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Update on the Application of Spectral Element Methods on Quench Simulation

Outline

- Numerical context of quench simulation
- Spectral Element Method in a nutshell
- Proof of concept
- Implementation notes
- Outlook

Problem

- Quench propagation:
  Multi-Physics
  (here: reduce to thermal problem)

- Magnet geometry:
  Multi-Scale, 3D
  (here: reduce to 1D cable simplification)

> ➢ Best way to reach fast and accurate simulation?

Main challenge:
Increase in simulated cable length
and/or increase in precision requirements
$\Rightarrow$ High increase in computational time

Problem

Approach A:

Find reasonable simplifications or models, e.g. for Comsol

Approach B:

Optimize numerical procedure which is internally used by solving programs

➢ Best way to reach fast and accurate simulation?

Ongoing research for finding numerical procedure best suitable for quench propagation in magnets*

My contribution: Implement solver for 1D thermal problem using a non-standard numerical solution procedure - SEM

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Reminder: Excerpt of Standard Simulation Workflow

PDE & model	*Temperature* $\theta(z)$
↓ (*Discretization method*)	↓
Discretizing PDE as algebraic (matrix) equation	*build matrix system* $Au = b$
↓	↓
Solving	*find unknown* u
↓	↓
Postprocessing: physical interpretation	*plot simulated* $\theta(z)$

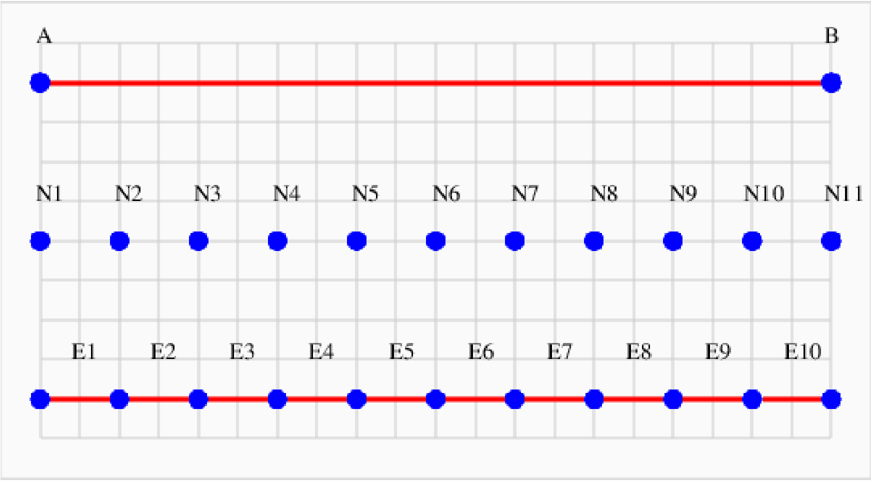
Comsol / Ansys

Discretization Method

Most wide-spread:

Finite Element Method (FEM)

- Discretize space by mesh nodes z_i
- Identify unknowns with temperature at nodes, i.e. $\theta(z_i) = u_i$
- Define matrix entries by dense mesh, low order polynomials



+ Arbitrary geometries

– Refinement comp. expensive

(Picture taken from John Burkardt, PostScript Graphics Creation PLOT_TO_PS, 2011, online: https://people.sc.fsu.edu/~jburkardt/f_src/plot_to_ps/plot_to_ps.html)

- Spectral Element Method as alternative approach?

