

# EXPLORING THE INTERIOR OF THE NUCLEON WITH TRANSVERSE MOMENTUM DEPENDENT PARTON DISTRIBUTIONS (TMDs)

Alessandro Bacchetta



# THANKS TO HADRONIC PHYSICS GROUP IN PAVIA

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**Valerio Bertone**



**Chiara Bissolotti**



**Giuseppe Bozzi**



**Francesco Celiberto**



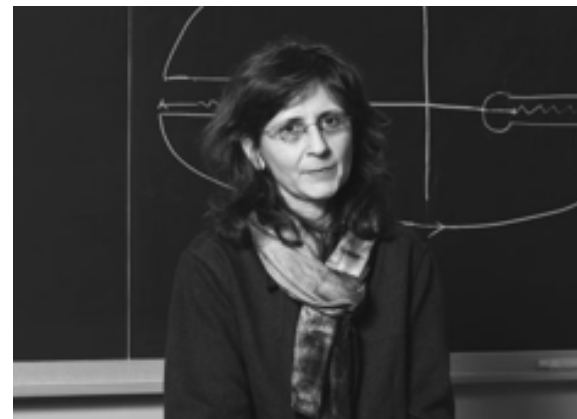
**Filippo Delcarro**



**Miguel G. Echevarria**



**Barbara Pasquini**



**Fulvio Piacenza**



**Cristian Pisano**



**Marco Radici**



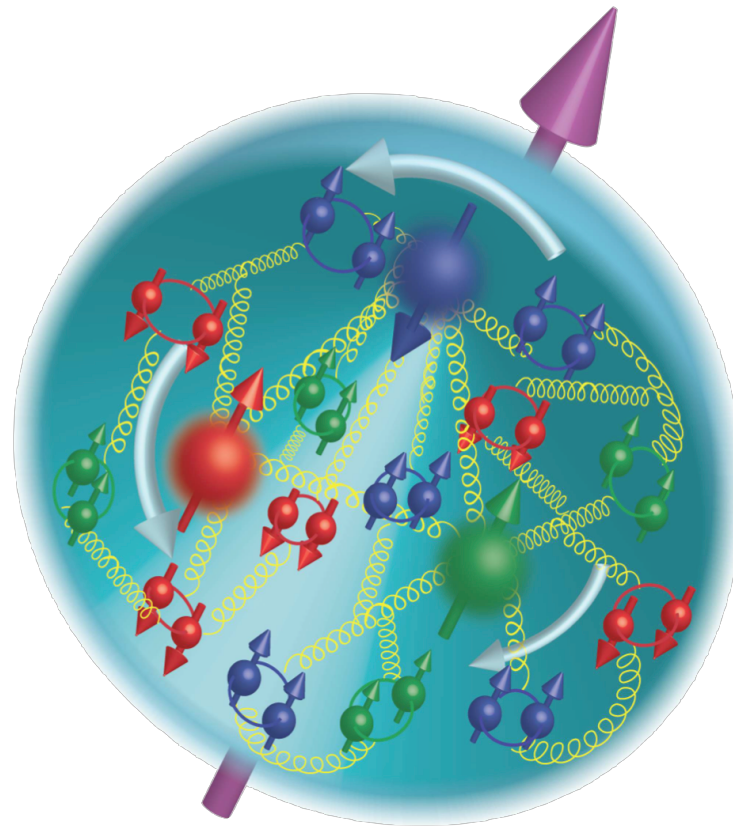
**Simone Rodini**





# WHY IS IT INTERESTING TO MAP THE NUCLEON?

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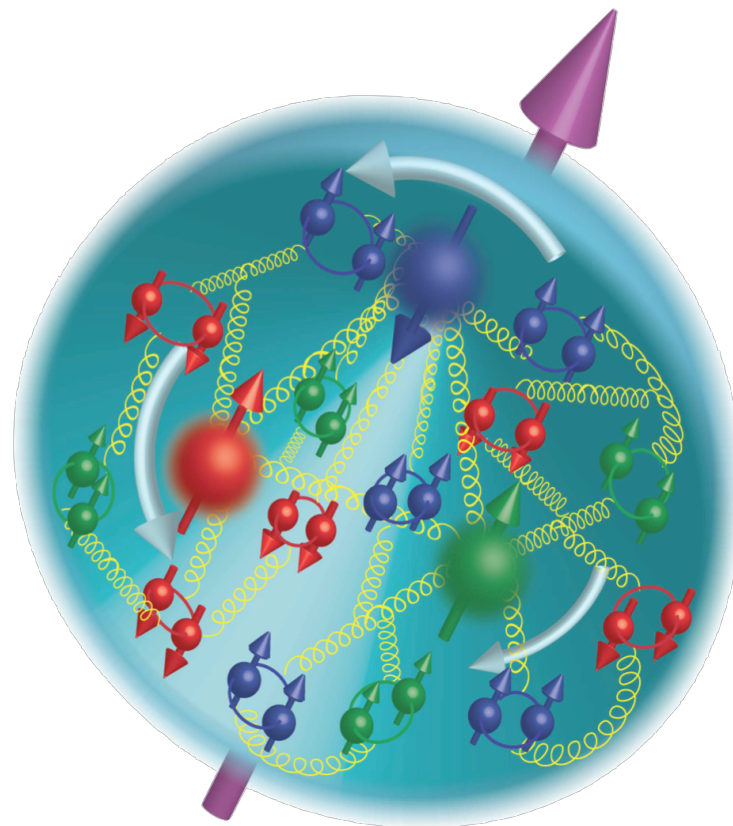
# WHY IS IT INTERESTING TO MAP THE NUCLEON?

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$$\mathcal{L}_{\text{QCD}} = \sum_q \bar{\psi}_q (i \not{\partial} - g \not{A} + m) \psi_q - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$



Check predictions

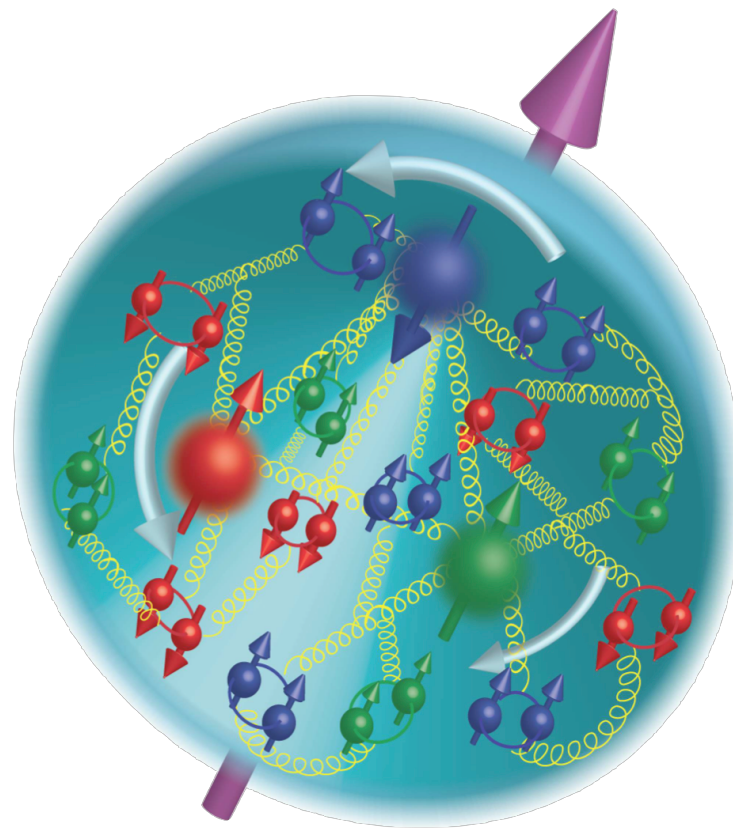


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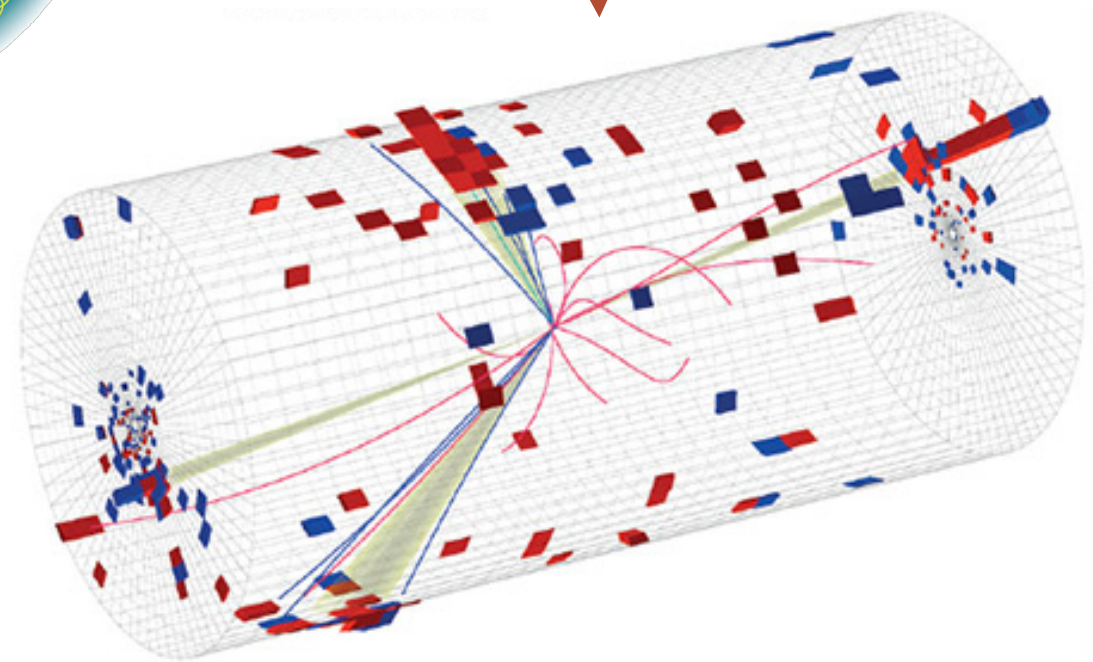
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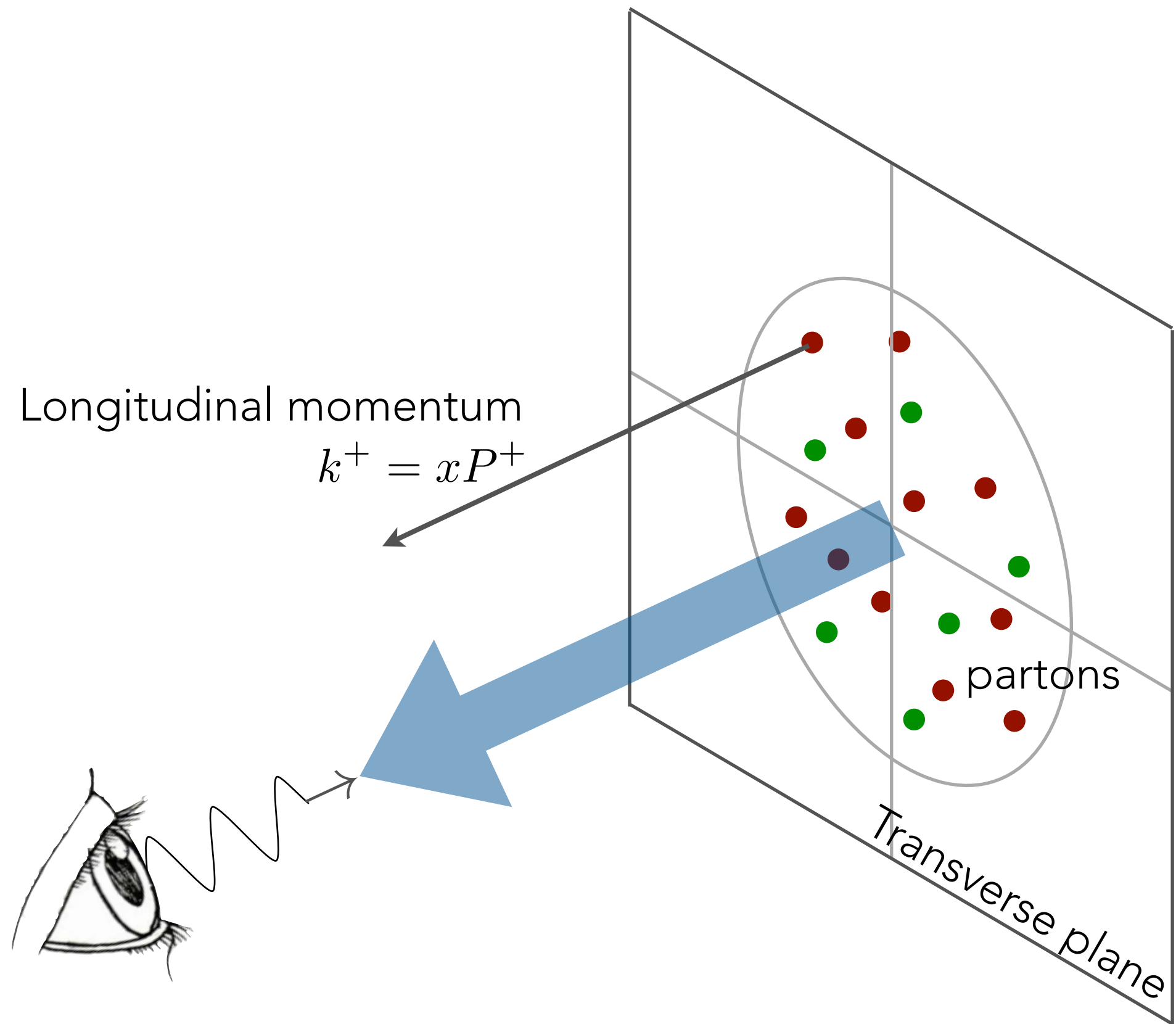
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Check predictions



Make predictions



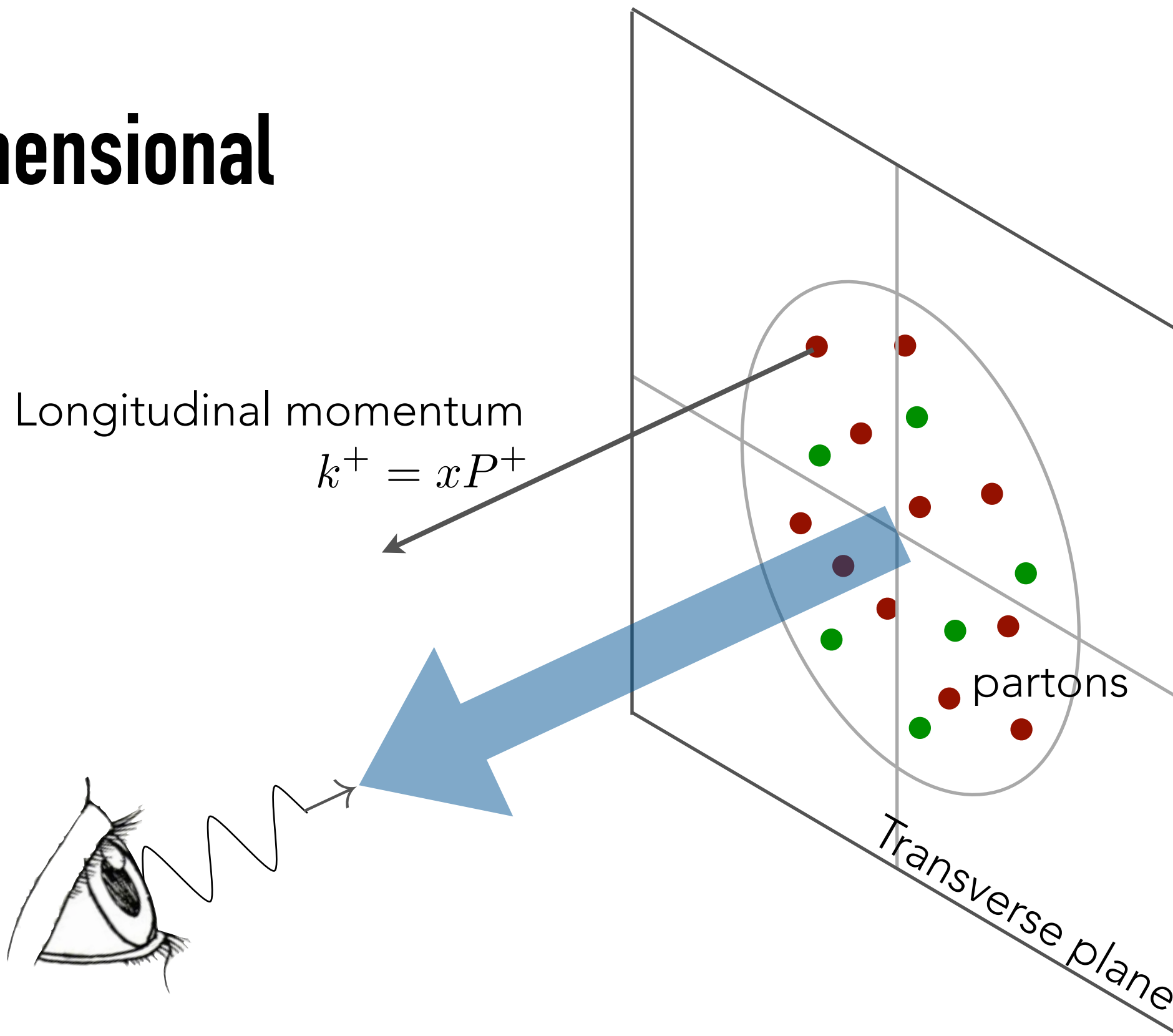




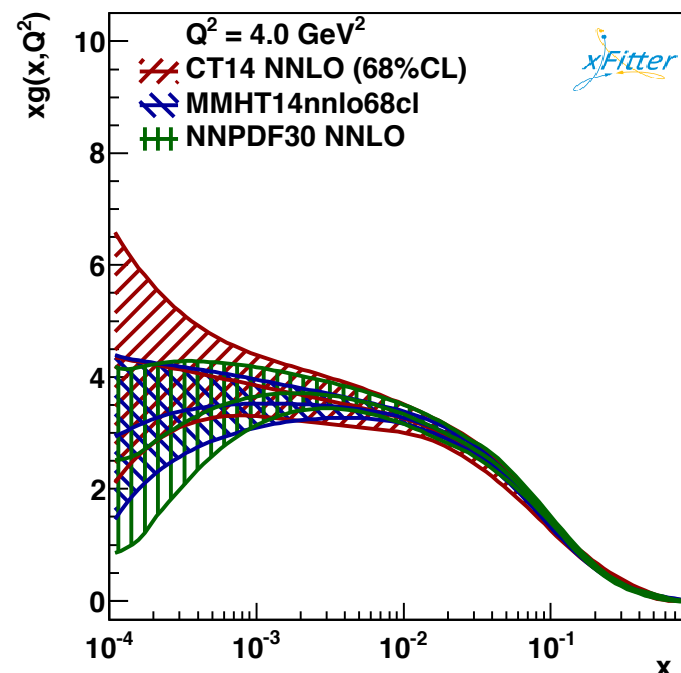
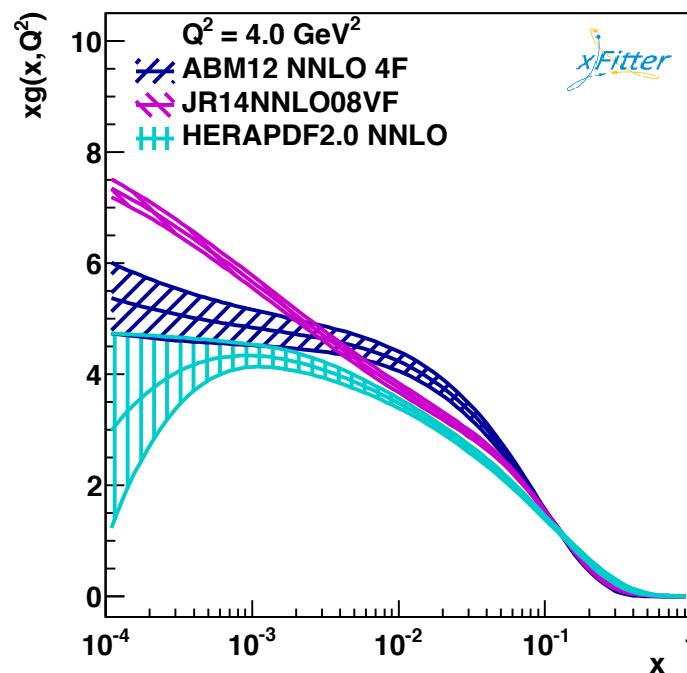
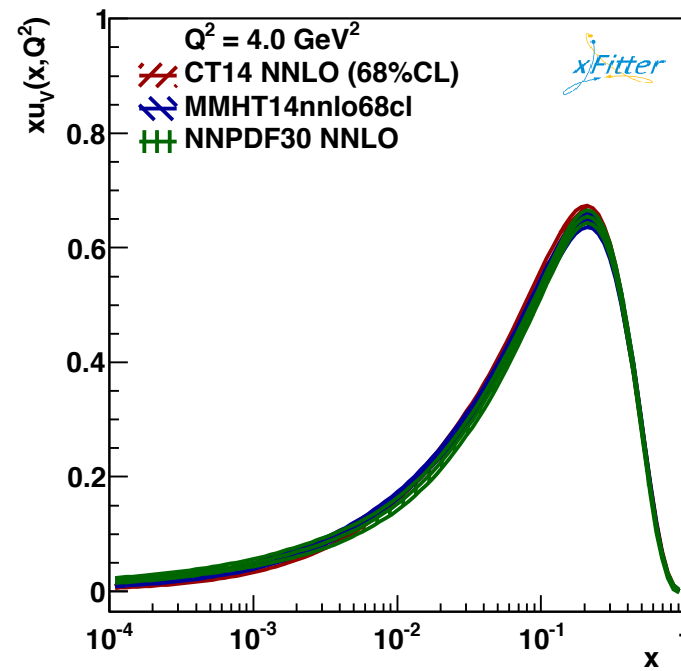
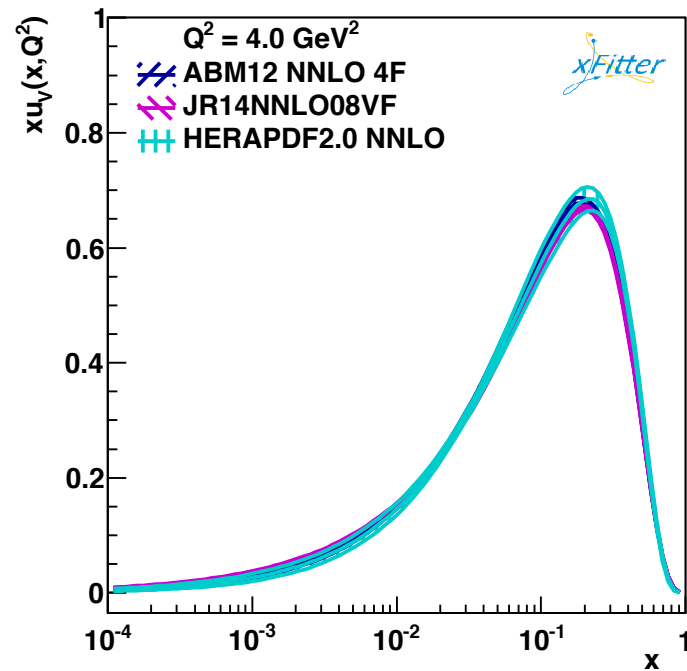
# Parton Distribution Functions

$$f(x)$$

## 1 dimensional

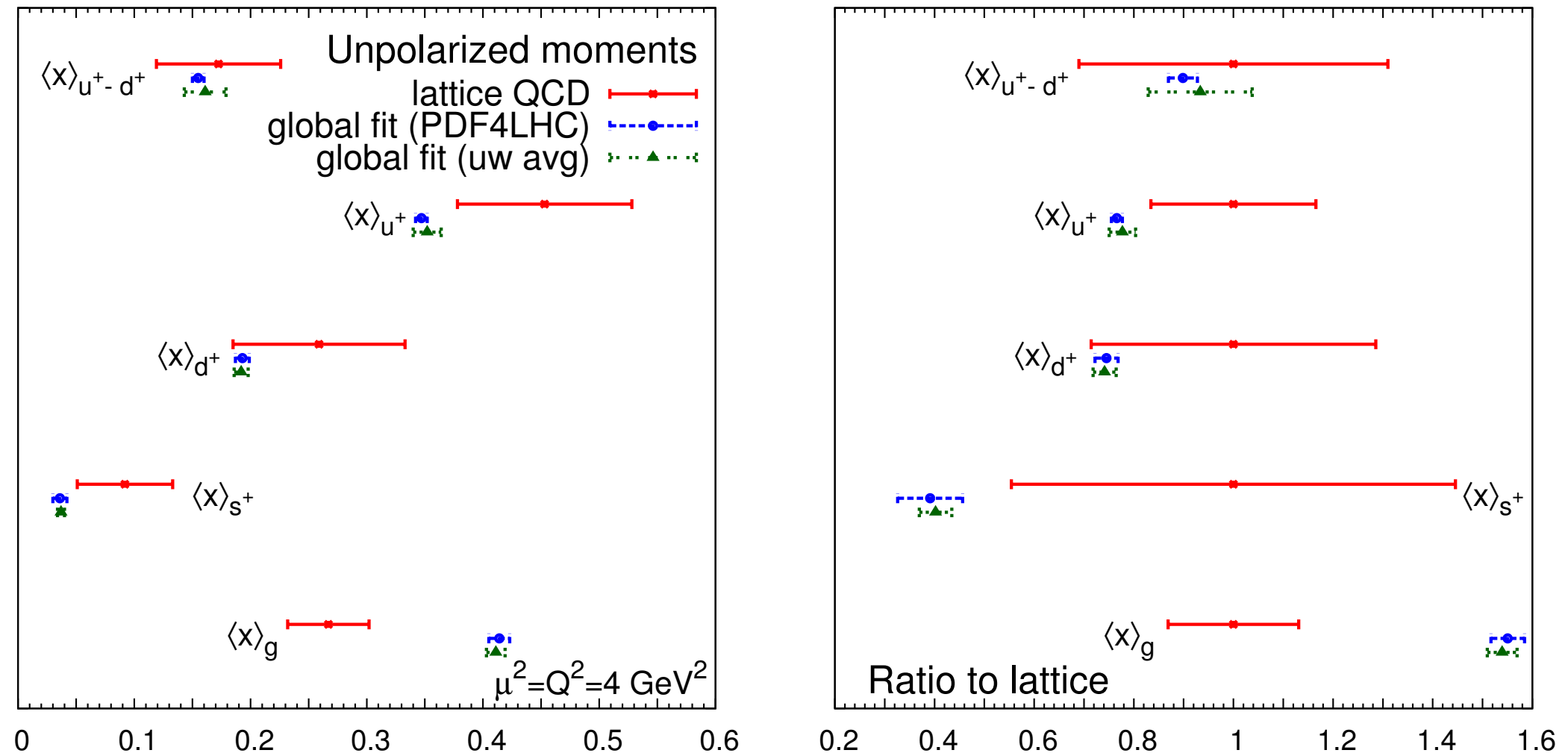


# STANDARD PARTON DISTRIBUTION FUNCTIONS



Standard collinear PDFs describe the distribution of partons in one dimension in momentum space. They are extracted through global fits

# UNPOLARIZED PDF MOMENTS AND LATTICE QCD

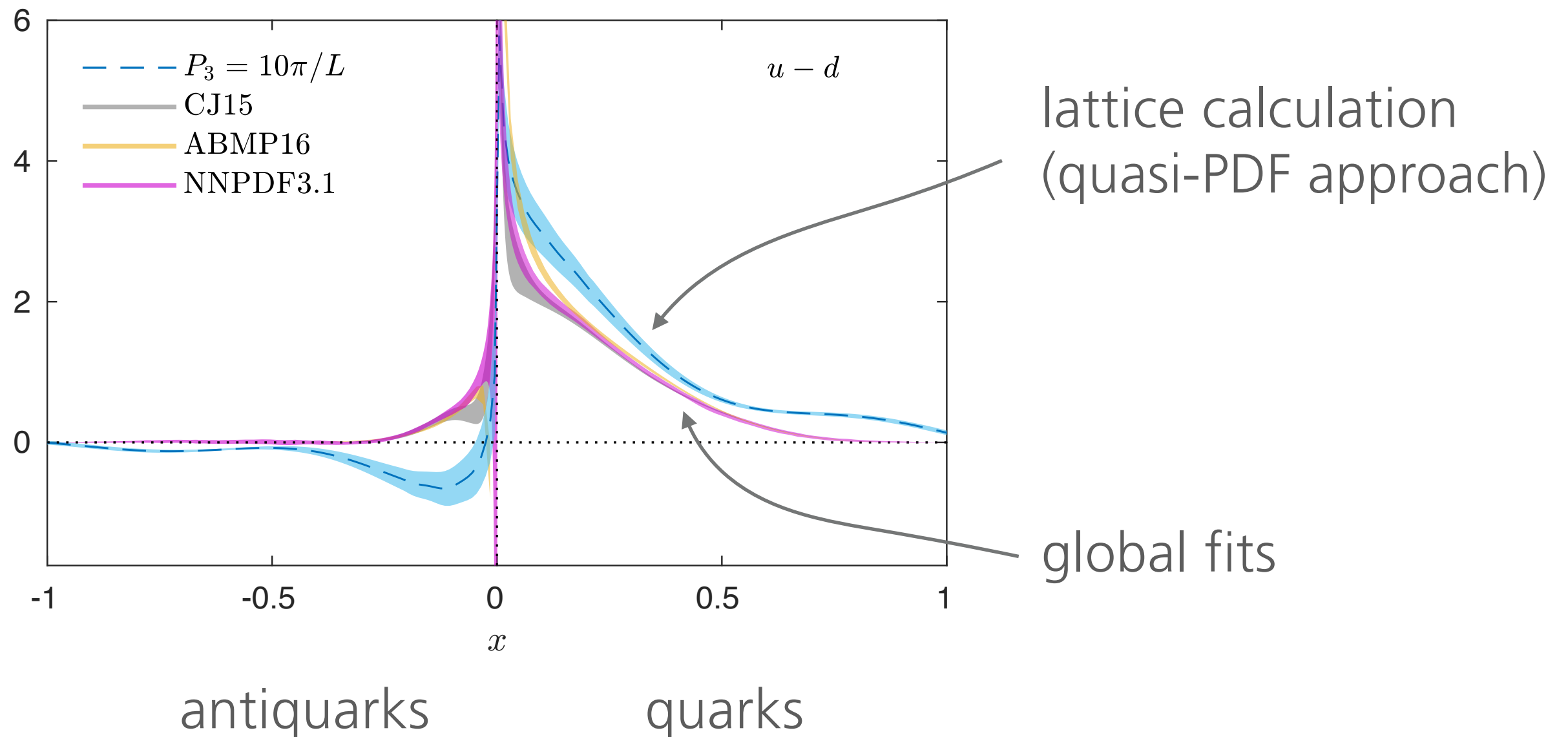


PDFLattice White Paper, arXiv:1711.07916

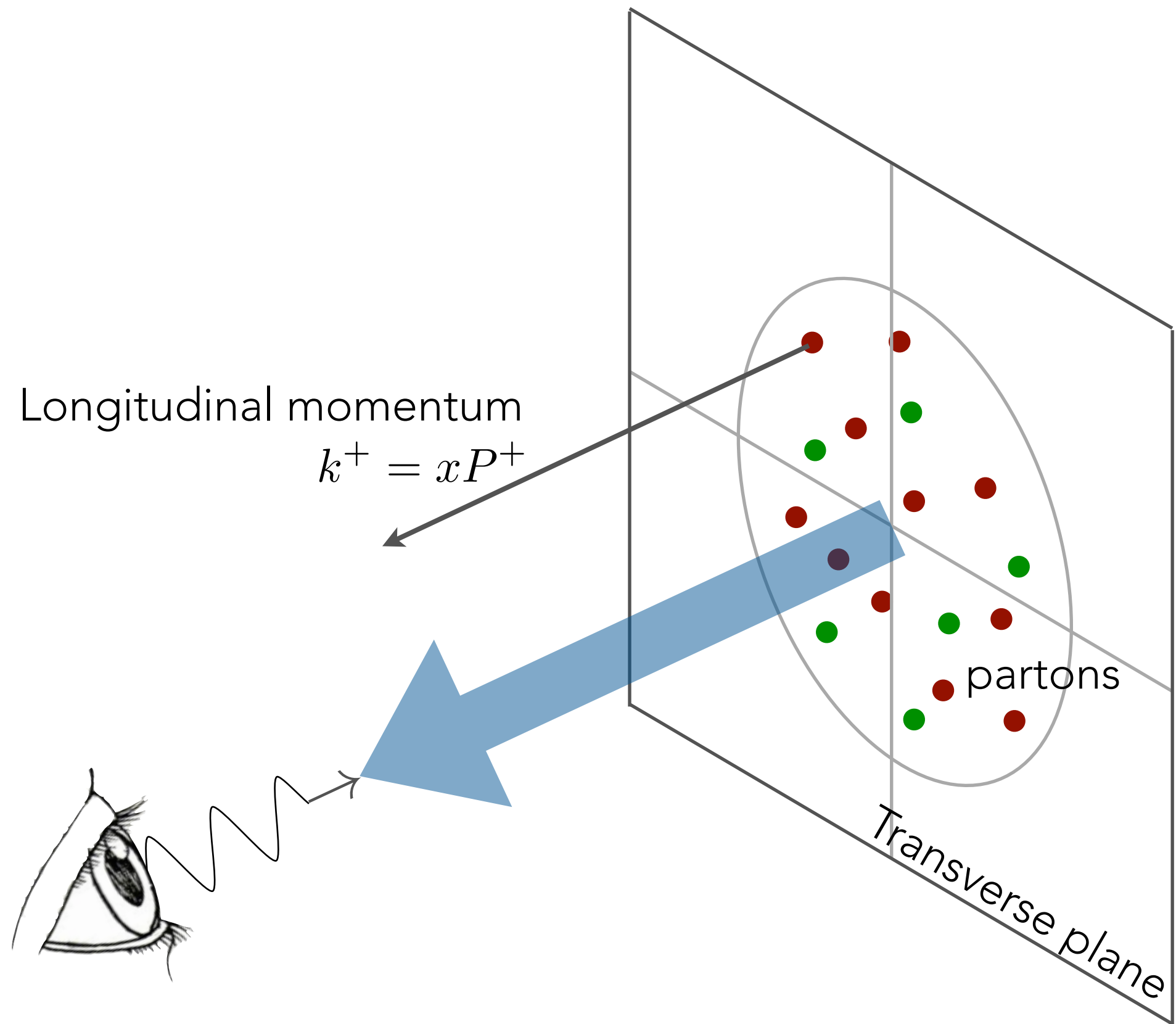
Fair agreement, but not perfect

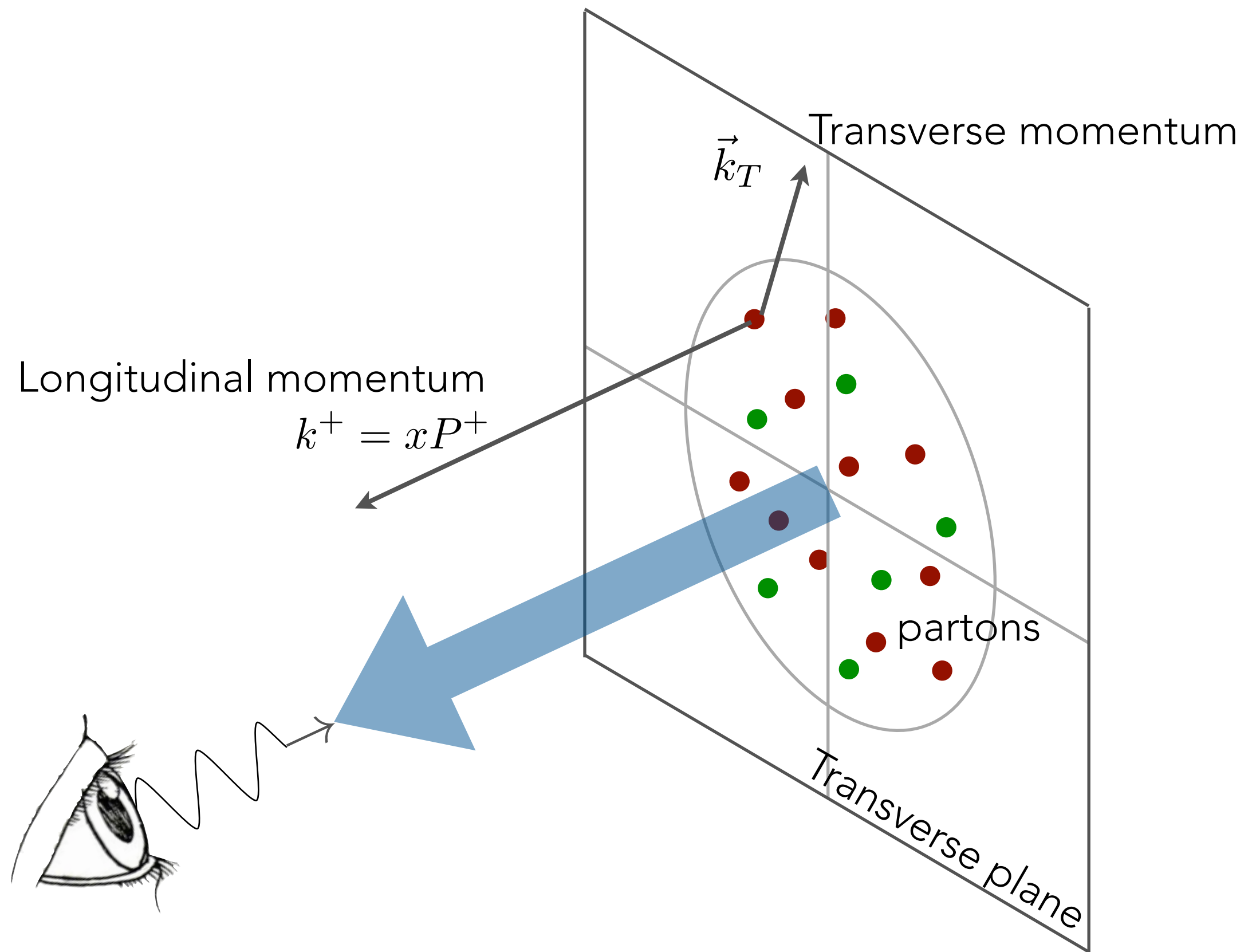
# FULL UNPOLARIZED PDF AND LATTICE QCD

Alexandrou, Cichy, Constantinou, Hadjiyiannakou, Jansen, Scapellato, Steffens, arXiv:1902.00587





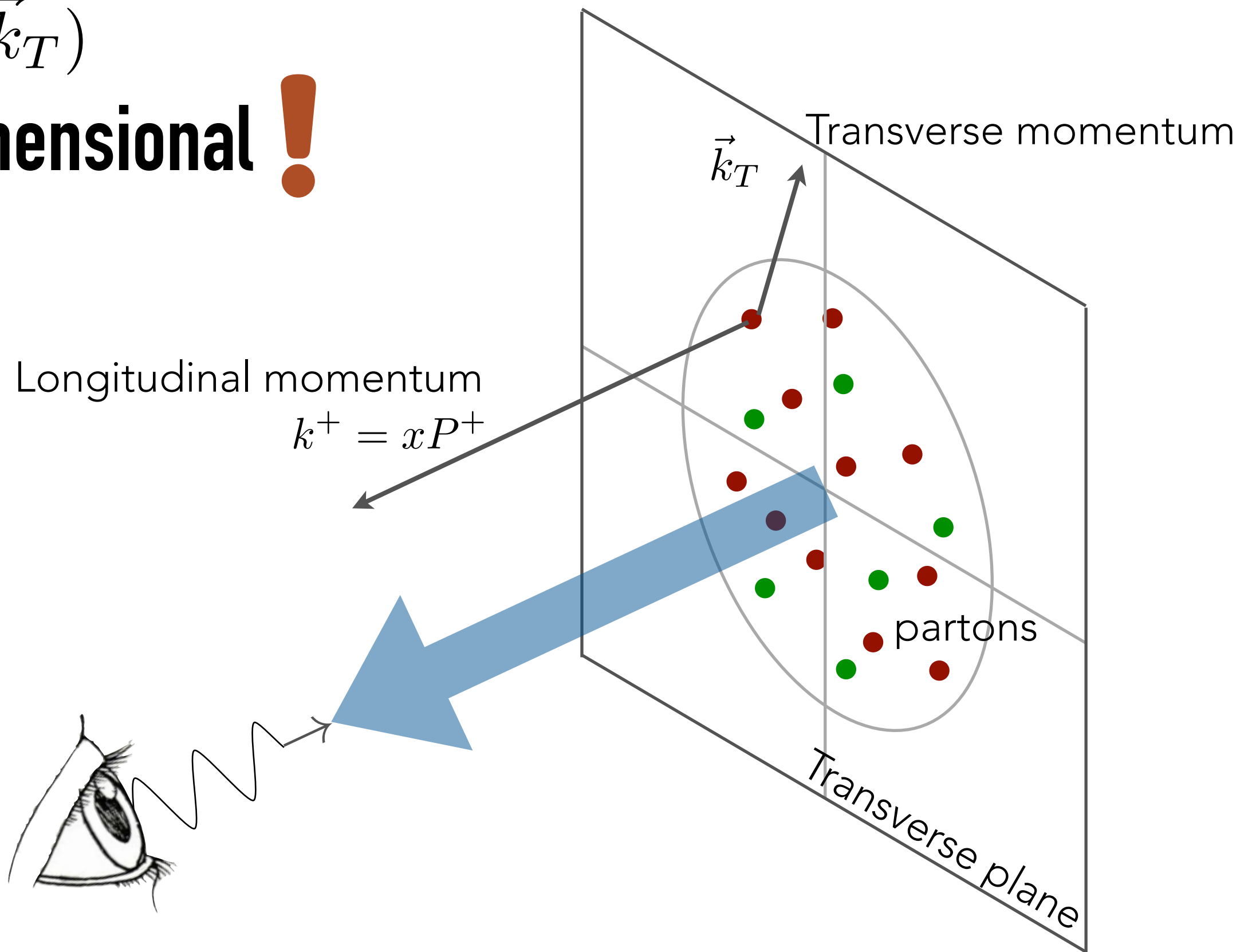




# Transverse-Momentum Distributions

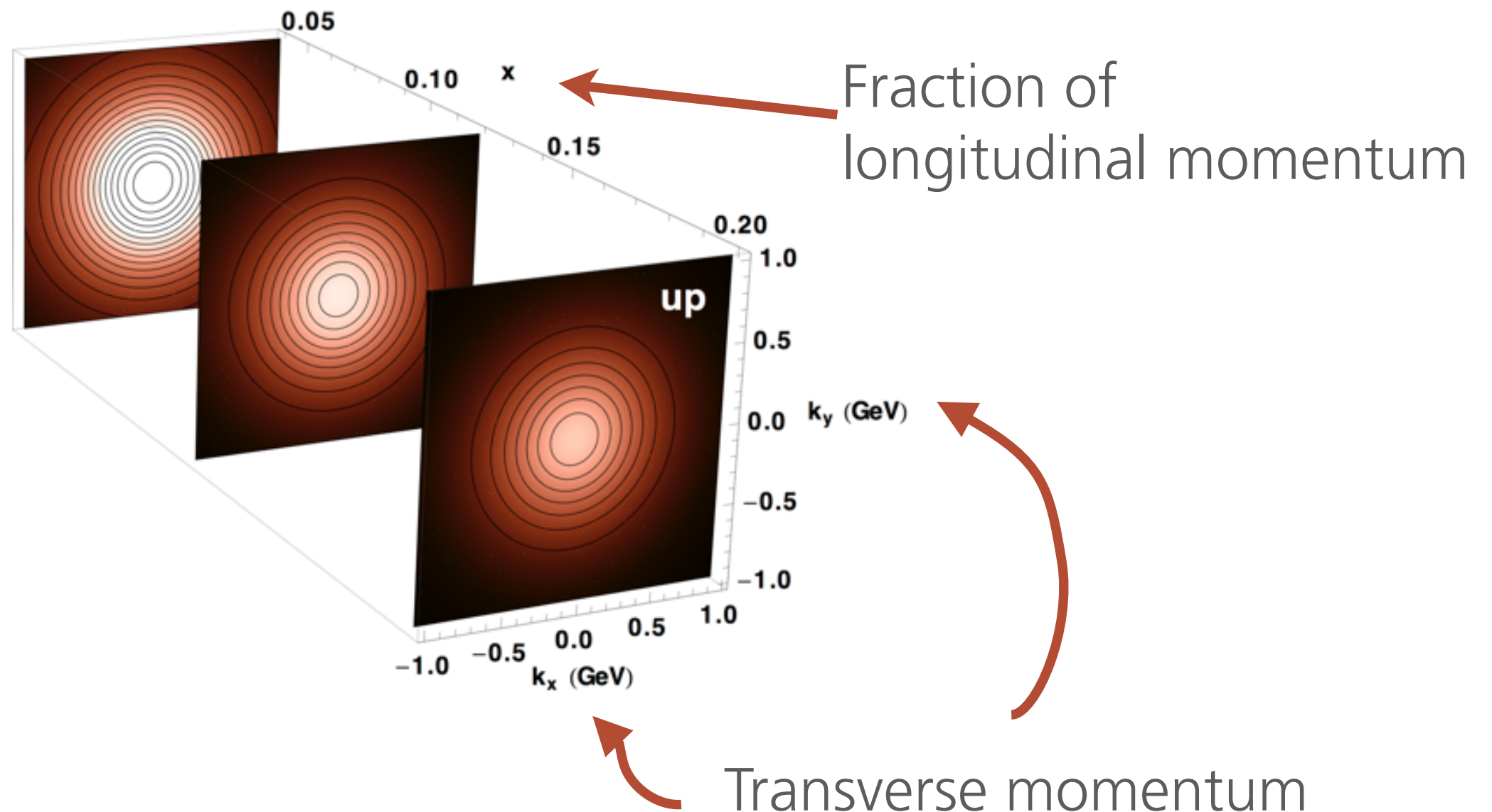
$$f(x, \vec{k}_T)$$

**3 dimensional !**



# TRANSVERSE MOMENTUM DISTRIBUTIONS

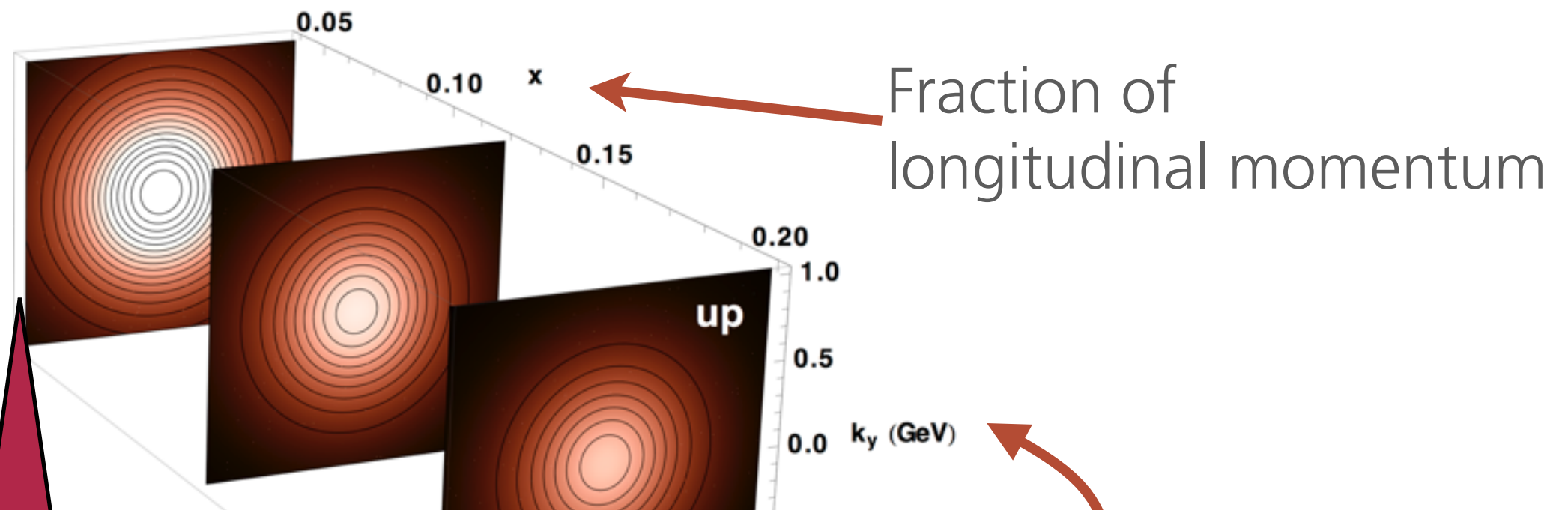
TMDs describe the distribution of partons in three dimensions in momentum space. They also have to be extracted through global fits.





# TRANSVERSE MOMENTUM DISTRIBUTIONS

TMDs describe the distribution of partons in three dimensions in momentum space. They also have to be extracted through global fits.



