Experimental techniques in high energy physics

April 2019
FYS3500
A. Read (U. Oslo)
Intro: Simplified anatomy of a typical HEP experiment

- Create a beam of particles, accelerate them, crash them into a target
- Observe the collisions in a detector, filter interesting collisions and store them
- Reconstruct the outgoing particles
- Filter the data, study (statistical) patterns in the data
- **MC** - Simulate known and potential new processes and compare with the data, check for anomalies, look for excesses above the known processes, measure process rates and particle properties
Plan

- To mention the «mission» of HEP and one of the recent huge milestones (Higgs)
- To touch very briefly all the elements of a «typical» HEP experiment (ATLAS at CERN Large Hadron Collider).
  - And take a quick look at other detectors
Quick peek at (high energy) particle physics
SM Before LHC (<2010)

- Standard Model from 60’s+70’s works like a dream (almost...)
- Neutrinos have small mass
- Separate theory from gravity
- Higgs not confirmed
- Many empirical constants
- Universe is full of dark matter (and energy) which isn’t SM particles
Higgs particle, the missing link of the SM

- Higgs particle couples to mass
  - Invented to give mass to fundamental particles, applied to the massive weak bosons W and Z (half a century ago!)
  - Regulated by empirical constants can give mass to SM fermions (electrons, quarks,...) as well (first results confirm this as well!)
- Must be at least 1 massive, spinless particle or theory is misleading/wrong (we found it at the LHC - 2012!)
- Fixes some problems, creates others
Beyond the Standard Model

- Is there a (super)symmetry between matter and force, fermions and bosons?
  - If there is, what hides the SUSY partners from us? What breaks the supersymmetry?

- Why is gravity SO much weaker than the other (strong, EM, weak) forces?

- Gravity leaks into extra dimensions? -> (unstable and harmless) microscopic black holes??

- CP, precision measurements, no time to make the full list...
The Large Hadron Collider (LHC)
Små partikkler - stort mikroskop

<table>
<thead>
<tr>
<th></th>
<th>$\lambda$</th>
<th>$E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Synlig lys</td>
<td>$\sim 500$ nm</td>
<td>2-3 eV</td>
</tr>
<tr>
<td>e-mikroskop</td>
<td>$10^{-13}$ m</td>
<td>MeV</td>
</tr>
<tr>
<td>partikkel-akselerator</td>
<td>$10^{-15}$-$10^{-18}$ m</td>
<td>GeV-TeV</td>
</tr>
</tbody>
</table>

$E=h*c/\lambda$ ...og ikke glem $E=mc^2$

<table>
<thead>
<tr>
<th>$e.g.$ $e^-$</th>
<th>$U=0$ V</th>
<th>$U=1$ M V</th>
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<tbody>
<tr>
<td>$E=e^*\Delta U=1$ MeV</td>
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ATLAS experiment
ATLAS-eksperimentet

Elektroner (stabile)
Myoner (stabile nok)
Tau-leptoner (fra henfallsprodukter)
Jets (fra kvarker og gluoner)

*Manglende transverse bev.mengde* (nøytrinoer, ny fysikk e.g. SUSY)

7000 tonn (~100 tomme Boeing 747er)

90 M 3-D piksler
400 “bilder”/s
Kaster 20
Mbilder/s
Noen 1000 TB/år
Made in Norway (+Uppsala)
New tracker 2026-

Fig. 1. The schedule of the LHC. The detector upgrades are completed during the long shutdowns (LS2 and LS3).

A new inner tracker for ATLAS

Figure 2: Left: ITk “Reference” layout, axial cross-section view. Right: Single-chip pixel module constructed for and installed in the ATLAS IBL. The size corresponds to the individual sensor size of 18.2 x 20.2 mm$^2$. 
Other detector types
“Fixed-target”

The LHCb Detector

Diagram of the LHCb Detector
ALICE TPC: Construction Parameters

- **Largest TPC:**
  - Length 5m
  - Diameter 5m
  - Volume 88m³
  - Detector area 32m²
  - Channels ~570 000

- **High Voltage:**
  - Cathode -100kV

- **Material $X_0$:**
  - Cylinder from composite materials from airplane industry ($X_0 = \sim 3\%$)
Aiming for groundbreaking discoveries

Origin of Matter
Could neutrinos be the reason that the universe is made of matter rather than antimatter? By exploring the phenomenon of neutrino oscillations, DUNE seeks to revolutionize our understanding of neutrinos and their role in the universe.

