OTOCs & CHaozzz

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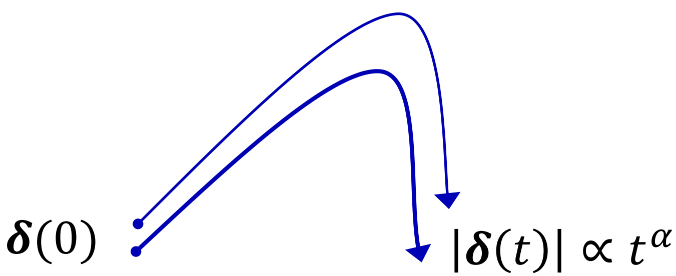
Outline:

1. Classical chaos
2. Quantum chaos
3. OTOCs (Otočky)
4. Examples

Classical chaos: Instability of trajectories

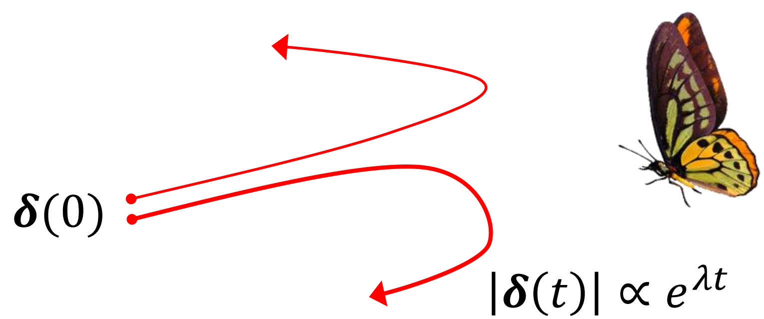
- Quantified by the **Lyapunov exponent** $$\lambda \equiv \max_{\boldsymbol{\delta}(0)} \lim_{t\to\infty} \frac{1}{t} \ln \frac{|\boldsymbol{\delta}(t)|}{|\boldsymbol{\delta}(0)|}$$

Stable (regular, quasiperiodic) trajectories



At most **polynomial** divergence

Unstable (chaotic) trajectories



Exponential divergence

(of a bunch of neigbouring trajectories)

Classical chaos: Instability of trajectories

- Quantified by the **Lyapunov exponent** $\lambda \equiv \max_{\delta(0)} \lim_{t\to\infty} \frac{1}{t} \ln \frac{|\boldsymbol{\delta}(t)|}{|\boldsymbol{\delta}(0)|}$

St ... ies

λt

e

Integrability

Hamiltonian: $H = H(q_1, \dots, q_f, p_1, \dots, p_f)$

Conservative system: $E = H = \text{const.}$

Integrals of motion: $I_j = I_j(q_1, \dots, q_f, p_1, \dots p_f) = \text{const.}$

