Transits of the QCD Critical Point

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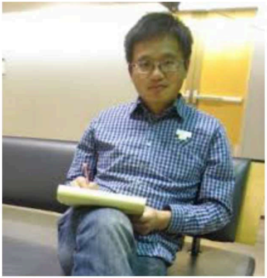
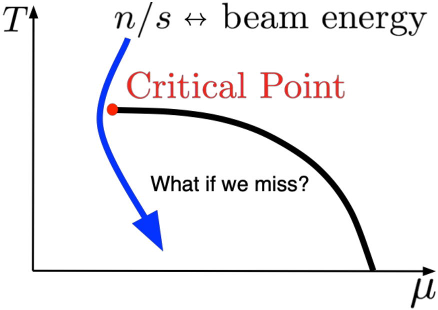
Yukinao Akamatsu, Derek Teaney, Yi Yin; arXiv:1811.05081v1





Transits of the critical point: two parameters



Finite expansion rate?
What if we miss?

- How does the finite expansion rate limit the critical flucts?

$$\epsilon \equiv \underbrace{\tau_o}_{\text{micro time}} \times \underbrace{\partial_\mu u^\mu}_{\text{expansion rate } 1/\tau_Q} = \frac{\tau_o}{\tau_Q}$$

- How does missing the critical point limit the critical flucts?

$$\Delta_s \equiv \frac{n_c}{s_c}\left(\frac{s}{n} - \frac{s_c}{n_c}\right)$$

What should we measure: baryon/entropy

see also Stephanov & Yin arXiv:1712.10305

$$C^{\hat{n}\hat{n}} \equiv \underbrace{\langle (\delta n - (n/s)\delta s)^2 \rangle}_{\text{flucts of } \delta\hat{n} \equiv s\delta(n/s)}$$

variance in n/s: not baryon number

- Why? It is a hydro eigenmode and *always* diverges fast

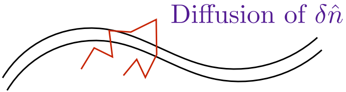
$$\underbrace{\langle (\delta n - (n/s)\delta s)^2 \rangle}_{\text{what we want}} \propto \underbrace{C_p}_{\text{specific heat}} \propto \underbrace{\chi_{\text{Ising}}}_{\text{Ising susceptibility}}$$

- Wavelength dependent model for the equilibrium correlator

$$C^{\hat{n}\hat{n}}(k,t) \equiv \langle \delta\hat{n}^*(k,t)\delta\hat{n}(-k,t) \rangle \propto \underbrace{\frac{\chi_{\text{Ising}}}{1+(k\xi)^{2-\eta}}}_{\text{Ising prediction}}$$

Question: What's the wavelength of fluctuations near the CP?

Estimate the wavelength of critical fluctuations



- The maximum wavelength that can be equilibrated by diffusion:

$$\underbrace{D_o}_{\text{diffusion coef}} \quad \times \quad \underbrace{\tau_Q}_{\text{the total time}} \quad = \quad \underbrace{\ell_{\rm max}^2}_{\text{the longest wavelength}}$$

- Here D_o is the (thermal) diffusion coefficient away from the CP:

$$D_0 \sim \frac{\ell_o^2}{\tau_o} \quad \Rightarrow \quad \ell_{\rm max} \sim \ell_o \epsilon^{-1/2} \qquad \ell_o \text{ is micro length}$$

The critical wavelength ℓ_{kz} must be in between ℓ_o and $\ell_{\rm max}$

$$\underbrace{\ell_o}_{\text{microlength}} \quad \ll \quad \underbrace{\ell_{\rm kz}}_{\text{typical critical wavelength}} \quad \ll \underbrace{\ell_o \epsilon^{-1/2}}_{\ell_{\rm max}} \qquad \epsilon \equiv \frac{\tau_0}{\tau_Q}$$

Slightly miss the critical point: constant n/s trajectories

- Ideal hydro conserves n/s

Entropy conservation: $u^\mu \partial_\mu s = -s \partial_\mu u^\mu \quad \Rightarrow \quad \partial_t s = -\frac{s}{\tau_Q}$

