EFT and ~~Higgs~~ fundamental physics

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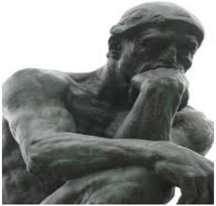

there's not much to say in 20 min

HC 2019, Oxford, 2 October 2019

Despite the striking fact that a large number of scientists are working , the vast stretches of the unknown and the unanswered and the unfinished still far outstrip our collective comprehension.

OK, what do we do in the meantime?

Friends, lend me your ears I come to praise EFT, not a model (or approximation), but a sequence of low-energy effective actions $\mathrm{S}_{\mathsf{eff}}(\Lambda)$, for all $\Lambda < \infty$, from SMEFT to GRSMEFT *

> The problem is not how to imagine wild scenarios, the problem is how to arrive to the correct scenario by making only small steps, without having to make unreasonable assumptions.

*A theory is aimed at a generalized statement aimed at explaining a phenomenon. A model, on the other hand, is a purposeful representation of reality more.

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How to proceed? (For fits see M&F's talks or move to Argonne)

① A step at the time (no desire to diminish the role), some with interesting results (Brivio:2019myy). Example of important questions: some SMEFT couplings (e.g. $\mathrm{H}\gamma\gamma$) formally arise at tree-level together with the leading corrections to the $\mathrm{HZZ}, \mathrm{HWW}$ couplings; is there any ***realistic*** SM-extension where this happens? $\mathscr{O}(150)$ loops?

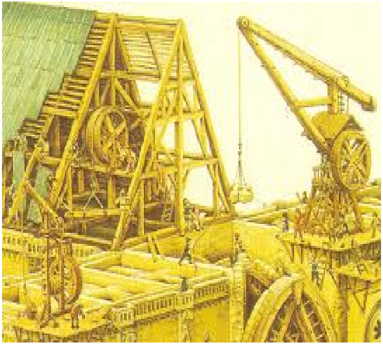
② Move up the timetable (***ab initio***), pay special attention to loops, mixing (i.e. proliferation of scalars), $\mathrm{dim} = 8$, interpretation and uncertainties introduced by going observable(s) $\to$ EFT coeff $\to$ BSM. (e.g. EFT-NLO introduces "large uncertainties" into the EFT-LO fit, not currently taken into consideration)

step ① is top priority, we are on the right track ; we should answer the "*Still missing: something we can all agree upon to use for general Higgs measurements*" cry. However, we don't want to

Miss The Forest For The Trees


I do not make any warranties about the completeness of this information

- Henning:2014wua, delAguila:2016zcb, Ellis:2016enq, Fuentes-Martin:2016uol, Ellis:2017jns, Donoghue:2017pgk, Henning:2016lyp, Gorbahn:2015gxa, Wells:2017aoy, Gabelmann:2018axh, Einhorn:2013kja, Gripaios:2015qya, Jiang:2016czg, Buchalla:2016bse, Henning:2017fpj, Brivio:2017vri, Barzinji:2018xvu, Criado:2018sdb, Hays:2018zze, Helset:2018dht, Gripaios:2018zrz, Jiang:2018pbd, Quevillon:2018mfl, Bakshi:2018ics, Criado:2019ugp, Brehmer:2015rna, Biekotter:2016ecg, Degrande:2016dqg, Boggia:2016asg, Boggia:2017hyq, Alte:2017pme,2018xgc, Ruhdorfer:2019qmk, Dawson:2019clf, Donoghue:2019fcb, Remmen:2019cyz, Corbett:2019cwl, Durieux:2019eor
- Brivio:2019myy, Durieux:2019lnv (CERN-LPCC-2019-2)



When people say ***by construction the SMEFT is always a valid QFT*** what do they mean?

Among other things, we need to make the SMEFT S-matrix UV (and IR) finite, including $\mathrm{dim} = 6$ operators and, at least, $\mathrm{dim} = 8$ operators (truncation uncertainty). The verification of any **claim** with **explicit computations** is of importance.

We will not discuss IR/coll, unitarity, stability and Ostrogradsky ghosts. We will not discuss EFT gauge anomalies and anomaly cancellation; perhaps, a deeper understanding of SMEFT is required, is SMEFT a low-energy limit of an underlying anomaly-free theory?

of any EFT ***theory*** (the length of the shortest description of the theory) where UV infinities must be absorbed into couplings/masses and (mixing of) Wilson coefficients; what is the connection between UV poles and symmetry?

- Use BFM

$$
\begin{aligned}
\mathscr{L}(\phi_{\rm c}+\phi) &= \mathscr{L}(\phi_{\rm c}) + \phi_i\,\mathscr{L}'_i(\phi_{\rm c}) + \frac{1}{2}\,\partial_\mu\,\phi_i\,\partial_\mu\,\phi_i \\
&+ \phi_i\,{\rm N}^{\mu}_{ij}\,\partial_\mu\,\phi_j + \frac{1}{2}\,\phi_i\,{\rm M}_{ij}(\phi_{\rm c})\,\phi_j + \mathscr{O}(\phi^3) + \text{total derivative}
\end{aligned}
$$

($\mathscr{L}'_i(\phi_{\rm c}) = 0$). All one loop diagrams are generated by $\mathscr{L}_2(\phi)$, the part quadratic in ϕ.

$$
\mathscr{L}_2(\phi) \quad \to \quad -\frac{1}{2}\,\left(\partial_\mu\,\phi\right)^2 + \phi\,{\rm N}^{\mu}\,\partial_\mu\,\phi + \frac{1}{2}\,\phi\,{\rm M}\,\phi
$$

$$
\begin{aligned}
\underset{\text{counter}\mathscr{L}}{\Delta\mathscr{L}} &= \frac{1}{8\,\pi^2\,(d-4)}\Big[a_0\,M^2 + a_1\,\left(\partial_\mu\,{\rm N}_\nu\right)^2 + a_2\,\left(\partial_\mu\,{\rm N}_\mu\right)^2 + a_3\,{\rm M}\,{\rm N}^2 \\
&+ a_4\,{\rm N}_\mu\,{\rm N}_\nu\,\partial_\mu\,{\rm N}_\nu + a_5\,\left({\rm N}^2\right)^2 + a_6\,\left({\rm N}_\mu\,{\rm N}_\nu\right)^2\Big]
\end{aligned}
$$

- However, define $X = \mathrm{M} - \mathrm{N}^{\mu}\,\mathrm{N}_{\mu}$ and see that $\mathscr{L}$ is invariant under (H) 't Hooft transformation, Λ antisymmetric

$$\phi' = \phi + \Lambda\,\phi \quad \mathrm{N}'_{\mu} = \mathrm{N}_{\mu} - \partial_{\mu}\Lambda + \left[\Lambda, \mathrm{N}_{\mu}\right] \quad X' = X + \left[\Lambda, X\right]$$

- Therefore $\Delta\mathscr{L}$ also will be invariant ($\mathrm{Tr}\,X$ is invariant)

$$\Delta\mathscr{L} = \frac{1}{\varepsilon}\,\mathrm{Tr}\left(a\,X^2 + b\,Y^{\mu\nu}\,Y_{\mu\nu}\right)$$

from 7 to 2 CTs

$$Y_{\mu\nu} = \partial_{\mu}\mathrm{N}_{\nu} - \partial_{\nu}\mathrm{N}_{\mu} + \mathrm{N}_{\mu}\,\mathrm{N}_{\nu} - \mathrm{N}_{\nu}\,\mathrm{N}_{\mu} \quad \text{transforms as } X$$

