B Anomalies after Moriond 2019

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3rd Red LHC

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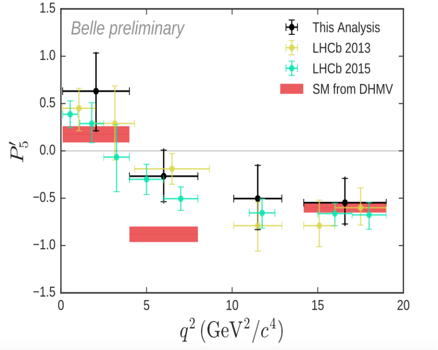
Outline

1. Experimental situation (after Moriond 2019)
2. Review of the theoretical framework
3. Global fits
4. Are we overlooking LFU?
4. Conclusions

Experimental Situation

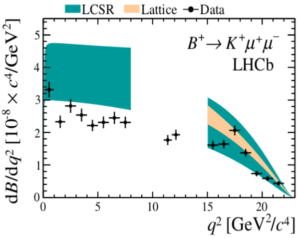
The P_5' anomaly

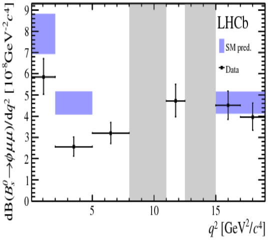
The field of semi-leptonic rare B decays has been providing some interesting anomalies.

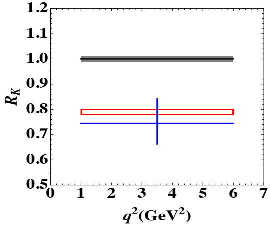


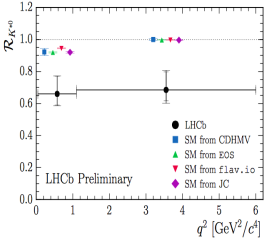
- 2013: 1fb^{-1} data-set LHCb found 3.7σ.
- 2015: 3fb^{-1} data-set LHCb found 3σ in 2 bins.
- Belle confirmed it in a bin [4,8] few months ago.

Other tensions beyond P_5' (2017)





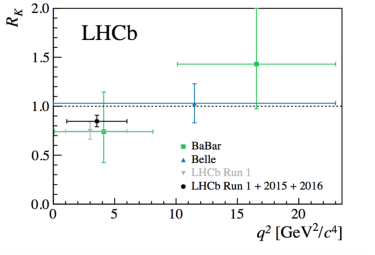


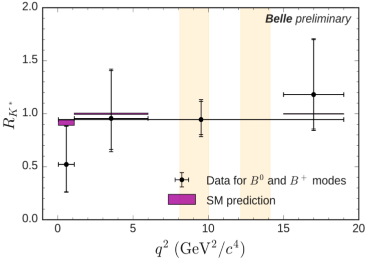


- $BR(B \to K\mu\mu)$ small compared to SM predictions.
- Deviations in $BR(B_s \to \phi\mu\mu)$.
- Several systematic low-recoil small tensions in BR_μ.
- LFUV ratios R_K & R_{K^*}.

LFUV $\equiv$ lepton flavour universality violation

News from Moriond 2019

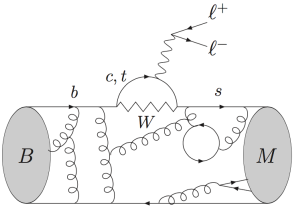




- New LHCb measurement of R_K.
- 2014 LHCb analysis: R_K at 2.6σ from the SM.
- Combination of LHCb 2014 & 2019 measurements: $R_K = 0.86^{+0.060+0.016}_{-0.054-0.014}$ still at 2.5σ from the SM, with the central value closer to 1.
- New Belle measurement of R_{K^*}.

Review of the theoretical framework

Effective Hamiltonian Approach



$\mathcal{A} \sim \mathcal{C}_i$ (short dist.)

$\times$ Hadronic Matrix Elements (long dist.)