Spectral Element Method (SEM) – I

- Polynomial approximation of function

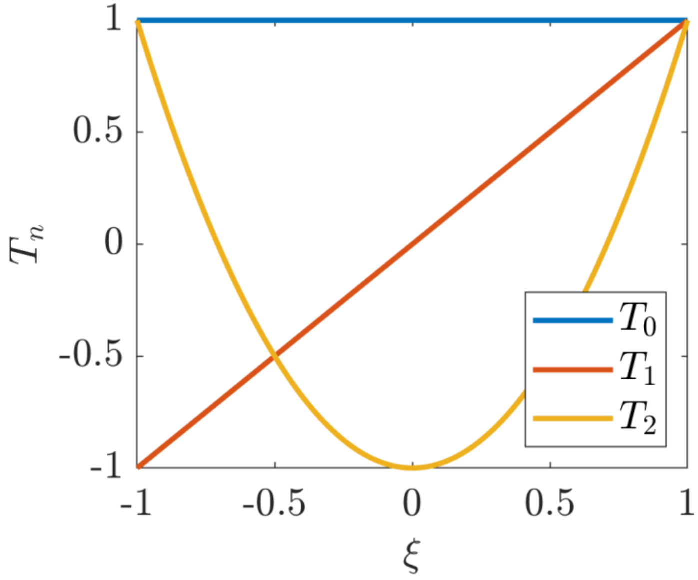
$$f(\xi) \approx \sum_{n=0}^{N} \widetilde{f}_n \, \xi^n$$

- Chebyshev-polynomials T_n :

$$T_0 = 1, \qquad T_1 = \xi,$$
$$T_2 = 2\,\xi^2 - 1, \ldots$$

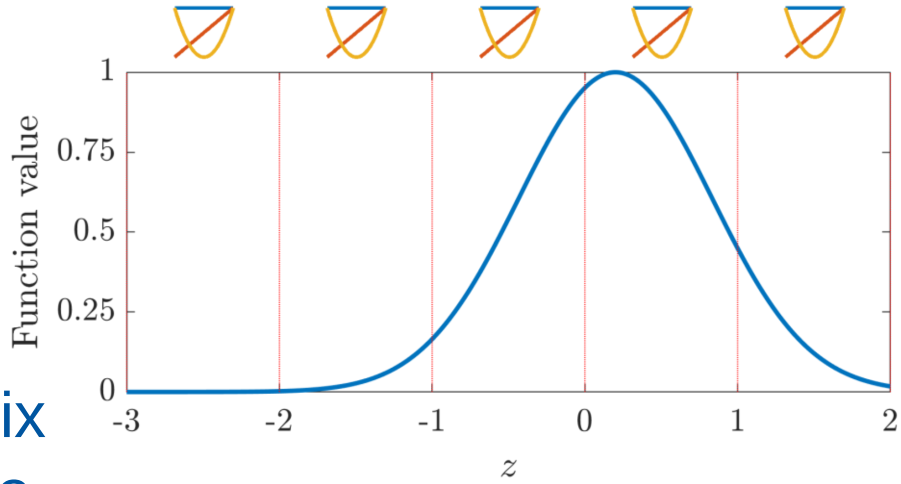
  - ➢ Orthogonality

$$\int_{-1}^{1} T_n T_m \, \omega_T \, d\xi = c_n \delta_{n,m}$$



Spectral Element Method (SEM) – II

- Discretization of space with mesh and polynomials



- Discretized PDE as matrix equation for element wise representation

$$\theta^e \approx \sum_{n=0}^{N} u_n^e \, T_n$$

(heat conduction PDE)

$$-\nabla \, (\, \lambda \, \nabla \, \theta \,) + c_v \, \partial_t \theta = j^2 \, \rho$$