- An EFT (e.g. SMEFT) including $\mathrm{dim} = 6, 8$ operators will have

$$\tfrac{1}{2}\,\partial_{\mu}\,\phi_i \;\; \mathrm{g}^{\mu\nu}_{ij}(\phi_{\mathrm{c}}) \;\; \partial_{\nu}\,\phi_j$$

matrix-valued metric tensor

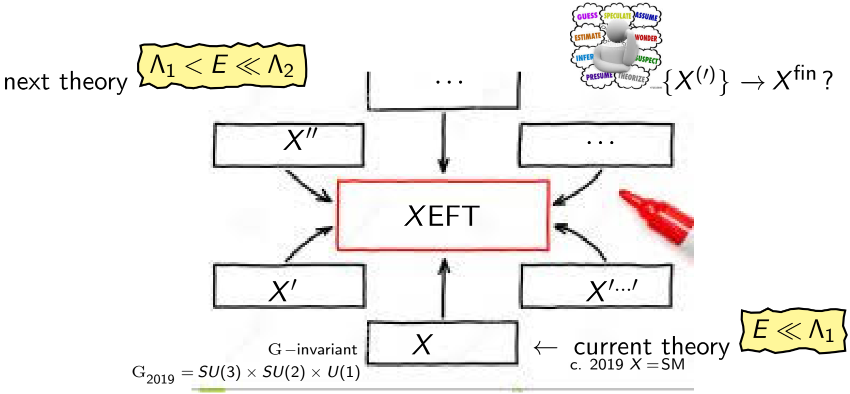
- $\mathrm{g} \mapsto$ corresponding Riemann tensors i.e. more invariants for $\Delta\mathscr{L}$, i.e. EFT is computationally more complex than QGR; the name of the game is to have the full $\Delta\mathscr{L}$, not the CTs for one/two processes.

Remark If $\mathscr{L}$ is (G) -invariant then the relation between G and (H) is crucial in proving closure under renormalization (not the same as strict renormalizability)

The X tree...

X' etc are UV completions of X or the next theory in a tower of theories

Scenarios: $\mathrm{Rep}_{\mathrm{G}} \supset$ heavy dof (including more scalars?) $\subset X'$ or X' is F-invariant and $\mathrm{G} \subset \mathrm{F}$



One of the next steps will be a concentric attack

$X' \to X\mathrm{EFT} \leftarrow X$ up to one loop, including combination of uncertainties

placing strong TH priors (causality, unitarity, etc.) on the parameter space of the SMEFT? ***controversial***

outline

① Mixing or SMEFTtools vs. SMEFTinterpretation vs. non-linear EFT

② Local, non-local, hard, soft and all that or why you should not forget ***loopy*** EFT †

③ Linear vs. quadratic EFT representation

} systematics . . . needed

Not covered

Do not expect something on different operator bases/conventions, the game is over. Observables related to production XS? If you do it right POs $\supset$ EFT, full stop. What is measurement, what is interpretation/intuitive meaning and how far they are is highly subjective. Correlations among observables should not bias measurements (***Gimme another talk*** or see my talk at HEFT2016).

† N.B. from *NONOtoNLO* of YR4 to *EFT structure becomes manifest at NLO* of LHCHXSWG, 10/12/18, i.e. Nemo propheta in patria

Mixing? Who ordered that?

- assume that there simply is no proliferation of Higgs bosons in nature (SM is particularly special)

Antithesis

- dark matter, inflation, flavor, generically predict that there should be many more particles (dozens of more Higgs bosons)

Example : singlet extension of the SM (RxSM)

weakly-coupled regime (linear EFT)

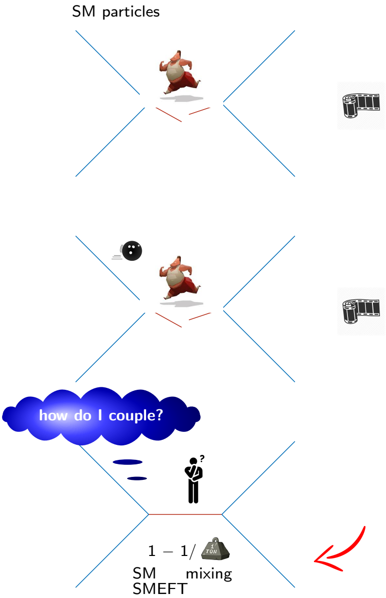
$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} h_2 + \sqrt{2}\,v + i\,\phi^0 \\ \sqrt{2}\,i\,\phi^- \end{pmatrix}$$

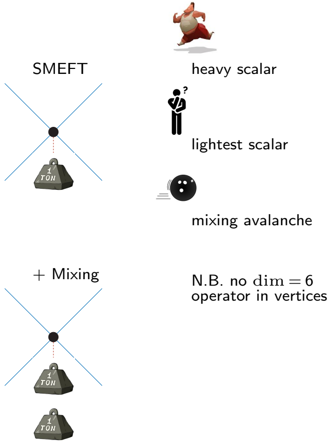
$$\chi = 1/\sqrt{2}\,(h_1 + v_s)$$

$$\Phi = \overbrace{\Big[\text{SM-like}\ \Phi_{h_2 \to h}\Big]}^{\text{not an SU(2) doublet}} + \frac{1}{\sqrt{2}}\Big[(\cos\alpha - 1)\,\overset{\text{light}}{h} + \sin\alpha\,\overset{\text{heavy}}{H}\Big]\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\sin\alpha \sim \frac{1}{\Lambda} \qquad \Lambda \equiv M_s = \frac{1}{2}\,g\,v_s \neq M_H$$

- X' at $E \ll \Lambda$ should be computed in the mass eigenbasis, not in the weak eigenbasis ($\mapsto$ mixing angles $= f(\Lambda)$) ; e.g. in LEFT the W/Z fields are integrated out, *not the $SU(2)$ fields*. This means parts of linear multiplets are integrated out, other states retained.
- A large number of $1/\Lambda^2$ terms comes **from** the expansion of mixing angles, **not from** the integration of heavy fields ($\Rightarrow$ dim $= 8$).





Simplified : decoupling cannot be obtained in terms of only one large scale and can only be achieved by imposing further assumptions on the couplings

Understanding $\mathscr{L}_{\text{RxSM}}$

- Working at $\mathcal{O}(1/\Lambda^2)$ we can split the total Lagrangian into

$$\mathscr{L}_{\mathrm{H}=0} \overset{\rightarrow}{\text{generates}} \mathscr{L}_{\mathrm{SM}}(\mathrm{h}) + \sum_{n=0,2} \Lambda^{2n-2}\,\delta\mathscr{L}_{6-2n}$$

$$\mathscr{L}_{\mathrm{H}} \overset{\rightarrow}{\text{generates}} \mathscr{L}^{\text{TG}} + \mathscr{L}^{\text{LG}} + \mathscr{L}^{\text{tad}}$$

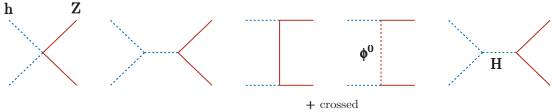
- The sum is due to the expansion of $\sin\alpha(\cos\alpha)$ in terms of Λ.
- After integrating out H only the SM-like operators acquire Wilson coefficients that are not Λ-suppressed (Appelquist-Carazzone theorem). tad poles, $\overline{\mathrm{MS}}$ and on-shell are a tricky business

- $\mathrm{h}_{1,2} \rightarrow \mathrm{h}, \mathrm{H}$

Example:

$$\mathcal{O}\underbrace{\left(\mathrm{D}_\mu\Phi\right)^\dagger\left(\mathrm{D}_\mu\Phi\right)}_{\mathrm{dim}=4} \rightarrow \mathcal{O}\Big[\text{SM-like}_{\mathrm{h}} - \sqrt{2}\,\sin^2\frac{\alpha}{2}\, g^{\mathrm{SM}}_{\mathrm{h}_2\mathrm{VV}}\underbrace{\mathrm{h\,V\,V}}_{1/\Lambda^2}$$

$$+ \frac{1}{\sqrt{2}}\sin\alpha\, g^{\mathrm{SM}}_{\mathrm{h}_2\mathrm{VV}}\,\mathrm{H\,V\,V} \rightarrow \underbrace{\text{EFTzation}}_{1/\Lambda^4}\Big]$$



$$\Downarrow$$

TG

RxSM color map

$$\sin\alpha = \lambda_2 \frac{M}{M_s} \qquad \cos\alpha = 1 - \frac{1}{2}\left(\lambda_2 \frac{M}{M_s}\right)^2 \qquad M_H^2 = \lambda_1 \left(1 + \lambda_2^2 \frac{M^2}{M_s^2}\right) M_s^2$$

$$A_{\mu\nu} = \left(1 - 2\,\lambda_2^2 \frac{M^2}{M_s^2}\right) A_{\mu\nu}^{MSM} \qquad T_{\mu\nu}^{sh} = M_Z^2\,\delta_{\mu\nu} + p_{1\mu}\,p_{2\nu}$$

$$+ \frac{g^2}{c_W^2}\,\lambda_2^2 \frac{M^2}{M_s^2}\left[\frac{1}{2}\left(1_{TG} - 1\right)\delta_{\mu\nu} + \frac{T_{\mu\nu}^{sh}}{t - M_Z^2} + + \frac{T_{\nu\mu}^{sh}}{u - M_Z^2}\right]$$

