



The Higgs Potential, Naturalness, and Oblique Corrections

Higgs Couplings
Oxford
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Stating a Well-Posed Question

To understand the origin and nature of the Higgs boson, we need to study how it behaves.

$$\begin{aligned}
 \mathcal{O}_T &= \frac{c_T}{2M^2} (H^\dagger \overleftrightarrow{D}^\mu H)^2 & \mathcal{O}_W &= \frac{ig c_W}{2M^2} (H^\dagger \sigma^a \overleftrightarrow{D}^\mu H) D^\nu W_{\mu\nu}^a & \mathcal{O}_{2B} &= -\frac{c_{2B}}{4M^2} (\partial_\rho B_{\mu\nu})^2 \\
 \mathcal{O}_{2G} &= -\frac{c_{2G}}{4M^2} (D_\rho G_{\mu\nu}^a)^2 & \mathcal{O}_\square &= \frac{c_\square}{M^2} |\square H|^2 & \mathcal{O}_{WW} &= \frac{g^2 c_{WW}}{M^2} |H|^2 W^{a\mu\nu} W_{\mu\nu}^a \\
 \mathcal{O}_B &= \frac{ig' c_B}{2M^2} (H^\dagger \overleftrightarrow{D}^\mu H) \partial^\nu B_{\mu\nu} & \mathcal{O}_6 &= \frac{c_6}{M^2} |H|^6 & \mathcal{O}_{GG} &= \frac{g_s^2 c_{GG}}{M^2} |H|^2 G^{a,\mu\nu} G_{\mu\nu}^a \\
 \mathcal{O}_{BB} &= \frac{g'^2 c_{BB}}{M^2} |H|^2 B^{\mu\nu} B_{\mu\nu} & \mathcal{O}_H &= \frac{c_H}{2M^2} (\partial^\mu |H|^2)^2 & \mathcal{O}_R &= \frac{c_R}{M^2} |H|^2 |D^\mu H|^2 \\
 \mathcal{O}_{2W} &= -\frac{c_{2W}}{4M^2} (D_\rho W_{\mu\nu}^a)^2 & \mathcal{O}_{WB} &= \frac{gg' c_{WB}}{M^2} H^\dagger \sigma^a H B^{\mu\nu} W_{\mu\nu}^a
 \end{aligned}$$

Operators like those above capture leading effects of heavy physics beyond the standard model. Probing them could reveal origins.

Organising Thoughts

Naïve dimensional analysis:

$$[H] = [A_\mu] = \frac{1}{LC} \quad , \quad [\psi] = \frac{1}{L^{3/2}C}$$

Fields carry not only dimension of inverse length, but also inverse coupling.

Fermi Scale

Interaction: $\mathcal{L} \sim \frac{\psi^4}{\Lambda^2}$

Dimension: $[\Lambda] = [G_F^{-1/2}] = \frac{[M_W]}{[g]}$

UV-completion

Coupling

Organising the UV

Higgs Only

$[g_*^0]$

$$\mathcal{O}_\square = \frac{c_\square}{M^2} |\square H|^2$$

$[g_*^2]$

$$\mathcal{O}_H = \frac{c_H}{2M^2} (\partial^\mu |H|^2)^2$$

$[g_*^4]$

$$\mathcal{O}_6 = \frac{c_6}{M^2} |H|^6$$

$$\mathcal{O}_T = \frac{c_T}{2M^2} (H^\dagger \overleftrightarrow{D}^\mu H)^2$$

$$\frac{c_R}{c_R} |H|^2 |D^\mu H|^2$$

Any new physics interacting primarily with Higgs and gauge sectors matches, at leading order, to these operators.

$$\mathcal{O}_{2G} = -\frac{c_{2G}}{4M^2} (D_\rho G_{\mu\nu}^a)^2$$

Mixed

$$\mathcal{O}_B = \frac{ig' c_B}{2M^2} (H^\dagger \overleftrightarrow{D}^\mu H) \partial^\nu B_{\mu\nu}$$

$$\mathcal{O}_W = \frac{ig c_W}{2M^2} (H^\dagger \sigma^a \overleftrightarrow{D}^\mu H) D^\nu W_{\mu\nu}^a$$

$$\mathcal{O}_{GG} = \frac{g_s^2 c_{GG}}{M^2} |H|^2 G^{a,\mu\nu} G_{\mu\nu}^a$$

$$\mathcal{O}_{WB} = \frac{gg' c_{WB}}{M^2} H^\dagger \sigma^a H B^{\mu\nu} W_{\mu\nu}^a$$