How “wide” is the distribution?  
Is there a difference between flavors?  
Does it get wider at low  $x$ ?

transverse momentum

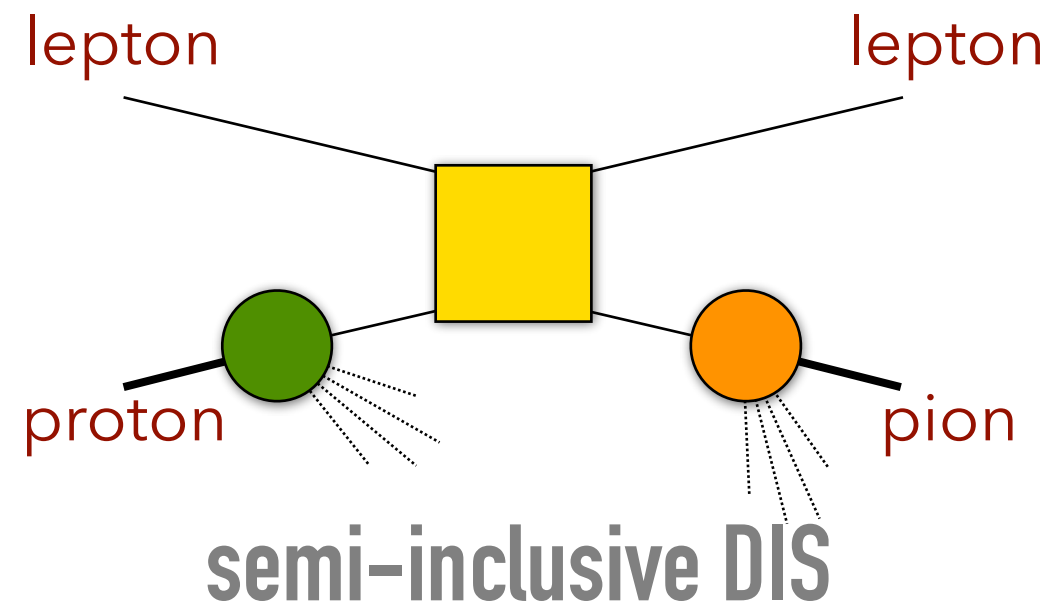
# UNPOLARISED QUARK TMDS

# UNPOLARISED QUARK TMDs

*see talk by M. Radici for polarized ones*

# FACTORIZATION AND UNIVERSALITY

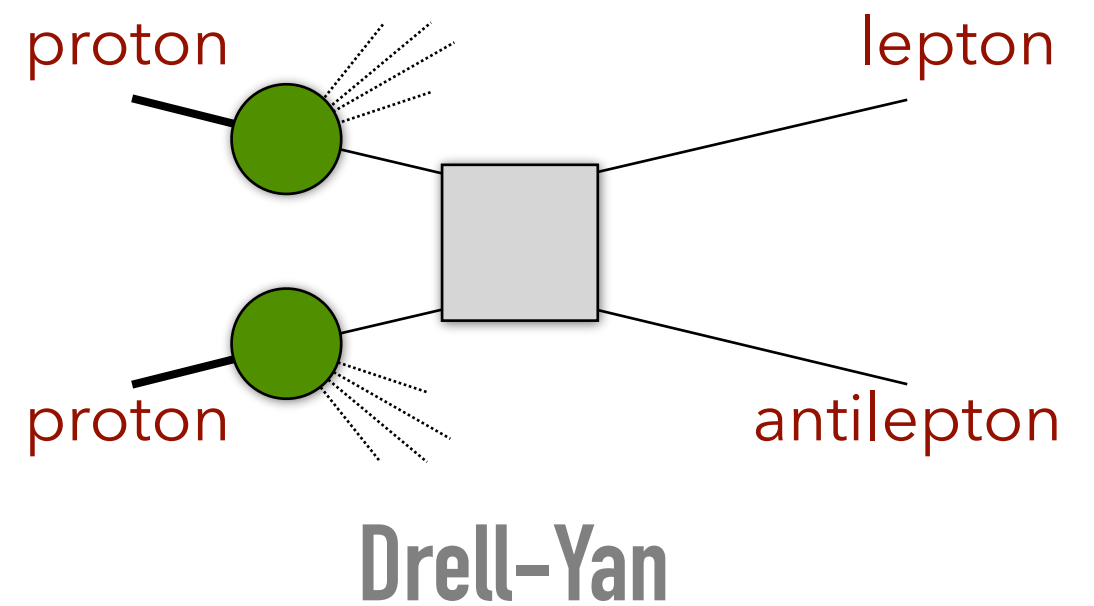
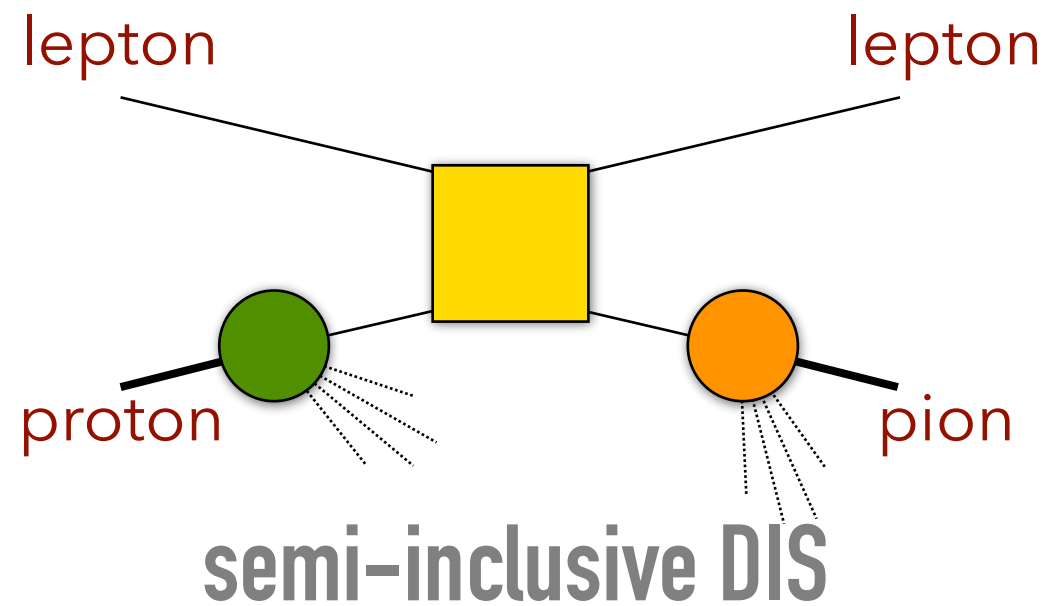
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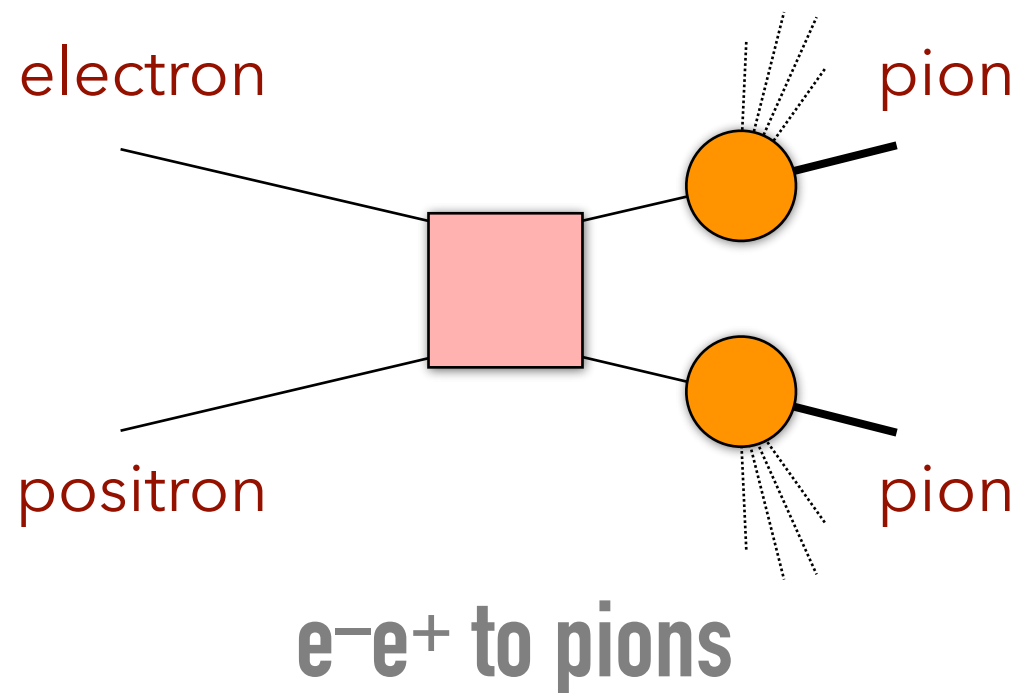
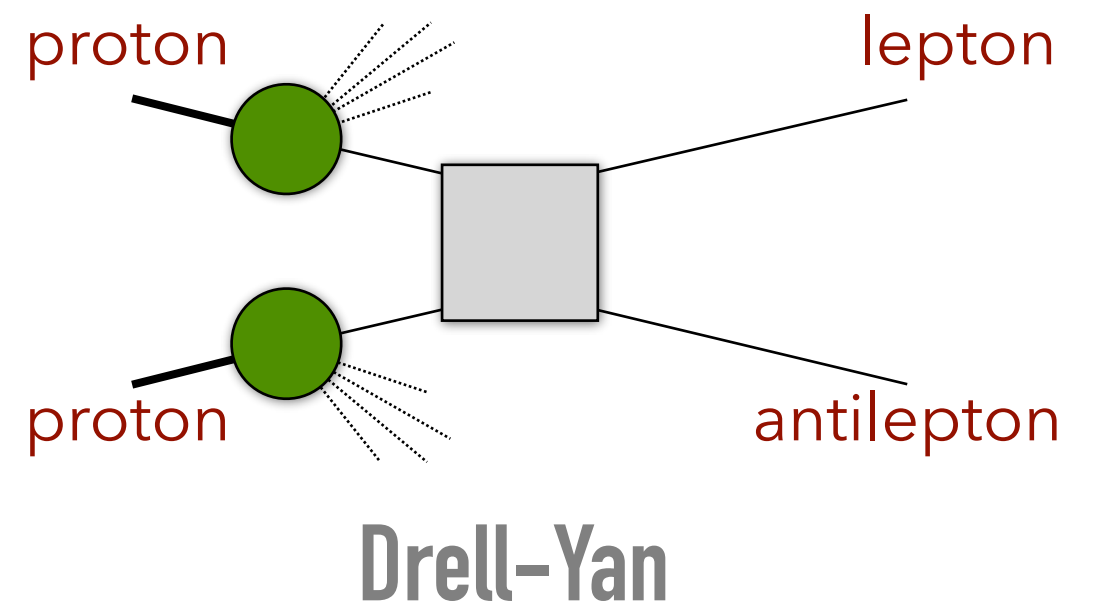
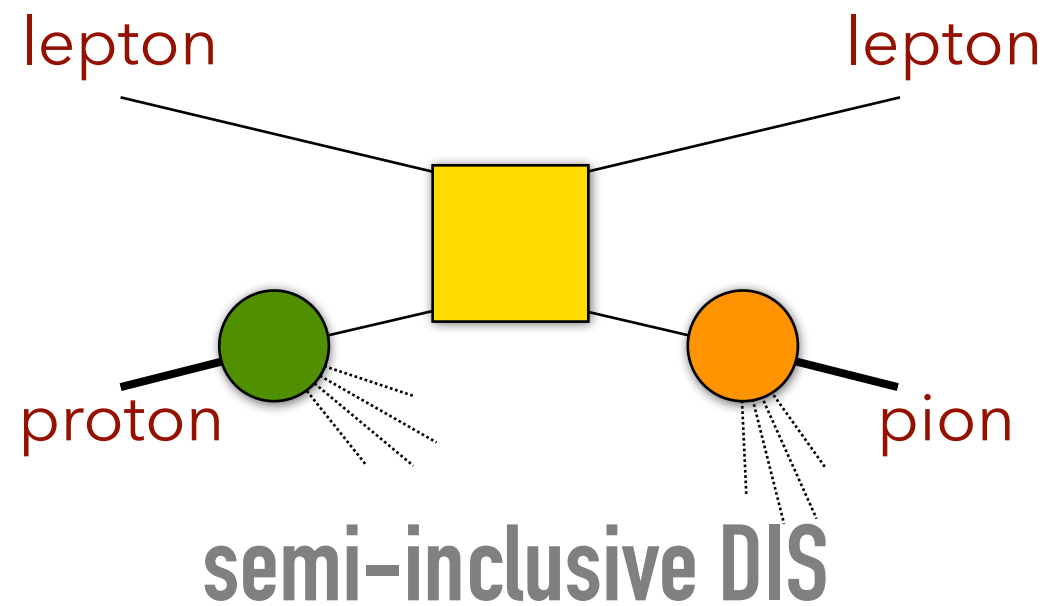
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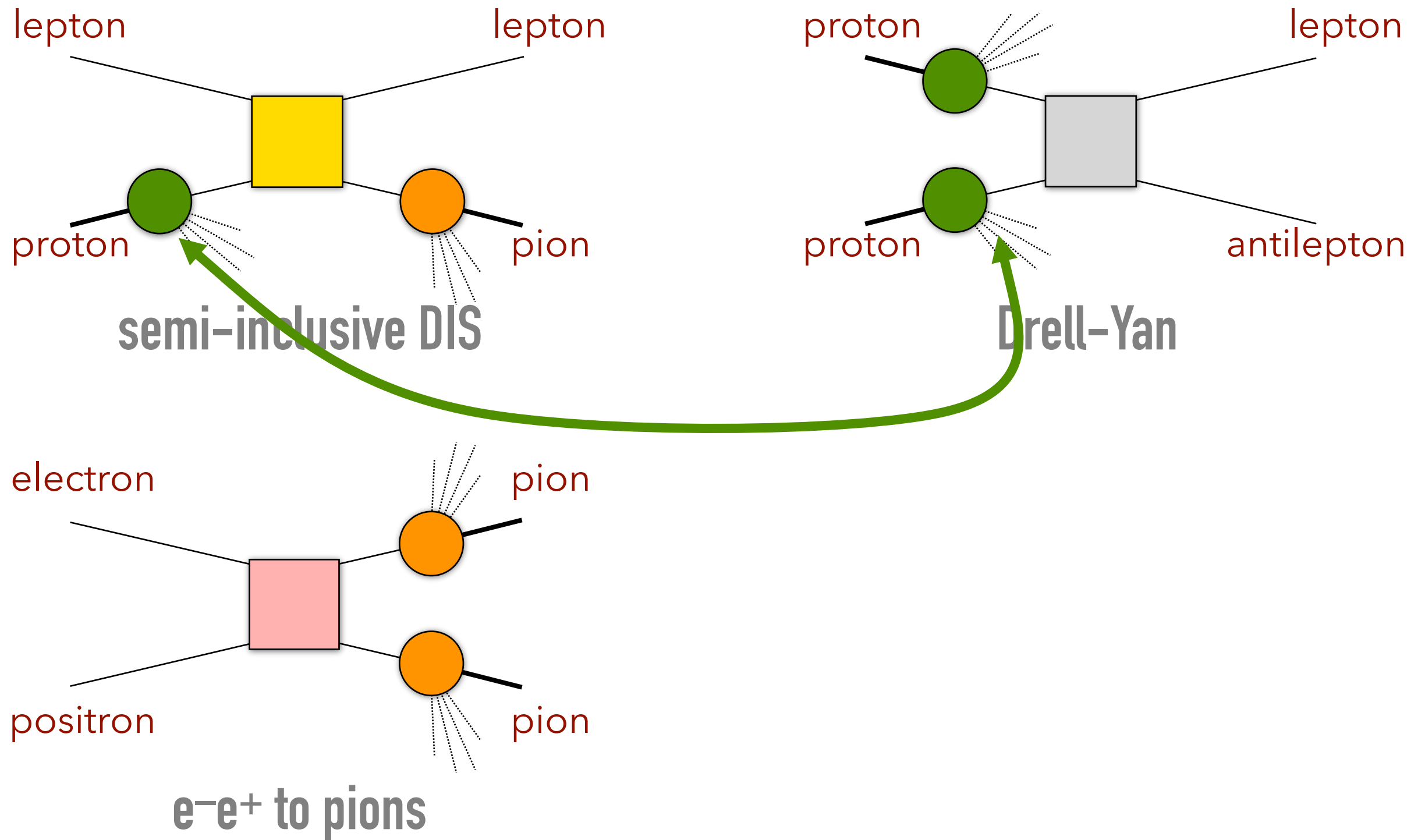
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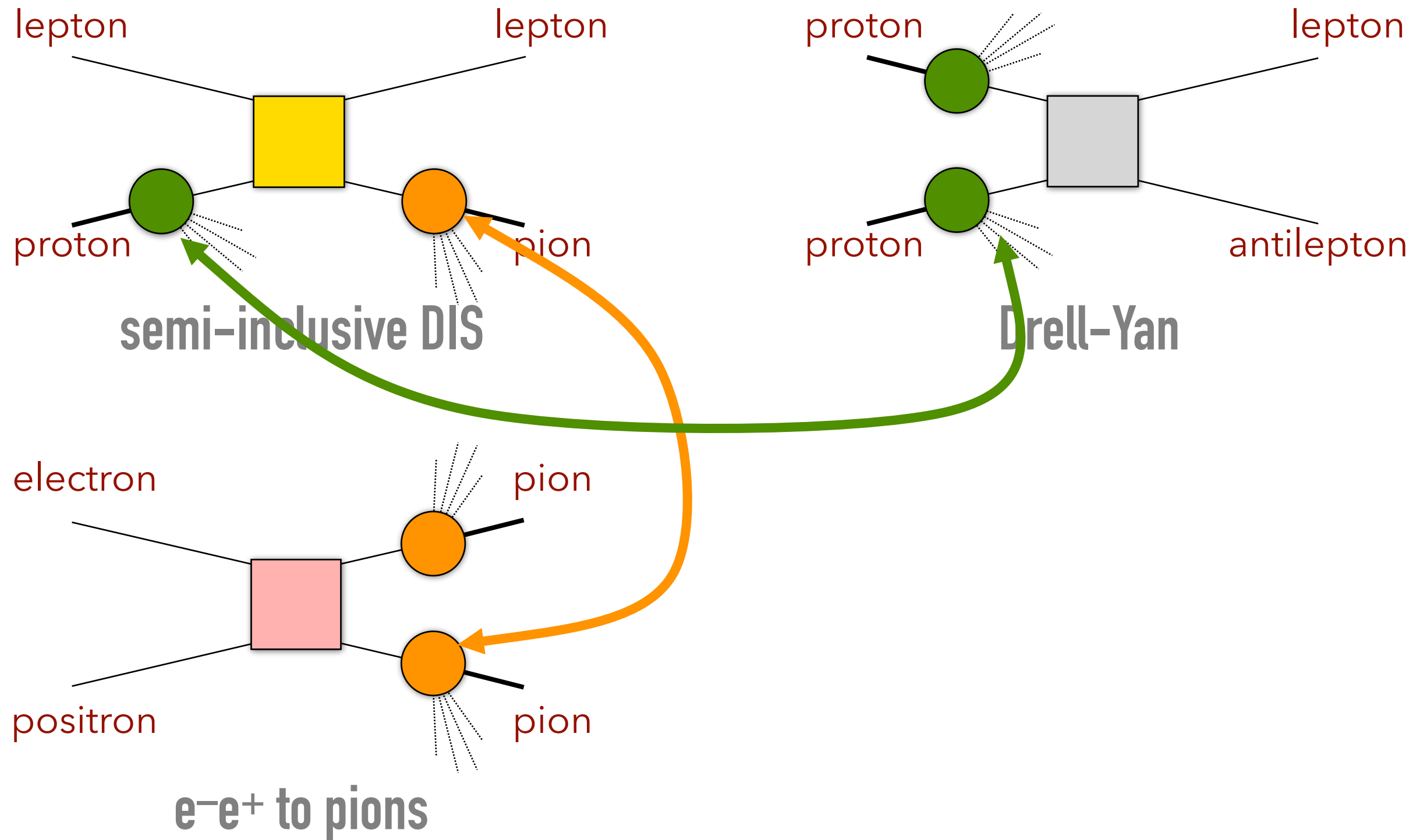
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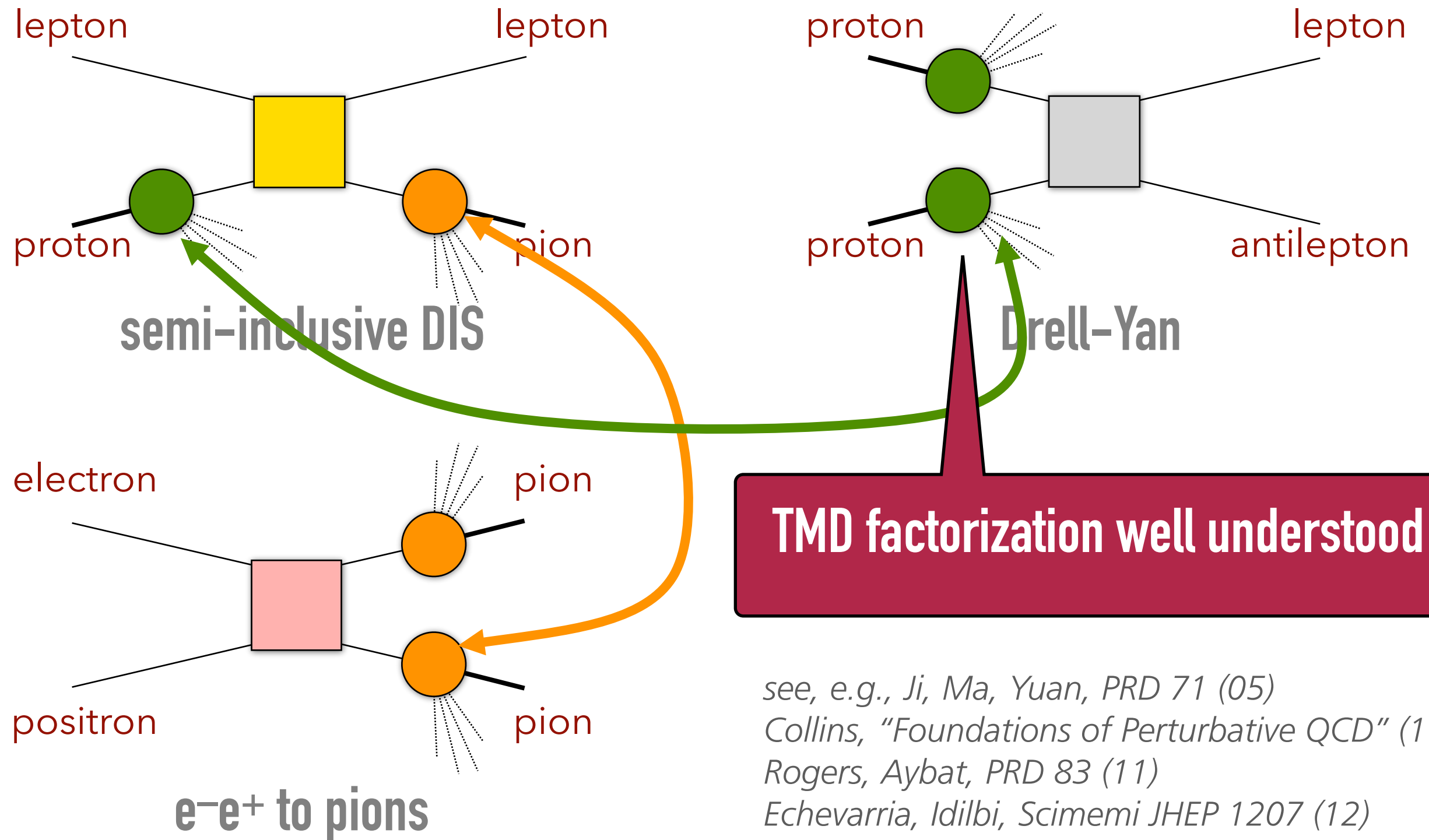


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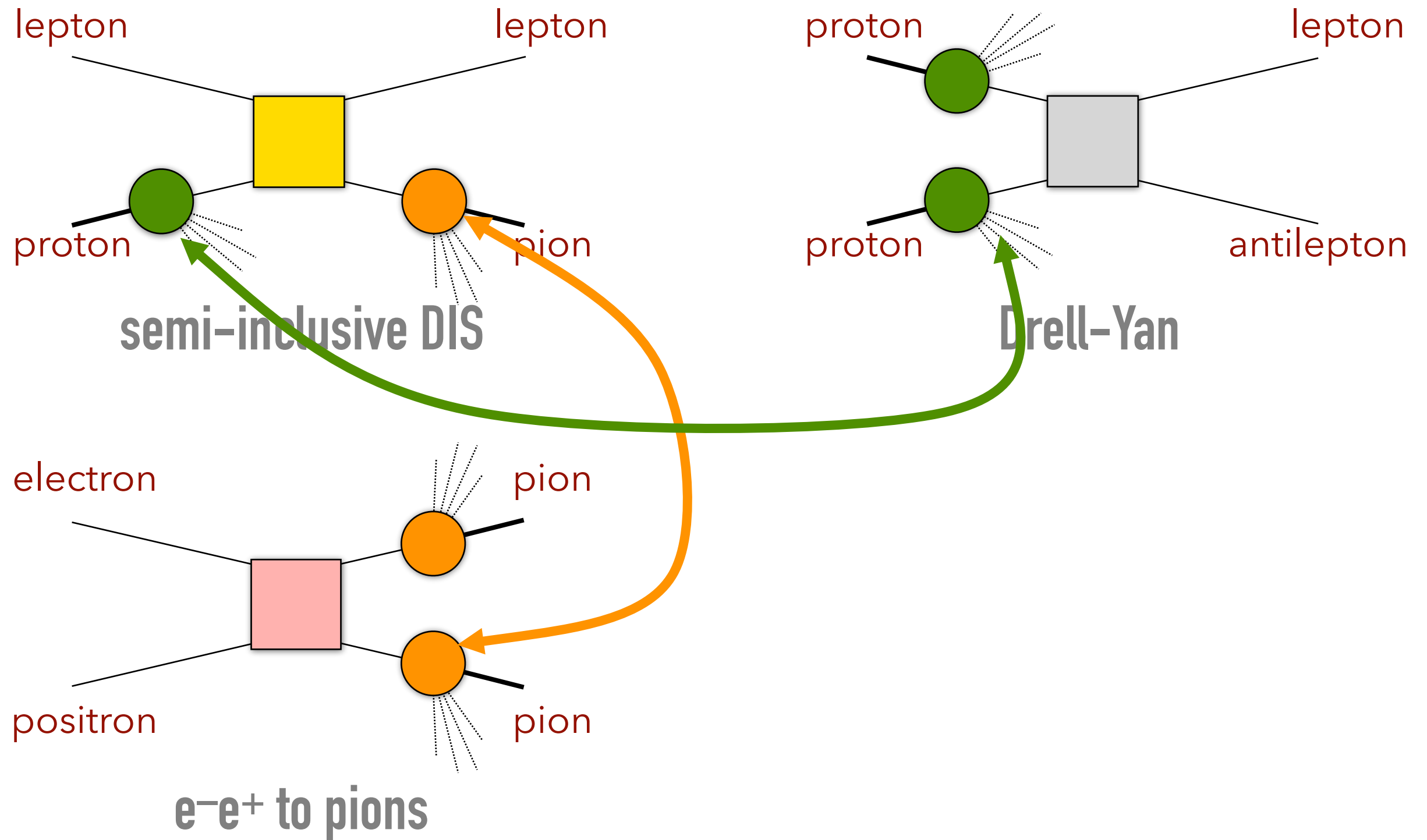


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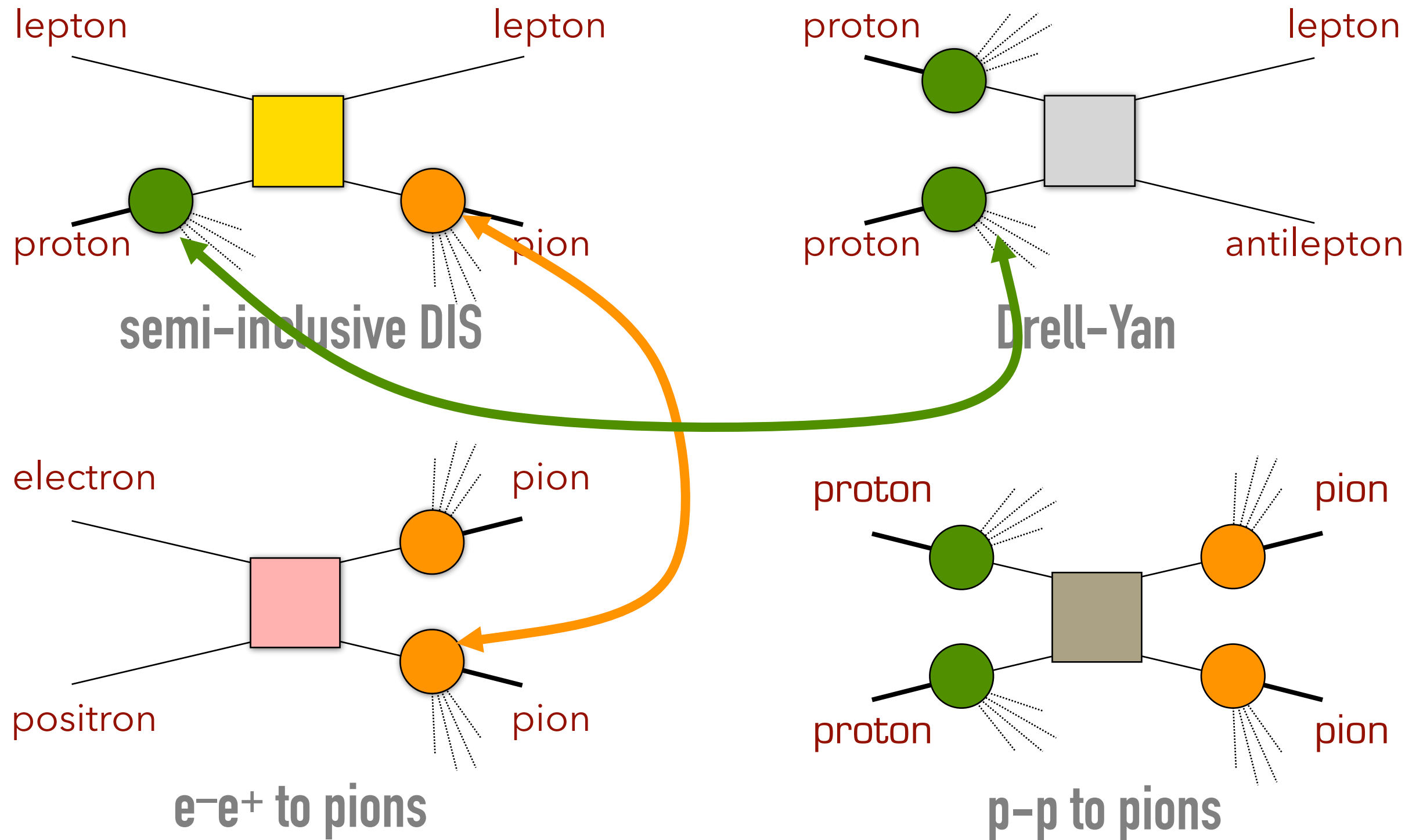
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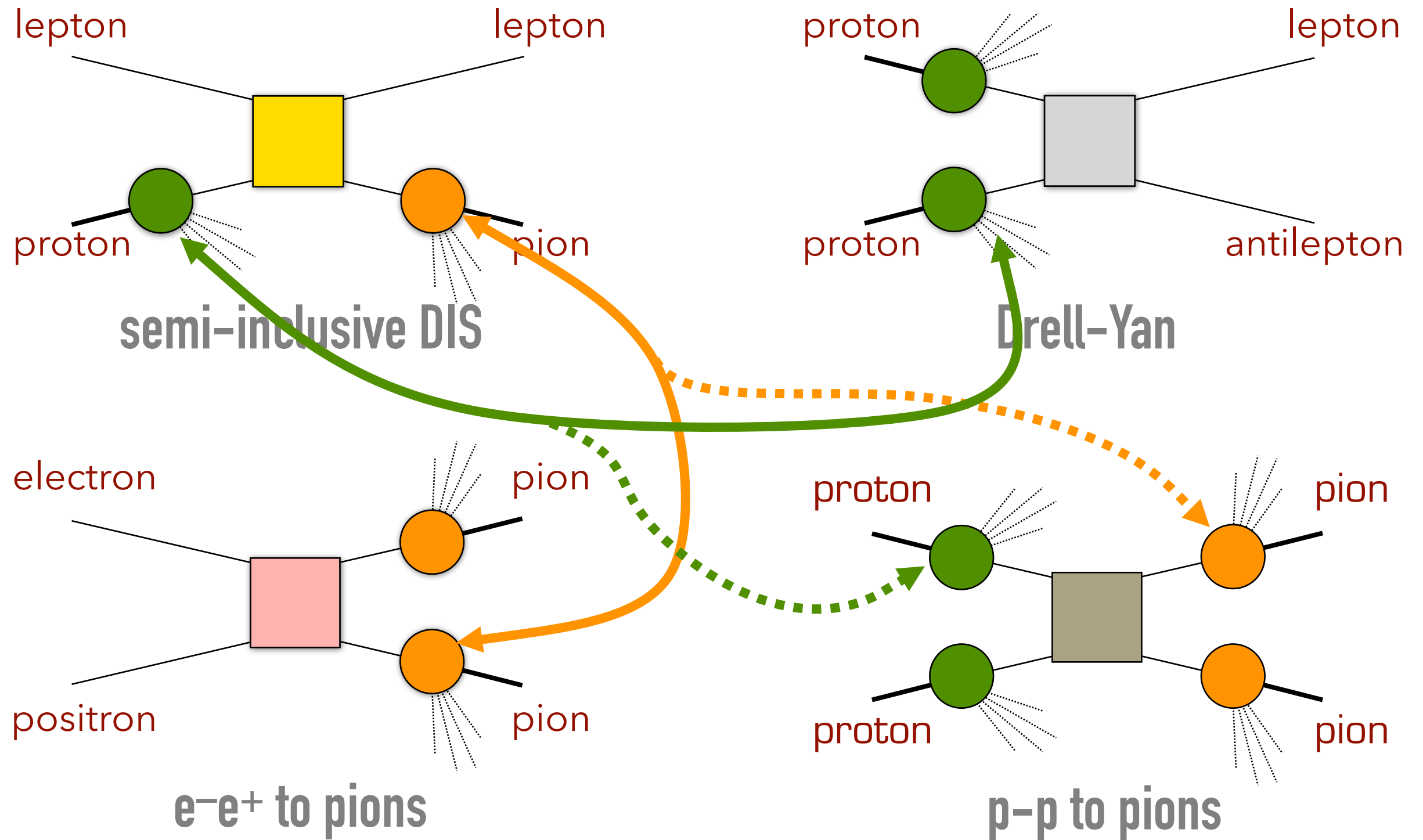
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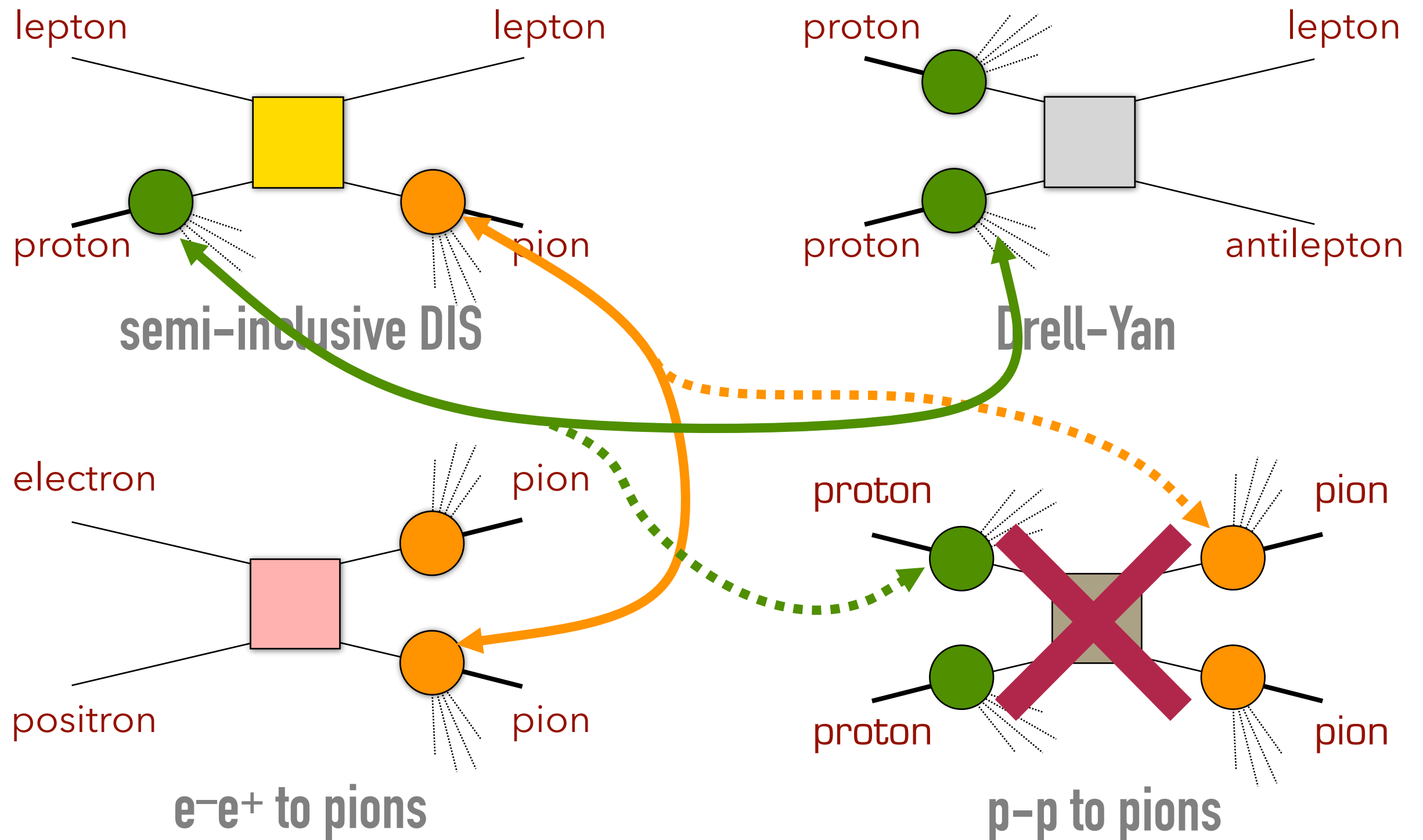


# FACTORIZATION AND UNIVERSALITY

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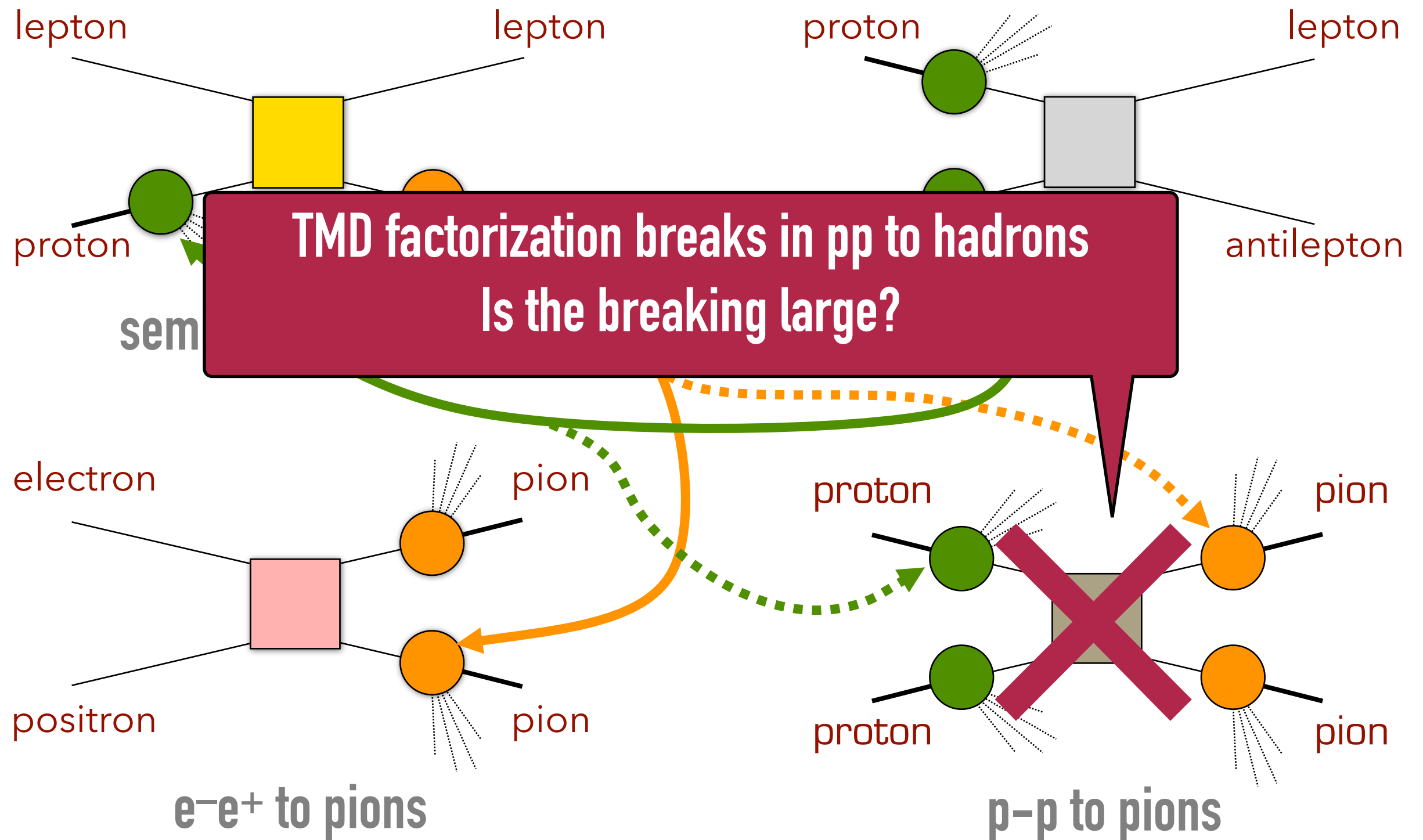
# FACTORIZATION AND UNIVERSALITY



see, e.g., Rogers, Mulders, PRD81 (10)

Buffing, Kang, Lee, Liu, arXiv:1812.07549 12

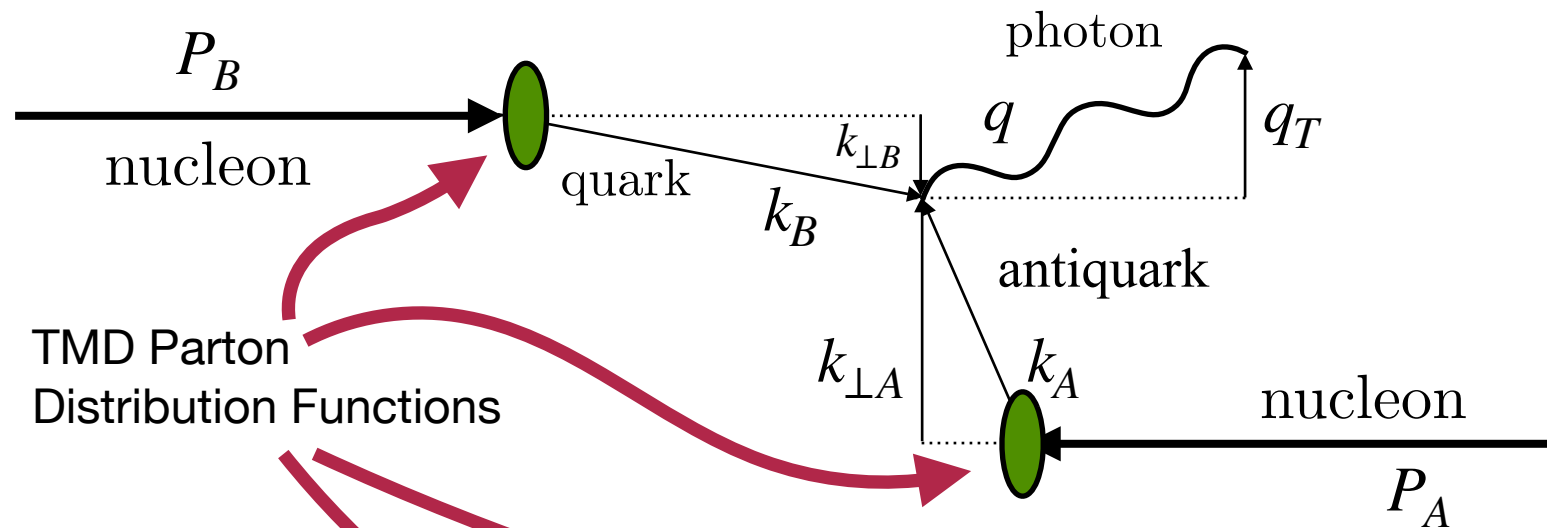
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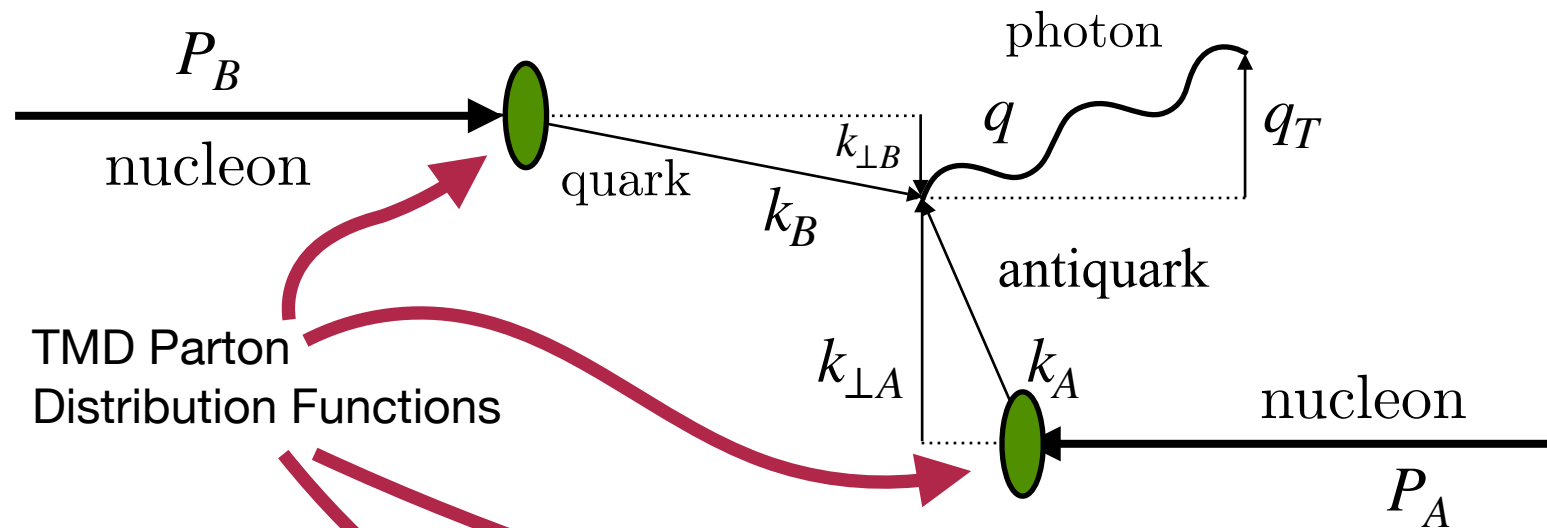
# TMDS IN DRELL-YAN PROCESSES



$$F_{UU}^1(x_A, x_B, \mathbf{q}_T^2, Q^2)$$

$$\approx \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int d^2\mathbf{k}_{\perp A} d^2\mathbf{k}_{\perp B} f_1^q(x_A, \mathbf{k}_{\perp A}^2; \mu^2) f_1^{\bar{q}}(x_B, \mathbf{k}_{\perp B}^2; \mu^2) \delta^{(2)}(\mathbf{k}_{\perp A} - \mathbf{q}_T + \mathbf{k}_{\perp B})$$

# TMDS IN DRELL-YAN PROCESSES

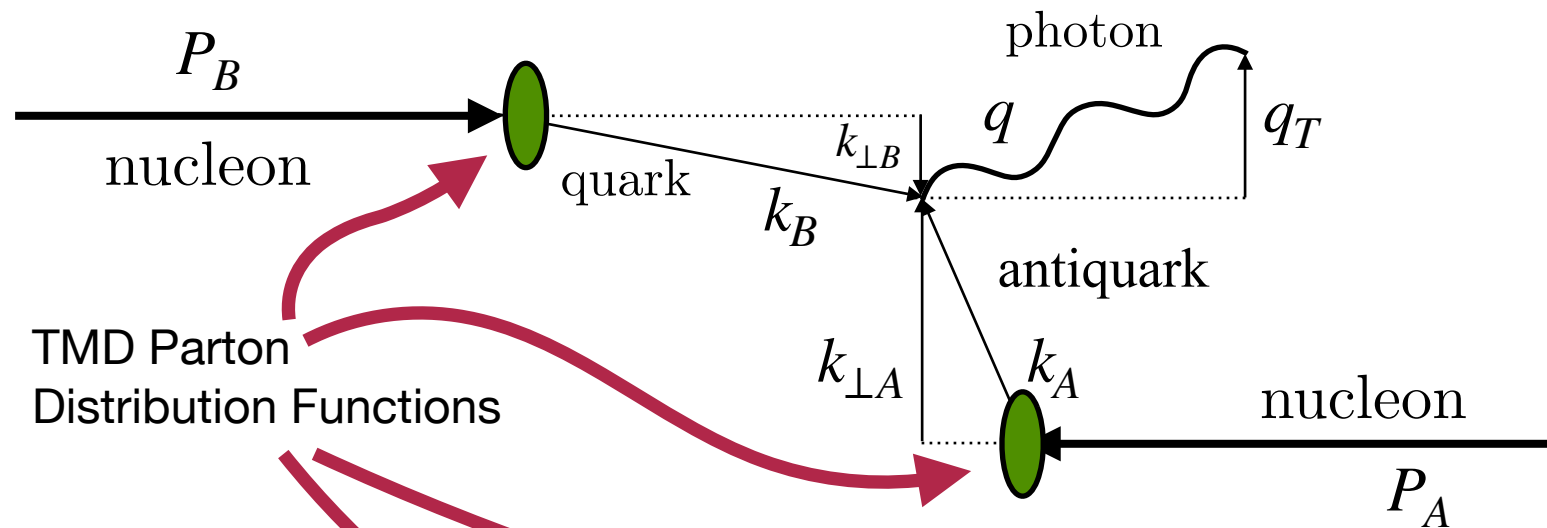


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At small transverse momentum, the dominant part is given by TMDs.

# TMDs IN DRELL-YAN PROCESSES



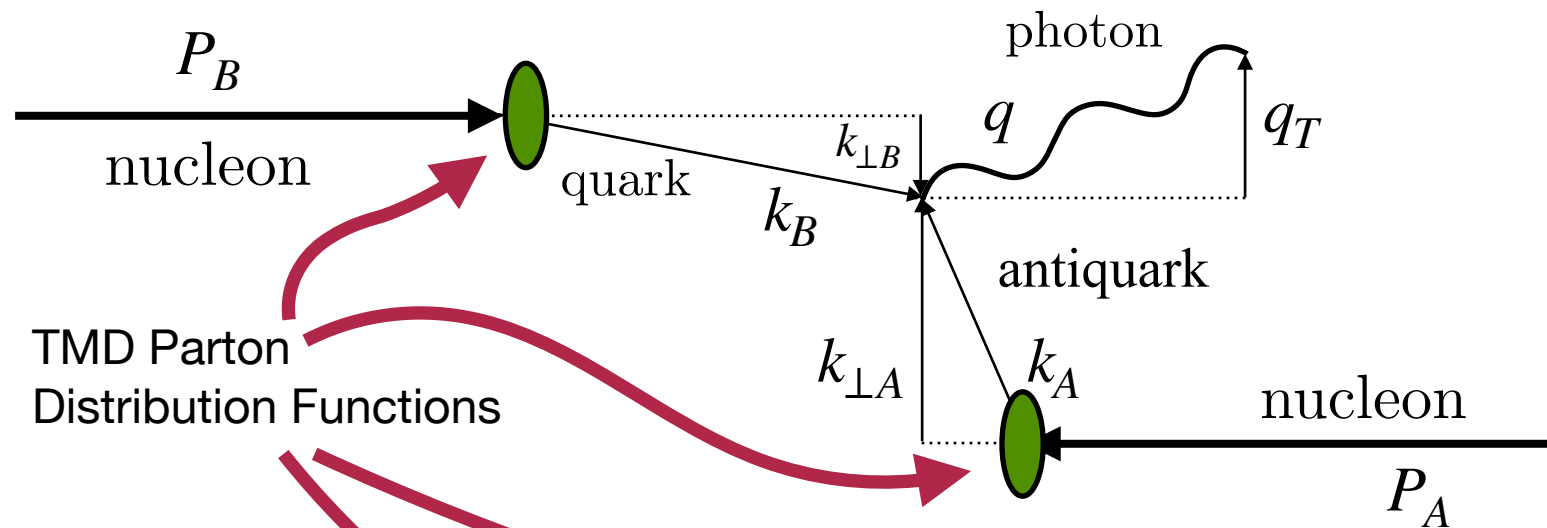
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$$= \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int db_T b_T J_0(b_T |\mathbf{q}_T|) \hat{f}_1^q(x_A, b_T^2; \mu^2) \hat{f}_1^{\bar{q}}(x_B, b_T^2; \mu^2)$$

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The analysis of is usually done in Fourier-transformed space

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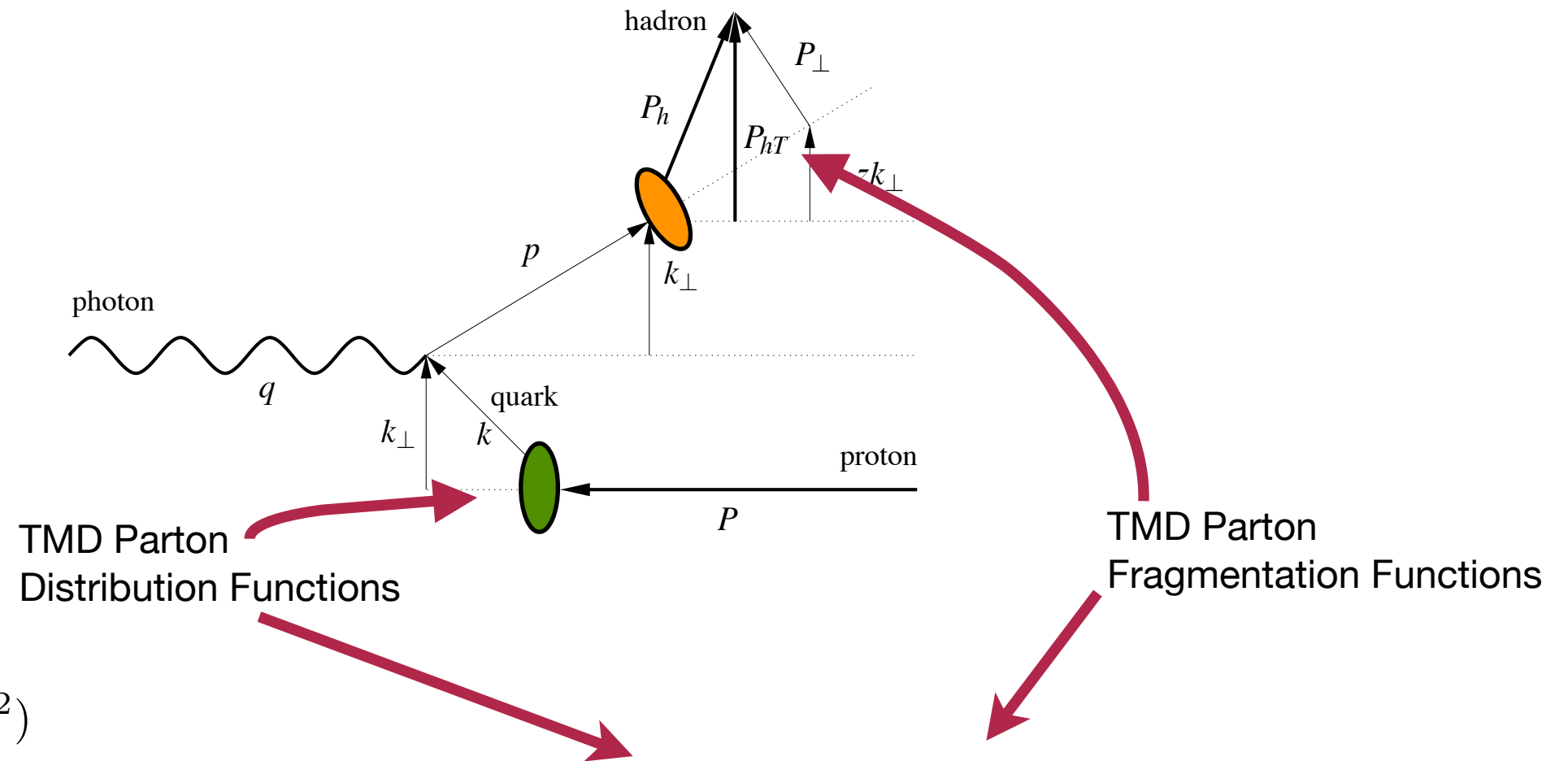
At small transverse momentum, the dominant part is given by TMDs.

The analysis of is usually done in Fourier-transformed space

TMDs formally depend on two scales, but usually they are set to be equal.



