Preserving physically important variables in optimal event selections

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Higgs Couplings
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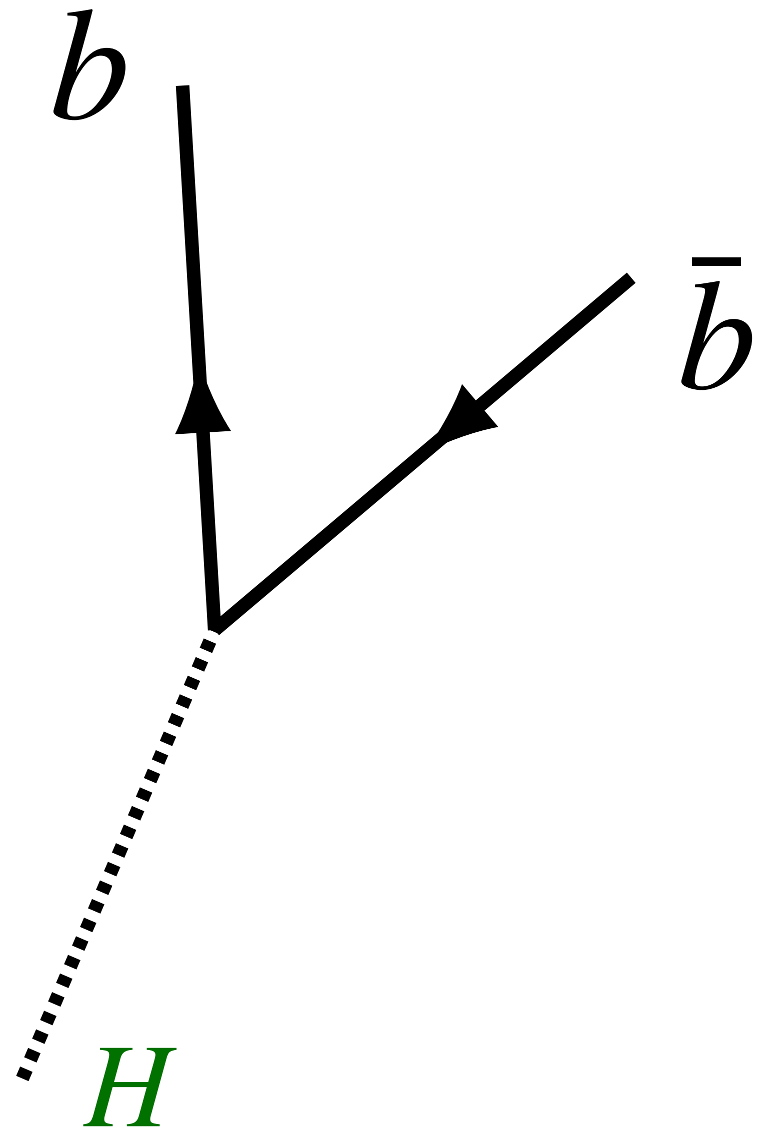

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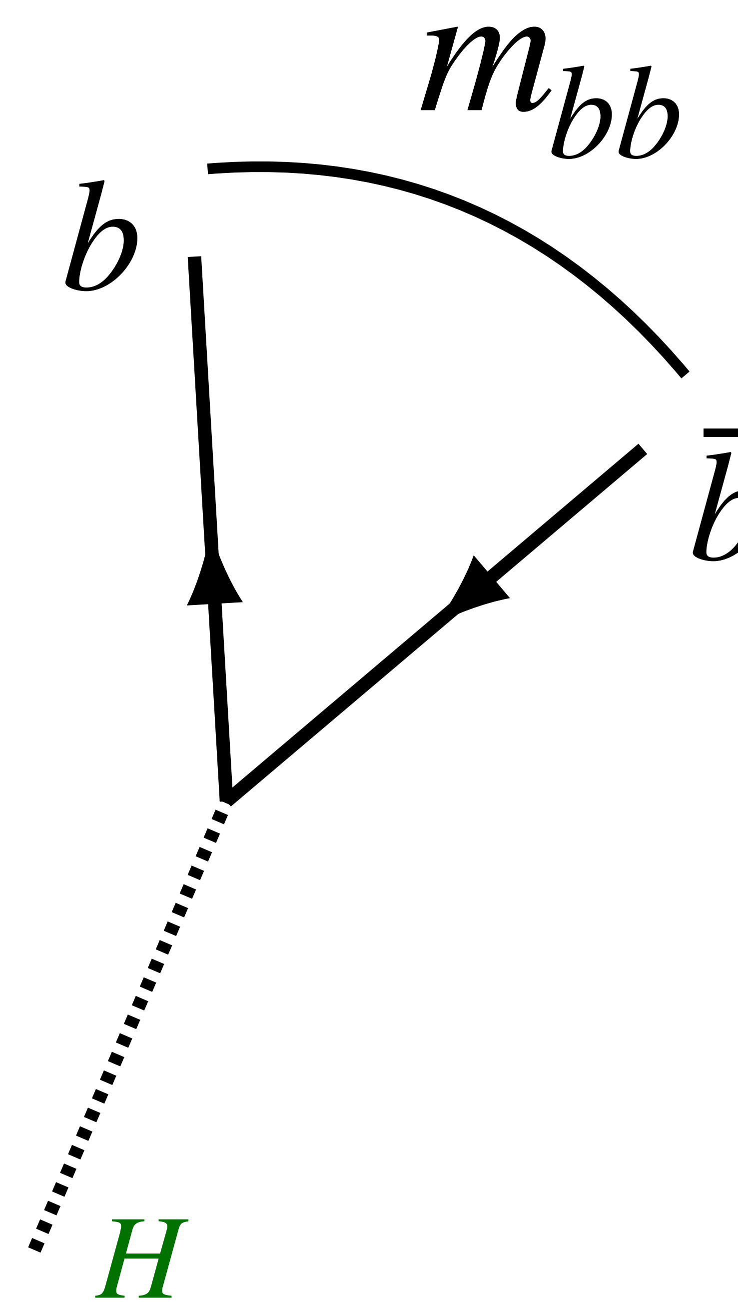
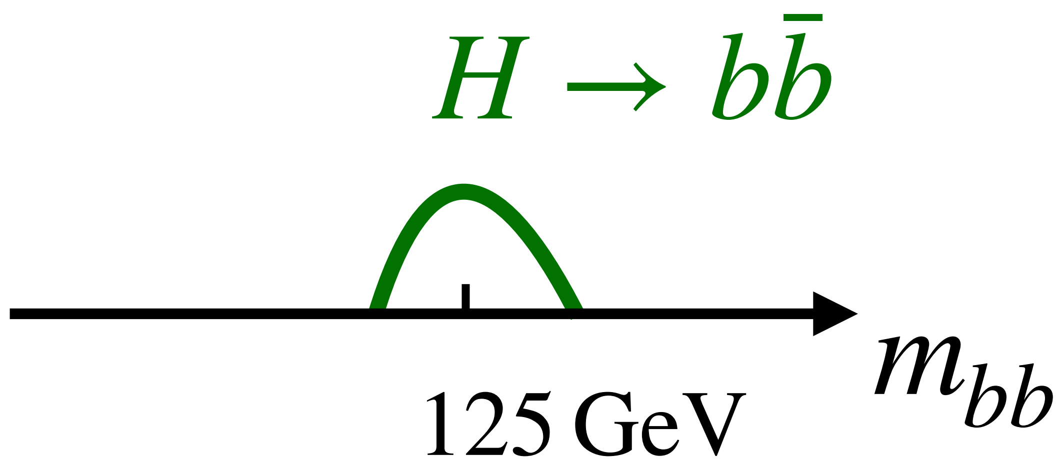
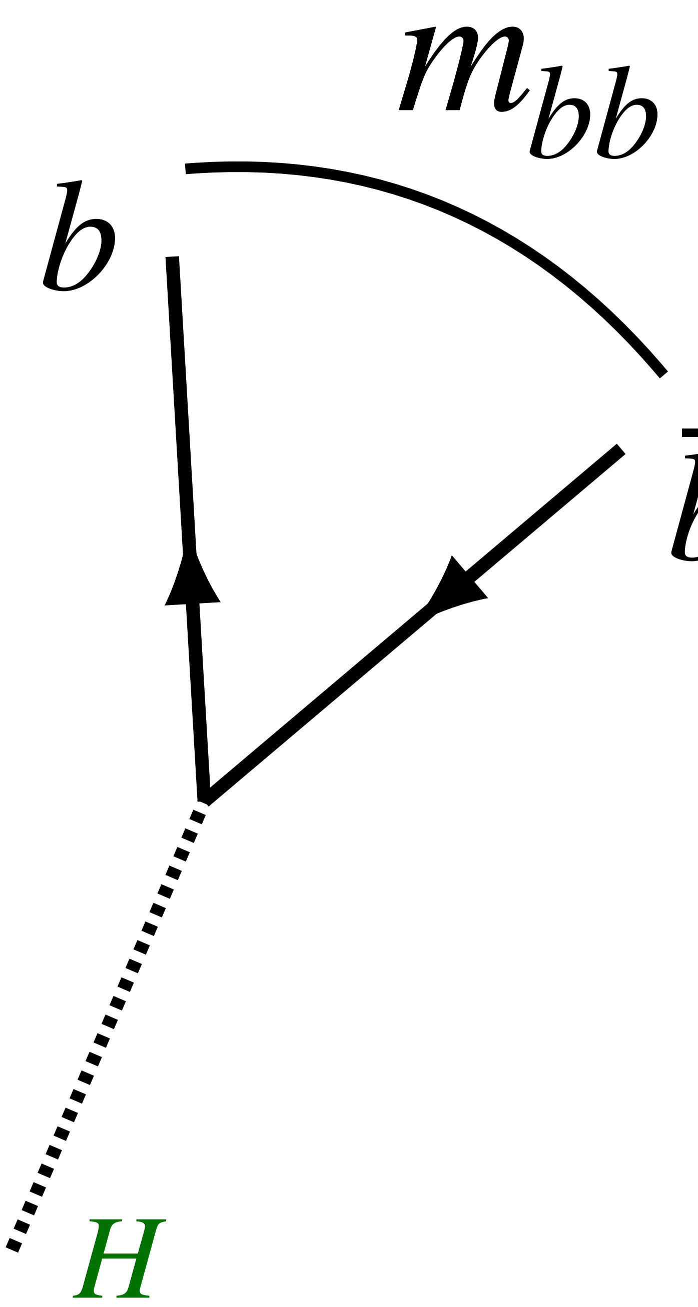


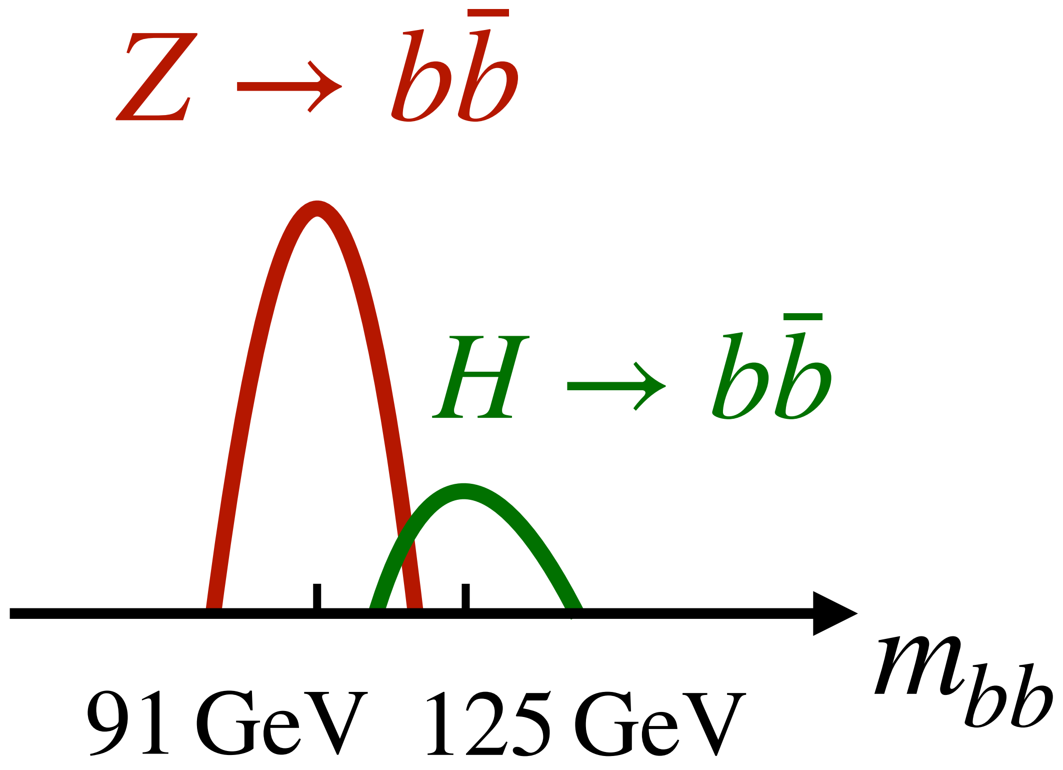
Preserving **physically** important variables in optimal event selections

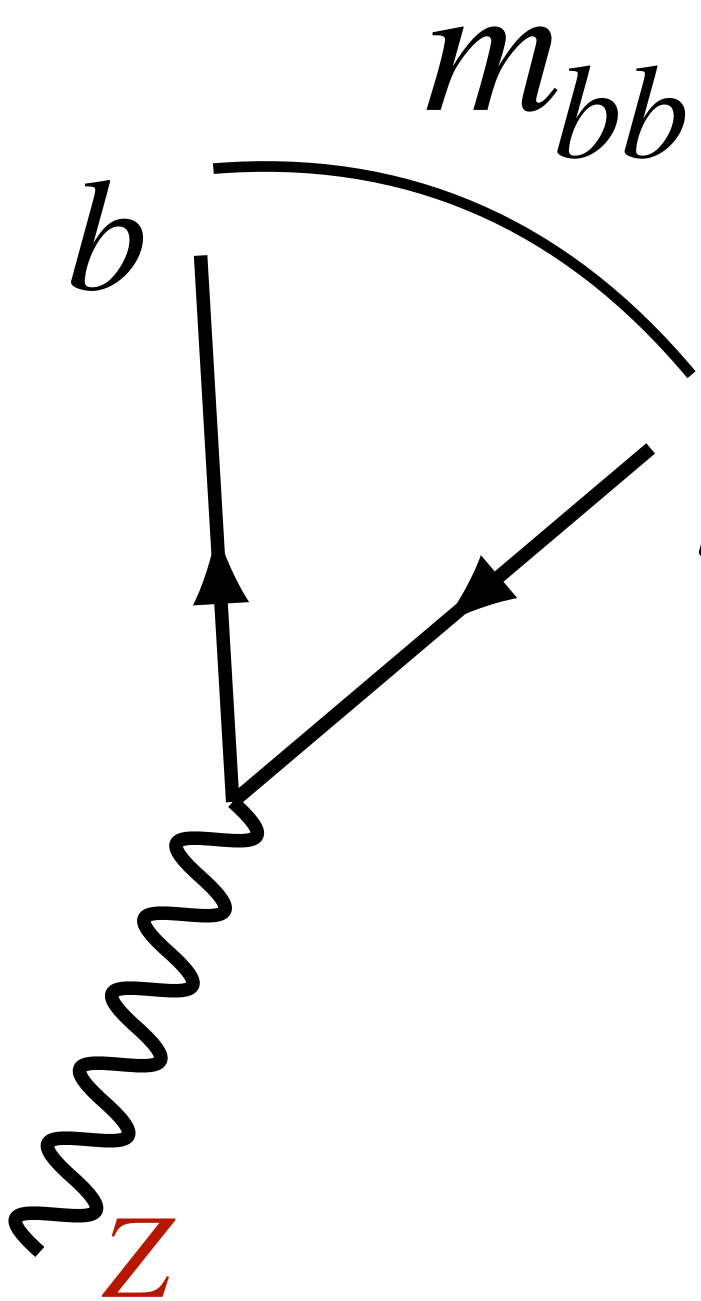




$\Longrightarrow \exists$ **physical state**

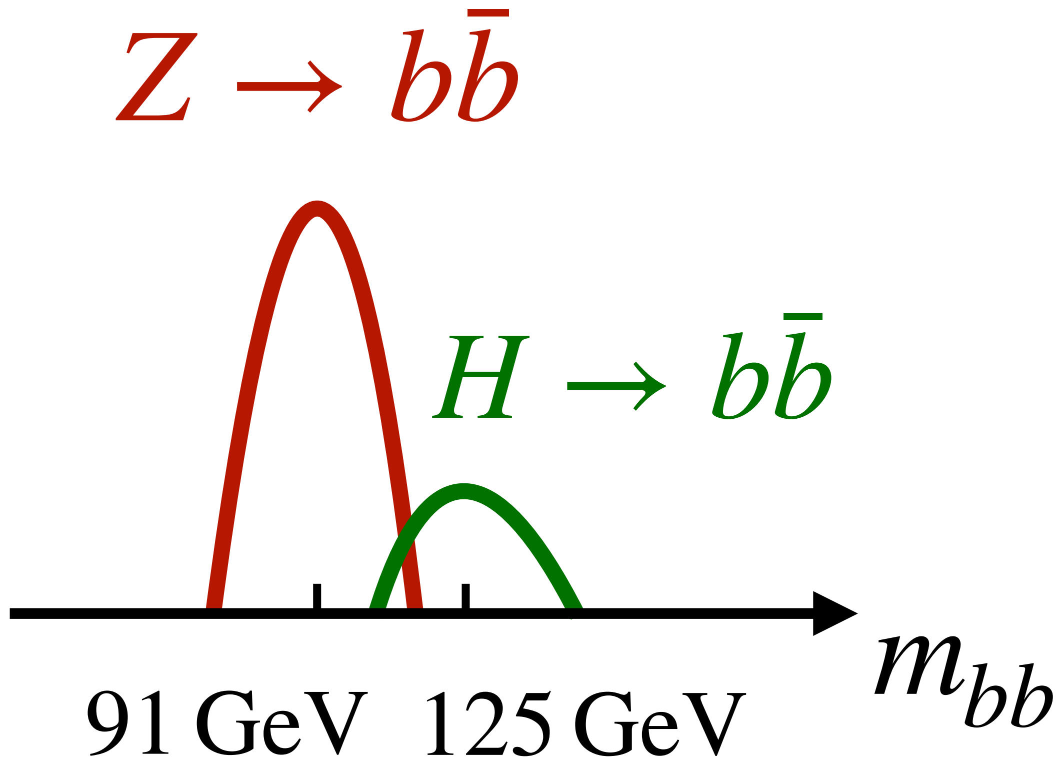
Preserving physically **important variables** in optimal event selections