$$\mathcal{O}_{WW} = \frac{g^2 c_{WW}}{M^2} |H|^2 W^{a\mu\nu} W_{\mu\nu}^a$$

$$\mathcal{O}_{BB} = \frac{g'^2 c_{BB}}{M^2} |H|^2 B^{\mu\nu} B_{\mu\nu}$$

Organising the UV

Higgs Only

$$\mathcal{O}_{\square} = \frac{c_{\square}}{M^2} |\square H|^2 \quad [g_*^0]$$

$$\begin{aligned} \mathcal{O}_H &= \frac{c_H}{2M^2} (\partial^\mu |H|^2)^2 \quad [g_*^2] \\ \mathcal{O}_T &= \frac{c_T}{2M^2} (H^\dagger \overleftrightarrow{D}^\mu H)^2 \\ \mathcal{O}_R &= \frac{c_R}{M^2} |H|^2 |D^\mu H|^2 \end{aligned}$$

$$\mathcal{O}_6 = \frac{c_6}{M^2} |H|^6 \quad [g_*^4]$$

Gauge Only

$$\mathcal{O}_{2G} = -\frac{c_{2G}}{4M^2} (D_\rho G_{\mu\nu}^a)^2$$

$$\mathcal{O}_{2W} = -\frac{c_{2W}}{4M^2} (D_\rho W_{\mu\nu}^a)^2$$

$$\mathcal{O}_{2B} = -\frac{c_{2B}}{4M^2} (\partial_\rho B_{\mu\nu})^2$$

Mixed

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$$\mathcal{O}_6 = \frac{c_6}{M^2} |H|^6$$

The highest
coupling-dimension
operator.

$$\mathcal{O}_\square = \frac{c_\square}{M^2} |\square H|^2$$

The lowest
coupling-dimension
Higgs-only operator.



$$\mathcal{O}_6 = \frac{c_6}{M^2} |H|^6$$

Parameterises
BSM deviations in sole
self-interaction of SM.

$$\mathcal{O}_\square = \frac{c_\square}{M^2} |\square H|^2$$

Parameterises
BSM deviations in how
the Higgs moves.



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Parameterises
BSM deviations in how
the Higgs moves.

These operators are
very special, both essentially
unexplored.



A Unique Operator

$$\mathcal{O}_6 = \frac{c_6}{M^2} |H|^6$$

is very very special, since:

$$[c_6] = C^4 \quad , \quad [\hbar] = C^{-2}$$

At one-loop we have:

$$[\hbar c_6] = C^2$$

Thus, if any other coupling enters the game, coupling dimension is too large to match any other dim-6 operator!

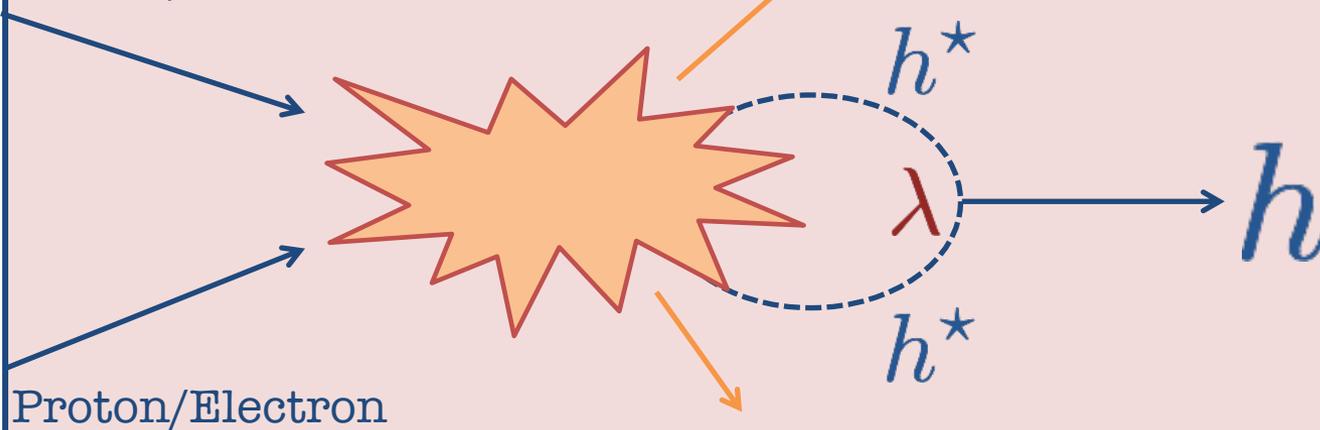
A Unique Operator

Observation:

$$\mathcal{O}_6 \xrightarrow{\text{One-loop running}} \mathcal{O}_6$$

This operator is a mountain-top in RG-space.