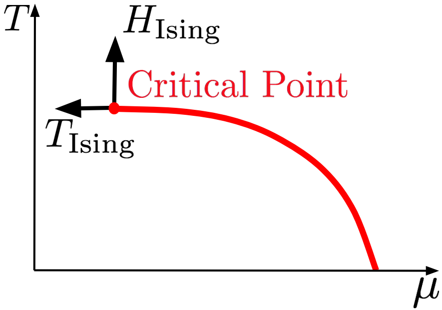
Baryon conservation: $u^\mu \partial_\mu n = -n \partial_\mu u^\mu \quad \Rightarrow \quad \partial_t n = -\frac{n}{\tau_Q}$

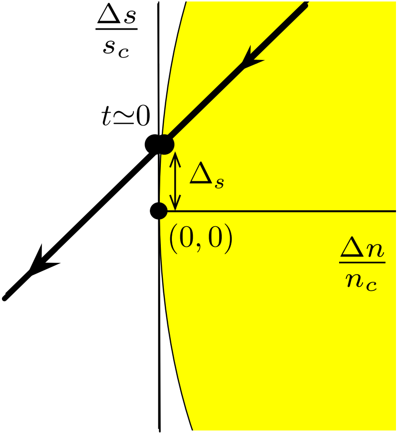
- Passing through critical point at $t = 0$ and expansion rate $1/\tau_Q$

$$\frac{\Delta n}{n_c} \approx -\frac{t}{\tau_Q} \qquad \frac{\Delta s}{s_c} \approx \underbrace{\Delta_s}_{\text{detuning}} - \frac{t}{\tau_Q}$$

Near the CP, the trajectories are controlled by the detuning parameter Δ_s and the expansion rate $\partial_\mu u^\mu \equiv 1/\tau_Q$

Straight line trajectories in $s - n$ plane





$-\Delta\mu_{\rm QCD} \longleftrightarrow \Delta T_{\rm Ising}$ $\qquad$ $\Delta s \longleftrightarrow \Delta M_{\rm Ising}$

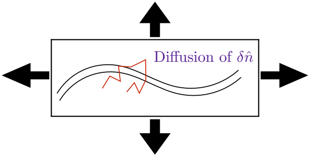
$\Delta T_{\rm QCD} \longleftrightarrow \Delta H_{\rm Ising}$ $\qquad$ $\Delta n \longleftrightarrow \Delta e_{\rm Ising}$

Hydrodynamic equation for $C^{\hat{n}\hat{n}}(k,t) = \langle \hat{n}(k,t)\hat{n}(-k,t)\rangle$

- Start from dissipative hydro with noise

$$\partial_\mu (T^{\mu\nu}_{\rm Ideal} + T^{\mu\nu}_{\rm diss} + \xi^{\mu\nu}) = 0$$

$$\partial_\mu (j^{\mu}_{\rm Ideal} + j^{\mu}_{\rm diss} + \xi^{\mu}) = 0$$



- Can derive time evolution equations for the correlators

$$C^{ee} = \langle \delta e^*(k,t)\delta e(-k,t)\rangle \quad C^{nn} = \langle \delta n^*(k,t)\delta n(-k,t)\rangle \quad , \text{etc}$$

- From C^{nn}, C^{ee} derive an equation for $C^{\hat{n}\hat{n}} = \langle (\delta n - (n/s)\delta s)^2 \rangle$

$$\partial_t C^{\hat{n}\hat{n}} = - \underbrace{\frac{\lambda_{\rm eff} k^2}{C_p}}_{\text{heat diffusion}} (C^{\hat{n}\hat{n}} - C_p)$$

From stochastic hydro find that $C^{\hat{n}\hat{n}}$ obeys a relaxation equation

Percentage changing rate of equilibrium

- The equilibrium correlator scales with correlation length

$$C_p \propto \chi_{\text{Ising}} \qquad \chi_{\text{Ising}} \propto \xi^{2-\eta}$$

- The correlation scales with Ising energy

$$\xi \propto (\Delta e_{\text{Ising}})^{-a\nu} \qquad a \equiv 1/(1-\alpha)$$

With the map $\Delta e_{\text{Ising}} \propto \Delta n/n_c = -t/\tau_Q$, we find time dependence:

$$\xi(t) = \ell_o \left(\frac{t}{\tau_Q}\right)^{-a\nu} \qquad \chi_{\text{Ising}}(t) = \chi_o \left(\frac{\xi(t)}{\ell_o}\right)^{2-\eta}$$