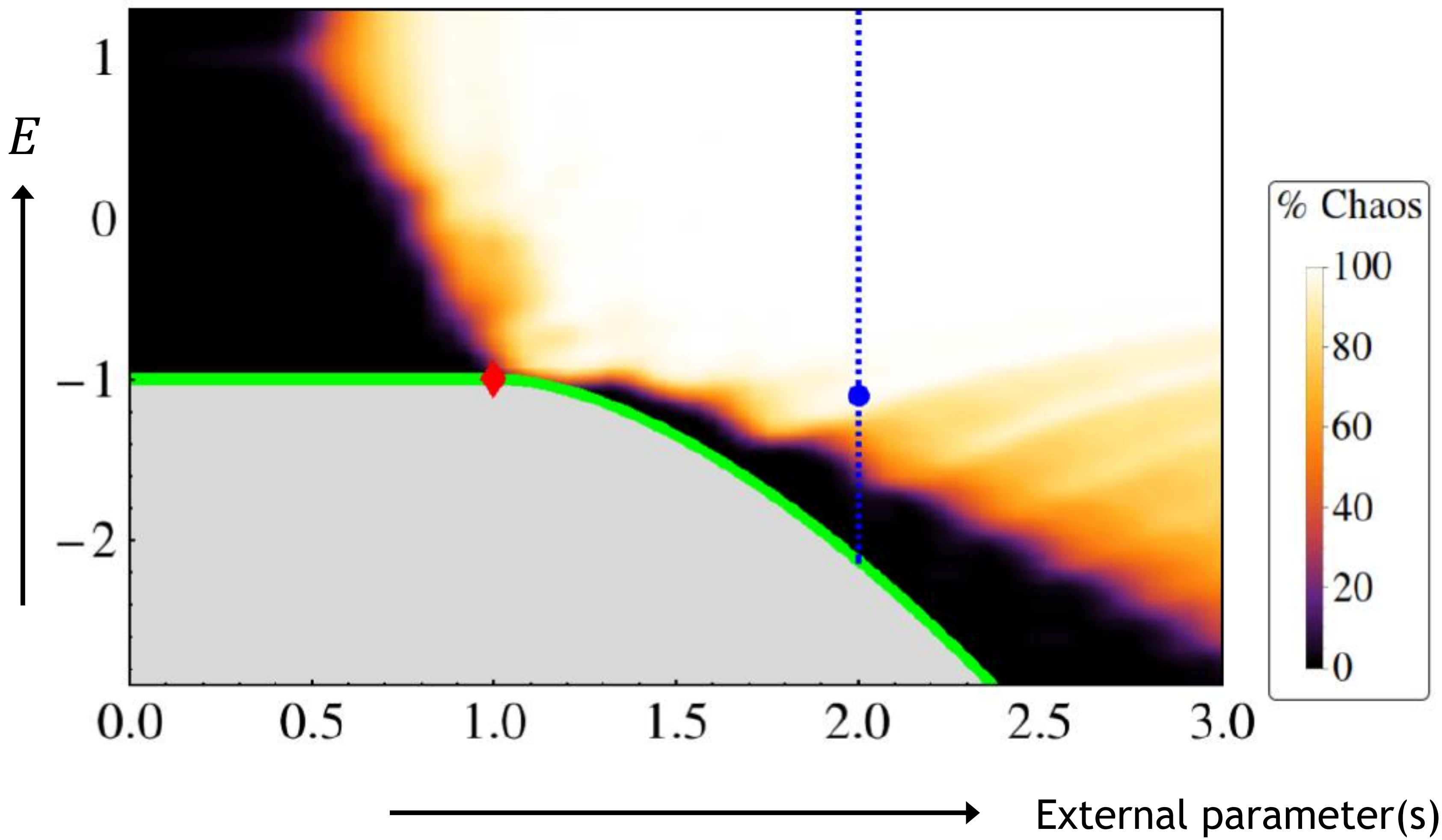
(connected with additional symmetries) $\{I_j, I_k\}_{\text{Poisson}} = 0$

Integrable system: Number of independent integrals of motion = number of degrees of freedom

All trajectories quasiperiodic

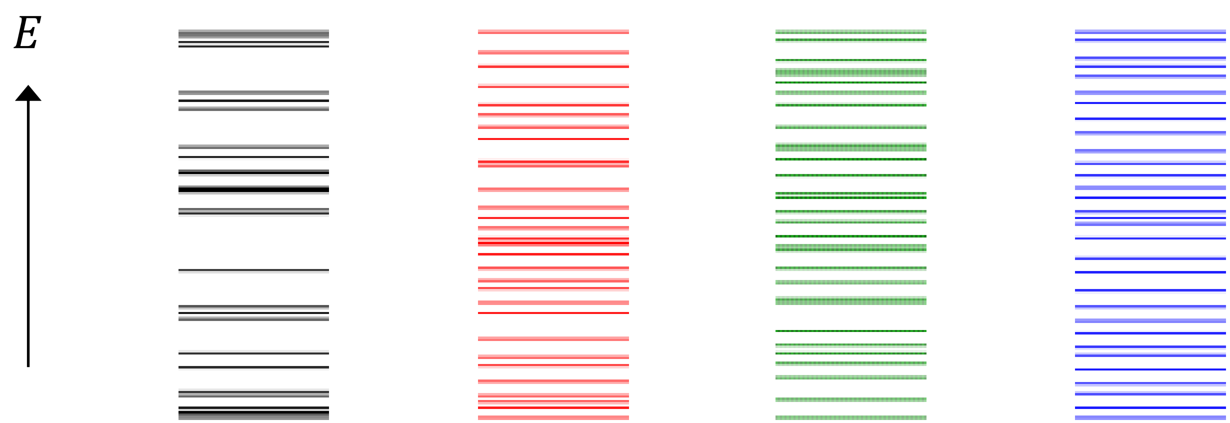
Map of chaos

(general behaviour in a low-dimensional classical system)