Proton/Electron



Proton/Electron

Insert into any one-loop diagram and no dim-6 counterterms will be required, result always finite!

A Unique Operator

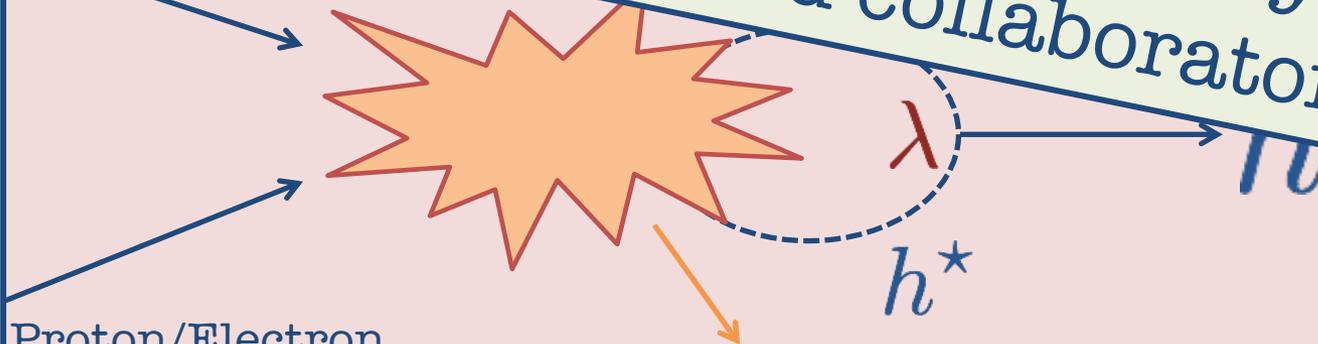
Observation:

$$\mathcal{O}_6 \xrightarrow{\text{One-loop running}} \mathcal{O}_6$$

...in-top in RG-space.

Can see where it lies in the space of Dim-6 operator RG space in papers by Jenkins, Manohar, Trott and collaborators...