❍ SM, one-doublet, Mixing, TG, $M_s^2 = 1/4\,g^2\,(\text{singlet VEV})^2$, depending on RxSM parameters

$$A^{\rm SMEFT}_{\mu\nu} = \Bigl[1 + \frac{1}{3\sqrt{2}\,G_F\,\Lambda^2}\,(6\,a_{\phi W} - a_{\phi D} + 10\,a_{\phi\Box})\Bigr]\,A^{\rm MSM}_{\mu\nu}$$

$$+ \quad \frac{1}{\sqrt{2}\,G_F\,\Lambda^2}\,\frac{g^2}{c^2_W}\,\Bigl[F_1\,\delta_{\mu\nu} + F_2\,T^{sh}_{\mu\nu} + F_3\,T^{sh}_{\nu\mu} + F_4\,T^{sZ}_{\mu\nu} + F_5\,T^{t}_{\mu\nu} + F_6\,T^{u}_{\mu\nu}\Bigr]$$

$$F_1 = 12\,\frac{M^2}{s - M^2_h}\,a_\phi + \frac{1}{4}\,\frac{s}{s - M^2_h}\,(a_{\phi D} - 4\,a_{\phi\Box}) - \frac{1}{6}\,(7\,a_{\phi D} - 4\,a_{\phi\Box})$$

$$F_2 = \frac{1}{6}\,\frac{1}{t - M^2_Z}\,(5\,a_{\phi D} - 8\,a_{\phi\Box}) \qquad F_3 = \frac{1}{6}\,\frac{1}{u - M^2_Z}\,(5\,a_{\phi D} - 8\,a_{\phi\Box})$$

$$F_4 = \frac{1}{M^2_Z}\,\Bigl(3\,\frac{M^2_h}{s - M^2_h} + 1\Bigr)\,a_{ZZ} \qquad F_5 = \frac{2}{t - M^2_Z}\,a_{ZZ} \qquad F_6 = \frac{2}{u - M^2_Z}\,a_{ZZ}$$

$$T^{sZ}_{\mu\nu} = p_{3\mu}\,p_{4\nu} + \Bigl(\frac{1}{2}\,s - M^2_Z\Bigr)\,\delta_{\mu\nu}$$

$$T^{t}_{\mu\nu} = (M^2_h - M^2_Z - t)\,\delta_{\mu\nu} - p_{1\mu}\,p_{4\nu} - p_{2\nu}\,p_{3\mu}$$

$$T^{u}_{\mu\nu} = (M^2_h - M^2_Z - u)\,\delta_{\mu\nu} - p_{1\nu}\,p_{3\mu} - p_{2\mu}\,p_{4\nu}$$

compare with SMEFT...

you can twist SMEFT for few processes only

extra terms not reproduced even in higher orders in $1/M_s$ $\Rightarrow$ $a_{ZZ} = 0$ etc. but a_{ZZ} controls hZZ

non-linear EFT? Compare one h-leg with two h-legs

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Enough variety of directions in EFT that allows it to cover other scenarios? Can any BSM model in nature be caught by using the EFT? ***"Covering" means that I have enough directions in any EFT to fit nearly everything but the underlying assumptions (one doublet, degenerate heavy scales, PTG/LG etc.) could affect the interpretation***.

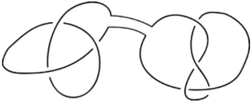
Can any SM-deviation be caught by using an EFT with many assumptions? ***Process-by-process Yes***
Will that lead me to what nature has in mind? ***No, but it will help***

But we need to find the deviations first. Then we have a lot of time to worry about their correct interpretation, no? ***No, do not create an EFT from UV models with wrong dictionaries, nevertheless SMEFT is a mandatory step***

Once again, consider the following scenario: SM , valid for $\mathrm{E} \ll \Lambda$, the corresponding EFT extension (say *SMEFT*) and *nextSM* , some UV completion of the SM (or the next theory in a tower of effective theories); we are interested in the low E limit of nextSM (at the loop level). We obtain:

- Non-local effects correspond to long distance propagation and hence to reliable predictions at low energy,
- local terms by contrast summarize the unknown effects from high energy:

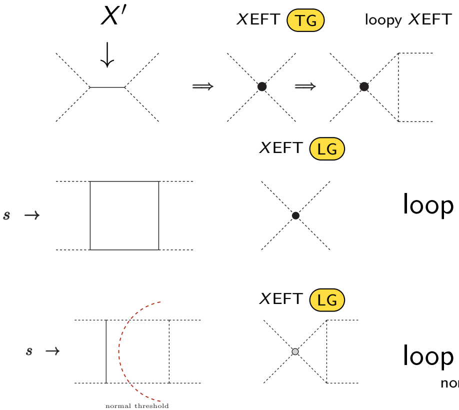
→ having both local and non-local terms allows us to implement the full (one-loop) EFT program.



$X' \to X\text{EFT}$

TG = tree generated

LG = loop generated



tree → loop

loop / local

loop / non-local

normal threshold

Different from: IF the collision probe does not reach the heavy mass THEN observable's dependence on that scale is simplified

The upshot of this is that

- $\mathscr{L}_{\rm EFT}$ mimics the unknown UV by matching the hard-local part of the loops, i.e. the terms having a bounded number of derivatives.
- soft-non local components in loops cancel on both sides of the (loopy) matching condition but they are not a throwaway.
- N.B. we could also introduce a non-local in space, one-loop effective, $\mathscr{L} = \int d^{\rm d}x\, d^{\rm d}y\, \phi(x)\, \mathrm{L}(x-y)\, \phi(y),\ \mathrm{L}(z) = \mathscr{F}\{\ln(p^2)\}$

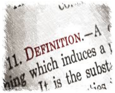
▷ Diagrams of X' with light external legs and heavy internal ones admit a local low-energy limit.