$\Longrightarrow$ **Signal (S) / background (B) discrimination**

Preserving physically important variables in optimal **event selections**



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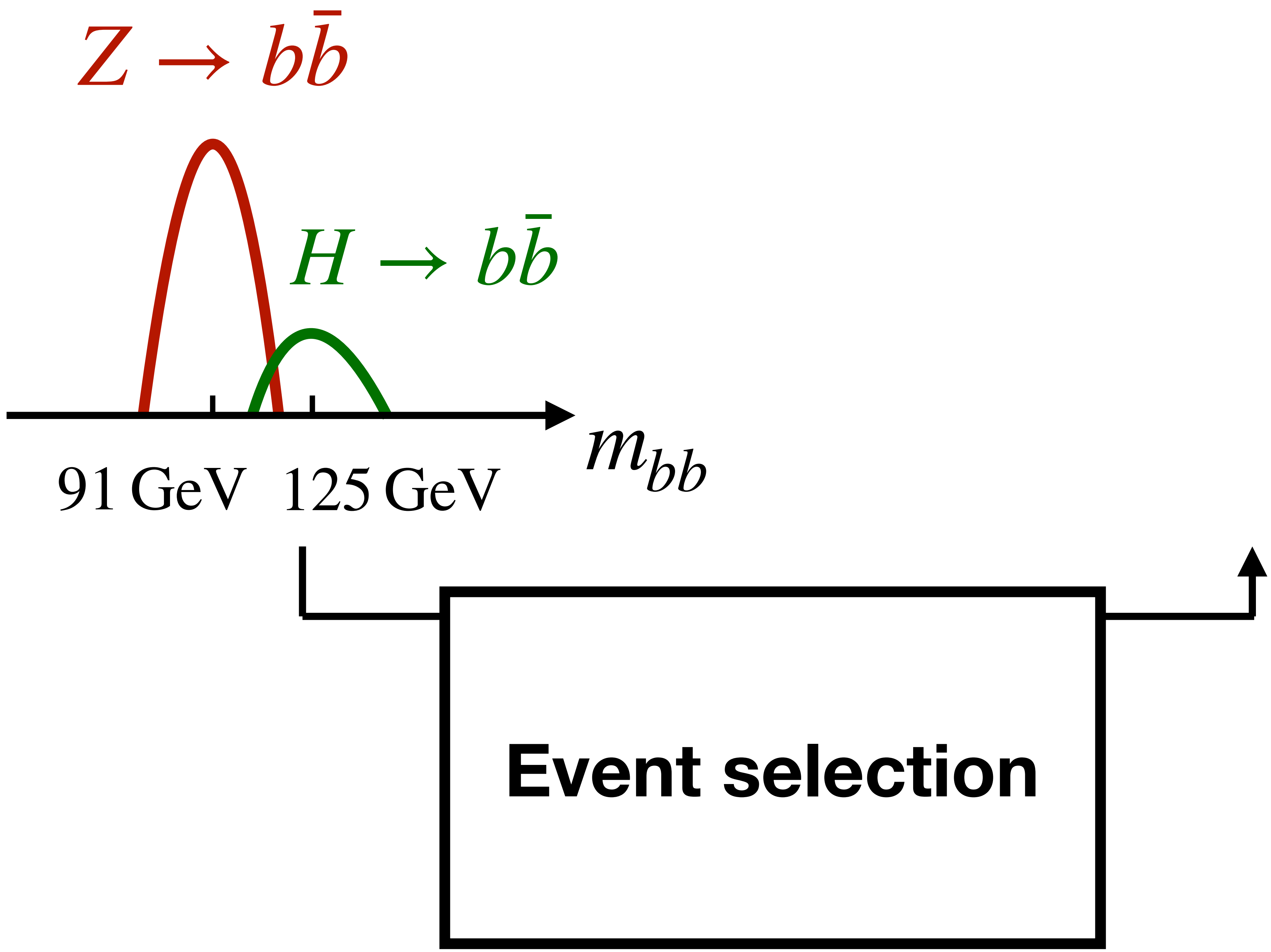

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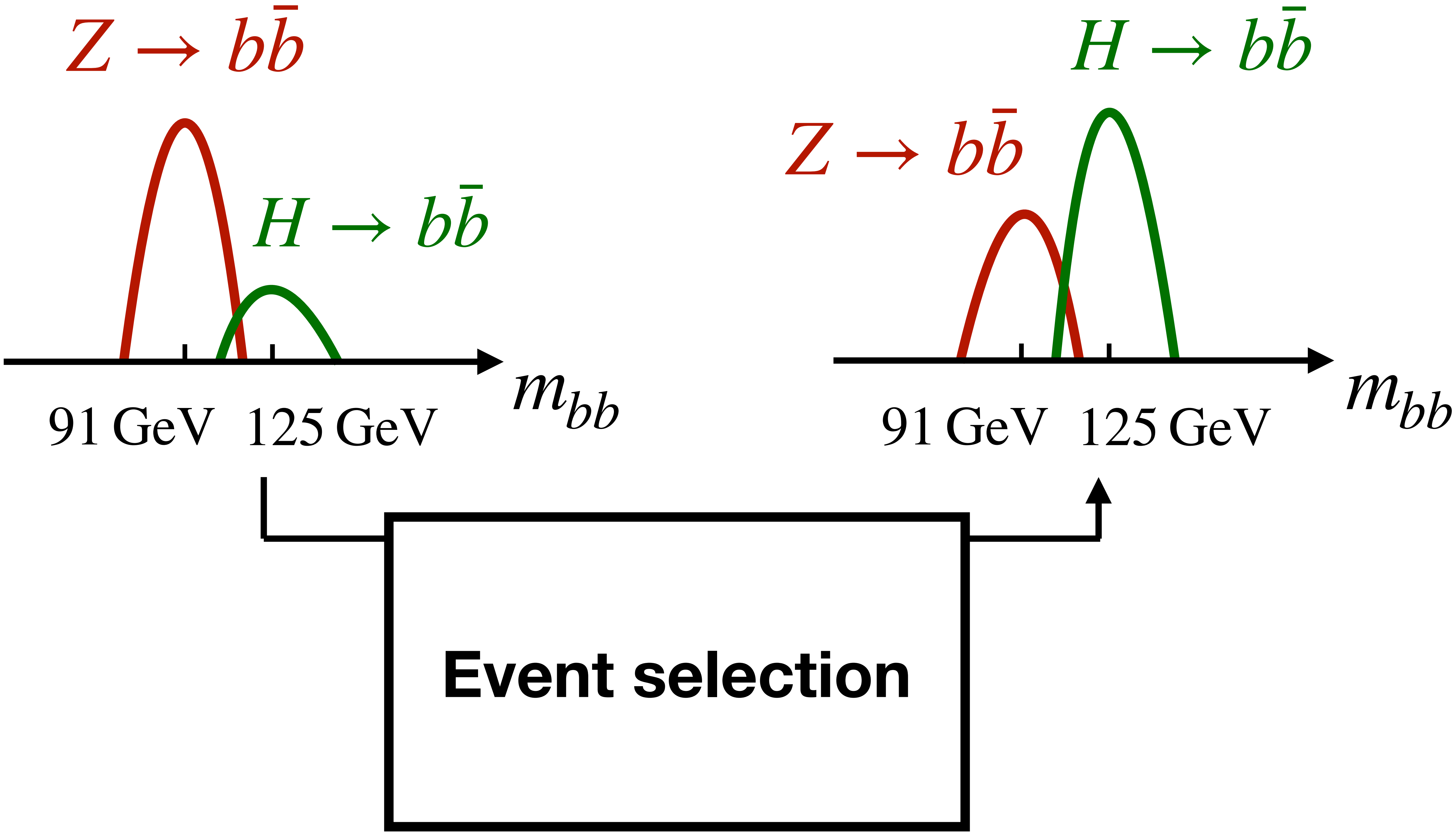

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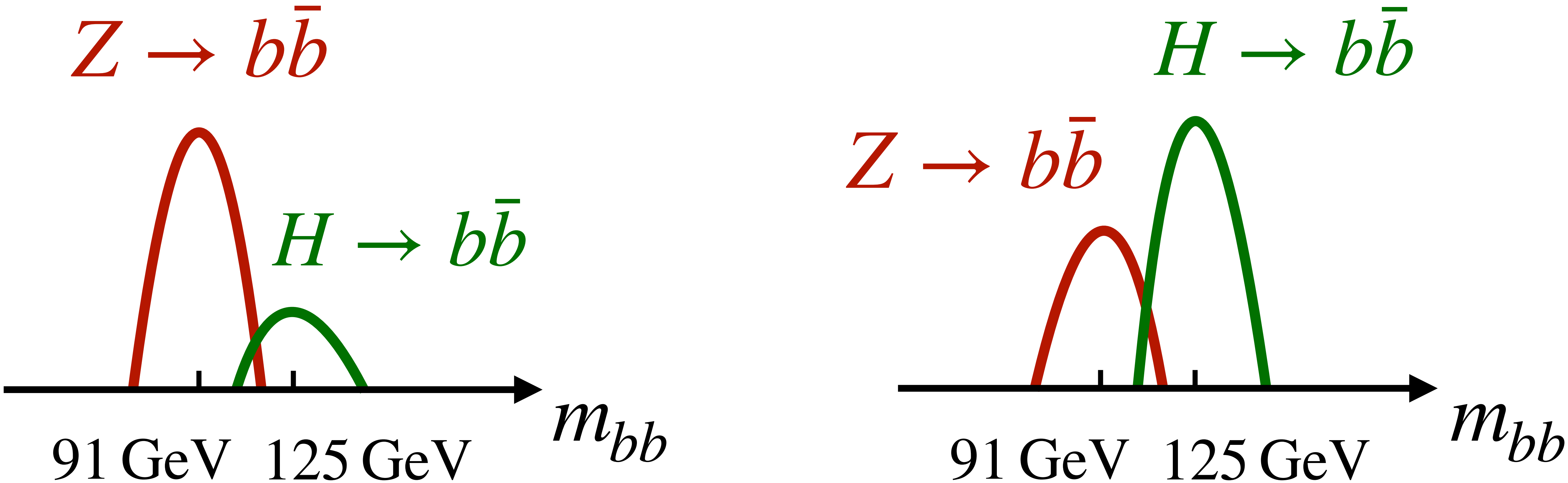

Likelihood ratio

$$L(\mathbf{e}) = \frac{p_S(\mathbf{e})}{p_B(\mathbf{e})} \gg 1$$

e… event

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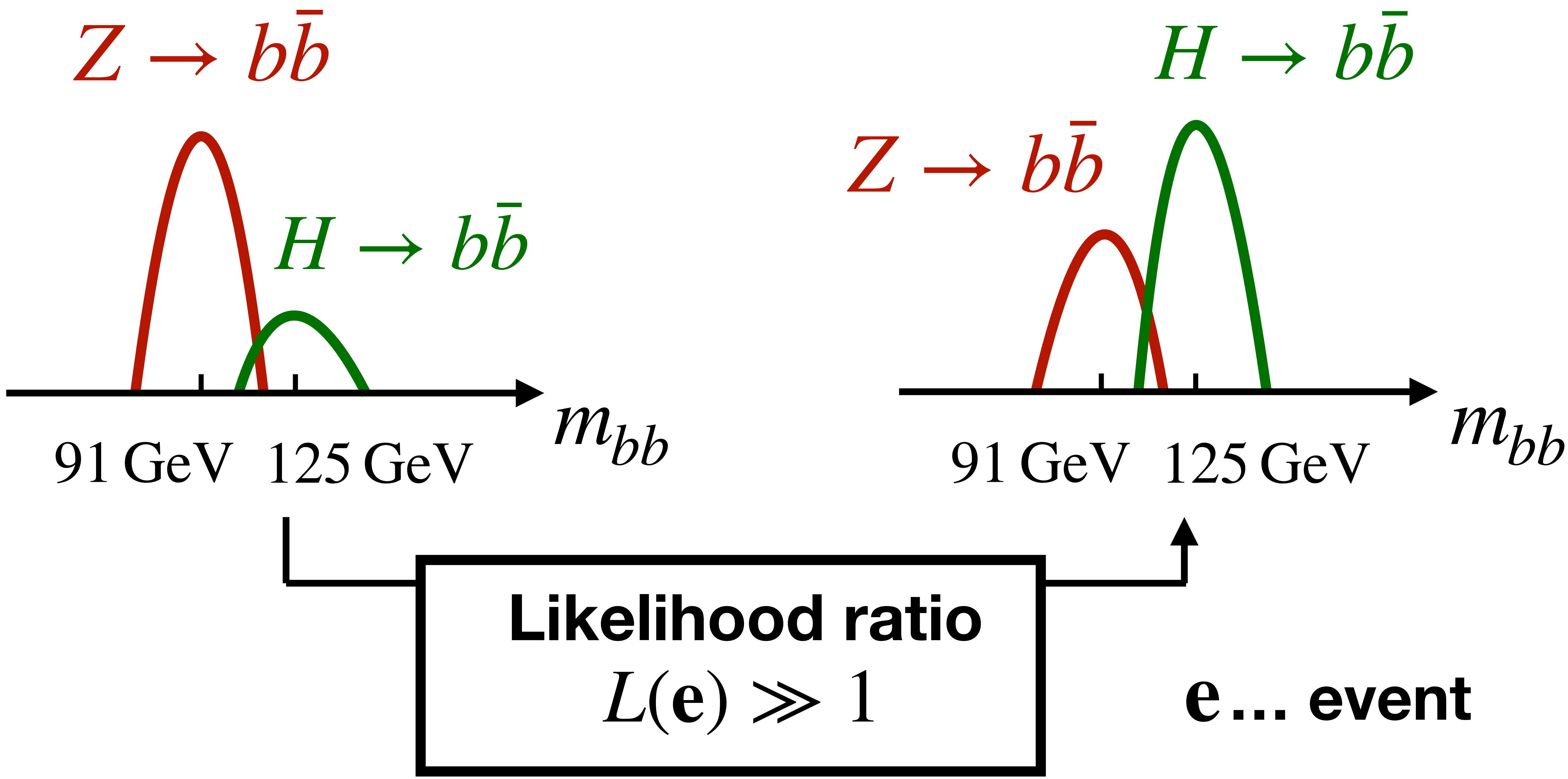