Proton/

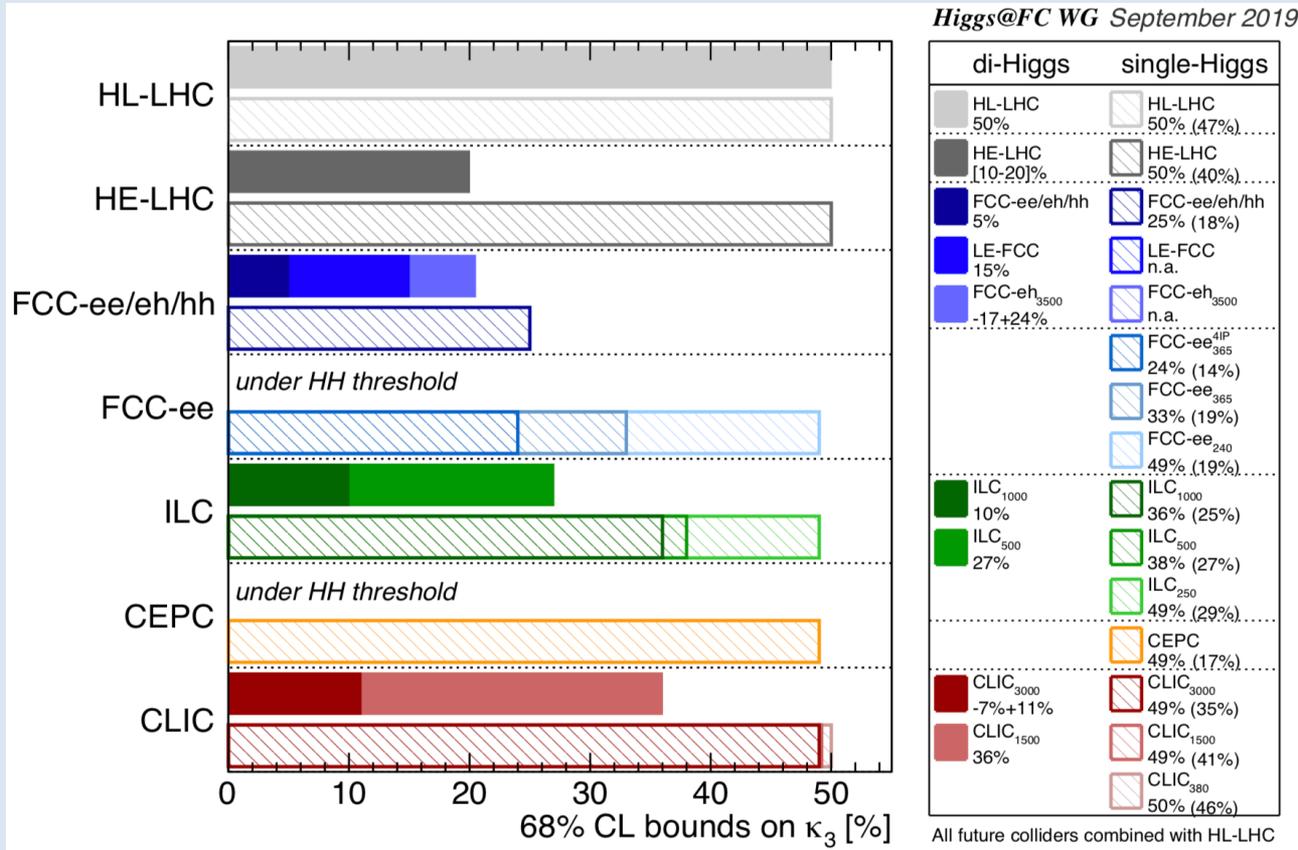


Proton/Electron

Insert into any one-loop diagram and no dim-6 counterterms will be required, result always finite!

A Unique Operator

Provides a calculable, complementary tool to explore the shape of the Higgs potential:



ECFA Higgs
Working
Group Report
1905.03764

Could play a useful role in the future...

Oblique Corrections

Oblique corrections have been a formidable toolkit in the effort to explore the electroweak sector.

- S-parameter
- T-parameter
- W-parameter
- Y-parameter

A Feynman diagram showing two external vector boson lines, labeled V , connected by a dashed internal line. A large 'X' is drawn over the dashed line, indicating a correction. Above the dashed line, the expression $\Delta_V(p^2)$ is written, representing the self-energy correction to the propagator.

The latter two contribute to amplitudes in an “energy-growing” manner:

$$\Delta_W(p^2) \approx \frac{1}{p^2 - M_W^2} - \frac{\hat{W}}{M_W^2}$$

Making these oblique parameters an excellent target for hadron colliders...

Oblique Corrections

Makes sense to extend to the Higgs sector. Especially since the Higgs can easily interact with new states...

• H-parameter:
$$H \text{ --- } \overset{\Delta_H(p^2)}{\times} \text{ --- } H$$

1903.07725

This also contributes to amplitudes in an “energy-growing” manner:

$$\Delta_H(p^2) \approx \frac{1}{p^2 - m_h^2} - \frac{\hat{H}}{m_h^2} + \dots$$

However, one needs to take the Higgs off-shell, which isn't easy...

Oblique Corrections

Makes sense to extend to the Higgs sector. Especially since the Higgs can easily interact with new states...

