# Higgs Boson Pair Production via Gluon Fusion: NLO QCD Corrections

Higgs Couplings 2019, Oxford

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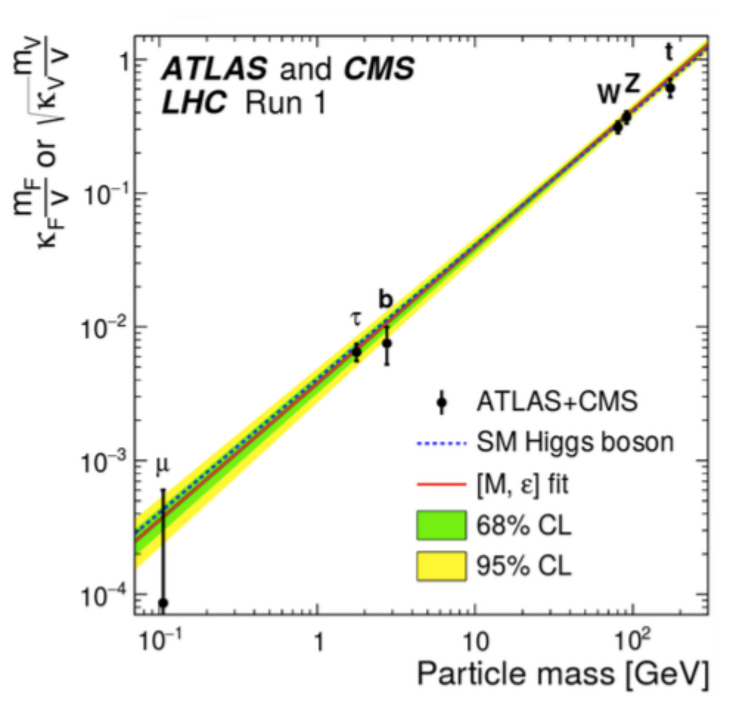
# Motivation

- Detection of a Higgs boson with a mass ~ 125 GeV
- Higgs mass, coupling strengths, spin and CP already determined
- *S*elf-coupling strength still unknown