$b \to s\gamma^{(*)}$ Effective Hamiltonian

$$\mathcal{H}^{\rm SM}_{\Delta F=1} \propto V^*_{ts} V_{tb} \sum_i \mathcal{C}_i \mathcal{O}_i$$

- $\mathcal{O}_{7(\prime)} = \frac{\alpha}{4\pi} m_b (\bar{s}\sigma_{\mu\nu} P_{R(L)} b) F^{\mu\nu}$
- $\mathcal{O}_{9(\prime)} = \frac{\alpha}{4\pi} (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \ell)$
- $\mathcal{O}_{10(\prime)} = \frac{\alpha}{16\pi} (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$

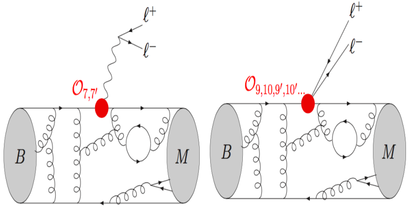
$\mathcal{C}^{\rm SM}_7(\mu_b) = -0.29 \qquad \mathcal{C}^{\rm SM}_9(\mu_b) = 4.1$

$\mathcal{C}^{\rm SM}_{10}(\mu_b) = -4.3 \qquad (\mu_b = m_b)$

$\Rightarrow$ In this picture, New Physics (NP) effects can enter through two mechanisms:

- Extra contributions to the WCs.
- Additional effective operators: $\mathcal{O}'_i$, $\mathcal{O}_S$, $\mathcal{O}_P$, $\mathcal{O}_T$,...

Effective Hamiltonian Approach



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Improved QCDF + Long Distance $c\bar{c}$ Loops

KMPW Form Factors $\Rightarrow \langle K^* | \mathcal{O}_i | B \rangle \sim F(q^2)$ $(i = 7, 9, 10)$ (KMPW Param.)

[Khodjamirian, Mannel, Pivovarov, Wang]

Improved QCDF (iQCDF) Approach: General decomposition of a full form factor (FF),

$$F^{\text{Full}}(q^2) = F^{\infty}(\xi_{\perp}(q^2), \xi_{\|}(q^2)) + \Delta F^{\alpha_s}(q^2) + \Delta F^{\Lambda}(q^2)$$

where F stands for any form factor, either from the helicity or transversity basis.

[Charles et al; Descotes-Genon, Hofer, Matias, Virto]

Factorisable Power Corrections:

- $\mathcal{O}(\alpha_s)$ corrections $\Rightarrow$ QCDF [Beneke, Feldman]
- $\mathcal{O}(\Lambda/m_B)$ corrections $\Rightarrow \Delta F^{\Lambda}(q^2) = a_F + b_F \frac{q^2}{m_B^2} + c_F \frac{q^4}{m_B^4}$

[Jäger & Camalich; Descotes-Genon, Hofer, Matias, Virto]

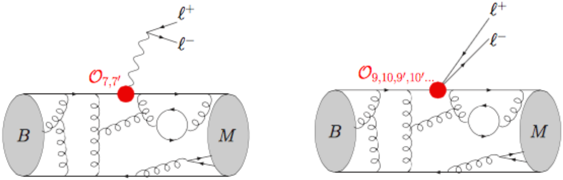
Non-Factorisable Power Corrections:

- Hard Scattering $\mathcal{O}(\alpha_s)$ corrections $\Rightarrow$ QCDF [Beneke, Feldman, Seidel]
- $c\bar{c}$ Loops $\mathcal{O}(\Lambda/m_B) \Rightarrow \mathcal{C}_{7,9}^{\text{eff}}(q^2) \to \mathcal{C}_{7,9}^{\text{eff}}(q^2) + s_i \delta\mathcal{C}_{7,9}^{\text{LD},i}(q^2)$ $(i = \perp, \|, 0)$

[Khodjamirian, Mannel, Pivovarov, Wang; Descotes-Genon, Hofer, Matias, Virto]

Hadronic corrections (diagrammatic depiction)

- Factorisable Corrections: *corrections that **can** be absorbed into the definition of the (full) form factors.*



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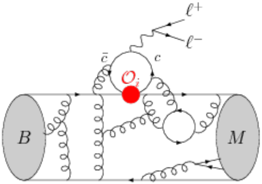


Global fits

List of observables in the fit (before 2017)

We perform a fit to all available data (except CPV obs.) $\Rightarrow$ 178 obs (2019).