# TMDS IN SEMI-INCLUSIVE DIS

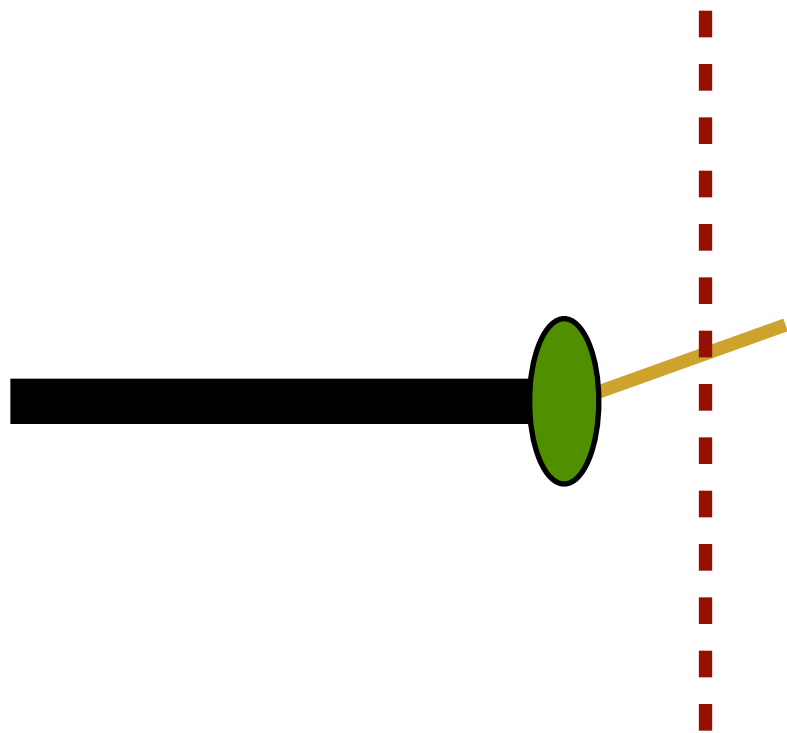


$$\begin{aligned}
 F_{UU,T}(x, z, \mathbf{P}_{hT}^2, Q^2) &= x \sum_q \mathcal{H}_{UU,T}^q(Q^2, \mu^2) \int d^2 \mathbf{k}_\perp d^2 \mathbf{P}_\perp f_1^a(x, \mathbf{k}_\perp^2; \mu^2) D_1^{a \rightarrow h}(z, \mathbf{P}_\perp^2; \mu^2) \delta(z \mathbf{k}_\perp - \mathbf{P}_{hT} + \mathbf{P}_\perp) \\
 &= x \sum_a \mathcal{H}_{UU,T}^a(Q^2, \mu^2) \int db_T b_T J_0(b_T |\mathbf{P}_{h\perp}|) \hat{f}_1^a(x, z^2 b_\perp^2; \mu^2) \hat{D}_1^{a \rightarrow h}(z, b_\perp^2; \mu^2)
 \end{aligned}$$

# DIFFERENT CONTRIBUTIONS TO TRANSVERSE MOMENTUM

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“intrinsic”  
transverse  
momentum



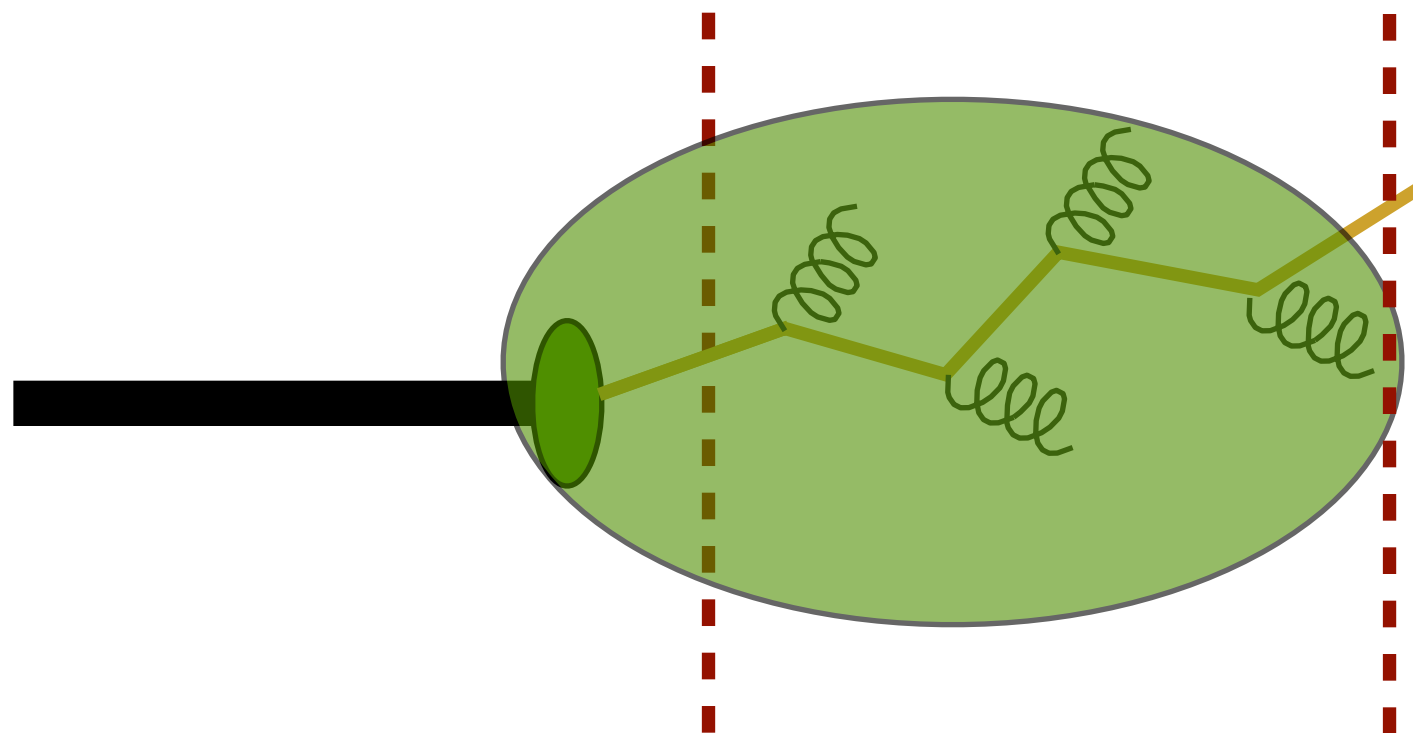
$$|k_{\perp}| \sim \Lambda_{\text{QCD}}$$

# DIFFERENT CONTRIBUTIONS TO TRANSVERSE MOMENTUM

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“intrinsic”  
transverse  
momentum

soft and collinear  
gluon radiation

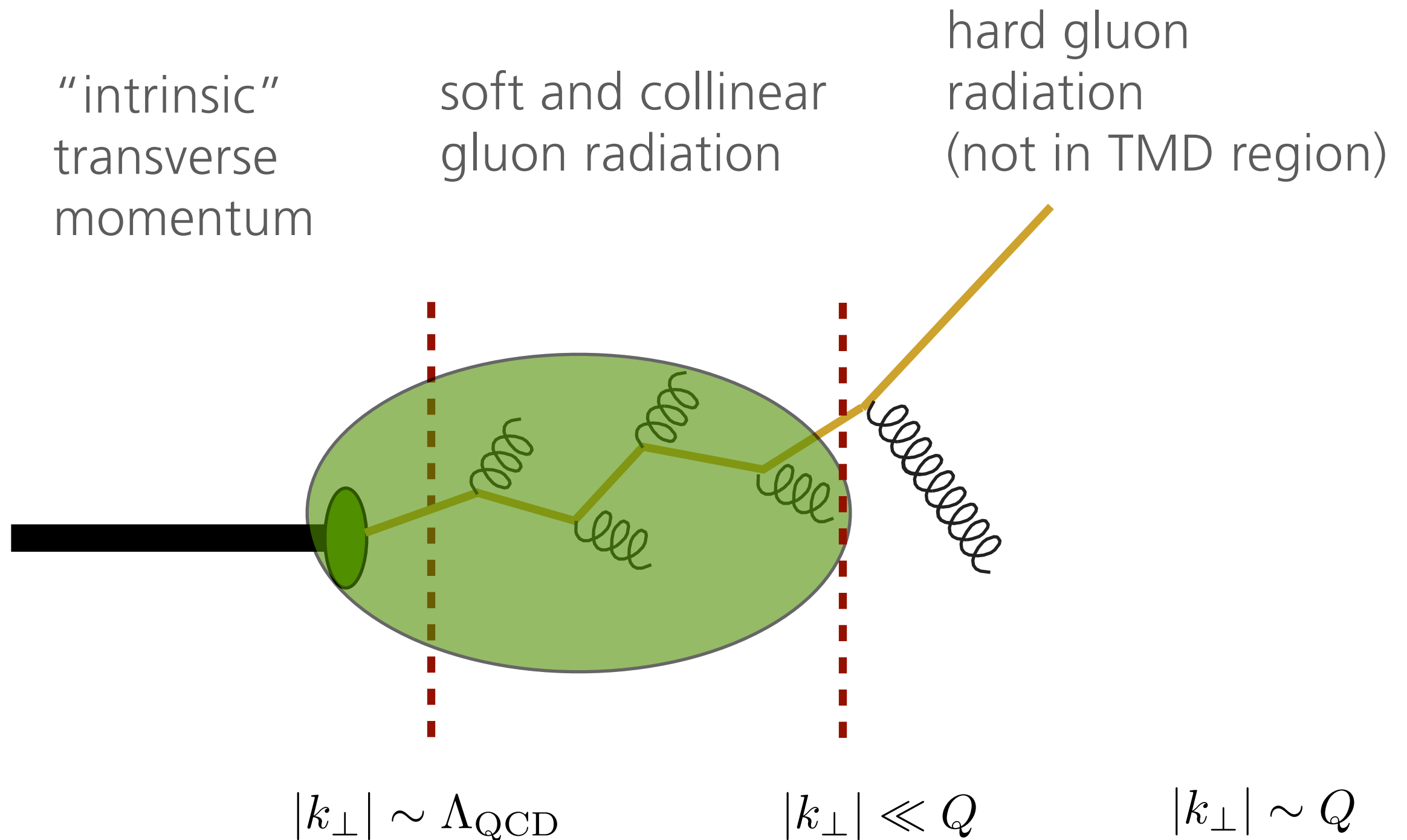


$$|k_{\perp}| \sim \Lambda_{\text{QCD}}$$

$$|k_{\perp}| \ll Q$$

# DIFFERENT CONTRIBUTIONS TO TRANSVERSE MOMENTUM

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# TMD FACTORIZATION

---

$$\hat{f}_1^q(x, b_T; \mu^2) = \int d^2 \mathbf{k}_\perp e^{i \mathbf{b}_T \cdot \mathbf{k}_\perp} f_1^q(x, \mathbf{k}_\perp^2; \mu^2)$$

---

see, e.g., Rogers, Aybat, *PRD* 83 (11),  
Collins, "Foundations of Perturbative QCD" (11)

other possible schemes, e.g.,  
Laenen, Sterman, Vogelsang, *PRL* 84 (00)  
Bozzi, Catani, De Florian, Grazzini, *NPB* 737 (06)  
Echevarria, Idilbi, Schaefer, Scimemi, *EPJ C* 73 (13)

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$$\hat{f}_1^q(x, b_T; \mu^2) = \sum_i (C_{qi} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\text{NP}}^q(x, b_T)$$

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$$\mu_b = \frac{2e^{-\gamma_E}}{b_*}$$

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matching coefficients (perturbative)      collinear PDF      perturbative Sudakov form factor      nonperturbative part of evolution      nonperturbative part of TMD

see, e.g., Rogers, Aybat, *PRD* 83 (11),  
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# LOGARITHMIC ACCURACY

---

Sudakov form factor

$$\text{LL} \quad \alpha_S^n \ln^{2n} \left( \frac{Q^2}{\mu_b^2} \right)$$

# LOGARITHMIC ACCURACY

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Sudakov form factor

LL  $\alpha_S^n \ln^{2n} \left( \frac{Q^2}{\mu_b^2} \right)$

NLL  $\alpha_S^n \ln^{2n} \left( \frac{Q^2}{\mu_b^2} \right), \quad \alpha_S^n \ln^{2n-1} \left( \frac{Q^2}{\mu_b^2} \right)$

# LOGARITHMIC ACCURACY

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$$C^0$$

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NLL'  $\alpha_S^n \ln^{2n} \left( \frac{Q^2}{\mu_b^2} \right), \quad \alpha_S^n \ln^{2n-1} \left( \frac{Q^2}{\mu_b^2} \right)$

$$\left( C^0 + \alpha_S C^1 \right)$$

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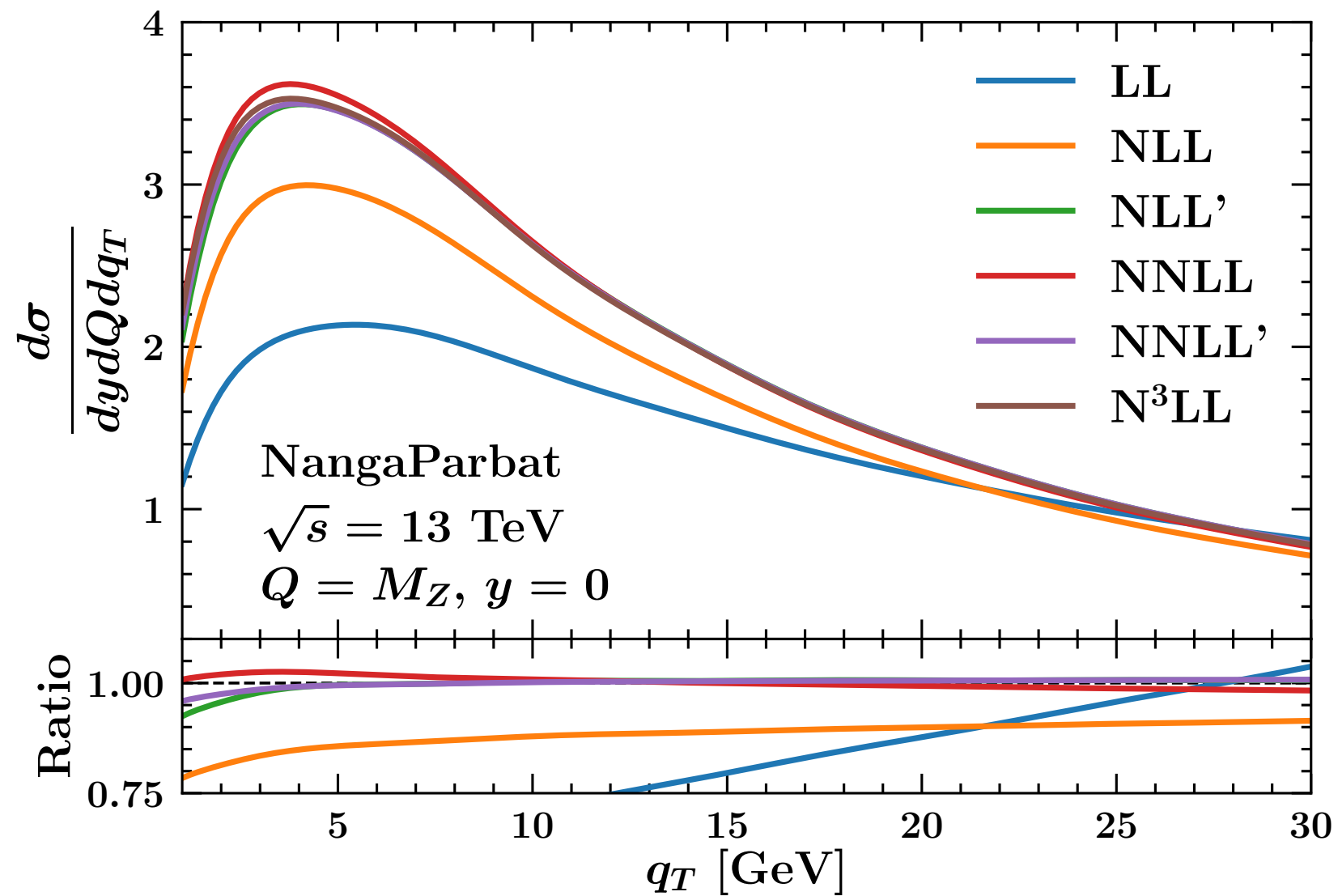
$$\left( C^0 + \alpha_S C^1 \right)$$

the difference between the two is NNLL

$$\alpha_S^n \ln^{2n-2} \left( \frac{Q^2}{\mu_b^2} \right)$$

# COMPARISON OF DIFFERENT ORDERS

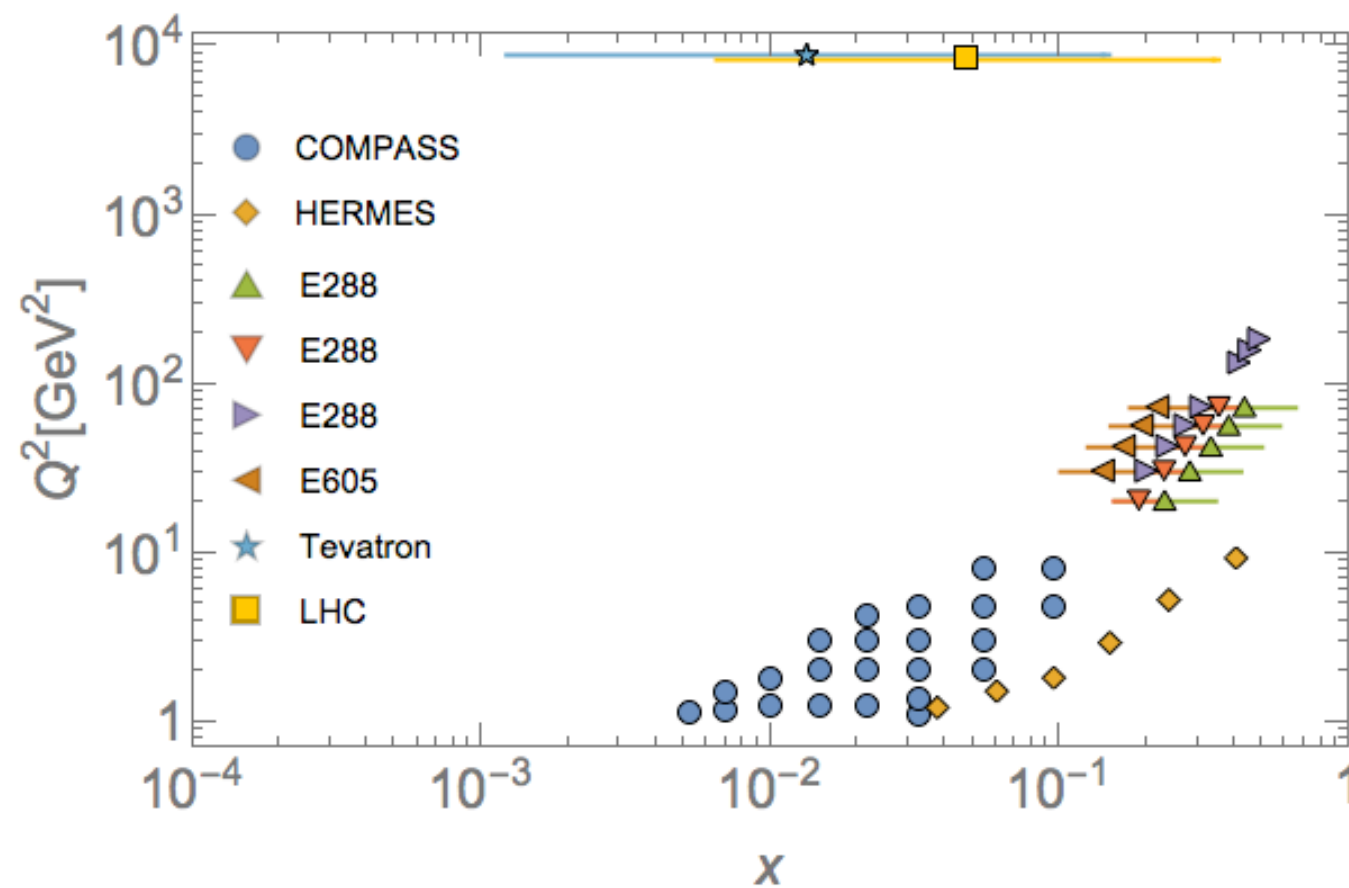
V. Bertone's talk at LHC EW WG General Meeting, Dec 2019  
<https://indico.cern.ch/event/849342/>



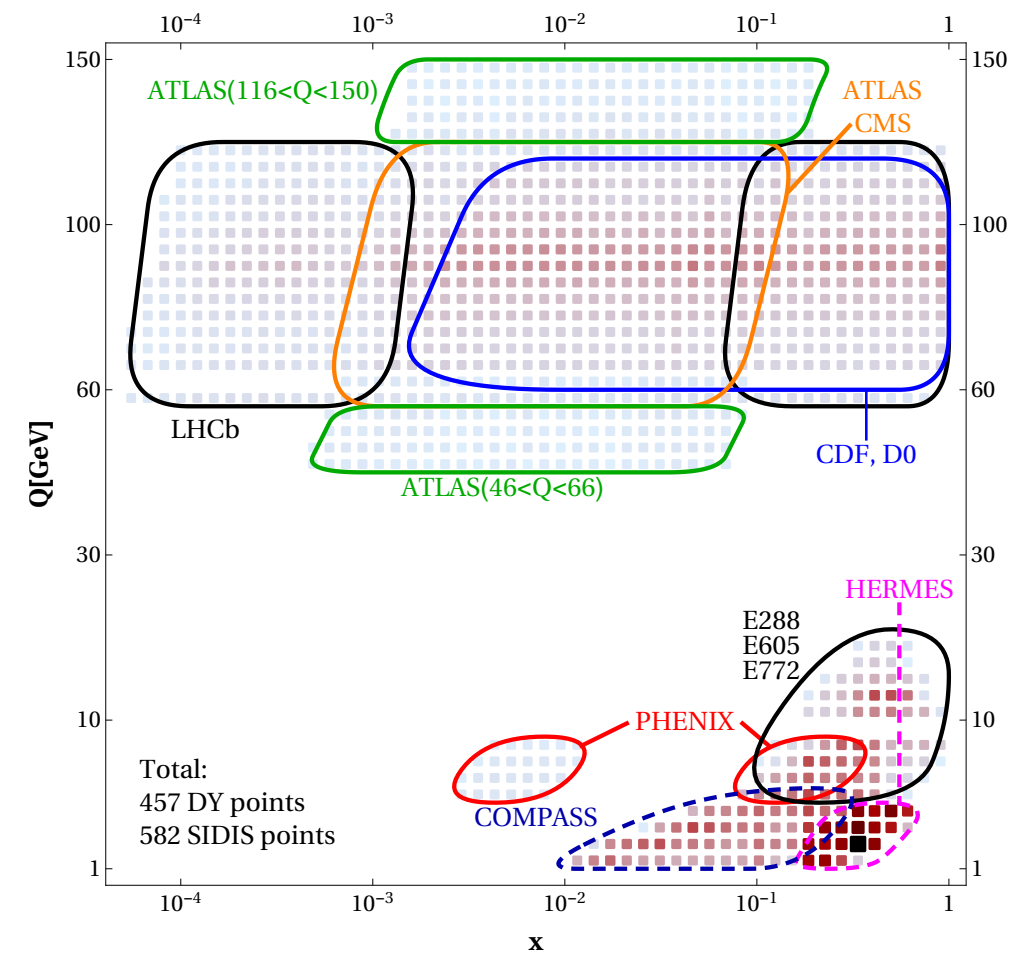
# RECENT TMD FITS OF UNPOLARIZED DATA

	Framework	HERMES	COMPASS	DY	Z production	N of points	$\chi^2/N_{\text{points}}$
Pavia 2017 <a href="#">arXiv:1703.10157</a>	NLL	✓	✓	✓	✓	8059	1.55
SV 2017 <a href="#">arXiv:1706.01473</a>	NNLL'	✗	✗	✓	✓	309	1.23
BSV 2019 <a href="#">arXiv:1902.08474</a>	NNLL'	✗	✗	✓	✓	457	1.17
SV 2019 <a href="#">arXiv:1912.06532</a>	NNLL'	✓	✓	✓	✓	1039	1.06
Pavia 2019 <a href="#">arXiv:1912.07550</a>	N <sup>3</sup> LL	✗	✗	✓	✓	353	1.02

# x-Q<sup>2</sup> COVERAGE



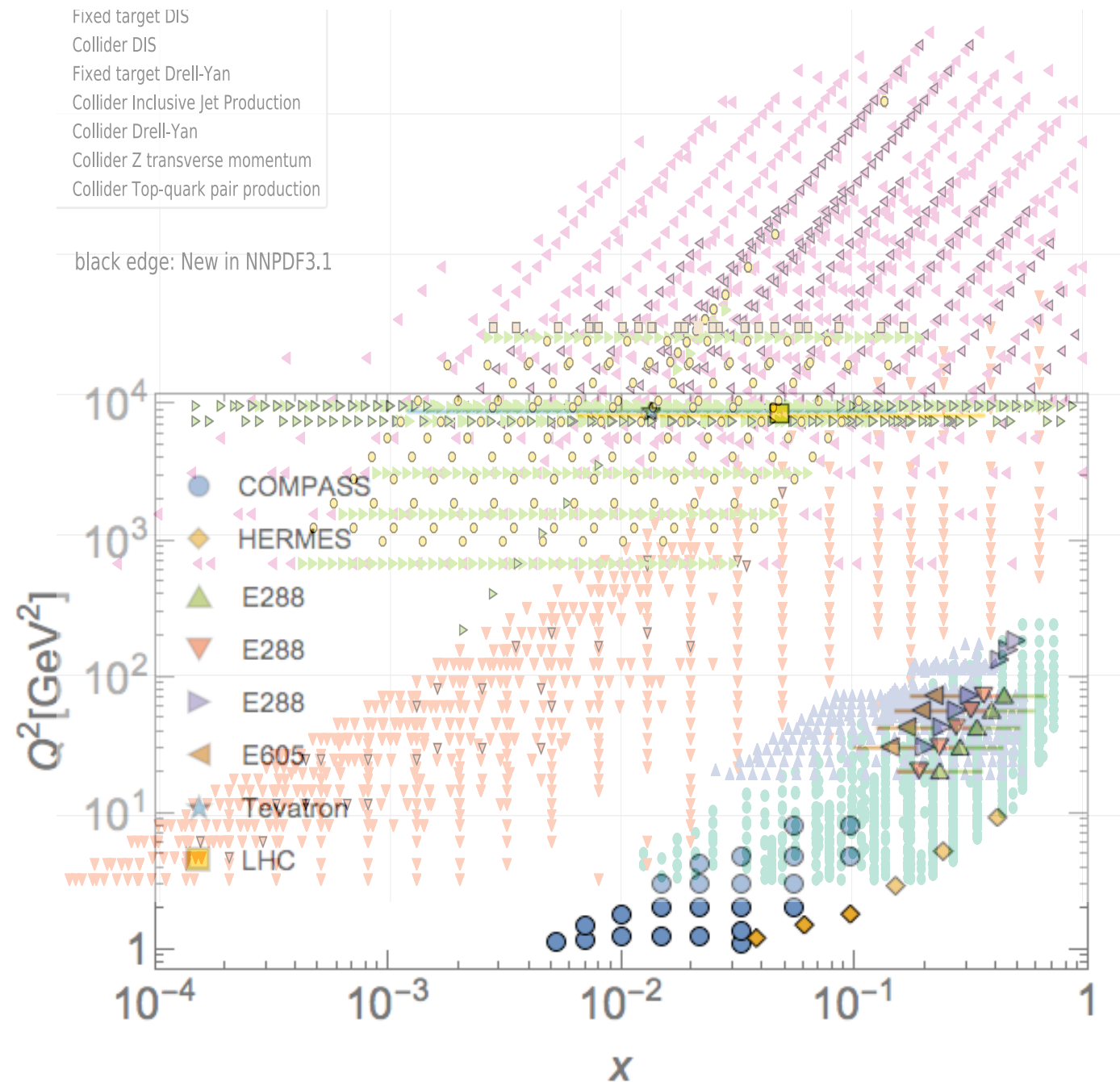
Bacchetta, Delcarro, Pisano, Radici,  
Signori, *arXiv:1703.10157*



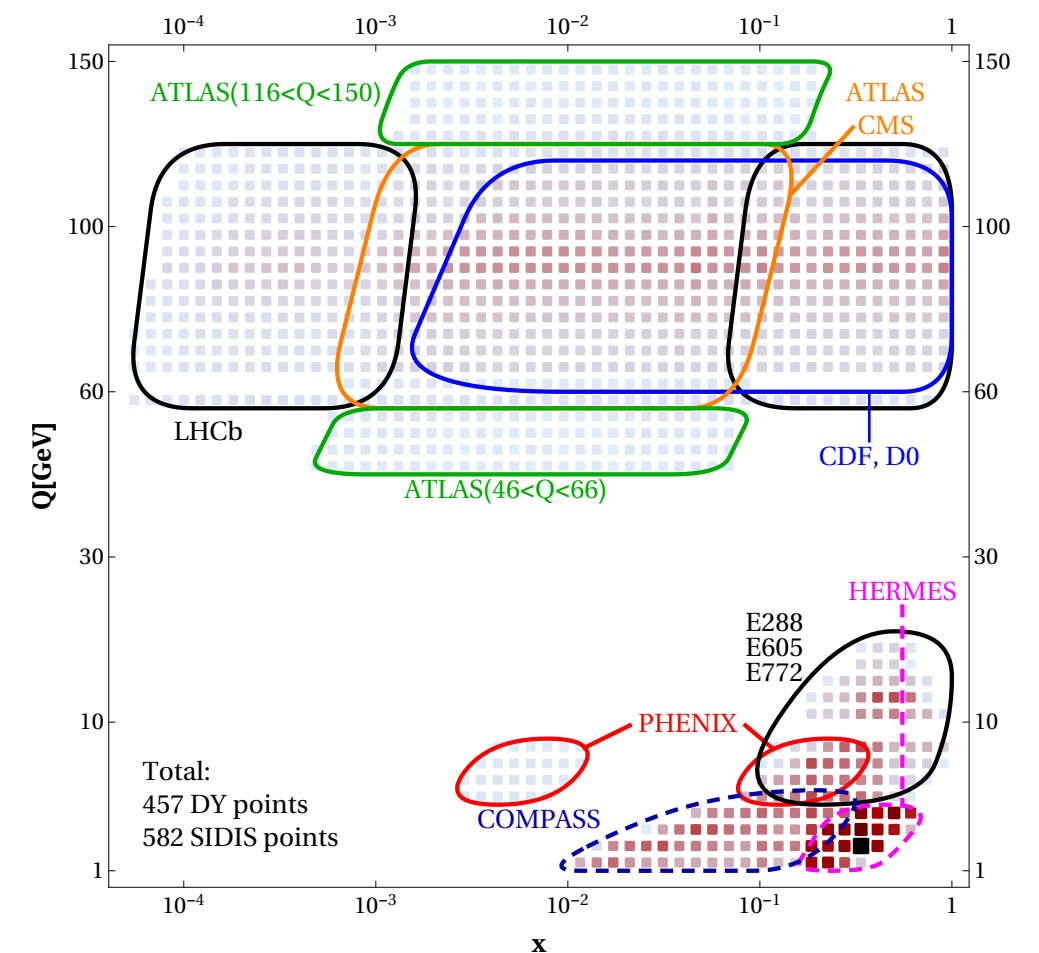
Scimemi, Vladimirov,  
*arXiv:1912.06532*



# x-Q<sup>2</sup> COVERAGE



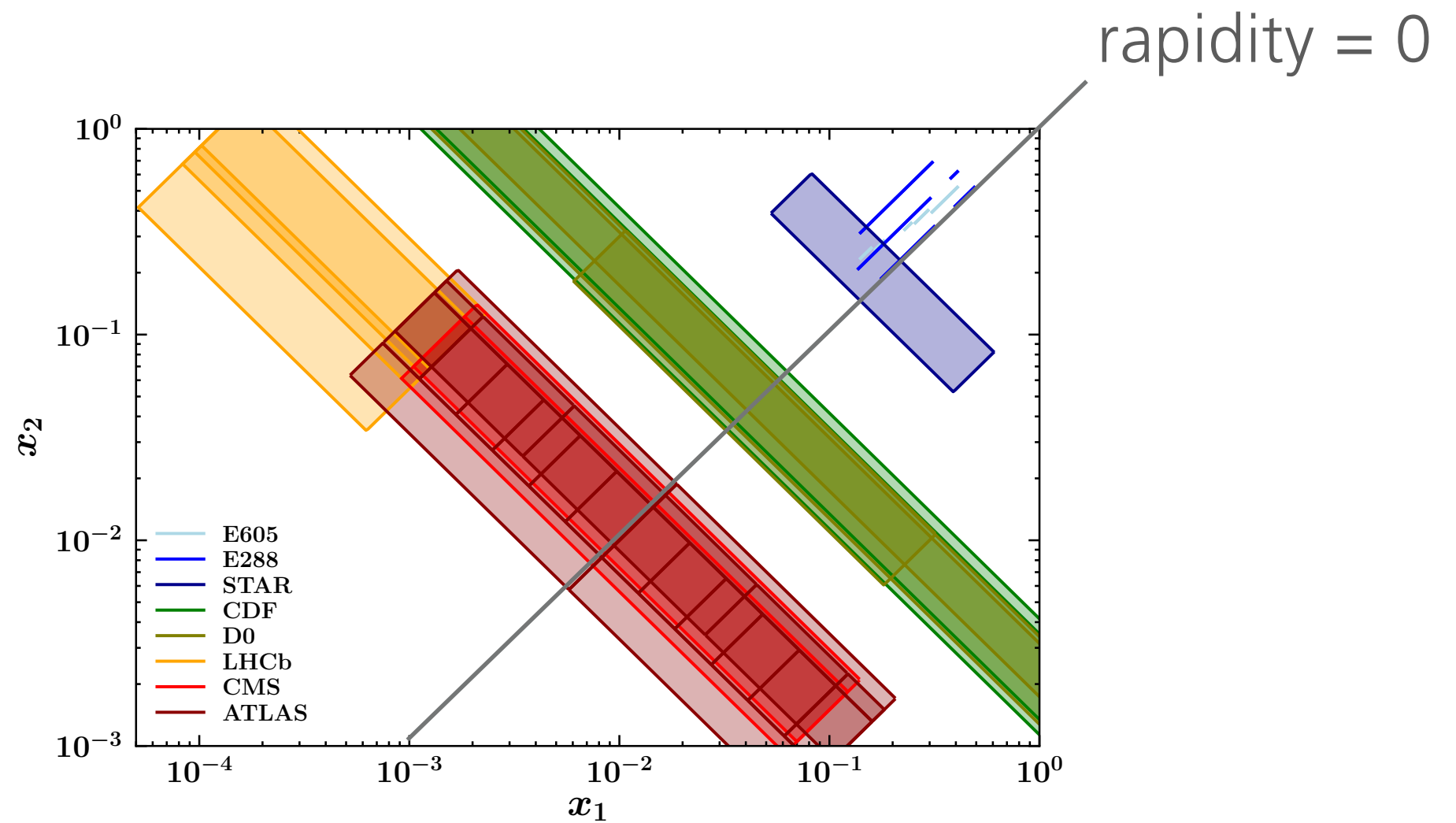
Bacchetta, Delcarro, Pisano, Radici,  
Signori, *arXiv:1703.10157*



Scimemi, Vladimirov,  
*arXiv:1912.06532*

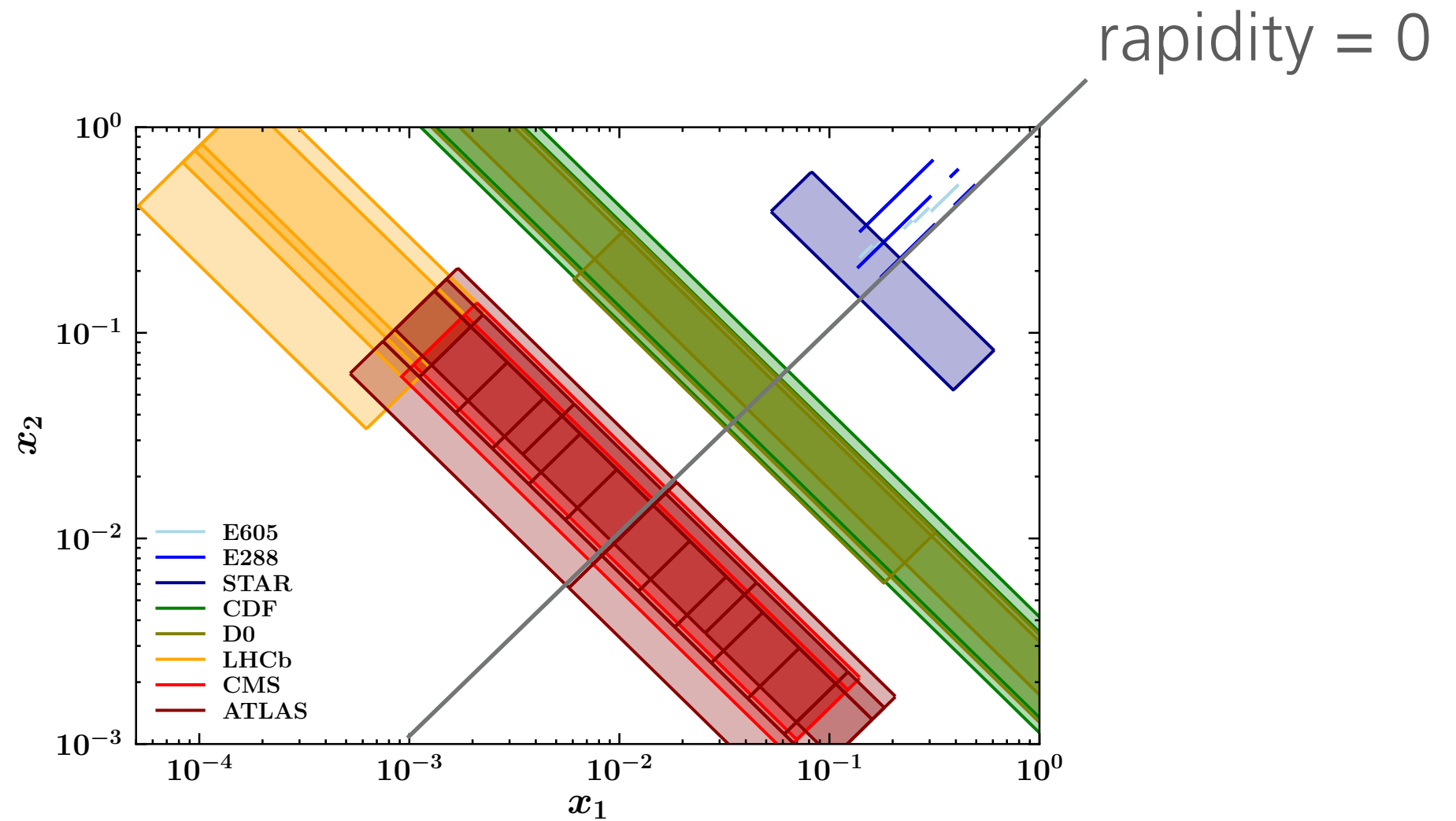
# $x_1$ $x_2$ COVERAGE

.....



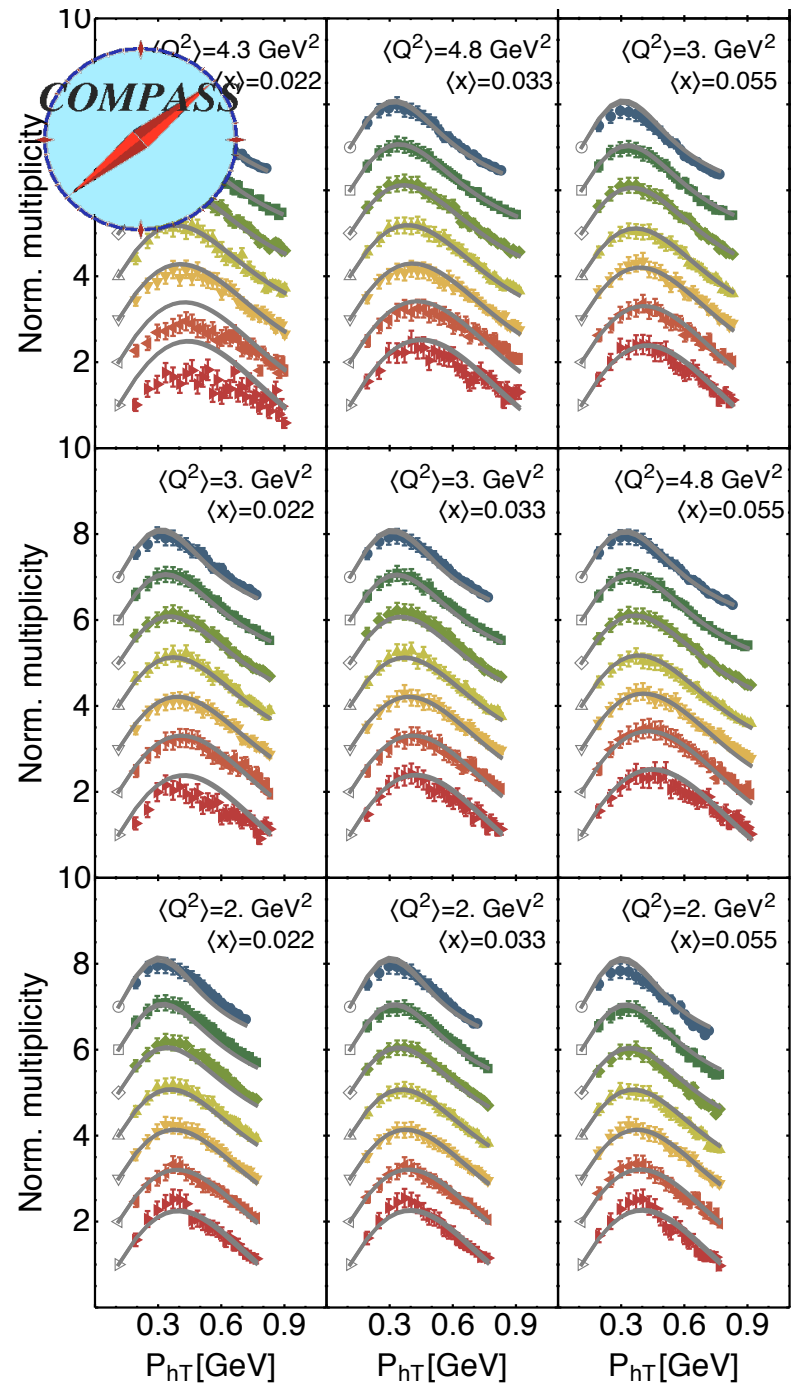
# $x_1$ $x_2$ COVERAGE

*Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550*



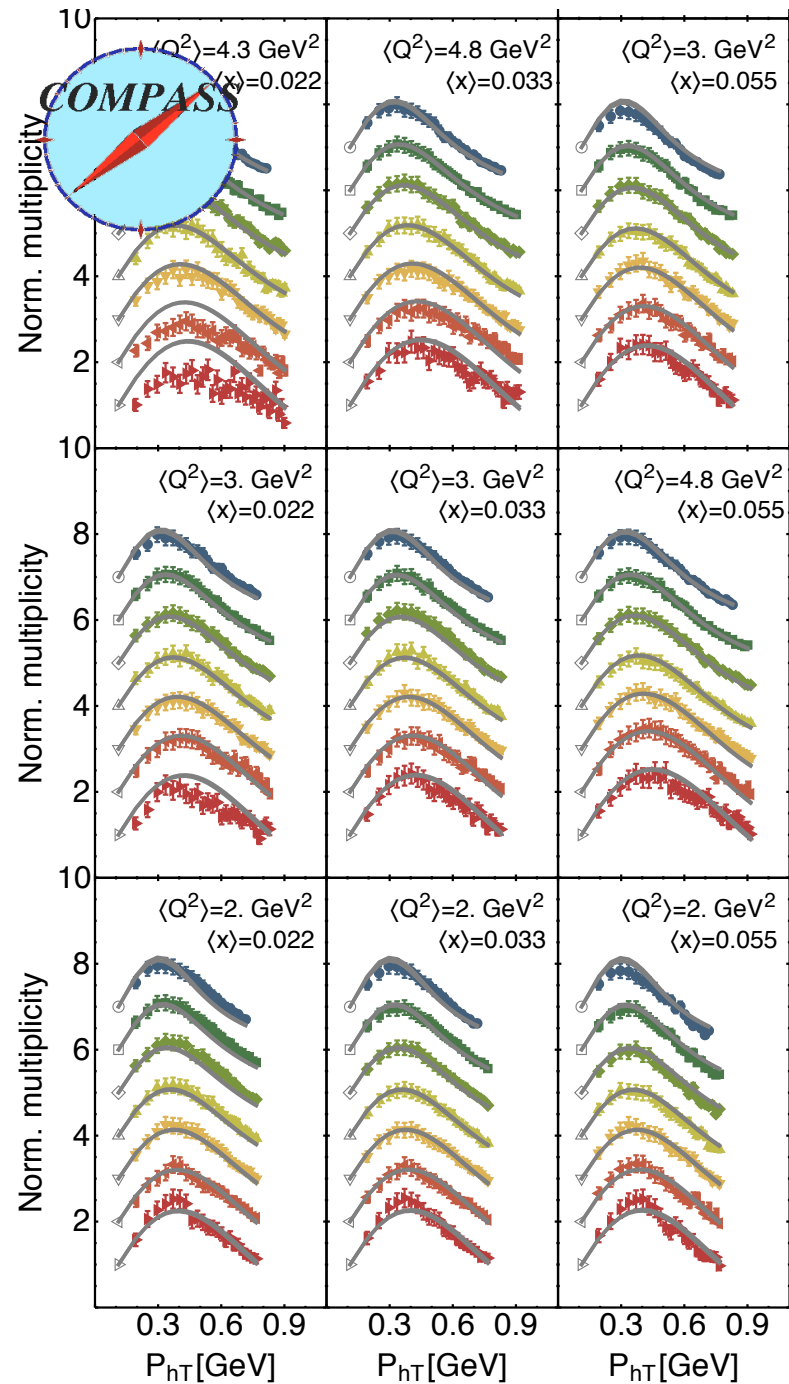
# THE PAVIA17 EXTRACTION

## SIDIS



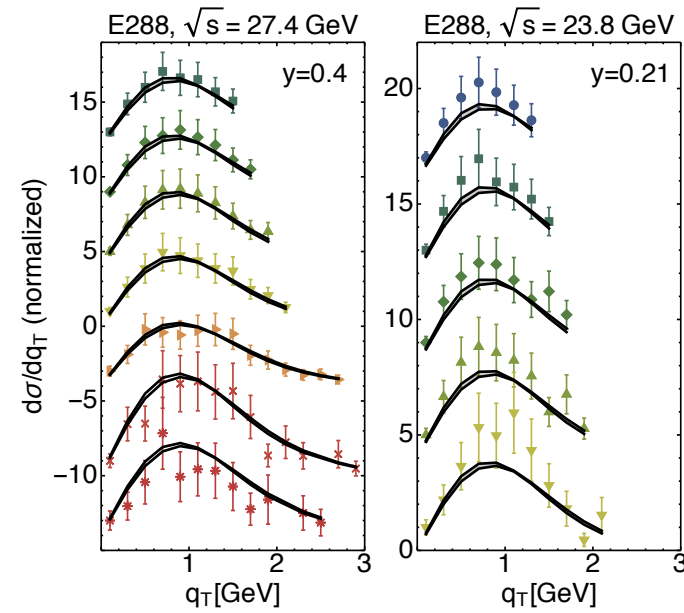
# THE PAVIA17 EXTRACTION

## SIDIS



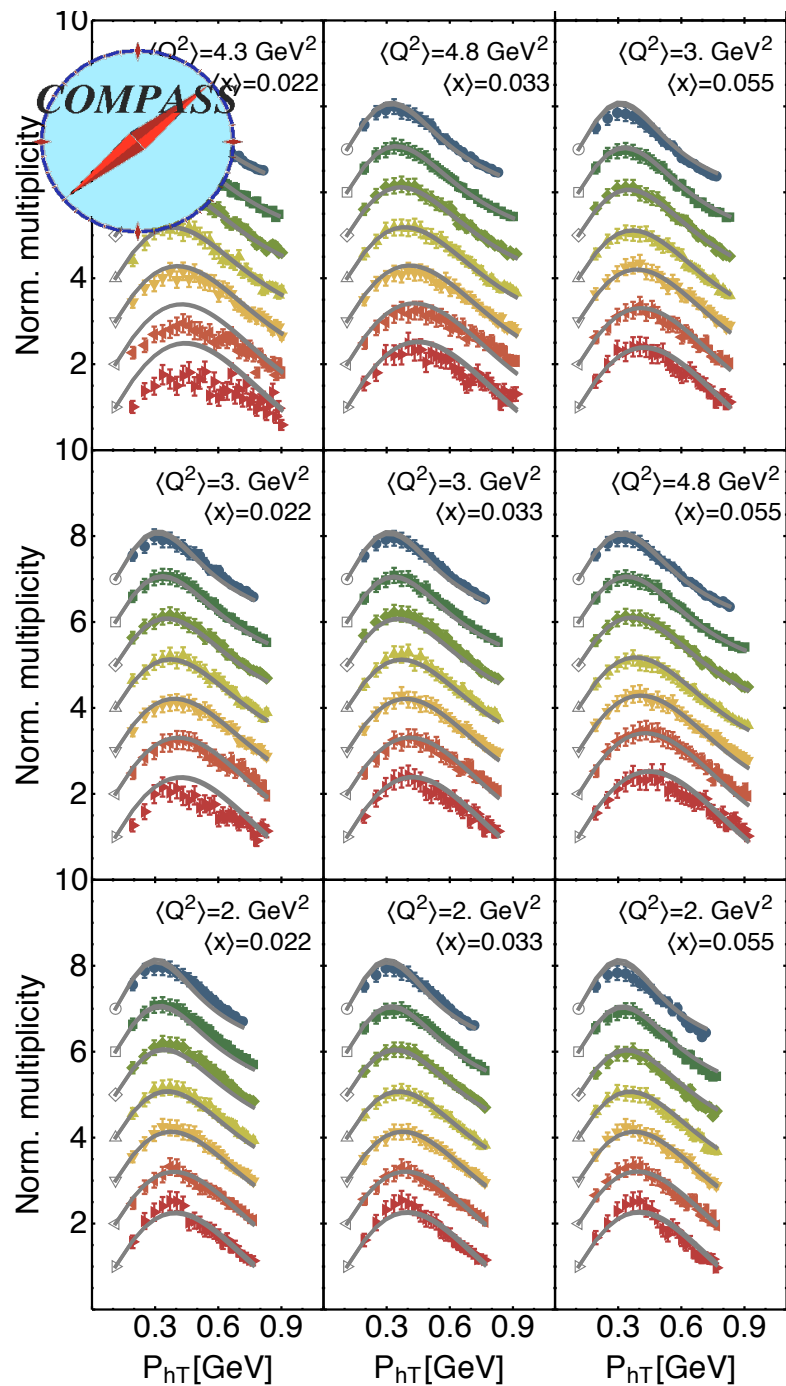
## Drell-Yan

Fermilab



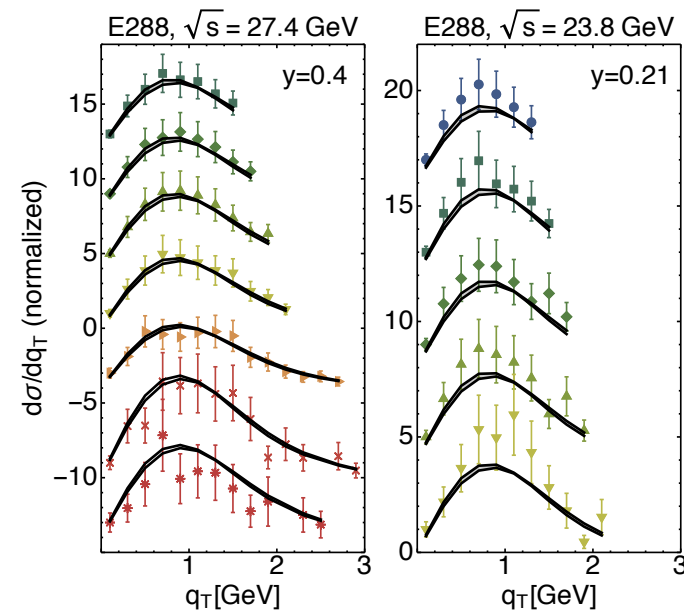
# THE PAVIA17 EXTRACTION

## SIDIS

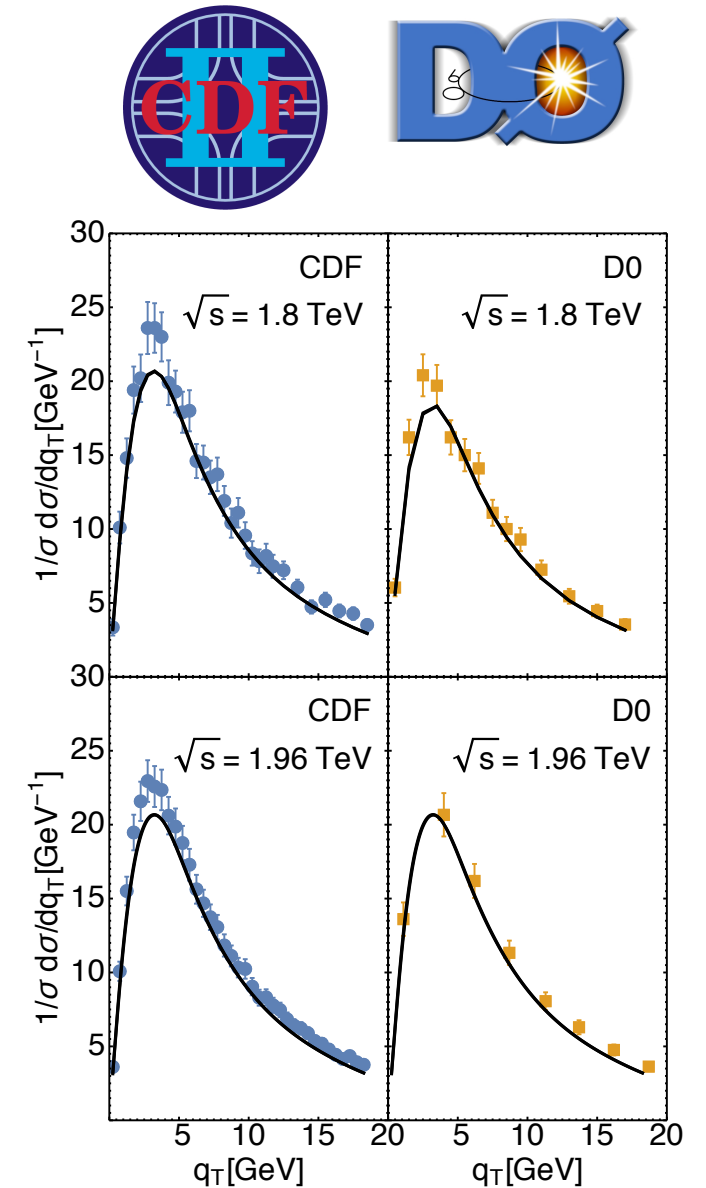


## Drell-Yan

Fermilab



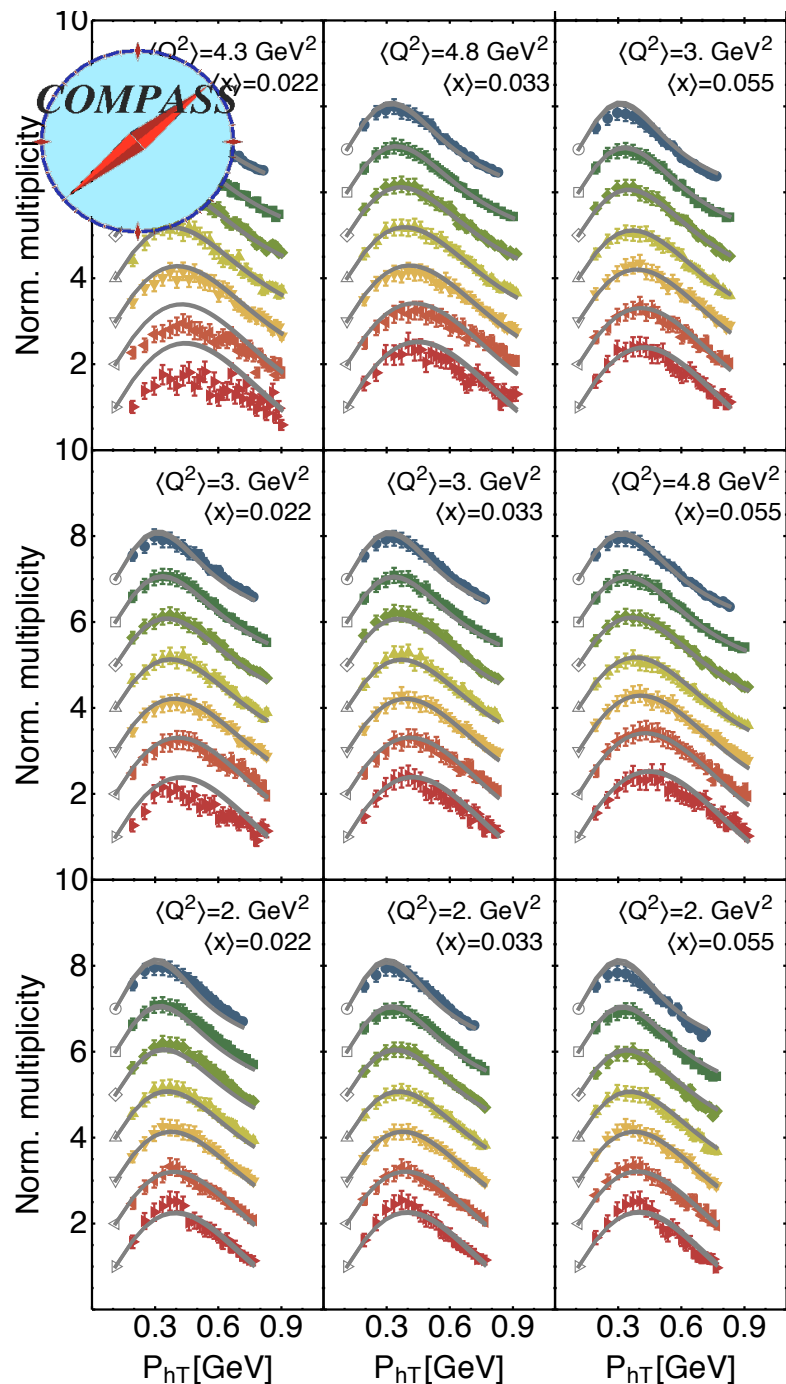
## Z production





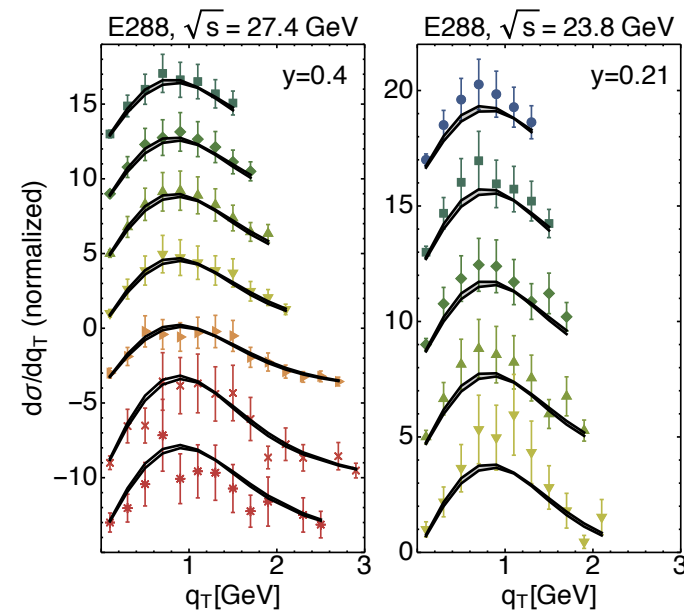
# THE PAVIA17 EXTRACTION

## SIDIS



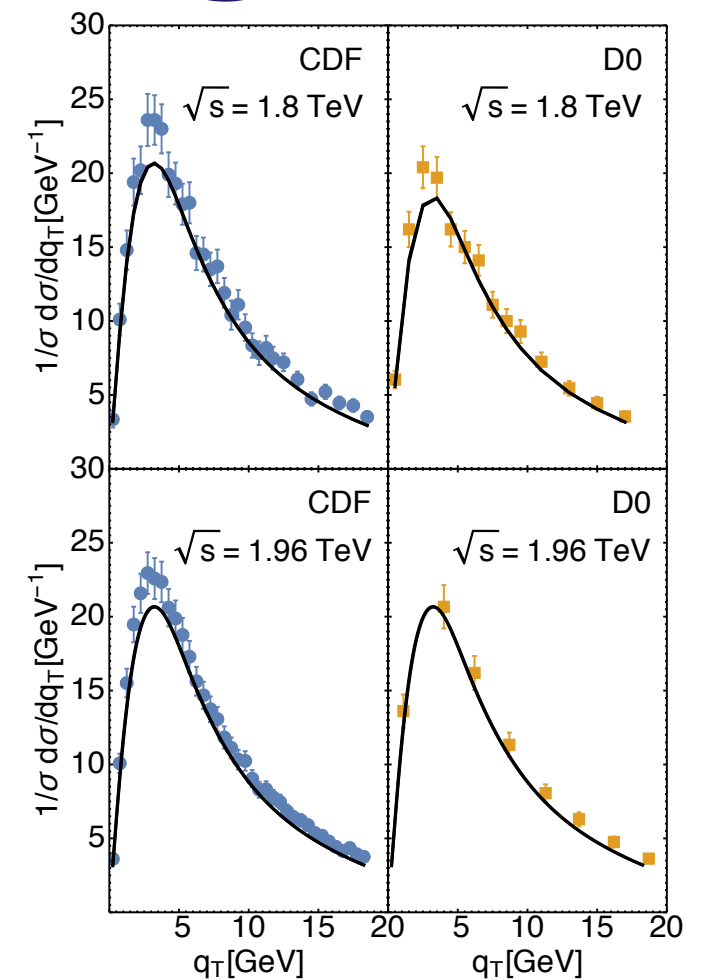
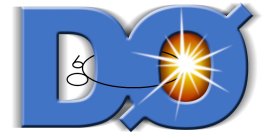
## Drell-Yan