Unification of Forces
With the world’s largest cryogenic particle detector located deep underground, DUNE can search for signs of proton decay. This could reveal a relation between the stability of matter and the Grand Unification of forces, moving us closer to realizing Einstein’s dream.

Black Hole Formation
DUNE’s observation of thousands of neutrinos from a core-collapse supernova in the Milky Way would allow us to peer inside a newly-formed neutron star and potentially witness the birth of a black hole.
Neutrino detector: DUNE

- Liquid argon (LAr) Time Projection Chamber (TPC) - 3D detection of ionising particles.
- The 10 kton LAr is first target then detector.
Higgs boson discovery
LHC challenge: We looked for about 10,000 Higgs bosons in a million billion proton-proton-collisions.
Results 4 July (*), 2012
H is electrically neutral, photon has only electromagnetic interactions. How can H, which “couples to mass”, decay to $\gamma\gamma$ ?!
Summary

- We took a superficial look at one of the current flagship experiments and accelerators in HEP today.
- We peeked inside the most well-known result at LHC, the discovery of the Higgs boson.
- We saw a few examples of other detector types and principles
Courses

- Particle (PP) and Nuclear physics (NP)
  - FYS3510 (PP), FYS3520 (NP) -> FYS3500 (SAP)
- Experimental nuclear and high energy physics: FYS4505
  - Particle and Heavy ion physics experiments ++
Maximum likelihood

- Ideal estimators of parameters are unbiased and efficient (minimum variance). Not always simultaneously achievable.

- Maximum likelihood (for convenience minimize \(-\ln(L)\) or even \(-2\ln(L)\)) is approximately unbiased, efficient for large data samples and widely applicable.

- Wald showed that for single parameter and «nested» functions

  \[-2 \ln \lambda(\mu) = \left(\frac{\mu - \hat{\mu}}{\sigma^2}\right)^2 + O(1/\sqrt{N})\]

- Wilks showed that if \(\hat{\mu}\) Gaus-distributed about \(\mu\) then

  \[-2 \ln \lambda(\mu) \rightarrow \chi^2\]
Nested functions

\[ P(m) = \frac{s \cdot S(m) + b \cdot B(m)}{s + b} \]

\[ y(x) = a + b \cdot x + c \cdot x^2 \]

- B(m) is nested in P(m) (s->0)
- line is nested in parabola (c->0)
Various likelihoods

\[ L(n|\mu) = \frac{e^{-\mu} \mu^n}{n!} \]  \quad \text{Poisson, counting (no background)}

\[ L(n|\mu s + b) = \frac{e^{-(\mu s+b)}(\mu s + b)^n}{n!} \]  \quad \text{Counting, known bkg}

\[ L(n, m|\mu s + b, \tau) = \frac{e^{-(\mu s+b)}(\mu s + b)^n}{n!} \frac{e^{-\tau}b(\tau b)^m}{m!} \]  \quad \text{Counting “on/off”}

\[ L(x|x_0, \sigma) = \frac{1}{\sqrt{2\pi}\sigma^2} e^{-\frac{(x-x_0)^2}{2\sigma^2}} \]  \quad \text{Gaussian}

Likelihood ratio of marked Poissons in combined channels

\[ Q = \frac{\prod_{i=1}^{N_{\text{chan}}} e^{-(s_i+b_i)}(s_i+b_i)^{n_i}}{n_i!} \frac{\prod_{j=1}^{n_i} s_i S_i(x_{ij})+b_i B_i(x_{ij})}{s_i+b_i} \frac{\prod_{i=1}^{N_{\text{chan}}} e^{-b_i b_i^{n_i}}}{n_i!} \frac{\prod_{j=1}^{n_i} B_i(x_{ij})}{n_i!} \]
Probability distribution functions (pdf)

\[
\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-x_0)^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2}}
\]

The red curve is the standard normal distribution.
k - degrees of freedom

- Fitting m parameters to n measurements gives $k=n-m$ degrees of freedom
- Classical example is mean of two numbers has $k=1$
Multivariate gaussian