$$V(\phi) = \frac{\lambda}{2}\left\{|\phi|^2 - \frac{v^2}{2}\right\}^2$$

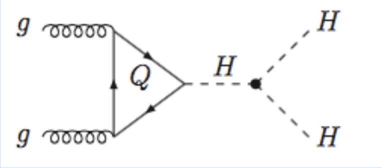
$$\lambda_{h^3} = 3\frac{m_h^2}{v}$$

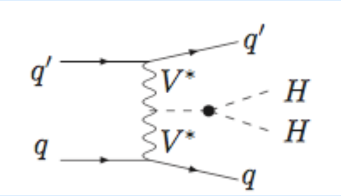
$$\lambda_{h^4} = 3\frac{m_h^2}{v^2}$$

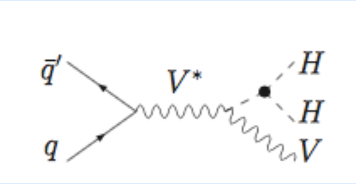
# Motivation

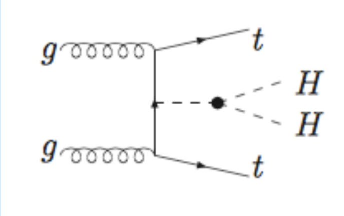
## Higgs boson pair production

### Production channels

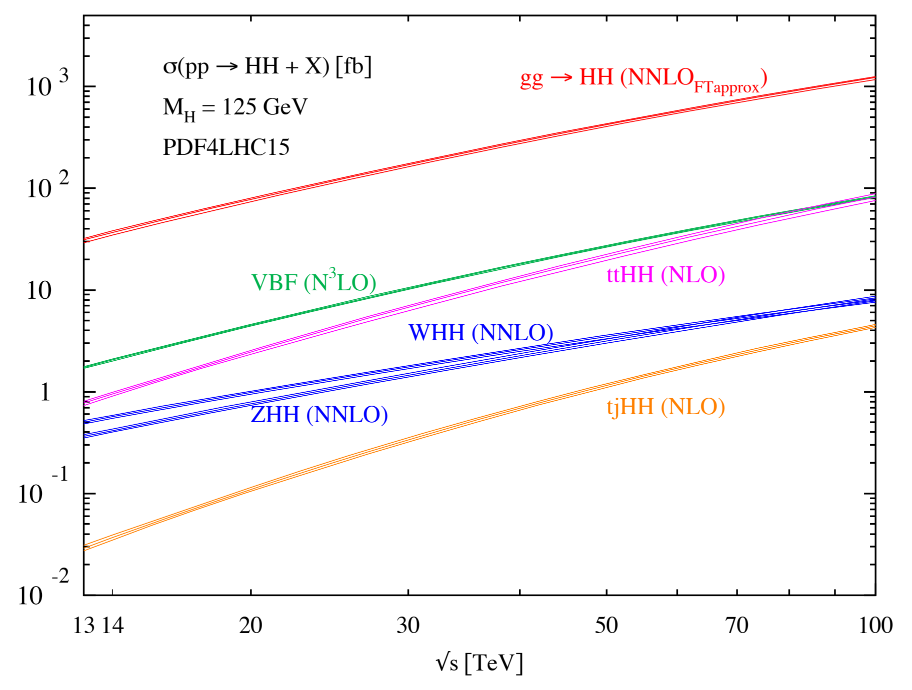








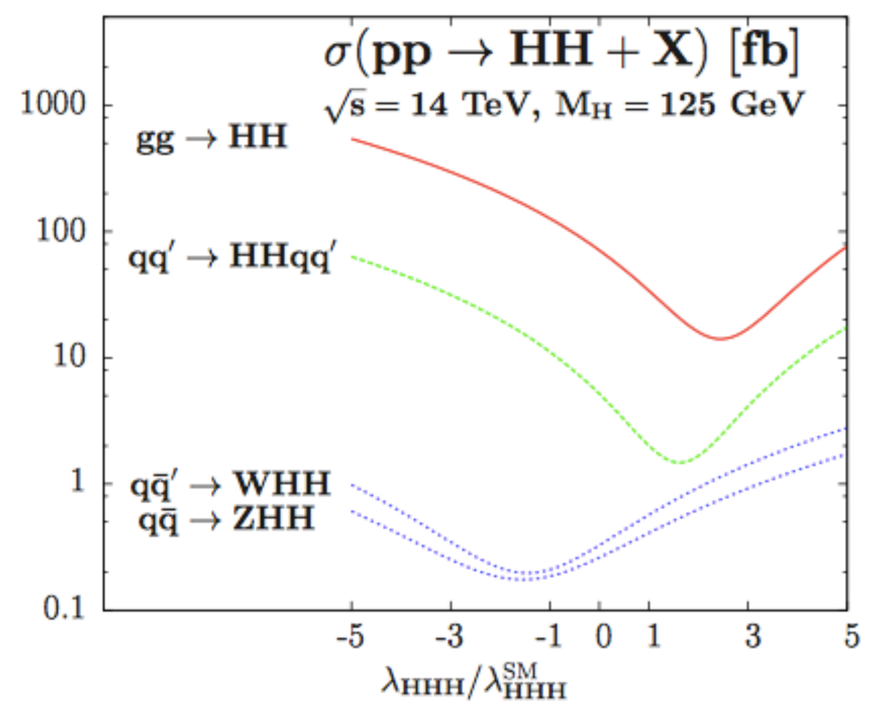
### Cross sections



HH White Paper

# Motivation

## Uncertainties:



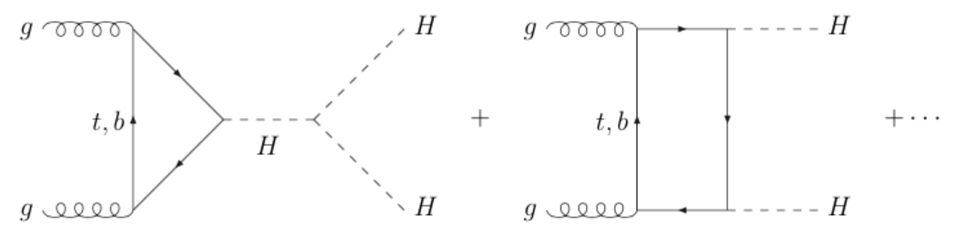
$$gg \to HH : \frac{\Delta\sigma}{\sigma} \sim -\frac{\Delta\lambda}{\lambda}$$

Baglio, Djouadi, Gröber,
Mühlleitner, Quevillon, Spira

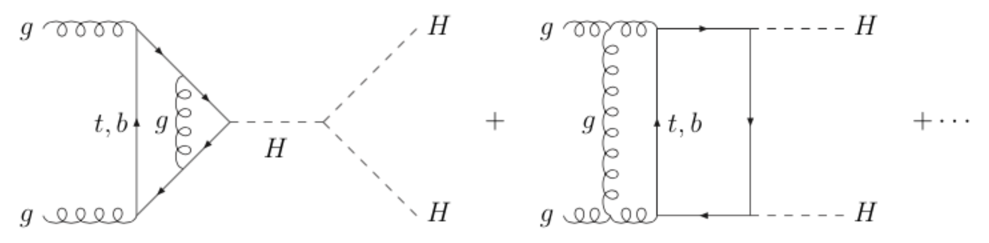
# NLO Corrections

$$\sigma_{\mathrm{NLO}}(pp \to HH + X) = \sigma_{\mathrm{LO}} + \Delta\sigma_{\mathrm{virt}} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}},$$

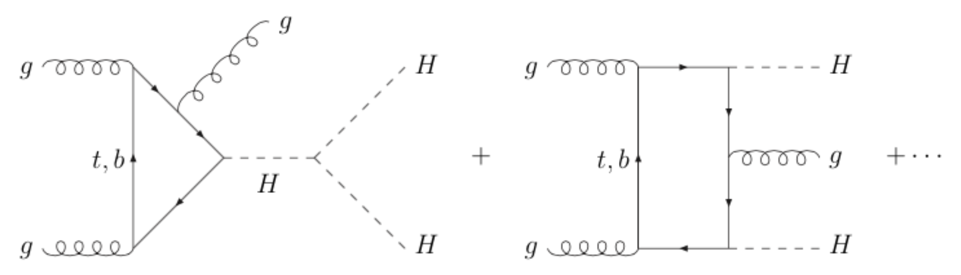
$\sigma_{\mathrm{LO}}$:



$\Delta\sigma_{\mathrm{virt}}$:



$\Delta\sigma_{ij}$:



# Current Status

- Virtual & real (N)NLO QCD corrections in large top mass limit (HTL): ~100%

Dawson,Dittmaier,Spira
de Florian,Mazzitelli
Grigo,Melnikov,Steinhauser

- Large top mass expansion: ~ ±10%

Grigo, Hoff, Melnikov, Steinhauser

- NLO mass effects of the real NLO correction alone ~ -10 %

Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Torrielli, Vryonidou, Zaro

- NLO QCD corrections including the full top mass dependence:
  - 15 % NLO mass effects

Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke

Baglio,Campanario,SG,Mühlleitner, Spira,Streicher

- New expansion/extrapolation methods:
  - $1/m_t^2$ expansion & conformal mapping & Padé approximants
  - $p_T^2$ expansion
  - high-energy

Gröber,Maier,Rauh

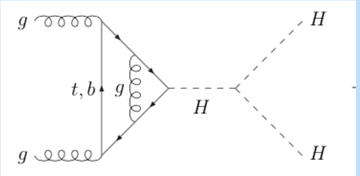
Bonciani,Degrassi,Giardino,Gröber

Davies,Mishima,Steinhauser,Wellmann

# Virtual Corrections

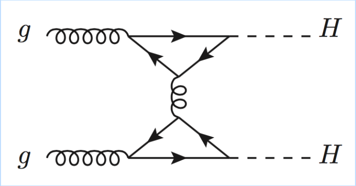
Triangular diagrams

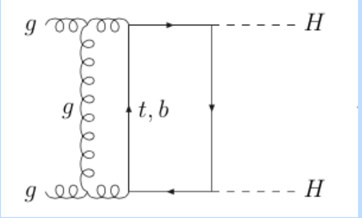
- Use existing results of single Higgs calculation



One-particle reducible diagrams

- Use existing results of $H \to Z\gamma$





Box diagrams

- Treat every diagram individually (no reduction to master integrals)
- Extract the ultraviolet divergences of the matrix elements using endpoint subtractions
- Extract the infrared and collinear divergences using a ‘proper’ subtraction of the integrand
- Integration by parts due to numerical instabilities above the thresholds where $m_{hh}^2 > 0,$ $m_{hh}^2 > 4m_t^2 \Rightarrow m_t^2 \to m_t^2(1 - i\bar{\epsilon})$ with $\bar{\epsilon} \ll 1$

# Virtual Corrections

## Total virtual corrections

- Numerical evaluation using Vegas (P. Lepage)

$$Q^2 \frac{d\Delta\sigma_{virt}}{dQ^2} = \tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{virt}(Q^2)\Big|_{\tau=\frac{Q^2}{s}}$$