Fermilab



Number of data points: 8059  
Global  $\chi^2/\text{dof} = 1.55$

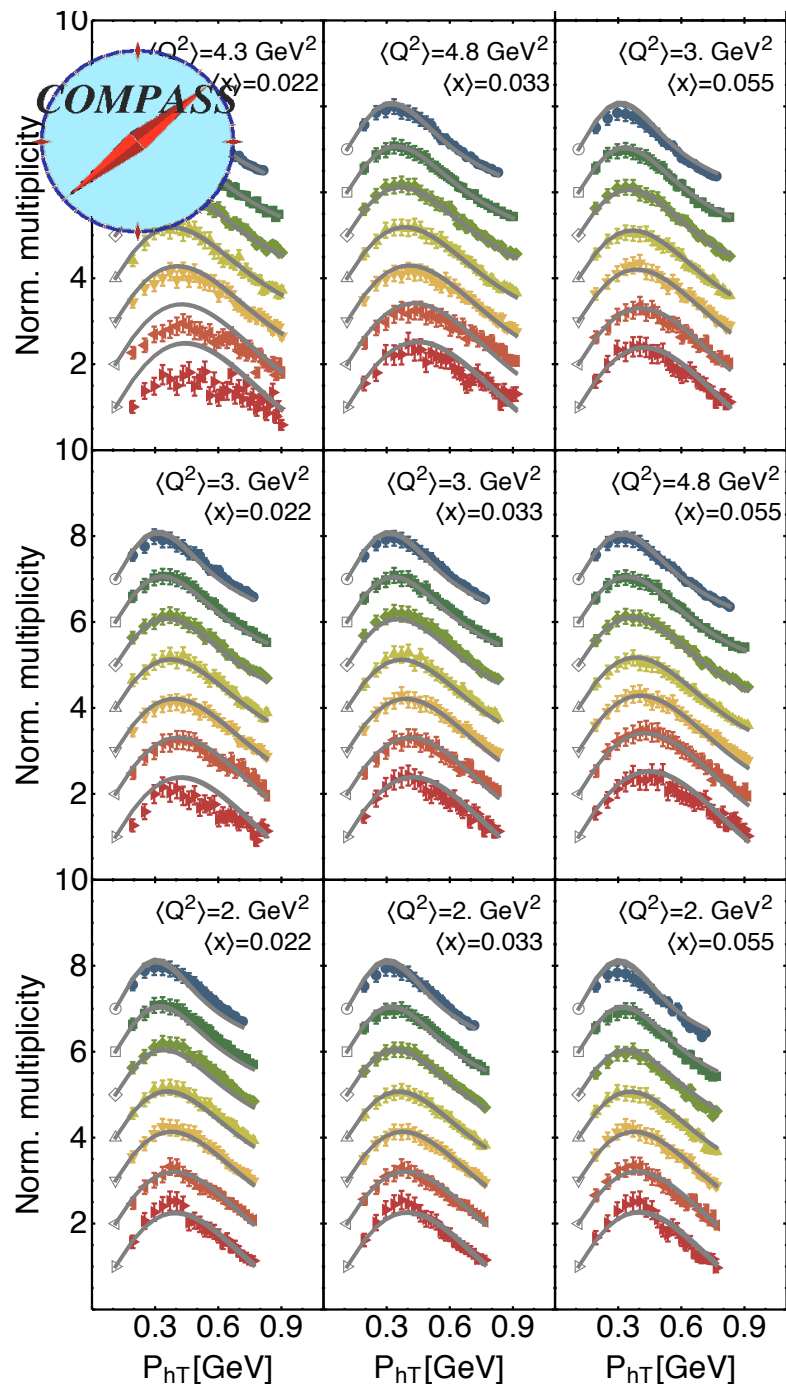
## Z production



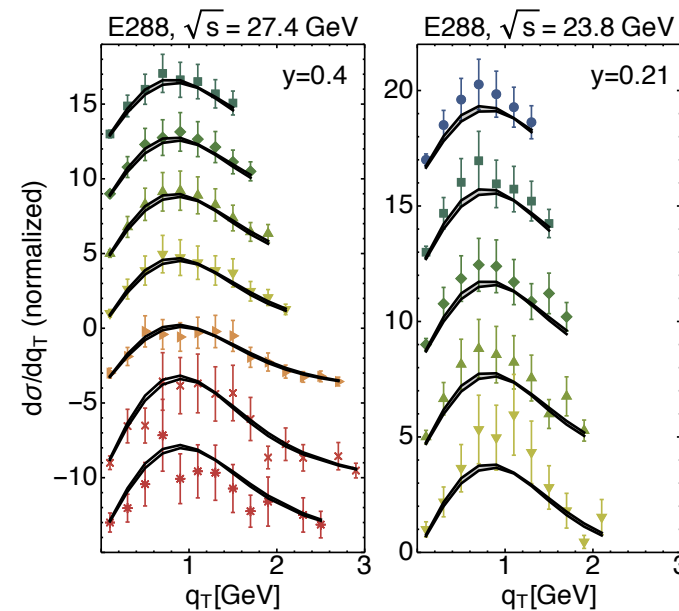
Bacchetta, Delcarro, Pisano, Radici, Signori, arXiv:1703.10157

# THE PAVIA17 EXTRACTION

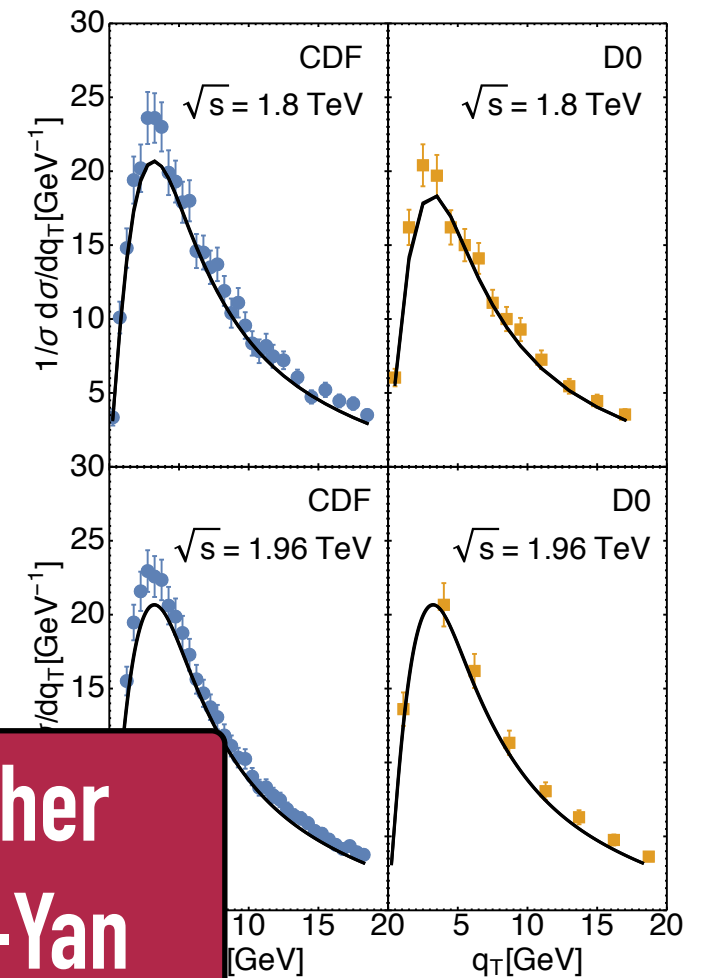
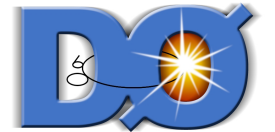
## SIDIS



## Drell-Yan



## Z production



Number of data points: 8059  
Global  $\chi^2/\text{dof} = 1.55$

**Pavia17: first fit putting together  
semi-inclusive DIS and Drell-Yan**

Bacchetta, Delcarro, Pisano, Radici, Signori, arXiv:1703.10157



# The TMD “eight-thousander” fit

.....8000 data points

*Nanga Parbat, Kashmir, 8126 m*



# The TMD “eight-thousander” fit

***Pavia 2017***

8000 data points

*Nanga Parbat, Kashmir, 8126 m*

# PV17 – RESULTING TMDS

---

*expression in  $b_T$  space*

$$\hat{f}_{\text{NP}}(x, b_T) = e^{-g_1(x) \frac{b_T^2}{4}} \left( 1 - \frac{\lambda g_1^2(x)}{1 + \lambda g_1(x)} \frac{b_T^2}{4} \right)$$

# PV17 – RESULTING TMDS

---

*Bacchetta, Delcarro, Pisano, Radici, Signori, arXiv:1703.10157*

*expression in  $b_T$  space*

$$\hat{f}_{\text{NP}}(x, b_T) = e^{-g_1(x) \frac{b_T^2}{4}} \left( 1 - \frac{\lambda g_1^2(x)}{1 + \lambda g_1(x)} \frac{b_T^2}{4} \right)$$

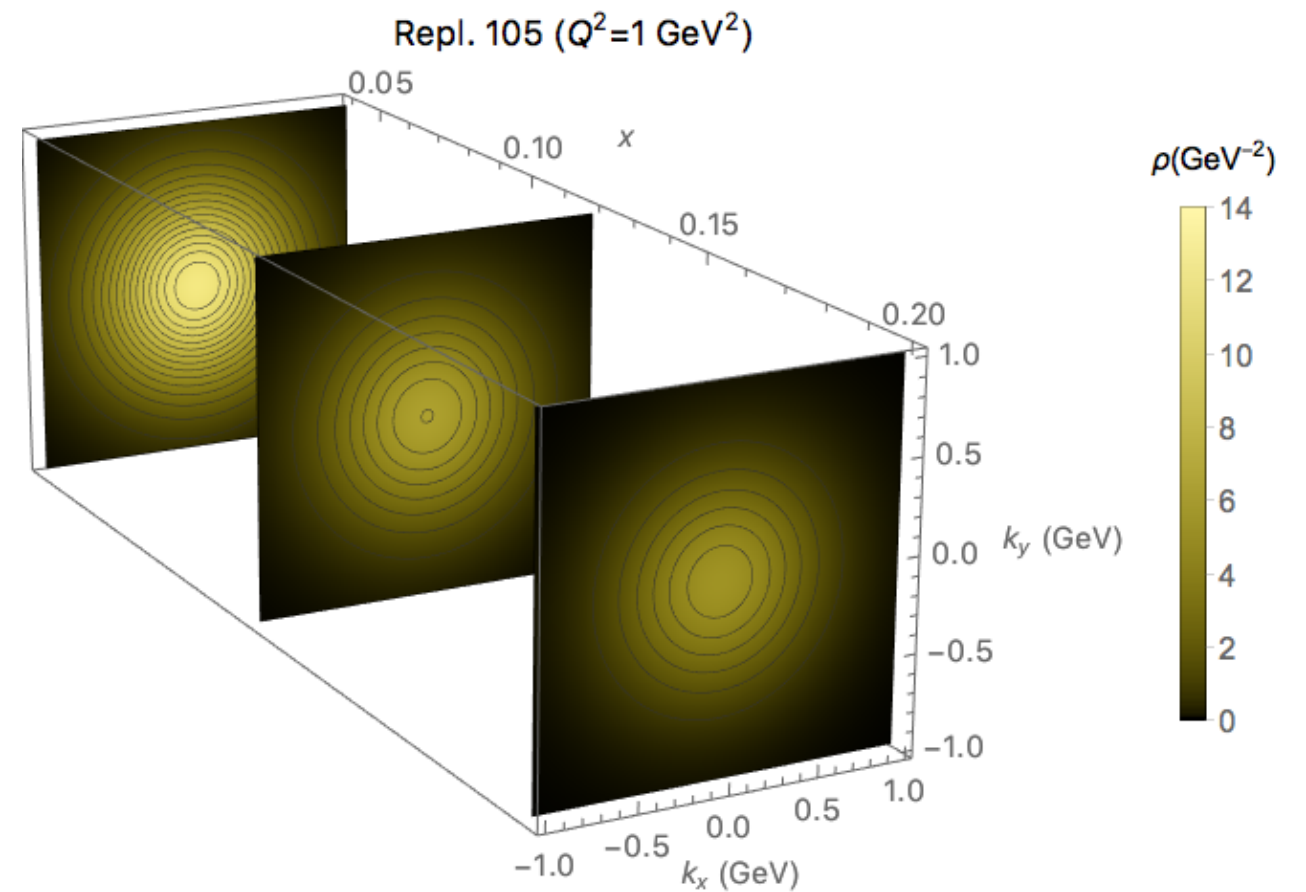
# PV17 – RESULTING TMDS

Bacchetta, Delcarro, Pisano, Radici, Signori, arXiv:1703.10157

expression in  $b_T$  space

$$\hat{f}_{\text{NP}}(x, b_T) = e^{-g_1(x) \frac{b_T^2}{4}} \left( 1 - \frac{\lambda g_1^2(x)}{1 + \lambda g_1(x)} \frac{b_T^2}{4} \right)$$

plot in  $k_\perp$  space





# PV17 – RESULTING TMDS

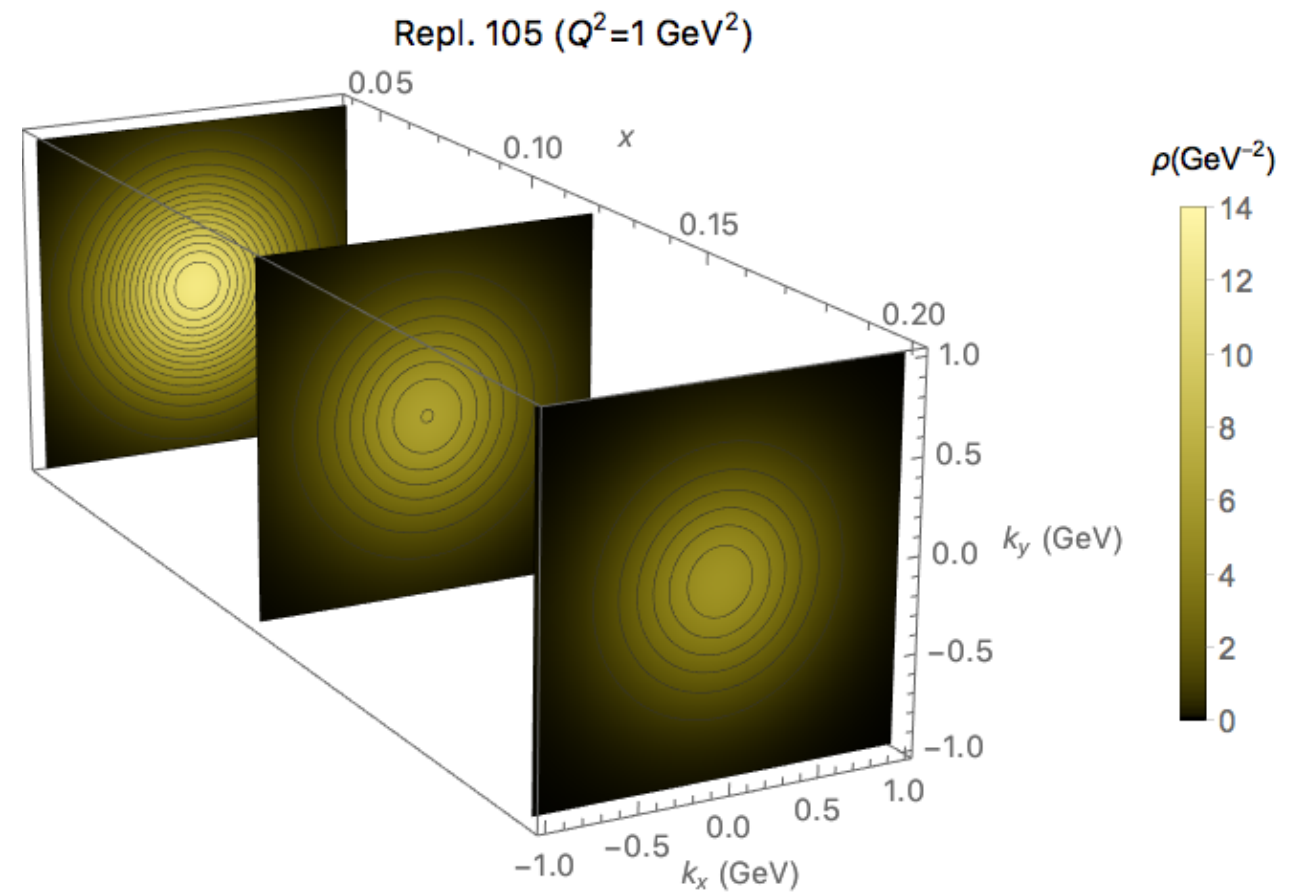
Bacchetta, Delcarro, Pisano, Radici, Signori, arXiv:1703.10157

expression in  $b_T$  space

$$\hat{f}_{\text{NP}}(x, b_T) = e^{-g_1(x) \frac{b_T^2}{4}} \left( 1 - \frac{\lambda g_1^2(x)}{1 + \lambda g_1(x)} \frac{b_T^2}{4} \right)$$

- Gaussian + weighted Gaussian

plot in  $k_\perp$  space



# PV17 – RESULTING TMDS

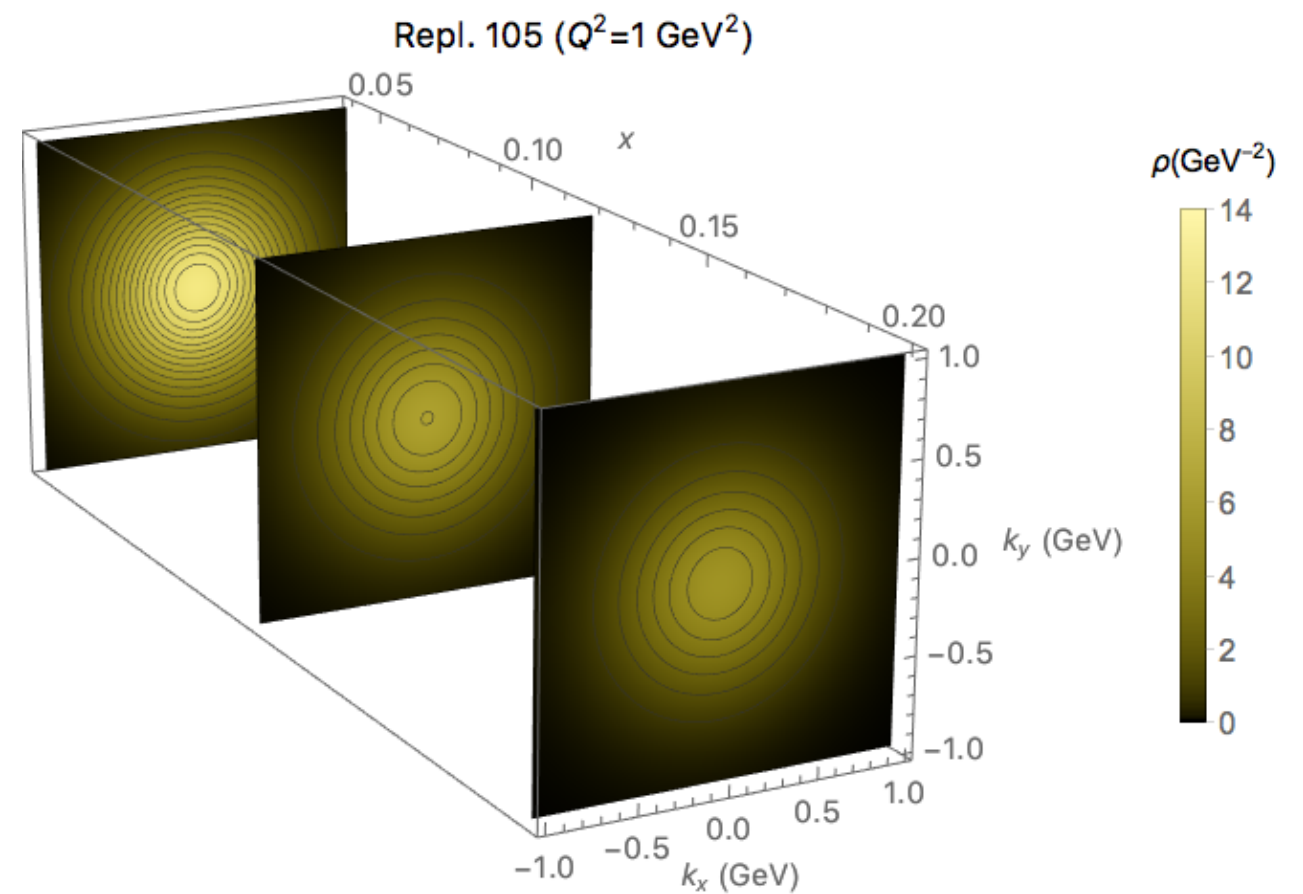
Bacchetta, Delcarro, Pisano, Radici, Signori, arXiv:1703.10157

expression in  $b_T$  space

$$\hat{f}_{\text{NP}}(x, b_T) = e^{-g_1(x) \frac{b_T^2}{4}} \left( 1 - \frac{\lambda g_1^2(x)}{1 + \lambda g_1(x)} \frac{b_T^2}{4} \right)$$

- Gaussian + weighted Gaussian
- nontrivial  $x$  dependence

plot in  $k_\perp$  space



# PV17 – RESULTING TMDS

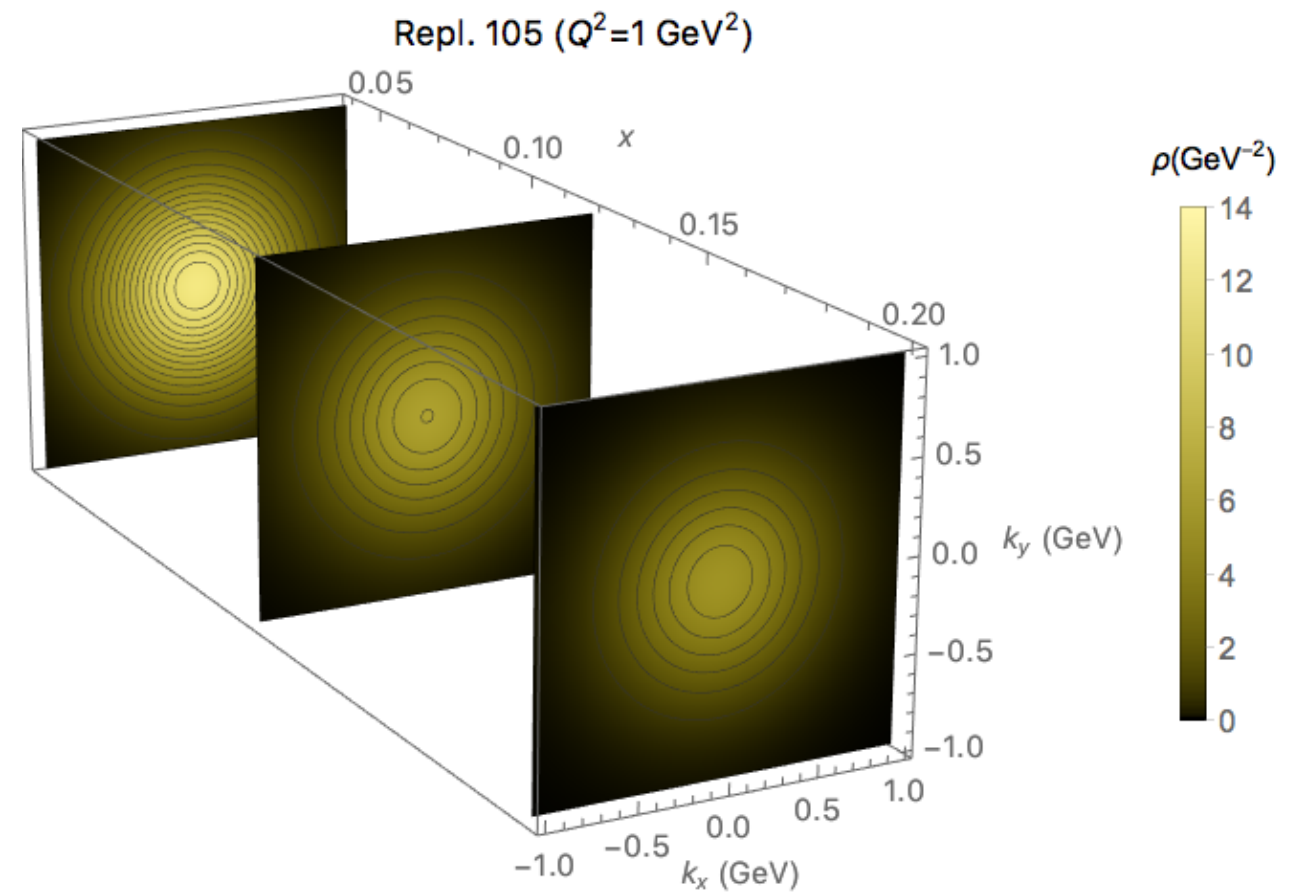
Bacchetta, Delcarro, Pisano, Radici, Signori, arXiv:1703.10157

expression in  $b_T$  space

$$\hat{f}_{\text{NP}}(x, b_T) = e^{-g_1(x) \frac{b_T^2}{4}} \left( 1 - \frac{\lambda g_1^2(x)}{1 + \lambda g_1(x)} \frac{b_T^2}{4} \right)$$

- Gaussian + weighted Gaussian
- nontrivial  $x$  dependence
- no flavor dependence

plot in  $k_\perp$  space





# PV17 – RESULTING TMDS

Bacchetta, Delcarro, Pisano, Radici, Signori, arXiv:1703.10157

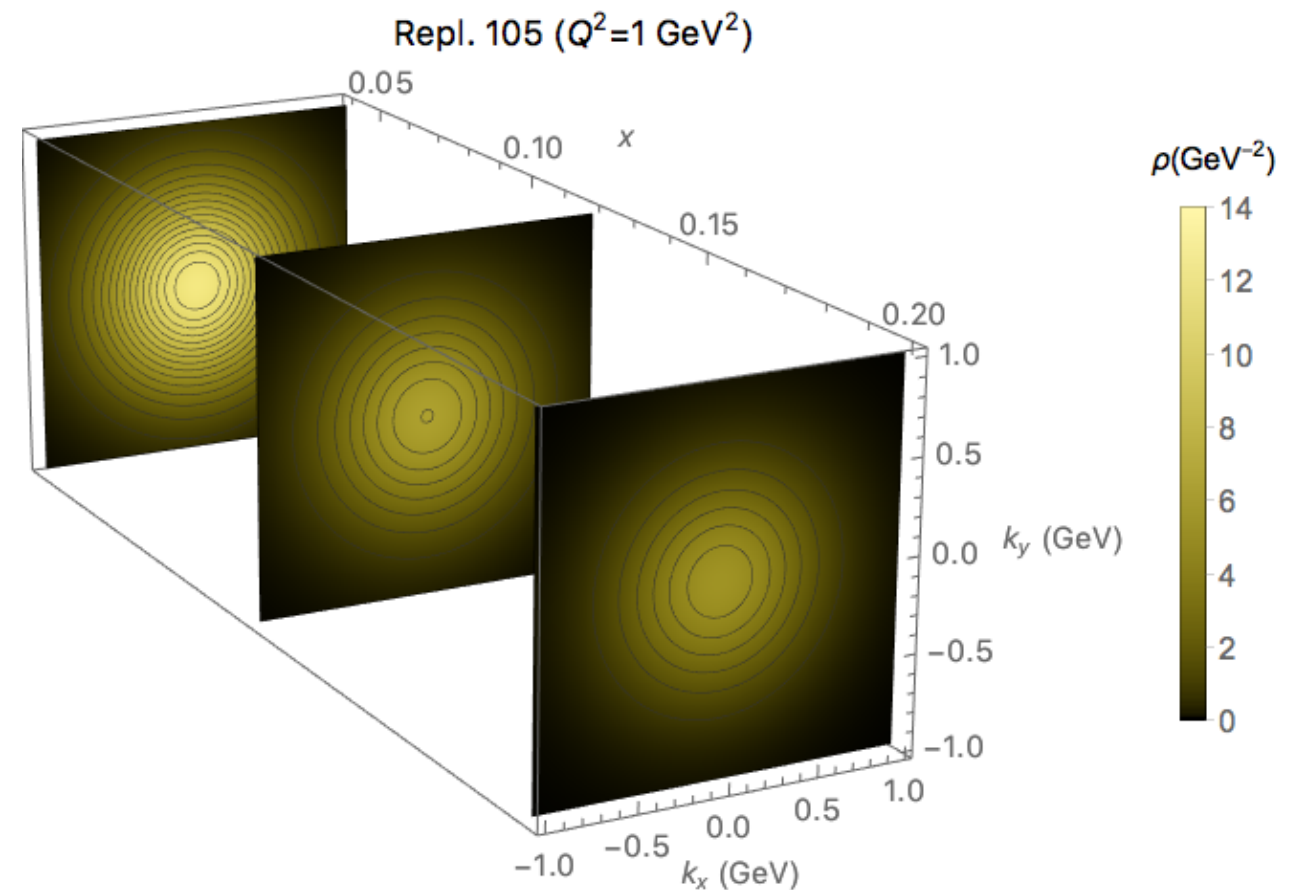
expression in  $b_T$  space

$$\hat{f}_{\text{NP}}(x, b_T) = e^{-g_1(x) \frac{b_T^2}{4}} \left( 1 - \frac{\lambda g_1^2(x)}{1 + \lambda g_1(x)} \frac{b_T^2}{4} \right)$$

- Gaussian + weighted Gaussian
- nontrivial  $x$  dependence
- no flavor dependence

$$g_K(b_T) = -\frac{g_2}{2} b_T^2 \quad \text{Gaussian}$$

plot in  $k_\perp$  space



# PV17 – RESULTING TMDS

Bacchetta, Delcarro, Pisano, Radici, Signori, arXiv:1703.10157

expression in  $b_T$  space

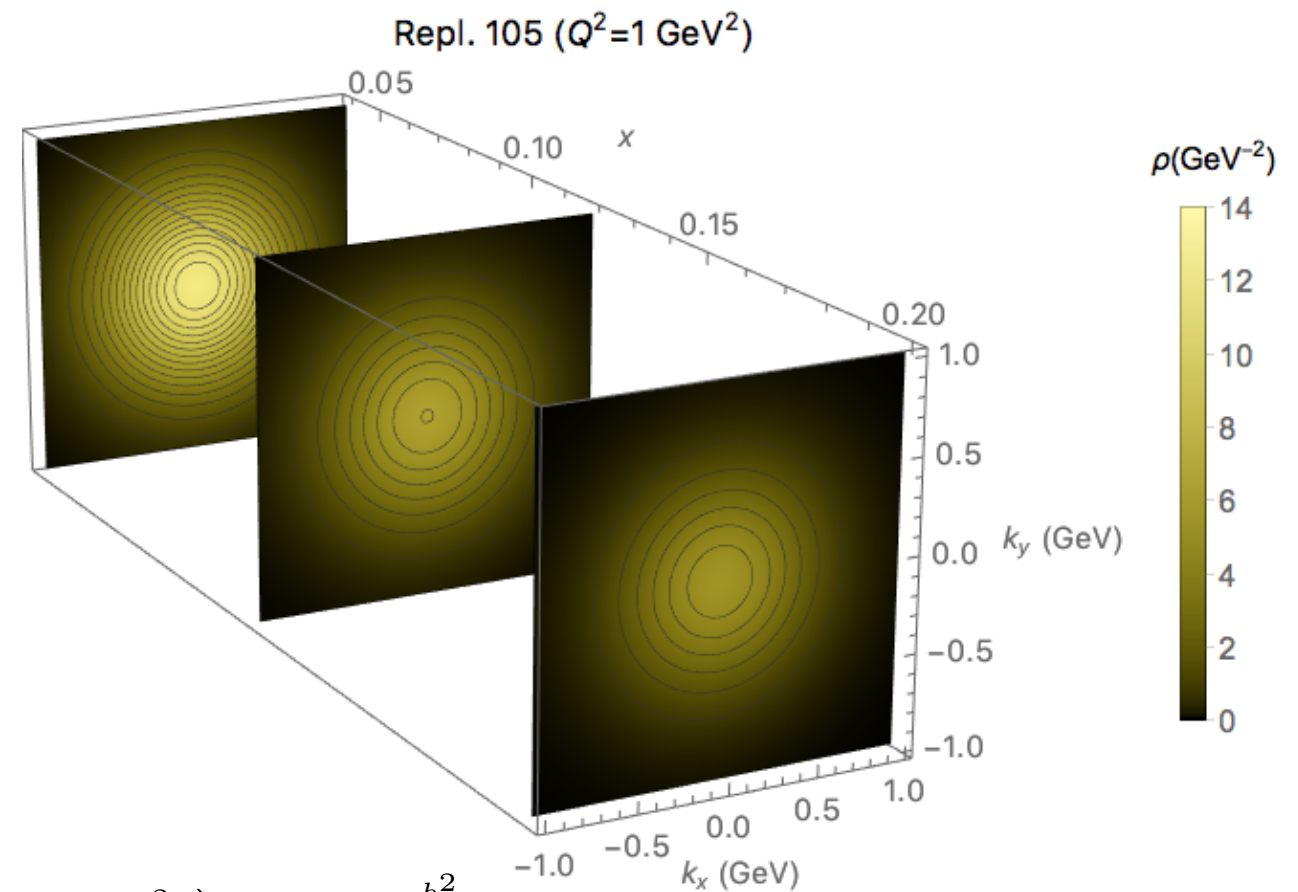
$$\hat{f}_{\text{NP}}(x, b_T) = e^{-g_1(x) \frac{b_T^2}{4}} \left( 1 - \frac{\lambda g_1^2(x)}{1 + \lambda g_1(x)} \frac{b_T^2}{4} \right)$$

- Gaussian + weighted Gaussian
- nontrivial  $x$  dependence
- no flavor dependence

$$g_K(b_T) = -\frac{g_2}{2} b_T^2 \quad \text{Gaussian}$$

$$\hat{D}_{\text{NP}}(z, b_T) = \frac{g_3(z) e^{-g_3(z) \frac{b_T^2}{4z^2}} + (\lambda_F/z^2) g_4^2(z) \left( 1 - g_4(z) \frac{b_T^2}{4z^2} \right) e^{-g_4^2(z) \frac{b_T^2}{4z^2}}}{z^2 \left( g_3(z) + (\lambda_F/z^2) g_4^2(z) \right)}$$

plot in  $k_\perp$  space



TMD Frag. Func.

# PV17 – RESULTING TMDS

Bacchetta, Delcarro, Pisano, Radici, Signori, arXiv:1703.10157

expression in  $b_T$  space

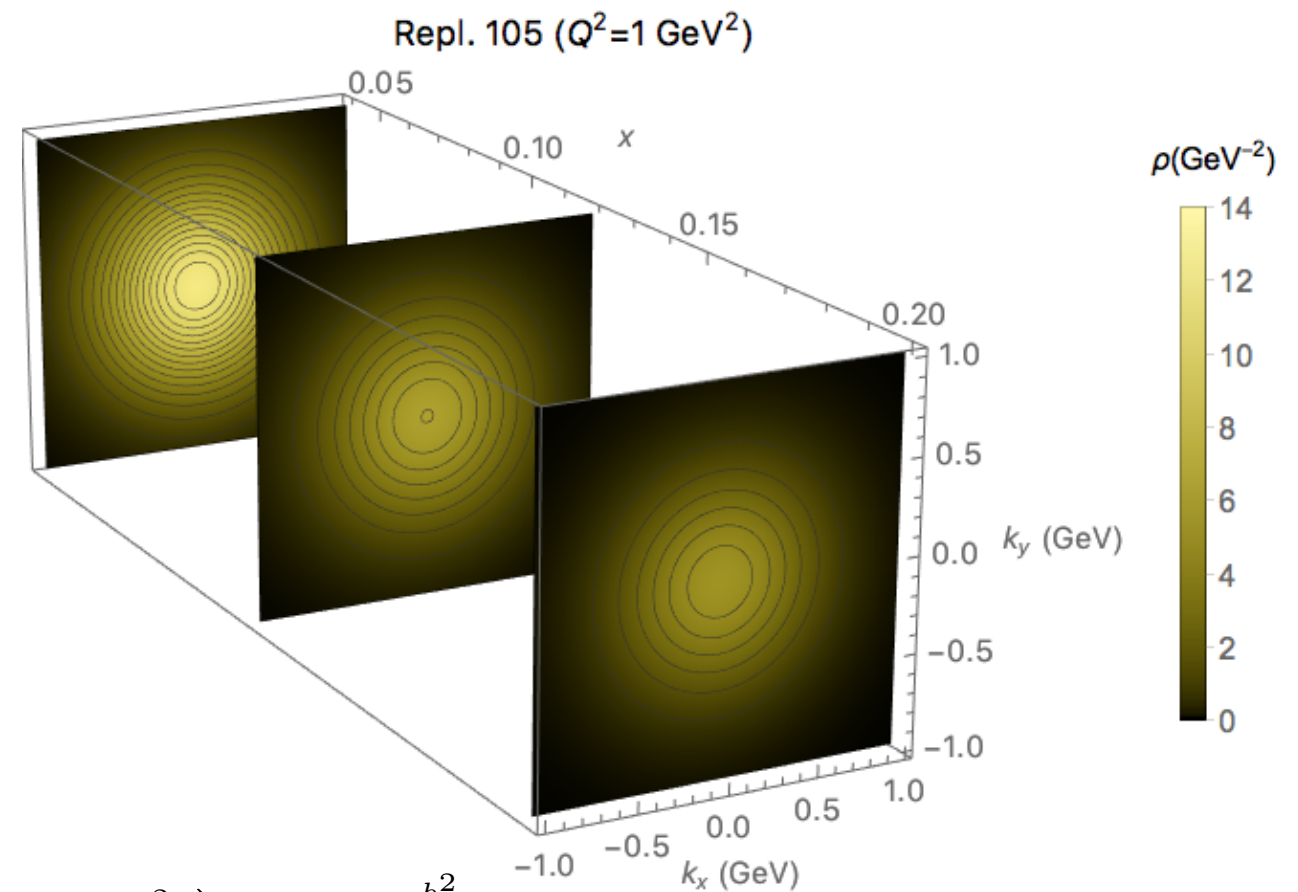
$$\hat{f}_{\text{NP}}(x, b_T) = e^{-g_1(x) \frac{b_T^2}{4}} \left( 1 - \frac{\lambda g_1^2(x)}{1 + \lambda g_1(x)} \frac{b_T^2}{4} \right)$$

- Gaussian + weighted Gaussian
- nontrivial  $x$  dependence
- no flavor dependence

$$g_K(b_T) = -\frac{g_2}{2} b_T^2 \quad \text{Gaussian}$$

$$\hat{D}_{\text{NP}}(z, b_T) = \frac{g_3(z) e^{-g_3(z) \frac{b_T^2}{4z^2}} + (\lambda_F/z^2) g_4^2(z) \left( 1 - g_4(z) \frac{b_T^2}{4z^2} \right) e^{-g_4^2(z) \frac{b_T^2}{4z^2}}}{z^2 \left( g_3(z) + (\lambda_F/z^2) g_4^2(z) \right)}$$

plot in  $k_\perp$  space

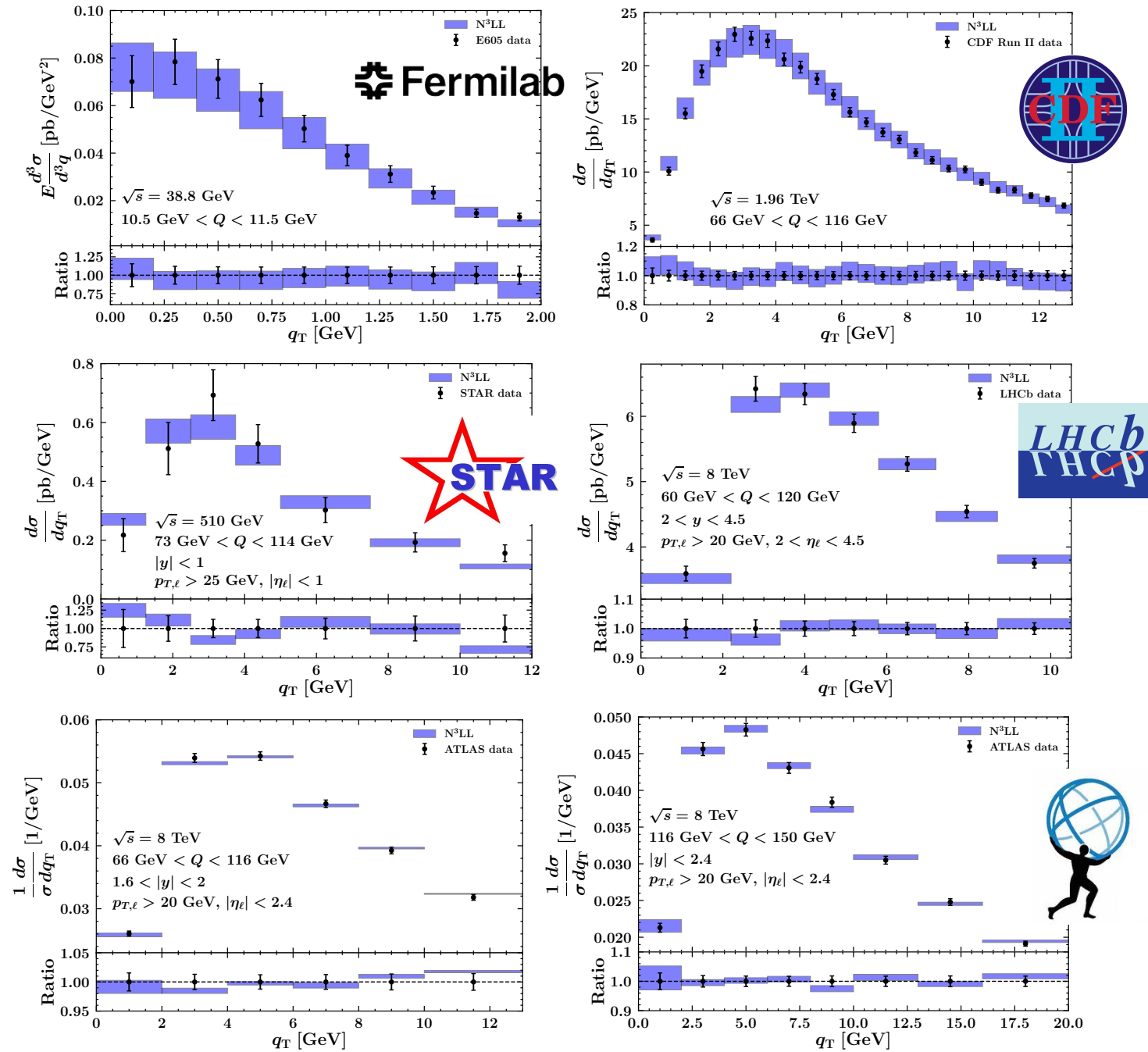


TMD Frag. Func.

**11 free parameters**

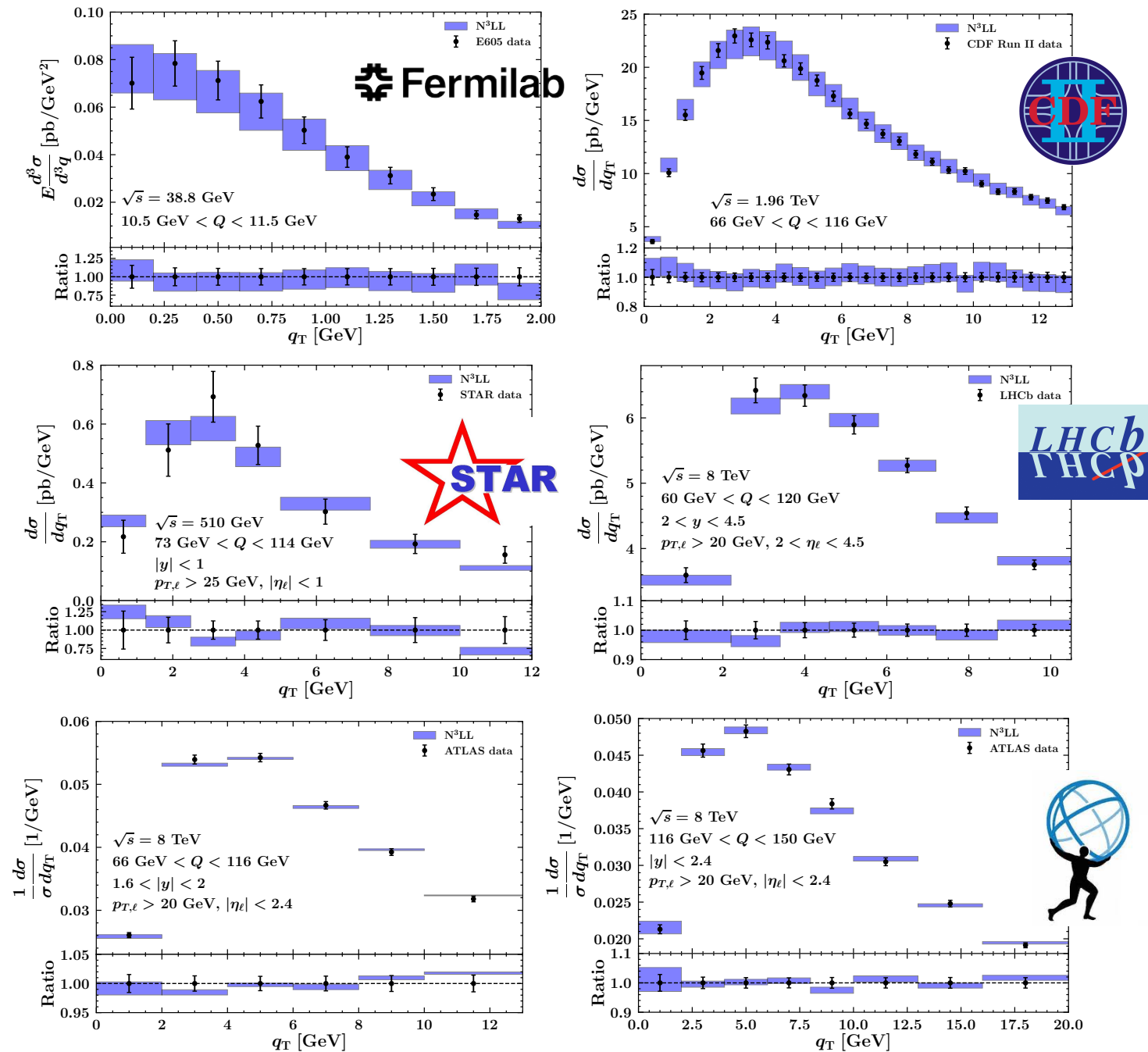
# THE PAVIA19 EXTRACTION

Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550



# THE PAVIA19 EXTRACTION

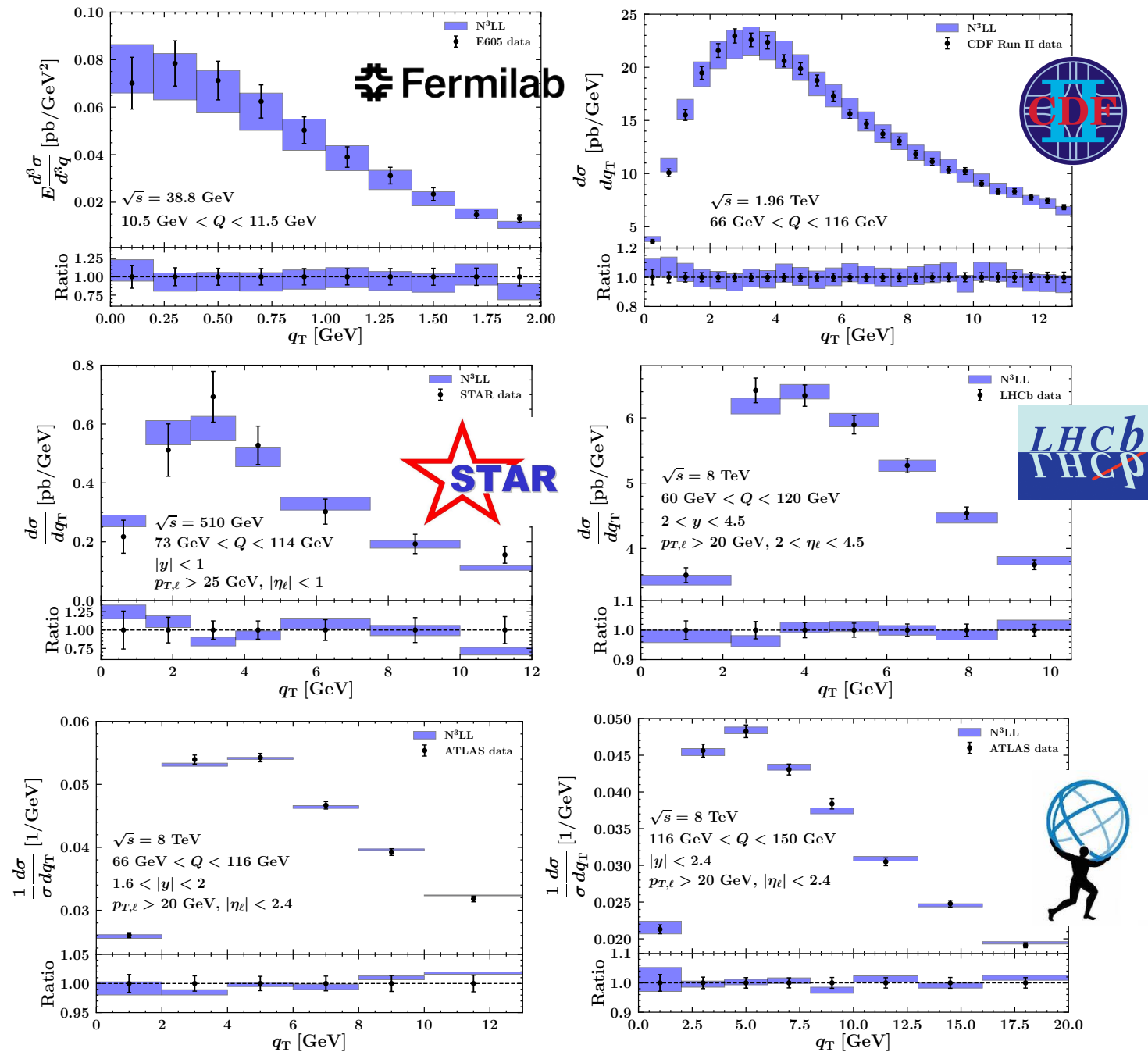
Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550



Data selection:  $q_T/Q < 0.2$

# THE PAVIA19 EXTRACTION

Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550



Data selection:  $q_T/Q < 0.2$

Number of data points: 353



# The TMD “Varzi” fit

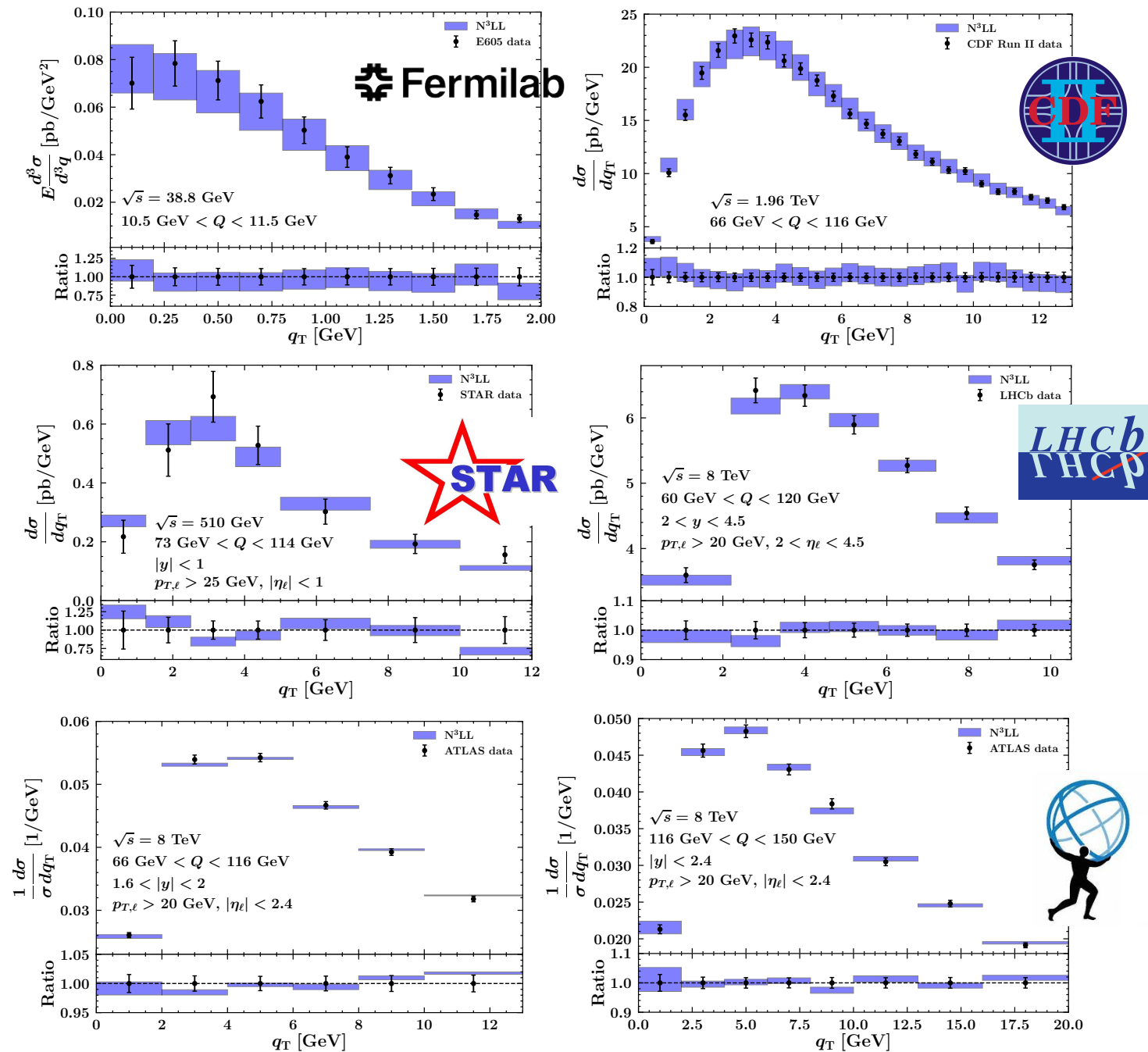


*Varzi, province of Pavia, Italy, 400 m*



# PV19 – DATA COMPARISON

Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550



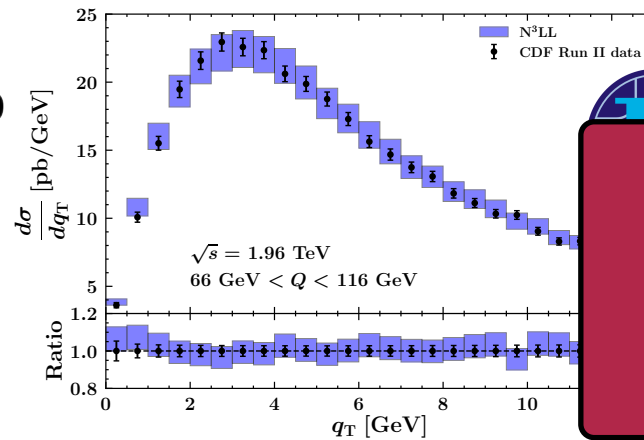
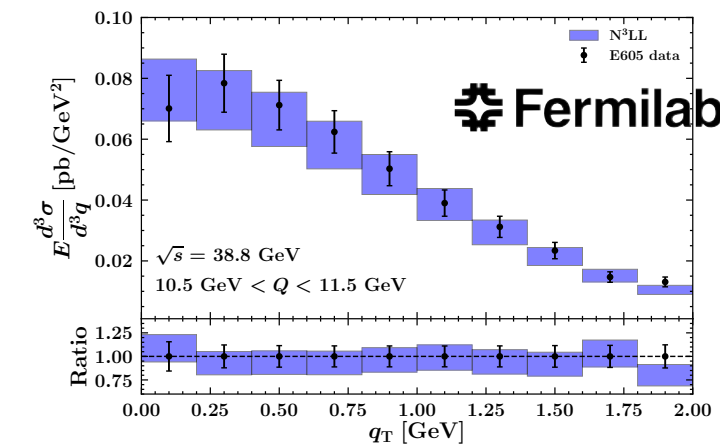
Data selection:  $q_T/Q < 0.2$