Quantum chaos: Correlations in spectra

- Schrödinger equation is linear → quantum suppression of chaos
- ✗ No trajectories, no phase space
- ✓ Energy spectra

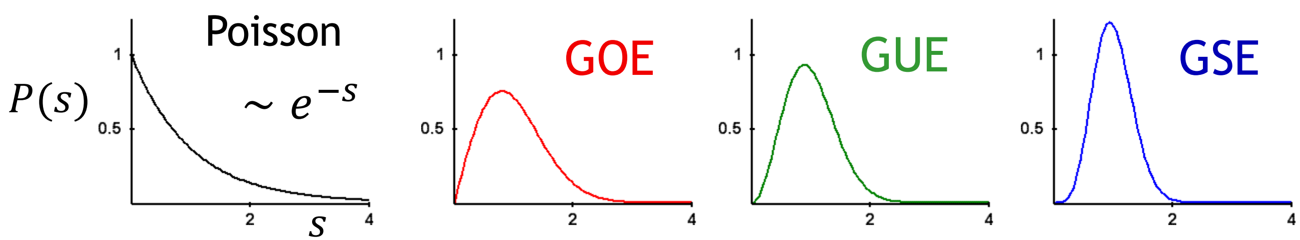


INTEGRABLE system

CHAOTIC systems

- Modeled by ensembles of Gaussian matrices
- High interaction between levels
- Spectral rigidity

Nearest-neigbour spacing distribution



$\sim s^{\alpha} e^{-s^2}$

Any connection?

Out-of-Time Ordered Commutator

$$C(t) \equiv -\left\langle \left[\hat{W}(t), \hat{V}(0)\right]^2 \right\rangle$$

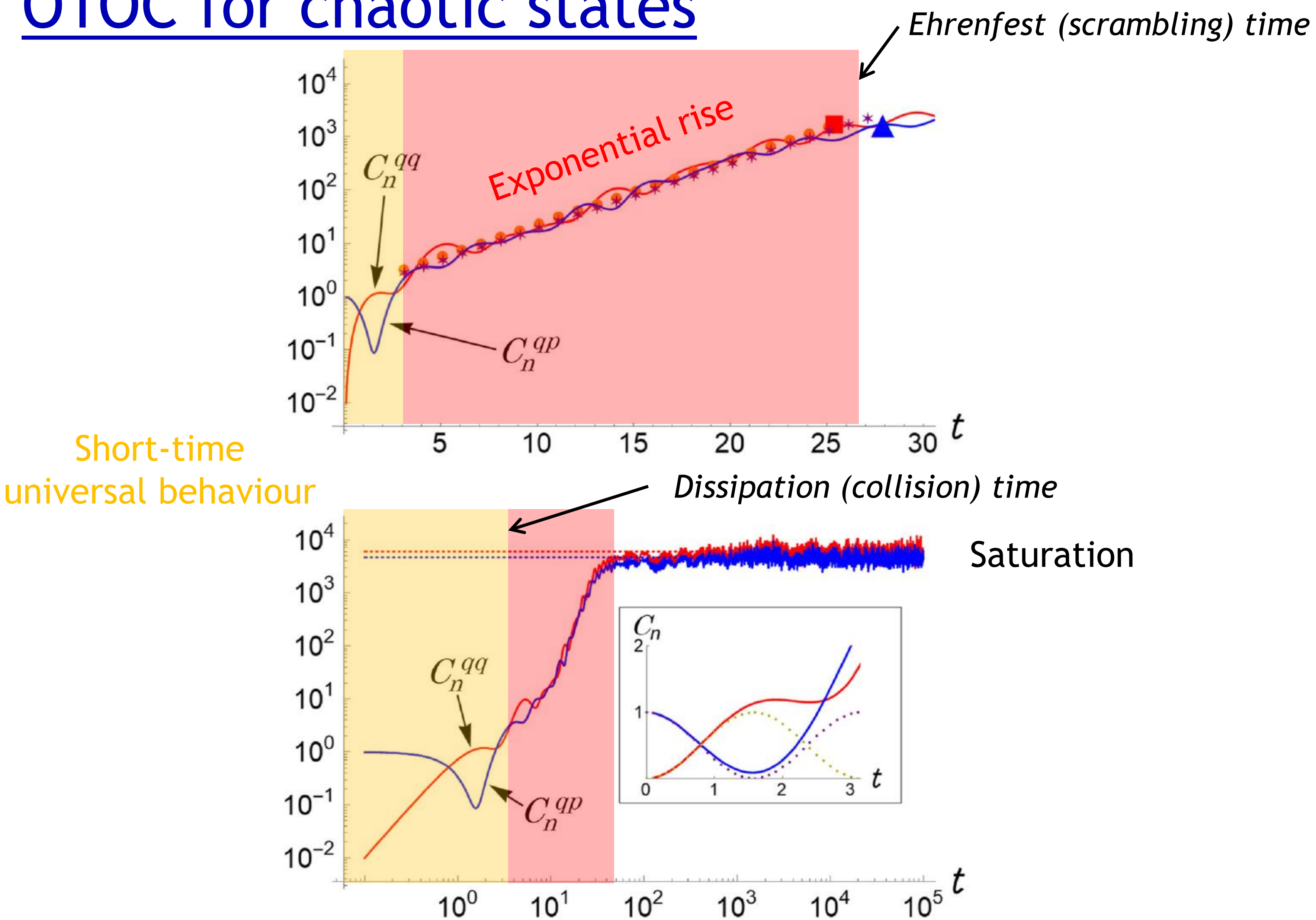
1. A special choice $\hat{W} \equiv \hat{q}$, $\hat{V} \equiv \hat{p}$ leads, performing the classical limit, to