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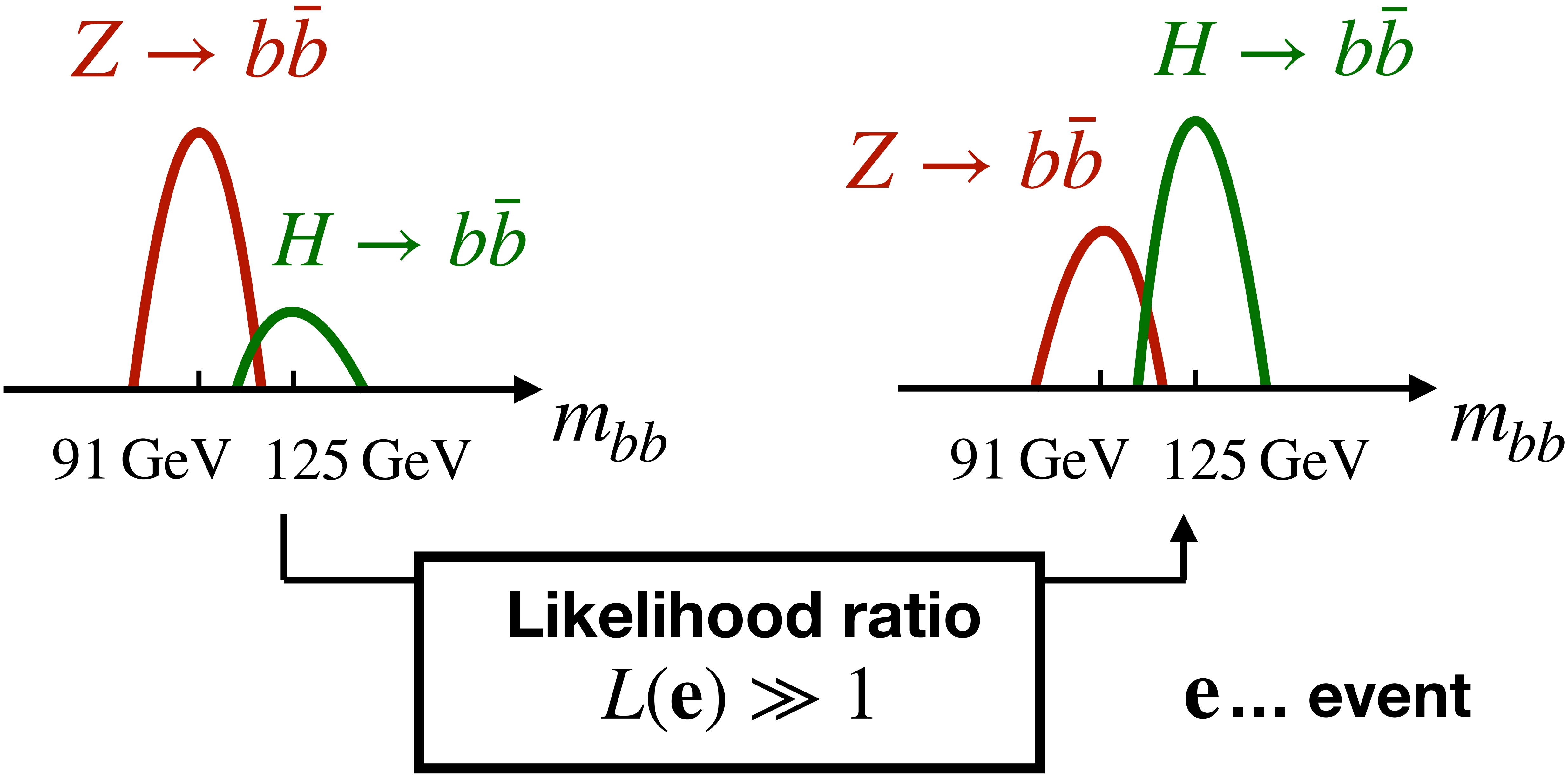

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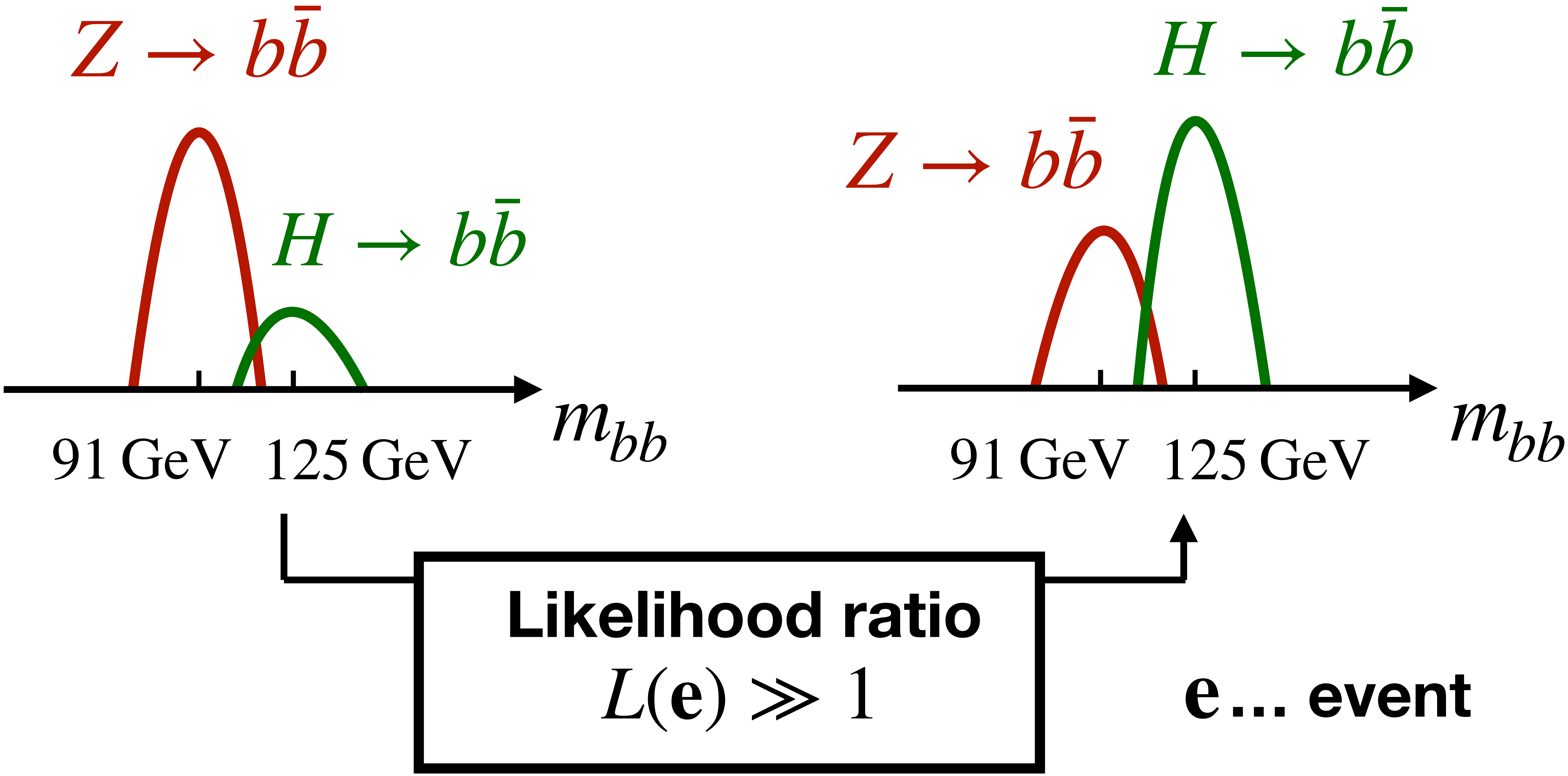

- $m_{bb}(\mathbf{e})$ and $L(\mathbf{e})$ strongly correlated $\longrightarrow$ **distortion**

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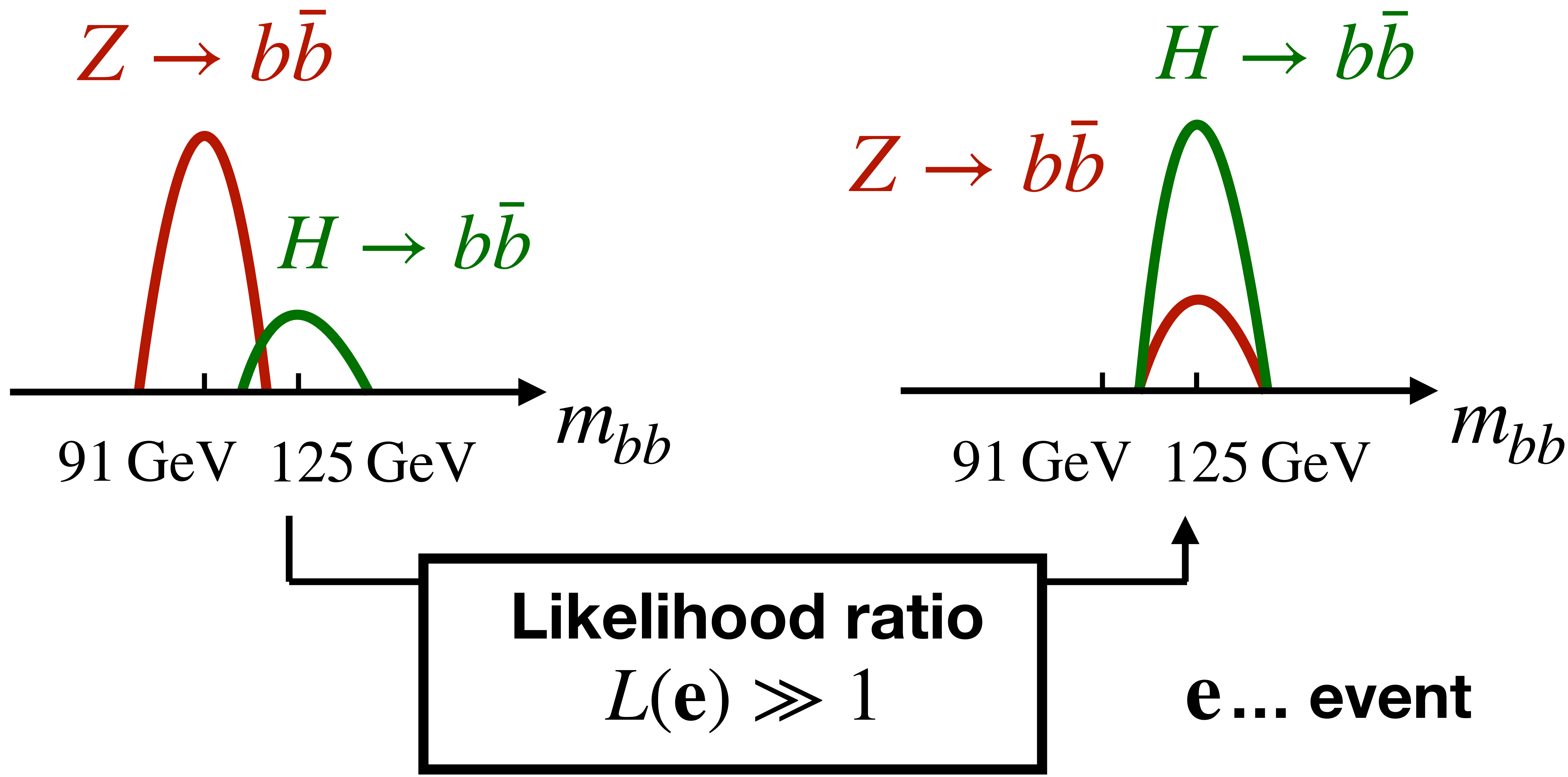

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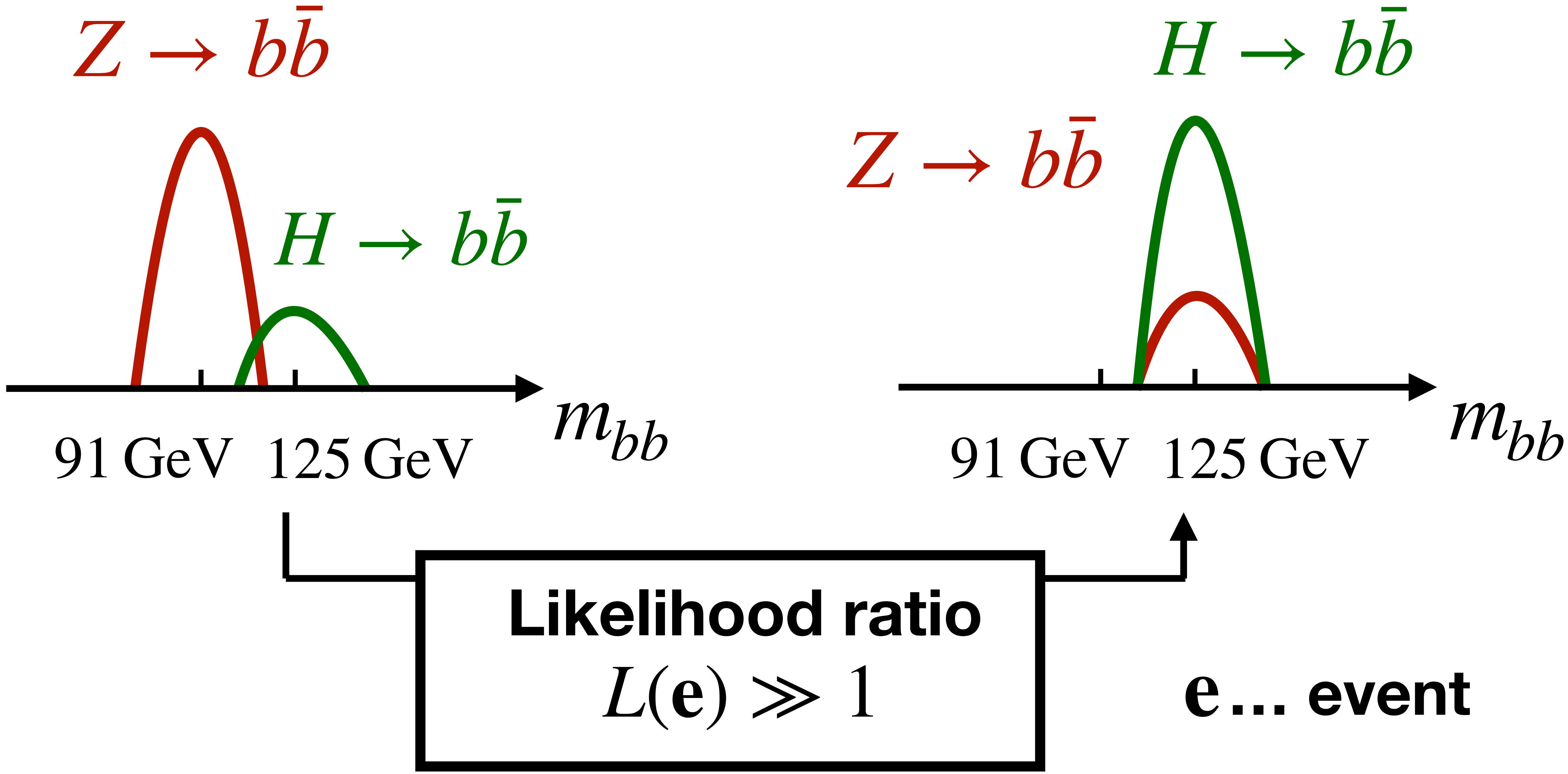