$(Q^2 = m^2_{HH})$

$\frac{\mathrm{d}\mathcal{L}^{gg}}{\mathrm{d}\tau} = \text{gluon luminosity}$

$\hat{\sigma}_{virt} = \text{virtual part of the partonic cross section}$

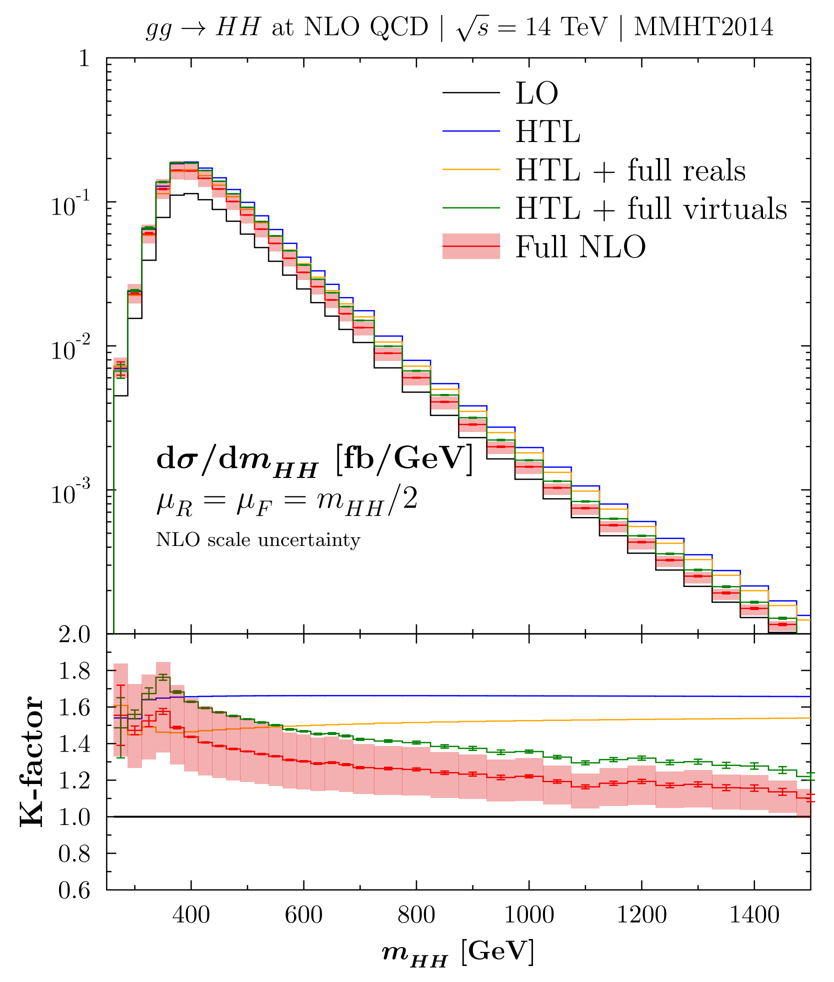
- Renormalization: $\alpha_s$ in $\overline{MS}$ with $N_F = 5$ and $m_t$ on shell (central value)
- Subtraction of HTL → IR finite top mass effects
- Numerical instabilities due to the small imaginary parts of the top mass: *Richardson extrapolation*

## Real corrections

- Full matrix elements generated with FeynArts and FormCalc
- Matrix element in the HTL (massive LO) subtracted → IR finite top mass effects

# Differential cross section

## Differential cross section



### K-factor

$$K = \frac{\sigma_{\mathrm{NLO}}}{\sigma_{\mathrm{LO}}}$$

# Total hadronic cross section

## Total hadronic cross section

| Energy | $m_t$ = 173 GeV | $m_t$ = 172.5 GeV |
|---|---|---|
| **13 TeV** | $27.80(9)^{+13.8\%}_{-12.8\%}$ fb | $27.73(7)^{+13.8\%}_{-12.8\%}$ fb |
| **14 TeV** | $32.91(10)^{+13.6\%}_{-12.6\%}$ fb | $32.78(7)^{+13.5\%}_{-12.5\%}$ fb |
| **27 TeV** | $127.7(2)^{+11.5\%}_{-10.4\%}$ fb | $127.0(2)^{+11.7\%}_{-10.7\%}$ fb |
| **100 TeV** | $1149(2)^{+10.8\%}_{-10.0\%}$ fb | $1140(2)^{+10.7\%}_{-10.0\%}$ fb |

HH White Paper (to appear soon)

# Scale dependence

## Factorisation / renormalisation scale dependence

varying both scales by a factor of two around central value of $\mu_F = \mu_R = m_{hh}/2$

Differential cross section:

$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg} \to \mathrm{HH})}{\mathrm{dQ}}\right|_{Q=300\,\mathrm{GeV}} = 0.02978(8)^{+15.3\%}_{-13.0\%}\,\mathrm{fb/GeV}$$

$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg} \to \mathrm{HH})}{\mathrm{dQ}}\right|_{Q=400\,\mathrm{GeV}} = 0.1609(4)^{+14.4\%}_{-12.8\%}\,\mathrm{fb/GeV}$$

$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg} \to \mathrm{HH})}{\mathrm{dQ}}\right|_{Q=600\,\mathrm{GeV}} = 0.03204(9)^{+10.9\%}_{-11.5\%}\,\mathrm{fb/GeV}$$

$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg} \to \mathrm{HH})}{\mathrm{dQ}}\right|_{Q=1200\,\mathrm{GeV}} = 0.000435(4)^{+7.1\%}_{-10.6\%}\,\mathrm{fb/GeV}$$

Total cross section:

$$\sigma(gg \to HH) = 32.78(7)^{+13.5\%}_{-12.5\%} \quad (\mathrm{PDF4LHC15})$$

$(\sqrt{s} = 14\,\mathrm{TeV})$

Baglio,Campanario,G,Mühlleitner,Spira,Streicher: 1811.05692

# Top mass uncertainty

## Uncertainty due to $m_t$: differential cross section

- uncertainty related to the scheme and scale choice of the top mass
- calculated the total NLO results for the differential cross section for the $\overline{MS}$ top mass at different scale choices
- $\overline{MS}$ top mass scale in the range [*Q*/4,*Q*], $m_t$

$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg}\to\mathrm{HH})}{\mathrm{dQ}}\right|_{Q=300\,\mathrm{GeV}} = 0.02978(7)^{+6\%}_{-34\%}\ \mathrm{fb/GeV}$$

$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg}\to\mathrm{HH})}{\mathrm{dQ}}\right|_{Q=400\,\mathrm{GeV}} = 0.1609(4)^{+0\%}_{-13\%}\ \mathrm{fb/GeV}$$

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$(\sqrt{s} = 14\,\mathrm{TeV})$

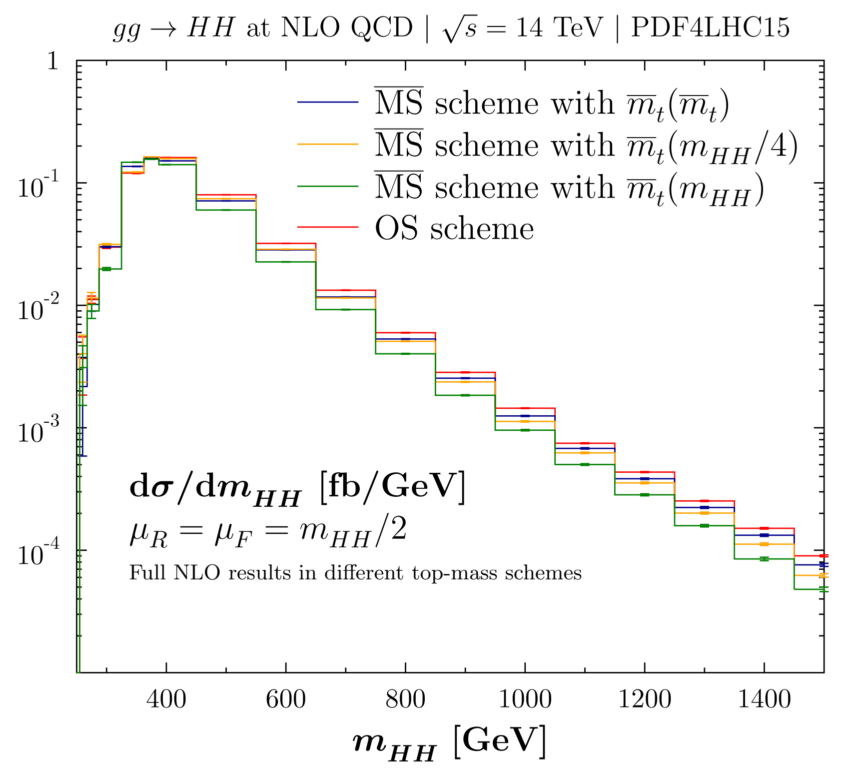
# Top mass uncertainty

## Uncertainty due to $m_t$: total hadronic cross section

Take for individual Q values the maximum / minimum differential cross section and integrate