$$\left(\frac{1}{i\hbar}[\hat{q}(t), \hat{p}(0)]\right)^2 \rightarrow \{q(t), p(0)\}_{\text{Poisson}} = \frac{\partial q(t)}{\partial q(0)} \sim e^{\lambda t}$$

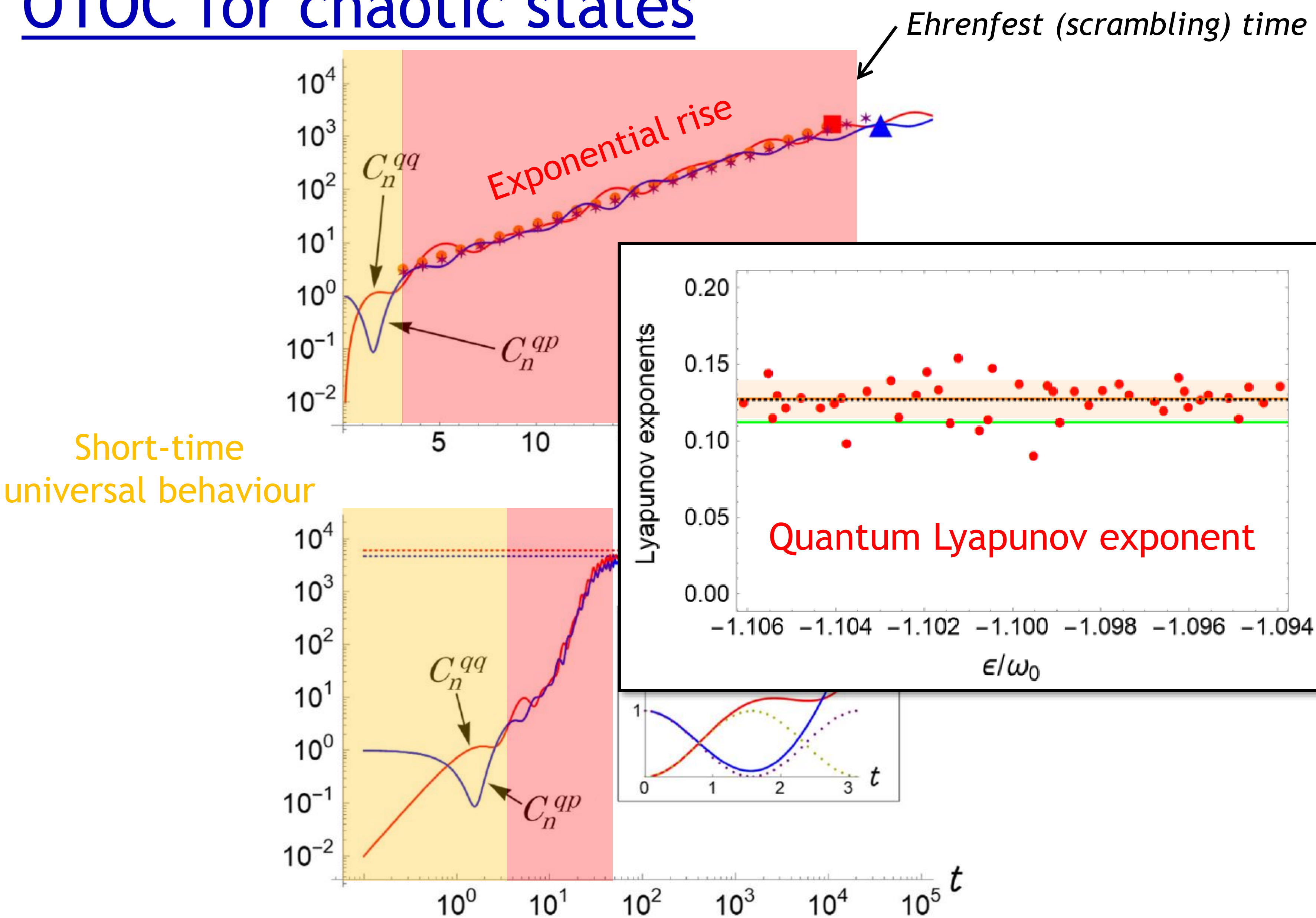
Exponential divergence of the OTOC in chaotic states expected.

A. Larkin, Y.N. Ovchinnikov, *Quasiclassical method in the theory of superconductivity,* J. Exp. Theor. Phys. **28**, 1200 (1969)