▷ Diagrams of X' with light external legs and mixed internal legs may show normal-threshold singularities in the low-energy region and give inherently non-local parts.

$$\text{non-local} \quad \Leftarrow \quad \Lambda^2 \gg s_{ij\dots k} = -(p_i + p_j + \cdots + p_k)^2 > (m_1 + m_2 + \cdots + m_n)^2 \text{ or } \Lambda^2 \gg \mid t \mid \gg m^2$$

Therefore The key advantage of including the non-local behavior is the appearance of some important kinematic dependence (important for tails of distributions)


INTERPRETATION
EXTENSION
ARBITRARY
CONSTANT
GENERALITY
STANDARD
REPRESENT
NATURAL
LETTER
FUNCTION
CONCEPTS
INFORMATION
EQUALITY
RELATION
DISCOURSE
SYNTACTIC
DEFINABLE



Higher order compensation of redundant operators

(see also last item on slide 25 and backup slide 41)

- Removing a redundant $\mathscr{O}^{(6)}$ with a Wilson coefficient $\mathrm{a}^6_{\mathrm{R}}$ will propagate $\mathrm{a}^6_{\mathrm{R}}$ into the Wilson coefficients of $\dim = 8$ operators.
- In the bottom-up approach it does not matter since we only "measure" combinations of Wilson coefficients, linear in the a^8_i and quadratic in $\mathrm{a}^6_{\mathrm{R}}$ ‡. **Thus, the shift due to the field redefinition can be absorbed into the coefficients of operators that are already present in the theory**.
- However, the low energy limit of X' may contain some $\mathscr{O}^{(8)}_{\mathrm{a}}$ as well as some $\mathscr{O}^{(6)}_{R}$ whose $\dim = 8$ compensations contain $\mathscr{O}^{(8)}_{\mathrm{a}}$; $\mathrm{a}^8_{\mathrm{a}}$ is now computable in terms of the parameters of X' **but what we have "measured" at low energy is not** $\mathrm{a}^8_{\mathrm{a}}$ (wrong eq. $\not\equiv$ systematics).

$$\begin{array}{ccc} X' & \longrightarrow & \mathscr{O}^{(8)}_{\mathrm{a}} \\ \downarrow & \nearrow & \\ \mathscr{O}^{(6)}_{R} & \longrightarrow & \mathscr{O}^{(6)}_{\mathrm{b}}, \mathscr{O}^{(6)}_{\mathrm{c}} \ldots \end{array}$$

‡ Indeed, when constructing the original EFT, one must include all possible operators consistent with the symmetries at every order in the $1/\Lambda$ expansion.

uncertainties arising in EFT $\mapsto$ underlying theory

① Experiments $\overset{\text{recommended}}{\overbrace{\Longrightarrow}}$ $\vec{\mathrm{a}}_{\mathrm{SMEFT}} \equiv$ fitted SMEFT Wilson coeff.

Remark If low energy measurements are not sensitive to a subset of operators, there would be a null result which could be interpreted as the impossibility of uncover the corresponding heavy sector while a new set of measurements could very well do it. Ex: mass-degenerated vector-like fermions with opposite hypercharge $\mapsto \mathscr{O}_{\mathrm{WB}}$ not generated, $\mathscr{O}_{\mathrm{W}} = \mathscr{O}\left(g^3/16\,\pi^2\right)$, i.e. measure even numbers of B-legs, i.e. find orthogonal directions, expand later the analysis

② How to ***compare*** with the low-energy limit of a theory (X) with mixing?

- Eventually fit (directly) $\vec{\mathrm{p}}_X$ ($\forall X$?) 
- Model independent parametrization of mixing 

Remark at high Q^2 there are $\dim = 8$ parameters with a greater impact than $\dim = 6$ parameters. Measurements of $\dim = 6$ parameters **will be different** if $\dim = 8$ is neglected, at least one should treat them as nuisance parameters and integrate over them to obtain a (truncation) uncertainty.

with A. David, *SMEFT − interpretation* $(\{X \in \mathrm{S} \mid X_{\mathrm{E} \ll \Lambda} \sim \mathrm{SMEFT}\})$ vs. *SMEFT − bookkeeping* $(X \notin \mathrm{S})$

In case deviations are observed, one needs to compare at the observable level (O) when interpreting the $X \notin \mathrm{S}$ parameters. Comparison at the O level implies:

① In X, with parameters $\vec{p}$, compute an observable $\mathrm{O}^i = \mathrm{O}^i_X(\vec{p})$.

② Perform a fit of the SMEFT coefficients $(\vec{\mathrm{a}})$ to a set of observables $(\vec{\mathrm{O}})$, that may but does not need to include O^i.

③ Take $(\hat{\vec{\mathrm{a}}})$ from ② and compute the SMEFT-predicted observable, $\hat{\mathrm{O}}^i{}_{\mathrm{SMEFT}} = \mathrm{O}^i_{\mathrm{SMEFT}}(\hat{\vec{a}})$, match with $X \in S$

④ Or perform a fit for the $X \notin S$ parameters $\vec{p}$ from the comparison to the SMEFT-predicted observable: $\hat{\mathrm{O}}^i{}_{\mathrm{SMEFT}} \sim \mathrm{O}^i_X(\vec{p})$. **Include systematics** (exact determination of the $1\,\sigma$ interval vs. EFT assumptions).

The result of a global SMEFT fit may yield $\hat{\vec{a}} \sim 0$. This can *only* be interpreted as no deviations from SM under the SMEFT assumptions . It is possible that the effects in the set observables used is such that the result *seems* to be null. This can come about via an averaging effect, with some observables pulling Wilson coefficients in opposite directions.

Proposition

You cannot construct S*-matrix elements at* $\mathscr{O}\left(1/\Lambda^4\right)$ *using a canonically transformed* $\mathscr{L}$ *truncated at* $\mathscr{O}\left(1/\Lambda^2\right)$*. At higher orders canonical normalization modifies the shape of differential distributions, not only their normalization.*

Linear vs. quadratic representation

- To summarize, the proper definition of quadraticEFT proceeds as follows: given a "truncated" Lagrangian

$$\mathscr{L} \quad = \quad \mathscr{L}^{(4)} + \frac{1}{\Lambda^2}\,\mathscr{L}^{(6)} + \frac{1}{\Lambda^4}\,\mathscr{L}^{(8)}$$

- we distinguish between redundant and non-redundant operators (select between $\mathscr{O}_i^{(n)} - \mathscr{O}_j^{(n)} = \mathrm{F}(\phi)\,\delta\mathscr{L}/\delta\phi$);

$$\mathscr{L}^{(6,8)} \quad = \quad \mathscr{L}^{(6,8)}_{\mathrm{NR}} + \sum_{i\in\mathrm{R}} \mathscr{O}_i^{(6,8)}\,\frac{\delta\mathscr{L}^{(4)}}{\delta\phi}$$

- redefine fields according to

$$\phi \quad \to \quad \phi - \sum_{n=2,4} \frac{1}{\Lambda^n} \sum_{i\in\mathrm{R}} \mathscr{O}_i^{(n+4)}$$

→ The corresponding shift in $\mathscr{L}$ will eliminate redundant operators leaving a (neglected) term

$$\Delta\mathscr{L} \quad = \quad -\frac{1}{\Lambda^4}\Big[\frac{\delta\mathscr{L}^{(4)}}{\delta\phi}\sum_{i\in\mathrm{R}}\mathscr{O}_i^{(8)} + \frac{\delta\mathscr{L}^{(6)}}{\delta\phi}\sum_{i\in\mathrm{R}}\mathscr{O}_i^{(6)} + \frac{1}{2}\,\frac{\delta^2\mathscr{L}^{(4)}}{\delta\phi^2}\sum_{i,j\in\mathrm{R}}\mathscr{O}_i^{(6)}\,\mathscr{O}_j^{(6)}\Big]$$

- We could do without elimination (overcomplete basis), indeed the S-matrix cannot distinguish between two equivalent operators $(\mathscr{O}, \mathscr{O}')$. Quadratic then means the full $\mathscr{O}\left(1/\Lambda^4\right)$ S-matrix. Different story is if we ask whether or not the underlying theory can generate $\mathscr{O}'$.