- Percentage change per time in equilibrium

$$\left|\frac{\partial_t \xi}{\xi}\right| = \frac{a\nu}{t} \sim \frac{1}{t}$$

The equilibrium changes infinitely fast at the CP where $t = 0$

Relaxation rate of $C^{\hat{n}\hat{n}}$

- Substitute $\chi_{\rm Ising} = \chi_o \left(\xi/\ell_o\right)^{2-\eta}$ into the relaxation equation

$$\partial_t C^{\hat{n}\hat{n}} = -\underbrace{\frac{\lambda_{\rm eff}}{\chi_o {\ell_o}^2 (\xi/\ell_o)^{4-\eta}}}_{\text{relaxation rate } \Gamma} \left(C^{\hat{n}\hat{n}} - \chi(k,t)\right)$$

- Define the relaxation rate

$$\Gamma \equiv \underbrace{\frac{\lambda_{\rm eff}}{\chi_o {\ell_o}^2}}_{1/\tau_o} \times \underbrace{\frac{1}{(\xi(t)/\ell_o)^{4-\eta}}}_{\text{goes to 0 at CP}}$$

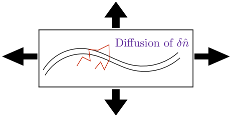
- Simplify Γ with $\xi(t) = \ell_o (\frac{t}{\tau_Q})^{-a\nu}$

$$\Gamma = \frac{1}{\tau_0}\left(\frac{t}{\tau_Q}\right)^{a\nu z} \quad \text{where} \quad z = 4 - \eta.$$

At CP, the hydo fluctuations relax infinitely slowly

Transiting the CP: Kibble-Zurek scales Berdnikov & Rajagopal; Mukherjee, Venugopalan, Yin

- When is this the changing rate of equilibrium comparable to the relaxation rate Γ ?



$$\left|\frac{\partial_t \xi}{\xi}\right| \sim \frac{1}{t} = \frac{(t/\tau_Q)^{a\nu z}}{\tau_o} = \Gamma$$

- The solution is defined to be Kibble-Zurek time t_{kz}

$$t_{kz} = \epsilon^{1/(a\nu z+1)}\tau_Q = \epsilon^{0.26}\tau_Q$$

- Kibble-Zurek length is defined to be

$$\ell_{kz} = \xi(t_{kz}) = \ell_o \epsilon^{-a\nu/(a\nu z+1)} = \ell_o \epsilon^{-0.19}$$

$$\underbrace{\ell_o}_{\text{micro-length}} \ll \underbrace{\ell_o \epsilon^{-0.19}}_{\text{kibble-zurek } \ell_{kz}} \ll \underbrace{\ell_o \epsilon^{-0.5}}_{\text{cutoff } \ell_{max}}$$

Transiting the critical point: Rescaled equation

- Measure t and k in KZ units

$$\bar{t} = t/t_{kz} \quad \bar{k} = k\ell_{kz}$$

- $C^{\hat{n}\hat{n}}$ is cut off at χ_{kz}

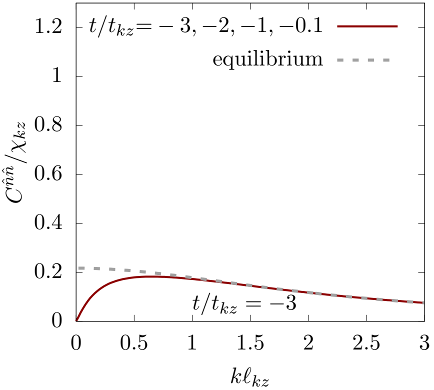
$$\bar{C}^{\hat{n}\hat{n}} = C^{\hat{n}\hat{n}}/\chi_{kz} \qquad \chi_{kz} \equiv \chi_{\rm Is}(t_{kz}) = \chi_o \epsilon^{-0.365}$$

- The rescaled equation becomes

$$\partial_{\bar{t}} \bar{C}^{\hat{n}\hat{n}} = -\frac{\bar{k}^2}{\bar{\chi}} \left(\bar{C}^{\hat{n}\hat{n}} - \bar{\chi}\right) \qquad \bar{\chi} = \frac{\bar{\chi}_{\rm Ising}}{1 + (\bar{k}\bar{\xi})^{2-\eta}}$$