- No discrimination power left in m_{bb} $\to$ ~~**physically important**~~

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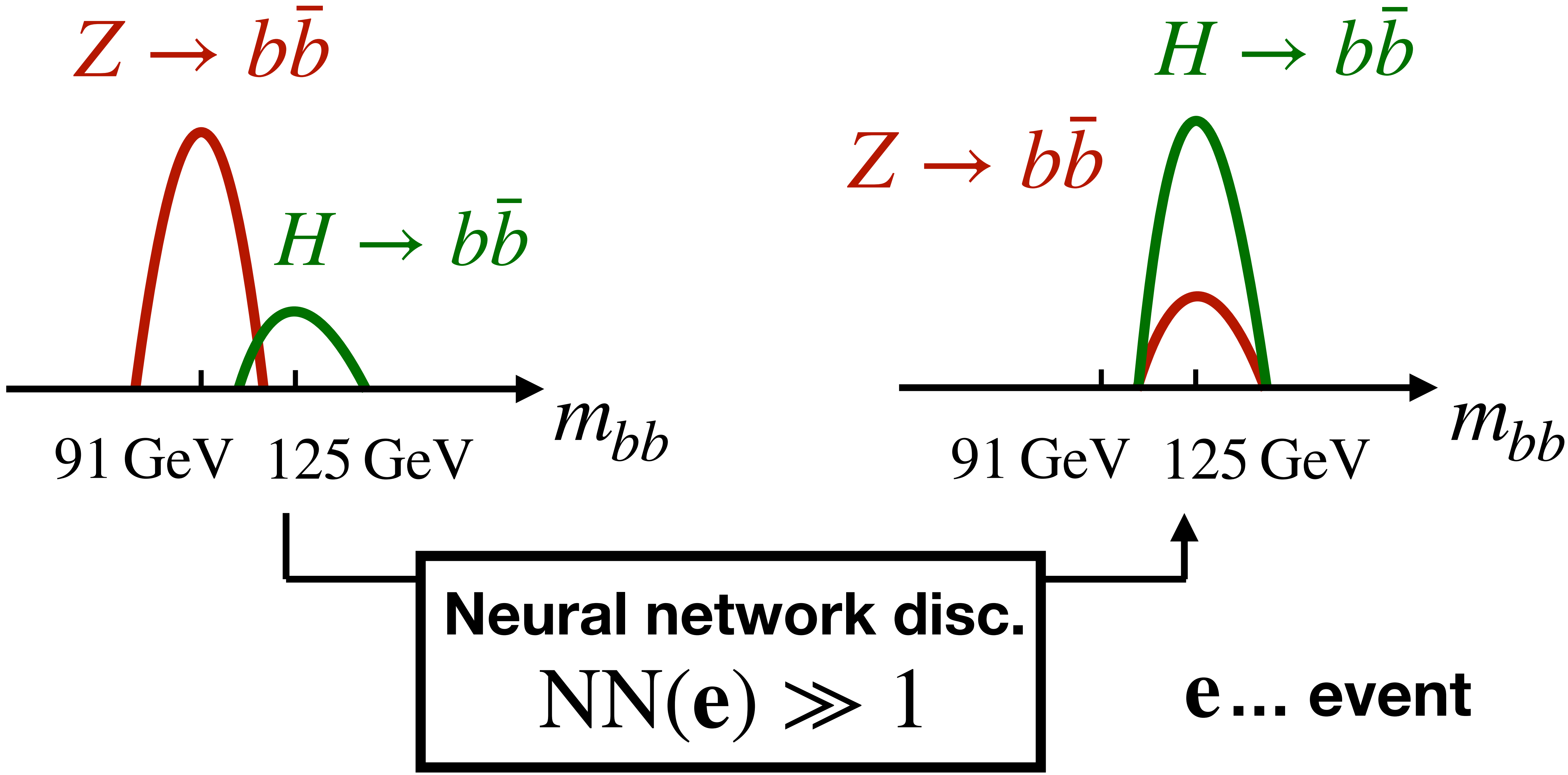

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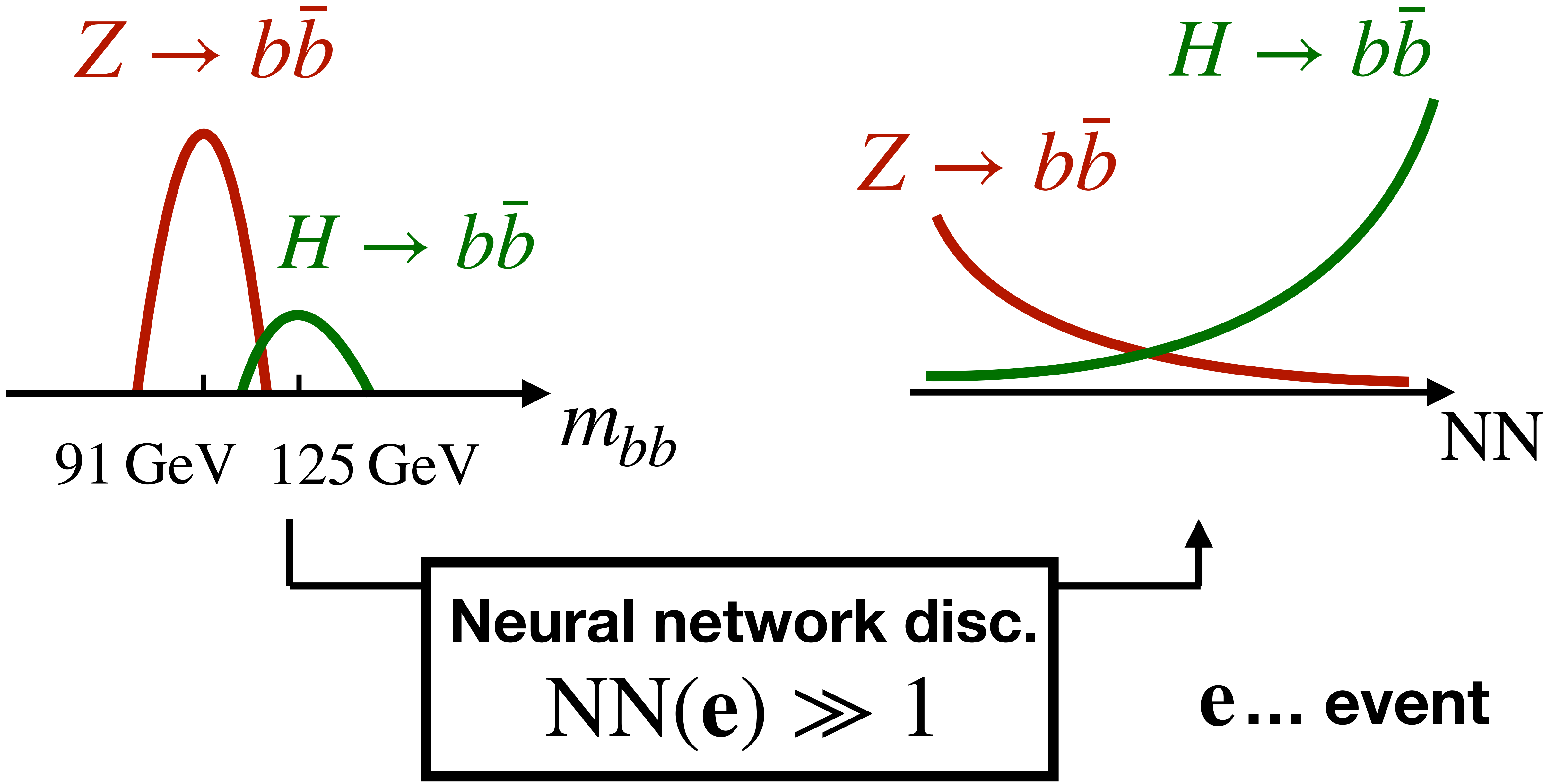

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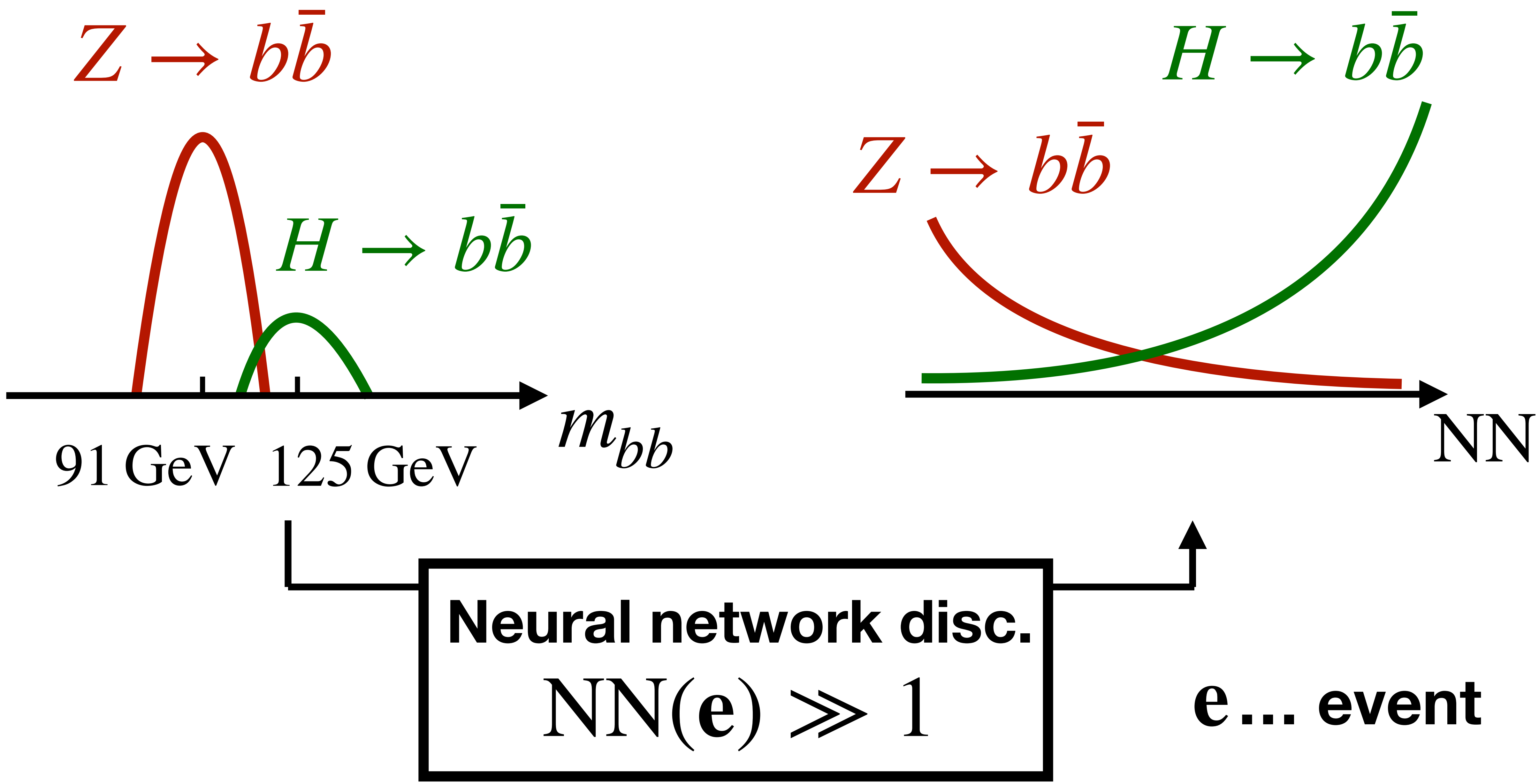

- $m_{bb}(\mathbf{e})$ vs. $\mathrm{NN}(\mathbf{e}) \leftrightarrow$ physics vs. statistics

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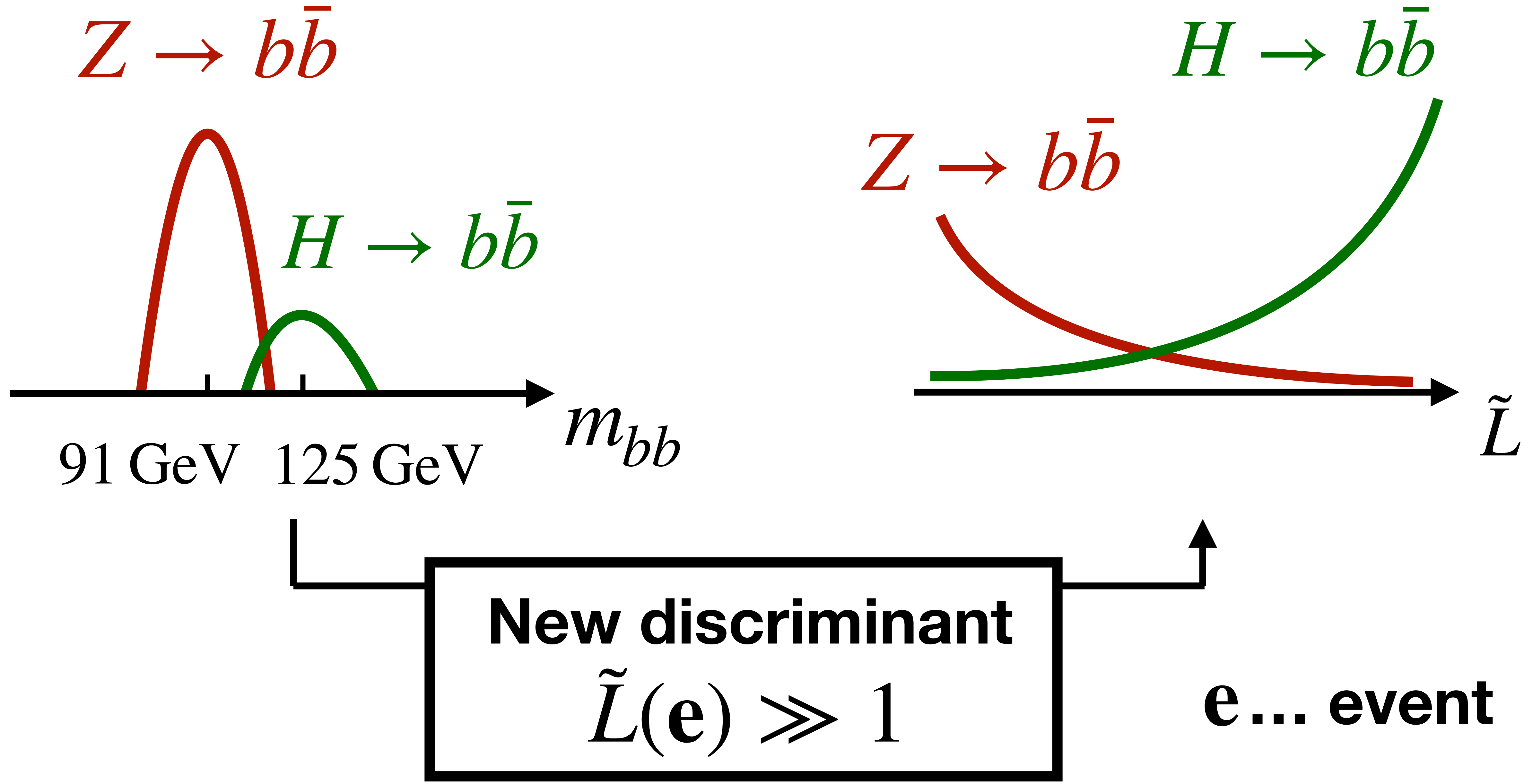

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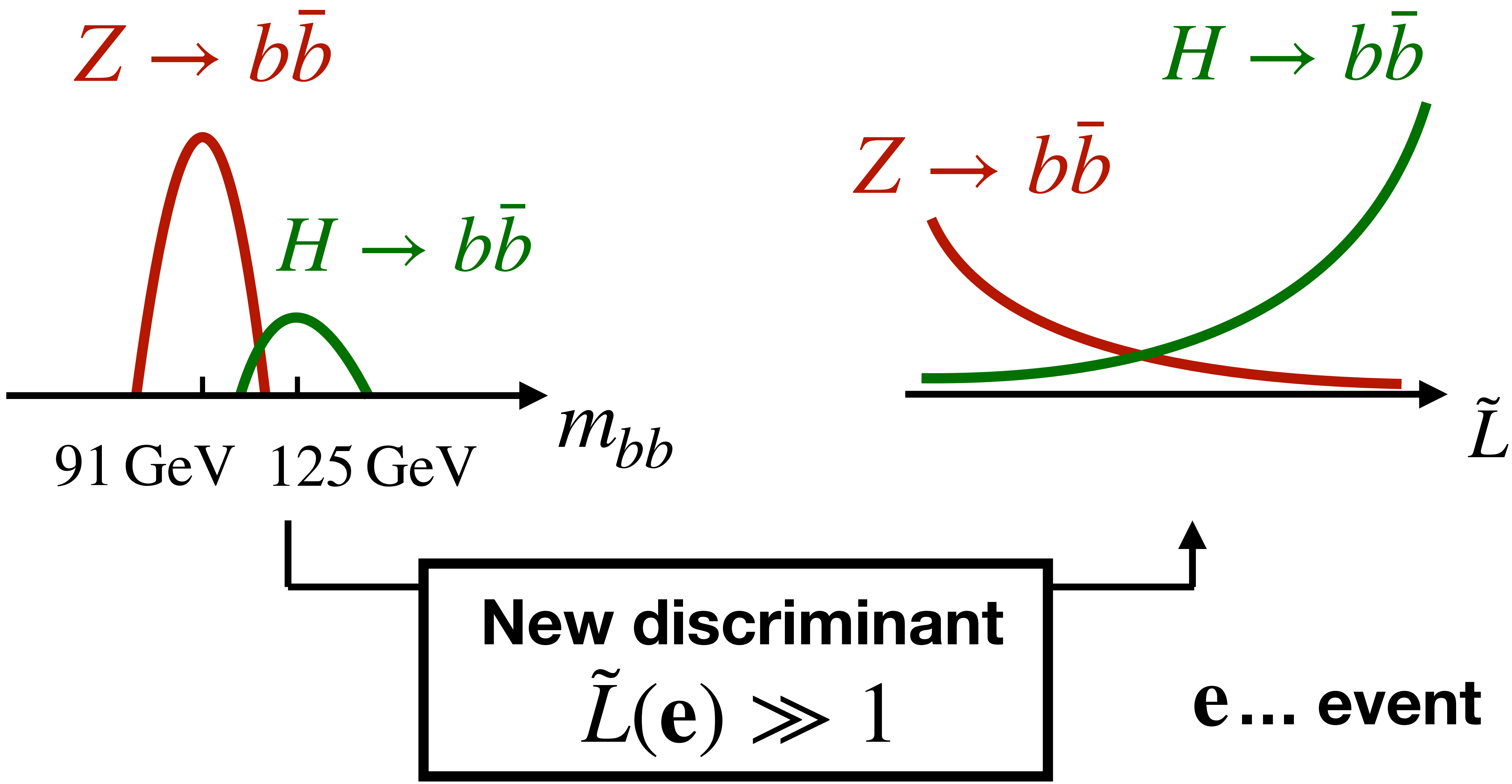