- Inclusive decays
  - $\Rightarrow B \to X_s \gamma$ (BR).
  - $\Rightarrow B \to X_s \mu^+ \mu^-$ (BR).
- Exclusive leptonic decays
  - $\Rightarrow B_s \to \mu^+ \mu^-$ (BR) (LHCb & CMS).
- Exclusive radiative/semileptonic decays
  - $\Rightarrow B \to K^* \gamma$ (BR, $S_{K^*\gamma}$, A_I).
  - $\Rightarrow B \to K \ell^+ \ell^-$ (BR_μ, R_K).
  - $\Rightarrow B \to K^* \ell^+ \ell^-$ (BR_μ, $P^{(\prime)\,\mu}_{1,2,4,5,6,8}$, F_L^μ, available electronic angular obs).
  - $\Rightarrow B_s \to \phi \mu^+ \mu^-$ (BR, $P^{(\prime}_{1,4,6}$), F_L).

List of observables in the fit (2017 & 2019 update)

- Updates 2017
  - $\Rightarrow$ $\mathcal{B}(B^0 \to K^{*0}\mu^+\mu^-)$ **(LHCb)**.
  - $\Rightarrow$ Isospin-averaged $P'^{\,e\mu}_{4,5}(B \to K^*\ell\ell)$ **(Belle)**.
  - $\Rightarrow$ $P^{(\prime)}_{1,4,5,6,8}, F_L(B^0 \to K^{*0}\mu^+\mu^-)$ in the large-recoil region **(ATLAS)**.
  - $\Rightarrow$ $P^{(\prime)}_{1,5}(B^0 \to K^{*0}\mu^+\mu^-)$ at large-recoil plus $[16, 19]$ GeV^2 bin **(CMS)**.
  - $\Rightarrow$ F_L, A_{FB} from 2015 and F_L, A_{FB}, BR from 2013 at 7 TeV **(CMS)**.
  - $\Rightarrow$ R_{K^*} in the bins $[0.045, 1.1]$, $[1.1, 6]$ GeV^2 **(LHCb)**.
- Updates 2019
  - $\Rightarrow$ $B_s \to \mu^+\mu^-$ (BR) (ATLAS, LHCb & CMS) $\Rightarrow$ Uncorr. average.
  - $\Rightarrow$ R_K **(LHCb)** $\Rightarrow$ LHCb 2014 & 2019 combination.
  - $\Rightarrow$ R_{K^*} in the bins $[0.045, 1.1]$, $[1.1, 6]$ & $[15, 19]$ GeV^2 **(Belle)**.

Statistical framework

We parametrize the Wilson coefficients as,

$$\mathcal{C}_i = \mathcal{C}_i^{\rm SM} + \mathcal{C}_i^{\rm NP} \quad (i = 7^{(\prime)}, 9^{(\prime)}, 10^{(\prime)},\ \mathcal{C}_i^{\rm NP} \in \mathbb{R} \Rightarrow \text{no CPV})$$

Standard frequentist fit to the NP contributions to the Wilson coefficients,

$$\chi^2(\mathcal{C}_i^{\rm NP}) = \left(\mathcal{O}^{\rm th}(\mathcal{C}_i^{\rm NP}) - \mathcal{O}^{\rm exp}\right)_i Cov_{ij}^{-1} \left(\mathcal{O}^{\rm th}(\mathcal{C}_i^{\rm NP}) - \mathcal{O}^{\rm exp}\right)_j$$