$$\sigma(gg \to HH) = 32.78(7)^{+4.0\%}_{-17\%}$$

*with PDF4LHC15*



# Conclusions

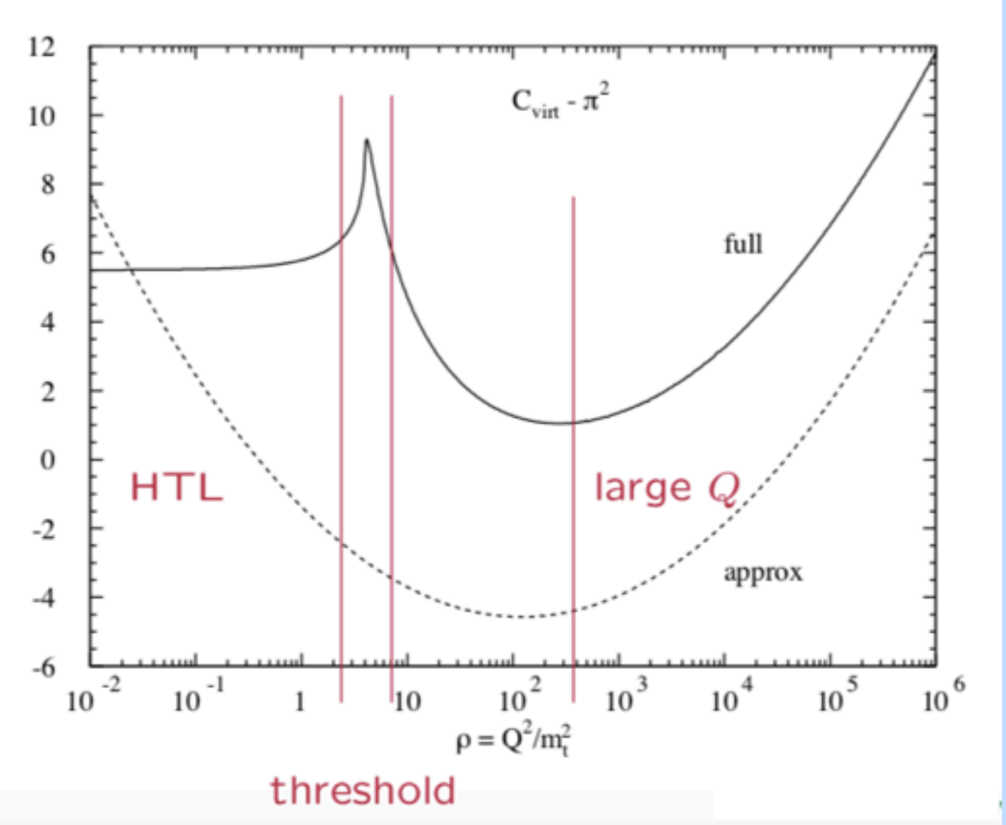
- Calculation of two-loop integral with three free parameter ratios
- NLO top mass effects of ~ -15% compared to HTL result
- Factorisation / renormalisation scale dependence: ~ 15% uncertainties
- Top mass scheme and scale uncertainties: $\lesssim$ 30%   (differential)

total cross section at 14 TeV: $\sigma(gg \to HH) = 32.78(7)^{+4.0\%}_{-17\%}$

# BACK-UP

# Top mass uncertainty

NNLO



Spira,Djouadi,Graudenz,Zerwas

# NLO Corrections

$$\sigma_{\text{NLO}}(pp \to HH + X) = \sigma_{\text{LO}} + \Delta\sigma_{\text{virt}} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}},$$

$$\begin{aligned}
\sigma_{\text{LO}} &= \int_{\tau_0}^{1} d\tau \, \frac{d\mathcal{L}^{gg}}{d\tau} \, \hat{\sigma}_{\text{LO}}(Q^2 = \tau s) \\
\Delta\sigma_{\text{virt}} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \, \frac{d\mathcal{L}^{gg}}{d\tau} \, \hat{\sigma}_{\text{LO}}(Q^2 = \tau s) \; C \\
\Delta\sigma_{gg} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \, \frac{d\mathcal{L}^{gg}}{d\tau} \int_{\tau_0/\tau}^{1} \frac{dz}{z} \, \hat{\sigma}_{\text{LO}}(Q^2 = z\tau s) \left\{ -zP_{gg}(z) \log \frac{M^2}{\tau s} \right. \\
&\qquad \left. + d_{gg}(z) + 6[1 + z^4 + (1-z)^4] \left( \frac{\log(1-z)}{1-z} \right)_+ \right\} \\
\Delta\sigma_{gq} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \sum_{q,\bar{q}} \frac{d\mathcal{L}^{gq}}{d\tau} \int_{\tau_0/\tau}^{1} \frac{dz}{z} \, \hat{\sigma}_{\text{LO}}(Q^2 = z\tau s) \left\{ -\frac{z}{2} P_{gq}(z) \log \frac{M^2}{\tau s (1-z)^2} \right. \\
&\qquad \left. + d_{gq}(z) \right\} \\
\Delta\sigma_{q\bar{q}} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \sum_{q} \frac{d\mathcal{L}^{q\bar{q}}}{d\tau} \int_{\tau_0/\tau}^{1} \frac{dz}{z} \, \hat{\sigma}_{\text{LO}}(Q^2 = z\tau s) \; d_{q\bar{q}}(z)
\end{aligned}$$

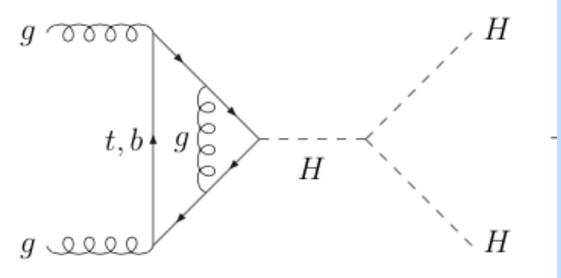
$$C \to \pi^2 + \frac{11}{2} + C_{\triangle\triangle}, \qquad d_{gg} \to -\frac{11}{2}(1-z)^3, \qquad d_{gq} \to \frac{2}{3}z^2 - (1-z)^2, \qquad d_{q\bar{q}} \to \frac{32}{27}(1-z)^3$$

