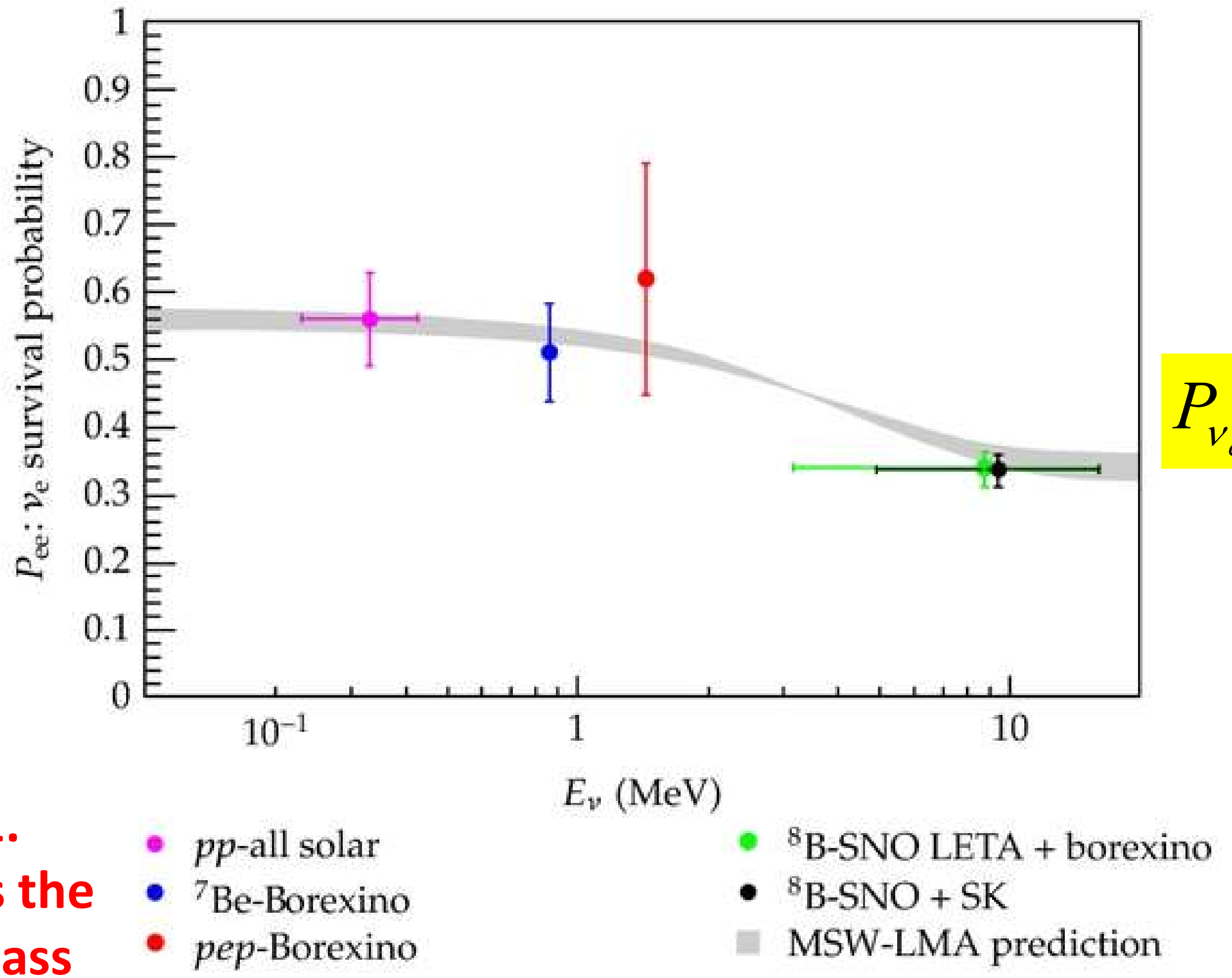
$$= e^{-i\int_0^x \Delta M_{21}^2(y)dy/4\hbar cE} \begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix} \Rightarrow P_{\nu_e \to \nu_e} = \sin^2\theta$$

$$P_{\nu_e \to \nu_e} = \sin^2\theta$$

Probability of solar neutrinos at the Earth

$$1-\frac{1}{2}\sin^2(2\theta)\cong 0.57$$

$$\Rightarrow \theta \cong 34.9^o$$

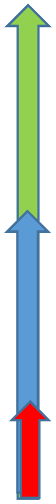


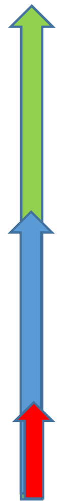
$$P_{\nu_e \to \nu_e} = \sin^2\theta$$

$$\sin^2(\theta)\cong 0.34$$

$$\Rightarrow \theta \cong 35.7^o$$

This measurement indicates that m2>m1. How to find that m3is the heaviest or lightest mass eigenstate?

