Update on the Application of Spectral Element Methods on Quench Simulation

Jonas Christ
Supervision: Lorenzo Bortot
Outline

- Numerical context of quench simulation
- Spectral Element Method in a nutshell
- Proof of concept
- Implementation notes
- Outlook
Problem

- Quench propagation: Multi-Physics
  *(here: reduce to thermal problem)*

- Magnet geometry: Multi-Scale, 3D
  *(here: reduce to 1D cable simplification)*

➢ Best way to reach fast and accurate simulation?

Main challenge:
Increase in simulated cable length
and/or increase in precision requirements
⇒ High increase in computational time
Problem

Approach A:
Find reasonable simplifications or models, e.g. for Comsol

Approach B:
Optimize numerical procedure which is internally used by solving programs

➢ Best way to reach fast and accurate simulation?

Ongoing research for finding numerical procedure best suitable for quench propagation in magnets*

My contribution: Implement solver for 1D thermal problem using a non-standard numerical solution procedure - SEM
Reminder: Excerpt of Standard Simulation Workflow

PDE & model

Discretization method

Discretizing PDE as algebraic (matrix) equation

Solving

Postprocessing: physical interpretation

Temperature $\theta(z)$

build matrix system

$Au = b$

find unknown $u$

plot simulated $\theta(z)$
Discretization Method

Most wide-spread:
Finite Element Method (FEM)

- Discretize space by mesh nodes $z_i$
- Identify unknowns with temperature at nodes, i.e. $\theta(z_i) = u_i$
- Define matrix entries by dense mesh, low order polynomials

- Arbitrary geometries
- Refinement comp. expensive

Spectral Element Method as alternative approach?

(Picture taken from John Burkardt, PostScript Graphics Creation PLOT_TO_PS, 2011, online: https://people.sc.fsu.edu/~jburkardt/f_src/plot_to_ps/plot_to_ps.html)
Spectral Element Method (SEM) – I

- Polynomial approximation of function
  \[ f(\xi) \approx \sum_{n=0}^{N} \tilde{f}_n \xi^n \]

- Chebyshev-polynomials \( T_n \):
  \[ T_0 = 1, \quad T_1 = \xi, \]
  \[ T_2 = 2 \xi^2 - 1, \ldots \]

  ➢ Orthogonality
  \[ \int_{-1}^{1} T_n T_m \omega_T \, d\xi = c_n \delta_{n,m} \]
Spectral Element Method (SEM) – II

- Discretization of space with mesh and polynomials
- Discretized PDE as matrix equation for element wise representation

\[ \theta^e \approx \sum_{n=0}^{N} u^e_n T_n \]

- Sparse mesh, high order polynomials
Benchmark: Proof of Concept

- 1D adiabatic thermal quench propagation in simplified LTS cable
- Cheby-SEM in Matlab vs. Comsol

➤ Results in good accordance
➤ Proof of functionality
Benchmark: Proof of Concept

<table>
<thead>
<tr>
<th>Solver</th>
<th>Runtime</th>
<th># DoF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comsol FEM, standard fine mesh</td>
<td>33 s</td>
<td>12,000</td>
</tr>
<tr>
<td>Comsol FEM, adaptive mesh</td>
<td>15 s</td>
<td>400 – 500</td>
</tr>
<tr>
<td>Matlab SEM, adaptive polynomial order</td>
<td>19 s</td>
<td>&lt; 200</td>
</tr>
</tbody>
</table>

More general:

Pro FEM
- Multi-purpose tool
- Steep changes
- Inhomogeneous materials

Pro SEM:
- Specialized tool
- Accuracy
- Less storage requirements
- Simple refinement
Implementation Notes

What it is:
- Matlab scripts
- Object-oriented:
  - 2 main classes
  - 15 methods
  - 30 basic unit and function tests
- Simulation driver:
  - Basic: ~ 50 LOC
  - Framework: ~ 150 LOC
  - Postprocessing: ~ 300 LOC
  + ~500 LOC in classes

Framework example:
- Resolution adaption over time reflecting quench front propagation
Summary

• Cheby-SEM and necessary framework has been implemented in Matlab
• Implementation has been validated for an academic example against FEM

• Clear advantages of SEM compared to FEM for quench propagation are shown:
  1. Simple refinement
     ➢ Obtain desired accuracy
  2. Less storage
     ➢ Cheaper application to larger geometries
  3. Local resolution
     ➢ Easy adaption to quench front
What’s next? – Background I

- Non-insulated (NI) HTS coils
- Wounded tapes
- Solenoid
- Quench tolerant (Self protection)

(Picture taken from Y Suetomi, 2019, https://doi.org/10.1088/1361-6668/ab016e)

(Picture taken from Seok Beom Kim, 2012, https://doi.org/10.1109/TASC.2011.2174559)
What’s next? – Background II

- Non-insulated (NI) HTS coils
  - Wounded tapes
  - Solenoid
  - Quench tolerant (Self protection)
- Planned application in fusion technology (cmp. e.g. tokamak energy)
- Application in accelerator technology?

What’s next? - Task

- Simulation of HTS tape peak temperature during quench
  - 1D simplified model
  - Current sharing btw. super- and normal-conducting domains
  - Equivalent resistance
- Mid-term:
  - Coolant (1D + 1D)
  - Turn-to-turn propagation

(Picture taken from Seok Beom Kim, 2012, https://doi.org/10.1109/TASC.2011.2174559)
Outlook: Numerical Aspects

• Coupling with magnetic problem?
• Advance to 3D simulation?
• Treatment of time-domain: solutions for multi-rate problem?

Only excerpt - focused on ongoing work in Darmstadt