# Virtual Corrections

47 two-loop box diagrams + 8 triangular diagrams + 2 one-particle reducible diagrams

Triangular Diagrams

⟵ single Higgs case

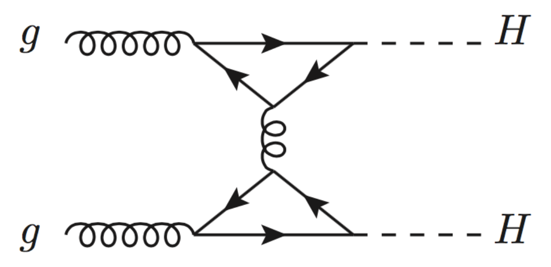


One-particle reducible diagrams

⟶ analytical results for $C_{\triangle\triangle}$

$(H \to Z\gamma)$

see e.g. Degrassi, Giardino, Gröber



# Virtual Corrections

## Box diagrams

- Generate matrix element form factors for all possible diagrams using Feynman rules (by hand → *Reduce, Mathematica, Form*)

- Perform Feynman parametrisation → additional 6-dimensional integrals

- Extract the ultraviolet divergences of the matrix elements using endpoint subtractions of the 6-dimensional Feynman integrals

$$\int_0^1 dx\, \frac{f(x)}{(1-x)^{1-\epsilon}} = \int_0^1 dx\, \frac{f(1)}{(1-x)^{1-\epsilon}} + \int_0^1 dx\, \frac{f(x)-f(1)}{(1-x)^{1-\epsilon}} = \frac{f(1)}{\epsilon} + \int_0^1 dx\, \frac{f(x)-f(1)}{1-x} + \mathcal{O}(\epsilon)$$

- Extract the infrared and collinear divergences using a ‘proper’ subtraction of the integrand (based on HTL calculation)

- Integration by parts due to numerical instabilities at the thresholds

$m_{hh}^2 > 4m_t^2 \Rightarrow m_t^2 \to m_t^2(1 - i\bar{\epsilon})$ with $\bar{\epsilon} \ll 1$

$$\int_0^1 \mathrm{dx}\, \frac{\mathrm{f(x)}}{(\mathrm{a}+\mathrm{bx})^3} = \frac{\mathrm{f(0)}}{2\mathrm{a}^2\mathrm{b}} - \frac{\mathrm{f(1)}}{2\mathrm{b}(\mathrm{a}+\mathrm{b})^2} + \int_0^1 \mathrm{dx}\, \frac{\mathrm{f}'(\mathrm{x})}{2\mathrm{b}(\mathrm{a}+\mathrm{bx})^2}$$

# Virtual Corrections

## Differential cross section

$$Q^2 \frac{d\Delta\sigma_{virt}}{dQ^2} = \tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{virt}(Q^2)\Big|_{\tau=\frac{Q^2}{s}} \qquad (Q^2 = m^2_{HH})$$

$$\frac{\mathrm{d}\mathcal{L}^{gg}}{\mathrm{d}\tau} = \text{gluon luminosity}$$

$$\hat{\sigma}_{virt} = \text{virtual part of the partonic cross section}$$

⟶ 7 dimensional integrals (6 Feynman and one phase space integration)

⟶ use Vegas for numerical integration (P. Lepage)

⟶ numerical instabilities due to the small imaginary parts of the top mass above the thresholds: Richardson extrapolation

# Virtual Corrections

## Renormalisation

$\alpha_s$ and $m_t$ need to be renormalised

→ $\alpha_s$ in $\overline{MS}$ with $N_F = 5$

→ $m_t$ on shell (→ central prediction)

$$\delta\sigma = \delta\alpha_s \frac{\delta\sigma_{LO}}{\delta\alpha_s} + \delta m_t \frac{\delta\sigma_{LO}}{\delta m_t}$$

Subtraction of the heavy-top limit → virtual mass effects only (infrared finite)

$$\Delta C_{mass} = C^0 - C^0_{HTL}$$

Adding back the results in the heavy-top limit (HPAIR)

$$C = \underset{\substack{\uparrow \\ \text{HPAIR}}}{C_{HTL}} + \Delta C_{mass}$$

# Virtual Corrections

## Box diagrams

- Generate matrix element form factors for all possible diagrams using Feynman rules (by hand → *Reduce, Mathematica*)
- Use dimensional regularisation: $D = 4 - 2\epsilon$
- Perform Feynman parametrisation → additional 6-dimensional integrals

$$\frac{1}{A_1^{\alpha_1}\cdots A_n^{\alpha_n}} = \frac{\Gamma(\alpha_1+\ldots+\alpha_n)}{\Gamma(\alpha_1)\cdots\Gamma(\alpha_n)}\int_0^1 du_1 \int_0^{1-u_1} du_2 \cdots \int_0^{1-u_1-\ldots-u_{n-2}} du_{n-1} \frac{u_1^{\alpha_1-1}\cdots u_{n-1}^{\alpha_n-1}(1-u_1-\ldots-u_{n-1})^{\alpha_n-1}}{[u_1A_1+\ldots+u_{n-1}A_{n-1}+(1-u_1-\ldots-u_{n-1})A_n]^{\alpha_1+\ldots+\alpha_n}},$$

- Substitution to obtain integrals from 0 to 1
- Evaluating momentum integrals using:

$$\int \frac{d^Dk}{(2\pi)^D}\frac{1}{(k^2-M^2+i\epsilon)^N} = i\frac{(-1)^N}{(4\pi)^{D/2}}\frac{\Gamma(N-\frac{D}{2})}{\Gamma(N)}\frac{1}{(M^2-i\epsilon)^{N-\frac{D}{2}}}$$

$$\int \frac{d^Dk}{(2\pi)^D}\frac{k_\mu k_\nu}{(k^2-M^2+i\epsilon)^N} = \frac{i}{2}\frac{(-1)^{N-1}}{(4\pi)^{D/2}}\frac{\Gamma(N-1-\frac{D}{2})}{\Gamma(N)}\frac{g_{\mu\nu}}{(M^2-i\epsilon)^{N-1-\frac{D}{2}}}$$

etc.