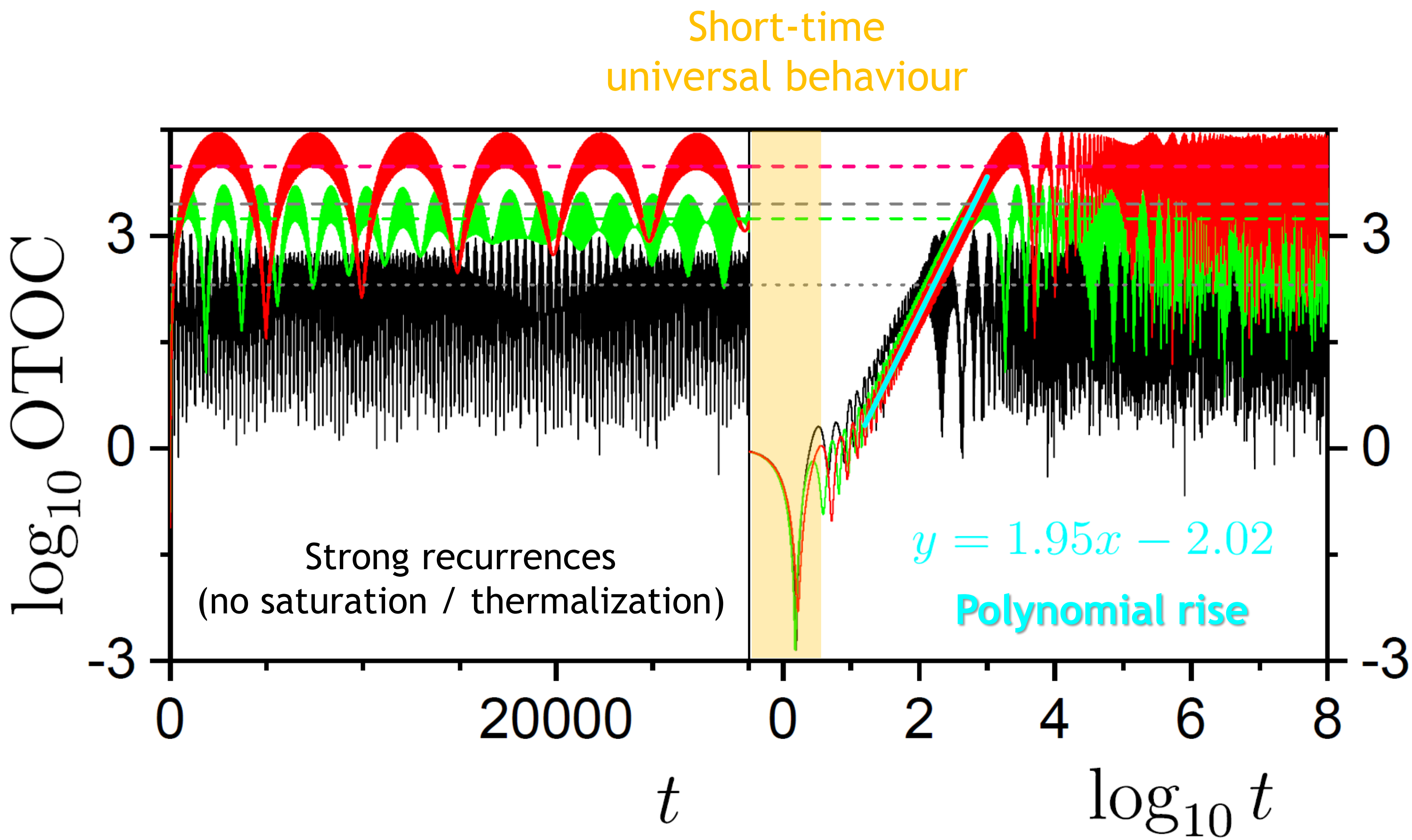
OTOC for chaotic states



OTOC for chaotic states



OTOC for regular states

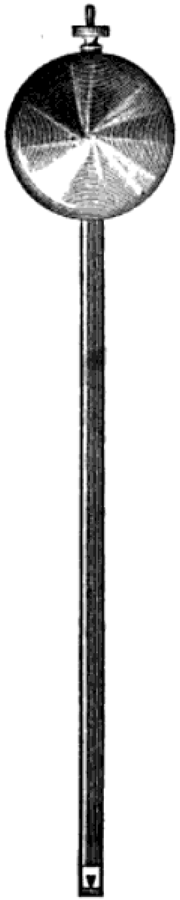


J. Chávez-Carlos, B. López-del-Caprio, M.A. Bastarrachea-Magnani, **P. Stránský**, S. Lerma-Hernández, L.F. Santos, J. Hirsch

Quantum and classical Lyapunov exponents in atom-field interaction systems

Physical Review Letters **122**, 024101 (2019)

Isolated unstable trajectories



Pendulum

All trajectories periodic except for the separatrix (having positive Lyapunov exponent)

Classical physics - impossible to prepare (measure) the unstable trajectory

Quantum physics – OTOC is sensitive to the unstable trajectory in a wider energy window!

E.B. Rozenbaum, L.A. Bunimovich, V. Galitski, *Quantum chaos in classically non-chaotic systems,* arXiv:1902.05466

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1. A special choice $\hat{W} \equiv \hat{q}$, $\hat{V} \equiv \hat{p}$ leads, performing the classical limit, to

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2. A special choice $\hat{W} \equiv e^{i\phi\hat{G}}$, $\hat{V} \equiv |\psi_0\rangle\langle\psi_0|$ leads to the **Fidelity OTOC**

$$F(t) \equiv \frac{C(t)}{\phi^2} = \text{var}\,\hat{G}(t)$$

- Experimentally accessible
- Easy to perform large-scale semiclassical calculations

R.J. Lewis-Swan, A. Safavi-Naini, J.J. Bollinger, A.M. Rey, *Unifying scrambling, thermalization and entanglement through measurement of fidelity out-of-time-order correlators in the Dicke model*, Nature Communications **10**, 1581 (2019)