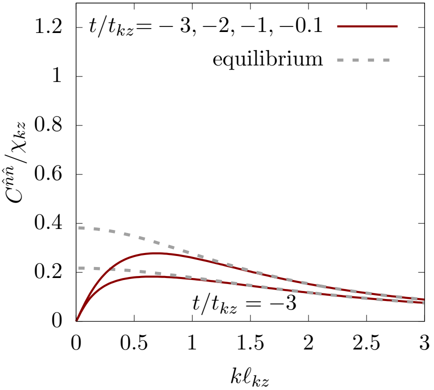
All dimensionful quantities are rescaled into the KZ-units

Solutions for $C^{\hat{n}\hat{n}}/\chi_{kz}$



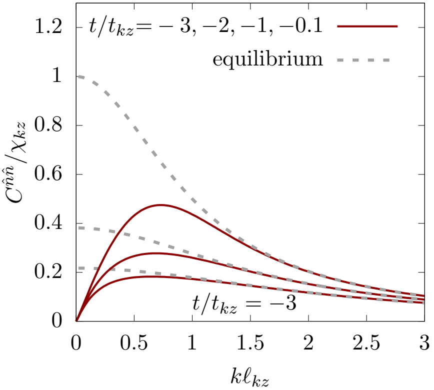
Before critical point

Solutions for $C^{\hat{n}\hat{n}}/\chi_{kz}$



Before critical point

Solutions for $C^{\hat{n}\hat{n}}/\chi_{kz}$



Before critical point

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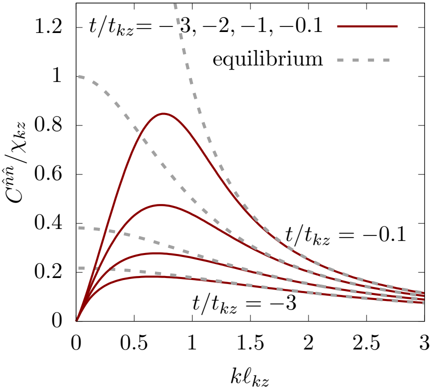
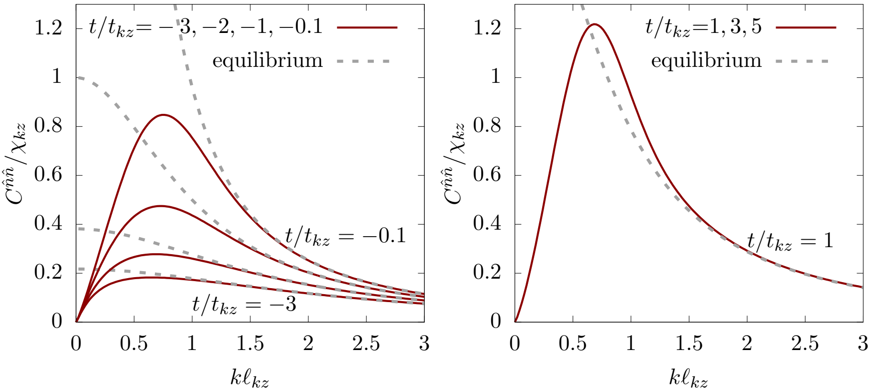


Before critical point

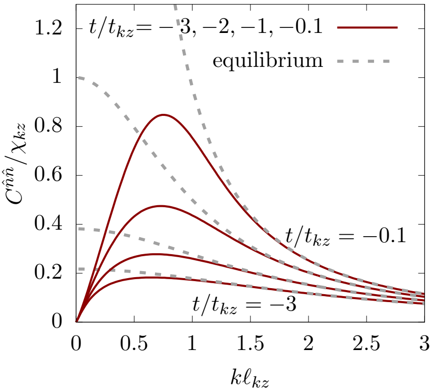
Solutions for $C^{\hat{n}\hat{n}}/\chi_{kz}$