- Both **theory and experiment** contribute to the covariance matrix,
  $\Rightarrow$ $Cov = Cov^{\rm th} + Cov^{\rm exp}$
- Experimental covariance,
  $\Rightarrow$ **Experimental correlations** between observables (if not provided, assumed uncorrelated). Assume gaussian errors (symmetrize if needed).
- Theoretical covariance,
  $\Rightarrow$ Compute the **theoretical correlations** by performing a multivariate gaussian scan over all nuisance parameters.
- In principle $Cov = Cov(\mathcal{C}_i)$,
  $\Rightarrow$ **Mild** dependency $\Rightarrow$ $Cov = Cov_{\rm SM} \equiv Cov(\mathcal{C}_i = 0)$.

[Descotes-Genon, Hofer, Matias, Virto; BC, Crivellin, Descotes-Genon, Matias, Virto]

Statistical framework

- Fit procedure:
  - ⇒ **Best fit points** (bfp): $\chi^2(\mathcal{C}_i^{\rm NP}) \to \chi^2_{\rm min} = \chi^2(\hat{\mathcal{C}}_{i\rm NP})$.
  - ⇒ **Confidence intervals**: $\chi^2(\mathcal{C}_i^{\rm NP}) - \chi^2_{\rm min} \leq Q^2$ $(1\sigma \to Q^2 = 1,\ 2\sigma \to Q^2 = 4,\ \ldots)$.
  - ⇒ Compute **pulls** (σ): $\text{Pull}_{\rm SM} = \sqrt{\chi^2_{\rm SM} - \chi^2_{\rm min,Sc}}$.
- Two types of fits
  - ⇒ *Canonical* fit: fit to **all data** (178 data points).
  - ⇒ LFUV fit: R_K, R_{K^*}, $P'^{\,e\mu}_{4,5}(B \to K^*\ell\ell)$ plus $b \to s\gamma$ (20 data points)
- Testing different **hypotheses**
  - ⇒ Hypotheses with NP only in one Wilson coefficient (**1D fits**).
  - ⇒ Hypotheses with NP in two Wilson coefficients (**2D fits**).
  - ⇒ Hypotheses with NP in the six Wilson coefficients (**6D fits**).

[Descotes-Genon, Hofer, Matias, Virto; BC, Crivellin, Descotes-Genon, Matias, Virto]

Analysing 1D NP hypotheses

2017	All			
1D Hyp.	Best fit	1σ	$\mathrm{Pull}_{\mathrm{SM}}$	p-value (%)
$\mathcal{C}_{9\mu}^{\mathrm{NP}}$	-1.11	$[-1.28, -0.94]$	5.8	68
$\mathcal{C}_{9\mu}^{\mathrm{NP}} = -\mathcal{C}_{10\mu}^{\mathrm{NP}}$	-0.62	$[-0.75, -0.49]$	5.3	58
$\mathcal{C}_{9\mu}^{\mathrm{NP}} = -\mathcal{C}'_{9\mu}$	-1.01	$[-1.18, -0.84]$	5.4	61
$\mathcal{C}_{9\mu}^{\mathrm{NP}} = -3\mathcal{C}_{9e}^{\mathrm{NP}}$	-1.07	[-1.24,-0.90]	5.8	70

2019	All			
1D Hyp.	Best fit	1σ	$\mathrm{Pull}_{\mathrm{SM}}$	p-value (%)
$\mathcal{C}_{9\mu}^{\mathrm{NP}}$	-1.02	$[-1.18, -0.85]$	5.8	65
$\mathcal{C}_{9\mu}^{\mathrm{NP}} = -\mathcal{C}_{10\mu}^{\mathrm{NP}}$	-0.49	$[-0.59, -0.40]$	5.4	56
$\mathcal{C}_{9\mu}^{\mathrm{NP}} = -\mathcal{C}'_{9\mu}$	-1.02	$[-1.18, -0.85]$	5.7	61
$\mathcal{C}_{9\mu}^{\mathrm{NP}} = -3\mathcal{C}_{9e}^{\mathrm{NP}}$	-0.92	[-1.08,-0.76]	5.7	63

$\Rightarrow$ The hierarchy of scenarios remains fundamentally the same.