- H-parameter:
$$H \text{ --- } \overset{\Delta_H(p^2)}{\times} \text{ --- } H$$

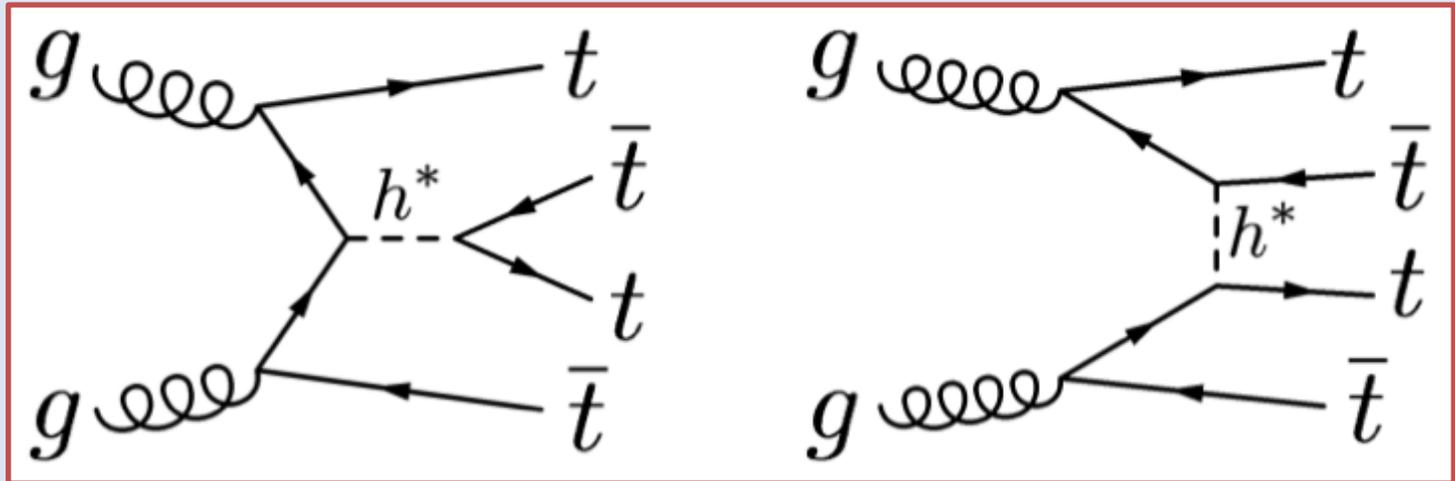
One can also translate basis to one in which this is a four-fermion operator and some more involving the Higgs

$$\mathcal{O} \propto \frac{\lambda^2 \hat{H}}{m_h^2} (\bar{\psi}\psi)^2$$

If new physics model interacts primarily with Higgs, then original basis may be better for interpretation purposes.

Oblique Corrections

Most promising avenue to take this Higgs off-shell is through four-top production:

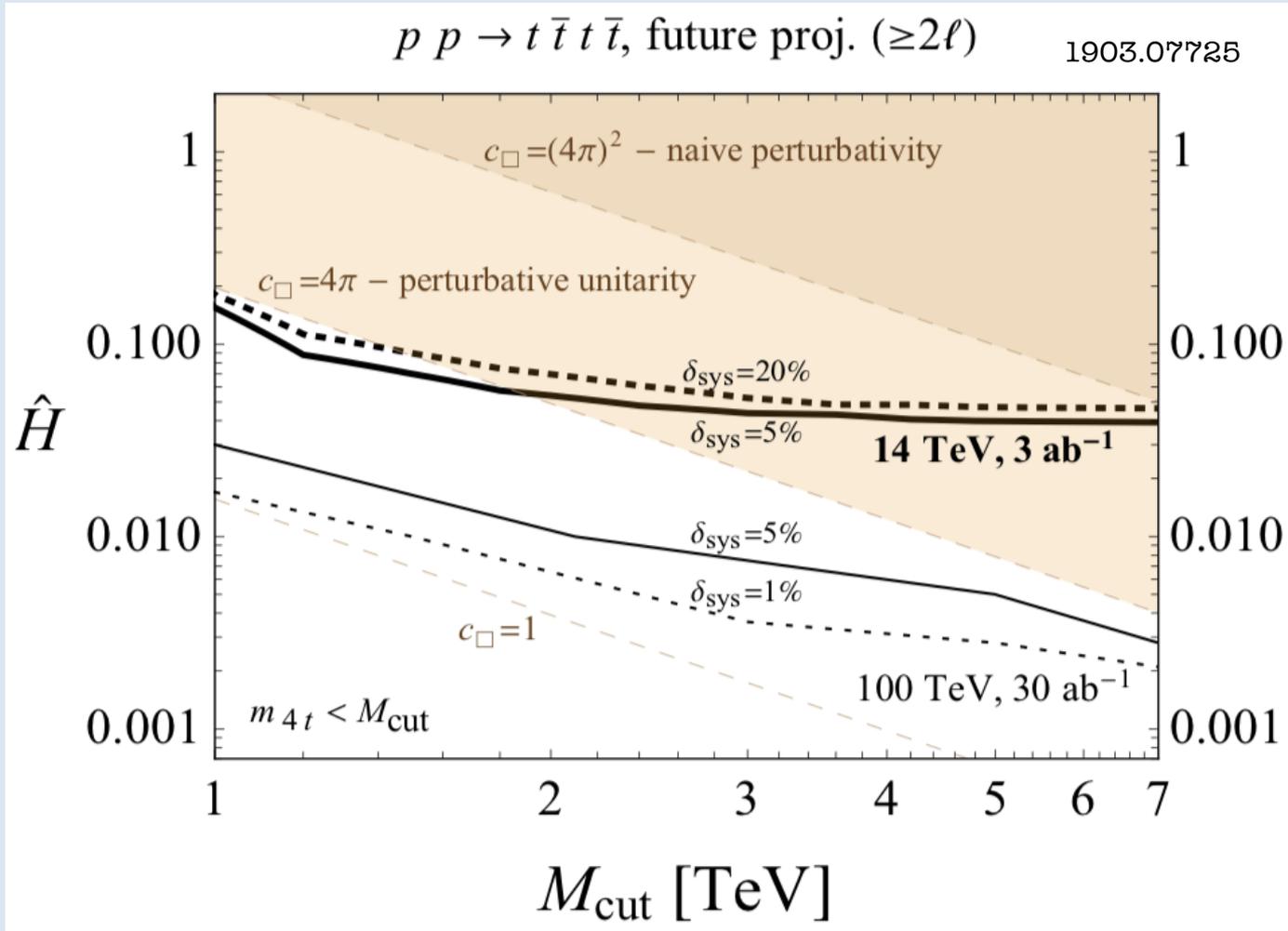


We may relate this Wilson coefficient to the scale of new physics as:

$$\frac{\hat{H}}{m_h^2} = \frac{c_{\square}}{M^2}$$

A Unique Operator

Our estimate suggests meaningful constraints challenging at the LHC:



A Unique Operator

CMS does better than our estimates:

Abstract

1908.06463

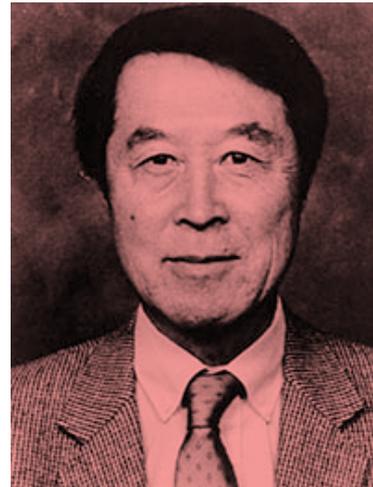
The standard model (SM) production of four top quarks ($t\bar{t}t\bar{t}$) in proton-proton collision is studied by the CMS Collaboration. The data sample, collected during the 2016–2018 data taking of the LHC, corresponds to an integrated luminosity of 137 fb^{-1} at a center-of-mass energy of 13 TeV. The events are required to contain two same-sign charged leptons (electrons or muons) or at least three leptons, and jets. The observed and expected significances for the $t\bar{t}t\bar{t}$ signal are respectively 2.6 and 2.7 standard deviations, and the $t\bar{t}t\bar{t}$ cross section is measured to be $12.6^{+5.8}_{-5.2} \text{ fb}$. The results are used to constrain the Yukawa coupling of the top quark to the Higgs boson, y_t , yielding a limit of $|y_t/y_t^{\text{SM}}| < 1.7$ at 95% confidence level, where y_t^{SM} is the SM value of y_t . They are also used to constrain the oblique parameter of the Higgs boson in an effective field theory framework, $\hat{H} < 0.12$. Limits are set on the production of a heavy scalar or pseudoscalar boson in Type-II two-Higgs-doublet and simplified dark matter models, with exclusion limits reaching 350–470 GeV and 350–550 GeV for scalar and pseudoscalar bosons, respectively. Upper bounds are also set on couplings of the top quark to new light particles.

A mountain goat with small horns stands on a rocky, grassy peak in the foreground. The background shows a vast valley with a large lake, a town, and distant mountains under a hazy sky. A speech bubble is positioned above the goat, containing the text: "What does all this have to do with naturalness?".

What does
all this have to do with
naturalness?