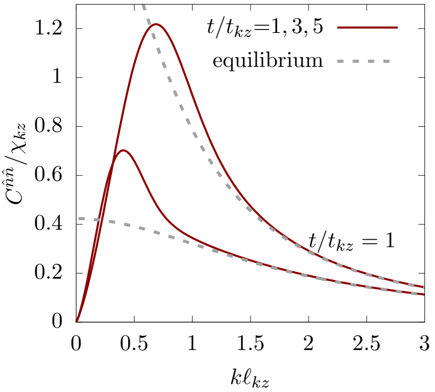


Before critical point

After critical point

Solutions for $C^{\hat{n}\hat{n}}/\chi_{kz}$

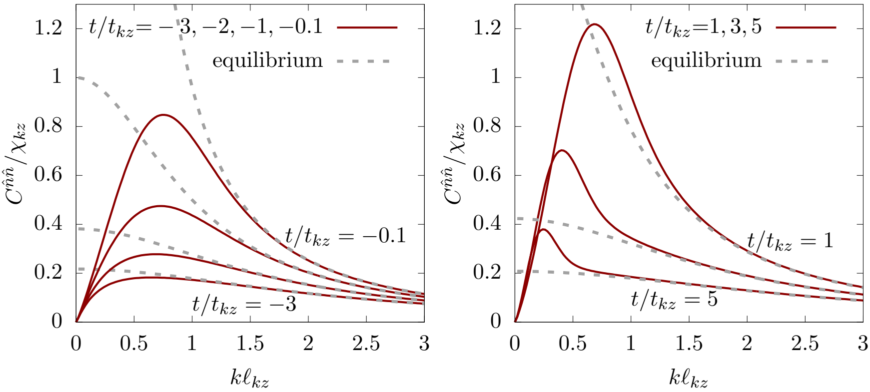




Before critical point

After critical point

Solutions for $C^{\hat{n}\hat{n}}/\chi_{kz}$



Before critical point

After critical point

What have we learnt so far?

- We have obtained order-1 plots after rescaling with KZ units

$$C^{\hat{n}\hat{n}}/\chi_{kz} \sim 1 \qquad k l_{kz} \sim 1 \qquad t/t_{kz} \sim 1$$

- The typical critical wavelength is ℓ_{kz}

$$\underbrace{\ell_o}_{\text{micro-length}} \ll \underbrace{\ell_o \epsilon^{-0.19}}_{\text{kibble-zurek } \ell_{kz}} \ll \underbrace{\ell_o \epsilon^{-0.5}}_{\text{cutoff } \ell_{\text{max}}}$$

Numerically these evaluate to with $\epsilon = 1/5$ and $\ell_0 = 1.2\,\text{fm}$

$$1.2\,\text{fm} \ll 1.6\,\text{fm} \ll 2.7\,\text{fm}$$

So the correlation is at most twice the interparticle spacing!
And the fluctuations are 80% larger than baseline:

$$\frac{C^{\hat{n}\hat{n}}}{\chi_0} \sim \left(\frac{\ell_{\text{kz}}}{\ell_0}\right)^{2-\eta} = \epsilon^{-0.365} \sim 1.8$$

$C^{\hat{n}\hat{n}}$ has length scale ℓ_{kz} and has limited growth of 80%

Slightly miss the critical point: scalings

- From Ising scaling, $\xi(\Delta e_{\text{Ising}}, \Delta M_{\text{Ising}})$, scales

$$\xi = \ell_o (\Delta e_{\text{Is}})^{-a\nu} f_\xi(\underbrace{\Delta e_{\text{Is}}/\Delta M_{\text{Is}}^{\frac{1-\alpha}{\beta}}}_{\text{scaling var}})$$

- Translating to QCD

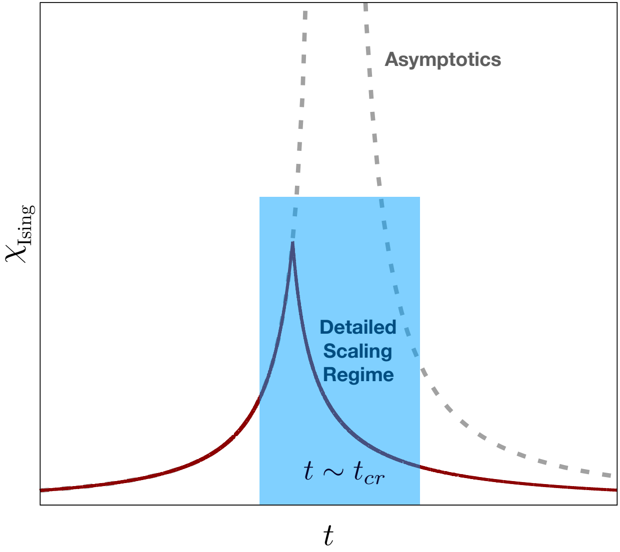
$$\Delta e_{\text{Ising}} \leftrightarrow \frac{\Delta n}{n_c} = -\frac{t}{\tau_Q} \qquad \Delta M_{\text{Ising}} \leftrightarrow \frac{\Delta s}{s_c} \sim \Delta_s$$