- i.e. mean of measurements (straightforward to extend to e.g. curve-fitting)

\[
L(x | \mu) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi \sigma_i^2}} e^{-\frac{(x_i - \mu)^2}{2\sigma_i^2}}
\]

\[-2 \log(L) = \text{constants} + \chi^2_{n-1}\]

\[
\chi^2_{n-1} = \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{\sigma_i^2}
\]

A. Read, U. Oslo
A graph of the normal distribution, showing 3 standard deviations on either side of the mean $\mu$. A five-sigma observation corresponds to data even further from the mean. Source: Wikimedia Commons/Mwtoews

Chances are, you heard this month about the discovery of a tiny fundamental physics particle that may be the long-sought Higgs boson. The phrase five-sigma was tossed about by scientists to describe the strength of the discovery. So, what does five-sigma mean?
p-value

p-value = probability that the «null» (baseline) hypothesis can produce a fluctuation at least as large as the one observed

A small p-value disfavors the null hypothesis in favor of the alternate hypothesis

This does not prove that the alternate hypothesis is correct!
1-sided p-values in large-sample limit

\[ N_{\sigma} \quad \Delta \chi^2 \quad \frac{1}{2} P(\chi^2 > c) \]

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.159</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2.3x10^{-2}</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>1.3x10^{-3}</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>3.2x10^{-5}</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>2.9x10^{-7}</td>
</tr>
</tbody>
</table>

\( q_{\mu} = -2 \ln \frac{L(\mu_{\text{test}})}{L(\hat{\mu})} = \chi^2_{\text{test}} - \chi^2_{\text{min}} \)
Extra material
p-value distribution

- If the hypothesis under test is true and the experiment is repeated many times, the p-value distribution will be....?
Uniform!
Why is p-value not always the full story (e.g. psychology, medisin)?
Exam question (Bob Cousins)

For most of this talk\(^1\), I assume familiarity with the ‘required reading’ for this workshop. But first, let’s review the root of the problem as I often explain it to students. (Imagine an oral exam.)

Suppose you have a particle ID detector. You take it to a test beam and measure:

- \(P(\text{counter says } \pi \mid \text{ particle is } \pi) = 90\%\)
- \(P(\text{counter says not } \pi \mid \text{ particle is } \pi) = 10\%\)
- \(P(\text{counter says } \pi \mid \text{ particle is not } \pi) = 1\%\)
- \(P(\text{counter says not } \pi \mid \text{ particle is not } \pi) = 99\%\)

Then you put the detector in your experiment. You select tracks which the detector says are pions.

Question: What fraction of these tracks are pions?
Related question: What is the probability that a particular track is a pion?
Nuisance parameters

1. Parameters fitted directly to the data but no real interest
   - E.g. parametric background; both shape and normalization uncertainty

2. Parameters from external estimates that incorporate systematic uncertainty
   - E.g. luminosity, signal theory, mass resolution, electron, muon and jet energy scales
Importance of nuisance parameters

- background, uncertainty, uncertainties among most frequent words in ATLAS Higgs boson discovery paper
Nuisance parameters

NP’s broaden the likelihood profile for the parameter of interest

\[
\frac{\partial \chi^2}{\partial \delta} = 0
\]

\[
\frac{\partial \chi^2}{\partial \mu} = 0
\]

\[
\frac{1}{\sigma^2_\mu} = \frac{1}{2} \frac{\partial^2 \chi^2}{\partial \mu^2}
\]

\[
\chi^2 = \frac{(n - (\mu + \delta))^2}{\sigma^2} + \frac{\delta^2}{\sigma_s^2}
\]

\[
\hat{\mu} = n, \delta = 0
\]

\[
\sigma_\mu = \sqrt{\sigma^2 + \sigma_s^2}
\]
LHCHCG Combination Procedures

Profile likelihood ratio: $p_0$ and $\hat{\mu}$

- $p_0$ to test background hypothesis
- $\hat{\mu}$ to estimate signal strength

Targeting $\mu$ to explore signal strength

\[ t_\mu = -2 \ln \left\{ \frac{\mathcal{L}(\mu, \hat{\theta}_\mu)}{\mathcal{L}(\hat{\mu}, \hat{\theta})} \right\} \]

\[ \chi^2 \]

\[ \int_{\text{LDL}}^{\text{SM}} = 2.05 \text{ fb}^{-1} \]