# Virtual Corrections

## Divergences

- Integration by parts due to numerical instabilities at the thresholds

$m_{hh}^2 > 4m_t^2 \Rightarrow m_t^2 \to m_t^2(1 - i\bar{\epsilon})$ with $\bar{\epsilon} \ll 1$

$$\int_0^1 \mathrm{dx}\,\frac{\mathrm{f(x)}}{(\mathrm{a}+\mathrm{bx})^3} = \frac{\mathrm{f}(0)}{2\mathrm{a}^2\mathrm{b}} - \frac{\mathrm{f}(1)}{2\mathrm{b}(\mathrm{a}+\mathrm{b})^2} + \int_0^1 \mathrm{dx}\,\frac{\mathrm{f}'(\mathrm{x})}{2\mathrm{b}(\mathrm{a}+\mathrm{bx})^2}$$

(more involved for second order polynomials)

further integration by parts not successful since new divergences are created (investigated further)

# Virtual Corrections

## Divergences

- Extract the ultraviolet divergences of the matrix elements using endpoint subtractions of the 6-dimensional Feynman integrals

$$\int_0^1 dx \, \frac{f(x)}{(1-x)^{1-\epsilon}} = \int_0^1 dx \, \frac{f(1)}{(1-x)^{1-\epsilon}} + \int_0^1 dx \, \frac{f(x)-f(1)}{(1-x)^{1-\epsilon}}$$

$$= \frac{f(1)}{\epsilon} + \int_0^1 dx \, \frac{f(x)-f(1)}{1-x} + \mathcal{O}(\epsilon)$$

- Extract the infrared and collinear divergences using a proper subtraction of the integrand

denominator: $N = ar^2 + br + c$ $\quad a = \mathcal{O}(\rho)$ $\quad \rho_s = \frac{\hat{s}}{m_Q^2}$

$N_0 = br + c$ $\quad b = 1 + \mathcal{O}(\rho)$

$c = -\rho_s x(1-x)(1-s)t$

$$\int_0^1 \mathrm{d}\vec{x}\mathrm{d}r \frac{rH(\vec{x},r)}{N^{3+2\epsilon}} = \int_0^1 \mathrm{d}\vec{x}\mathrm{d}r \left\{ \left( \frac{rH(\vec{x},r)}{N^{3+2\epsilon}} - \frac{rH(\vec{x},0)}{N_0^{3+2\epsilon}} \right) + \frac{rH(\vec{x},0)}{N_0^{3+2\epsilon}} \right\}$$

↑ Taylor expansion in $\epsilon$ ↑ analytical r-integration

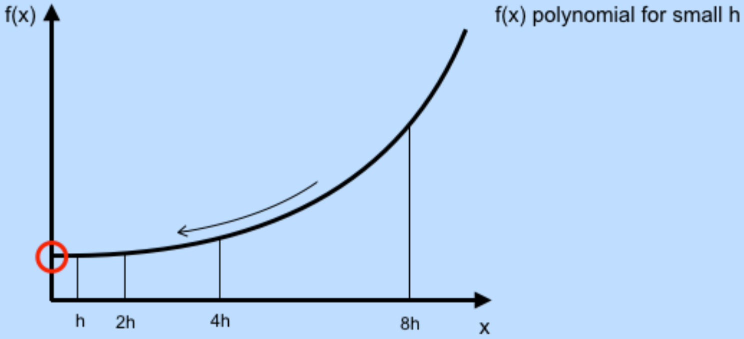
# Numerical Analysis

## Richardson extrapolation

⟶ sequence acceleration method to obtain a better convergence behaviour

Approximation polynomial

$$M_{i+1}(h) = \frac{t^{k_i} M_i(\frac{h}{t}) - M_i(h)}{t^{k_i} - 1}$$



*h* and *h/t* the two step sizes and $k_i$ the truncation error

$$M_2[f(h), f(2h)] = 2f(h) - f(2h) \qquad = f(0) + \mathcal{O}(h^2)$$

$$M_4[f(h), f(2h), f(4h)] = (8f(h) - 6f(2h) + f(4h))/3 \qquad = f(0) + \mathcal{O}(h^3)$$

$$M_8[f(h), f(2h), f(4h), f(8h)] = (64f(h) - 56f(2h) + 14f(4h) - f(8h))/21 \quad = f(0) + \mathcal{O}(h^4)$$

In our case $h = \bar{\epsilon}$ and $\bar{\epsilon}_n = 0.05 \times 2^n \quad n = 0 \ldots 9$

Theoretical error from Richardson extrapolation estimated by the difference of the fifth and the

# Numerical Analysis

## Numerical Instabilities

1. due to phase-space integration over Mandelstam variable *t*

   → cut-off at $t = 10^{-8}$ for individual diagrams (total sum finite)

   → logarithmic substitution with $y = \log\frac{t - t_-}{m_t^2}$

2. due to the small imaginary parts $\bar{\epsilon}$ of the top mass above the thresholds, where $m_t^2 \to m_t^2(1 - i\bar{\epsilon})$

   need value in narrow width approximation where $\bar{\epsilon} \to 0$

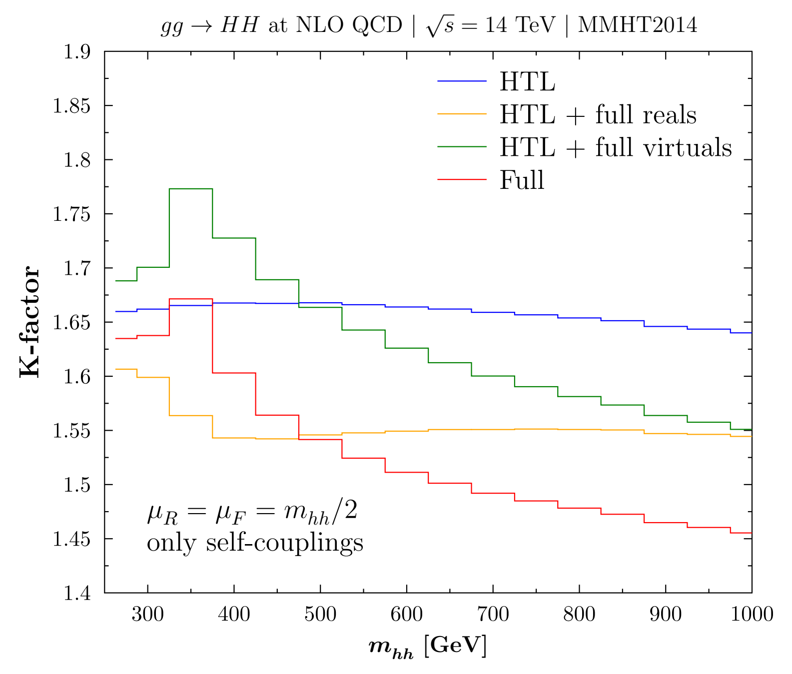
   calculate partonic cross section for different $\bar{\epsilon}$ → Richardson extrapolation