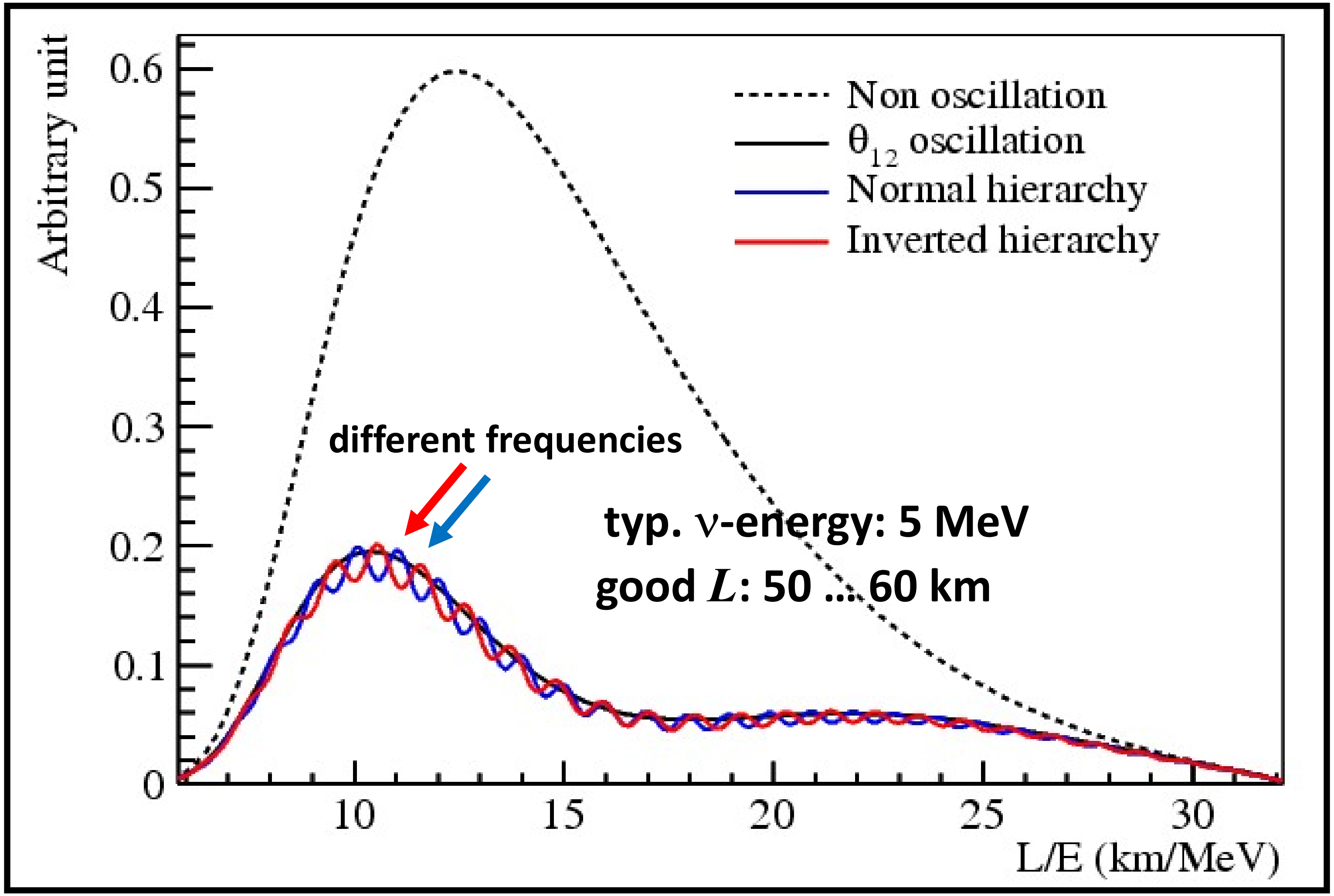




$$A_{\nu_e \to \nu_e}(x) = e^{-i\frac{m_1^2}{2\hbar c}\frac{x}{E}}\left|U_{e1}\right|^2 + e^{-i\frac{m_2^2}{2\hbar c}\frac{x}{E}}\left|U_{e2}\right|^2 + e^{-i\frac{m_3^2}{2\hbar c}\frac{x}{E}}\left|U_{e3}\right|^2$$

$$= e^{-i\frac{m_1^2}{2\hbar c}\frac{x}{E}}\left(\left|U_{e1}\right|^2 + e^{-i\frac{m_2^2-m_1^2}{2\hbar c}\frac{x}{E}}\left|U_{e2}\right|^2 + e^{-i\frac{m_3^2-m_1^2}{2\hbar c}\frac{x}{E}}\left|U_{e3}\right|^2\right)$$

Expected spectrum in future JUNO experiment



Týmy z ÚČJF jsou součástí neutrinových experimentů
Daya Bay a JUNO v Číně – reaktorová antineutrina
NOvA a DUNE v USA – urychlovačová neutrina

Přeji Vám mnoho zdaru v bádání a děkuji za pozornost