Number of data points: 353

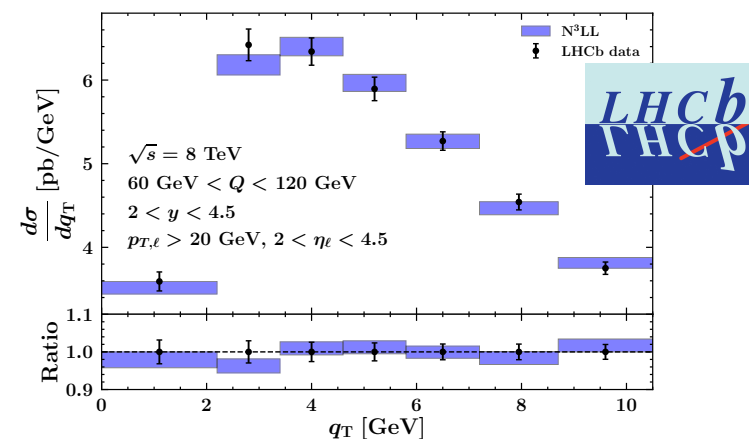
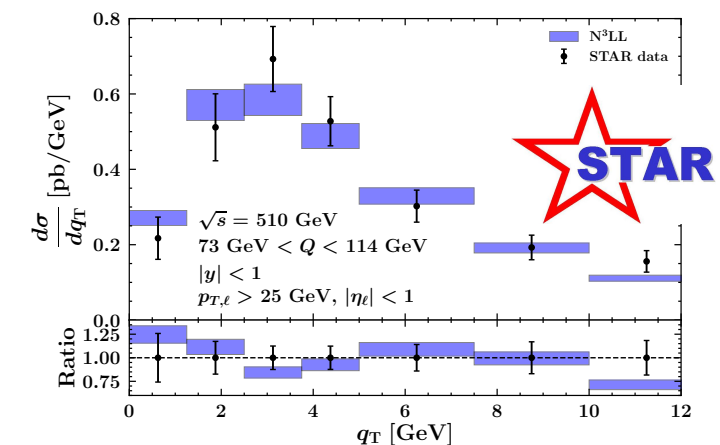


# PV19 – DATA COMPARISON

Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550

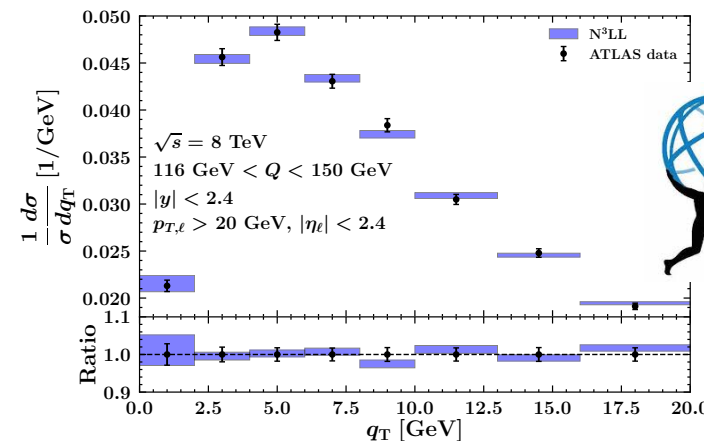
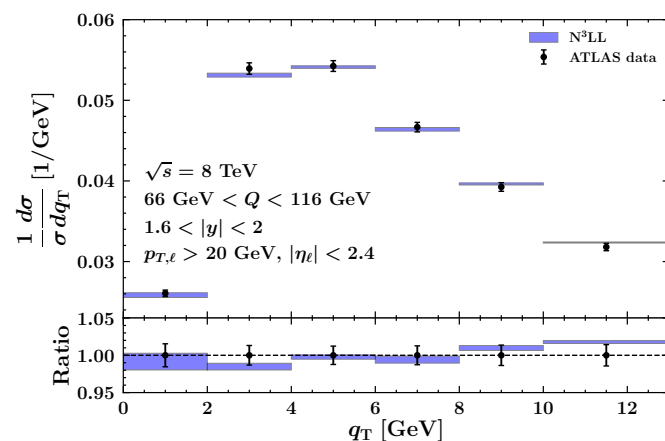


**Pavia19: first DY fit at N<sup>3</sup>LL,  
exactly reproduces normalization**



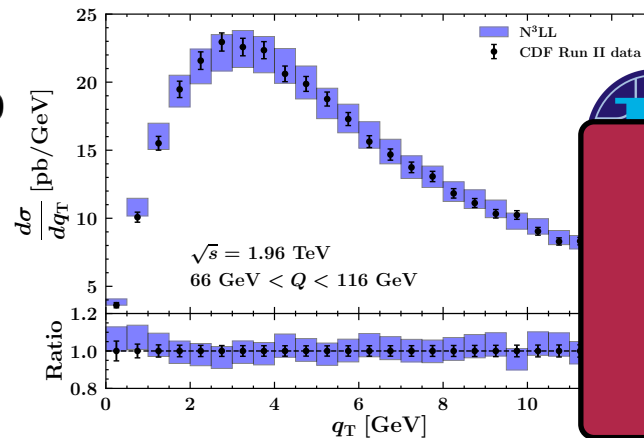
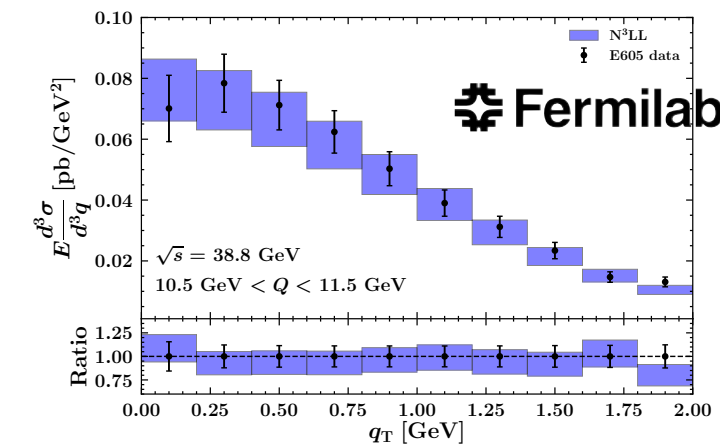
Data selection:  $q_T/Q < 0.2$

Number of data points: 353

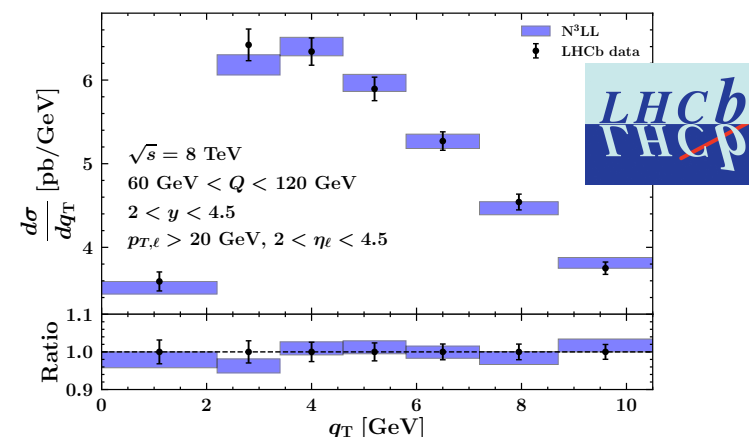
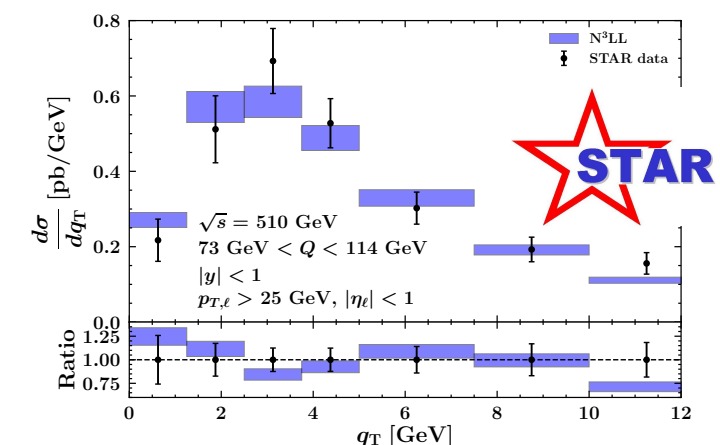


# PV19 – DATA COMPARISON

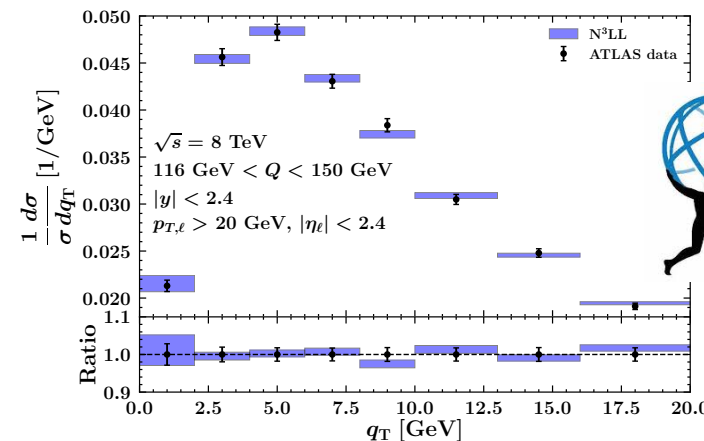
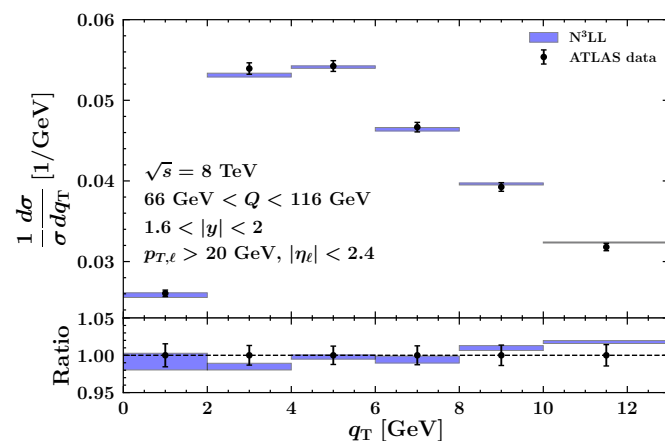
Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550



**Pavia19: first DY fit at N<sup>3</sup>LL,  
exactly reproduces normalization**



Data selection:  $q_T/Q < 0.2$



Number of data points: 353  
 Global  $\chi^2/\text{dof} = 1.02$

# PV19 – RESULTING TMDS

---

*expression in  $b_T$  space*

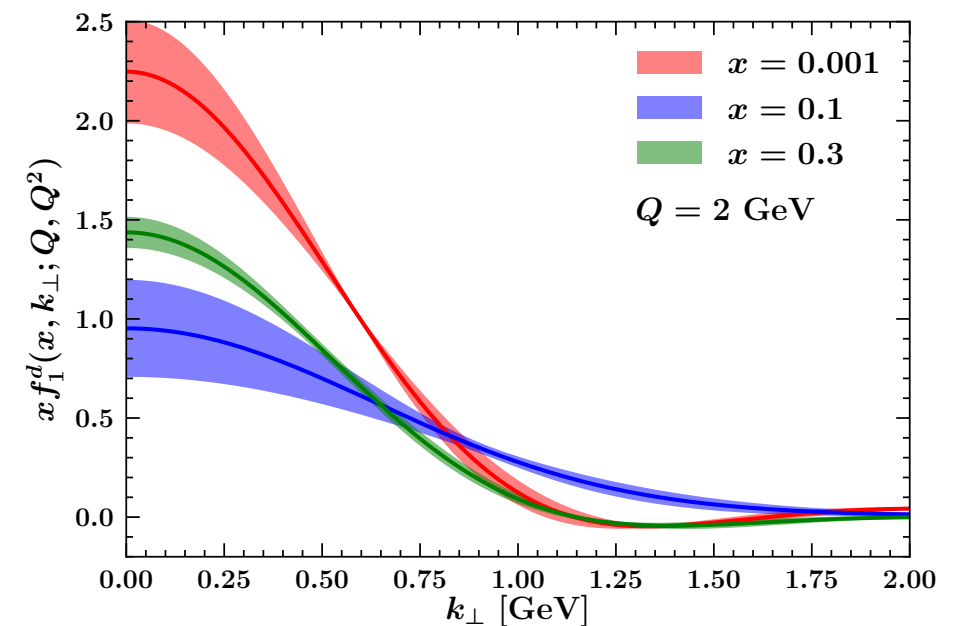
$$f_{\text{NP}}(x, b_T, \zeta) = \left[ \frac{1 - \lambda}{1 + g_1(x) \frac{b_T^2}{4}} + \lambda \exp \left( -g_{1B}(x) \frac{b_T^2}{4} \right) \right] \\ \times \exp \left[ - (g_2 + g_{2B} b_T^2) \ln \left( \frac{\zeta}{Q_0^2} \right) \frac{b_T^2}{4} \right],$$

# PV19 – RESULTING TMDS

*expression in  $b_T$  space*

$$f_{\text{NP}}(x, b_T, \zeta) = \left[ \frac{1 - \lambda}{1 + g_1(x) \frac{b_T^2}{4}} + \lambda \exp \left( -g_{1B}(x) \frac{b_T^2}{4} \right) \right] \\ \times \exp \left[ - (g_2 + g_{2B} b_T^2) \ln \left( \frac{\zeta}{Q_0^2} \right) \frac{b_T^2}{4} \right],$$

*plot in  $k_\perp$  space*



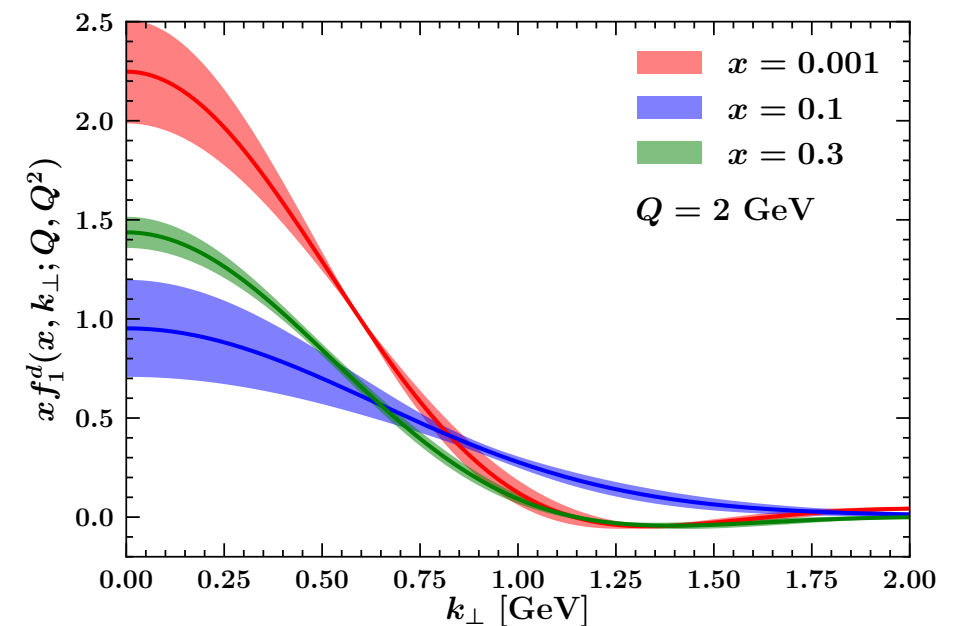
# PV19 – RESULTING TMDS

*expression in  $b_T$  space*

$$f_{\text{NP}}(x, b_T, \zeta) = \left[ \frac{1 - \lambda}{1 + g_1(x) \frac{b_T^2}{4}} + \lambda \exp \left( -g_{1B}(x) \frac{b_T^2}{4} \right) \right] \\ \times \exp \left[ - (g_2 + g_{2B} b_T^2) \ln \left( \frac{\zeta}{Q_0^2} \right) \frac{b_T^2}{4} \right],$$

- q-Guassian + Gaussian

*plot in  $k_\perp$  space*



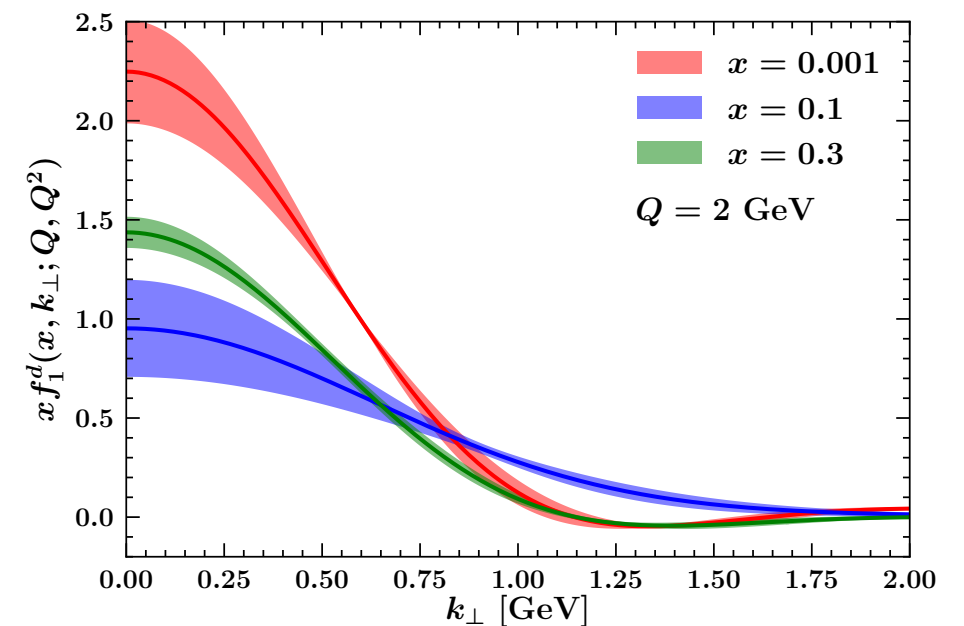
# PV19 – RESULTING TMDS

*expression in  $b_T$  space*

$$f_{\text{NP}}(x, b_T, \zeta) = \left[ \frac{1 - \lambda}{1 + g_1(x) \frac{b_T^2}{4}} + \lambda \exp \left( -g_{1B}(x) \frac{b_T^2}{4} \right) \right] \\ \times \exp \left[ - (g_2 + g_{2B} b_T^2) \ln \left( \frac{\zeta}{Q_0^2} \right) \frac{b_T^2}{4} \right],$$

- q-Guassian + Gaussian
- nontrivial  $x$  dependence

*plot in  $k_\perp$  space*



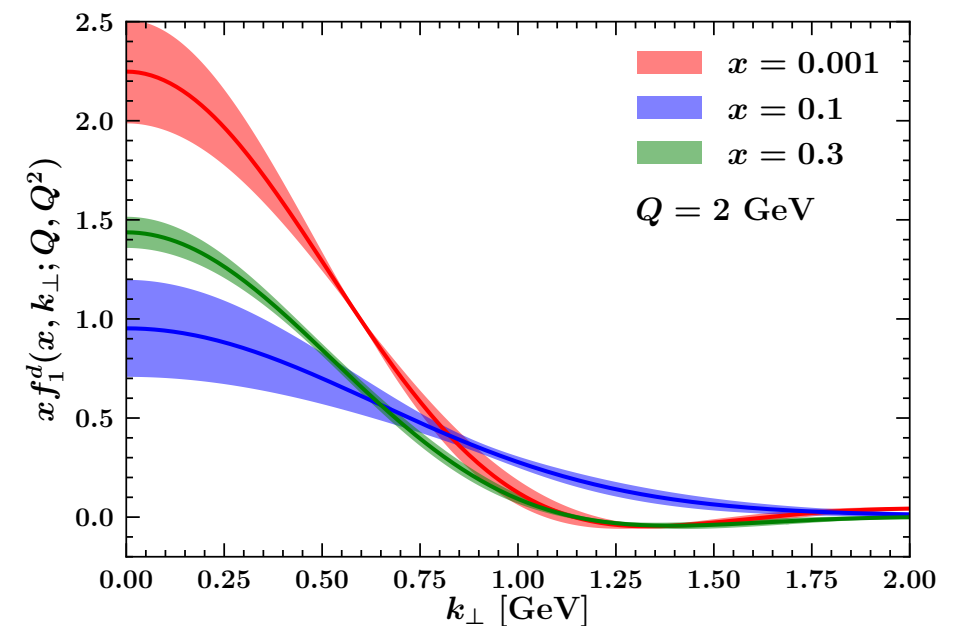
# PV19 – RESULTING TMDS

*expression in  $b_T$  space*

$$f_{\text{NP}}(x, b_T, \zeta) = \left[ \frac{1 - \lambda}{1 + g_1(x) \frac{b_T^2}{4}} + \lambda \exp \left( -g_{1B}(x) \frac{b_T^2}{4} \right) \right] \\ \times \exp \left[ - (g_2 + g_{2B} b_T^2) \ln \left( \frac{\zeta}{Q_0^2} \right) \frac{b_T^2}{4} \right],$$

- q-Gaussian + Gaussian
- nontrivial  $x$  dependence
- no flavor dependence

*plot in  $k_\perp$  space*

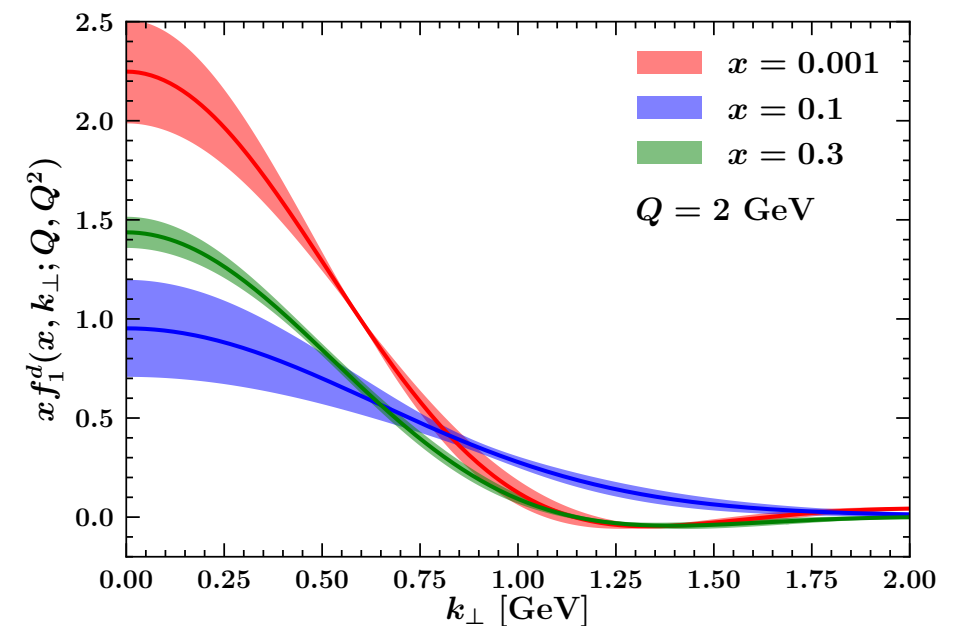


# PV19 – RESULTING TMDS

*expression in  $b_T$  space*

$$f_{\text{NP}}(x, b_T, \zeta) = \left[ \frac{1 - \lambda}{1 + g_1(x) \frac{b_T^2}{4}} + \lambda \exp \left( -g_{1B}(x) \frac{b_T^2}{4} \right) \right] \\ \times \exp \left[ - (g_2 + g_{2B} b_T^2) \ln \left( \frac{\zeta}{Q_0^2} \right) \frac{b_T^2}{4} \right],$$

*plot in  $k_\perp$  space*



- q-Gaussian + Gaussian
- nontrivial  $x$  dependence
- no flavor dependence
- non-Gaussian nonperturbative TMD evolution

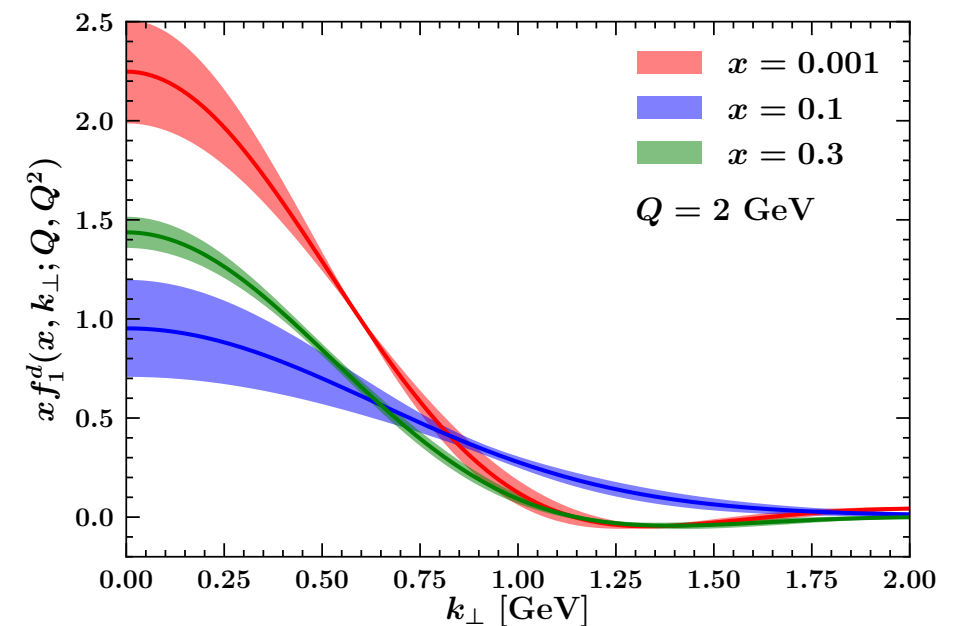


# PV19 – RESULTING TMDS

*expression in  $b_T$  space*

$$f_{\text{NP}}(x, b_T, \zeta) = \left[ \frac{1 - \lambda}{1 + g_1(x) \frac{b_T^2}{4}} + \lambda \exp \left( -g_{1B}(x) \frac{b_T^2}{4} \right) \right] \\ \times \exp \left[ - (g_2 + g_{2B} b_T^2) \ln \left( \frac{\zeta}{Q_0^2} \right) \frac{b_T^2}{4} \right],$$

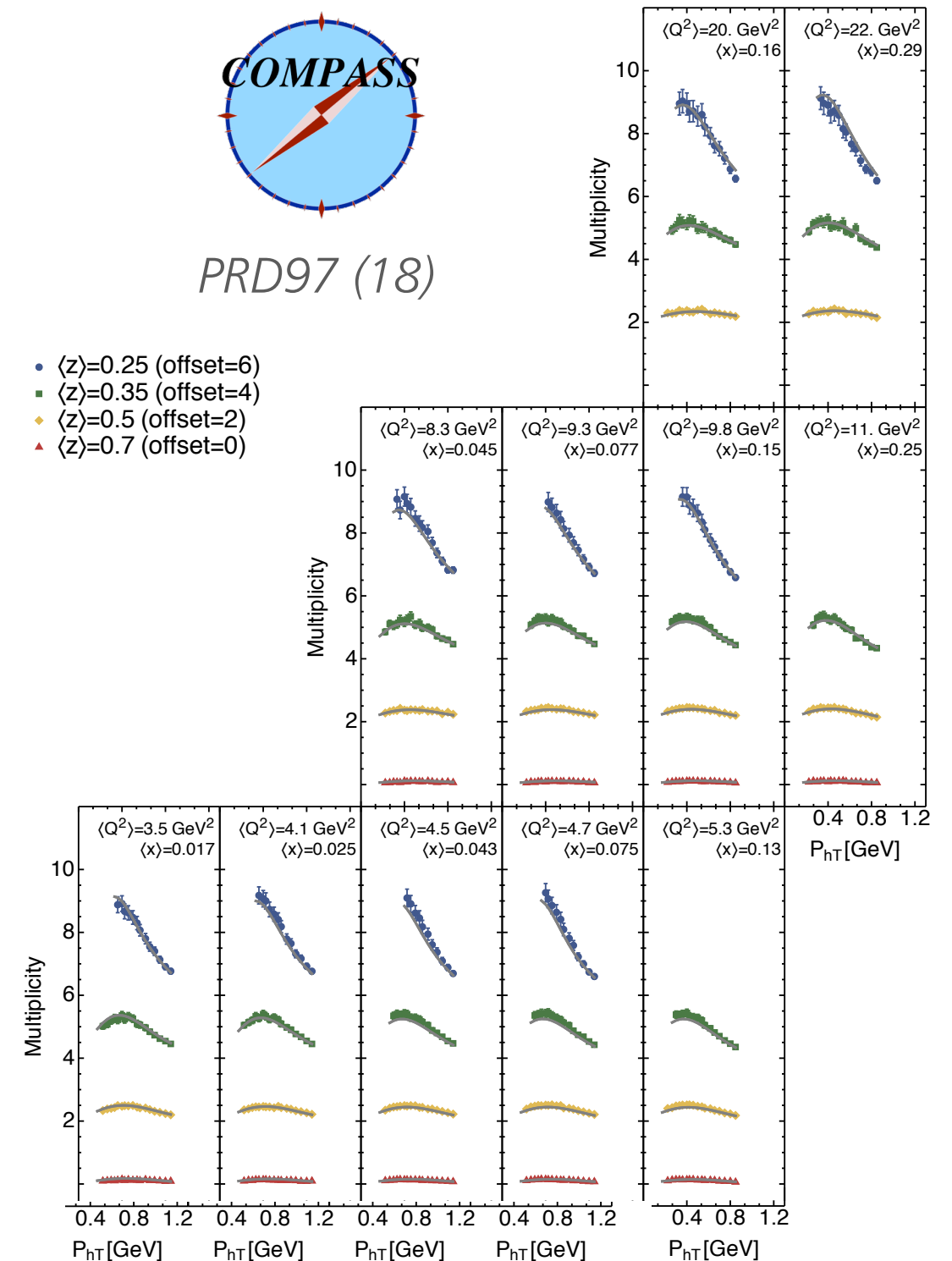
*plot in  $k_\perp$  space*



- q-Gaussian + Gaussian
- nontrivial  $x$  dependence
- no flavor dependence
- non-Gaussian nonperturbative TMD evolution

**9 free parameters**

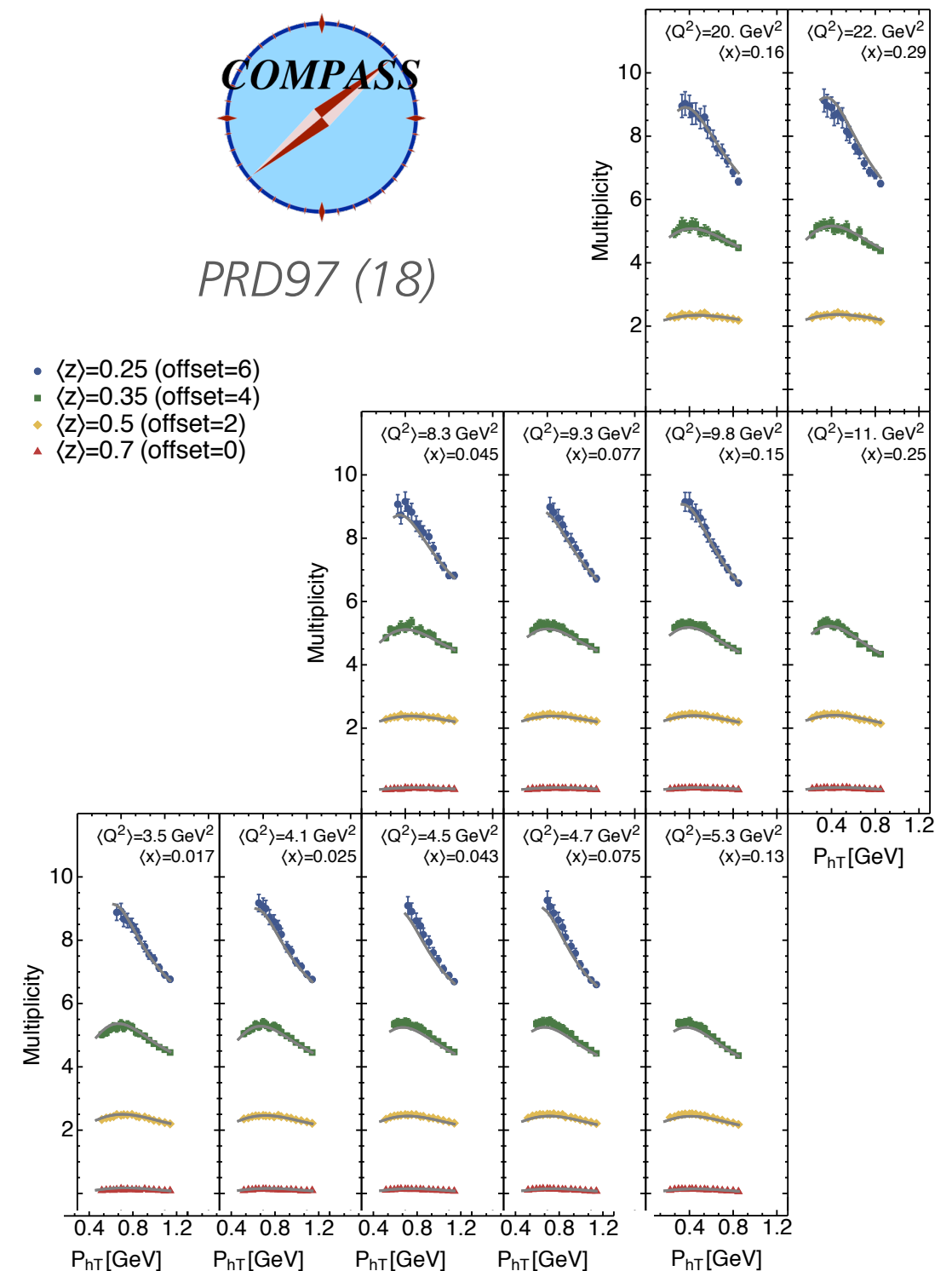
# PROBLEMS WITH SIDIS NORMALIZATION



from F. Piacenza's PhD thesis

# PROBLEMS WITH SIDIS NORMALIZATION

Comparing the PV17 extraction with the new COMPASS data, without normalization factors, at NLL the agreement is very good

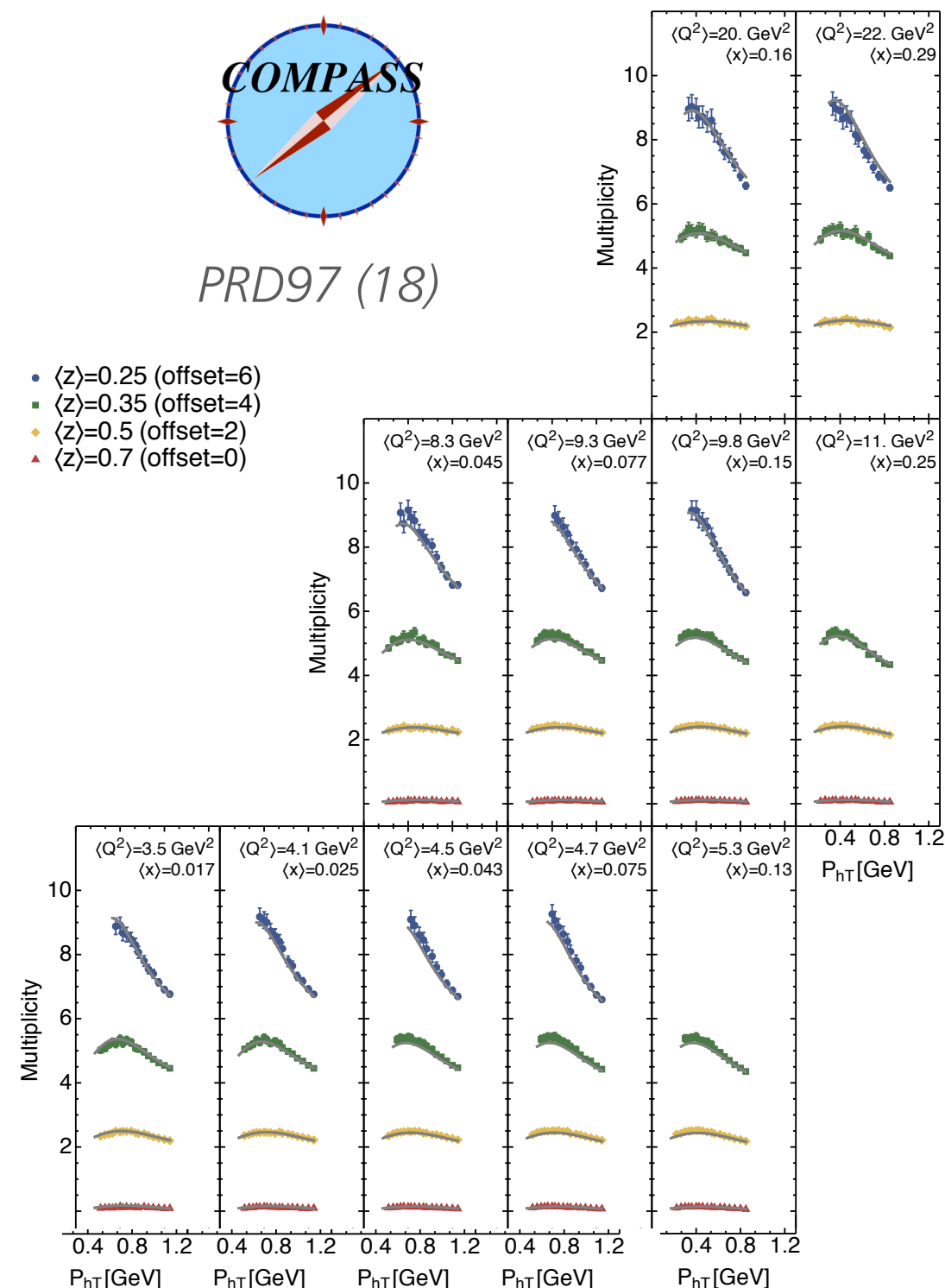


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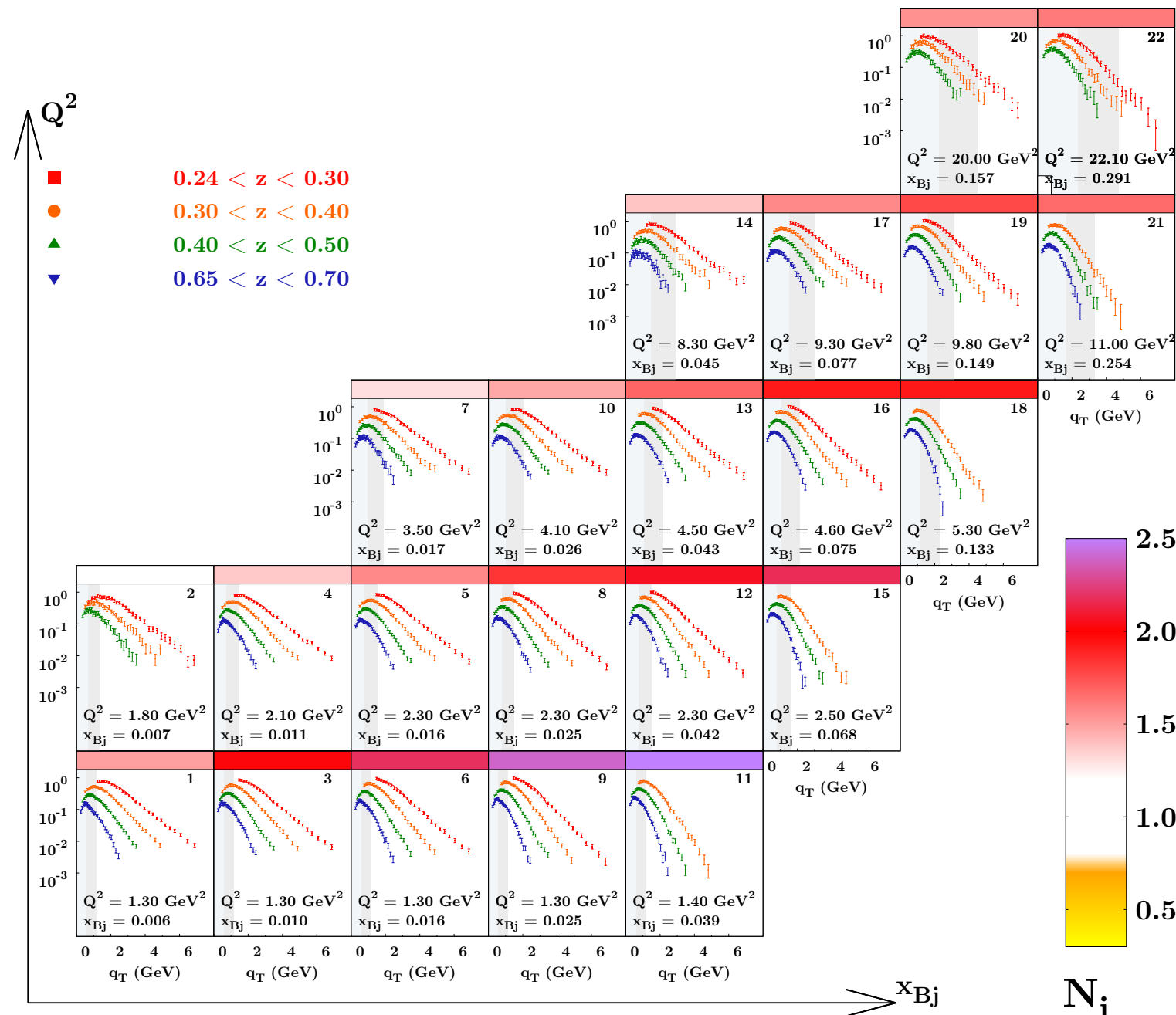
**Going to NLL' or NNLL the situation dramatically worsens!**



from F. Piacenza's PhD thesis

# PROBLEMS WITH SIDIS NORMALIZATION

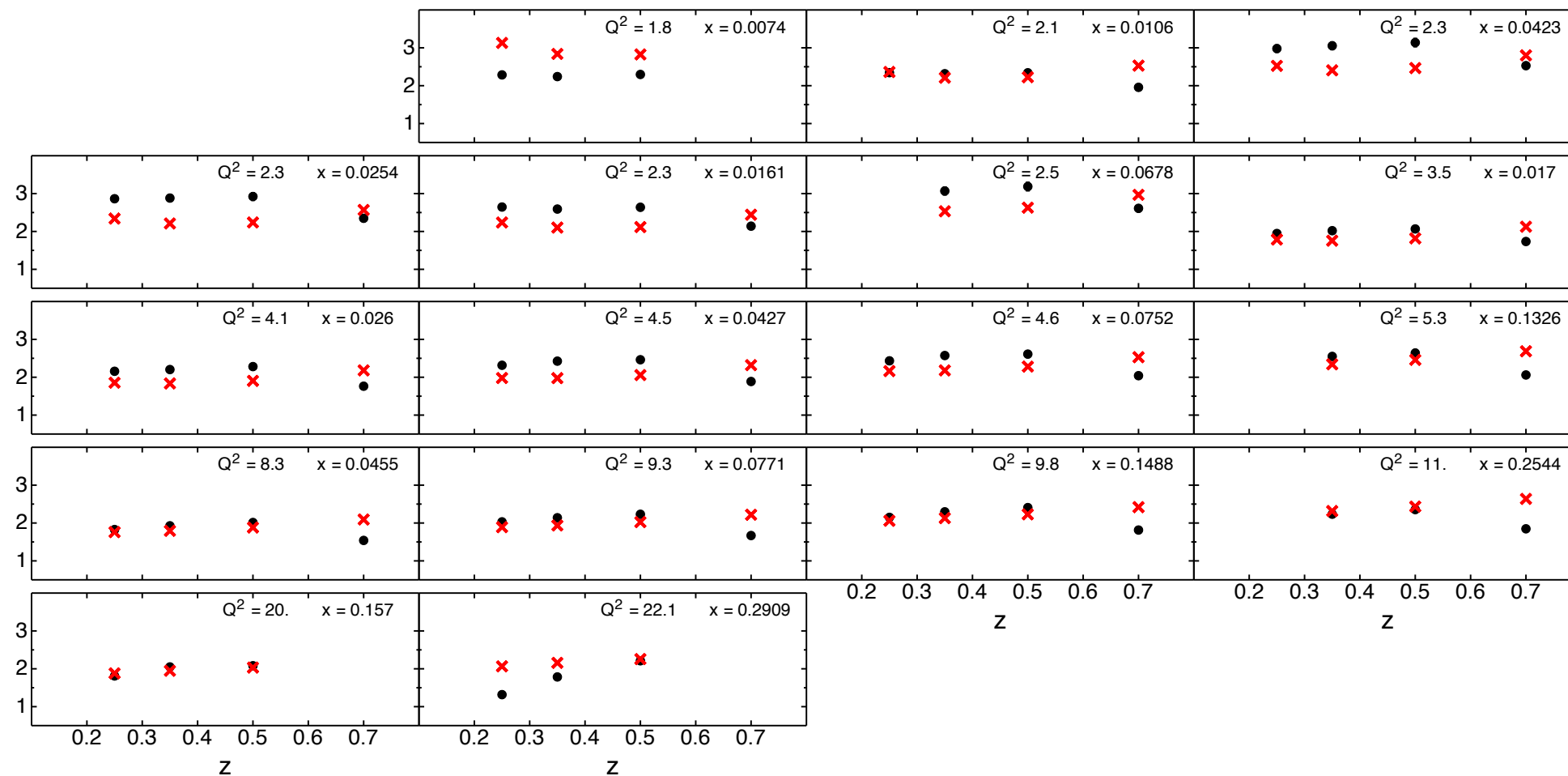
talk by O. Gonzalez at DIS2019



Torino's group also confirmed that large normalisation factors have to be introduced to describe COMPASS data

# PROBLEMS WITH SIDIS NORMALIZATION

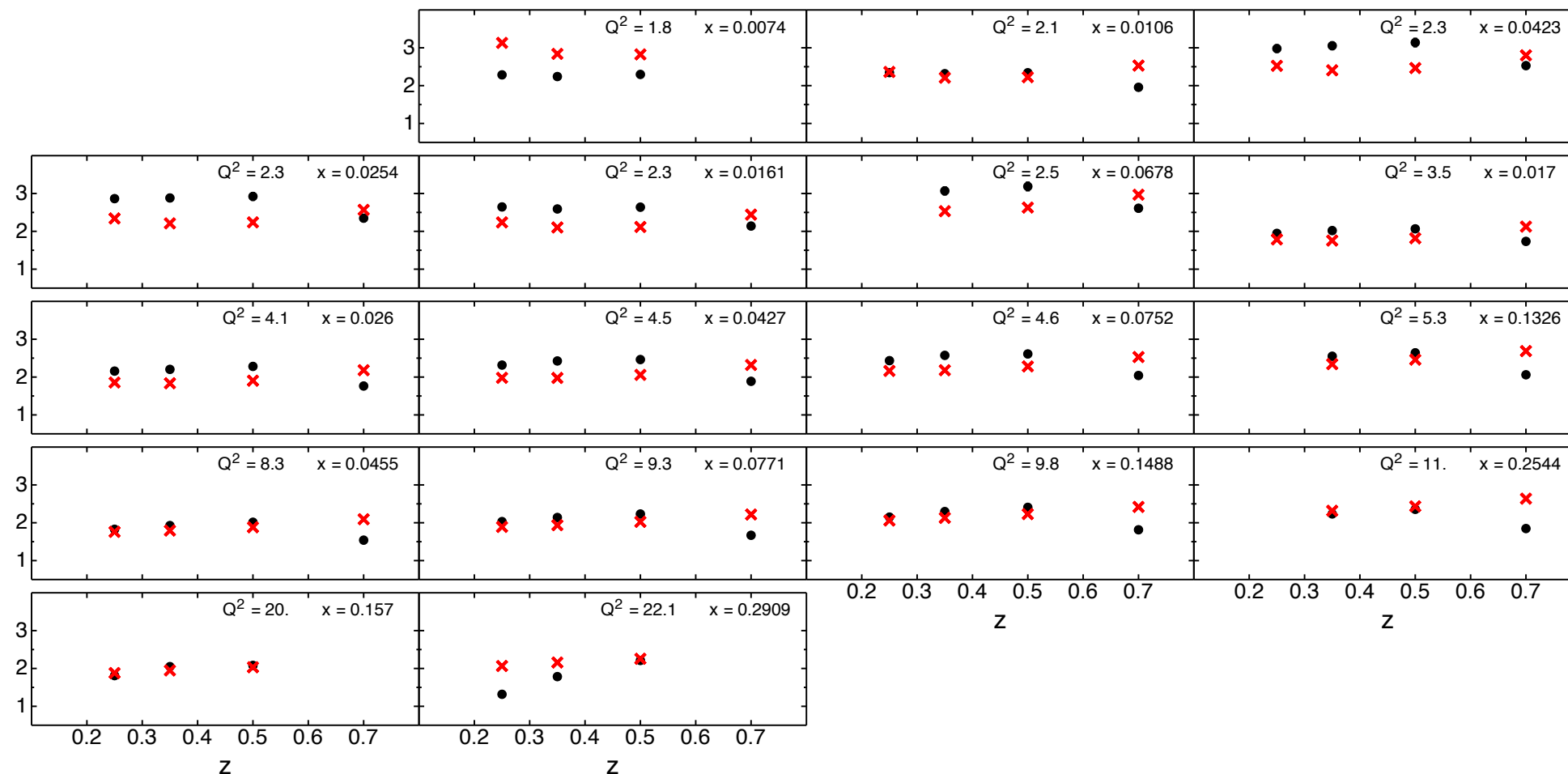
*from F. Piacenza's PhD thesis*



Black dots: large normalisation factors  
required to fit COMPASS multiplicities at NLL'

# PROBLEMS WITH SIDIS NORMALIZATION

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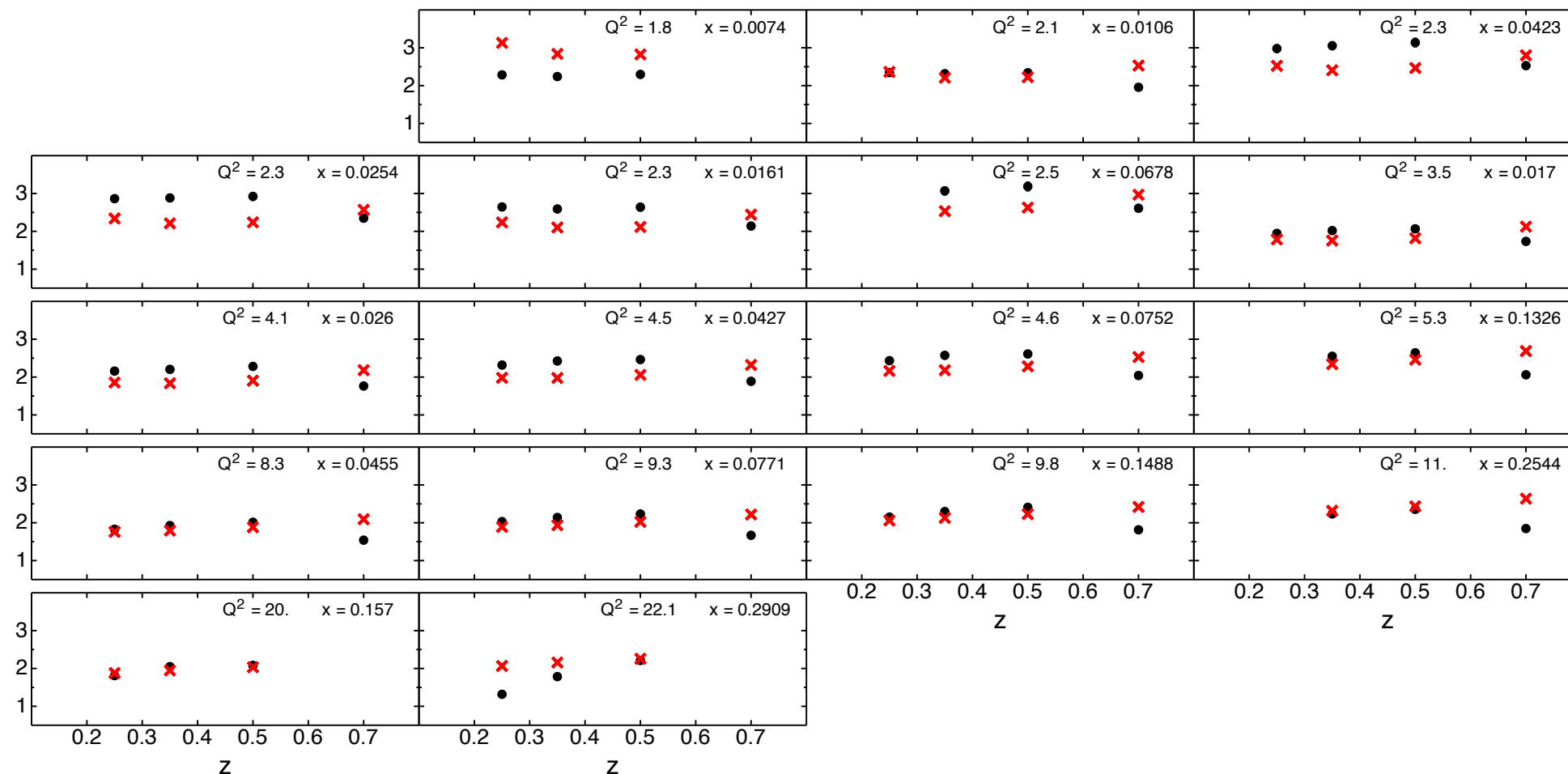
Black dots: large normalisation factors  
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**BAD**