$\tilde{L}$ … **best discriminant that limits distortion of** m_{bb}

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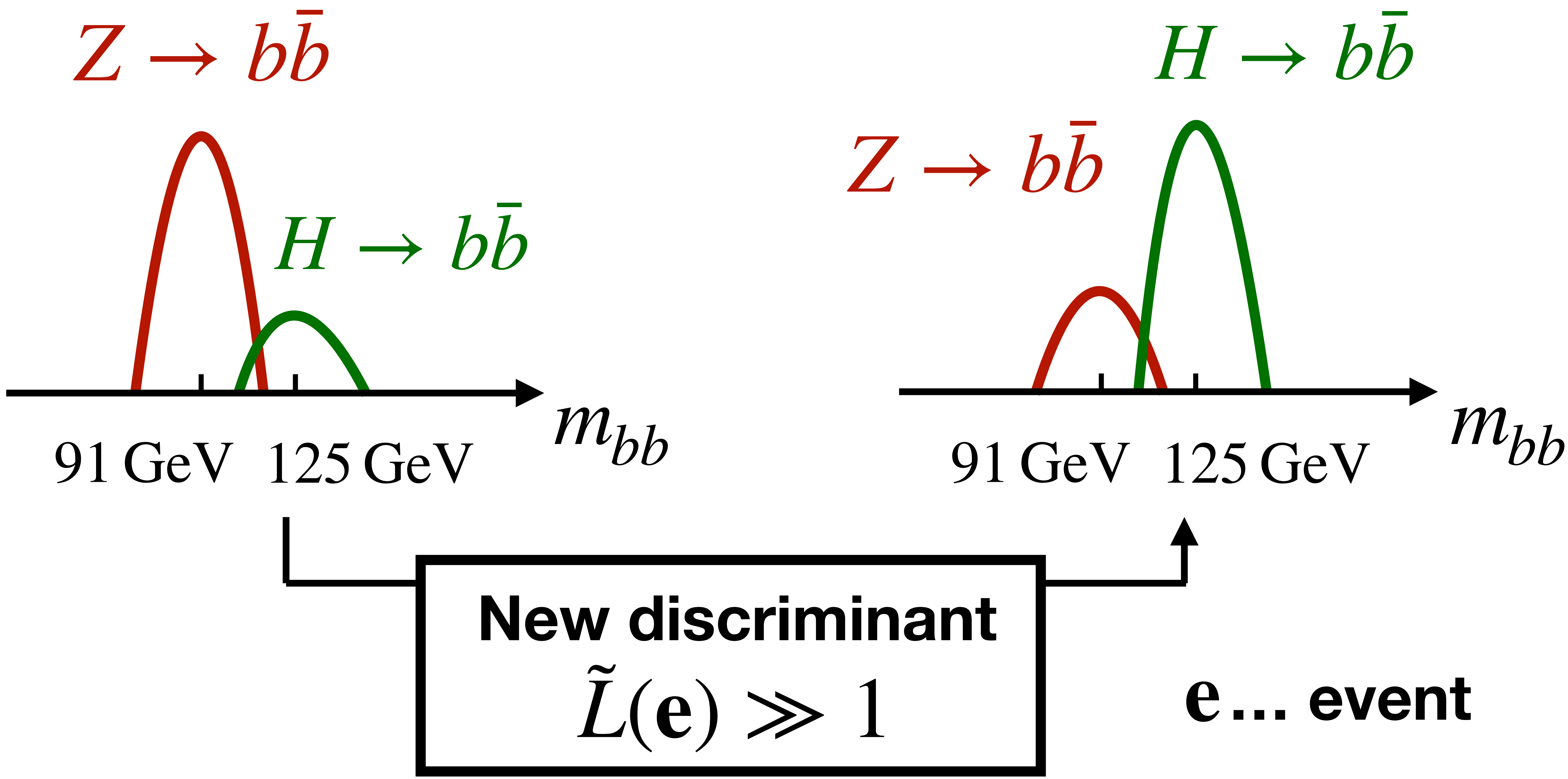

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Towards a solution

Likelihood ratio = maximally powerful discriminant

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$$\frac{p_S(\mathbf{e})}{p_B(\mathbf{e})} =: L(\mathbf{e})$$

Towards a solution

Likelihood ratio = maximally powerful discriminant

$$\frac{p_S(\mathbf{e})}{p_B(\mathbf{e})} =: L(\mathbf{e}) := \arg\max_f \left(\mathscr{P}[f] \right)$$

performance measure

candidate discriminant

Towards a solution

Likelihood ratio = maximally powerful discriminant

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$\tilde{L}$: control distortion of m_{bb} by event selection

Towards a solution

$$L(\mathbf{e}) := \arg\max_f \left(\ \mathscr{P}[f] \ \right)$$

limit correlation of m_{bb} and $\tilde{L}$ $\leftrightarrow$ $\tilde{L}$: control distortion of m_{bb} by event selection

Towards a solution

$$L(\mathbf{e}) := \arg\max_f \left(\ \mathscr{P}[f] \ \right)$$

Constraint:

limit correlation of m_{bb} and $\tilde{L}$

$\leftrightarrow$

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Towards a solution

$$\tilde{L}(\mathbf{e}) := \arg\max_f \Big(\ \mathscr{P}[f]$$

Constraint:

limit correlation of

m_{bb} and $\tilde{L}$

Towards a solution

$$\tilde{L}(\mathbf{e}) := \arg\max_f \Big(\mathscr{P}[f] + C(f, m_{bb}) \Big)$$

Constraint:

limit correlation of m_{bb} and $\tilde{L}$

Towards a solution

$$\tilde{L}(\mathbf{e}) := \arg\max_f \Big(\ \mathscr{P}[f] \quad + \quad C(f, m_{bb}) \ \Big)$$

↑ constraint

Constraint:

limit correlation of

m_{bb} and $\tilde{L}$

Towards a solution

$$\tilde{L}(\mathbf{e}) := \arg\max_f \Big(\mathscr{P}[f]$$

$$+ \lambda \cdot \; C(f, m_{bb}) \Big)$$

Lagrange multiplier *(constraint strength)* → λ

constraint → C

Constraint:

limit correlation of m_{bb} and $\tilde{L}$

Towards a solution

$$\tilde{L}(\mathbf{e}) := \arg\max_f \Big(\mathcal{P}[f]$$

$$- \lambda \cdot \quad \rho\left(f, m_{bb}\right) \Big)$$

Lagrange multiplier

(constraint strength)

Pearson correlation $\rightarrow 0$

Constraint:

limit correlation of

m_{bb} and $\tilde{L}$

Towards a solution

$$\tilde{L}(\mathbf{e}) := \arg\max_f \Big(\mathscr{P}[f] - \lambda \cdot \rho(f, m_{bb}) \Big)$$

Lagrange multiplier *(constraint strength)* → λ

Pearson correlation $\to 0$ → $\rho(f, m_{bb})$

$\implies \tilde{L}, m_{bb}$ linearly uncorrelated

Constraint:

limit correlation of m_{bb} and $\tilde{L}$

Towards a solution

$$\tilde{L}(\mathbf{e}) := \arg\max_f \Big(\mathscr{P}[f] - \lambda \cdot \rho(f, m_{bb}) \Big)$$

Lagrange multiplier

(constraint strength)

Pearson correlation $\to 0$

$\implies \tilde{L}, m_{bb}$ linearly uncorrelated nonlinearly related

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limit correlation of m_{bb} and $\tilde{L}$

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Pearson correlation $\to 0$

Constraint:

limit correlation of
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Towards a solution

*http://cern.ch/go/zKv8

$$\tilde{L}(\mathbf{e}) := \arg\max_f \Big(\mathscr{P}[f] - \lambda \cdot \mathrm{MI}(f, m_{bb}) \Big)$$

Lagrange multiplier *(constraint strength)*

Mutual information* $\rightarrow 0$

Constraint:

limit correlation of m_{bb} and $\tilde{L}$

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$$\tilde{L}(\mathbf{e}) := \arg\max_f \left(\mathscr{P}[f] - \lambda \cdot \mathrm{MI}(f, m_{bb}) \right)$$

Lagrange multiplier
(constraint strength)

Mutual information* $\to 0$

$\implies \tilde{L}, m_{bb}$ independent

Constraint:

limit correlation of m_{bb} and $\tilde{L}$

Towards a solution

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Lagrange multiplier
(constraint strength)

Mutual information* $\to 0$

$$\implies p(\tilde{L}, m_{bb}) = p(\tilde{L}) \cdot p(m_{bb})$$

Constraint:

limit correlation of
m_{bb} and $\tilde{L}$

Towards a solution

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$$\tilde{L}(\mathbf{e}) := \arg\max_f \Big(\mathscr{P}[f]$$

$$- \lambda \cdot \mathrm{MI}(f, m_{bb}) \Big)$$

Lagrange multiplier

(constraint strength)

Mutual information* $\rightarrow 0$

$$\Longrightarrow p(\tilde{L}, m_{bb}) = p(\tilde{L}) \cdot p(m_{bb})$$

Constraint:

limit correlation of m_{bb} and $\tilde{L}$ $\longleftrightarrow$ $\tilde{L}$: control distortion of m_{bb} by event selection

The final discriminant

$$\tilde{L}(\mathbf{e}) := \arg\max_f \Big(\mathscr{P}[f] - \lambda \cdot \mathbf{MI}(f, m_{bb}) \Big)$$

Into practice

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Into practice

candidate discriminant: **neural network**

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$$\tilde{L}(\mathbf{e}) := \arg\max_f \Big(\mathscr{P}[f] - \lambda \cdot \mathrm{MI}(f, m_{bb}) \Big)$$

numerical optimisation

gradient descent

Into practice

candidate discriminant: **neural network**

$$\tilde{L}(\mathbf{e}) := \arg\max_f \left(\mathscr{P}[f] - \lambda \cdot \mathrm{MI}(f, m_{bb}) \right)$$

numerical optimisation

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loss function

Into practice

candidate discriminant: **neural network**

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numerical optimisation

gradient descent

free parameter

loss function

Into practice

Evaluation: toy analysis, VH/bb

Into practice

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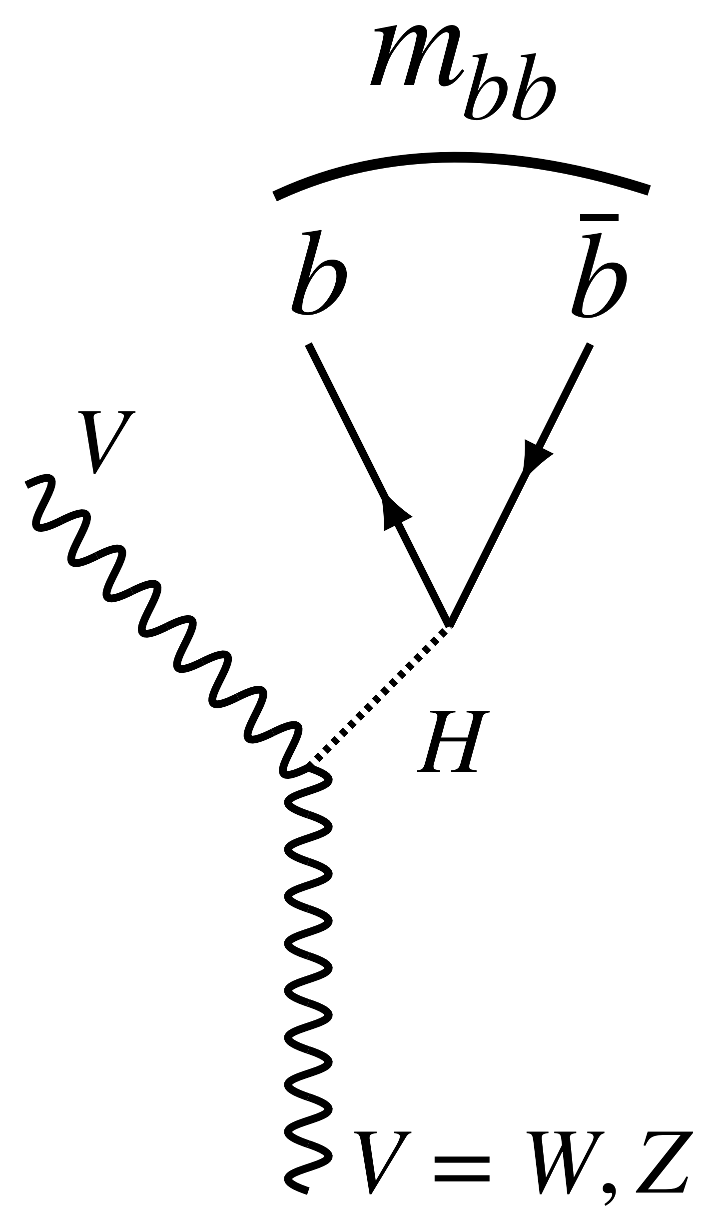


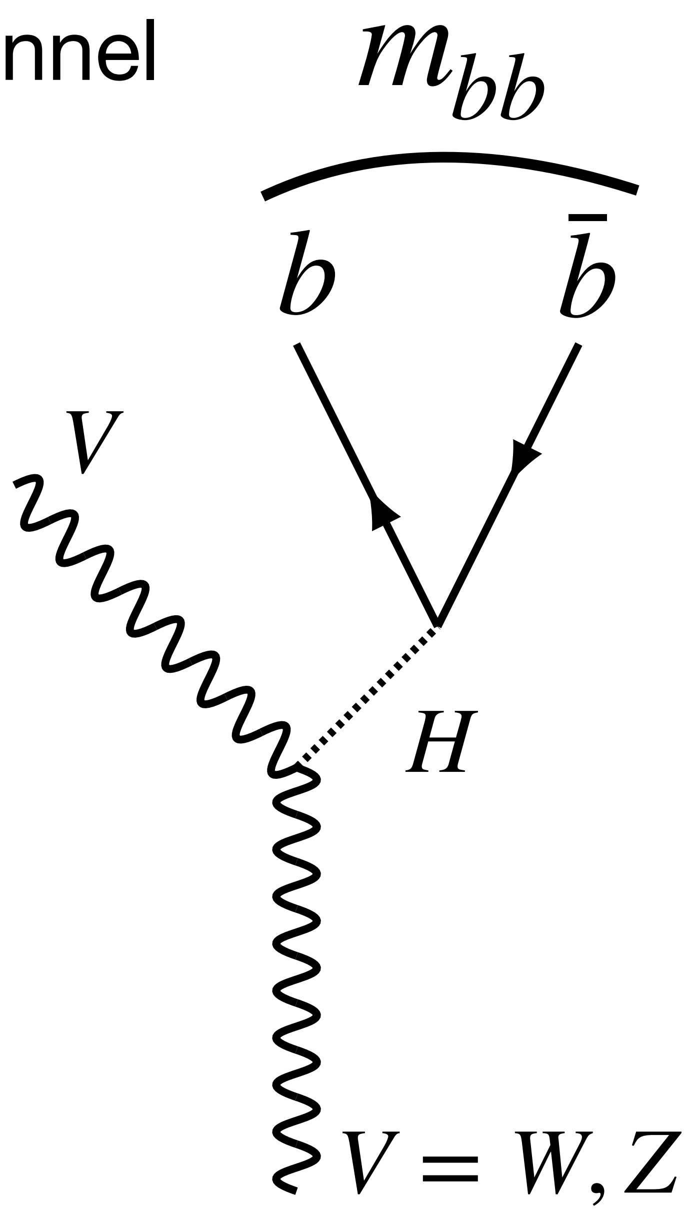
Into practice

Evaluation: **toy analysis**, VH/bb, 0-lepton channel

VH

$\hookrightarrow b\bar{b}$

$\hookrightarrow Z \to \nu\bar{\nu}, W \to \ell\nu$



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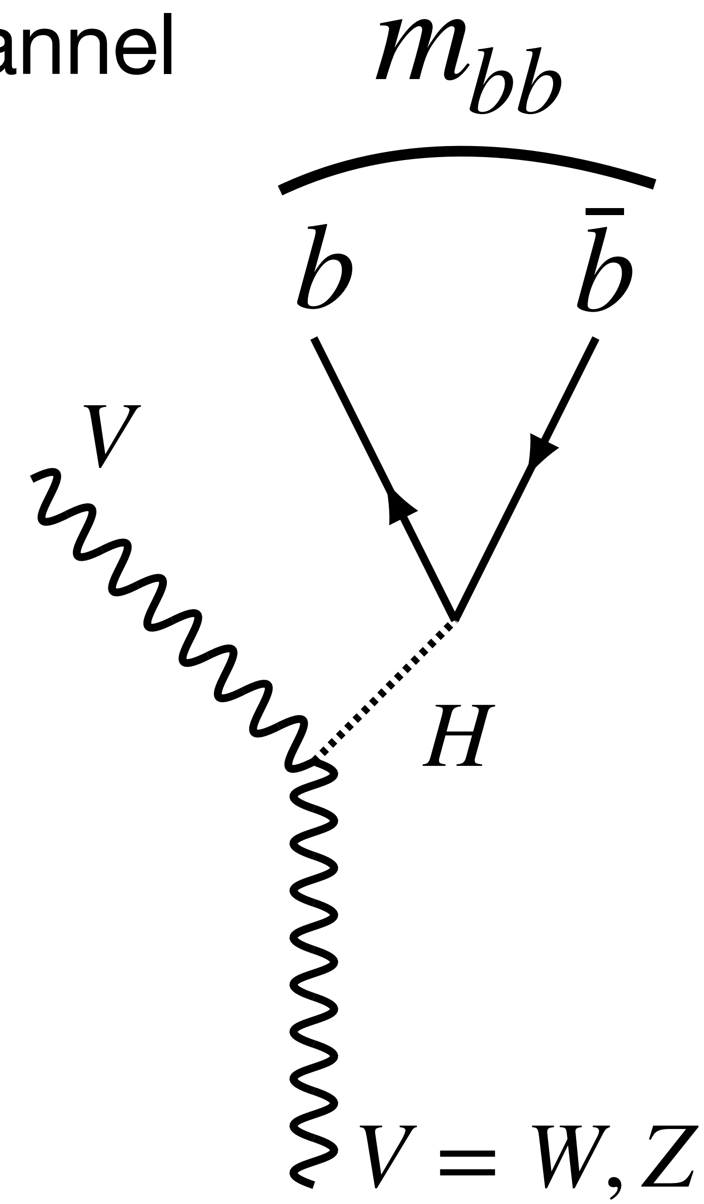