\( \sqrt{s} = 7 \text{ TeV} \)

2011 Data

Observed $p_0$

Data 2011, $\sqrt{s} = 7$ TeV

- SM $H \to \gamma\gamma$ expected $p_0$
  \[ \int_{\text{LDL}}^{\text{SM}} = 4.9 \text{ fb}^{-1} \]

ATLAS Preliminary

Aspen 2C
Profile likelihood ratio: CL$_s$ and $\mu_{95}^{up}$

Profile likelihood ratio:

$t_\mu = -2 \ln \left\{ \frac{\mathcal{L}(\mu, \hat{\mu})}{\mathcal{L}(\hat{\mu}, \hat{\theta})} \right\}$

$CL_s$: test signal + background

$\chi^2$: test signal

$CL_s$: ~ test signal

$\mu_{95}^{up} = \mu(\text{CL}_s = 0.05)$
Combined Results

\begin{align*}
L(m_H, \mu, \bar{\theta}) &= \prod_i L_i(m_H, \mu, \bar{\theta}_i) \\
\bar{x} &= \frac{\sum_{i=1}^{n} x_i / \sigma_i^2}{1/\sigma^2} \\
\frac{1}{\sigma^2} &= \frac{1}{\sum_{i=1}^{n} 1/\sigma_i^2}
\end{align*}

\[ t_\mu = -2 \ln \left\{ \frac{L(\mu, \hat{\theta}_\mu)}{L(\hat{\mu}, \hat{\theta})} \right\} \]
From exclusion to discovery to measurement

Release 1 by 1 the model assumptions in the statistical model used in the search, e.g.

<table>
<thead>
<tr>
<th>Background (scan m&lt;sub&gt;H&lt;/sub&gt;)</th>
<th>( \lambda(\mu = 0, m_H) = \frac{L(\mu = 0, m_H, \hat{\theta})}{L(\hat{\mu}, m_H, \hat{\theta})} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal (scan m&lt;sub&gt;H&lt;/sub&gt;)</td>
<td>( \lambda(\mu, m_H) = \frac{L(\mu, m_H, \hat{\theta})}{L(\hat{\mu}, m_H, \hat{\theta})} )</td>
</tr>
<tr>
<td>Mass consistency</td>
<td>( \lambda(m_H) = \frac{L(m_H, \hat{\mu}<em>1, \hat{\mu}<em>2, \hat{\theta})}{L(m</em>{1H}, m</em>{2H}, \hat{\mu}_1, \hat{\mu}_2, \hat{\theta})} )</td>
</tr>
<tr>
<td>Mass</td>
<td>( \lambda(m_H) = \frac{L(m_H, \hat{\mu}_1, \hat{\mu}_2, \hat{\theta})}{L(\hat{m}_H, \hat{\mu}_1, \hat{\mu}_2, \hat{\theta})} )</td>
</tr>
<tr>
<td>Signal and mass</td>
<td>( \lambda(\mu, m_H) = \frac{L(\mu, m_H, \hat{\theta}_\mu)}{L(\hat{\mu}, \hat{m}<em>H, \hat{\theta}</em>\mu)} )</td>
</tr>
</tbody>
</table>
Mass measurements

Compatible combination

Aspen 2013 - Higgs Quo Vadis 35
Other interesting subjects

- Details of signal exclusion/favoring the null hypothesis
- «Look-elsewhere»/trials factors
- Parameterized likelihoods
- Data-driven background estimation
- Multivariate analysis: 2-d correlations, many-d w/boosted decision trees, artificial neural networks
- Precision measurements of the Higgs boson+++
- Detector and accelerator technologies, astroparticle physics experiments
2 main statistical approaches

- **Bayesian** - probability(\(\theta|\mathbf{x}\))
  - well-defined accounting for beliefs
  - prior-probability for the theory **must** be given
  - prior-dependence should be studied

- **Frequentist/classical** - probability(\(\mathbf{x}|	heta\))
  - says nothing about probability of theory
  - typically used in HEP to report experimental results objectively (as possible)
  - can lead to subset of individual results which are obviously wrong but consistent with methodology
Bayes vs. freq.