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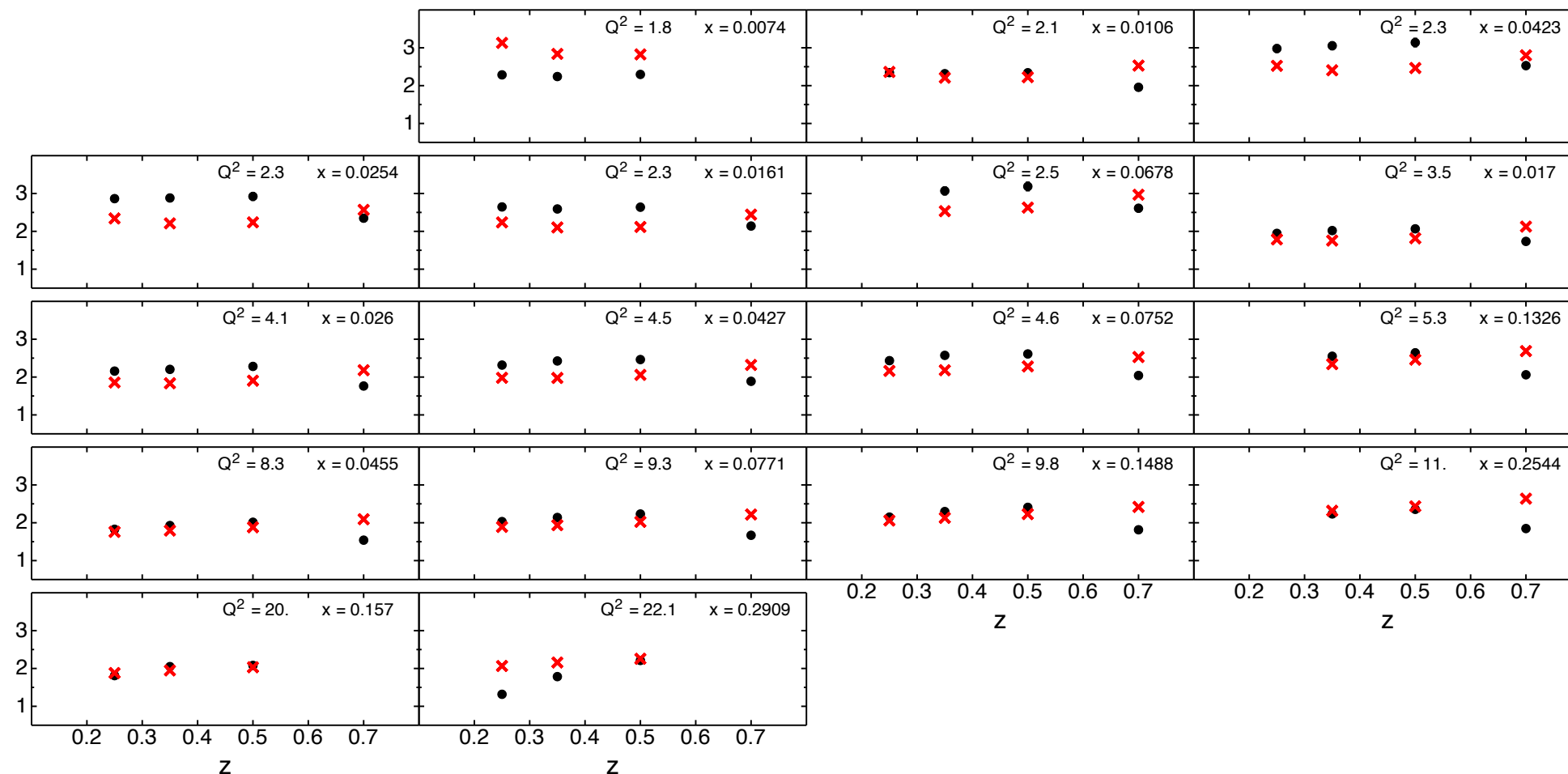
Black dots: large normalisation factors required to fit COMPASS multiplicities at NLL'

**BAD**

Red dots: ratio between collinear formula and integral of TMD part at order  $\alpha_s$

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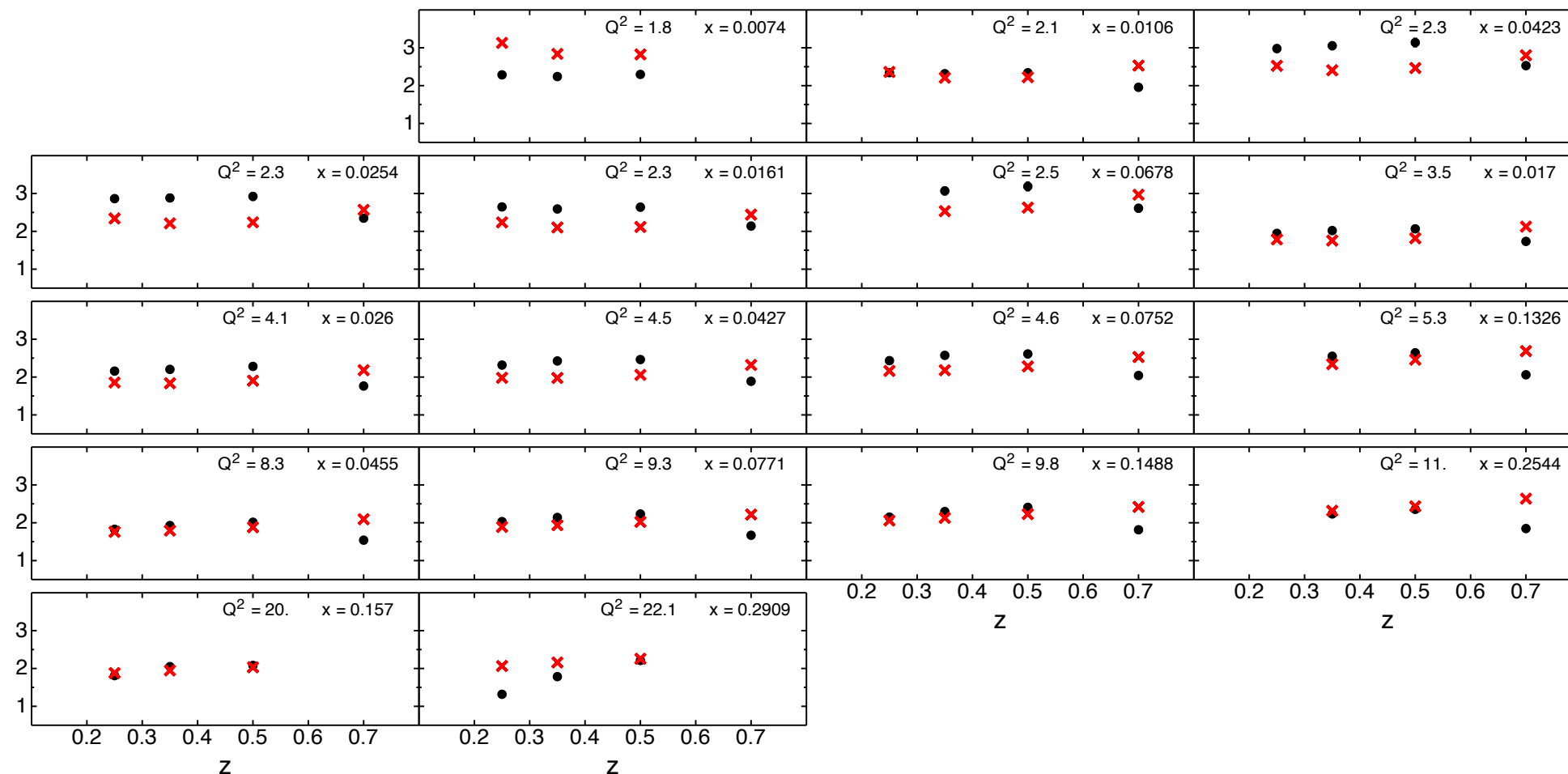
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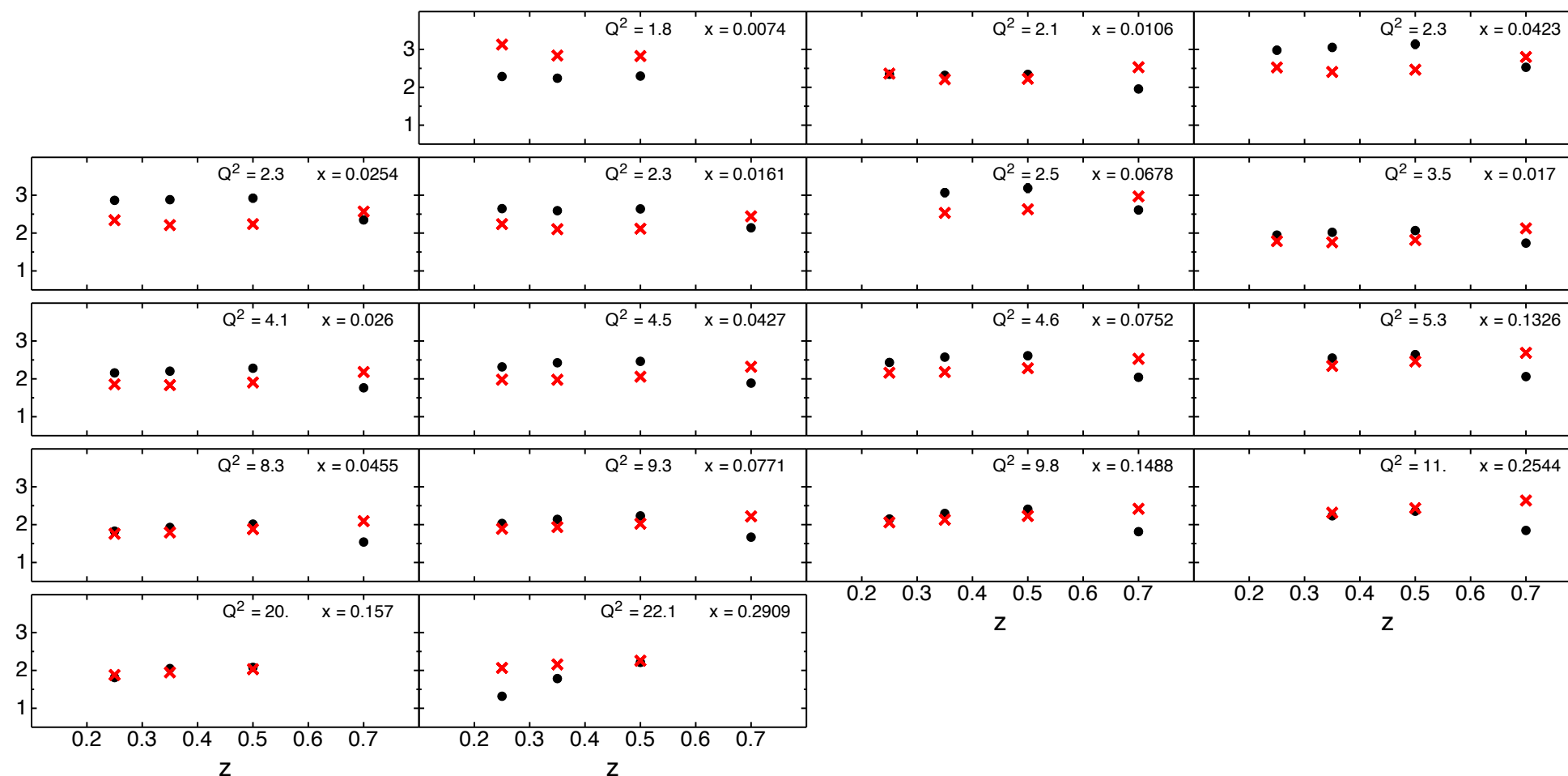
Black and red dots are similar

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from F. Piacenza's PhD thesis



Black dots: large normalisation factors required to fit COMPASS multiplicities at NLL'

**BAD**

Black and red dots are similar

Red dots: ratio between collinear formula and integral of TMD part at order  $\alpha_s$

**BAD**

**GOOD?**

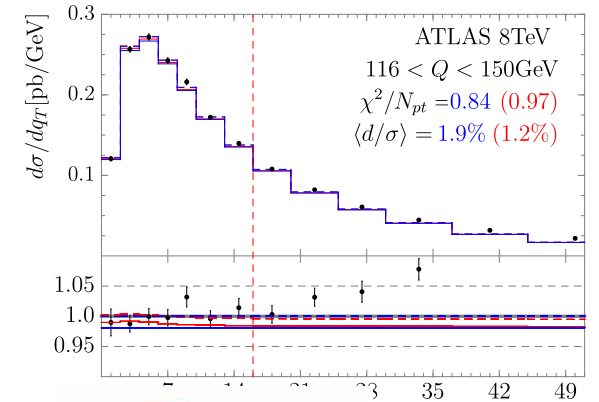
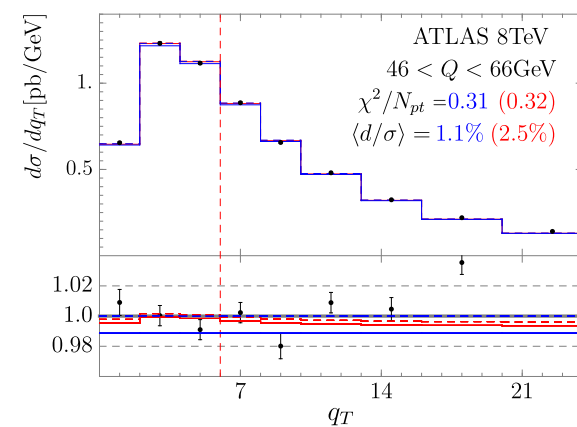
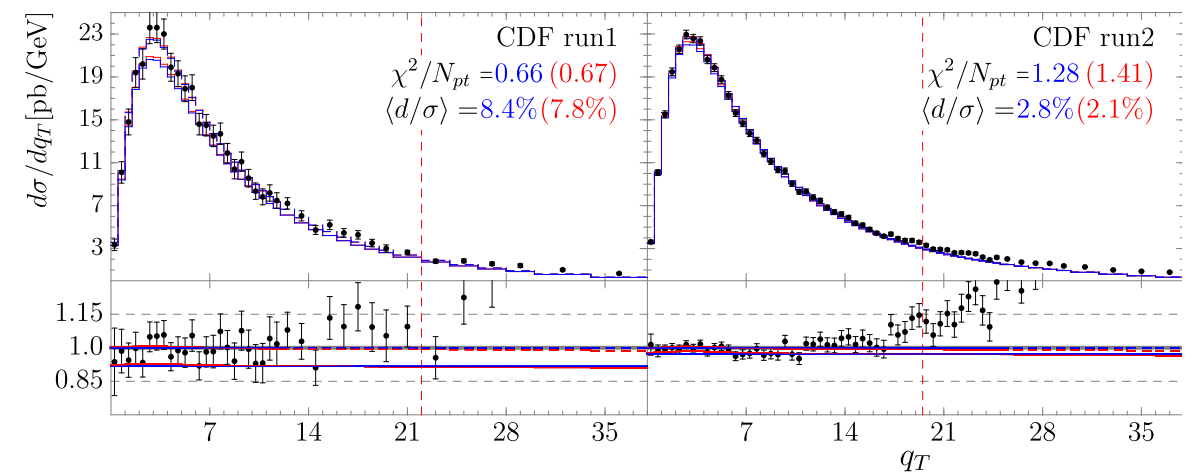
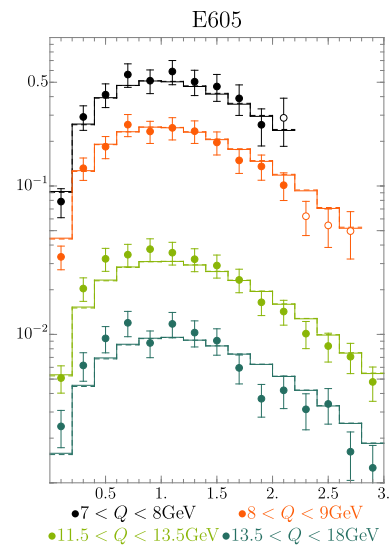
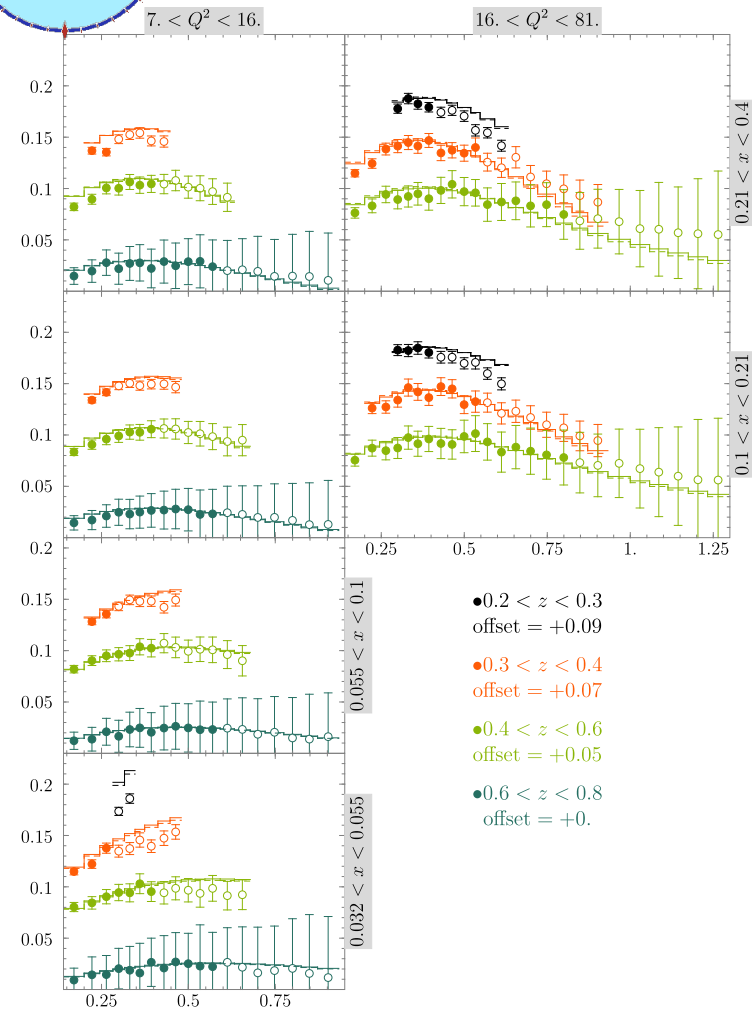
# THE SCIMEMI-VLADIMIROV 19 EXTRACTION

Scimemi, Vladimirov, arXiv:1912.06532



$$z^2 \times M(z, p_T)$$

$$d \rightarrow h^+$$



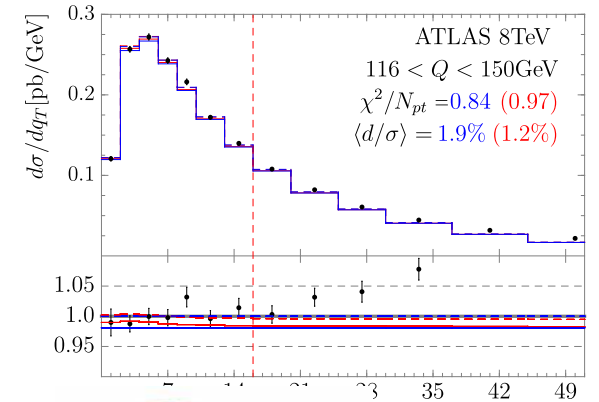
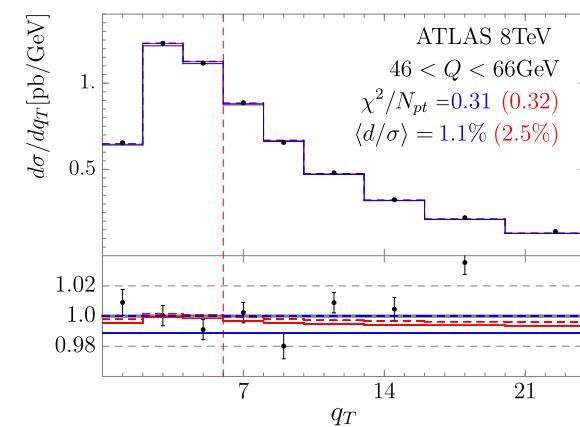
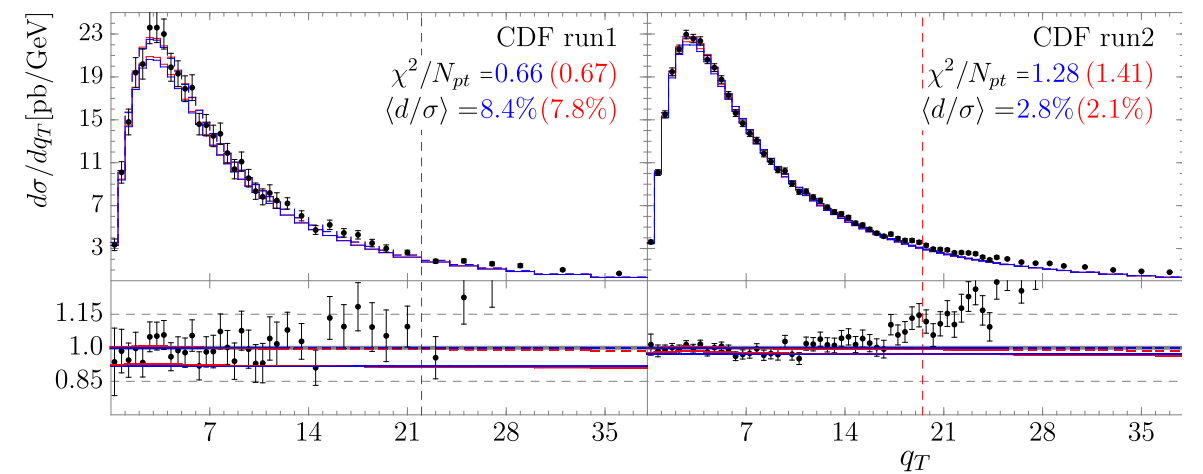
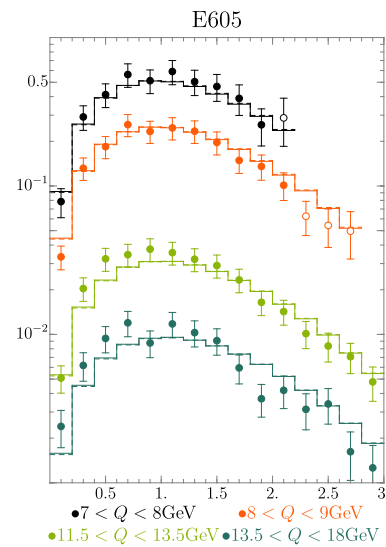
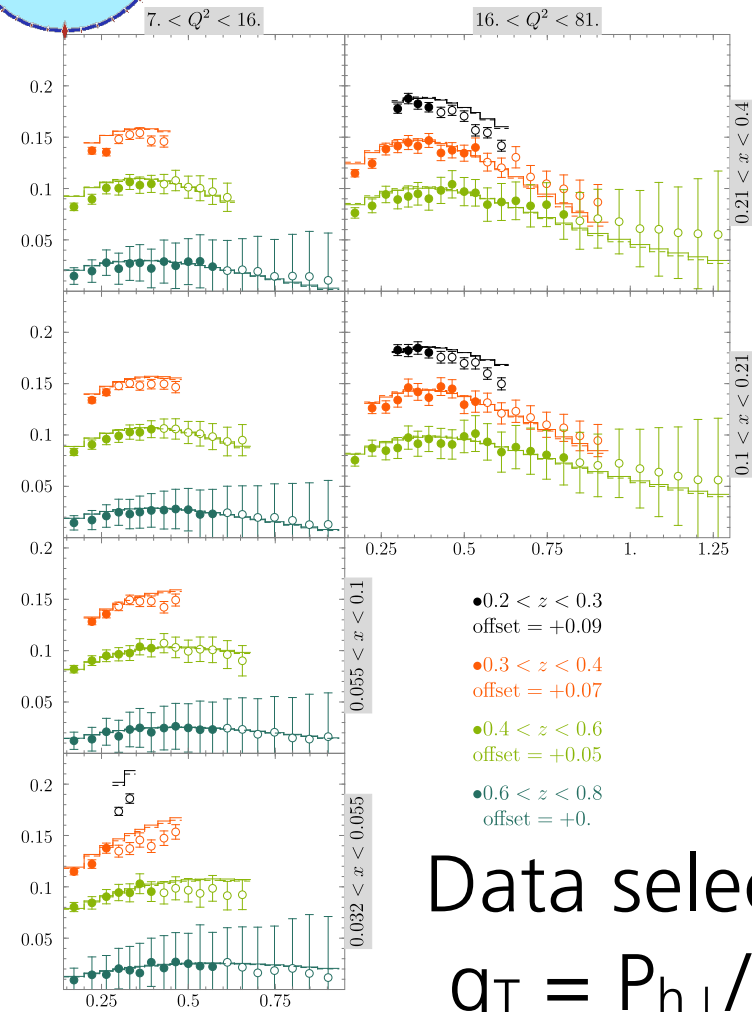
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Data selection:  
 $q_T = P_{h\perp}/z < 0.25 Q$



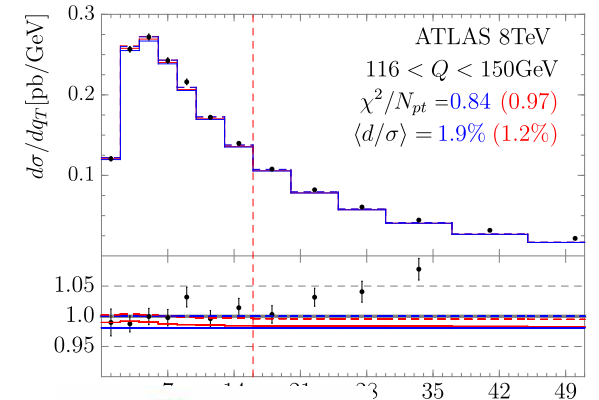
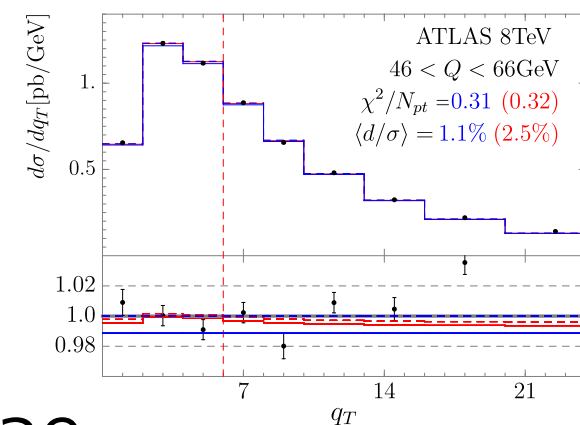
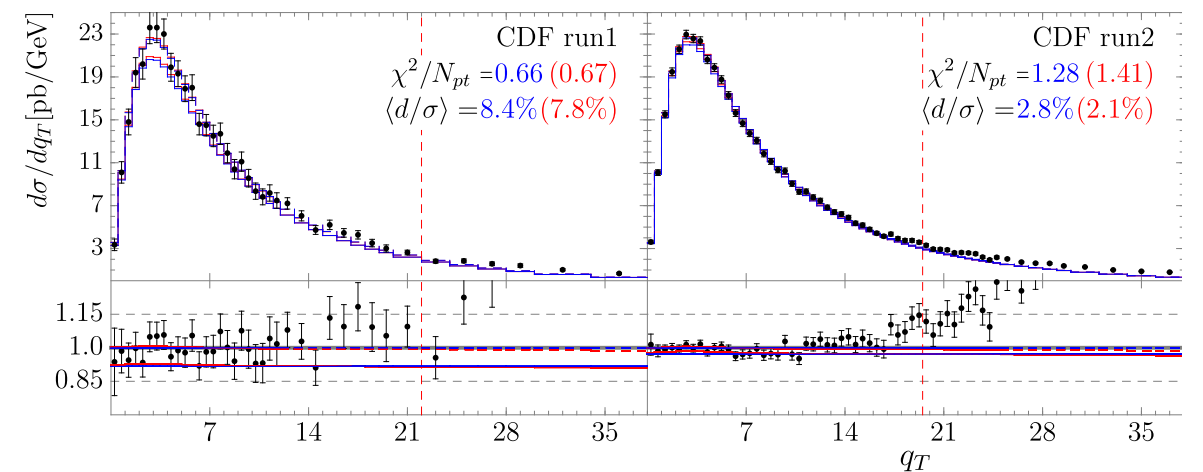
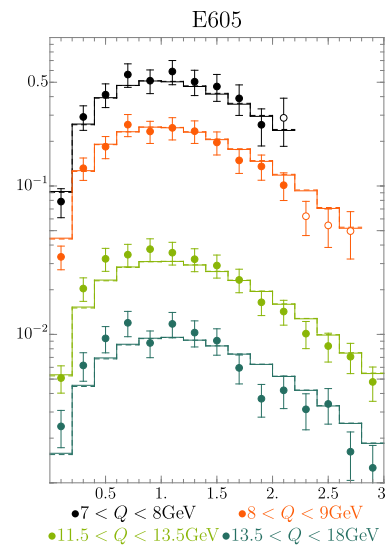
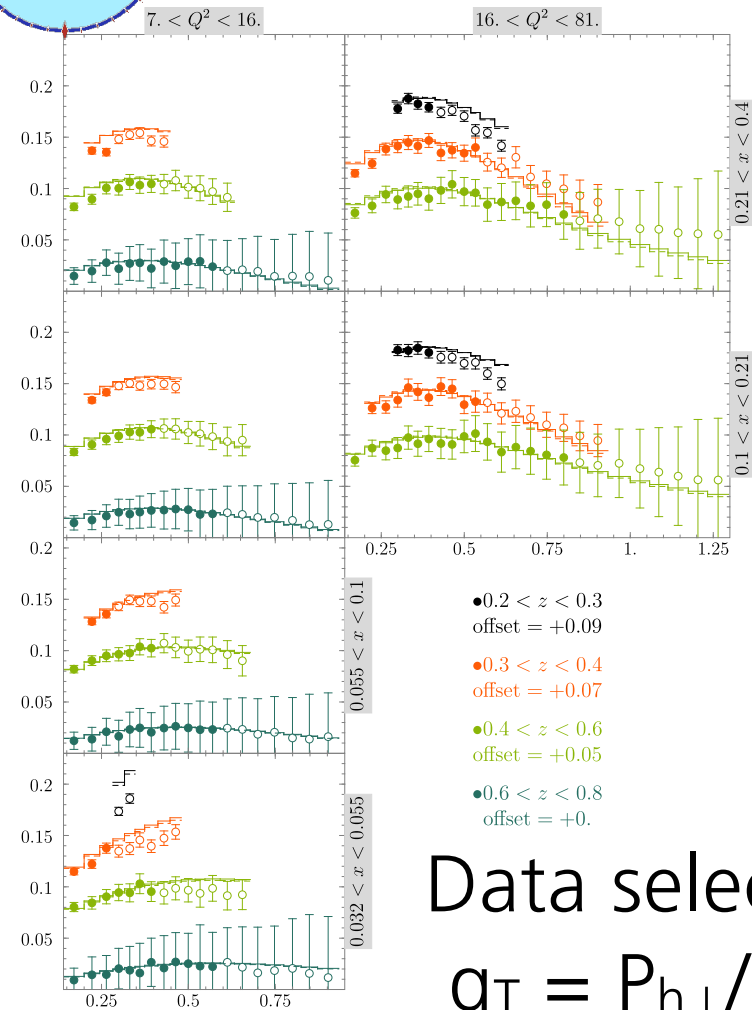
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$$q_T = P_{h\perp}/z < 0.25 Q$$

Number of data points: 1039

$$\text{Global } \chi^2/\text{dof} = 1.06$$





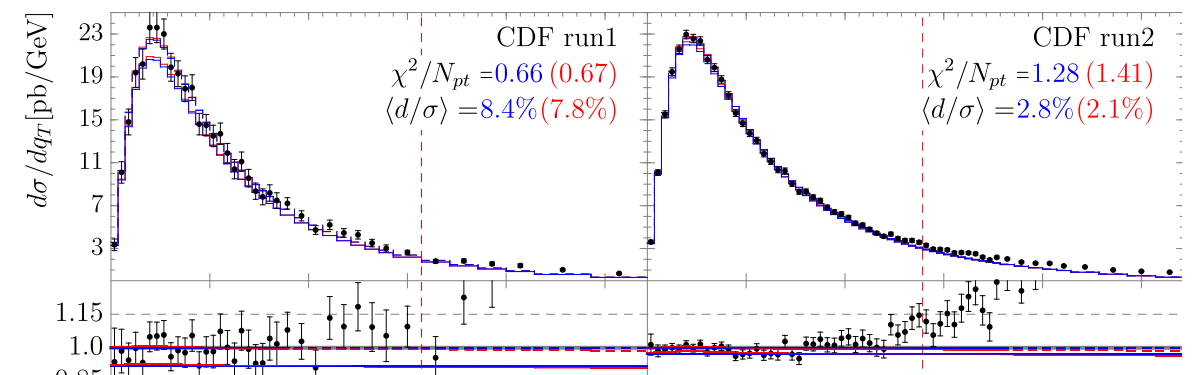
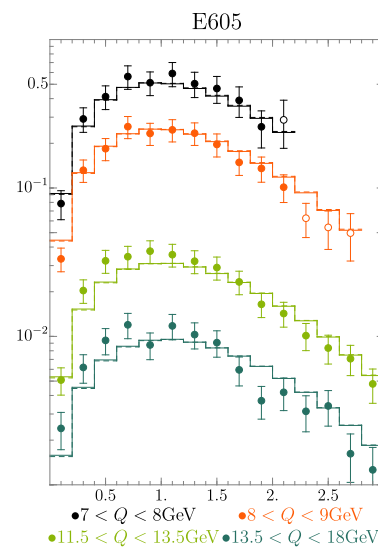
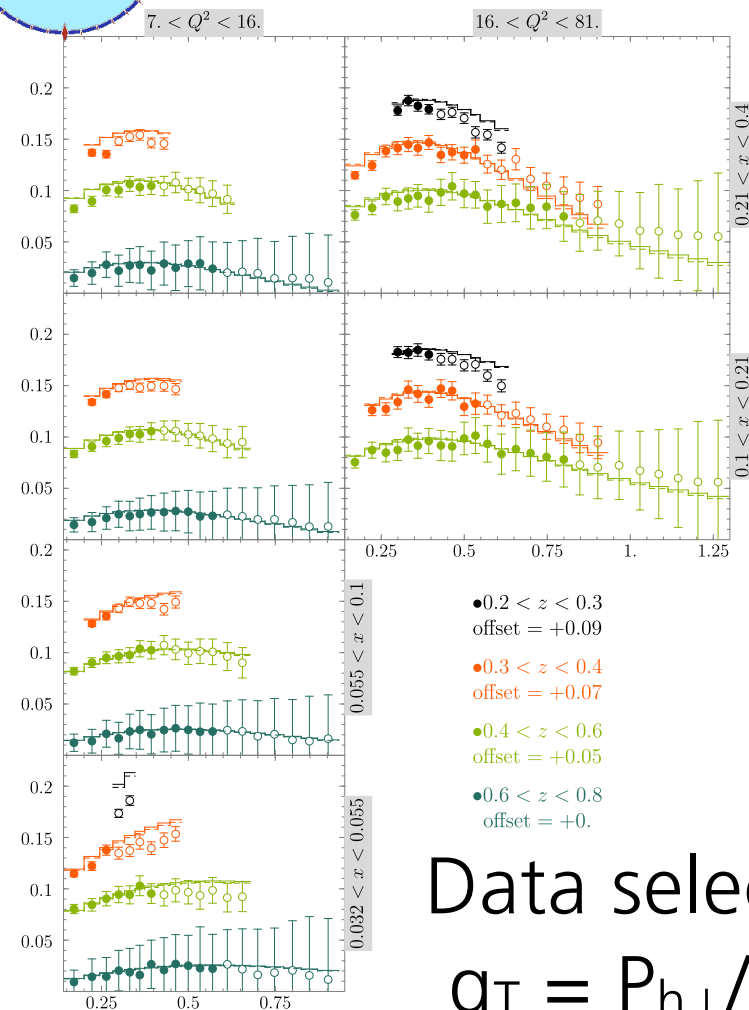
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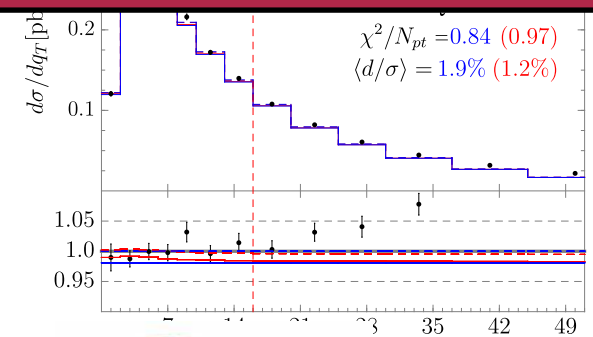
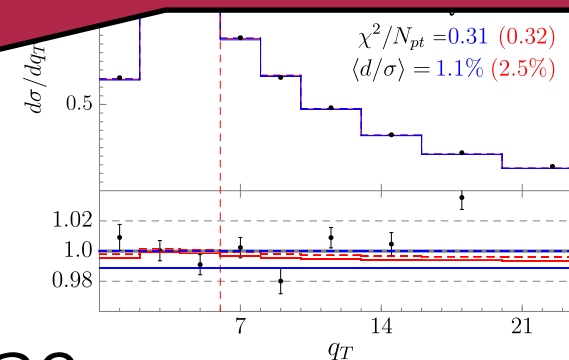


$$z^2 \times M(z, p_T)$$

$$d \rightarrow h^+$$



**SV19: first SIDIS+DY fit at NNLL, without normalization problems!**



Data selection:

$$q_T = P_{h\perp}/z < 0.25 Q$$

Number of data points: 1039

$$\text{Global } \chi^2/\text{dof} = 1.06$$



# SV19 – RESULTING TMDS

---

*Scimemi, Vladimirov, arXiv:1912.06532*

*expression in  $b_T$  space*

$$f_{NP}(x, b) = \exp \left( - \frac{\lambda_1(1-x) + \lambda_2 x + x(1-x)\lambda_5}{\sqrt{1 + \lambda_3 x^{\lambda_4} b^2}} b^2 \right)$$

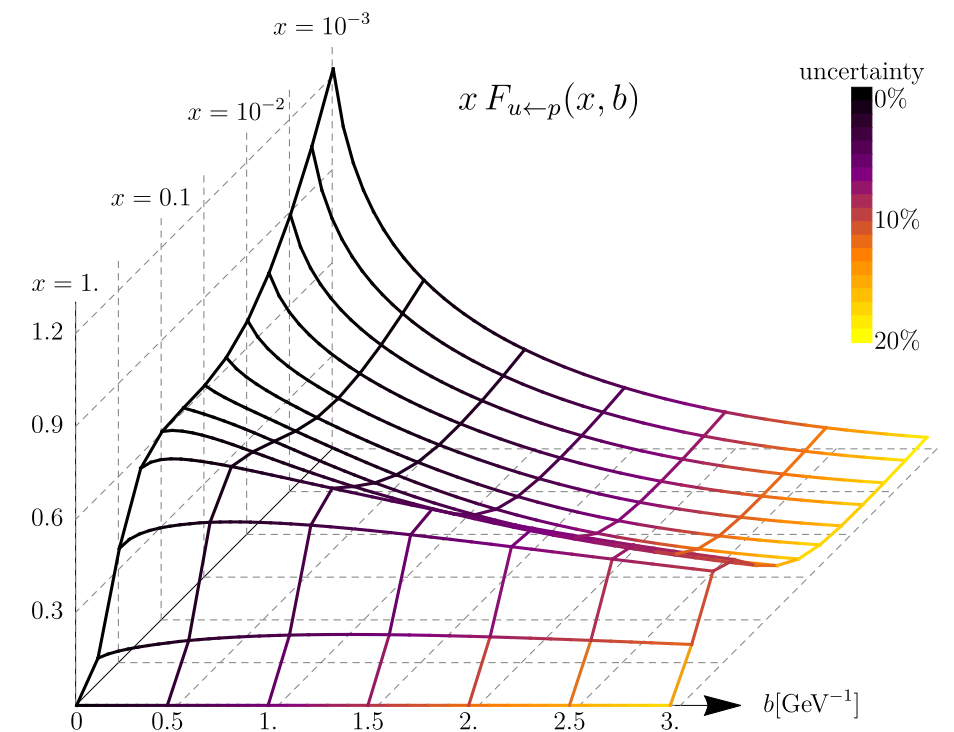
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plot in  $b_T$  space



# SV19 – RESULTING TMDS

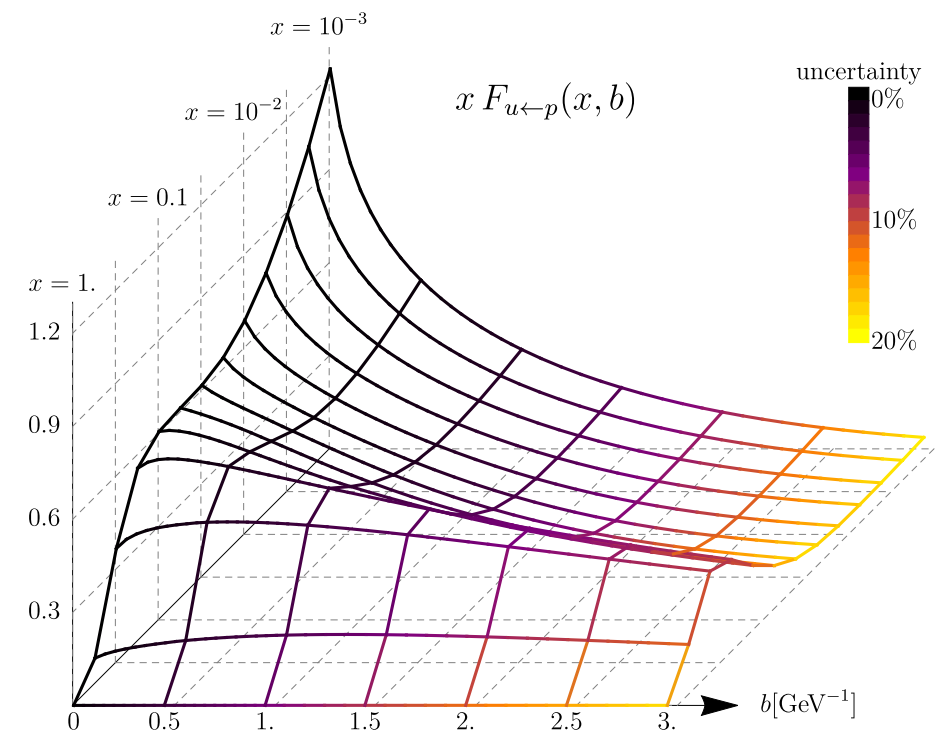
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- Gaussian at low  $b_T$ , exponential at high  $b_T$

plot in  $b_T$  space



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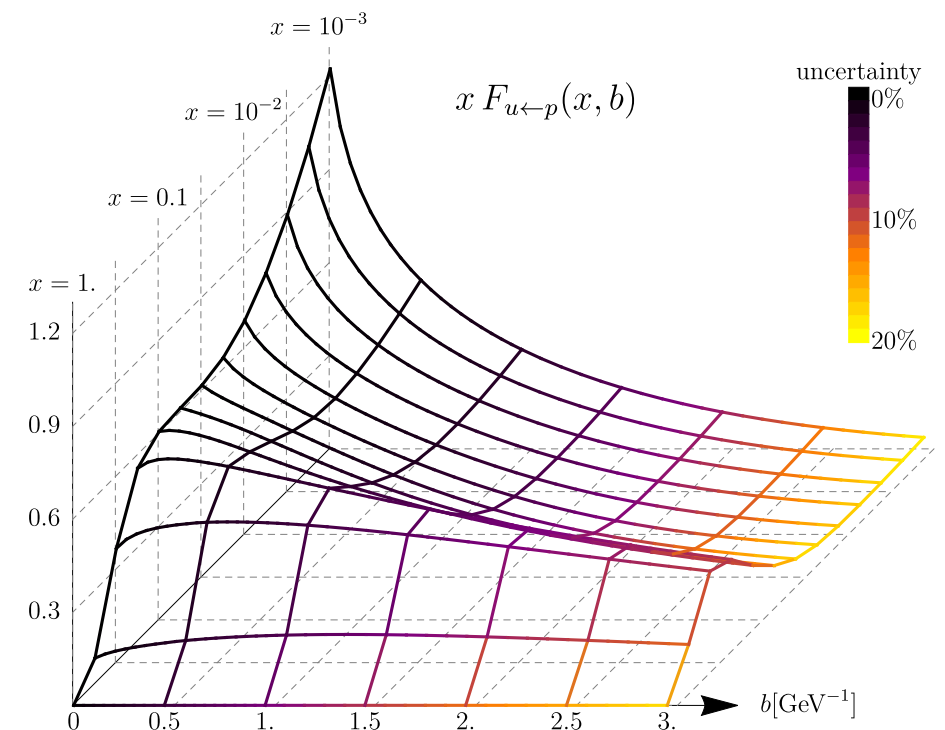
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- nontrivial  $x$  dependence

plot in  $b_T$  space



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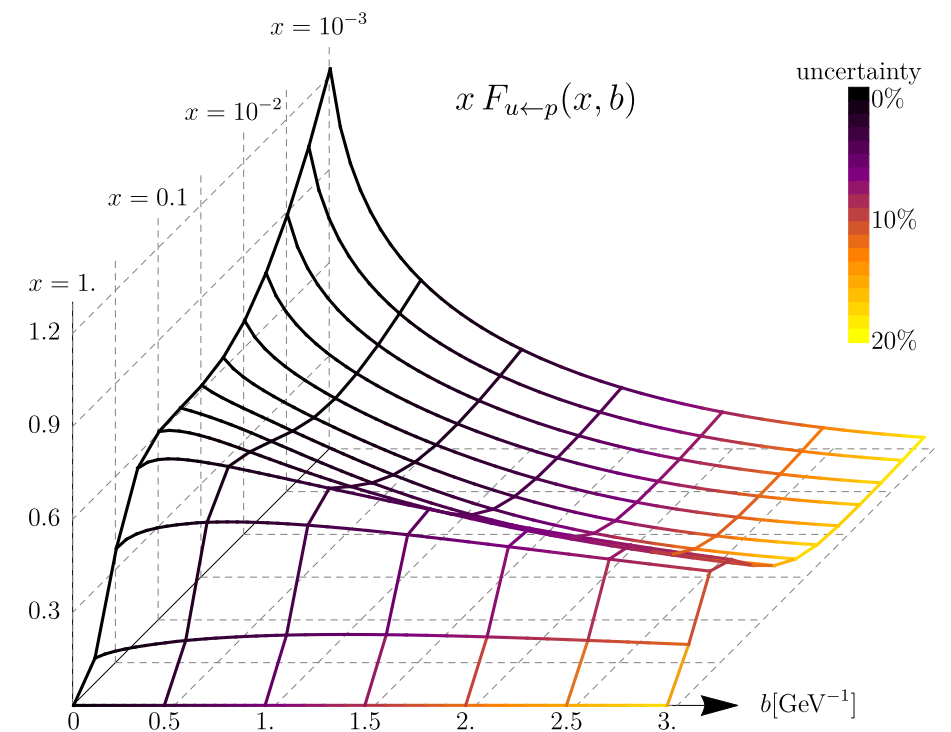
Scimemi, Vladimirov, arXiv:1912.06532

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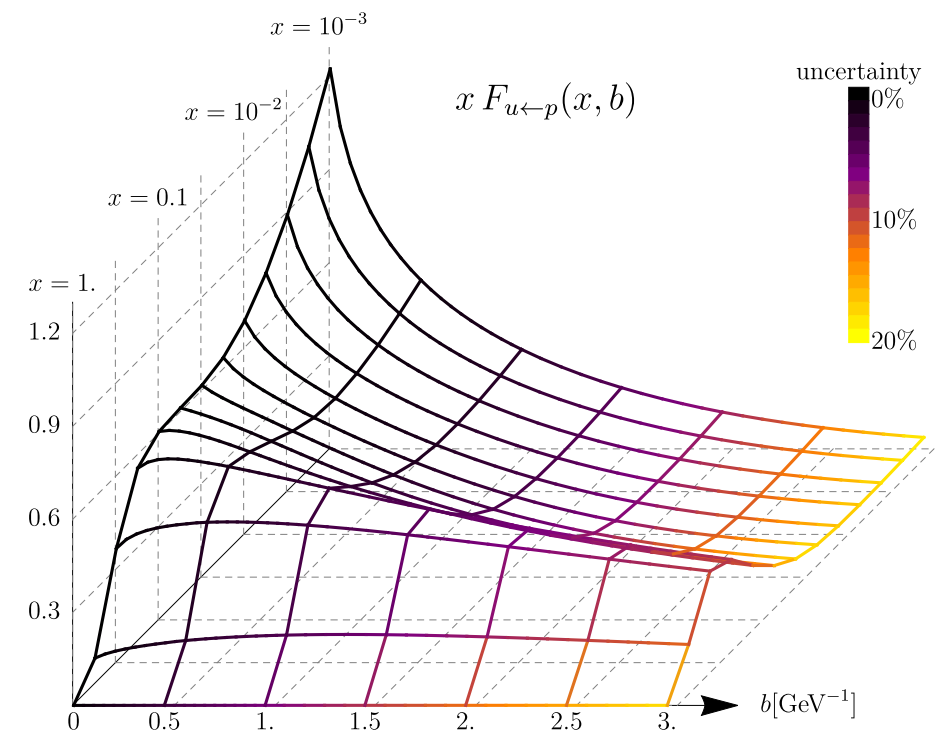
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- Gaussian at low  $b_T$ , exponential at high  $b_T$
- nontrivial  $x$  dependence
- no flavor dependence
- Rapidity anomalous dimension (related to nonperturbative TMD evolution)

$$\mathcal{D}(\mu, b) = \mathcal{D}_{\text{resum}}(\mu, b^*(b)) + c_0 b b^*(b),$$

plot in  $b_T$  space





# SV19 – RESULTING TMDS

Scimemi, Vladimirov, arXiv:1912.06532

expression in  $b_T$  space

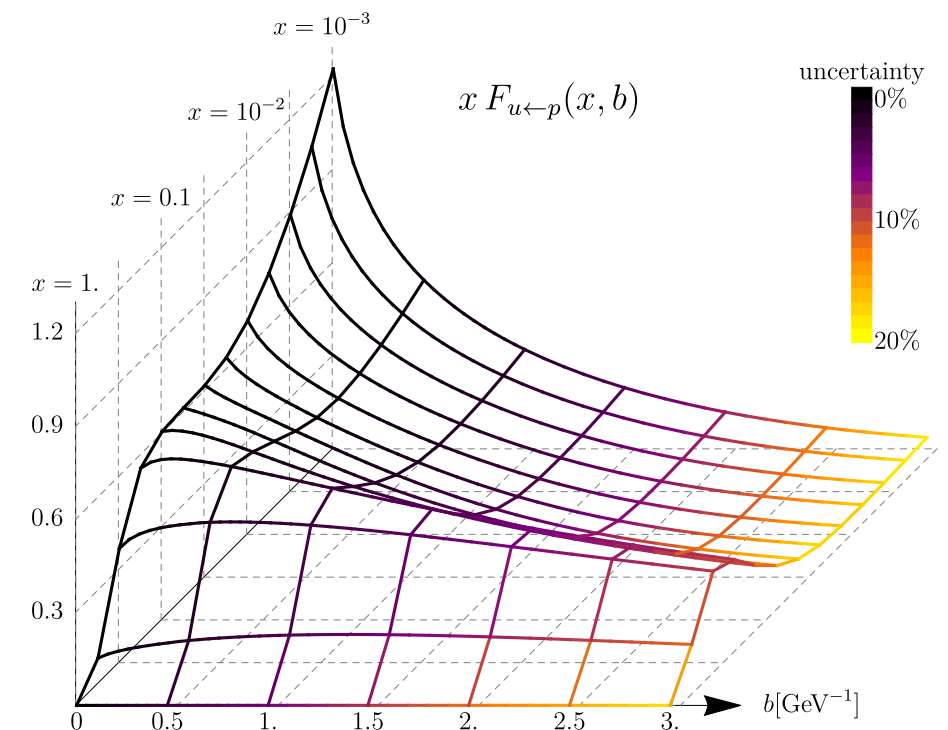
$$f_{NP}(x, b) = \exp \left( - \frac{\lambda_1(1-x) + \lambda_2 x + x(1-x)\lambda_5}{\sqrt{1 + \lambda_3 x^{\lambda_4} b^2}} b^2 \right)$$

- Gaussian at low  $b_T$ , exponential at high  $b_T$
- nontrivial  $x$  dependence
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- Rapidity anomalous dimension (related to nonperturbative TMD evolution)

$$\mathcal{D}(\mu, b) = \mathcal{D}_{\text{resum}}(\mu, b^*(b)) + c_0 b b^*(b),$$

$$D_{NP}(x, b) = \exp \left( - \frac{\eta_1 z + \eta_2(1-z)}{\sqrt{1 + \eta_3 (b/z)^2}} \frac{b^2}{z^2} \right) \left( 1 + \eta_4 \frac{b^2}{z^2} \right),$$

plot in  $b_T$  space



TMD Frag. Func.

# SV19 – RESULTING TMDS

Scimemi, Vladimirov, arXiv:1912.06532

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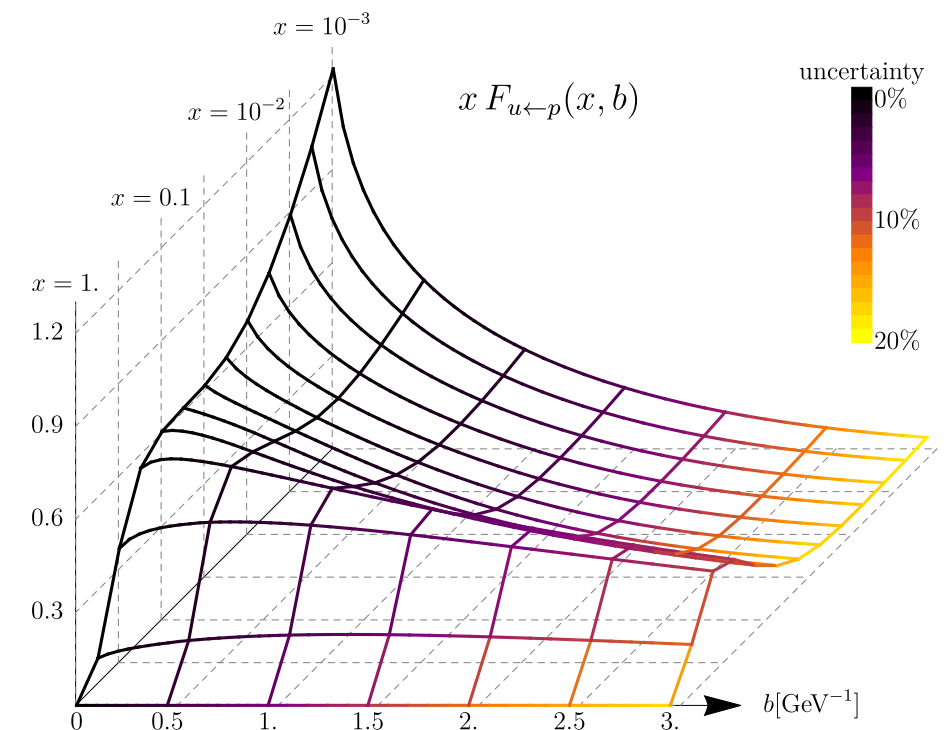
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plot in  $b_T$  space



TMD Frag. Func.

**11 free parameters**

# GENERAL CONSIDERATIONS

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- Not easy to perform direct comparison due to different formalisms employed

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# GENERAL CONSIDERATIONS

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- Not easy to perform direct comparison due to different formalisms employed
- In all extractions, simple Gaussians are not sufficient
- Nontrivial  $x$ -dependence is required
- No flavor dependence is needed for the moment (note however that some flavor dependence is already generated by the collinear PDFs)



# AVAILABLE TOOLS: NANGA PARBAT

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*<https://github.com/vbertone/NangaParbat>*



## Nanga Parbat: a TMD fitting framework

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Nanga Parbat is a fitting framework aimed at the determination of the non-perturbative component of TMD distributions.

### Download

---

You can obtain NangaParbat directly from the github repository:

<https://github.com/vbertone/NangaParbat/releases>

For the last development branch you can clone the master code:

```
git clone git@github.com:vbertone/NangaParbat.git
```

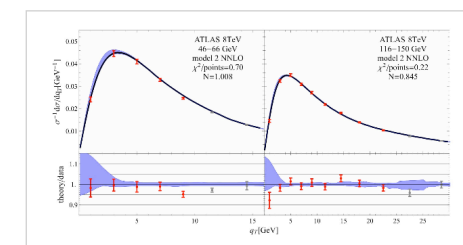
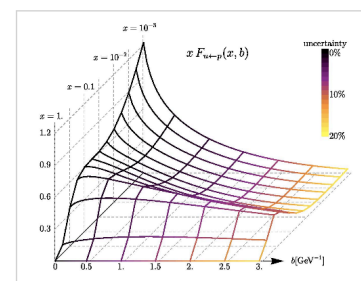
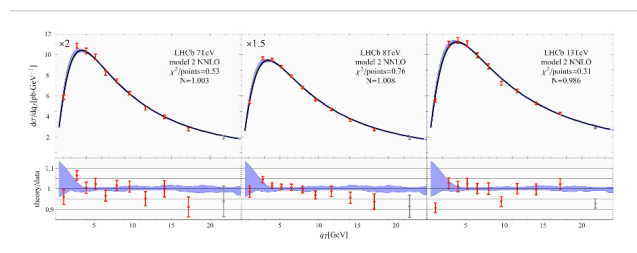
If you instead want to download a specific tag:

---

# AVAILABLE TOOLS: ARTEMIDE

<https://teorica.fis.ucm.es/artemide/>

## arTeMiDe



### News



**12 Dec 2019:** Version 2.02 released (+manual update).

**23 Feb 2019:** Version 1.4 released (+manual update).

**21 Jan 2019:** Artemide now has a [repository](#).

[Archive of older links/news.](#)

### Articles, presentations & supplementary materials



[Extra pictures for the paper arXiv:1902.08474](#)

[Seminar of A.Vladimirov in Pavia 2018 on TMD evolution.](#)

[Link to the text in Inspire.](#)

[Archive of older links/news.](#)

### Download



**[Recent version/release can be found in repository.](#)**

### About us & Contacts



If you have found mistakes, or have suggestions/questions, please, contact us.