Setup:

Backgrounds:

- W+jets, Z+jets
- ttbar
- di-boson (ZZ, WZ)

Simulation: MadGraph (LO) + Pythia8 + Delphes

Into practice

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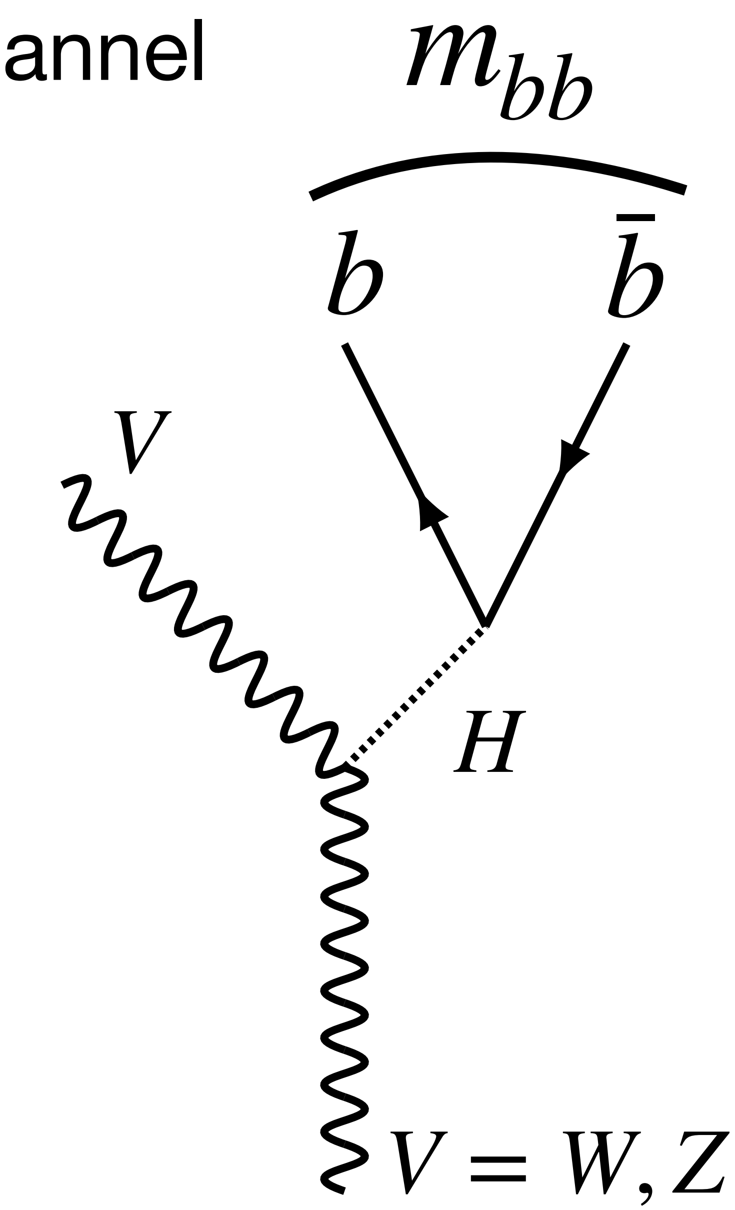


Setup:

Backgrounds:

- W+jets, Z+jets
- ttbar
- di-boson (ZZ, WZ)

Simulation: MadGraph (LO) + Pythia8 + Delphes

Event selection: 2 or 3 jets, 2 b-tags, MET, no leptons

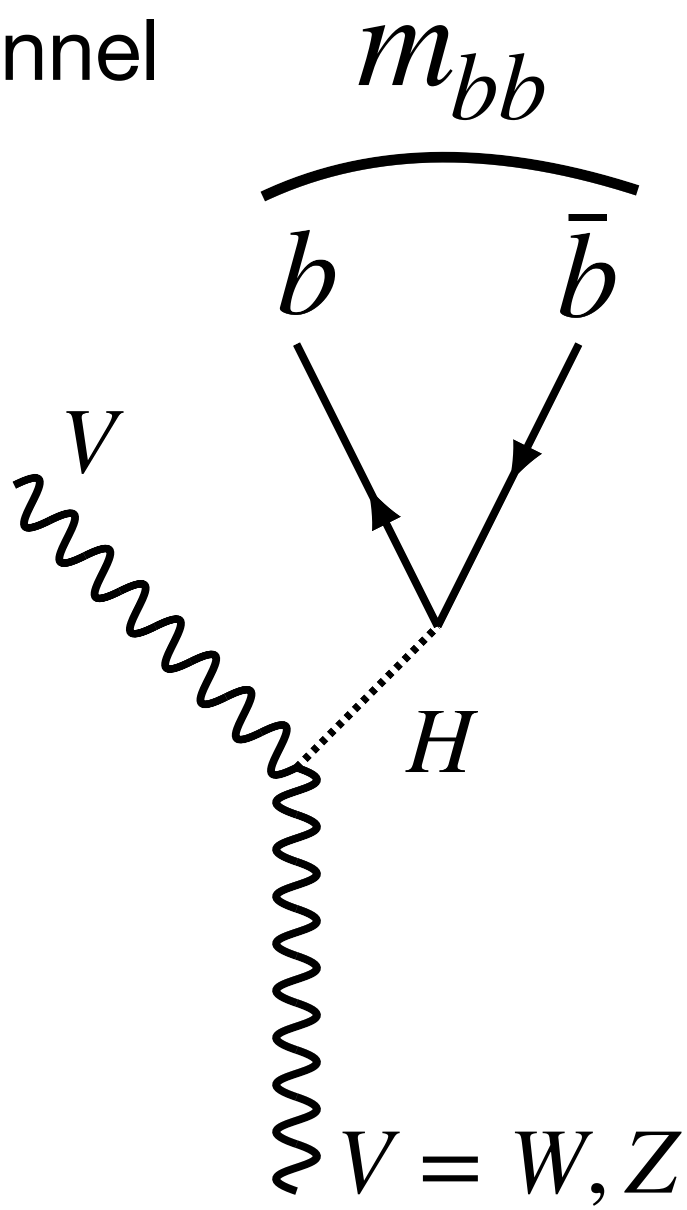
Into practice

Evaluation: toy analysis, VH/bb, 0-lepton channel

VH

$\quad \hookrightarrow b\bar{b}$

$\hookrightarrow Z \to \nu\bar{\nu}, W \to \ell\nu$



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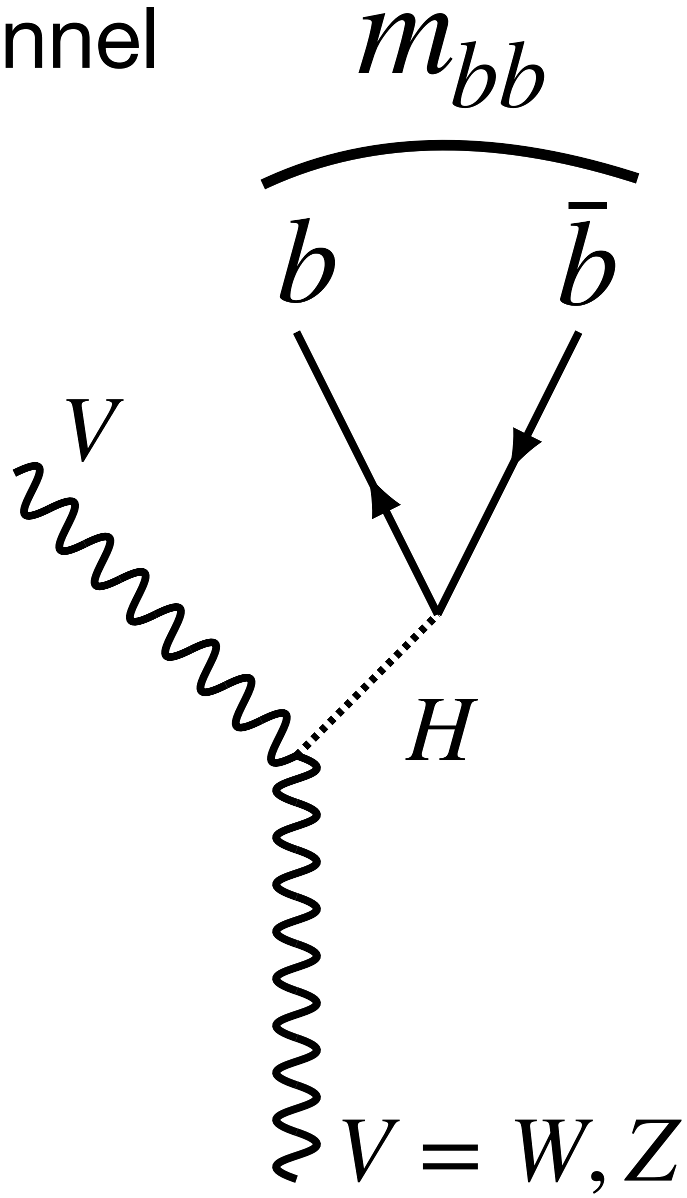

Signal regions:

	2-jet	3-jet
$\tilde{L}$ "tight"	**SR 1** m_{bb}	**SR 3** m_{bb}
$\tilde{L}$ "loose"	**SR 2** m_{bb}	**SR 4** m_{bb}

Into practice

Evaluation: **toy analysis**, VH/bb, 0-lepton channel

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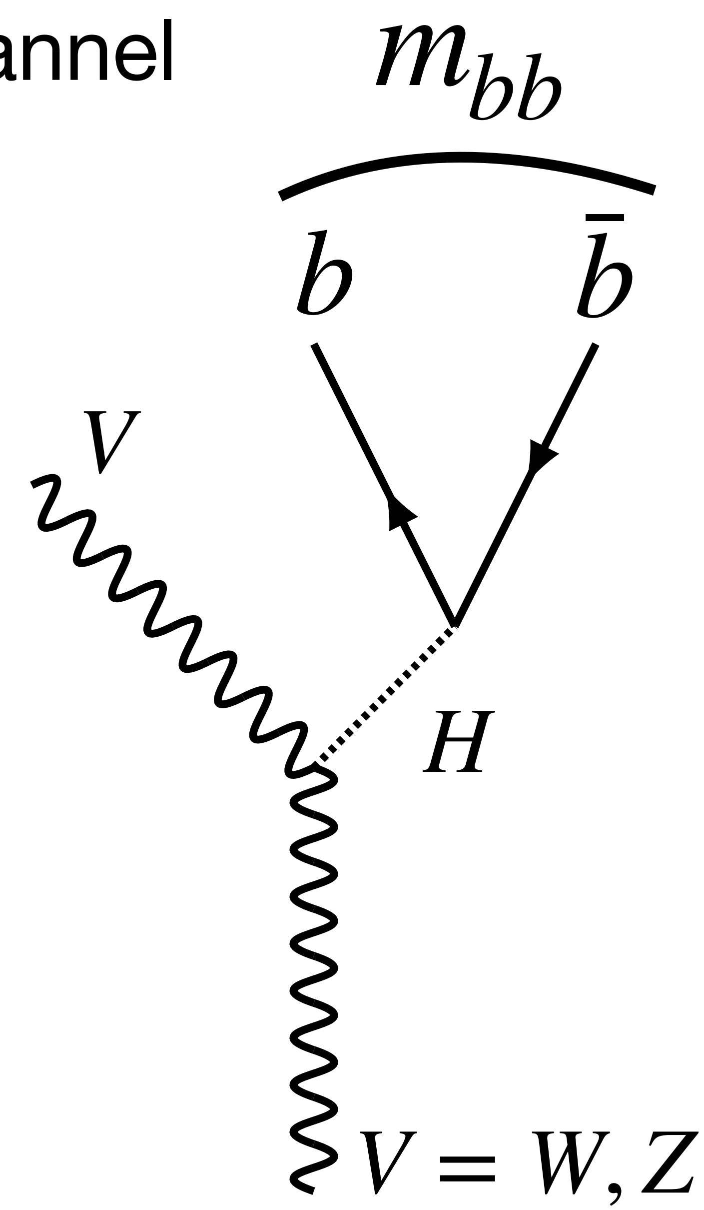

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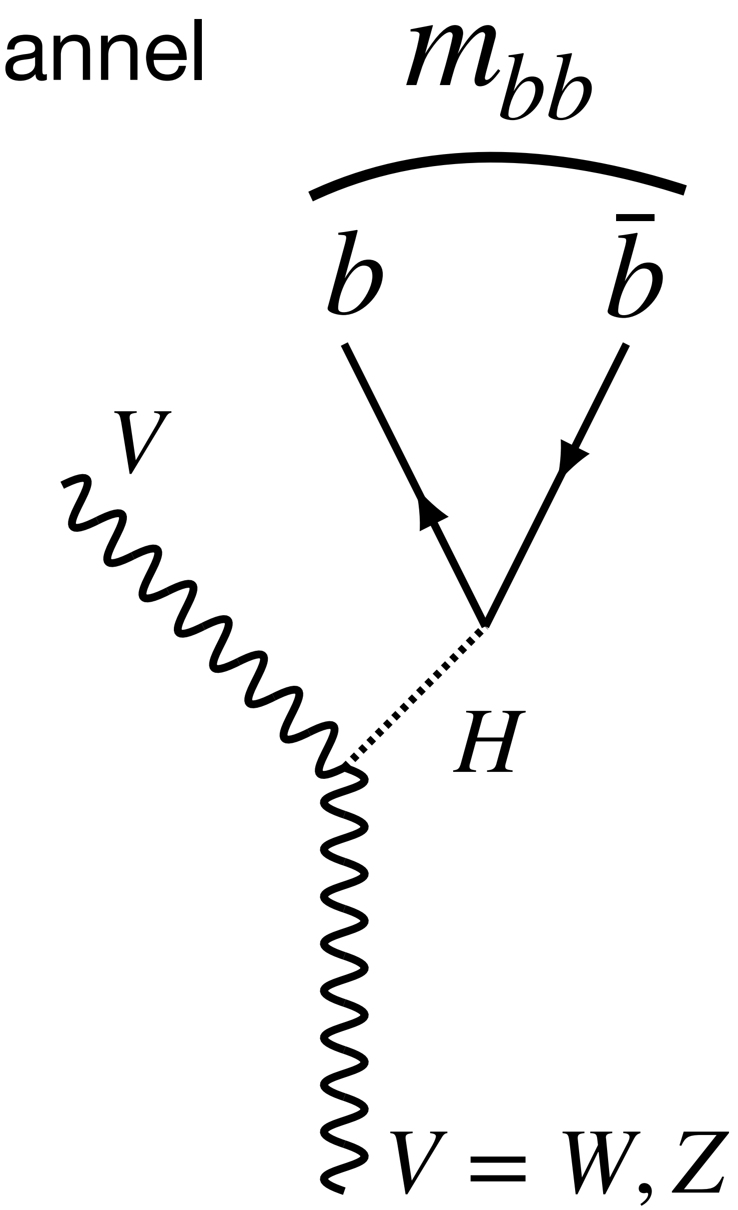
Figure of merit: Asimov fit, signal and all backgrounds floating