In many data-dominated situations hardly any difference in reported results, eg. $M_Z = 91.1876 \pm 0.0021$ GeV

But interpretation is not the same!
Which is B and which is F?
1) $P(|M_Z - 91.1876| < 0.0021) = 68\%$
2) 68% of such intervals contain the true $M_Z$

Small data samples, physical boundaries typically lead to differences

Doing both analyses and studying the differences can give insights
Parameterized signal and/or background models
e.g. ATLAS H-→γγ search
9 categories of unbinned likelihood

Parameterized signal model from fits to MC

Background model: **selected** functions with unconstrained nuisance parameters

<table>
<thead>
<tr>
<th>Name</th>
<th>Criteria</th>
</tr>
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<tbody>
<tr>
<td>CP1</td>
<td>unconverted central low $p_{Tt}$</td>
</tr>
<tr>
<td>CP2</td>
<td>unconverted central high $p_{Tt}$</td>
</tr>
<tr>
<td>CP3</td>
<td>unconverted non-central low $p_{Tt}$</td>
</tr>
<tr>
<td>CP4</td>
<td>unconverted non-central high $p_{Tt}$</td>
</tr>
<tr>
<td>CP5</td>
<td>converted central low $p_{Tt}$</td>
</tr>
<tr>
<td>CP6</td>
<td>converted central high $p_{Tt}$</td>
</tr>
<tr>
<td>CP7</td>
<td>converted non-central low $p_{Tt}$</td>
</tr>
<tr>
<td>CP8</td>
<td>converted non-central high $p_{Tt}$</td>
</tr>
<tr>
<td>CP9</td>
<td>converted transition</td>
</tr>
</tbody>
</table>

4/9 categories
Various terms in $\mathcal{L}$

$L$ per event in a category

$$\mathcal{L}_c(\mu, \theta_c) = e^{-N_c} \prod_{n=1}^{N_c} \mathcal{L}_{c,n}(m_{\gamma\gamma}(n); \mu, \theta_c)$$

Mass distribution

$$\mathcal{L}_{c,n}(m_{\gamma\gamma}(n); \mu, \theta_c) = N_{s,c}(\mu, \theta_c^{\text{norm}}) f_{s,c}(m_{\gamma\gamma}; \theta_c^{\text{shape}}) + N_{bkg,c} f_{bkg,c}(m_{\gamma\gamma}; \theta_c^{\text{bkg}}),$$

Signal normalization

$$N_{s,c}(\mu, \theta_c^{\text{norm}}) = \mu \left[ N_c^{ggH,SM}(\theta_c^{ggH}) + N_c^{VBF,SM}(\theta_c^{VBF}) + N_c^{WH,SM}(\theta_c^{WH}) + N_c^{ZH,SM}(\theta_c^{ZH}) + N_c^{ttH,SM}(\theta_c^{ttH}) \right] 
\cdot K_{BR}(\theta_{BR}) K_{lumi}(\theta_{lumi}) K_{eff}(\theta_{eff}) K_{isol}(\theta_{isol}) K_{pile-up}(\theta_{pile-up}) K_{EScale}(\theta_{EScale}) K_{pile-up,c}(\theta_{pile-up,c}) K_{mat,c}(\theta_{mat}) + \sigma_{spurious,c} \theta_{spurious,c}. \quad (8.12)$$
Distinguish signal from spurious signal

**Best fit background model**

**True (but unknown!) background distribution**
Model tests (on MC)

- 9 categories
- No CPU time for full simulation
- 3 MC generators, don’t expect them to perfectly reproduce the background data
- Select parameterizations which can incorporate shape uncertainty in unconstrained nuisance parameters without producing false signals
BG model selection