$\Rightarrow$ $\mathcal{C}_{9\mu}^{\mathrm{NP}}$ is the strongest signal for the "All fit". $\mathcal{C}_{9\mu}^{\mathrm{NP}} = -\mathcal{C}_{10\mu}^{\mathrm{NP}}$ dominates in the LFUV fit.

$\Rightarrow$ $\mathcal{C}_{10\mu}^{\mathrm{NP}}$ solution shows a significance of $\sim 4\sigma$.

$\Rightarrow$ $\mathcal{C}_{9\mu}^{\mathrm{NP}} = -\mathcal{C}'_{9\mu}$ significances increases, since this predicts $R_K \simeq 1$.

[BC, Crivellin, Descotes-Genon, Matias, Virto; Algueró, BC, Crivellin, Descotes-Genon, Masjuan, Matias, Virto]

Analysing 2D NP hypotheses

	All		
2017	Best fit	$\mathrm{Pull}_{\mathrm{SM}}$	p-value (%)
$(\mathcal{C}_{9\mu}^{\mathrm{NP}}, \mathcal{C}_{10\mu}^{\mathrm{NP}})$	(-1.01,0.29)	5.7	72
$(\mathcal{C}_{9\mu}^{\mathrm{NP}}, \mathcal{C}_{9'\mu})$	(-1.15,0.41)	5.6	71
$(\mathcal{C}_{9\mu}^{\mathrm{NP}}, \mathcal{C}_{10'\mu})$	(-1.22,-0.22)	5.7	72
$(\mathcal{C}_{9\mu}^{\mathrm{NP}}, \mathcal{C}_{9e}^{\mathrm{NP}})$	(-1.00,0.42)	5.5	68
Hyp. 1	(-1.16,0.38)	5.7	73
2019	Best fit	$\mathrm{Pull}_{\mathrm{SM}}$	p-value (%)
$(\mathcal{C}_{9\mu}^{\mathrm{NP}}, \mathcal{C}_{10\mu}^{\mathrm{NP}})$	(-0.95,0.20)	5.7	70
$(\mathcal{C}_{9\mu}^{\mathrm{NP}}, \mathcal{C}_{9'\mu})$	(-1.13,0.54)	5.9	75
$(\mathcal{C}_{9\mu}^{\mathrm{NP}}, \mathcal{C}_{10'\mu})$	(-1.17,-0.34)	6.1	78
$(\mathcal{C}_{9\mu}^{\mathrm{NP}}, \mathcal{C}_{9e}^{\mathrm{NP}})$	(-1.04,-0.11)	5.5	65
Hyp. 1	(-1.09,0.28)	6.0	76
Hyp. 4	(-0.52,0.11)	5.2	59
Hyp. 5	(-1.17,0.24)	6.1	78

Hyp 1: $(\mathcal{C}_{9\mu}^{\mathrm{NP}} = -\mathcal{C}_{9'\mu}, \mathcal{C}_{10\mu}^{\mathrm{NP}} = \mathcal{C}_{10'\mu})$, 4: $(\mathcal{C}_{9\mu}^{\mathrm{NP}} = -\mathcal{C}_{10\mu}^{\mathrm{NP}}, \mathcal{C}_{9'\mu} = -\mathcal{C}_{10'\mu})$ and 5: $(\mathcal{C}_{9\mu}^{\mathrm{NP}}, \mathcal{C}_{9'\mu} = -\mathcal{C}_{10'\mu})$.

⇒ New data (mainly R_K) allows more space for right-handed currents (RHCs).