# Results

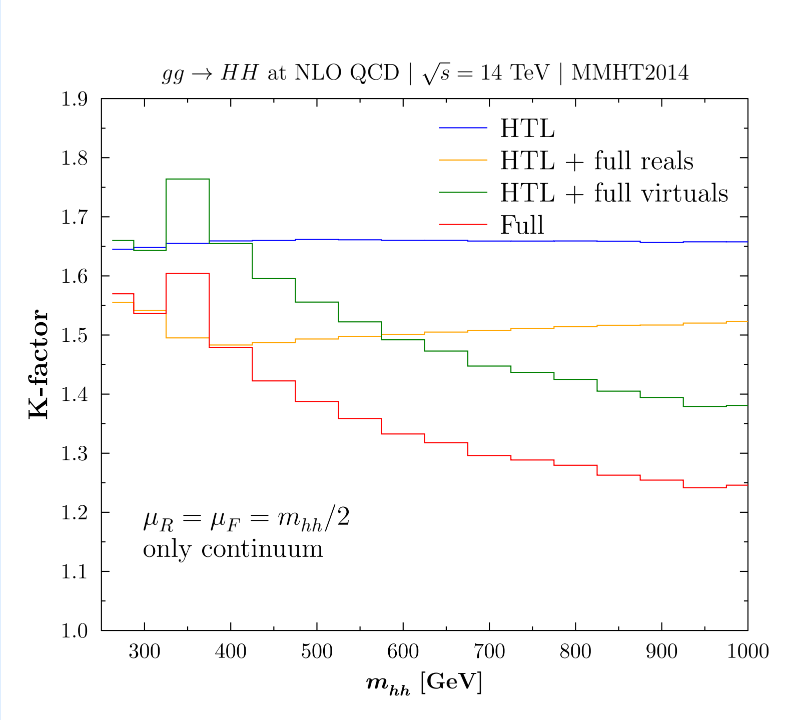
## K-factor distribution

$$K = \frac{\sigma_{\text{NLO}}}{\sigma_{\text{LO}}}$$

Triangular contributions

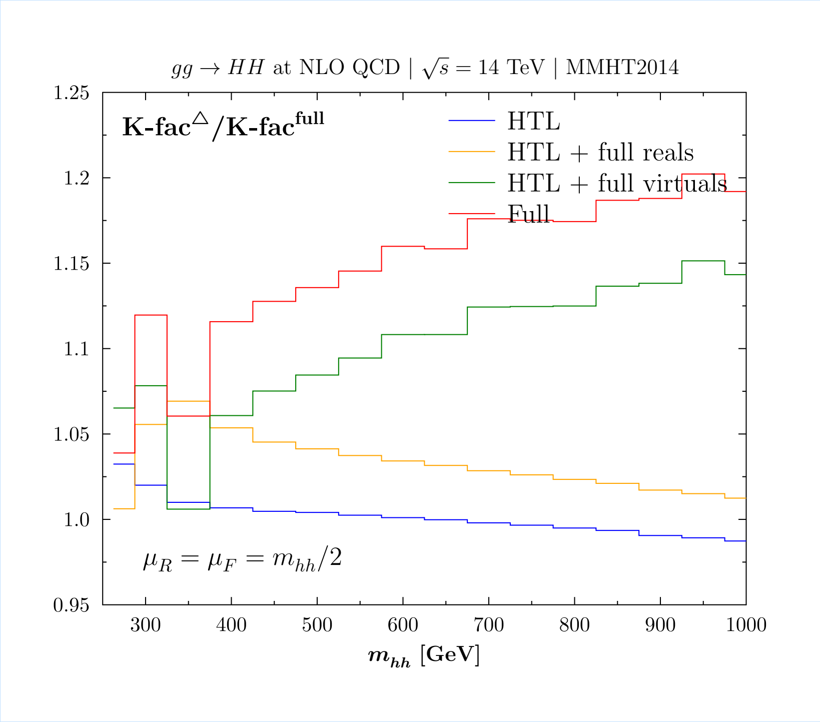


Box contributions

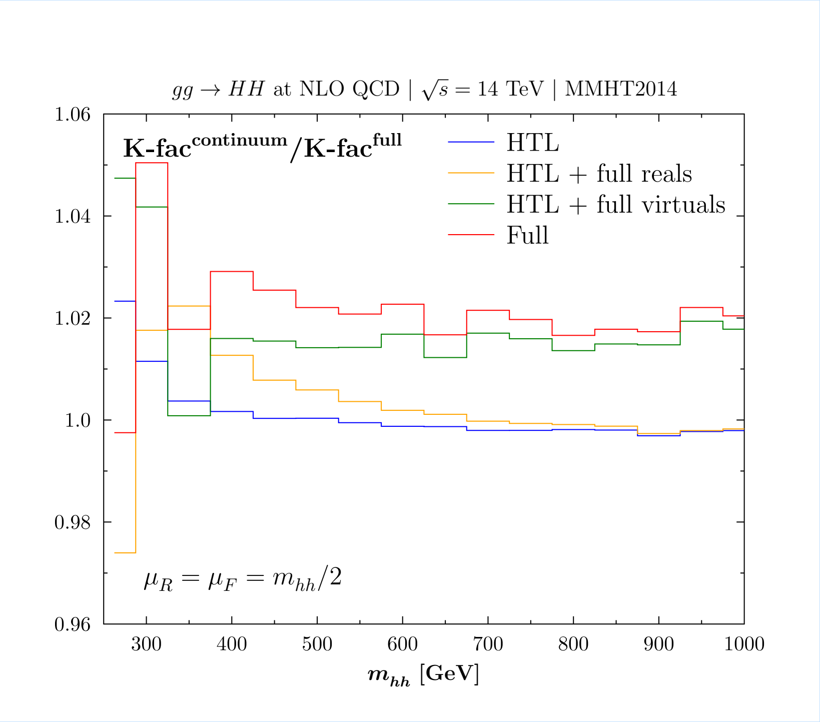


# Results

## Triangular contributions



## Box contributions



# Results

## Uncertainty due to $m_t$ for single Higgs

→ $\overline{MS}$ top mass in the range [Q/4,Q]

$$\sigma(gg \to H)\Big|_{m_H=125\,\mathrm{GeV}} = 42.17^{+0.4\%}_{-0.5\%}\ \mathrm{pb}$$

$$\sigma(gg \to H)\Big|_{m_H=300\,\mathrm{GeV}} = 9.85^{+7.5\%}_{-0.3\%}\ \mathrm{pb}$$

$$\sigma(gg \to H)\Big|_{m_H=400\,\mathrm{GeV}} = 9.43^{+0.1\%}_{-0.9\%}\ \mathrm{pb}$$

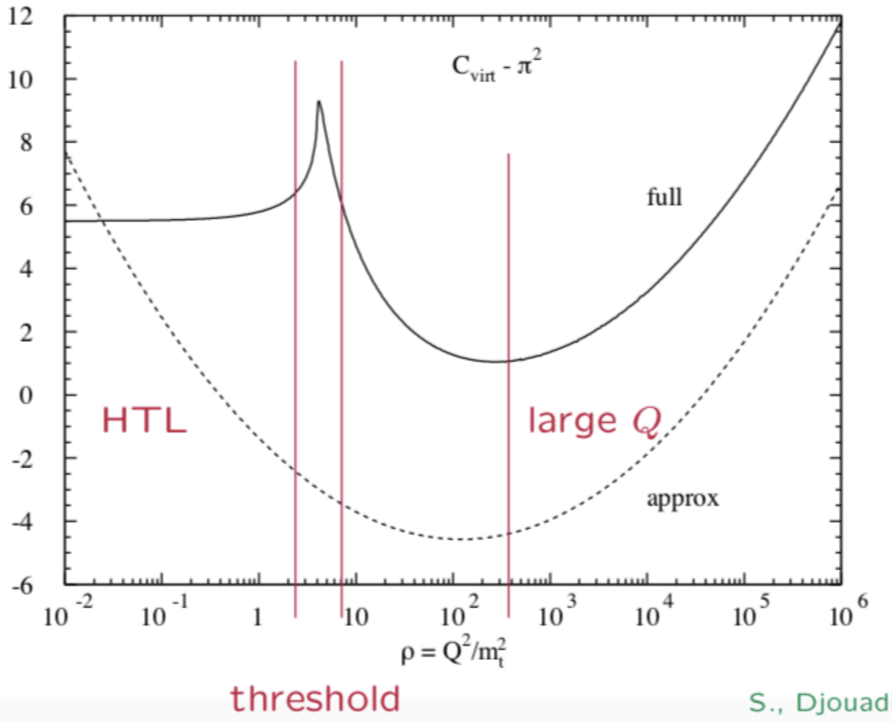
$$\sigma(gg \to H)\Big|_{m_H=600\,\mathrm{GeV}} = 1.97^{+0.0\%}_{-15.9\%}\ \mathrm{pb}$$