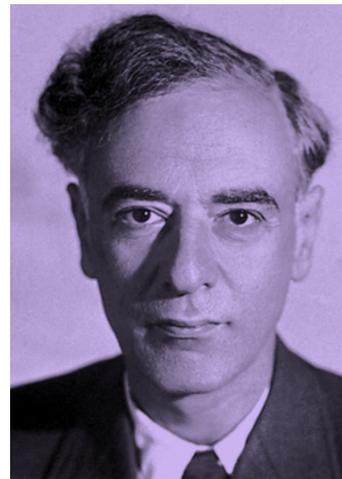
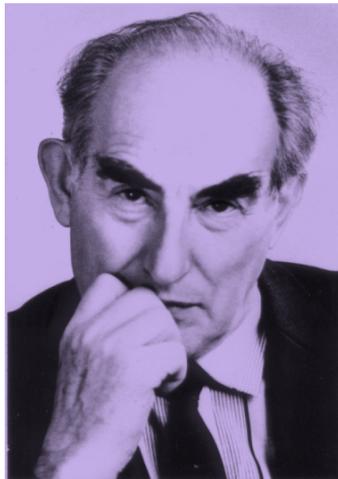
Naturalness

Much as the pion mass, couplings, and so on can be calculated from the microscopic parameters of QCD,



Naturalness

Much as the pion mass, couplings, and so on can be calculated from the microscopic parameters of QCD, and the origin of the scalar potential for the Landau-Ginzburg scalar field may be calculated from the microscopic parameters of BCS theory,



Naturalness

Much as the pion mass, couplings, and so on can be calculated from the microscopic parameters of QCD, and the origin of the scalar potential for the Landau-Ginzburg scalar field may be calculated from the microscopic parameters of BCS theory, naturalness concerns the quest to determine the microscopic origins of the Higgs sector.



Composite Higgs/SUSY

Higgs Only

$$\mathcal{O}_{\square} = \frac{c_{\square}}{M^2} | \square H |^2 \quad [g_*^0]$$

$$\begin{aligned} \mathcal{O}_H &= \frac{c_H}{2M^2} (\partial^\mu |H|^2)^2 \quad [g_*^2] \\ \mathcal{O}_T &= \frac{c_T}{2M^2} (H^\dagger \overleftrightarrow{D}^\mu H)^2 \\ \mathcal{O}_R &= \frac{c_R}{M^2} |H|^2 |D^\mu H|^2 \end{aligned}$$

$$\mathcal{O}_6 = \frac{c_6}{M^2} |H|^6 \quad [g_*^4]$$

Gauge Only

$$\mathcal{O}_{2G} = -\frac{c_{2G}}{4M^2} (D_\rho G_{\mu\nu}^a)^2$$

$$\mathcal{O}_{2W} = -\frac{c_{2W}}{4M^2} (D_\rho W_{\mu\nu}^a)^2$$

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Mixed

$$\begin{aligned} \mathcal{O}_B &= \frac{ig' c_B}{2M^2} (H^\dagger \overleftrightarrow{D}^\mu H) \partial^\nu B_{\mu\nu} \\ \mathcal{O}_W &= \frac{ig c_W}{2M^2} (H^\dagger \sigma^a \overleftrightarrow{D}^\mu H) D^\nu W_{\mu\nu}^a \end{aligned}$$

$$\begin{aligned} \mathcal{O}_{GG} &= \frac{g_s^2 c_{GG}}{M^2} |H|^2 G^{a,\mu\nu} G_{\mu\nu}^a \\ \mathcal{O}_{WB} &= \frac{gg' c_{WB}}{M^2} H^\dagger \sigma^a H B^{\mu\nu} W_{\mu\nu}^a \\ \mathcal{O}_{WW} &= \frac{g^2 c_{WW}}{M^2} |H|^2 W^{a\mu\nu} W_{\mu\nu}^a \\ \mathcal{O}_{BB} &= \frac{g'^2 c_{BB}}{M^2} |H|^2 B^{\mu\nu} B_{\mu\nu} \end{aligned}$$

But, until we know what's going on with the Higgs, let's not overlook the outlier operators...

Higgs Only

$[g_*^0]$

$$\mathcal{O}_\square = \frac{c_\square}{M^2} |\square H|^2$$

$[g_*^2]$

$$\mathcal{O}_H = \frac{c_H}{2M^2} (\partial^\mu |H|^2)^2$$

$$\mathcal{O}_T = \frac{c_T}{2M^2} (H^\dagger \overleftrightarrow{D}^\mu H)^2$$

$$\mathcal{O}_R = \frac{c_R}{M^2} |H|^2 |D^\mu H|^2$$

$[g_*^4]$

$$\mathcal{O}_6 = \frac{c_6}{M^2} |H|^6$$

which determine the dynamics of the Higgs, from how it moves to the shape of the Higgs potential.