Linear vs. quadratic representation

- Once again, $\Delta\mathscr{L}$ will never generate terms that are not present in $\mathscr{L}^{(8)}$ (symmetry). however, we will see a difference when interpreting "fitted" Wilson coefficients in terms of the low-energy behavior of some X'. Furthermore, assembling the used terms

$$
\begin{aligned}
\mathscr{L} &= -\frac{1}{2} Z_\phi^{ij}\, \partial_\mu \phi_i \,\partial_\mu \phi_j - \frac{1}{2} Z_m^{ij}\, \phi_i \phi_j + \mathscr{L}_{\text{rest}} \\
Z_\phi^{ij} &= \delta^{ij} + \frac{1}{\Lambda^2}\,\delta Z_\phi^{(6)\,;\,ij} + \frac{1}{\Lambda^4}\,\delta Z_\phi^{(8)\,;\,ij} \\
Z_m^{ij} &= m_i^2\,\delta^{ij} + \frac{1}{\Lambda^2}\,\delta Z_m^{(6)\,;\,ij} + \frac{1}{\Lambda^4}\,\delta Z_m^{(8)\,;\,ij}
\end{aligned}
$$

- We rescale fields and masses (and possibly couplings) in order to reestablish canonical normalization.
  - ▷ This additional transformation will affect $\mathscr{L}_{\text{rest}}$
- Actually, this is not the end of the story since we have to link the Lagrangian parameters to a given set of experimental data.
  - ▷ These relations will, once again, change $\mathscr{L}_{\text{rest}}$

Linear vs. quadratic representation

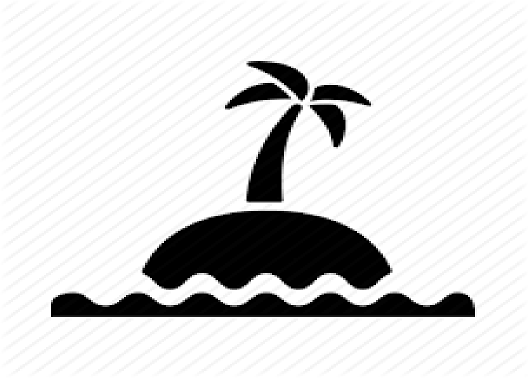
- Furthermore, a given $\mathrm{A}_{\mathrm{tree}}$ containing terms up to $\mathscr{O}\left(1/\Lambda^4\right)$ has single and double insertions of $\dim = 6$ operators in the tree diagrams $\in \mathscr{L}^{(4)}$ (plus set of diagrams having new structures, $\notin \mathscr{L}^{(4)}$).
- Once we have the Lagrangian (up to $\mathscr{O}\left(1/\Lambda^4\right)$) we can obtain Feynman rules and amplitudes. Given

$$\mathrm{A} \quad = \quad \mathrm{A}^{(4)} + \frac{1}{\Lambda^2}\mathrm{A}^{(6)} + \frac{1}{\Lambda^4}\mathrm{A}^{(8)}$$

- **linear** means including the interference between $\mathrm{A}^{(4)}$ and $\mathrm{A}^{(6)}$,
- **quadratic** "currently" means including the square of $\mathrm{A}^{(6)}$ and **Not**
- the **complete inclusion** of all terms giving $1/\Lambda^4$ (before considering $\mathrm{A}^{(8)}$).

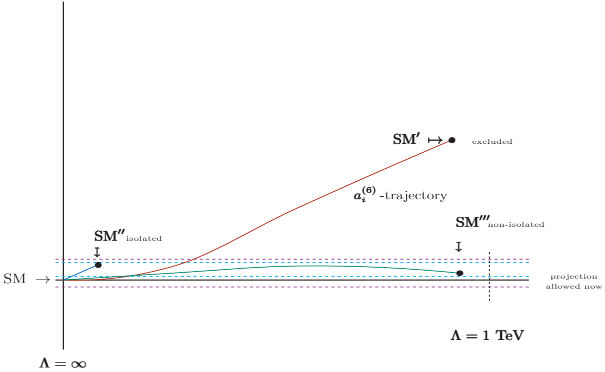
N.B. heavy-light turn on $\mathrm{Im}\,\mathrm{A}^{(6)}$, i.e. π^2 terms in "quadratic"

EOI: towards a descendant of the blue -band?

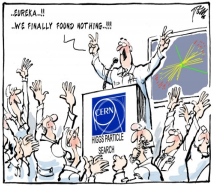


Jon Butterworth: ***If nothing beyond the SM shows up at the LHC, we will in a sense have established that the SM is isolated.***

SM extensions with heavy new particles that couple to any SM particle with $\mathcal{O}(< 1)$ couplings mapped into SMEFT(Λ)



Higgs boson was the last piece of 20th century particle physics. Approach based on mathematical consistency. LHC has found nothing else but many key questions remain to be answered



(SM)EFT, we know what we have to do No one said it would be easy

Those who ignore history are condemned to repeat it; don't sell the sizzle, sell the steak! Ths can insure that no progress is achieved if they quarrel or are indifferent. But we must be clear that "if Exs go smeft, we will come".

We choose to go beyond the SM straits. We choose to do that and do the other things, not because they are easy, but because they are hard, because that goal will serve to organize and measure the best of our energies and skills, because that challenge is one that we are willing to accept, one we are unwilling to postpone, and one which we intend to win, and the others, too.

For alternative views *If we don't see a deviation that's a much, much bigger gauntlet thrown down at the feet of theorists to try to figure out what is happening*. In any case, *The questions raised by the Higgs discovery go to the heart of our understanding of space-time and QM* see `https://www.math.columbia.edu/~woit/wordpress/?p=11312`

Thank you for your attention

More than

for $Mixing$

$\bar{q}q \to HZ$

There are 9 Wilson coeff. 3 of them are LG (divided by $16\,\pi^2$)

RxSM At LO there is a simple rescaling of SM predictions

SMEFT **left** $a_{AZ}(\text{LG}) = -1$, rest of Wilson coeff. is 0.

right $a_{\phi dV}(\text{TG}) = 1$, rest of Wilson coeff. is -1.

$p_\perp$[GeV]	L/SM	Q/SM
10	−0.0259	−0.0239
59	−0.0259	−0.0239
108	−0.0260	−0.0240
157	−0.0261	−0.0240
206	−0.0262	−0.0241
255	−0.0263	−0.0242
304	−0.0263	−0.0242
353	−0.0264	−0.0243
402	−0.0265	−0.0243
451	−0.0265	−0.0243
500	−0.0266	−0.0244

$p_\perp$[GeV]	L/SM	Q/SM
10	−0.098	−0.083
59	−0.124	−0.099
108	−0.188	−0.129
157	−0.288	−0.149
206	−0.425	−0.127
255	−0.601	−0.017
304	−0.814	+0.235
353	−1.064	+0.692
402	−1.352	+1.426
451	−1.677	+2.518
500	−2.040	+4.056

linear is negative →

quadratic exploding

for ***Expansion***

$$\mathscr{L}_{\rm SM} - \frac{1}{2}\,\mathrm{M}_s^2\,\mathrm{S}^2 + \mu_s\,\Phi^\dagger\Phi\mathrm{S}$$

expansions, loopy EFT

- $p_i^2 = -\mathrm{M}_{\rm H}^2$ and $s = -(p_1+p_2)^2$.

$$\mathrm{I} \;=\; \mu_{\rm R}^{\varepsilon}\int d^{\rm d}q\,\frac{1}{(q^2+\mathrm{M}_{\rm H}^2)((q+p_1)^2+\mathrm{M}_s^2)((q+p_1+p_2)^2+\mathrm{M}_{\rm H}^2)}$$

① $$\rightarrow \quad \frac{1}{(q+p_1)^2+\mathrm{M}_s^2} = \frac{1}{\mathrm{M}_s^2}\,(\overbrace{1}^{\rm dim=4} - \overbrace{\frac{(q+p_1)^2}{\mathrm{M}_s^2}}^{\rm dim=6} + \dots)$$

$$\mathrm{I} \;\sim\; \frac{i\pi^2}{\mathrm{M}_s^2}\,(\underbrace{\frac{1}{\bar{\varepsilon}} - \ln\frac{\mathrm{M}_{\rm H}^2}{\mu_{\rm R}^2}}_{+\text{EFT C.T.}} + \underbrace{2-\beta\,\ln\frac{\beta+1}{\beta-1}}_{\text{soft}} + \dots)$$

$$\underbrace{\mathrm{M}_s^2 \gg |\,q^2\,| \sim |\,p_i^2\,|}_{\text{cancels out in the matching}}$$