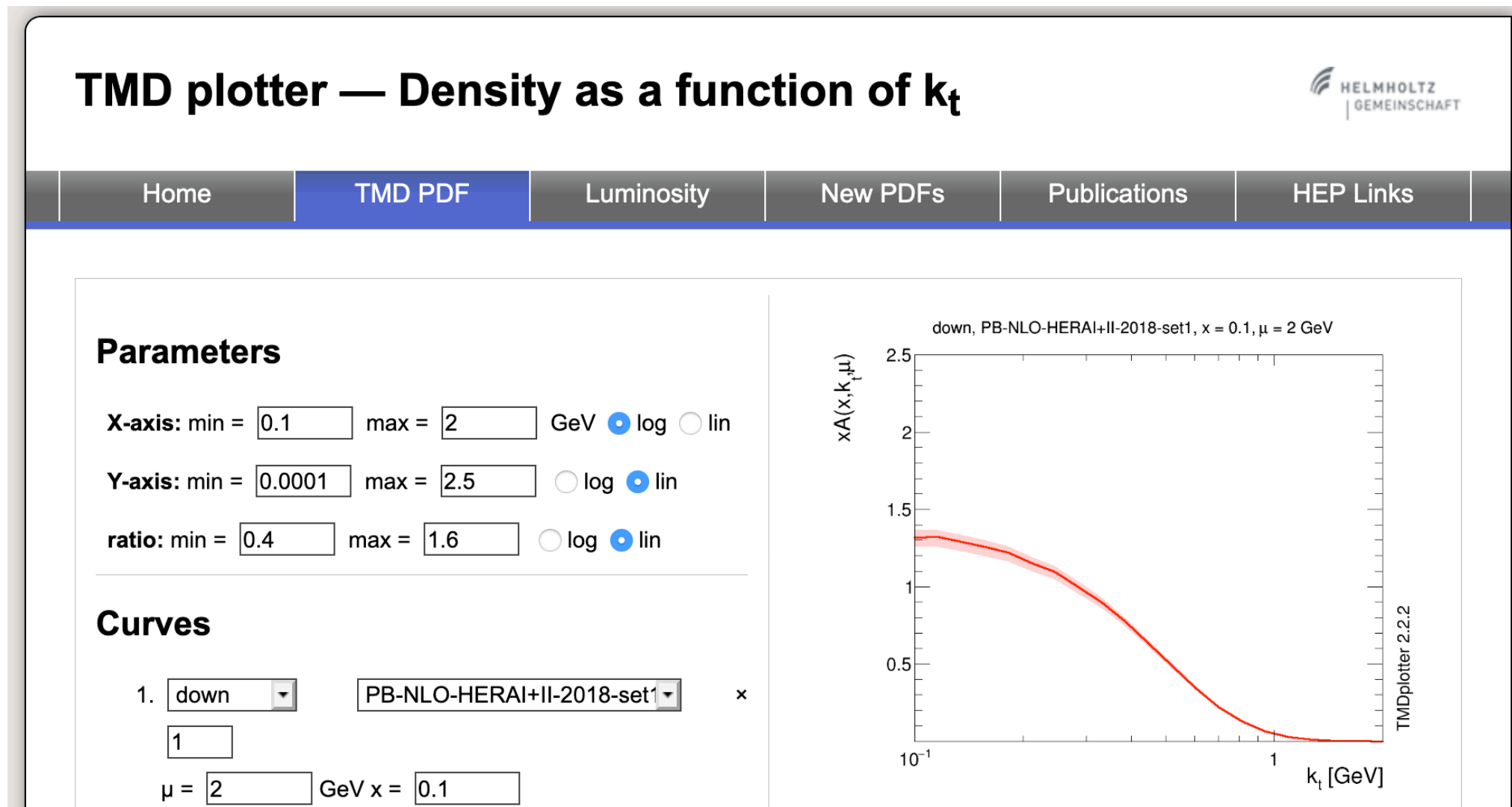
Some extra materials can be found on [Alexey's web-page](#)

**Alexey Vladimirov** [Alexey.Vladimirov@physik.uni-regensburg.de](mailto:Alexey.Vladimirov@physik.uni-regensburg.de)

**Ignazio Scimemi** [ignazios@fis.ucm.es](mailto:ignazios@fis.ucm.es)

# TMDLIB AND TMDPLOTTER

<https://tmdlib.hepforge.org/>



Soon more TMD parametrisation will be available

# TOOLS USED FOR DRELL-YAN PREDICTIONS

---

*V. Bertone's talk at LHC EW WG General Meeting, Dec 2019*  
<https://indico.cern.ch/event/849342/>

SCETlib

[<https://confluence.desy.de/display/scetlib>]

CuTe

[<https://cute.hepforge.org>]

DYRes/DYTURBO

[<https://gitlab.cern.ch/DYdevel/DYTURBO>]

ReSolve

[<https://github.com/fkhorad/reSolve>]

RadISH

[<https://arxiv.org/pdf/1705.09127.pdf>]

PB-TMD

[<https://arxiv.org/pdf/1906.00919.pdf>]

NangaParbat

[<https://github.com/vbertone/NangaParbat>]

arTeMiDe

[<https://teorica.fis.ucm.es/artemide/>]

SCET

qT-res.

PB

TMD

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arTeMiDe

[<https://teorica.fis.ucm.es/artemide/>]

SCET

q<sub>T</sub>-res.

PB

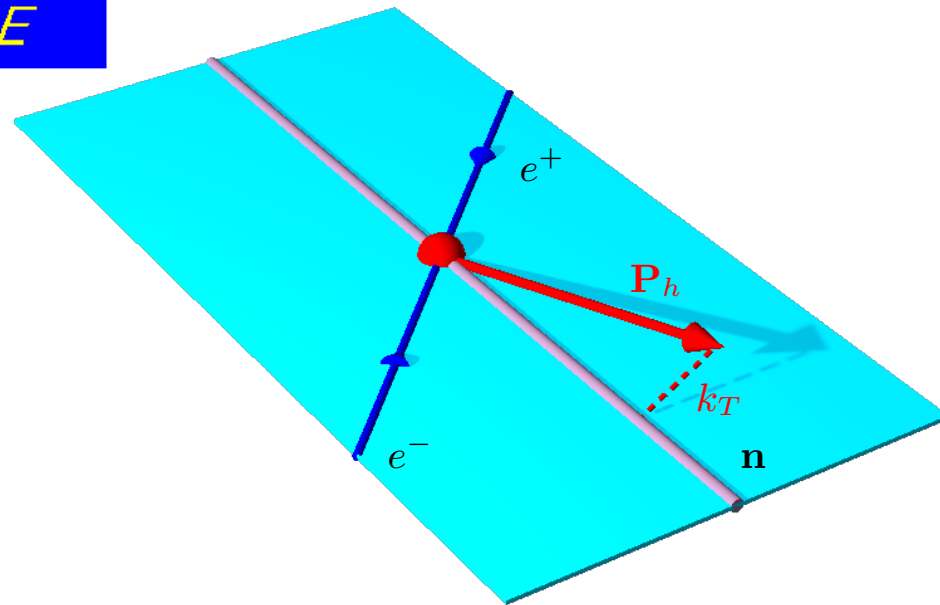
TMD

There is an entire industry of tools that make predictions for observables related to TMDs.

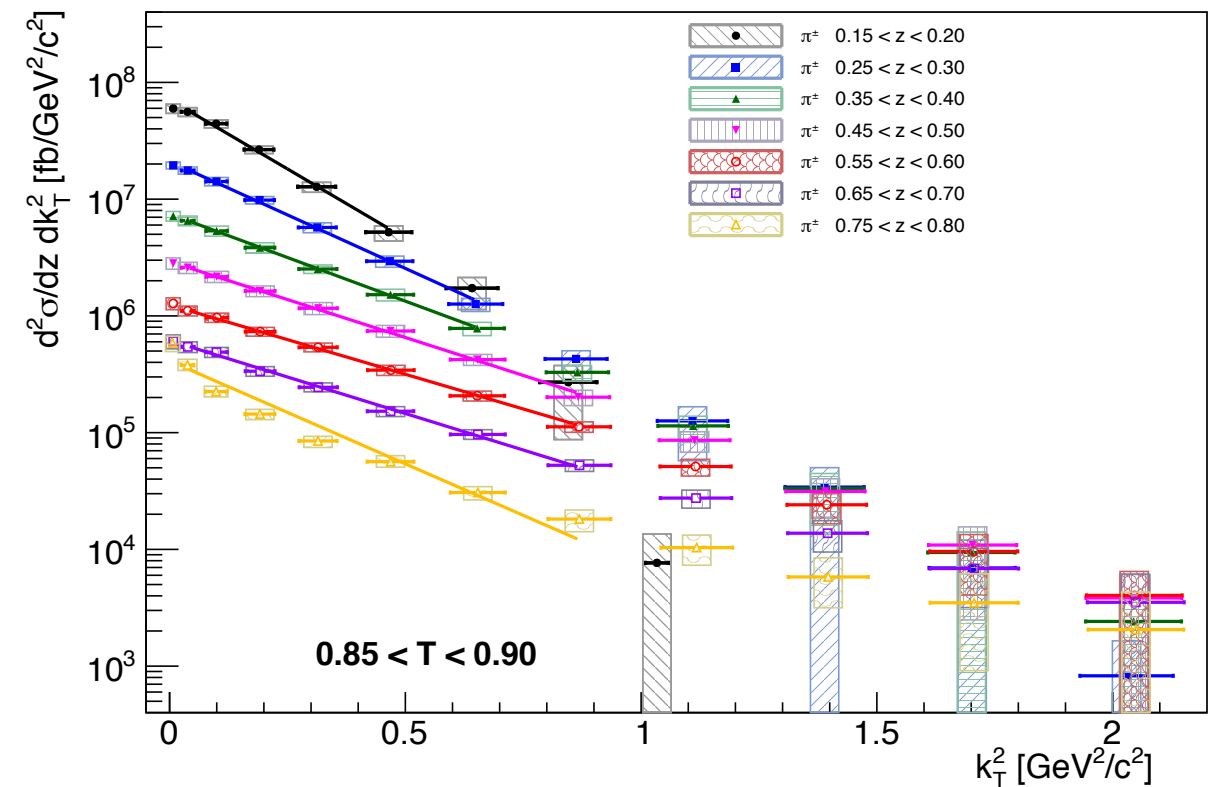
Most of them neglect SIDIS and the important effects coming from nonperturbative TMD components.

**OPEN ISSUES**

# TRANSVERSE MOMENTUM IN FRAGMENTATION FUNCTIONS



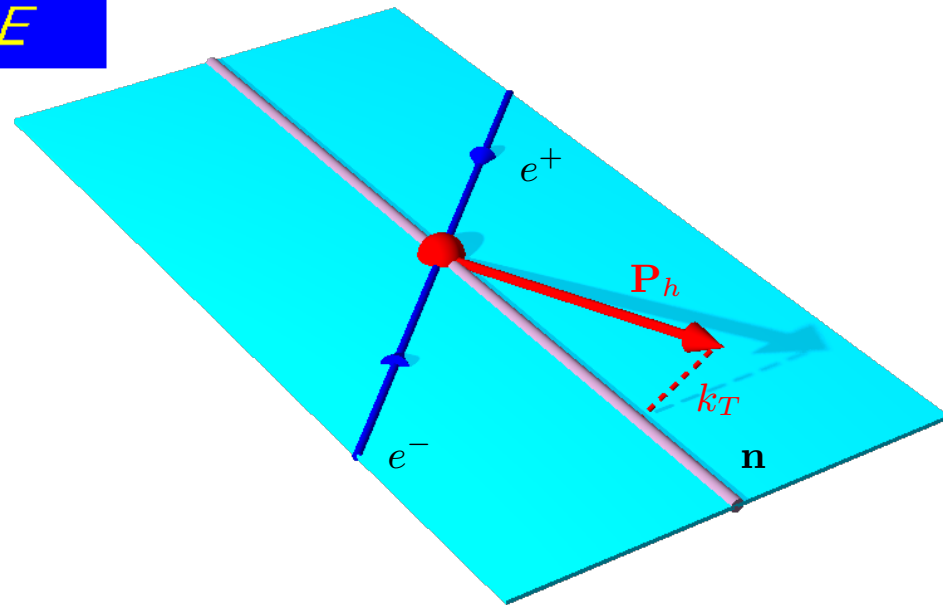
Seidl et al., arXiv:1807.02101



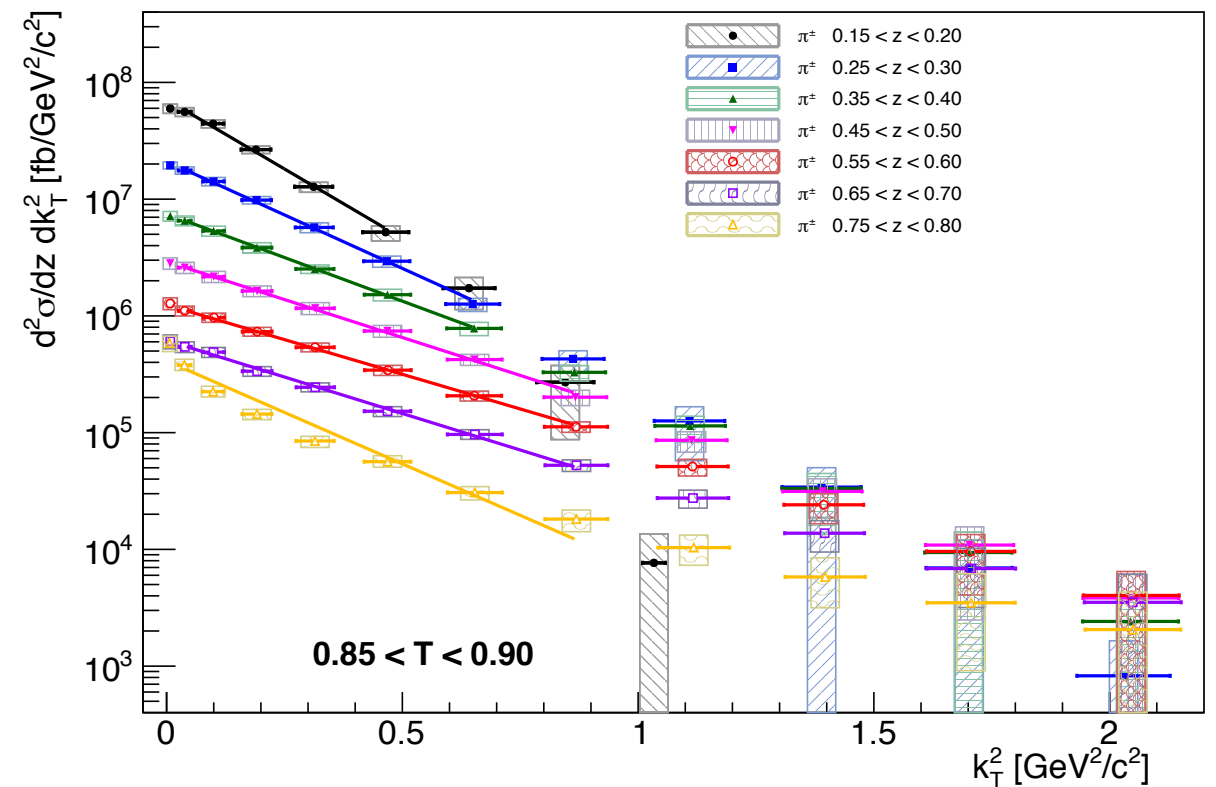
First direct measurement of TMD effects in fragmentation functions  
Makes use of thrust axis: the formalism should take it into account



# TRANSVERSE MOMENTUM IN FRAGMENTATION FUNCTIONS



Seidl et al., arXiv:1807.02101



First direct measurement of TMD effects in fragmentation functions  
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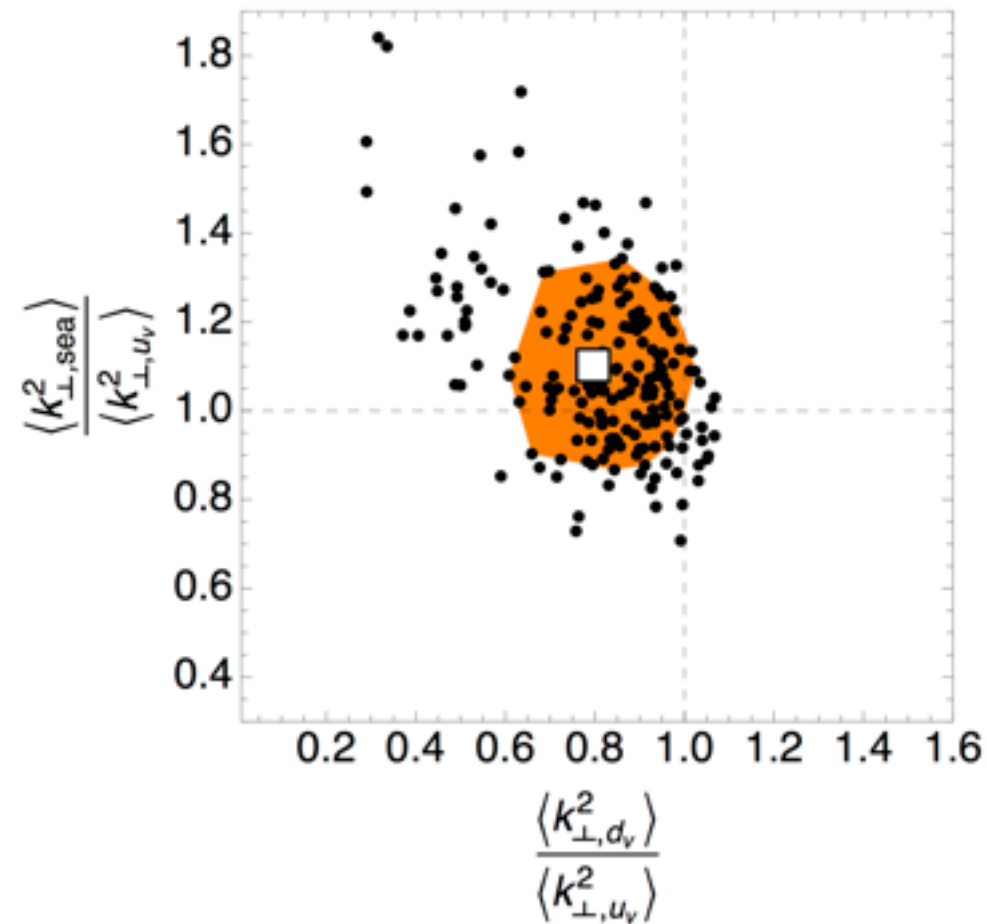
Parton-model attempt to extract TMDFFs: arXiv:1907.12294



# FLAVOR DEPENDENCE OF TMDs

.....  
*Signori, Bacchetta, Radici, Schnell JHEP 1311 (13)*

Ratio of width of sea /  
width of up valence

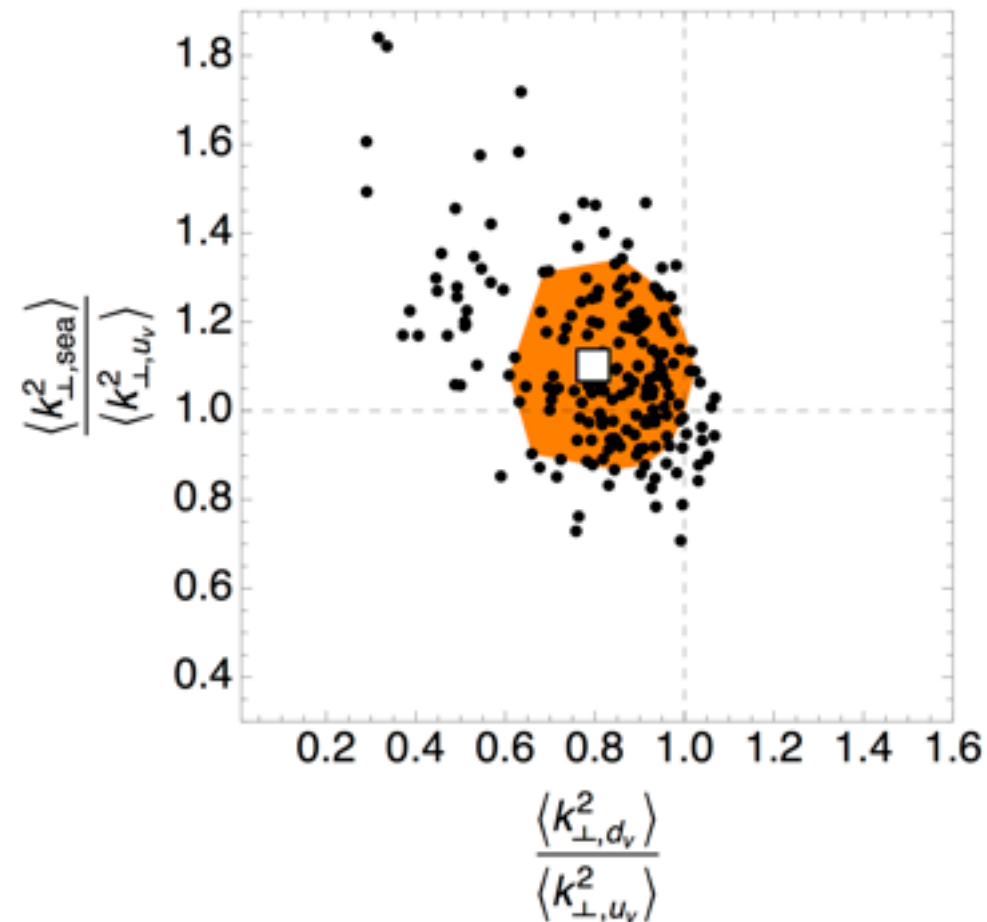


Ratio width of down valence/  
width of up valence

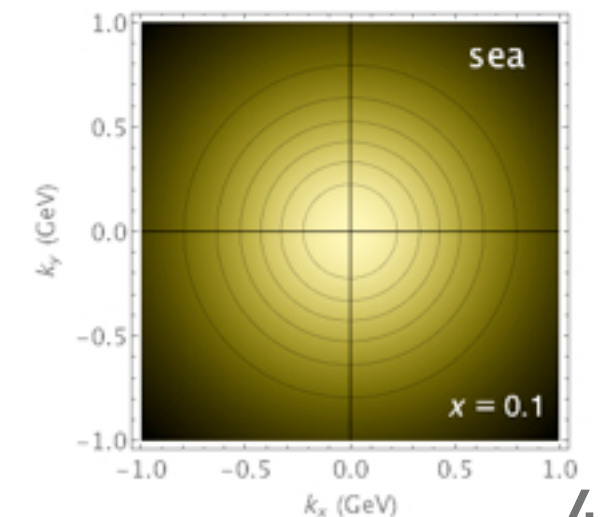
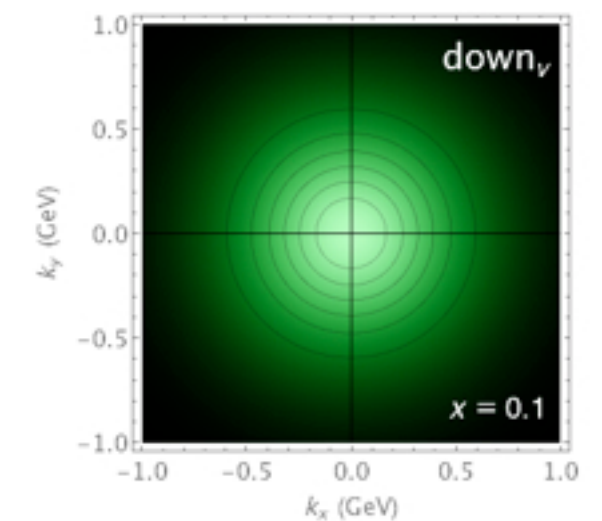
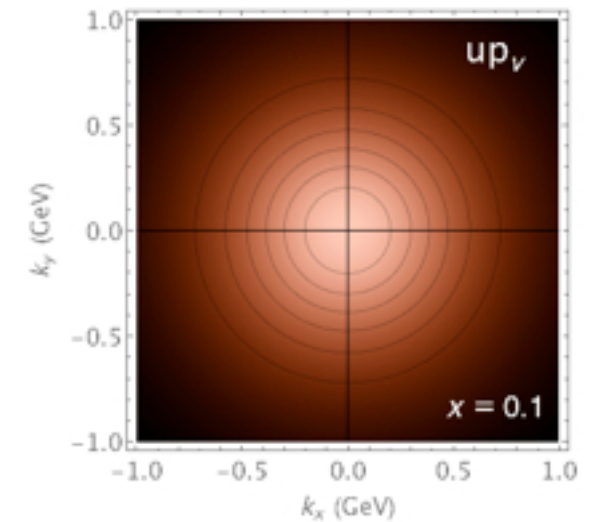
# FLAVOR DEPENDENCE OF TMDs

*Signori, Bacchetta, Radici, Schnell JHEP 1311 (13)*

Ratio of width of sea /  
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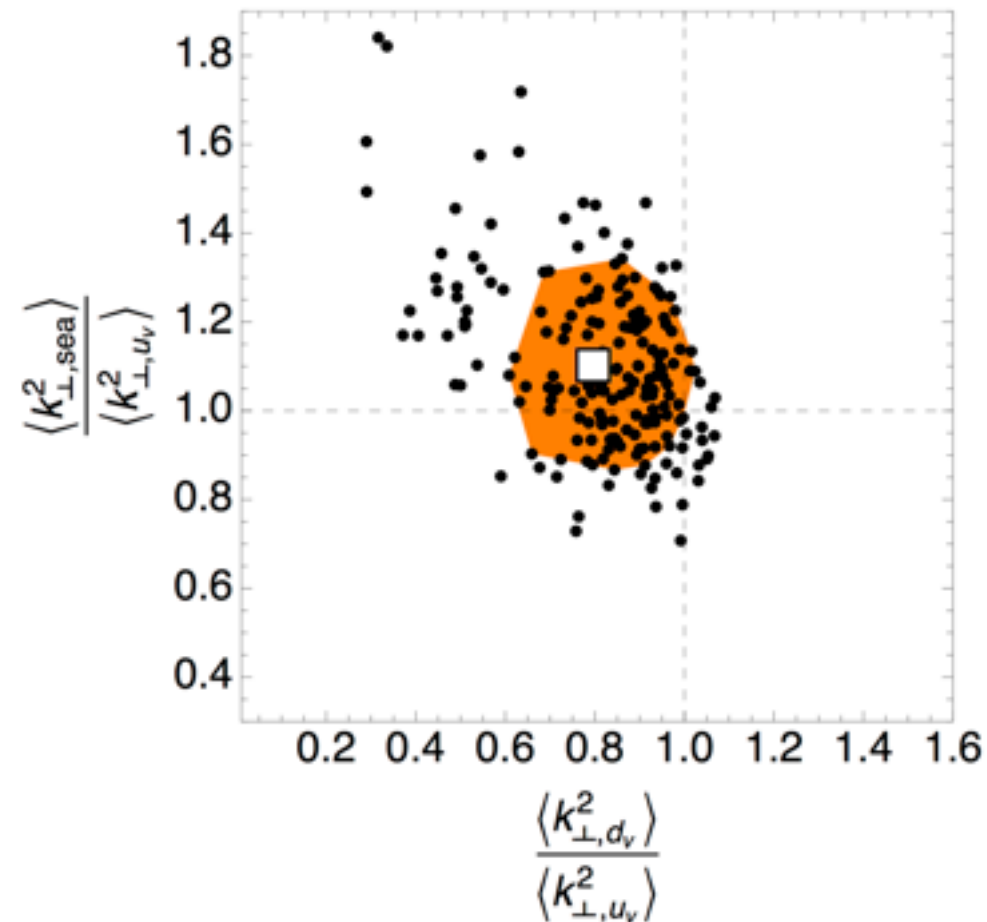
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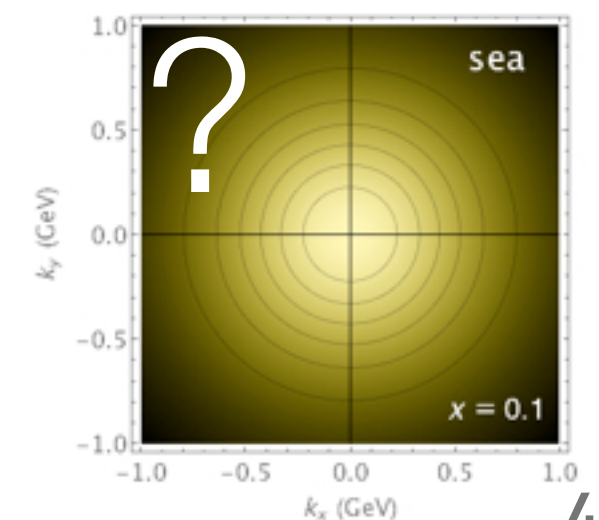
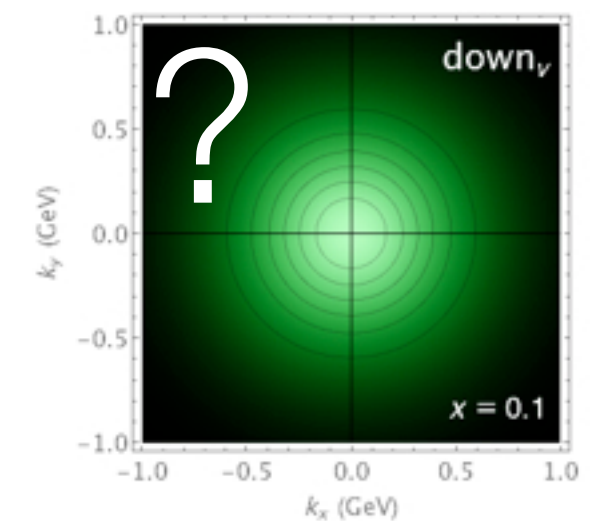
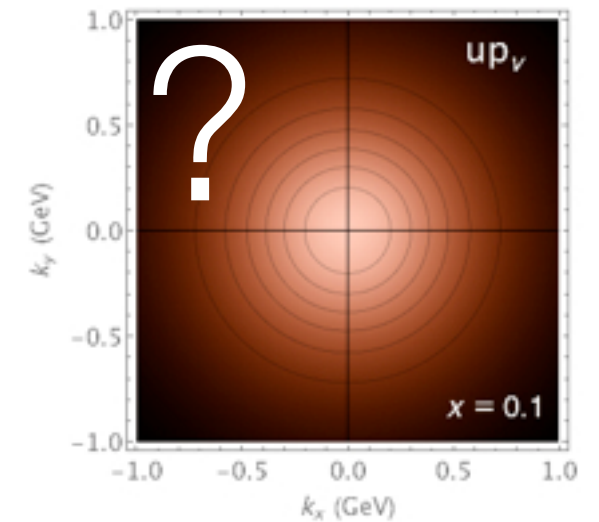
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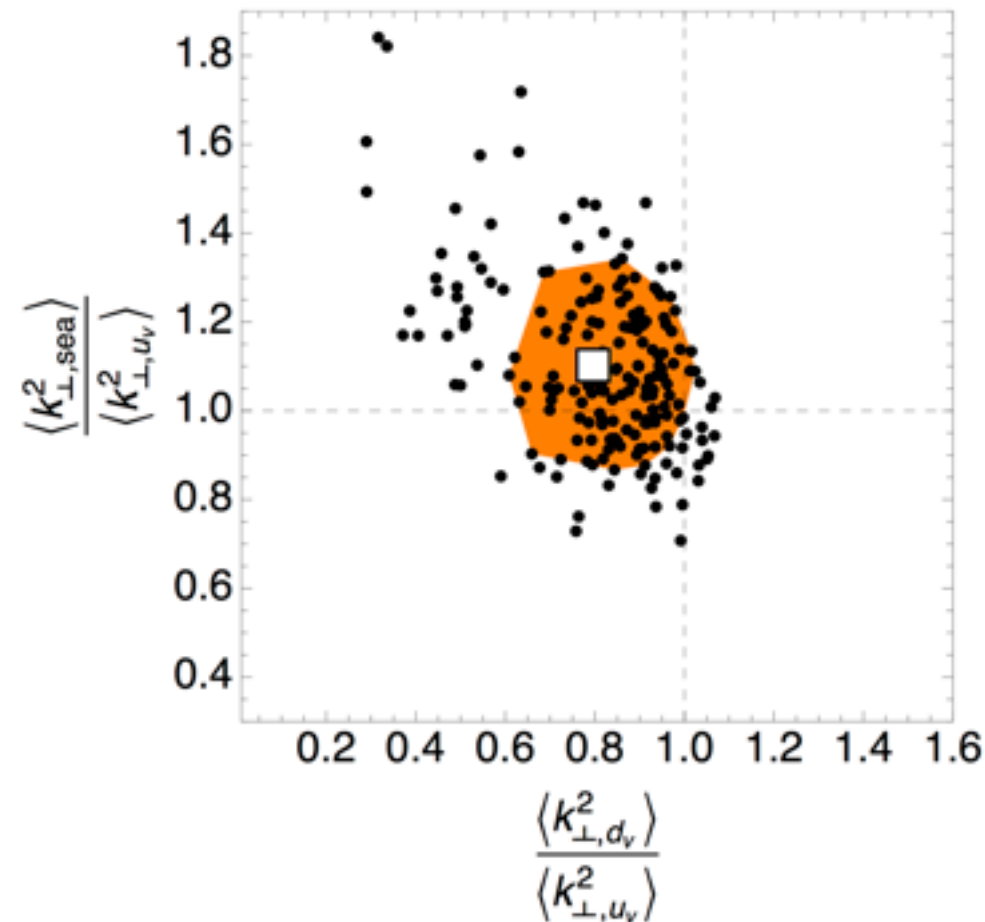
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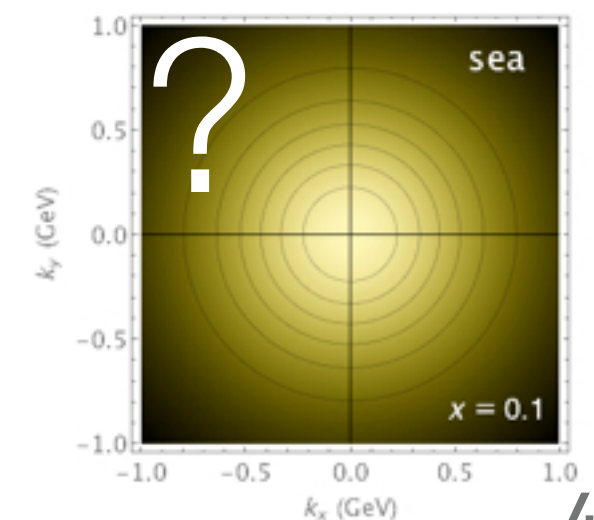
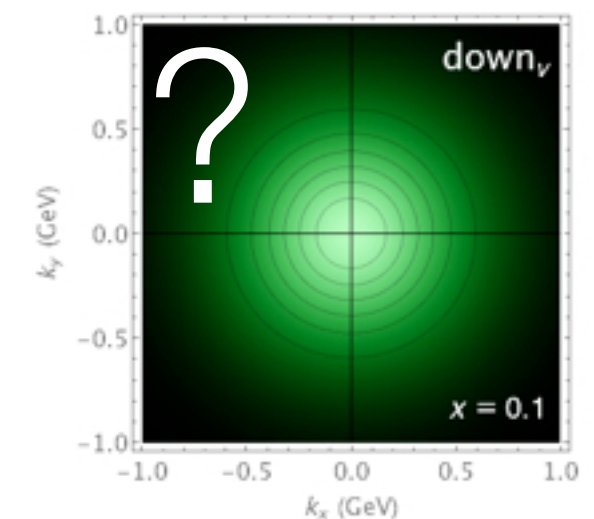
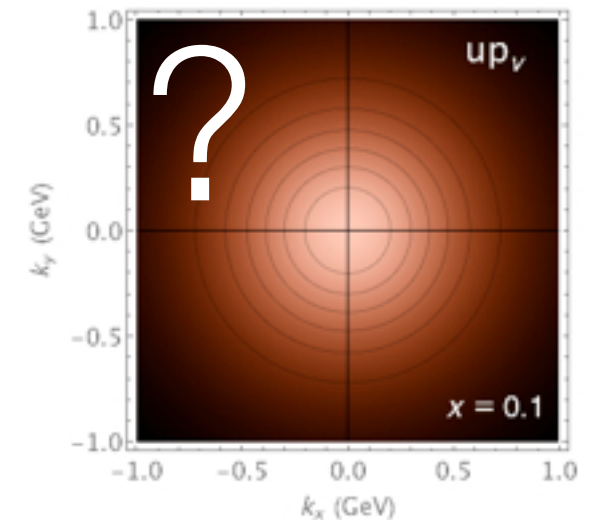
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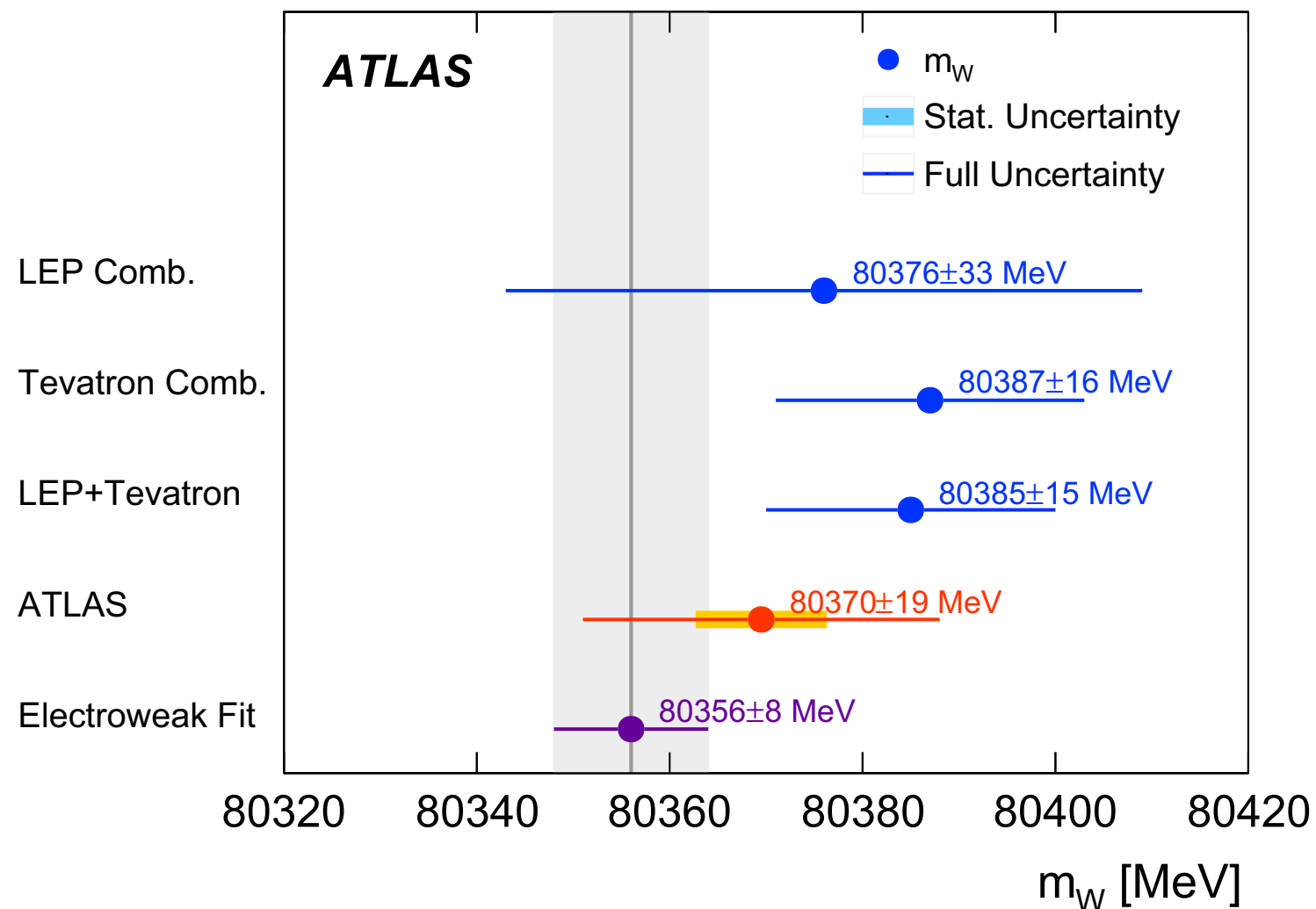
Ratio width of down valence/  
width of up valence

There is room for flavour dependence,  
but we don't control it well



# IMPACT ON W MASS DETERMINATION

ATLAS Collab. *arXiv:1701.07240*

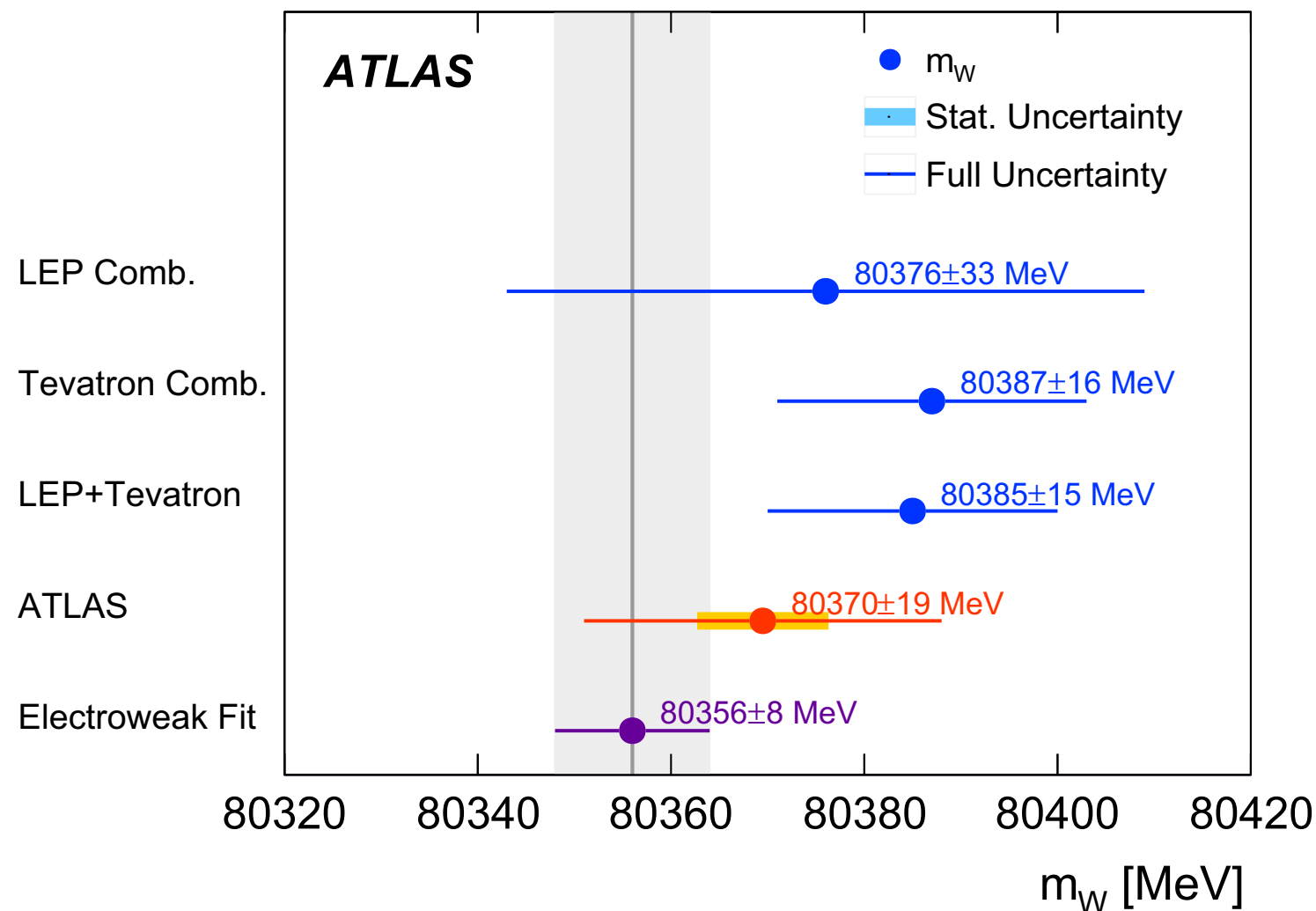


$$\begin{aligned}
 m_W &= 80370 \pm 7 \text{ (stat.)} \pm 11 \text{ (exp. syst.)} \pm 14 \text{ (mod. syst.) MeV} \\
 &= 80370 \pm 19 \text{ MeV,}
 \end{aligned}$$

$$m_{W^+} - m_{W^-} = -29 \pm 28 \text{ MeV.}$$

# IMPACT ON W MASS DETERMINATION

ATLAS Collab. arXiv:1701.07240



All analyses assume that  
TMDs are not flavor  
dependent.  
What happens if they are?

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Set	$u_v$	$d_v$	$u_s$	$d_s$	$s$
1	0.34	0.26	0.46	0.59	0.32
2	0.34	0.46	0.56	0.32	0.51
3	0.55	0.34	0.33	0.55	0.30
4	0.53	0.49	0.37	0.22	0.52
5	0.42	0.38	0.29	0.57	0.27



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narrow, medium, large  
narrow, large, narrow  
large, narrow, large  
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	$\Delta M_{W^+}$		$\Delta M_{W^-}$	
Set	$m_T$	$p_{T\ell}$	$m_T$	$p_{T\ell}$
1	0	-1	-2	3
2	0	-6	-2	0
3	-1	9	-2	-4
4	0	0	-2	-4
5	0	4	-1	-3

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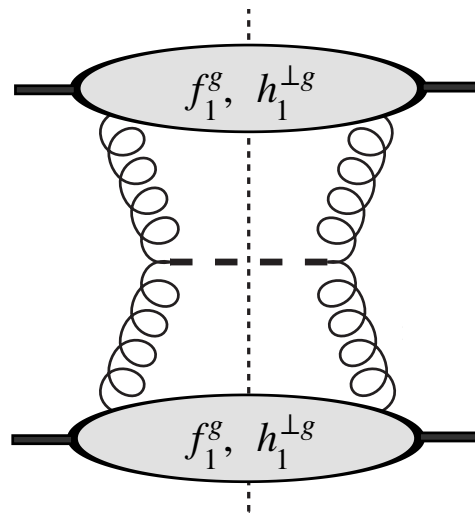
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**Not taking into account the  
 flavour dependence of TMDs  
 can lead to errors in the  
 determination of the W mass**

# GLUON TMDS

## Higgs production

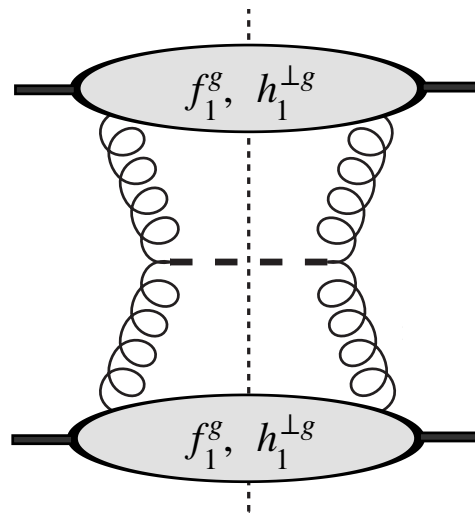
*Gutierrez-Reyes, Leal-Gomez, Scimemi,  
Vladimirov, arXiv:1907.03780*



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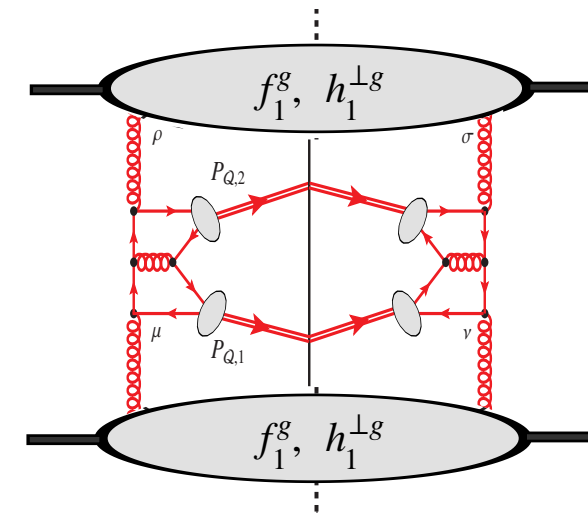
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*Gutierrez-Reyes, Leal-Gomez, Scimemi,  
Vladimirov, arXiv:1907.03780*



## Quarkonium-pair production

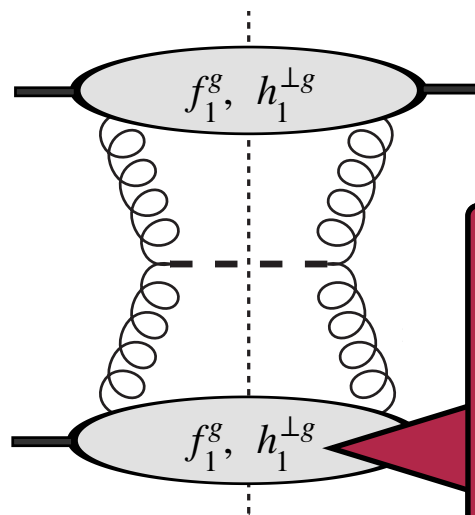
*Scarpa, Boer, Echevarria, Lansberg,  
Pisano, Schlegel, arXiv:1909.05769*



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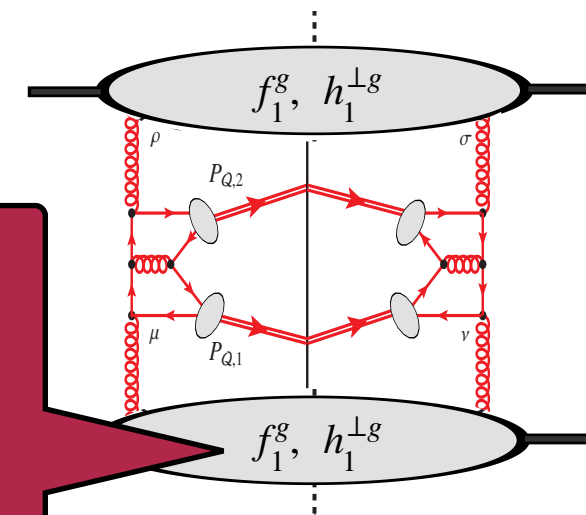
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**Also linearly polarized  
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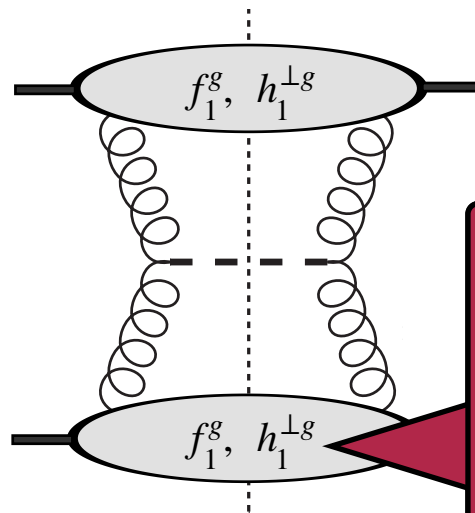
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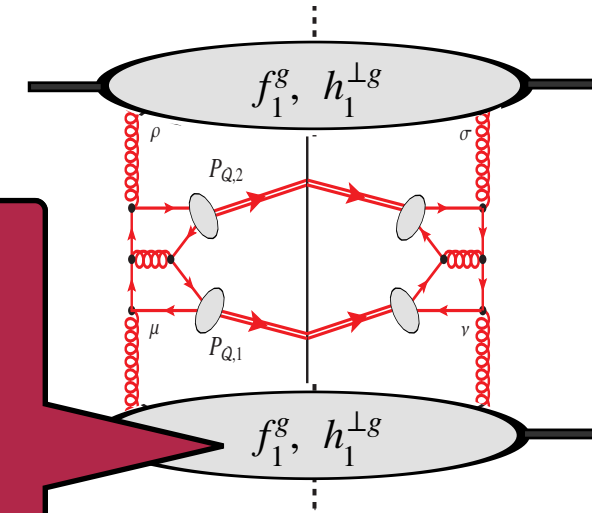
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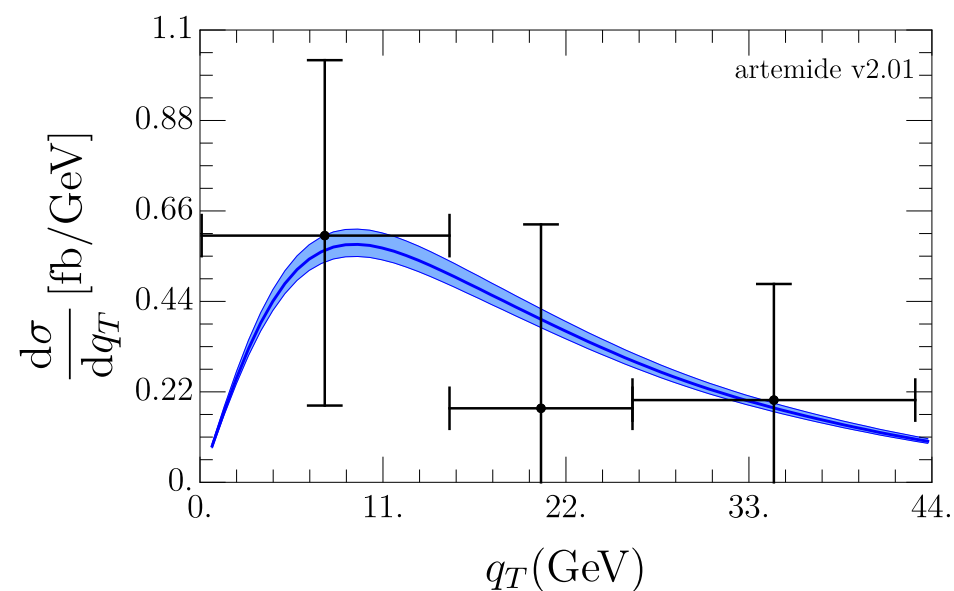
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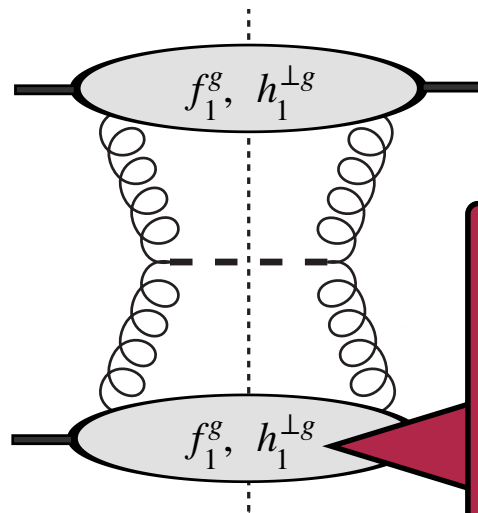
$$pp \rightarrow H(\rightarrow \gamma\gamma) + X$$