| Category | Function | Max $|S_{SP}|$ ($m_{H}[\text{GeV}]$) | $\%\sqrt{S}$ ($N_{S}$) | $\%\sigma_{0}$ ($\sigma_{0}$) | $\sigma_{N_{S}}$ | $\sigma_{S_{SP}}$ | Pass | Pass_{all} |
|----------|----------|----------------------------------|-----------------|----------------|----------------|----------------|-------|------------|
| CP1      | Exp      | -4.7 (126)                       | -45 (11)        | -35 (14)        | 0.78           | -0.35         | ✔    | ✔          |
| CP1      | Epoly2   | 2.1 (117)                        | 18 (12)         | 13 (16)         | 0.70           | 0.13          | ✔    | ✔          |
| CP2      | Exp      | -0.23 (110)                      | -15 (1.5)       | -6.4 (3.5)      | 0.43           | -0.064        | ✔    | ✔          |
| CP3      | Exp      | 12 (117)                         | 50 (23)         | 35 (33)         | 0.71           | 0.33          | ✔    | ✔          |
| CP3      | Epoly2   | 9.2 (112)                        | 41 (23)         | 26 (36)         | 0.64           | 0.26          | ✔    | ✔          |
| CP3      | Epoly3   | 3.4 (111)                        | 15 (22)         | 8.8 (38)        | 0.59           | 0.088         | ✔    | ✔          |
| CP3      | Bernl3   | 5.8 (111)                        | 26 (22)         | 16 (36)         | 0.62           | 0.16          | ✔    | ✔          |
| CP3      | Bernl4   | 2.8 (111)                        | 13 (22)         | 7.1 (40)        | 0.56           | 0.071         | ✔    | ✔          |
| CP4      | Exp      | 0.5 (132)                        | 19 (2.6)        | 7.2 (0.9)       | 0.38           | 0.072         | ✔    | ✔          |
| CP5      | Exp      | -4.4 (126)                       | -64 (6.8)       | -34 (13)        | 0.54           | -0.34         | ✔    | ✔          |
| CP5      | Epoly2   | 1.6 (117)                        | 22 (7.4)        | 10 (16)         | 0.47           | 0.10          | ✔    | ✔          |
| CP5      | Exp      | -0.27 (110)                      | -27 (0.98)      | -8.0 (3.4)      | 0.29           | -0.080        | ✔    | ✔          |
| CP7      | Exp      | 8.5 (122)                        | 20 (22)         | 18 (37)         | 0.60           | 0.17          | ✔    | ✔          |
| CP7      | Epoly2   | 5.8 (122)                        | 26 (22)         | 14 (40)         | 0.56           | 0.14          | ✔    | ✔          |
| CP7      | Epoly3   | -0.3 (110)                       | -29 (22)        | -13 (48)        | 0.46           | -0.13         | ✔    | ✔          |
| CP7      | Bernl3   | -6.3 (110)                       | -29 (22)        | -14 (46)        | 0.47           | -0.14         | ✔    | ✔          |
| CP7      | Bernl4   | -4.5 (110)                       | -20 (22)        | -8.8 (50)       | 0.43           | -0.088        | ✔    | ✔          |
| CP8      | Exp      | 0.45 (134)                       | 18 (2.5)        | 5.7 (7.9)       | 0.32           | 0.057         | ✔    | ✔          |
| CP9      | Exp      | -1.6 (150)                       | -179 (9.1)      | -50 (28)        | 0.33           | -0.59         | ✔    | ✔          |
| CP9      | Epoly2   | -3.2 (110)                       | -33 (9.9)       | -8.3 (39)       | 0.26           | -0.083        | ✔    | ✔          |

- the exponential function
  \[ N e^{-\beta m_{\gamma\gamma}}, \]  
  where \( N \) and \( \beta \) were the fitted parameters – the normalization and slope of the exponential, respectively;

- the exponential polynomial of order \( n \) (orders 2 and 3 were used)
  \[ \sum_{i=0}^{n} \beta_{i} m_{\gamma\gamma}^{i}, \]  
  where \( \beta_{i} \) were the fitted parameters. Note that the latter \( i \) is not an index, but the power \( m_{\gamma\gamma} \) is raised to. The normalization, \( N \), is described by the first term, \( e^{\beta_{0}} \);

- the Bernstein polynomial of order \( n \) (orders 3 - 7 were used)
  \[ b_{n}(t) = \sum_{i=0}^{n} \beta_{i} \binom{n}{i} t^{i}(1-t)^{n-i}, \]  
  where \( t = \frac{m_{\gamma\gamma}[\text{GeV}]-100}{60} \), and where \( \beta_{i} \) were fitted parameters.

