Double-Higgs boson production at NLO

Combine numerical evaluation and analytic high energy approximation

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[arXiv:1907.06408]
\[ \lambda_{HHH} \text{ in the Standard Model} \]

Higgs potential

\[ V(H) = \frac{1}{2} m_H^2 H^2 + \lambda_{HHH} v H^3 + \frac{1}{4} \lambda_{HHHH} H^4 \]

in SM:

\[ \lambda_{HHH} = \frac{m_H^2}{2v^2} = 0.13 \ldots \]

(not directly measured)

[CMS: arXiv:1811.09689]:

\[-11.8 < \lambda / \lambda_{SM} < 18.8\]

[ATL-PHYS-PUB-2019-009]:

\[-3.2 < \lambda / \lambda_{SM} < 11.9\]
Summary/Conclusions

**HL-LHC**: potential for new physics discoveries and precision measurements in the Higgs sector:

- Few per-cent level precision on most Higgs cross sections and couplings
- Significance of about $2.6\sigma$ for HH production $\rightarrow$ triple self coupling
- Higgs width measurable to within 1 MeV
- Sensitivity to BSM effects in Higgs physics derived

Many inclusive measurements limited by systematic uncertainties $\rightarrow$ work needed from theoretical and experimental side

An exciting journey ahead!

N. De Filippis

Sept 30 - Oct 4, Higgs couplings 2019
The simplest process is Higgs pair production.
Previous works

exact analytic@LO
[Eboli, Marques, Novaes, Natale, ’87, Glover, van der Bij ’88, Plehn, Spira, Zerwas, ’96]

Born-improved HEFT@NLO
[Dawson, Dittmaier Spira, ’98]

FT$\text{approx}$, FT$'\text{approx}$
[Maltoni, Vryonidou, Zaro, ’14]

HEFT@NNLO with 1/mt corr.
[Grigo, Hoff, Melnikov, Steinhauser, ’13,
Grigo, Melnikov, Steinhauser, ’14,
Grigo, Hoff, Steinhauser, ’15, Degrassi, Giardino, Gröber, ’16]

exact numerical@NLO
[Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Zicke, ’16,
Baglio, Campanario, Glaus, Mühlleitner, Spria, Streicher, ’18]

Padé approximation using the large top-mass and the threshold expansion@NLO
[Gröber, Maier Rauh, ’17]

small $p_T$ expansion@NLO
[Bonciani, Degrassi, Giardino, Gröber, ’18]
Our work

two-loop integrals: high-energy approximation and numerical evaluation
**high-energy approximation**


Expand each Feynman diagrams by means of the method of region. [Beneke, Smirnov ’97, Smirnov ’02, Jantzen ’11]

\[
\sum \quad = \sum_{n_h=0,1} \left( \frac{m_h^2}{s} \right)^{n_h} \sum_{n_t} \left( \frac{m_t^2}{s} \right)^{n_t} c_{n_h,n_t}
\]

Cross section is expressed as a series in \( m_t \) (we have obtained up to \( m_t^{32} \)).

\[
\delta \sigma_{\text{virtual}}^{\text{NLO}} = \sum_{n_h=0,1} \left( \frac{m_h^2}{s} \right)^{n_h} \sum_{n_t} \left( \frac{m_t^2}{s} \right)^{n_t} d_{n_h,n_t}
\]

Apply the Padé approximation with respect to \( m_t \).

\[ f_0 + f_1 x + \cdots + f_{n+m} x^{n+m} \rightarrow \frac{a_0 + a_1 x + \cdots + a_n x^n}{1 + b_1 x + \cdots + b_m x^m} \]

\[ \text{Re} \left[ G_{59}(1,1,1,1,1,1,-2,0) \right] \quad \text{at}\ 	ext{O}(\epsilon^0) \]

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Double-Higgs boson production at NLO

Go Mishima: Karlsruhe Institute of Technology (KIT), Higgs Coupling 2019, Sep. 30 - Oct. 4, Oxford
numerical evaluation


Numerically evaluated two-loop integrals (virtual correction) combined with parton showers within the POWHEG-BOX-V2 and MG5_aMC@NLO frameworks.

talk slide by Keith Hamilton on 1st Oct.

We are trying to improve this!