Into practice

Evaluation: toy analysis, VH/bb, 0-lepton channel

VH
- $\to b\bar{b}$
- $\to Z \to \nu\bar{\nu}, W \to \ell\nu$



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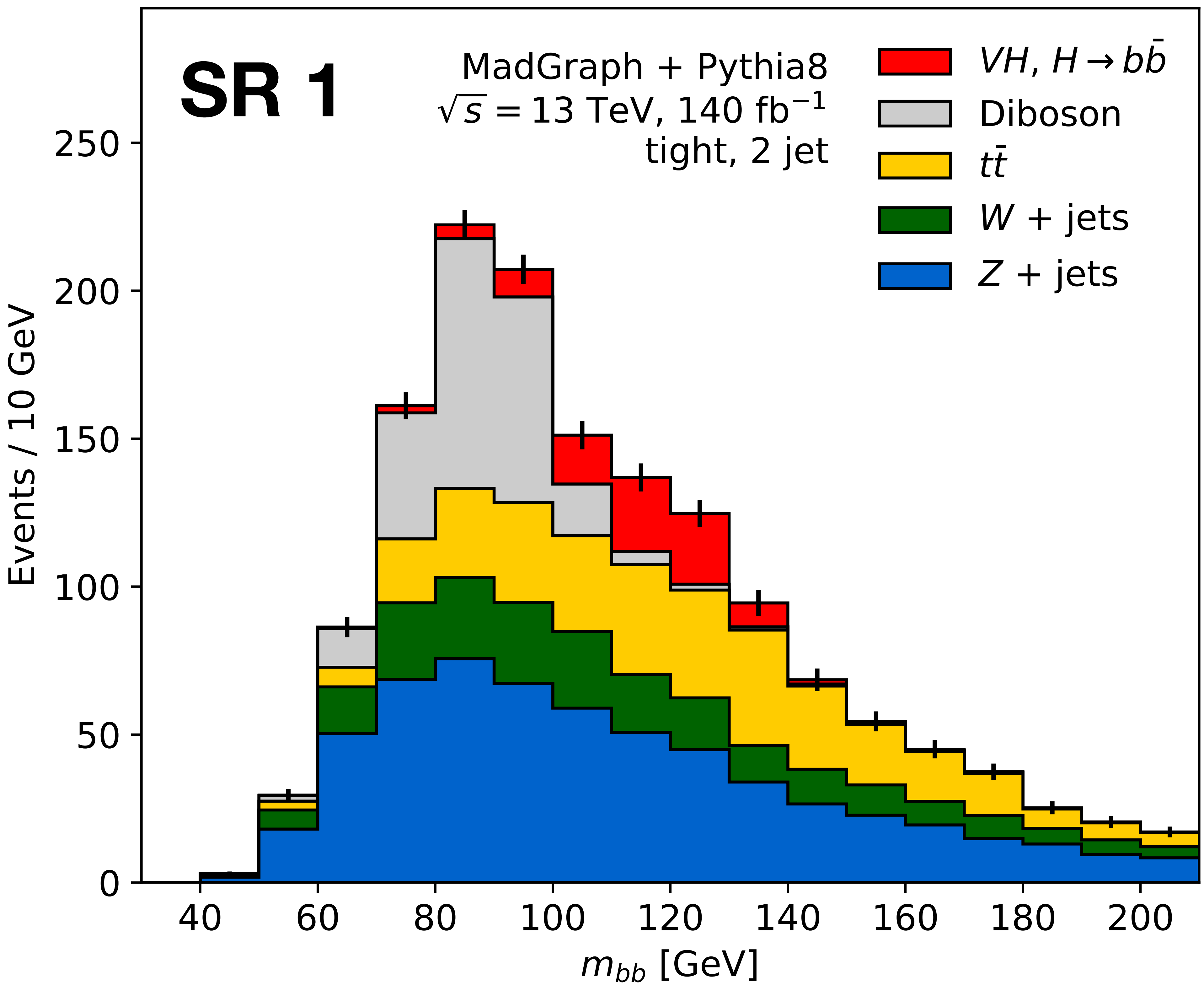

Into practice

$$\tilde{L}(\mathbf{e}) = \arg\max_f \left(\mathscr{P}[f] - \lambda \cdot \mathbf{MI}(f, m_{bb}) \right)$$

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$\lambda = 0$ $\lambda \gg 0$

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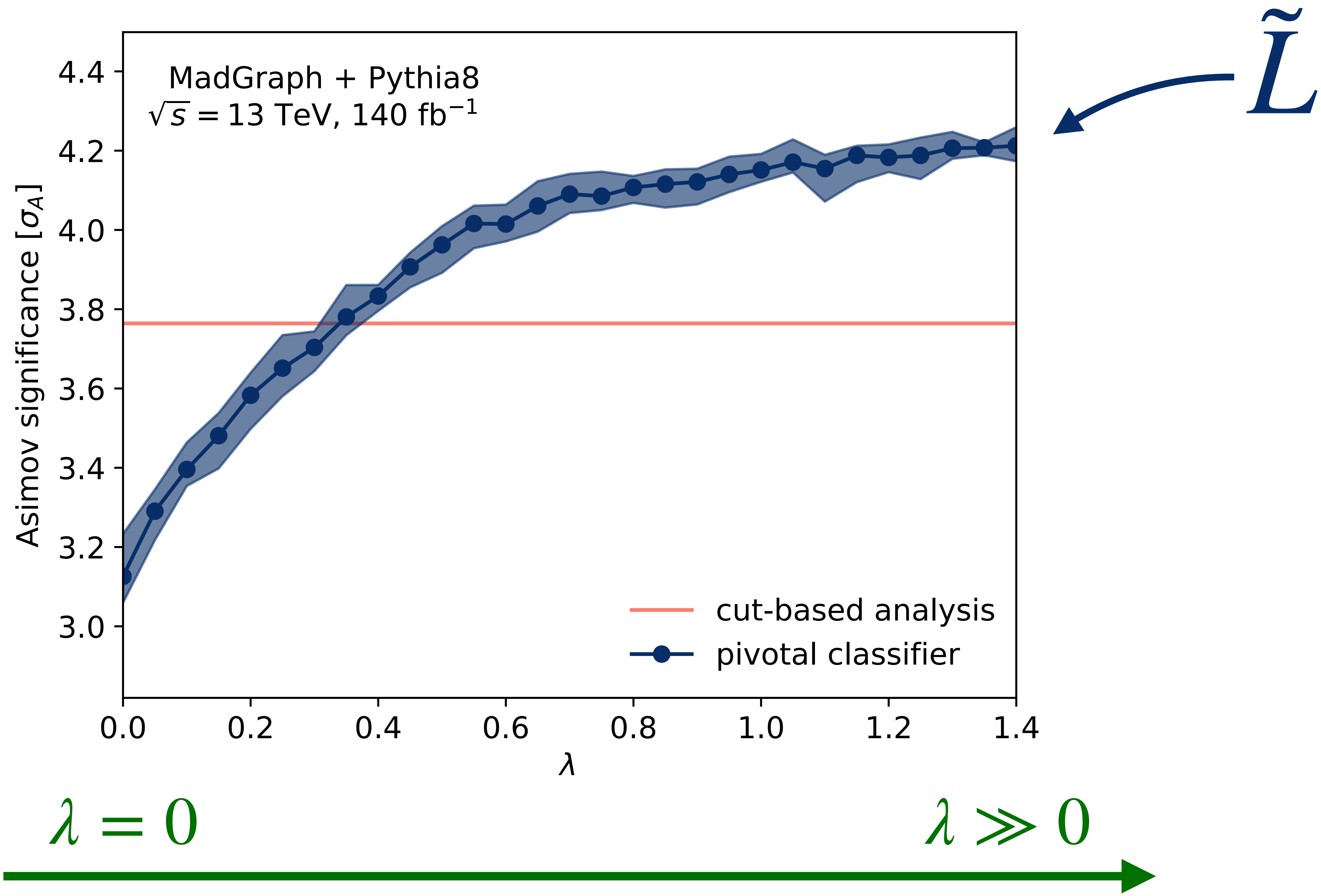

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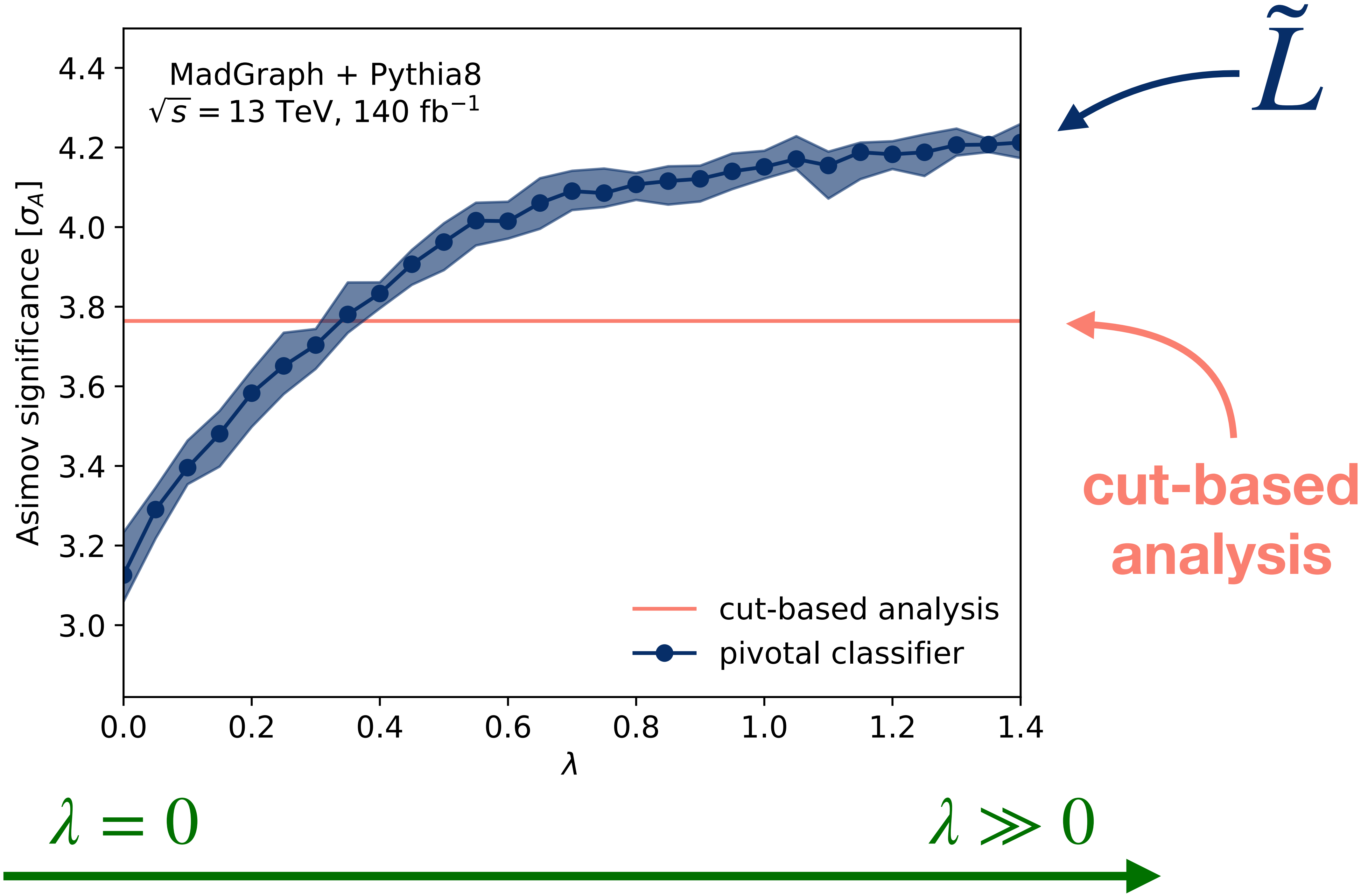

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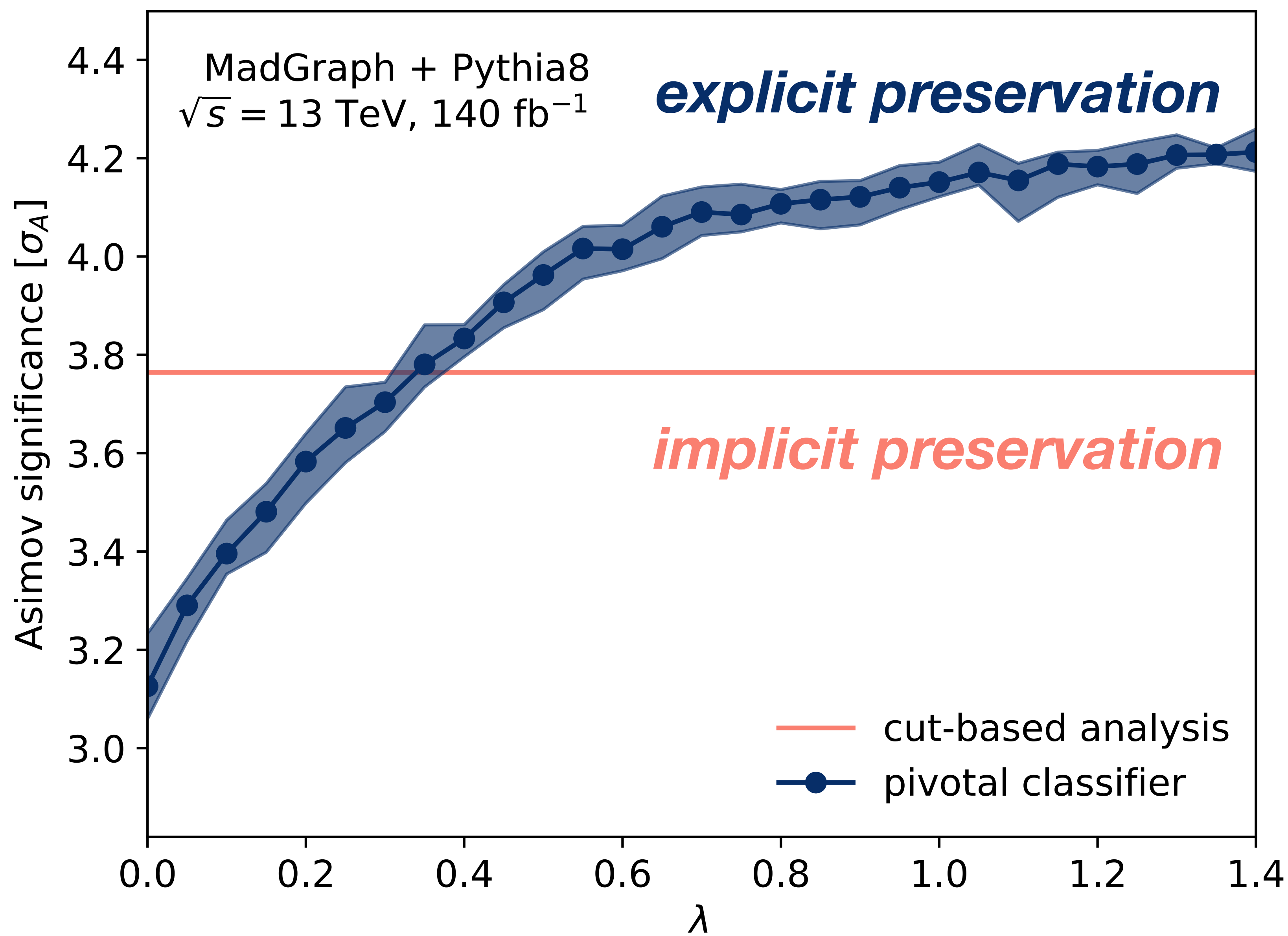

$\tilde{L}$

cut-based analysis

$\lambda = 0$ $\lambda \gg 0$

Summary

- Event selections *distort* distributions
    - Physical observables lose interpretation

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- Construct discriminants that obey auxiliary constraints
  - Require independence $\longleftrightarrow$ *preserve* shapes

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- Event selections *distort* distributions
  - Physical observables lose interpretation

- Construct discriminants that obey auxiliary constraints
  - Require independence $\longleftrightarrow$ *preserve* shapes

- **Ready to be used in practice**

More information:

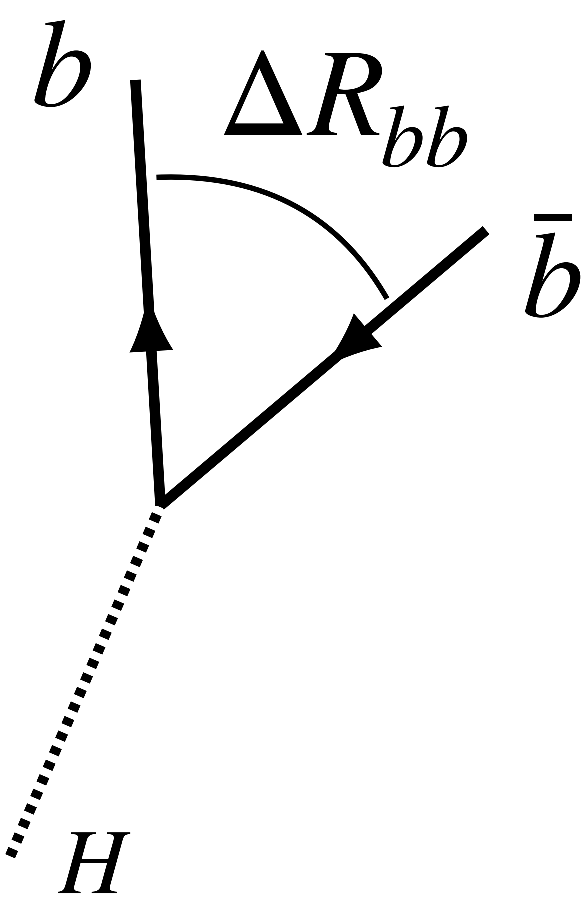
arXiv:1907.02098

Backup

Cut-based analysis

Signal regions: (inspired by a published ATLAS analysis)

	2-jet	3-jet
MET > 250 GeV $\Delta R_{bb} < 1.2$	**SR 1** m_{bb}	**SR 3** m_{bb}
250 > MET > 150 GeV $\Delta R_{bb} < 1.8$	**SR 2** m_{bb}	**SR 4** m_{bb}



Optimise cuts to maximise Asimov significance

“Pivotal classifier” analysis

Signal regions:

		2-jet	3-jet
$\tilde{L}$	“tight”	**SR 1** m_{bb}	**SR 3** m_{bb}
$\tilde{L}$	“loose”	**SR 2** m_{bb}	**SR 4** m_{bb}

- Cuts on $\tilde{L}$ designed to achieve the same signal efficiency in each region

Mutual Information

$\mathbf{f, m_{bb}}$ **independent** $\Longrightarrow$ $\mathbf{f, m_{bb}}$ **linearly uncorrelated**

Mutual Information

$$\mathrm{MI} = 0$$

$$\Downarrow$$

no nonlinear relationship

Pearson correlation

$$\rho = 0$$

$$\Downarrow$$

no linear relationship

Mutual Information

Two random variables:

$$X \qquad\qquad Y$$

Objects that characterise them:

$$p_X(x) \qquad\qquad p_Y(y)$$

$p_{(X,Y)}(x, y)$ … joint probability distribution

Mutual Information:

$$\text{MI}(X, Y) = D_{\text{KL}}\left(p_{(X,Y)} \,||\, p_X \cdot p_Y\right)$$

Kullback-Leibler divergence

"distance" between densities

Mutual Information

Kullback-Leibler divergence:

$$D_{\mathrm{KL}}(P\,||\,Q) = \int \mathrm{d}x\, p(x)\, \log\left(\frac{p(x)}{q(x)}\right)$$

"Distance" between probability distributions p and q.

Mutual information:

$$\mathrm{MI}(X,Y) = D_{\mathrm{KL}}\left(p_{(X,Y)}\,||\,p_X p_Y\right)$$

"Distance" between joint PDF and product of marginal PDFs.

$$\mathrm{MI}(X,Y) = \int \mathrm{d}x\,\mathrm{d}y\, p_{(X,Y)}(x,y)\, \log\left(\frac{p_{(X,Y)}(x,y)}{p_X(x)\,p_Y(y)}\right)$$

How to compute Mutual Information?

MI can recognise arbitrary complicated dependencies between random variables…

$$\mathrm{MI}(X, Y) = \int \mathrm{d}x\, \mathrm{d}y\, p_{(X,Y)}(x, y)\, \log\left(\frac{p_{(X,Y)}(x, y)}{p_X(x)\, p_Y(y)}\right)$$

… but requires knowledge about distributions.

MI also admits a functional definition:

$$\mathrm{MI}(X, Y) = \sup_{T \in \mathscr{F}} \langle T \rangle_{p(X,Y)} - \langle e^{T-1} \rangle_{p(X)p(Y)}$$

How to compute Mutual Information?

$$\tilde{L}(\mathbf{e}) := \arg\max_f \Big(\mathscr{P}[f] - \lambda \cdot \mathrm{MI}(f, m_{bb}) \Big)$$

neural network ($\mathscr{P}[f]$)

loss function ($\mathscr{P}[f] - \lambda \cdot \mathrm{MI}(f, m_{bb})$)

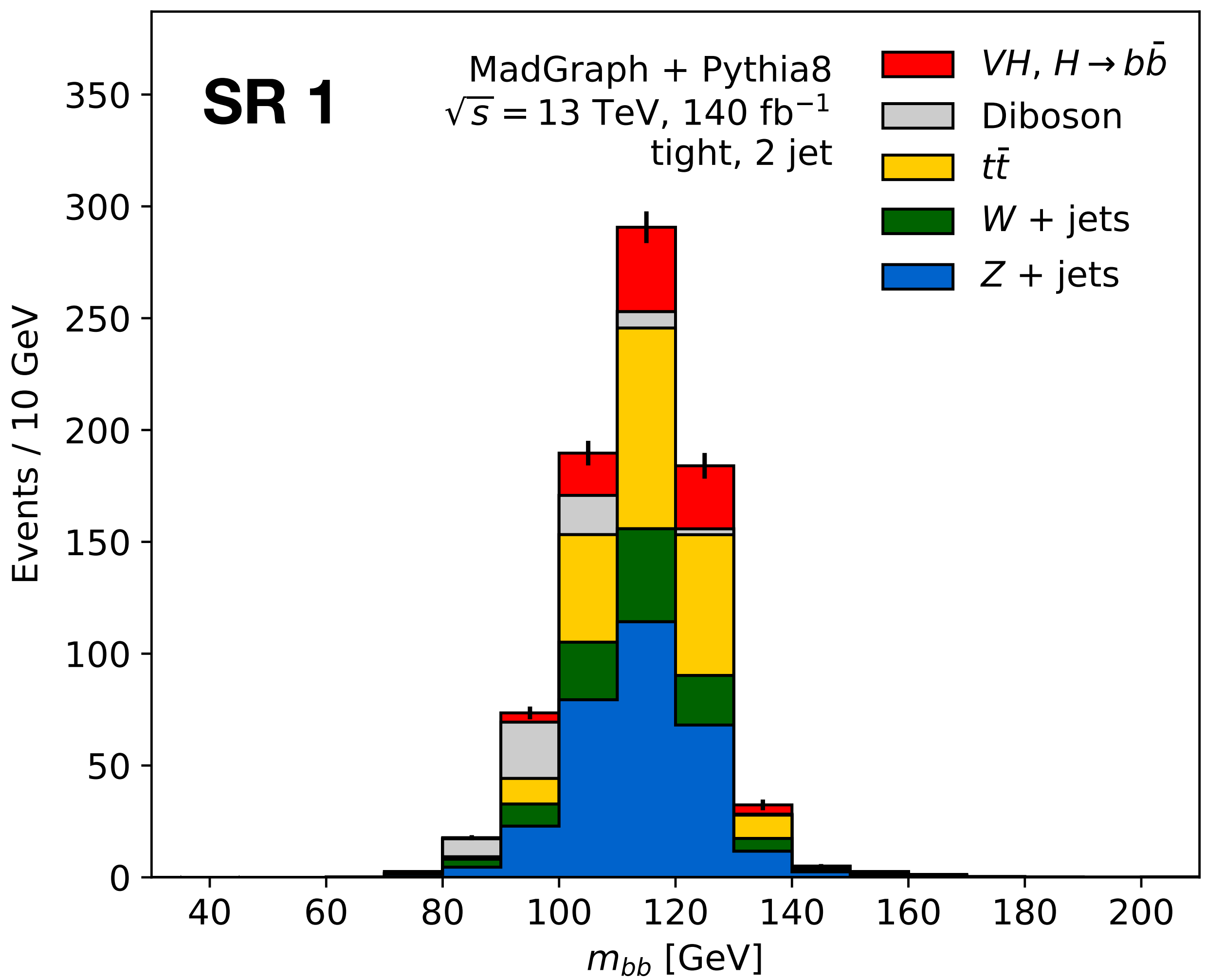
must be *differentiable* w.r.t. f

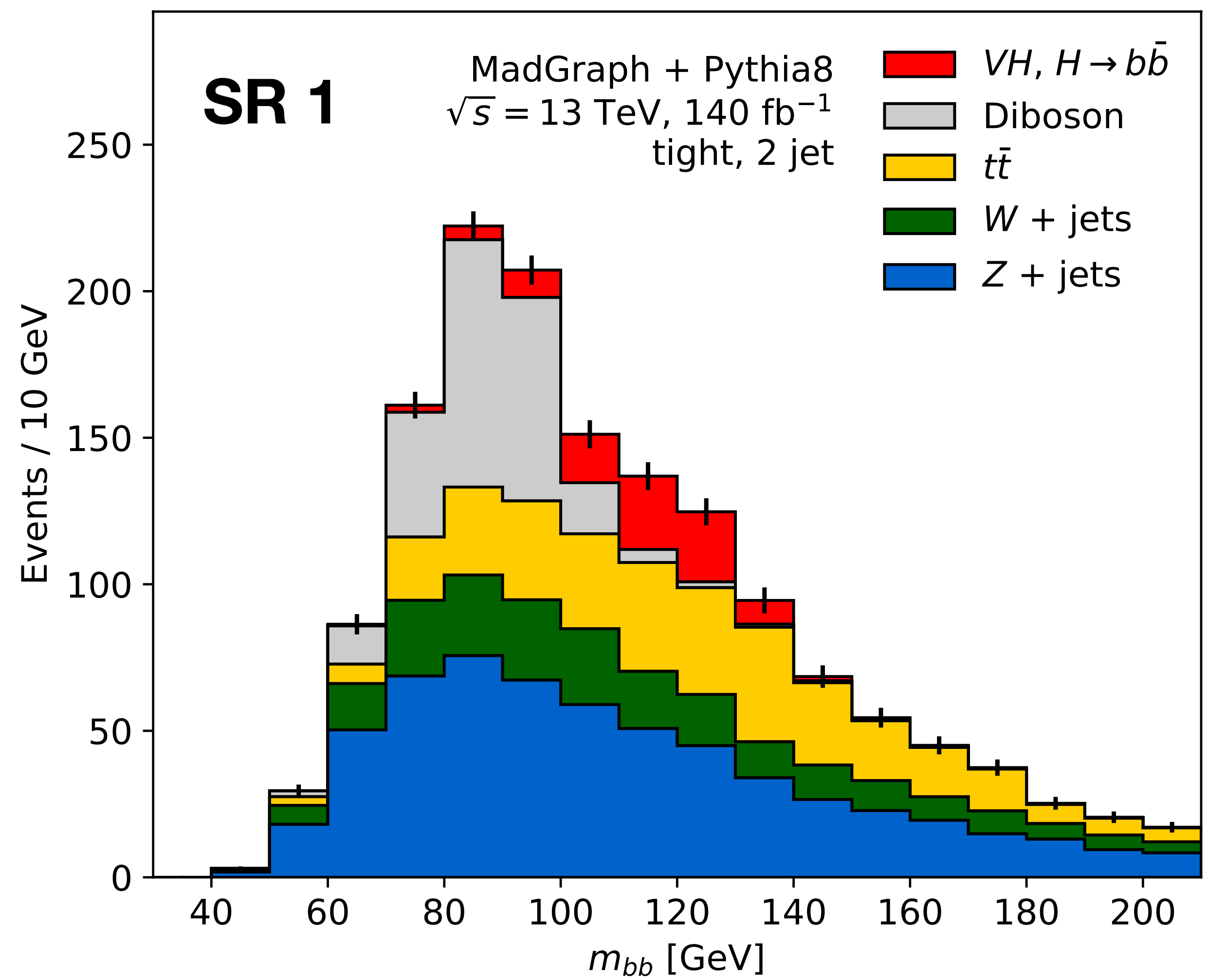
$$\mathrm{MI}(X,Y) = \sup_{T \in \mathscr{F}} \left(\langle T \rangle_{p(X,Y)} - \langle e^{T-1} \rangle_{p(X)p(Y)} \right)$$

adversarial **neural network** (T)

Into practice

$$\tilde{L}(\mathbf{e}) = \arg\max_f \left(\mathscr{P}[f] - \lambda \cdot \mathrm{MI}(f, m_{bb}) \right)$$

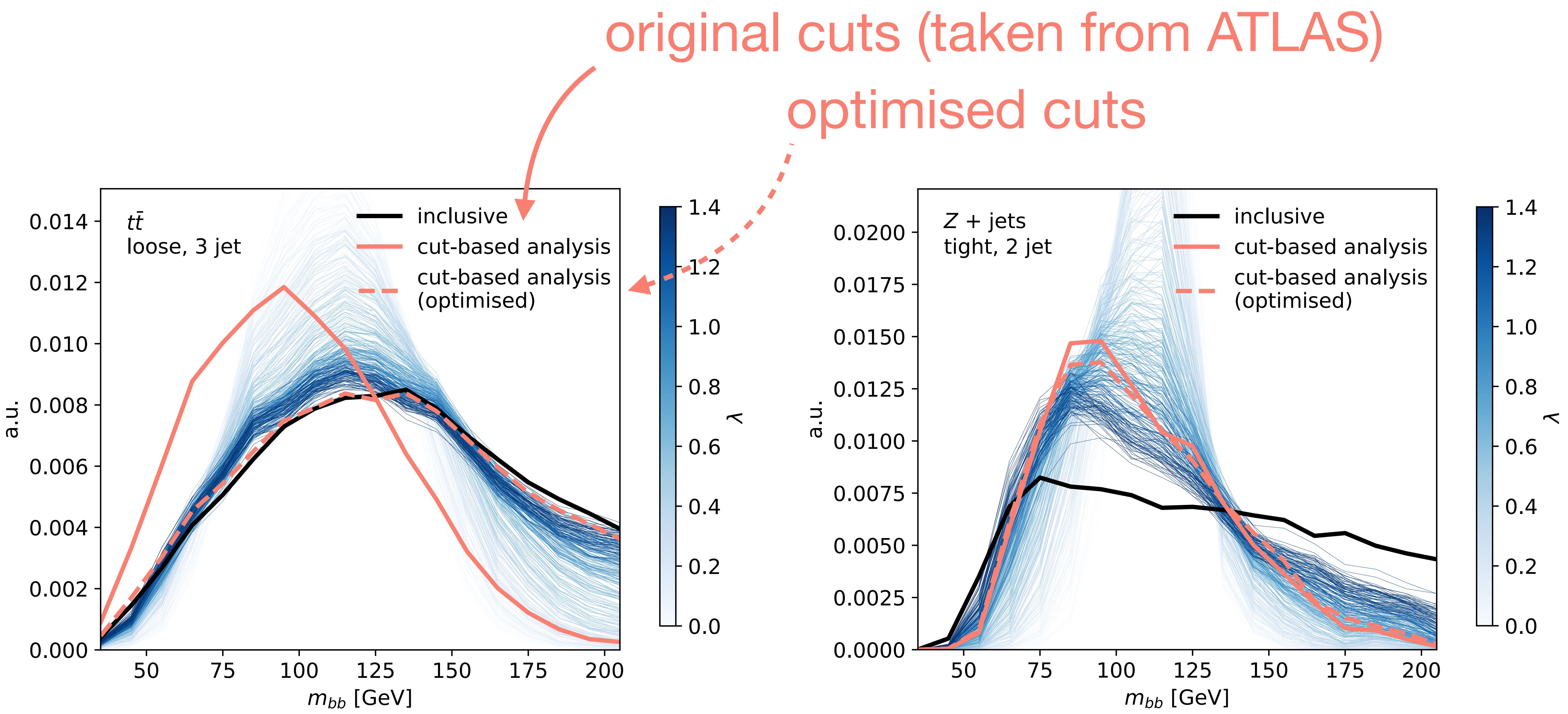




$\lambda = 0$ → $\lambda \gg 0$

More performance plots

Comparison: cut-based vs. pivotal classifier, mass shapes



More performance plots

Comparison: cut-based vs. pivotal classifier, significances