Out-of-Time Ordered Commutator

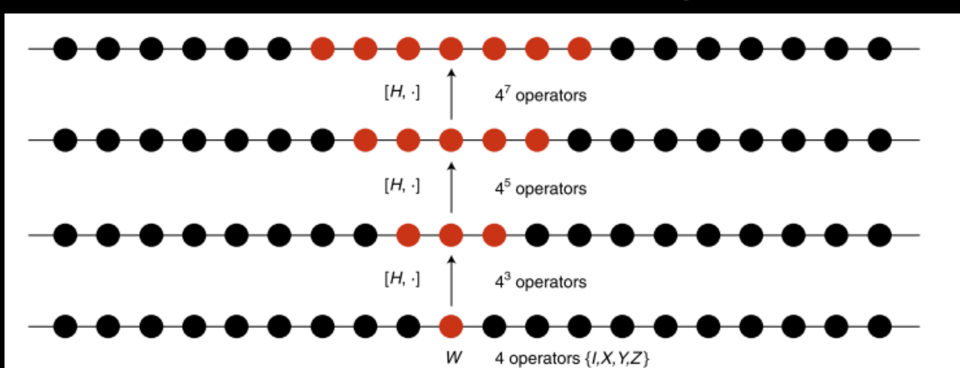
$$C(t) \equiv -\left\langle \left[\hat{W}(t), \hat{V}(0)\right]^2 \right\rangle$$

Out-of-Time-Ordered Correlator

$$c(t) \equiv \left\langle \hat{W}^+(t)\hat{V}^+(0)\hat{W}(t)\hat{V}(0) \right\rangle$$

Overlap between two vectors: $|\psi_1(t)\rangle = \hat{W}(t)\hat{V}(0)|\psi_0\rangle$

$$|\psi_2(t)\rangle = \hat{V}(0)\hat{W}(t)|\psi_0\rangle$$



$$W(t) = e^{i\hat{H}t}\hat{W}e^{-i\hat{H}t} = \sum_{l=0}^{\infty} \frac{(il)^l}{l!} [\hat{H}, \dots, [\hat{H}, \hat{W}], \dots]$$

B. Swingle, *Unscrambling the physics of OTOCs*, Nature Physics **14**, 988 (2018)

Fundamental OTOC literature

- S.H. Shenker, D. Stanford, *Black holes and the butterfly effect,* JHEP 2014:067

  - For operators separated in space $c(t,x) \sim e^{\left[\lambda\left(t-t^{*}-\frac{|x|}{v_B}\right)\right]}$ Butterfly velocity

  Gauge gravity duality: Black holes in asymptotically AdS spaces are dual to strongly coupled many-body quantum systems. Chaotic nature of quantum many-body systems characterized by OTOCs are related to the collision of shock waves close to the black hole horizon.

- P. Hosur, X.L. Qi, D.A. Roberts, B. Yoshida, *Chaos in quantum channels,* JHEP 2016:004
- J. Maldacena, S.H. Shenker, D. Stanford, *A bound on chaos,* JHEP 2016:106

  Bound for Lyapunov exponent: $\lambda \leq \frac{2\pi}{\beta}$

- K. Hashimoto, K. Murata, R. Yoshii, *OTOCs in quantum mechanics,* JHEP 2017:138
- E.B. Rozenbaum, S. Ganeshan, V. Galitski, *Lyapunov exponent and OTOC's growth rate in a chaotic system,* Physical Review Letters **118**, 086801 (2017)
- R. Fan, P. Zhang, H. Shen, H. Zhai, *OTOC for many-body localization,* Science Bulletin **62**, 707 (2017)

  Relation between Rényi entropy of a subsystem and OTOC:

  $$S_A^{(2)} = Tr_B \hat{\rho}_{AB}^2 \qquad e^{-S_A^{(2)}} = \sum_{\hat{M}\in B} \langle \hat{M}(t)\hat{V}(0)\hat{M}(t)\hat{V}(0)\rangle \qquad \hat{V} = \hat{O}e^{-\beta\hat{H}}\hat{O}^{+}$$ an arbitrary operator

- E.B. Rozenbaum, S. Ganeshan, V. Galitski, *Universal Level Statistics of the OTOC,* arXiv:1801.10591

  Lyapunovian: operator with GOE level statistics and OTOC expectation value