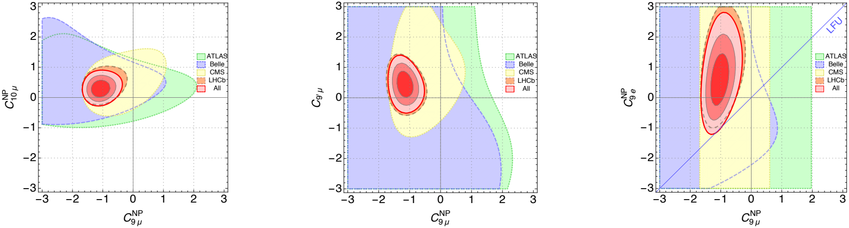
⇒ $\mathcal{C}_{9\mu}^{\mathrm{NP}}$ (vector) structure preferred over $\mathcal{C}_{9\mu}^{\mathrm{NP}} = -\mathcal{C}_{10\mu}^{\mathrm{NP}}$ (left-handed) with RHCs of the type $\mathcal{C}_{9'\mu} = -\mathcal{C}_{10'\mu}$ ⇒ Hyp. 4 vs Hyp. 5

[BC, Crivellin, Descotes-Genon, Matias, Virto; Algueró, BC, Crivellin, Descotes-Genon, Masjuan, Matias, Virto]

Analysing 2D NP hypotheses

Confidence regions plots (2017):

[BC, Crivellin, Descotes-Genon, Matias, Virto]



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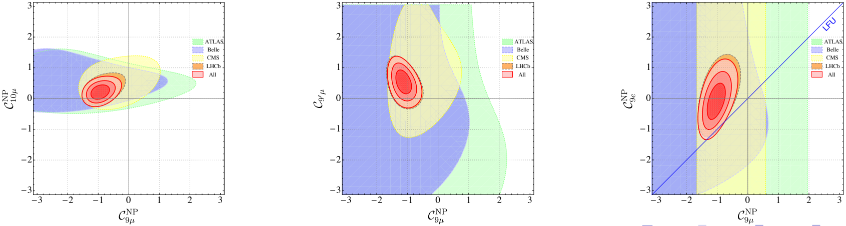

Are we overlooking Lepton Flavour Universal (LFU) NP?

NP scenarios with LFU NP

⇒ The distinction between lepton flavour dependent (LFD) observables and LFUV observables suggests a new (and natural) basis,

$$\mathcal{C}_{i\ell}^{\rm NP} = \mathcal{C}_{i\ell}^{\rm V} + \mathcal{C}_{i}^{\rm U}$$

with $i = 9, 10, 9', 10'$ and $\ell = e, \mu$ (trivial extension to $\ell = \tau$).

⇒ The NP parameter space can be equally described with $\{\mathcal{C}_{i\mu}^{\rm NP}, \mathcal{C}_{ie}^{\rm NP}\}$ or $\{\mathcal{C}_{i\mu}^{\rm V}, \mathcal{C}_{i}^{\rm U}\}$ ($\mathcal{C}_{ie}^{\rm V} = 0$).

⇒ The LFU vs LFUV language generates non-obvious NP scenarios in the μ vs e language,

$$\left\{ \begin{array}{l} \mathcal{C}_{9\mu}^{\rm V} = -\mathcal{C}_{10\mu}^{\rm V} \\ \mathcal{C}_{9}^{\rm U} \end{array} \right. \Rightarrow \left\{ \begin{array}{l} \mathcal{C}_{9\mu}^{\rm NP} = -\mathcal{C}_{10\mu}^{\rm NP} + \mathcal{C}_{9e}^{\rm NP} \\ \mathcal{C}_{9e}^{\rm NP} \end{array} \right.$$

[Algueró, BC, Descotes-Genon, Masjuan, Matias; Algueró, BC, Crivellin, Descotes-Genon, Masjuan, Matias, Virto]