- Pioneering 2 → 2 two-loop calculations with internal and external mass scales
- NLO corr's to HH with finite \( m_t \) are large and not captured by \( m_t \rightarrow \infty \) K-factor
- NLOPS simulations implemented; variable \( \lambda_{HH} \) recently added in POWHEG
- NLO corr's to HJ with finite \( m_t \) are large but are captured by \( m_t \rightarrow \infty \) K-factor

Two-loop integrals: evaluated points are increased: 3398 → 6320
Complementarity of HE approximation and numerics

Double-Higgs boson production at NLO

**Go Mishima**: Karlsruhe Institute of Technology (KIT), Higgs Coupling 2019, Sep. 30 - Oct. 4, Oxford
In the following we discuss di...

For our analysis we use the parton distribution functions from the improved version of the grid and the solid lines are based on the Padé-improved high-energy expansion.

On the basis of this observation we extend the grid provided in [42] as follows: We increase the number of points computed using the full NLO result from 3398 to 6320. The new points are sampled according to the distribution of unweighted events and, therefore, populate the same kinematic regime as the original points.

We now define a criterion which provides a prescription for the improvement of the grid [42]. In order to have guidance we show in Fig. 6 the relative uncertainty of the Padé results in the grid [42]. The solid lines are based on the Padé-improved differential distributions, and in the lower panels we display the ratio of the different versions of the NLO prediction, all of which contain larger values of the uncertainties of for fixed values of the centre-of-mass energies.

For high values of the centre-of-mass energies,

$$s \approx 1 \frac{g_{NLO} \times \delta_{\text{virtual}}}{s_{\text{NLO}}}$$

We choose different values of the centre-of-mass energies.

The LO values are shown in black and the corresponding uncertainty bars are obtained from the grid [42]. The solid lines are based on the Padé-improved high-energy expansion. For high values of the centre-of-mass energies, the uncertainty grows towards lower values. Still, even for relatively quickly. Still, even for

$$s \approx 1 \frac{g_{NLO} \times \delta_{\text{virtual}}}{s_{\text{NLO}}}$$

the kinematic boundary is obtained from the requirement that

$$1 \frac{g_{NLO} \times \delta_{\text{virtual}}}{s_{\text{NLO}}}$$

is around a few percent for most values of the kinematic boundary. Note that the kinematic boundary is obtained from the requirement that

$$1 \frac{g_{NLO} \times \delta_{\text{virtual}}}{s_{\text{NLO}}}$$

is small.

The data points and the theoretical predictions are compared in Fig. 8. We use the parton distribution functions of [51].
Combine HE approximation and numerics

- For $p_s < 700$ GeV and $p_T > 200$ GeV we add points from the Padé approximation.
- The boundary above which we include points from the Padé approximation is denoted as a yellow line in Fig. 6. We note here that if one reproduces Figs. 5 and 6 using the 6320 points described above the behaviour is qualitatively the same and we therefore refrain from showing them in this paper.
- In Fig. 7 we compare the Padé results to the improved version of the grid, which provides precise results in the whole relevant phase space. We note that the wiggly behaviour and the deviation of the grid data points from the Padé approximation for larger values of $p_s$ and smaller values of $p_T$ could be improved by including further data points from the Padé approximation. This behaviour would then be pushed to higher values of $p_s$.
- We judge the performance of the grid as displayed by Fig. 7 to be sufficient for the phenomenological applications of this paper, and further improvements of the grid not to be necessary. This improved grid can be downloaded from [42].
New grid

Figure 7: Scatter plots of $\Delta \sigma_{\text{virtual}}$ (NLO) and $\Delta \sigma_{\text{virtual}}$ (LO) as a function of the centre-of-mass energy $\sqrt{s}$ for different charged hadronic $p_T$ values. The data points are obtained from the improved version of the grid and the solid lines are based on the Padé-improved differential distributions w.r.t. the Higgs boson pair invariant mass. The colour codes correspond to different values of $m_H$: black (100 GeV), red (150 GeV), yellow (200 GeV), magenta (250 GeV), cyan (300 GeV), green (350 GeV), and blue (400 GeV). The errors shown are statistical, and uncertainties due to the higher-order factorization and renormalization scales are discussed in the text.
Result: $p_T$ distribution at 14 TeV

![Graph showing $p_T$ distribution at 14 TeV](image)

**Legend:**
- LO
- NLO FTapprox
- NLO Grid
- NLO Grid + Padé

**Note:**
- The graph shows the differential cross section $d\sigma/dp_{T,h}$ in [fb/GeV] for a hadronic centre-of-mass energy $p_s = 14$ TeV.
- The curves represent different theoretical predictions at NLO.
- Each curve corresponds to a different scheme for incorporating virtual corrections:
  - **Black Curve (LO):** Incorporates only the LO and the virtual corrections.
  - **Blue Curve (NLO FTapprox):** Computes the virtual corrections in the infinite top quark mass limit and rescales by the exact LO prediction.
  - **Red Curve (NLO Grid):** Based on the grid constructed in Ref. [13] but improved by increasing the number of points from 3398 to 6320.
  - **Green Curve (NLO Grid + Padé):** Combination of the NLO Grid and Padé approximants.