② $$\rightarrow \quad \frac{1}{(q+p_1)^2+\mathrm{M}_s^2} = \frac{1}{q^2+\mathrm{M}_s^2}\,(1 - \frac{p_1^2+2\,p_1\cdot q}{q^2+\mathrm{M}_s^2} + \dots)$$ respects UV at one loop

$$\mathrm{I} \;\sim\; \frac{i\pi^2}{\mathrm{M}_s^2}\,(\underbrace{1+\ln\frac{\mathrm{M}_s^2}{\mathrm{M}_{\rm H}^2} - \beta\,\ln\frac{\beta+1}{\beta-1}}_{\text{soft + hard}} + \dots)$$

$$|\,q^2\,| \sim \mathrm{M}_s^2 \gg |\,p_i^2\,|$$

à la Mellin-Barnes $$\rightarrow \quad \mathrm{I} \sim \frac{i\pi^2}{\mathrm{M}_s^2}\,(1+\ln\frac{\mathrm{M}_s^2}{\mathrm{M}_{\rm H}^2} - \beta\,\ln\frac{\beta+1}{\beta-1} + \dots)$$

for *Canonical normalization*

Canonical normalization is more than "normalization"

- field normalization, $\mathrm{H} \to \left[1 - \frac{1}{4}\,\frac{M^2}{\Lambda^2}\,(\mathrm{a}_{\phi\mathrm{D}} - 4\,\mathrm{a}_{\phi\Box})\right]\mathrm{H}$
- Process, $\overline{\mathrm{u}}(x_1\,p_1) + \mathrm{u}(x_2\,p_2) \to \mathrm{H}(-p_{\mathrm{H}}) + \mathrm{Z}(-p_{\mathrm{Z}})$
- Invariants, $\hat{s} = -2\,x_1 x_2\,p_1 \cdot p_2$ and $\hat{t} = 2\,x_1\,p_1 \cdot p_{\mathrm{Z}} + \mathrm{M}_{\mathrm{Z}}^2$
- Look for $\mathrm{a}_{\phi\Box}$ effects:

$$\sum_{\mathrm{spin}} |\,\mathrm{A}\,|^2 = \frac{3}{4}\,\frac{g^4\,\mathrm{v}_{\mathrm{u}}^2}{\mathrm{c}_{\mathrm{W}}^4}\,\mathrm{M}_{\mathrm{Z}}^2\,\hat{s}\,|\,\Delta_{\mathrm{Z}}\,|^2\,(1(=\text{LO SM}) + 2\,\frac{g_6}{\sqrt{2}}\,\mathrm{a}_{\phi\Box}\,)$$

$$+ \frac{g^4}{\mathrm{c}_{\mathrm{W}}^4}\,g_6^2\,\mathrm{a}_{\phi\Box}\,\left[\mathrm{A}_2(\hat{s},\hat{t})\,|\,\Delta_{\mathrm{Z}}\,|^2 + \mathrm{A}_1(\hat{s},\hat{t})\,\mathrm{Re}\,\Delta_{\mathrm{Z}}\,)\right]$$

$$\mathrm{v}_{\mathrm{u}} = 1 - \frac{8}{3}\,\mathrm{s}_{\mathrm{W}}^2 \qquad \Delta_{\mathrm{Z}} = \frac{1}{\hat{s} - \mathrm{M}_{\mathrm{Z}}^2} \qquad g_6 = \frac{1}{\sqrt{2}\,G_{\mathrm{F}}\,\Lambda^2}$$

- At $\mathscr{O}\left(1/\Lambda^2\right)$ the Wilson coefficient $\mathrm{a}_{\phi\Box}$ modifies the normalization of the $\hat{s}$ distribution; at $\mathscr{O}\left(1/\Lambda^4\right)$ the shape of the $\hat{t}$-distribution is modified.

for ***field redefinition***

The two facets of field redefinition

$$\mathscr{L} = -\frac{1}{2} Z_\phi \partial_\mu \phi \partial_\mu \phi + \mathscr{L}_{\rm rest}$$

$$Z_\phi = 1 + \sum_{n=2,4} \frac{1}{\Lambda^n} \delta Z_\phi^{(n+2)}$$

LSZ

$$Z_J J \frac{1}{Z_\phi (p^2+m^2)} Z_J J$$

field redefinition or source normalization is a matter of taste, the crucial point is residue $=1$. Be careful beyond LO, this is not a massless theory.

① Work directly with the S-matrix, e.g. with on-shell methods. A basis for $\mathscr{O}^{(6,8)}$ is not strictly needed, i.e. it is equally reasonable not to eliminate $\mathscr{O}_R$ (overcomplete basis). Explicit absence of $\mathscr{O}$ redundancies, the $\dim = 8$ $S_{\rm SMEFT}$-matrix will contain terms linear in $\dim = 8$ Wilson coefficients and quadratic in the $\dim = 6$ ones.

② Select a basis through field redefinition $\mapsto$ higher order compensations. Compute the SMEFT S-matrix.

☛ Compare observables with underlying UV X-theory, $O^i_{\rm SMEFT} \sim O^i_X(\vec{p})$. No ambiguity in both approaches (once higher order compensations are properly taken into account in ②).

You play

for $\dim = 8$

$$X \text{ is } \mathscr{L}^{(4)} = -\frac{1}{2}\,\partial_\mu\phi\,\partial_\mu\phi - \frac{1}{2}\,m^2\phi^2 - \frac{1}{4\,!}\,g\phi^4$$

X' gives

$$\mathscr{L}^{(6)} = \frac{1}{\Lambda^2}\left[g^4\,\mathrm{a}_0^6\,\phi^6 + \mathrm{a}_1^6\,\phi\,\Box^2\phi + g^2\,\mathrm{a}_2^6\,\phi^3\,\Box\phi\right]$$

$$\mathscr{L}^{(8)} = \frac{1}{\Lambda^4}\Big\{g^6\,\mathrm{a}_0^8\,\phi^8 + g^4\,\mathrm{a}_1^8\,\phi^5\,\Box\phi + \mathrm{a}_2^8\,(\Box\phi)\,\Box^2\phi$$

$$+ \quad g^2\left[\mathrm{a}_3^8\,\phi^3\,\Box^2\phi + \mathrm{a}_4^8\,\phi^2\,(\partial_\mu\partial_\nu\phi)\,(\partial_\mu\partial_\nu\phi) + \mathrm{a}_5^8\,\phi^2\,(\Box\phi)^2\right]\Big\}$$

Not the same basis used for fits $\mapsto$ eliminate all the operators containing $\Box^n\phi$

$$\mathscr{L} = \ldots + \frac{g^2}{\Lambda^4}\,\phi^2\,(\partial_\mu\partial_\nu\phi)^2\left[\mathrm{a}_4^8 + 6\,\mathrm{a}_3^8 + \mathrm{a}_2^8 - 9\,\mathrm{a}_1^6\,\mathrm{a}_2^6 - 2\,(\mathrm{a}_1^6)^2\right]$$

- In fitting the data we constrain the combinations of coefficients appearing in $\mathscr{L}$; after that the Wilson coefficients are the pseudo-data
- When interpreting the results we should remember that the coefficient of $\phi^2 (\partial_\mu \partial_\nu \phi)^2$ is not a_4^8, etc.