$$\sigma(gg \to H)\Big|_{m_H=900\,\mathrm{GeV}} = 0.230^{+0.0\%}_{-22.3\%}\ \mathrm{pb}$$

$$\sigma(gg \to H)\Big|_{m_H=1200\,\mathrm{GeV}} = 0.0402^{+0.0\%}_{-26.0\%}\ \mathrm{pb}$$

- triangles for $Q^2 \gg 4M_t^2$:

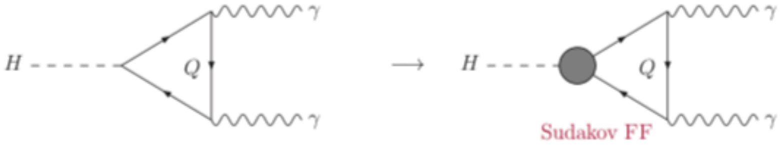
$$C \to \frac{C_A - C_F}{12}\left[\log\frac{Q^2}{m_t^2} - i\pi\right]^2 - C_F\left[\log\frac{Q^2}{m_t^2} - i\pi\right] + 3C_F\log\frac{\mu_t^2}{m_t^2} + \mathcal{O}(1)$$



S., Djouadi, Graudenz, Zerwas

## resummation for large $Q$

- Abelian logs ($C_F$): $H \to \gamma\gamma$

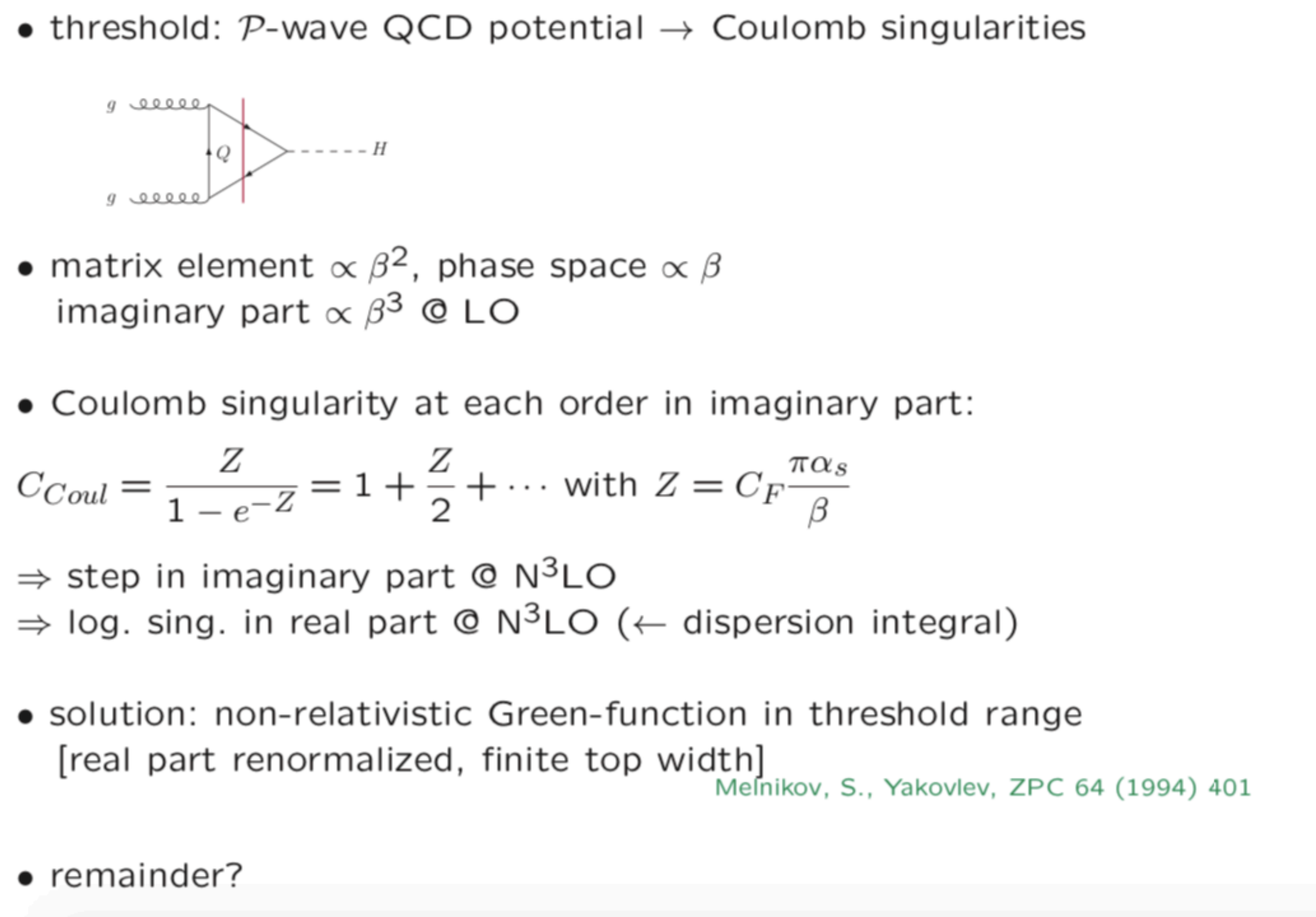


Kotsky, Yakovlev, PLB 418 (1998) 335 (LL)
Akhoury, Wang, Yakovlev, PRD 64 (2001) 113008 (NLL)

- non-Abelian logs ($C_A$): LL related to IR singularities → exponentiate

Liu, Penin, PRL 119 (2017) no.26, 262001; JHEP 1811 (2018) 158

- non-Abelian NLL?
- remainder (NNLL)?
- boxes? (more scales)



- threshold: $\mathcal{P}$-wave QCD potential → Coulomb singularities
- matrix element $\propto \beta^2$, phase space $\propto \beta$
  imaginary part $\propto \beta^3$ @ LO
- Coulomb singularity at each order in imaginary part:

$$C_{Coul} = \frac{Z}{1-e^{-Z}} = 1 + \frac{Z}{2} + \cdots \text{ with } Z = C_F \frac{\pi \alpha_s}{\beta}$$

⇒ step in imaginary part @ $N^3LO$
⇒ log. sing. in real part @ $N^3LO$ (← dispersion integral)

- solution: non-relativistic Green-function in threshold range
  [real part renormalized, finite top width]

Melnikov, S., Yakovlev, ZPC 64 (1994) 401

- remainder?