Analysing LFU NP Fits

2017 - Scenario		Best-fit point	1σ	Pull_{SM}	p-value
Scenario 6	$\mathcal{C}_{9\mu}^{\text{V}} = -\mathcal{C}_{10\mu}^{\text{V}}$	-0.64	$[-0.77, -0.51]$	6.0	79 %
	$\mathcal{C}_{9}^{\text{U}} = \mathcal{C}_{10}^{\text{U}}$	-0.44	$[-0.58, -0.29]$		
Scenario 7	$\mathcal{C}_{9\mu}^{\text{V}}$	-1.57	$[-2.14, -1.06]$	5.7	72 %
	$\mathcal{C}_{9}^{\text{U}}$	$+0.56$	$[+0.01, +1.15]$		
Scenario 8	$\mathcal{C}_{9\mu}^{\text{V}} = -\mathcal{C}_{10\mu}^{\text{V}}$	-0.42	$[-0.57, -0.27]$	5.8	74 %
	$\mathcal{C}_{9}^{\text{U}}$	-0.67	$[-0.90, -0.42]$		
2019 - Scenario		**Best-fit point**	**1σ**	**Pull_{SM}**	**p-value**
Scenario 6	$\mathcal{C}_{9\mu}^{\text{V}} = -\mathcal{C}_{10\mu}^{\text{V}}$	-0.52	$[-0.64, -0.41]$	5.8	71.5 %
	$\mathcal{C}_{9}^{\text{U}} = \mathcal{C}_{10}^{\text{U}}$	-0.37	$[-0.52, -0.22]$		
Scenario 7	$\mathcal{C}_{9\mu}^{\text{V}}$	-0.91	$[-1.25, -0.58]$	5.5	65.3 %
	$\mathcal{C}_{9}^{\text{U}}$	-0.08	$[-0.46, +0.31]$		
Scenario 8	$\mathcal{C}_{9\mu}^{\text{V}} = -\mathcal{C}_{10\mu}^{\text{V}}$	-0.33	$[-0.45, -0.22]$	5.9	74.1 %
	$\mathcal{C}_{9}^{\text{U}}$	-0.72	$[-0.93, -0.47]$		

⇒ $\{\mathcal{C}_{9\mu}^{\text{V}}, \mathcal{C}_{9}^{\text{U}}\}$ largely correlated ⇒ $\mathcal{C}_{9\mu}^{\text{V}} + \mathcal{C}_{9}^{\text{U}} = -0.98$ vs $\mathcal{C}_{9\mu}^{\text{NP}} = -1.02$.

⇒ Left-handed structure $\mathcal{C}_{9\mu}^{\text{V}} = -\mathcal{C}_{10\mu}^{\text{V}}$ preferred over the vector structure $\mathcal{C}_{9\mu}^{\text{V}}$, with the presence of $\mathcal{C}_{9\mu}^{\text{U}}$.

[Algueró, BC, Descotes-Genon, Masjuan, Matias; Algueró, BC, Crivellin, Descotes-Genon, Masjuan, Matias, Virto]

Conclusions

Conclusions

- Performing our analyses with the new measurements of R_K, R_{K^*} and $B_s \to \mu\mu$:
  $\Rightarrow$ $\mathcal{C}_{9\mu}$ is (still) the most strong signal of NP.
  $\Rightarrow$ Hierarchy of scenarios unaltered.
- Stronger signals coming from RHC:
  $\Rightarrow$ With a signature $\mathcal{C}_{9'\mu} > 0$ and $\mathcal{C}_{10'\mu} < 0$.
  $\Rightarrow$ Hyp. 5 ($\mathcal{C}_{9\mu}^{\rm NP}, \mathcal{C}_{9'\mu} = -\mathcal{C}_{10'\mu}$) emerges as one of the scenarios with higher significance (6.1σ).
- The interplay between LFU and LFUV NP might be instrumental for finding new directions both for phenomenological analyses and models.
  $\Rightarrow$ Overcoming the hierarchy between $\mathcal{C}_{9\mu}^{\rm V} = -\mathcal{C}_{10\mu}^{\rm V}$ and $\mathcal{C}_{9\mu}^{\rm V}$ by introducing LFU NP ($\mathcal{C}_{9\mu}^{\rm U}$).

Thank you