- The scaling of the Ising EOS implies a scaling in time

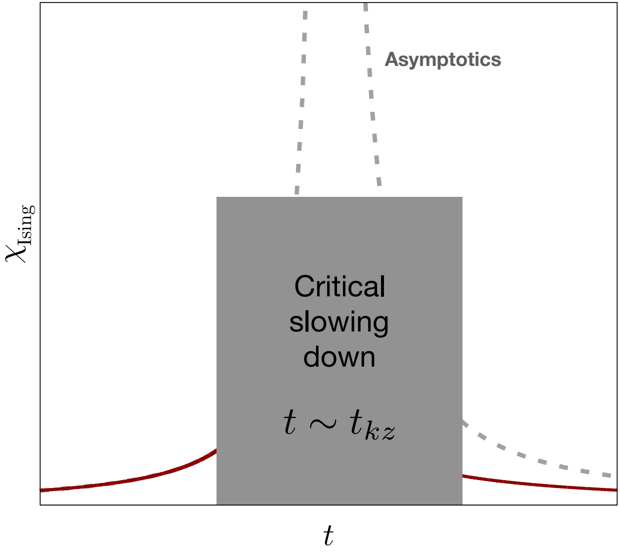
$$\xi = \ell_o \left(\frac{t}{\tau_Q}\right)^{-a\nu} \times \underbrace{f_\xi(t/t_{cr})}_{\text{scaling func}} \qquad t_{cr} \equiv \Delta_s^{\frac{1-\alpha}{\beta}} \tau_Q$$

t_{cr} is a new time scale that quantifies the missing of CP

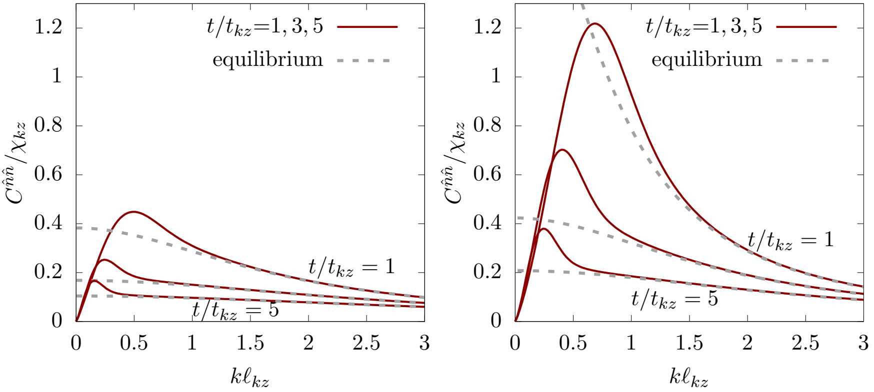
Detailed scaling regime happens when $t \sim t_{kz}$



Detailed scaling regime is obscured by KZ dynamics if $t_{kz} \gg t_{cr}$



Missing the critical point further limits the flucts



$t_{cr}/t_{kz} = 1$ $\qquad$ $t_{cr}/t_{kz} = 0$

Conclusions

- There are two scales t_{kz} and t_{cr}, they compete with each other

$$t_{kz} = \epsilon^{0.26}\tau_Q \qquad \text{vs} \qquad t_{cr} = {\Delta_s}^{2.72}\tau_Q$$

Numerically with $\tau_Q = 10$ fm, $\epsilon = 0.2$, $\Delta_s = 0.3$

$$t_{kz} = 6.58 \text{ fm} \qquad \gg \qquad t_{cr} = 0.38 \text{ fm}$$

So Kibble-Zurek dynamics is more important than detailed scaling

- $C^{\hat{n}\hat{n}}$ is a non-flow, and is quite local near CP

$$\underbrace{\ell_o}_{\substack{1.2\,\text{fm} \\ \text{micro-length } \ell_o}} \ll \underbrace{\ell_o\epsilon^{-0.19}}_{\substack{1.6\,\text{fm} \\ \text{Kibble-Zurek } \ell_{kz}}} \ll \underbrace{\ell_o\epsilon^{-0.5}}_{\substack{2.7\,\text{fm} \\ \text{cutoff } \ell_{\max}}} \ll \underbrace{R}_{\substack{15\,\text{fm} \\ \text{nucleus size}}}$$