  $\therefore$ caution should be used in constructing the coefficients in the $\dim = 8$ part of the basis if we want to extract the parameters of the high-energy theory from the pseudo-data

for ***Non-local***

$X =$ **sigma-model** from Donoghue:2017pgk

- Low-energy behavior of $\mathrm{A}_{\mathrm{full}}(\pi^+\pi^0 \to \pi^+\pi^0)$

$$\mathrm{A}_{\mathrm{full}} \mapsto \frac{t}{\mathrm{v}^2} + \frac{1}{\mathrm{v}^4}\,\mathrm{Polynomial}(s\,,t\,,u) - \frac{1}{96\,\pi^2\,\mathrm{v}^4}\left[3\,t^2\ln\frac{-t}{\mathrm{M}_\sigma^2} + s\,(s-u)\ln\frac{-s}{\mathrm{M}_\sigma^2} + u\,(u-s)\ln\frac{-u}{\mathrm{M}_\sigma^2}\right]$$

- $\mathrm{A}_{\mathrm{EFT}}$ computed using $\Sigma_{\mu\nu} = \partial_\mu \mathrm{U}\,\partial_\nu \mathrm{U}^\dagger$ and

$$\mathscr{L}_{\mathrm{EFT}} = \frac{\mathrm{v}^2}{4}\,\mathrm{Tr}\,\Sigma^\mu_\mu + \mathrm{a}_1\left(\mathrm{Tr}\,\Sigma^\mu_\mu\right)^2 + \mathrm{a}_2\left(\mathrm{Tr}\,\Sigma_{\mu\nu}\right)^2$$

- Match "full" and EFT, obtained by including one-loop bubbles (loopy EFT);

$$\mathrm{A}_{\mathrm{EFT}} = \frac{t}{\mathrm{v}^2} + \frac{1}{\mathrm{v}^4}\,\mathrm{Polynomial}(s\,,t\,,u\,;\mathrm{a}_1\,,\mathrm{a}_2) - \frac{1}{96\,\pi^2\,\mathrm{v}^4}\left[3\,t^2\ln\frac{-t}{\mathrm{M}_\sigma^2} + s\,(s-u)\ln\frac{-s}{\mathrm{M}_\sigma^2} + u\,(u-s)\ln\frac{-u}{\mathrm{M}_\sigma^2}\right]$$

sigma-model Cont'd

- derive (renormalized) Wilson coefficients

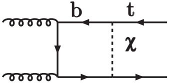
$$a_1 = \frac{1}{8}\frac{v^2}{M_\sigma^2} + \frac{1}{384\,\pi^2}\left(\ln\frac{M_\sigma^2}{\mu_R^2} - \frac{35}{6}\right) \qquad a_2 = \frac{1}{192\,\pi^2}\left(\ln\frac{M_\sigma^2}{\mu_R^2} - \frac{11}{6}\right)$$

- Compare with the tree-level matching and conclude that we have taken into account an important kinematic feature,

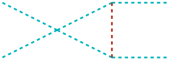
  the logarithmic dependence upon the characteristic momentum transfer in the problem

  $\mathscr{L}_{\mathrm{EFT}}$ and A_{EFT} have a different meaning;
  the Lagrangian is local (as it should), the amplitude generates long-distance kinematic logarithms. However, it is a question of language: nothing prevents us from introducing a one-loop, effective , non-local $\mathscr{L}$ including all processes up to a given order.

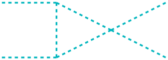
A more difficult case: CxSM



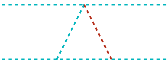
a



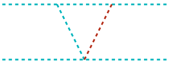
b



c



d



e

$\mathrm{gg} \to \overline{\mathrm{t}}\,\mathrm{t}$ reducible to 4 scalars

After $1/M^2$ Mellin-Barnes expansion

χ-heavy line

a,c) generate $\mathrm{Li}_2(\beta_-^{-1}), \mathrm{Li}_2(\beta_+^{-1})$

$$\beta_\pm = \frac{1}{2}\,(1 \pm \beta) \quad \beta^2 = 1 - \frac{4\,m_\mathrm{b}^2}{s}$$

b) generates $\ln \frac{m_\mathrm{b}^2}{\mu_\mathrm{R}^2}\,, \beta \ln \frac{\beta+1}{\beta-1}$

d,e) generate $\ln \frac{m_\mathrm{b}^2}{\mu_\mathrm{R}^2}$

But $\mathrm{gg} \to \overline{\mathrm{b}}\,\mathrm{b}$ is non-local only for $s > 4\,m_\mathrm{t}^2$

for $\boldsymbol{EFT\ representations}$

Linear vs. Quadratic

$$\mathscr{L} = -\frac{1}{2}\,(1 + \frac{M^2}{\Lambda^2}\,\delta Z_H^6 + \boxed{\frac{M^4}{\Lambda^4}\,\delta Z_H^8}\,)\,\partial_\mu H\,\partial_\mu H + \dots + \frac{1}{\Lambda^2}\Big[\overbrace{a\,M^3\,H\,Z_\mu\,Z_\mu}^{\text{pick at random}} + \dots\Big] + \boxed{\frac{1}{\Lambda^4}\sum_i a_i^8\,\mathscr{O}_i^{(8)}}$$

where the frame box indicates that the terms are not available.

$$H = (1 + \frac{M^2}{\Lambda^2}\,\eta_H^6 + \frac{M^4}{\Lambda^4}\,\eta_H^8)\,\hat{H},$$

$$\eta_H^6 = -\frac{1}{2}\,\delta Z_H^6, \qquad \eta_H^8 = \frac{3}{8}\,[\delta Z_H^6]^2,$$

$$\hat{\mathscr{L}} = -\frac{1}{2}\,\partial_\mu \hat{H}\,\partial_\mu \hat{H} + a\,\frac{M^3}{\Lambda^2}\,(1 - \frac{1}{2}\,\frac{M^2}{\Lambda^2}\,\delta Z_H^6\,)\,\hat{H}\,Z_\mu\,Z_\mu + \dots$$

where the round box gives terms that are neglected in the "naive" quadratic approach.

for *gauge invariance*

RxSM gauge transformation

$$h \to h + \frac{1}{2} g \cos\alpha \left(\frac{\Gamma^Z}{c_W} \phi^0 + \Gamma^- \phi^+ + \Gamma^+ \phi^- \right)$$

$$H \to H + \frac{1}{2} g \sin\alpha \left(\frac{\Gamma^Z}{c_W} \phi^0 + \Gamma^- \phi^+ + \Gamma^+ \phi^- \right)$$

$$\phi^0 \to \phi^0 - \frac{1}{2} g \frac{\Gamma_Z}{c_W} \left(\cos\alpha\, h + \sin\alpha\, H + 2 \frac{M_W}{g} \right) + \frac{i}{2} g \left(\Gamma^- \phi^+ - \Gamma^+ \phi^- \right)$$

$$\phi^- \to \phi^- - \frac{1}{2} g \Gamma^- \left(\cos\alpha\, h + \sin\alpha\, H + 2 \frac{M_W}{g} + i \phi^0 \right)$$

$$+ \frac{i}{2} g \left[(c_W^2 - s_W^2) \frac{\Gamma_Z}{c_W} + 2 s_W \Gamma^A \right] \phi^-$$

RxSM gauge transformation Cont'd

$$\Phi^{\dagger}_{h} = (h, \phi^0, \phi^-, \phi^+) \qquad \Gamma^{\dagger} = (0, \frac{\Gamma_Z}{c_W}, \Gamma^-, \Gamma^+)$$

$$t_1 = \begin{pmatrix} \Gamma^-\Gamma^+ & \\ 0 & 0 \end{pmatrix} \qquad t_2 = \begin{pmatrix} \Gamma^- & \Gamma^+ \\ i\,\Gamma^- & -i\,\Gamma^+ \end{pmatrix}$$

$$T_d = \begin{pmatrix} i\,\frac{\Gamma_Z}{c_W}\,\tau_2 & t_2 \\ -t_2^{\dagger} & -i\,X\,\tau_3 \end{pmatrix} \qquad T_{nd} = \begin{pmatrix} i\,\frac{\Gamma_Z}{c_W}\,\tau_2 & t_1 \\ -t_1^{\dagger} & 0 \end{pmatrix}$$

RxSM gauge transformation Cont'd

$$X = (c_W^2 - s_W^2)\Gamma_Z/c_W + 2 s_W \Gamma_A$$

$$\sin\alpha = \frac{\Delta_s}{\Lambda} + \mathcal{O}\left(\Lambda^{-2}\right) \qquad \cos\alpha = 1 - \frac{1}{2}\frac{\Delta_s^2}{\Lambda^2} + \mathcal{O}\left(\Lambda^{-4}\right)$$