$$\Downarrow$$

$$K_\lambda \, u + M_{c_v} \, \partial_t \, u = q$$

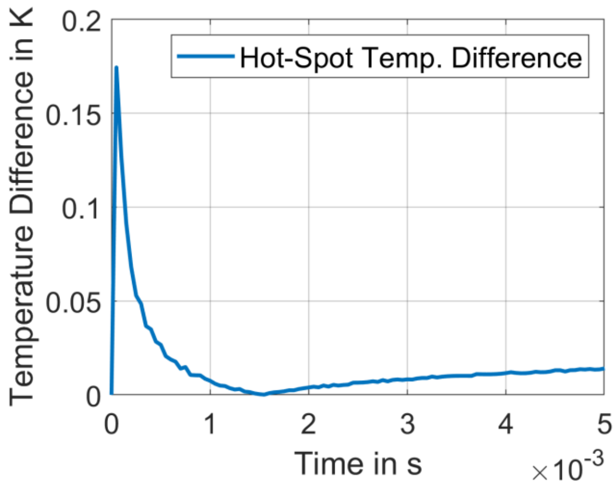
$$\Downarrow$$

$$A \, u = b$$

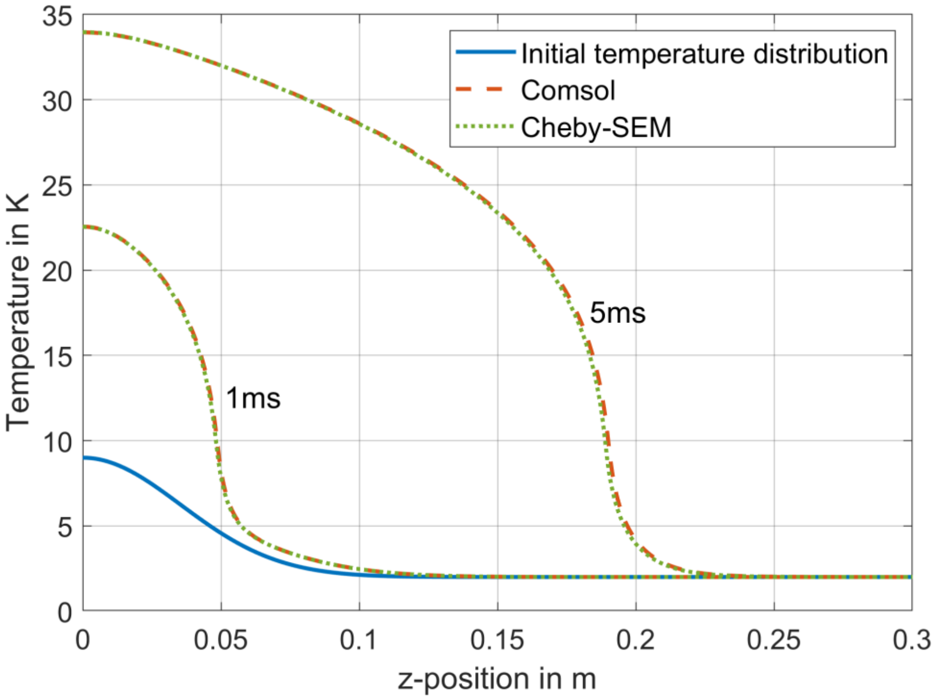
➢ Sparse mesh, high order polynomials

Benchmark: Proof of Concept

- 1D adiabatic thermal quench propagation in simplified LTS cable
- Cheby-SEM in Matlab vs. Comsol



- ➢ Results in good accordance
- ➢ Proof of functionality



Benchmark: Proof of Concept

	Solver	Runtime	# DoF
Comsol	FEM, standard fine mesh	33 s	12.000
Comsol	FEM, adaptive mesh	15 s	400 – 500
Matlab	SEM, adaptive polynomial order	19 s	< 200

➢ More general:

Pro FEM

- Multi-purpose tool
- Steep changes
- Inhomogeneous materials

Pro SEM:

- Specialized tool
- Accuracy
- Less storage requirements
- Simple refinement

Implementation Notes

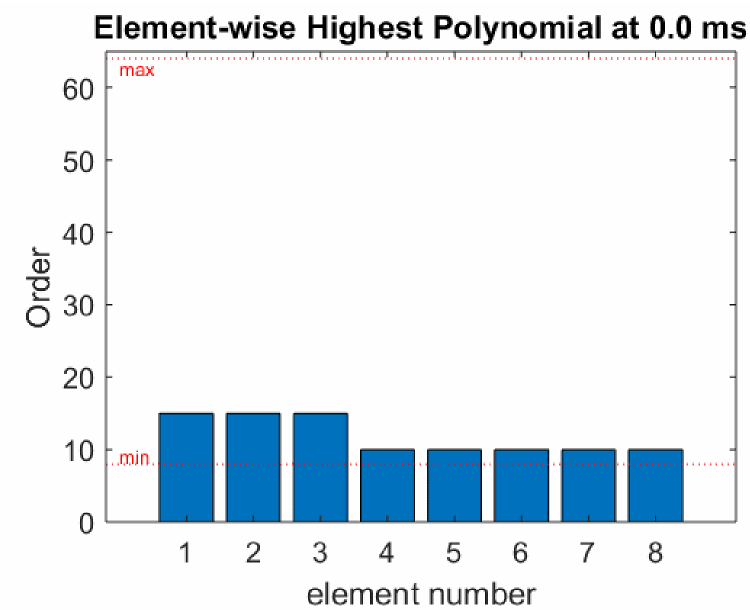
What it is:

- Matlab scripts
- Object-oriented:
  - 2 main classes
  - 15 methods
  - 30 basic unit and function tests
- Simulation driver:
  - Basic: ~ 50 LOC
  - Framework: ~ 150 LOC
  - Postprocessing: ~ 300 LOC

\+ ~500 LOC in classes

Framework example:

- Resolution adaption over time reflecting quench front propagation



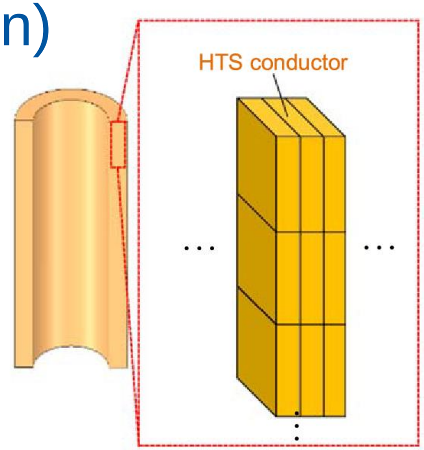
Summary

- Cheby-SEM and necessary framework has been implemented in Matlab
- Implementation has been validated for an academic example against FEM

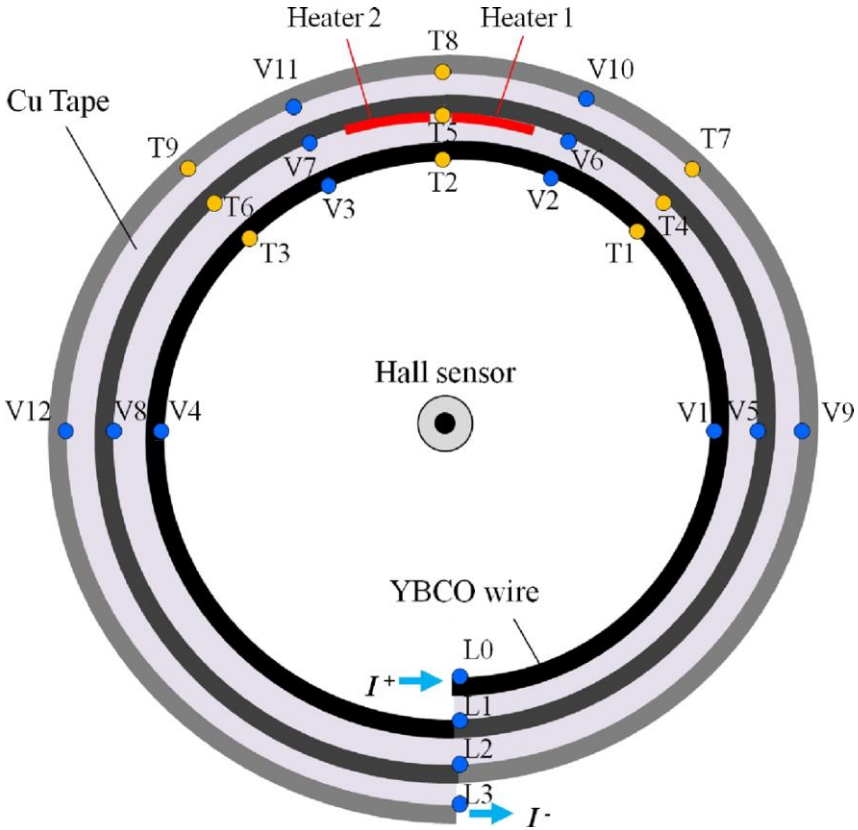
- Clear advantages of SEM compared to FEM for quench propagation are shown:
  1. Simple refinement
     - Obtain desired accuracy
  2. Less storage
     - Cheaper application to larger geometries
  3. Local resolution
     - Easy adaption to quench front