**Discussion:**
- The green curve is considered the best prediction.
- The grey and green bands represent the uncertainty due to variations in $\mu_R$ and $\mu_F$.
- The $K$ factor is reduced from $K_\pi \approx 1.7$ to $K_\pi \approx 1.5$ by including high-energy results in the grid.

**Observation:**
- For small $m_{hh}$ and $p_{T,h}$, there is perfect agreement between the red and green curves.
- For higher values, a difference is observed, with the red curve falling within the green uncertainty band.
- For $p_{T,h} = 2000$ GeV, the $K$ factor is reduced from $K_\pi \approx 1.7$ to $K_\pi \approx 1.5$.

**Conclusion:**
- The results highlight the importance of using accurate theoretical predictions, especially at high energies.
- Variations in the virtual corrections and the implementation of these corrections have a significant impact on the final predictions.
Result: $m_{hh}$ distribution at 14 TeV

LHC 14 TeV
PDF4LHC15 NLO
\( \mu = m_{hh}/2 \)

$K$ factor

$\frac{d\sigma}{d m_{hh}}$ [fb/GeV]

$m_{hh}$ [GeV]
**Result:** $p_T$ distribution at 100 TeV

- **Legend:**
  - LO
  - NLO FTapprox
  - NLO Grid
  - NLO Grid + Padé

- **Axes:**
  - $d\sigma/dp_{T,h}$ [fb/GeV]
  - $p_{T,h}$ [GeV]

- **Data Points:**
  - FCC 100 TeV
  - PDF4LHC15 NLO
  - $\mu = m_{hh}/2$

- **K factor:**
  - 0.5
  - 1.0
  - 1.5
  - 2.0

**Conclusions**

We provide optimized predictions for the NLO corrections to Higgs boson pair production by combining the exact numerical results with analytic expressions for the form factors obtained in a high-energy expansion. For the latter the region of convergence is significantly improved by constructing Padé approximants, which are validated at the level of master integrals. Furthermore, we identify regions in the phase space where both the exact numerical evaluations and the Padé results provide precise predictions and find good agreement. We thus combine both approaches and generate a new grid which is available from [42]. The analytic expressions for the high-energy expansion of the form factors are available from [48].

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Figure 9: $m_{hh}$ and $p_{T,h}$ distributions for a hadronic centre-of-mass energy $\sqrt{s} = 100$ TeV.

The improved grid for phenomenological analyses, if one wishes to consider large values of $m_{hh}$ or $p_{T,h}$, even for $\sqrt{s} = 14$ TeV. In these regions the predictions based on "FTapprox" deviate significantly from the green curve.

Conclusions

We provide optimized predictions for the NLO corrections to Higgs boson pair production by combining the exact numerical results with analytic expressions for the form factors obtained in a high-energy expansion. For the latter the region of convergence is significantly improved by constructing Padé approximants, which are validated at the level of master integrals. Furthermore, we identify regions in the phase space where both the exact numerical evaluations and the Padé results provide precise predictions and find good agreement. We thus combine both approaches and generate a new grid which is available from [42]. The analytic expressions for the high-energy expansion of the form factors are available from [48].

We apply the improved grid to phenomenological studies of the Higgs boson pair invariant mass and Higgs boson transverse momentum distributions at LHC energies and for $\sqrt{s} = 100$ TeV. We show that at high energies the improvements are noticeable and we recommend to use the updated grid for phenomenological studies, even for $\sqrt{s} = 14$ TeV.

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Summary

• We have improved the NLO virtual corrections to the Higgs pair production cross section via gluon fusion by combining numerical evaluation and the high-energy approximation.

• The two method agree when $200 \text{ GeV} < p_T < 400 \text{ GeV}, \sqrt{s} < 800 \text{ GeV}$

• Padé improved high-energy approximation provides reasonable results even down to $p_T \approx 150 \text{ GeV}$

• The updated grid is available at https://github.com/mppmu/hhgrid