- At $\mathcal{O}\left(1/\Lambda^2\right)$ we obtain

$$\Phi_{(h)} \rightarrow \overbrace{\Phi_{(h)} - M_W \Gamma + \frac{g}{2} T_d \Phi_{(h)}}^{\text{doublet}} - \frac{g}{4}\frac{\Delta_s^2}{\Lambda^2} T_{nd} \Phi_{(h)} - \frac{g}{2}\frac{\Delta_s}{\Lambda}\Gamma (H)$$

$$(H) \rightarrow (H) - \frac{g}{2}\frac{\Delta_s}{\Lambda}\Gamma^\dagger \Phi_{(h)}$$

$$\mathscr{L} = \mathscr{L}^{(4)} + \frac{1}{\Lambda^2}\mathscr{L}^{(6)} + \frac{1}{\Lambda^4}\mathscr{L}^{(8)} \quad \text{truncated}$$

$$\phi_i \rightarrow \phi_i + \lambda_{ij}\,\phi_j \quad \mathrm{G}\text{-invariance}$$

$$\phi_i = \hat{\phi}_i + \frac{1}{\Lambda^2}\eta^{(6)}_{ij}\,\hat{\phi}_j + \frac{1}{\Lambda^4}\eta^{(8)}_{ij}\,\hat{\phi}_j \quad \text{can. norm.}$$

$$\mathscr{L} \rightarrow \hat{\mathscr{L}} \quad \text{truncated}$$

shifted invariance up to $\mathscr{O}\left(1/\Lambda^2\right)$ $\mathscr{O}\left(1/\Lambda^4\right)$

$$\hat{\phi}_i \rightarrow \hat{\phi}_i + \hat{\lambda}_{ij}\,\hat{\phi}_j \quad \text{order-by-order shifted } \mathrm{G}\text{-invariance}$$

Abelian Higgs model at arbitrary dim

$$
\begin{aligned}
\mathscr{L}_2 \;=\;& -\frac{1}{2} X_{\mathrm{H}}\, \partial_\mu \mathrm{H}\, \partial_\mu \mathrm{H} - \frac{1}{2} X_\chi\, \partial_\mu \chi\, \partial_\mu \chi - \frac{1}{4} X_{\mathrm{A}}\, \mathrm{F}_{\mu\nu}\, \mathrm{F}_{\mu\nu} + M\, X_t\, \mathrm{A}_\mu\, \partial_\mu \chi \\
& - \frac{1}{2} X_{M_{\mathrm{H}}}\, M_{\mathrm{H}}^2\, \mathrm{H}^2 - \frac{1}{2} X_{\mathrm{M}}\, M^2 \chi^2 - \frac{1}{2} X_{M_{\mathrm{A}}}\, M^2\, \mathrm{A}_\mu\, \mathrm{A}_\mu \\
& - \frac{M}{g} (\beta_h + X_\beta)\, \mathrm{H} - \frac{1}{2} \beta_h\, (\mathrm{H}^2 + \chi^2)
\end{aligned}
$$

- X_{M} and X_β start at $\mathscr{O}\left(1/\Lambda^2\right)$ while ther remaining X_i start at $\mathscr{O}(1)$

$$
\mathrm{H} = \mathrm{Z}_{\mathrm{H}}^{1/2}\, \hat{\mathrm{H}} \qquad \chi = \mathrm{Z}_\chi^{1/2}\, \hat{\chi} \qquad \mathrm{A}_\mu = \mathrm{Z}_{\mathrm{A}}^{1/2}\, \hat{\mathrm{A}}_\mu
$$

$$
M = \mathrm{Z}_{\mathrm{M}}\, \hat{M} \qquad M_{\mathrm{H}} = \mathrm{Z}_{M_{\mathrm{H}}}\, \hat{M}_{\mathrm{H}} \qquad g = \mathrm{Z}_g\, \hat{g}
$$

- Furthermore, $\beta_h = \mathrm{Z}_\beta\, \hat{\beta}_h + \Delta\beta$.

- The general definition of canonical normalization is as follows: $\Delta\beta = -\hat{M}^2\, X_{\rm M}\, {\rm Z}_{\rm M}^2$ and

$$
{\rm Z}_{\rm H} = X_{\rm H}^{-2} \qquad {\rm Z}_{\chi} = X_{\chi}^{-2} \qquad {\rm Z}_{\rm A} = X_{\rm A}^{-2}
$$

$$
{\rm Z}_{\rm M}^2 = \frac{X_{\rm A}}{X_{M_{\rm A}}} \qquad {\rm Z}_{M_{\rm H}}^2 = \Big(\frac{\hat{M}^2}{\hat{M}_{\rm H}^2}\, \frac{X_{\rm M}}{X_{M_{\rm A}}}\, X_{\rm A} + X_{\rm H}\Big)\, X_{M_{\rm H}}^{-1}
$$

- the shifted-transformation law for the fields can be written as:

$$
\begin{aligned}
\hat{\rm H} &\to \hat{\rm H} - \alpha\,\Big(\frac{{\rm Z}_{\chi}}{{\rm Z}_{\rm H}}\Big)^{1/2} {\rm Z}_g\, \hat{g}\, \hat{\chi} \\
\hat{\chi} &\to \hat{\chi} + \alpha\,\Big[\Big(\frac{{\rm Z}_{\rm H}}{{\rm Z}_{\chi}}\Big)^{1/2} {\rm Z}_g\, \hat{g} + \frac{{\rm Z}_{\rm M}}{{\rm Z}_{\chi}^{1/2}}\Big]\, \hat{\rm H} \\
\hat{\rm A}_{\mu} &\to \hat{\rm A}_{\mu} + {\rm Z}_{\rm A}^{-1/2}\, \partial_{\mu}\alpha
\end{aligned}
$$

for *multi heavy scales*

Two scales scenario: $m \ll M_i$ but $0 < | M_1^2 - M_2^2 | \ll M_1^2 + M_2^2$

$$\mathscr{L} = -\frac{1}{2}\partial_\mu\phi\,\partial_\mu\phi - \frac{1}{2}m^2\phi^2 + \mathscr{L}_{\rm int}(\phi) - \frac{1}{2}\sum_{i=1,2}(\partial_\mu\chi_i\,\partial_\mu\chi_i + M_i^2\chi_i^2)$$
$$- \phi^2(\lambda_1\chi_1^2 + \lambda_2\chi_2^2) - \lambda_{12}\chi_1^2\chi_2^2 - \lambda_3\chi_1^4 - \lambda_4\chi_2^4 - \lambda_5\phi^2\chi_1\chi_2$$

Let $M_\pm^2 = 1/2(M_1^2 \pm M_2^2)$, $Y_3 = \tau_3 Y$, and

$$Y = -\begin{pmatrix} 2\lambda_1 & \lambda_5 \\ \lambda_5 & 2\lambda_2 \end{pmatrix}\phi^2$$

$$\mathscr{L}{\rm eff} = \frac{1}{32\pi^2}\sum_{n=0}^{\infty}(-1)^n 2^{n-4}(M_+^2)^{2-n}\,{\rm a}_n\,{\rm Tr}\,\mathscr{O}_n$$

where $\mathscr{O}_0 = 1$ and

$$\mathscr{O}_1 = -Y \quad \mathscr{O}_2 = \frac{1}{2}Y^2 - M_-^2\,Y_3$$
$$\mathscr{O}_3 = -\frac{1}{3!}Y^3 + \frac{1}{12}\partial_\mu Y\,\partial_\mu Y + \frac{1}{2}M_-^2\,Y_3\,Y \;\ldots$$

Ad Libitum

- The price one has to pay is that EFTs are only valid in a limited domain. This prompts the important question whether there is a last fundamental theory in this tower of EFTs which supersede each other with rising energies. Some people conjecture that this deeper theory could be a string theory, i.e., a theory which is not a field theory any more. Or should one ultimately expect from physics theories that they are only valid as approximations and in a limited domain? Hartmann (2001) and Castellani (2002) discuss the fate of reductionism vis-à-vis EFTs.

  Kuhlmann, Meinard, "Quantum Field Theory", The Stanford Encyclopedia of Philosophy (Winter 2018 Edition), Edward N. Zalta (ed.)

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