Maximum spurious signal amplitude
Residual (unknown!) bias: Spurious signal term in likelihood

\[ \chi^2 = \frac{(n - (\mu + \delta))^2}{\sigma^2} + \frac{\delta^2}{\sigma_s^2} \]

\[ \hat{\mu} = n, \delta = 0 \]

\[ \sigma_\mu = \sqrt{\sigma^2 + \sigma_s^2} \]
Data-driven methods

- HEP depends heavily on Monte Carlo calculations of physics processes and detector response for both signals (known and hypothetical) and backgrounds.

- Sometime we just don’t know and/or have reason not to trust the MC results.

- Various data-driven methods used to estimate background in signal region.
  - Fits (unbinned, many bins) with sidebands
  - Variations of “on-off”: ABCD, Matrix method, fit to shapes derived from well-understood (signal-free) control regions, ...
Other data-driven methods (ABCD)
(Variations of on-off and sideband fits)

- Known small backgrounds (e.g. electroweak processes):

\[ \mu_{A,B,C,D}^K \]

- Poorly known ("Unknown") backgrounds:

- Naively:

\[ \mu_{U}^A \]
\[ \mu_{U \tau_B}^B \]
\[ \mu_{U \tau_C}^C \]
\[ \mu_{U \tau_B \tau_C}^D \]

- Correlations should be accounted for as well...

"Let’s write down the likelihood function”

\[
L(n_A, n_B, n_C, n_D | \mu, \theta_\mu) = \Pi_{i=A,B,C,D} \frac{e^{-\mu_i} \mu_i^{n_i}}{n_i!}
\]

\[
\begin{align*}
\mu_A &= \mu + \mu_K^A + \mu_U \\
\mu_B &= b \mu + \mu_K^B + \mu_U \tau_B \\
\mu_C &= c \mu + \mu_K^C + \mu_U \tau_C \\
\mu_D &= d \mu + \mu_K^D + \mu_U \tau_B \tau_C
\end{align*}
\]
Look-elsewhere effect (LEE)

- Rule of thumb for trials factor used before LHC
- Eilam and Ofer discovered that trials factor grows with significance $Z$ (ROT ~OK for $Z=3$)

$$TF = \frac{p_{0}^{\text{global}}}{p_{0}^{\text{local}}}$$

$$TF \sim \frac{\Delta m}{\sigma m}$$

$$TF \simeq 1 + \sqrt{\frac{\pi}{2}} \mathcal{N} Z$$
Look-elsewhere effect (LEE)

\[ P_0^{\text{global}} \approx P_0^{\text{local}} + \langle N(q_{\text{ref}}) \rangle e^{-(q-q_{\text{ref}})/2} \]

\[ TF = \frac{P(q(\hat{m}) > Z^2)}{P(q(m) > Z^2)} \approx 1 + \mathcal{N} \frac{P(\chi_2^2 > Z^2)}{P(\chi_1^2 > Z^2)} \]

1M fits
Fit to background toy

background only

background + signal@0.5

background + floating signal
2 examples toy fits
138 Mfits - it all checks out

Delta 2NLL for fitted gaussian versus background

\[ \chi_1^2 \]

\[ \chi_2^2 \]

Fitted regions

Extrapolations
Testing $J^P$ - 2 point hyp. test

-2 ln(Q)

Null OK
Alt OK

!!?!!
Mass measurements

Compatibility, combination

A. Read, U. Oslo
Bayesian credible intervals

Posterior density for parameter

$$p(\theta|x) = \frac{L(x|\theta)\pi(\theta)}{\int L(x|\theta')\pi(\theta')\,d\theta'}$$

Marginalizing nuisance parameters (e.g. data-driven backgrounds, systematics)

$$p(\theta|x) = \int p(\theta,\nu|x)\,d\nu$$

Minimum interval

$$1 - \alpha = \int_{\theta_{lo}}^{\theta_{up}} p(\theta|x)\,d\theta$$

Highest density Physical boundary (e.g. m≥0)
Confidence intervals (Neyman construction)

- Need to know the ensemble for every \( \theta_0 \)
- Multi-dimensional space with nuisance parameters more complicated (ugh)