What’s next? – Background I

- Non-insulated (NI) HTS coils
  - Wounded tapes
  - Solenoid
  - Quench tolerant (Self protection)



(Picture taken from Y Suetomi, 2019, https://doi.org/10.1088/1361-6668/ab016e)



(Picture taken from Seok Beom Kim, 2012, https://doi.org/10.1109/TASC.2011.2174559)

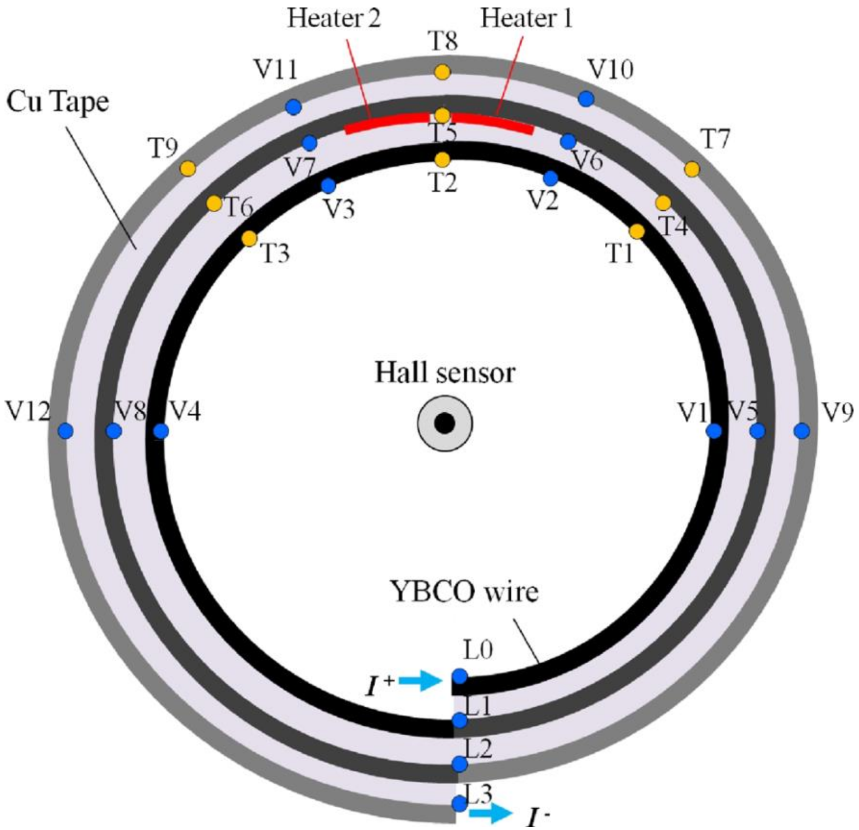
What's next? – Background II

- Non-insulated (NI) HTS coils
  - Wounded tapes
  - Solenoid
  - Quench tolerant (Self protection)
- Planned application in fusion technology (cmp. e.g. tokamak energy)
- Application in accelerator technology?

(Picture taken from tokamak energy, WAM-HTS presentation, 2019, https://indico.cern.ch/event/775529/contributions/3334053/attachments/1829923/3003215/20190412_GB_Stability_and_quench_dynamic_behaviour_of_Tokamak_Energy_REBCO_QA_coils_Indico.pdf#search=van%20nugteren%20AND%20EventID%3A775529)

What’s next? - Task

- Simulation of HTS tape peak temperature during quench
  - 1D simplified model
  - Current sharing btw. super- and normal-conducting domains
  - Equivalent resistance
- Mid-term:
  - Coolant (1D + 1D)
  - Turn-to-turn propagation



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Outlook: Numerical Aspects

- Coupling with magnetic problem?
- Advance to 3D simulation?
- Treatment of time-domain: solutions for multi-rate problem?

Only excerpt - focused on ongoing work in Darmstadt

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