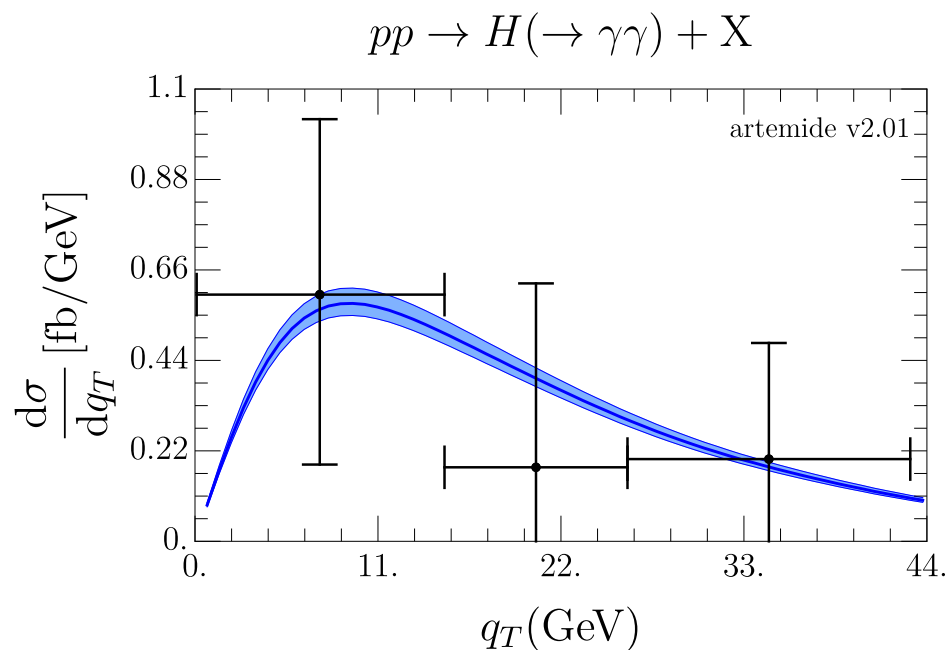
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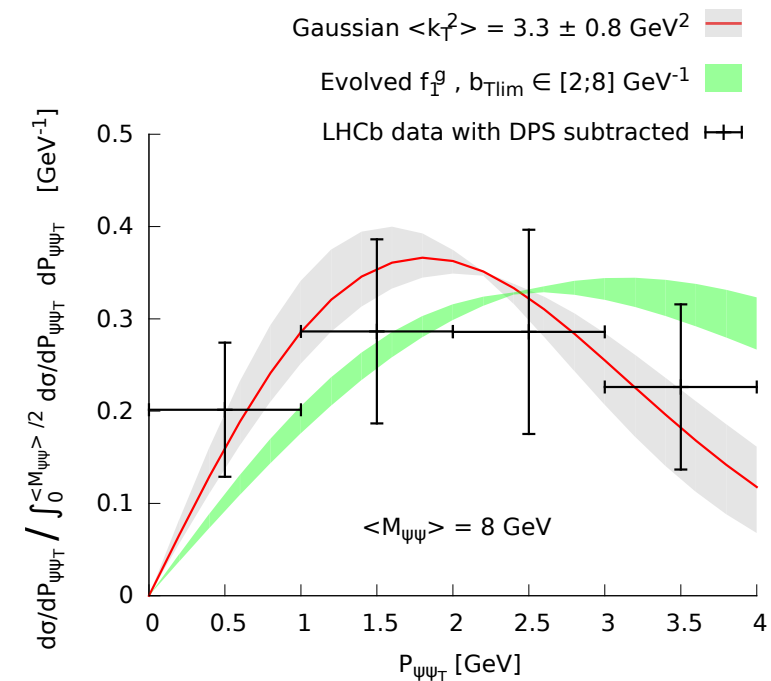
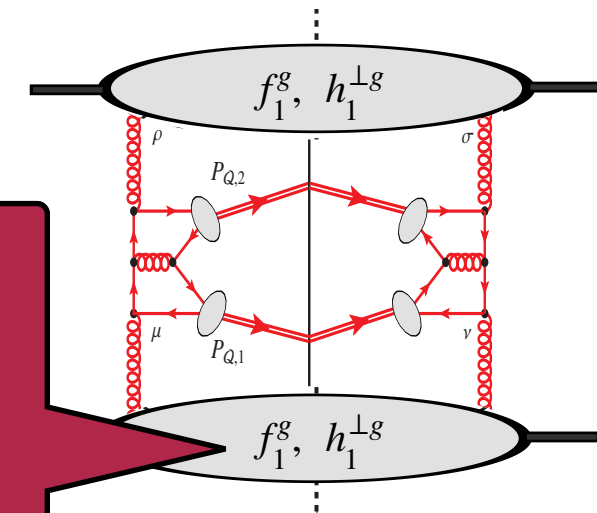


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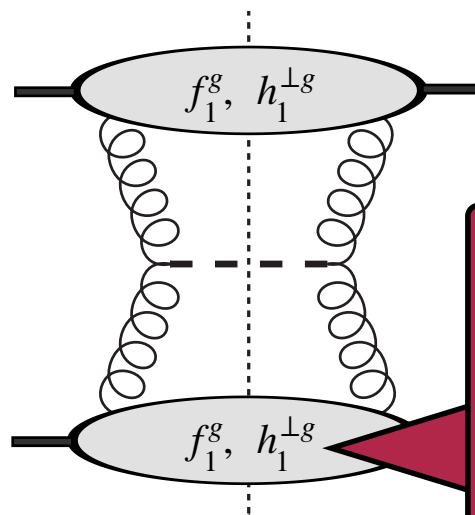




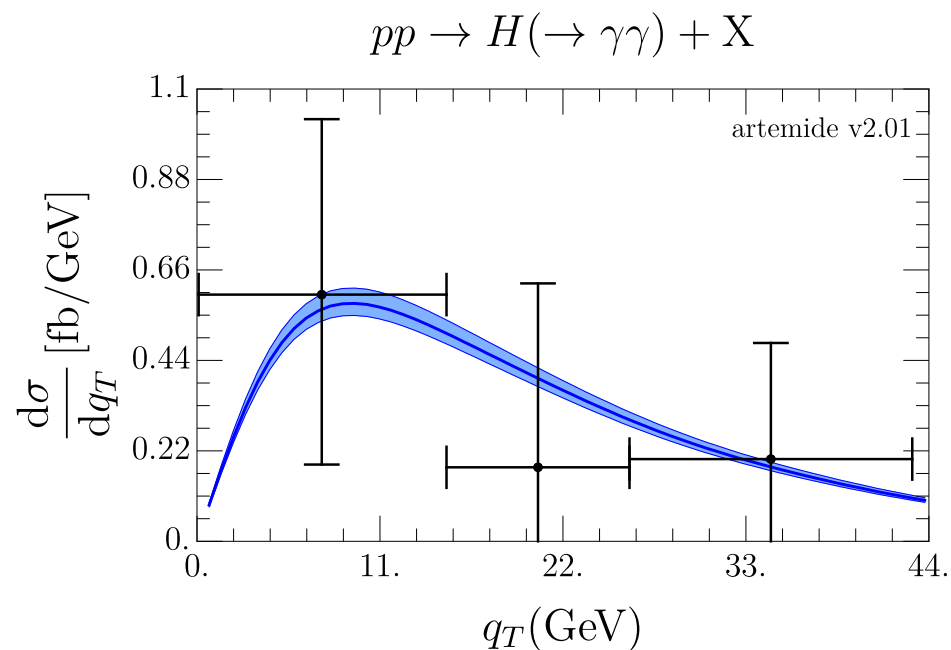
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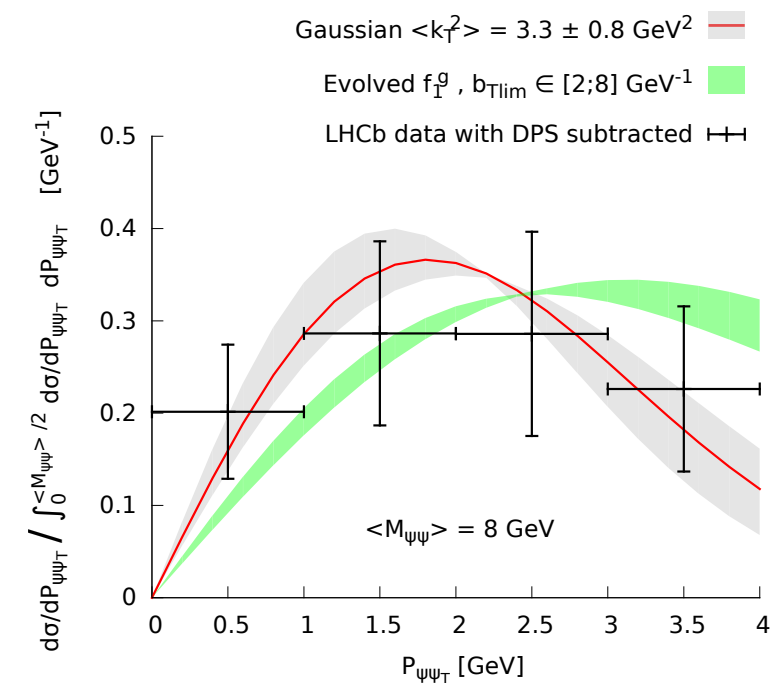
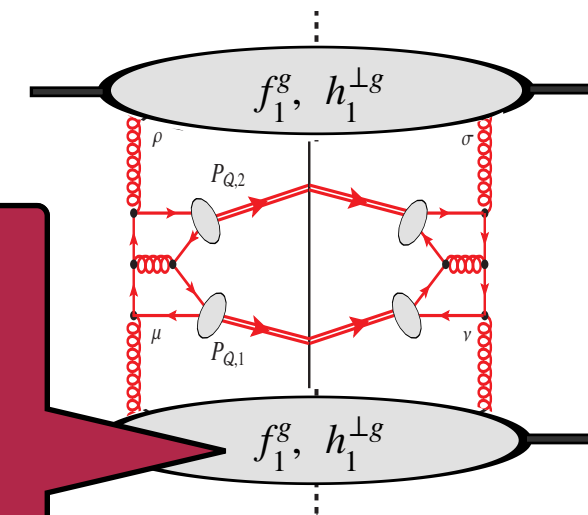


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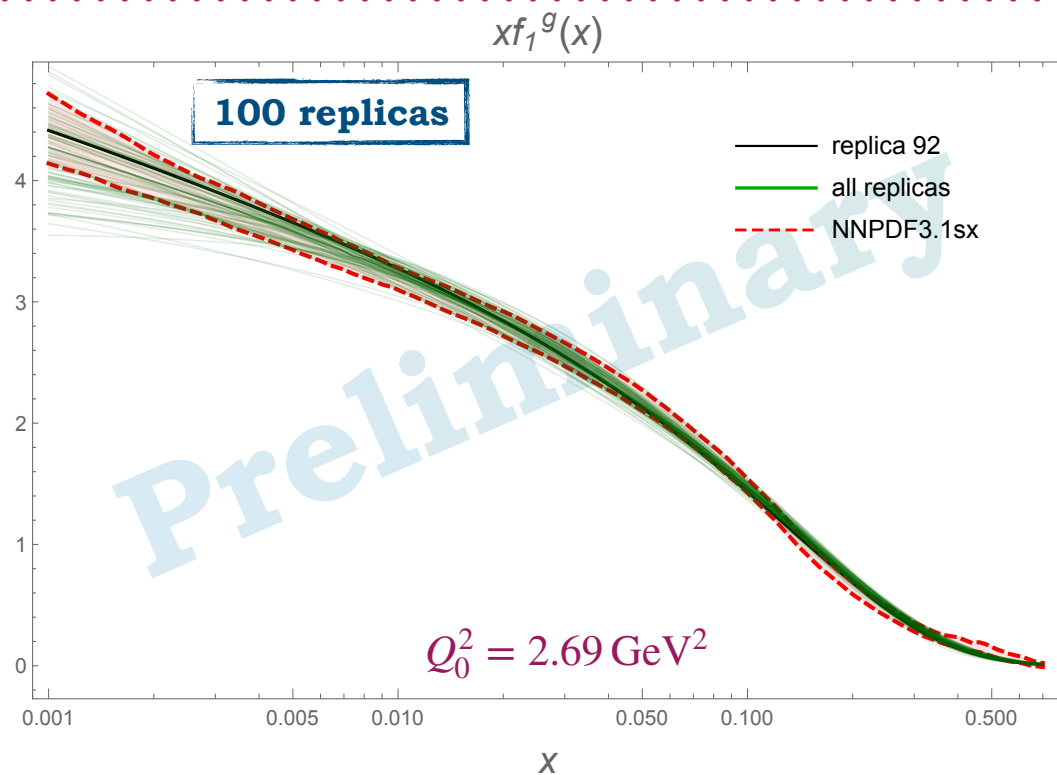
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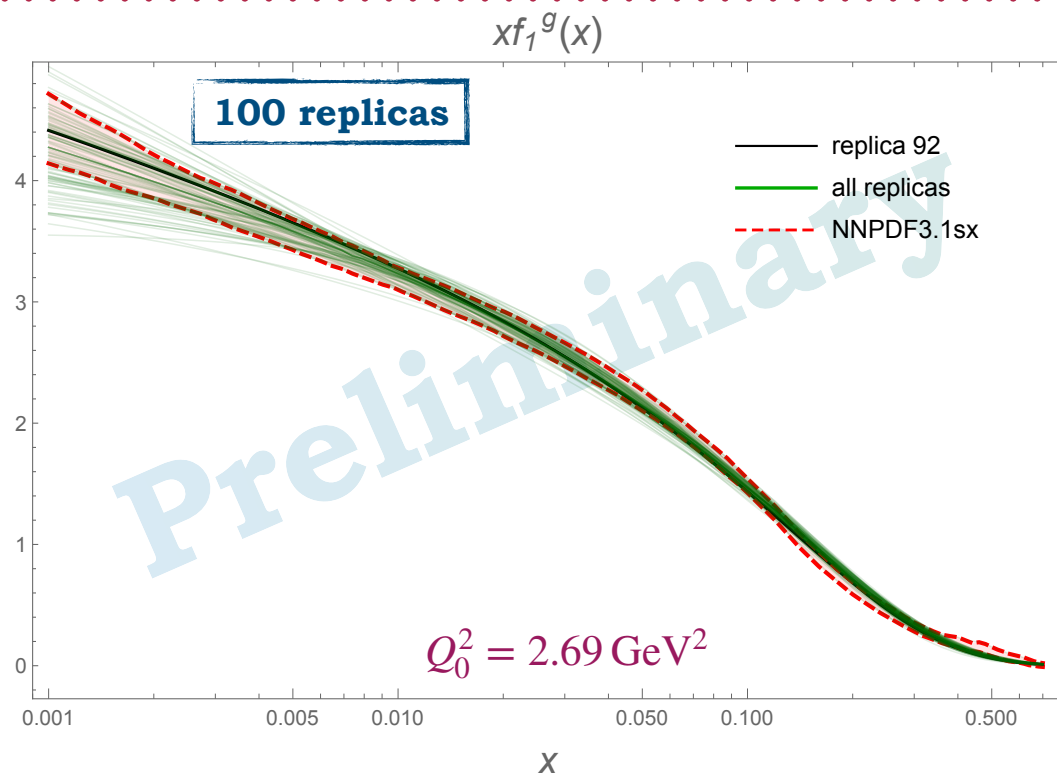
see also talk by Raj Kishore for other process

# MODEL FOR GLUON TMDs



see talk by F. Celiberto at REF2019  
<https://agenda.infn.it/event/17749>

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Spectator model with spectral  
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Reproduces collinear gluon PDFs

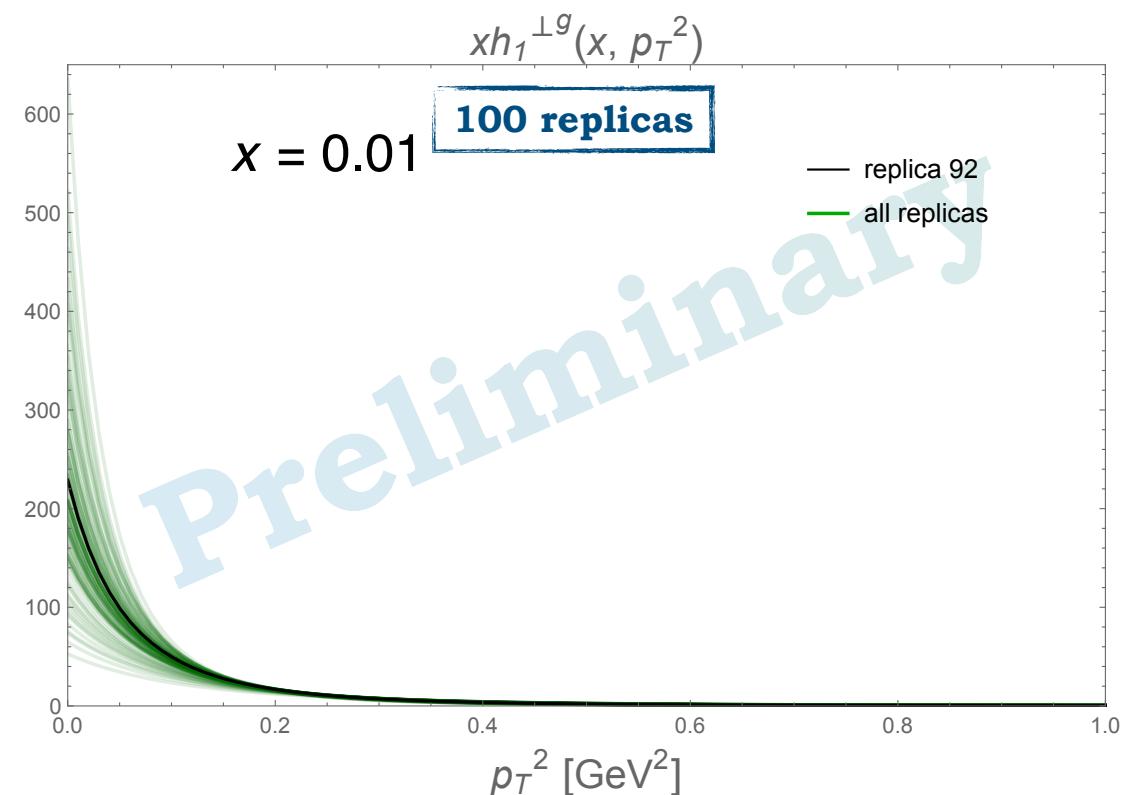
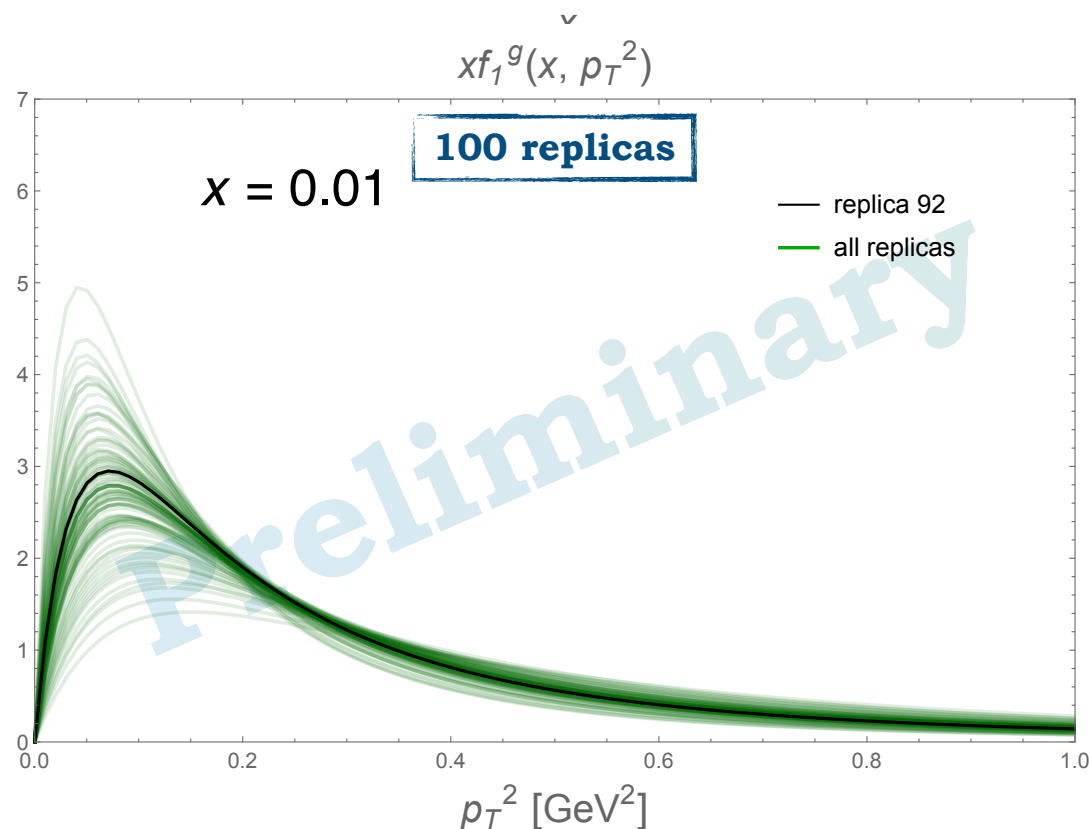
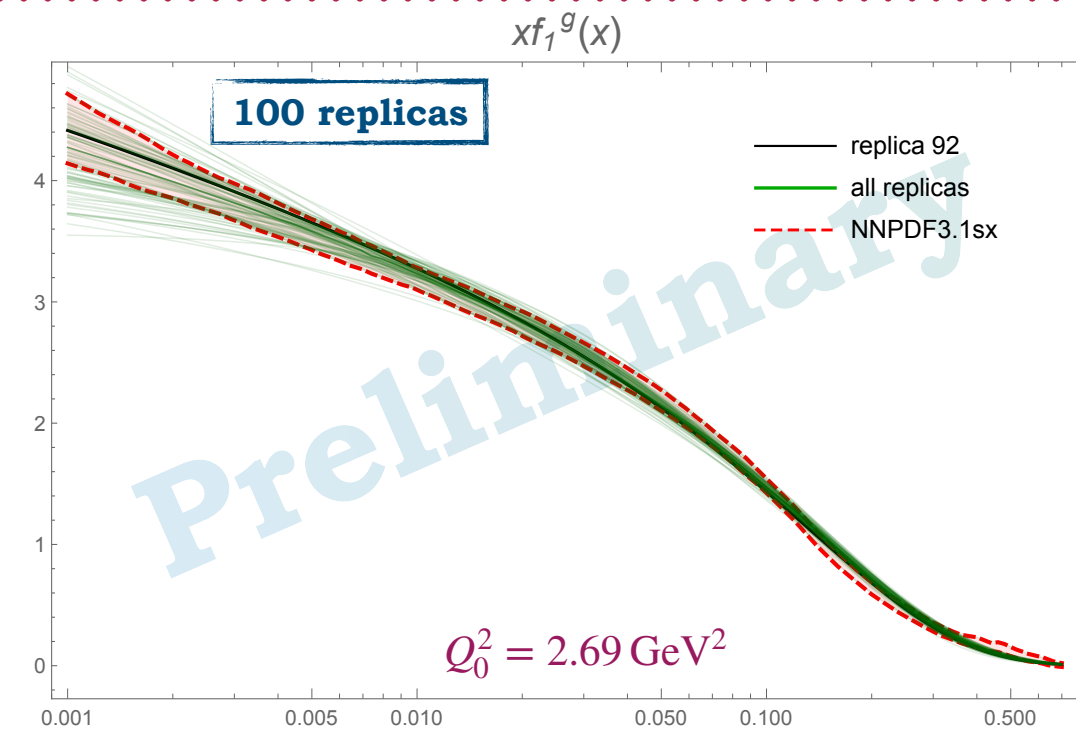
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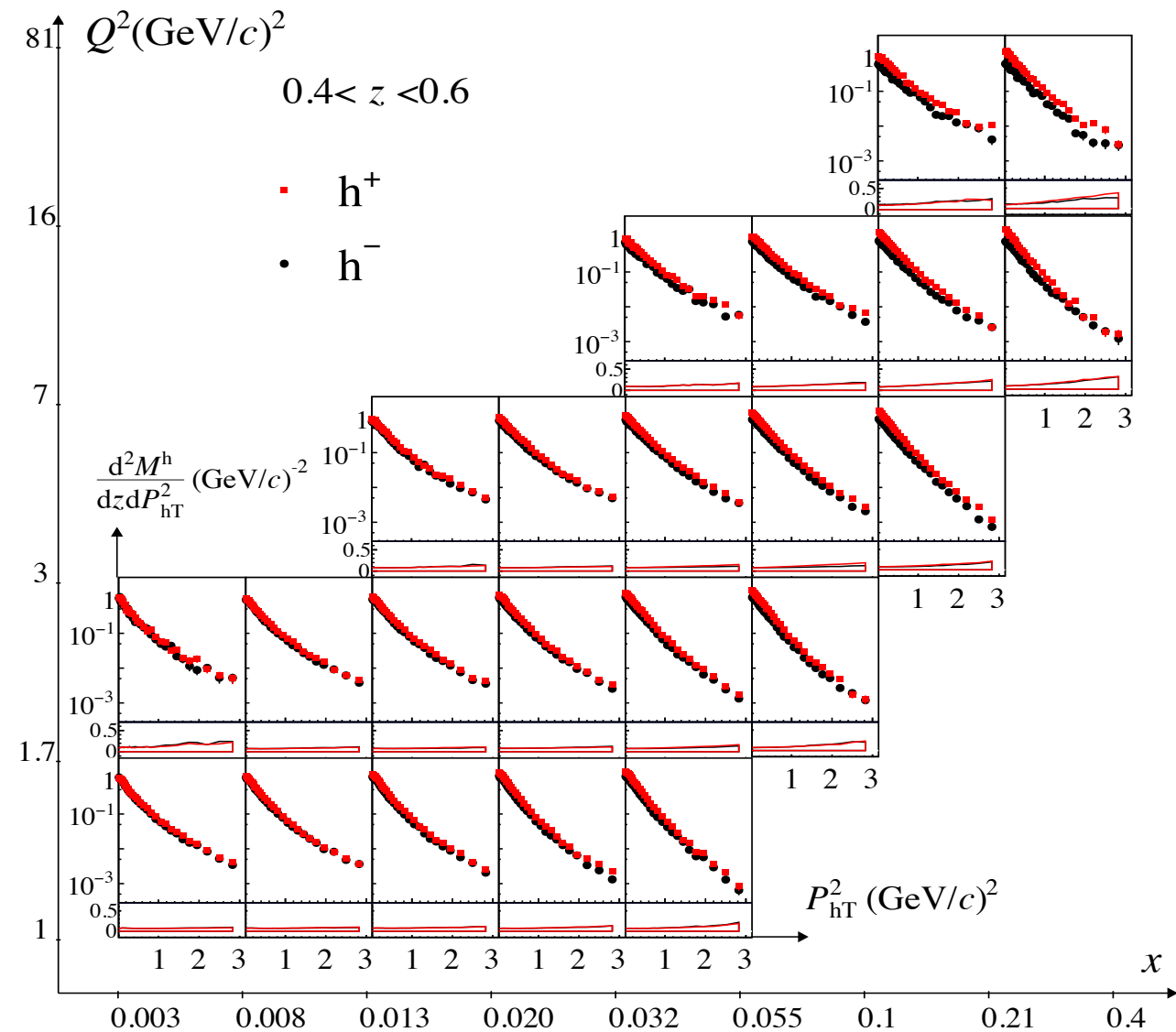
Generates nontrivial and widely different TMDs



**THE FUTURE**

# NEW DATA FROM COMPASS

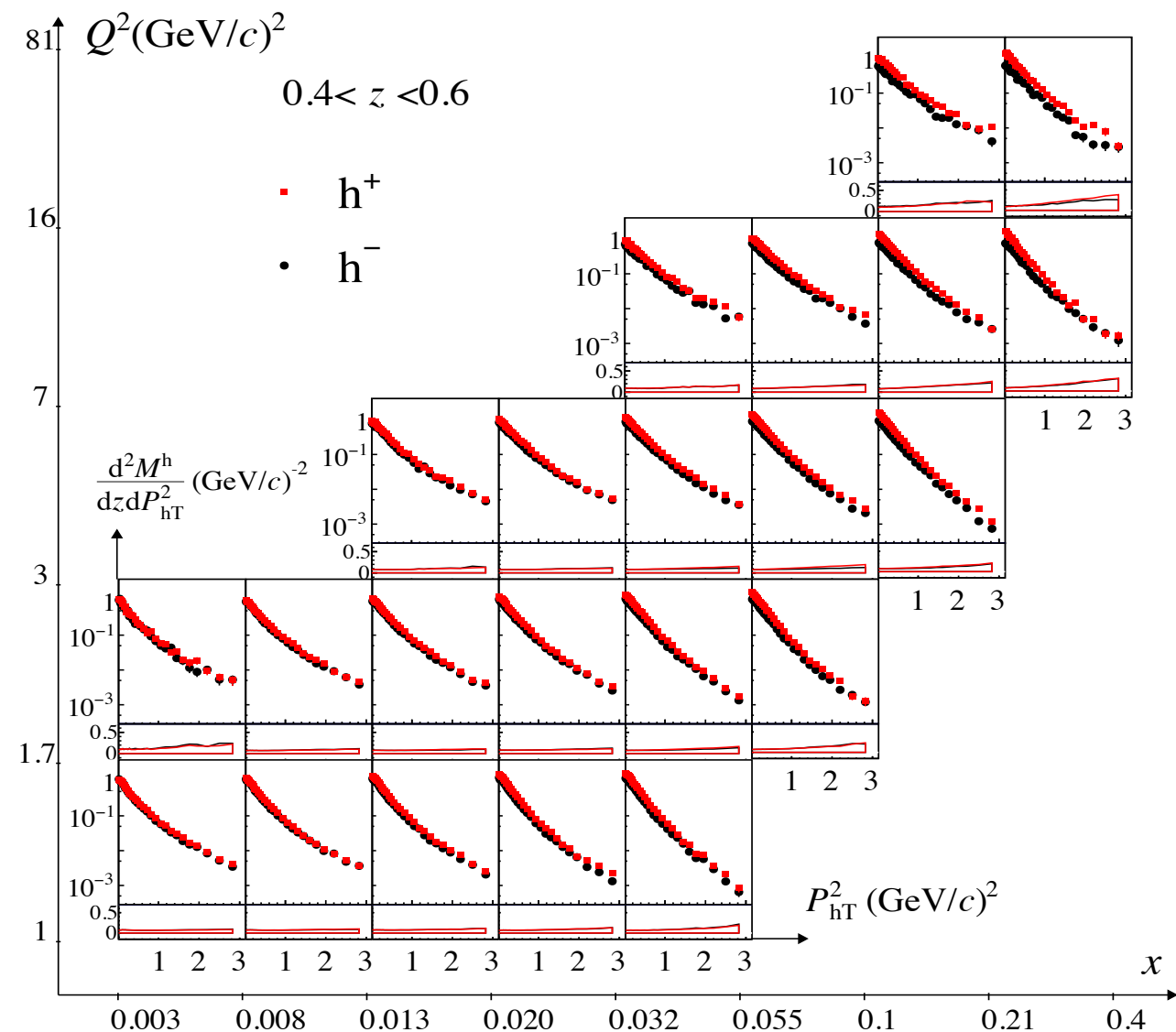
Multidimensional  
binning



COMPASS Collab., arXiv:1709.07374

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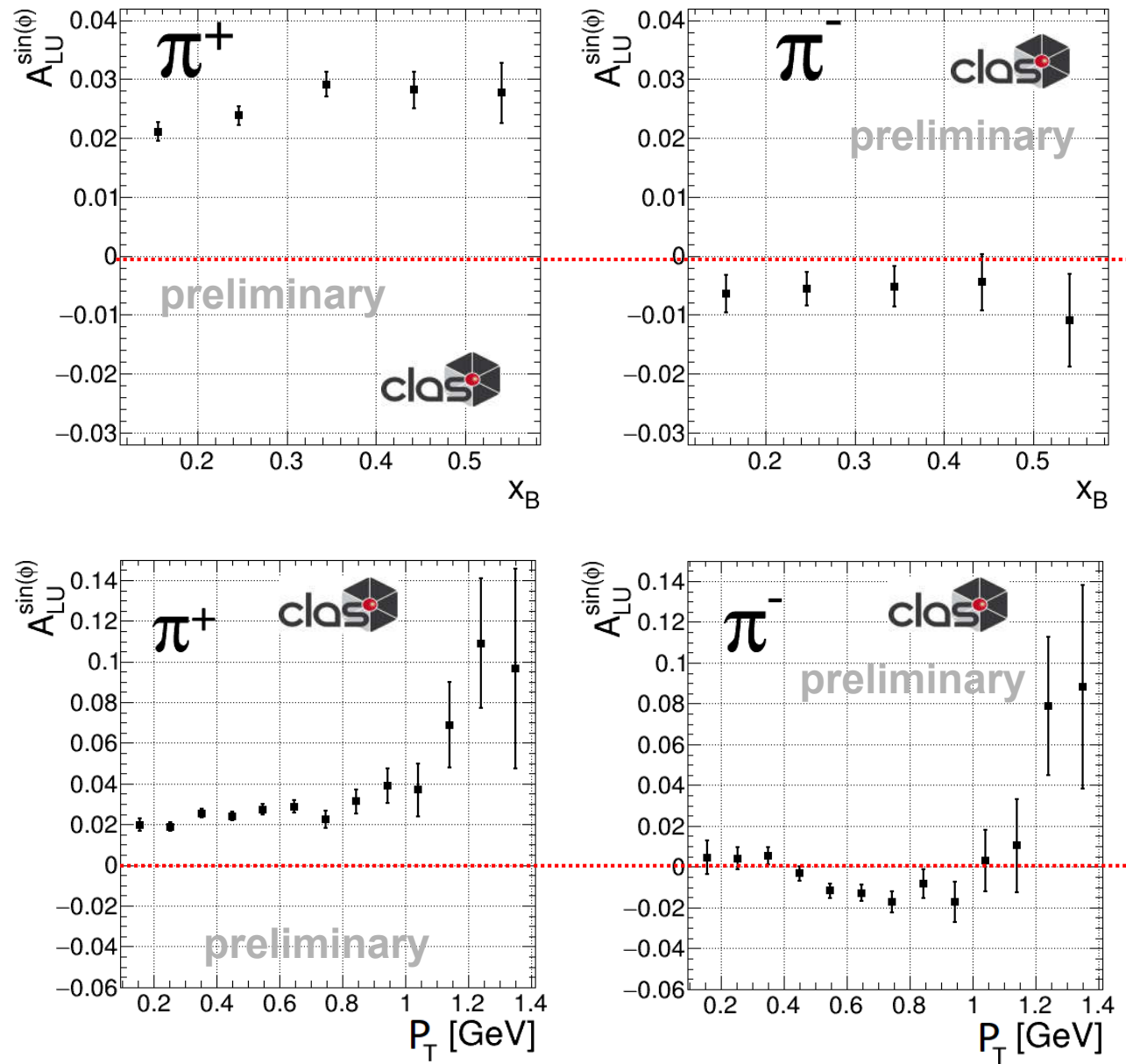
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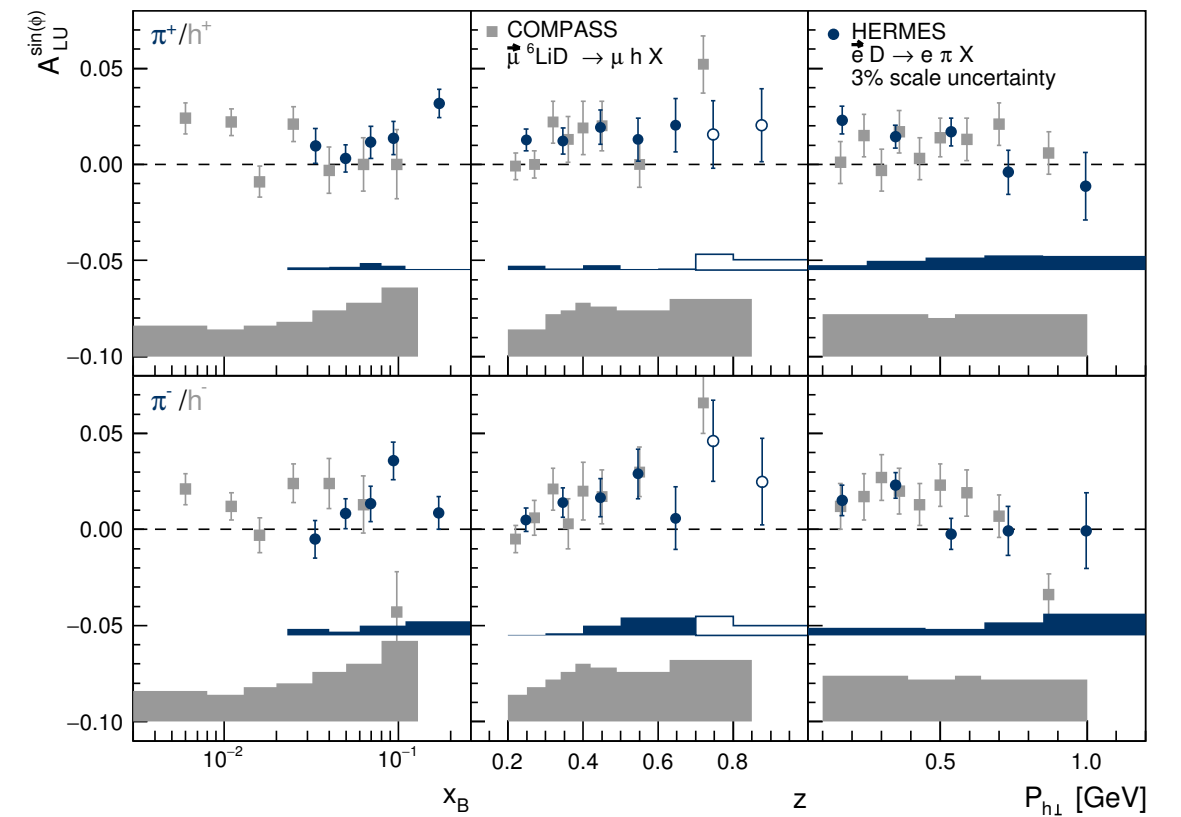
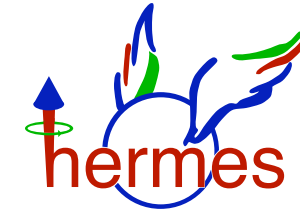
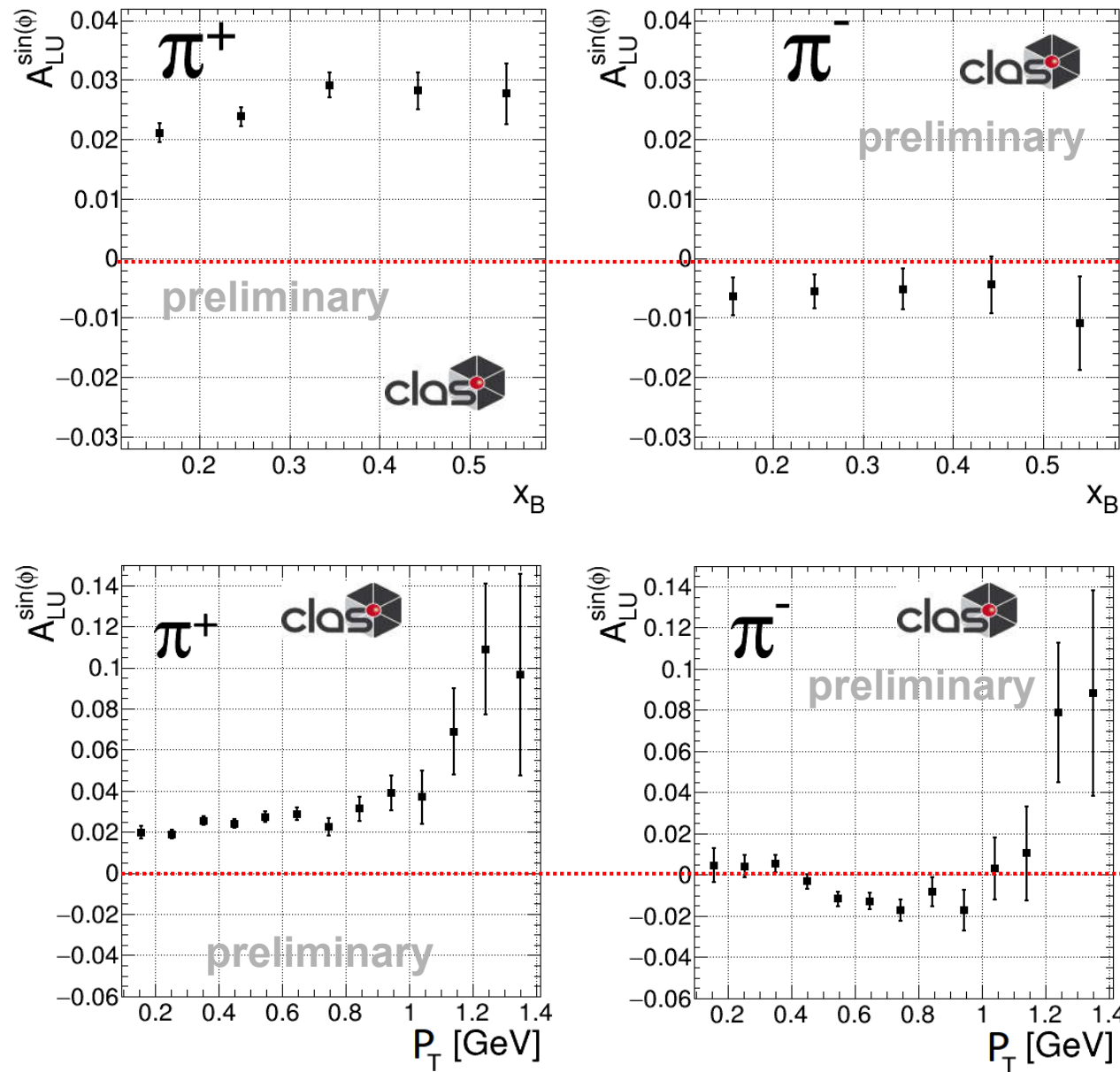
COMPASS is in “full swing” mode.  
Proton-target data are also expected

# FIRST JLAB PRELIMINARY DATA

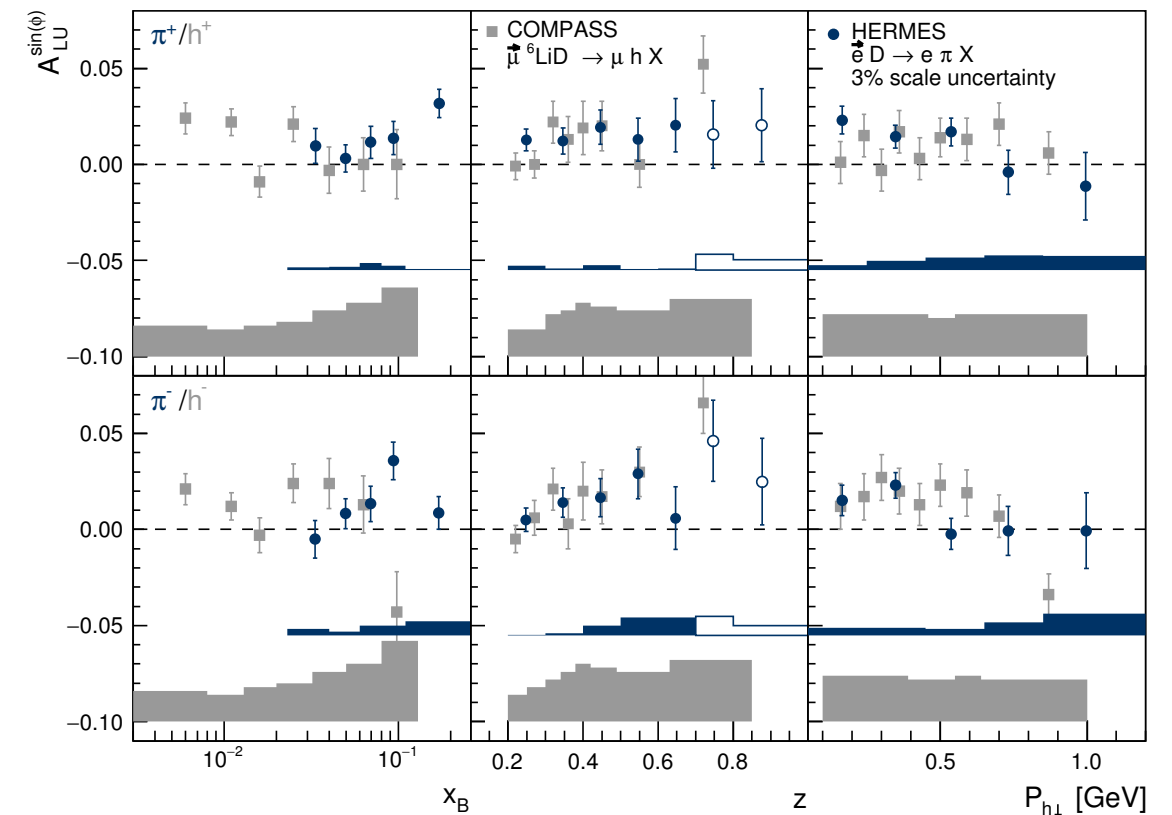
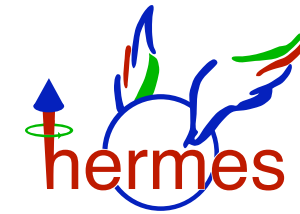
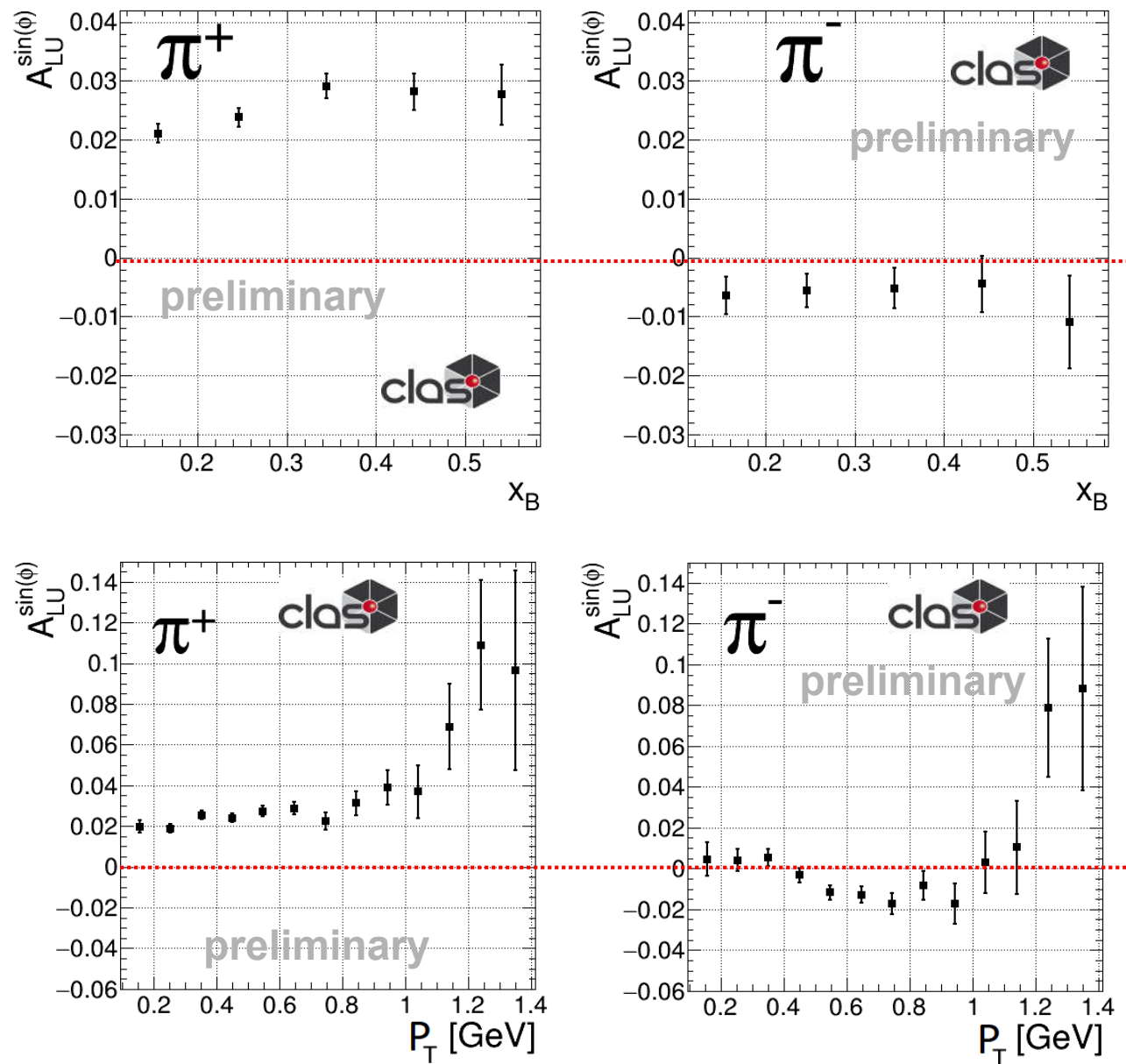




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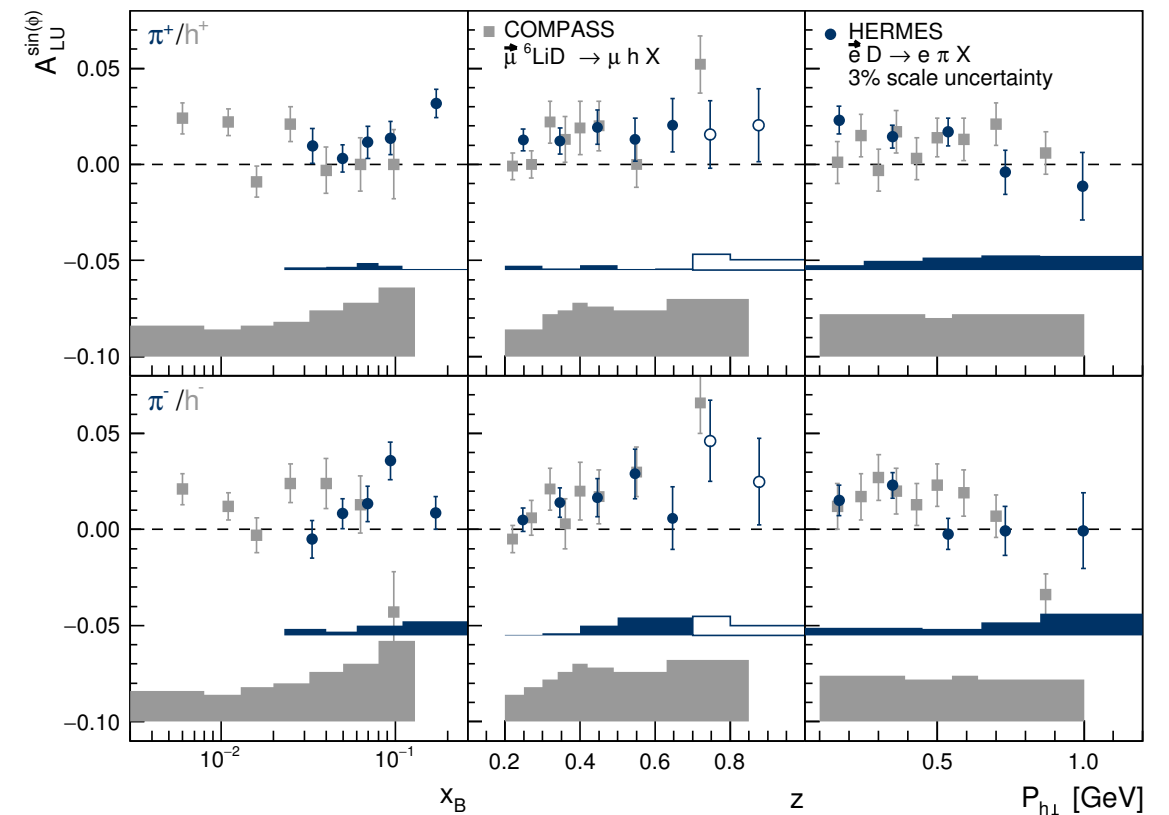
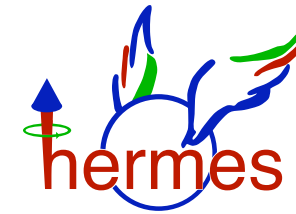
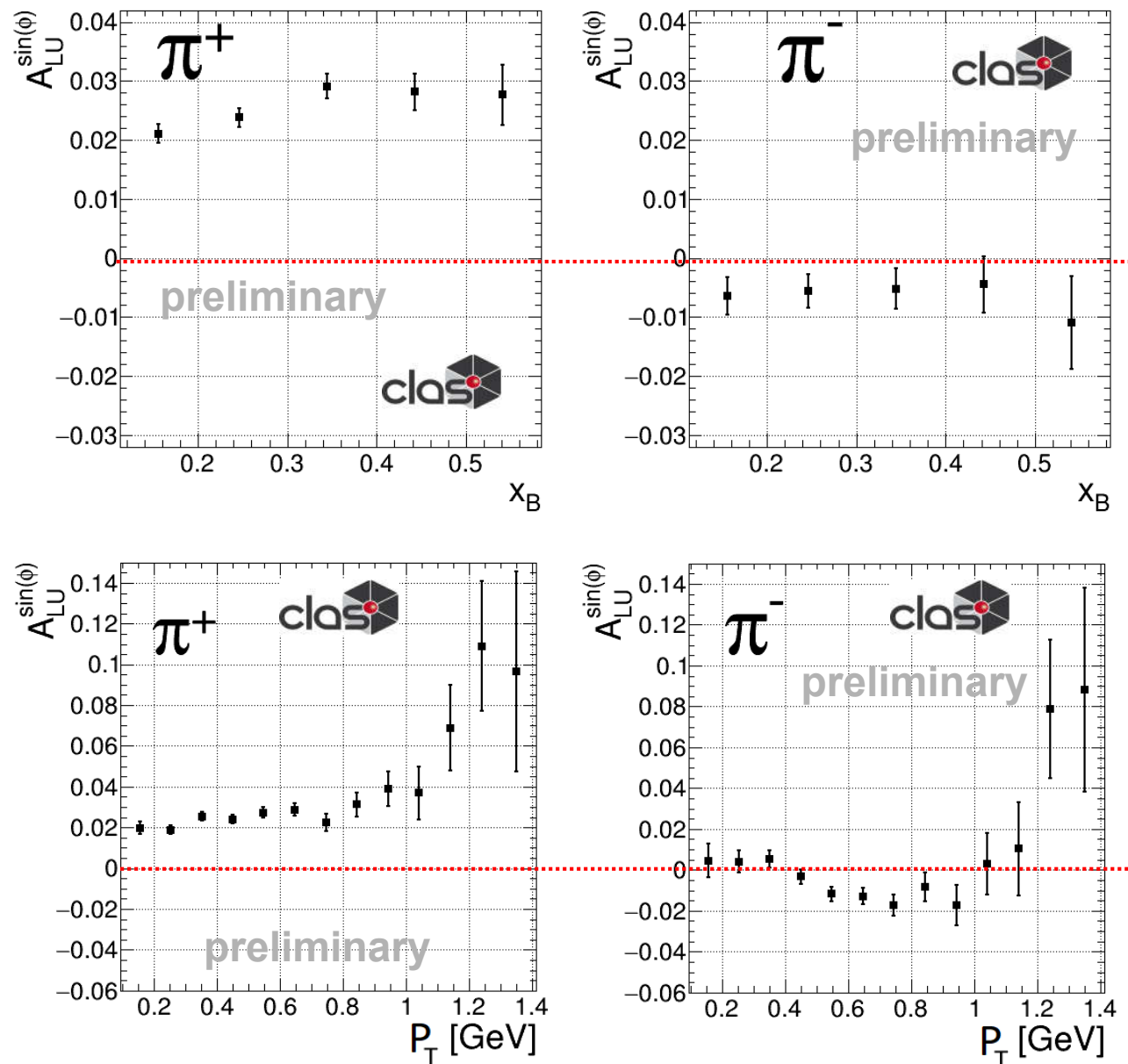


# FIRST JLAB PRELIMINARY DATA



Only 2% of approved data taking

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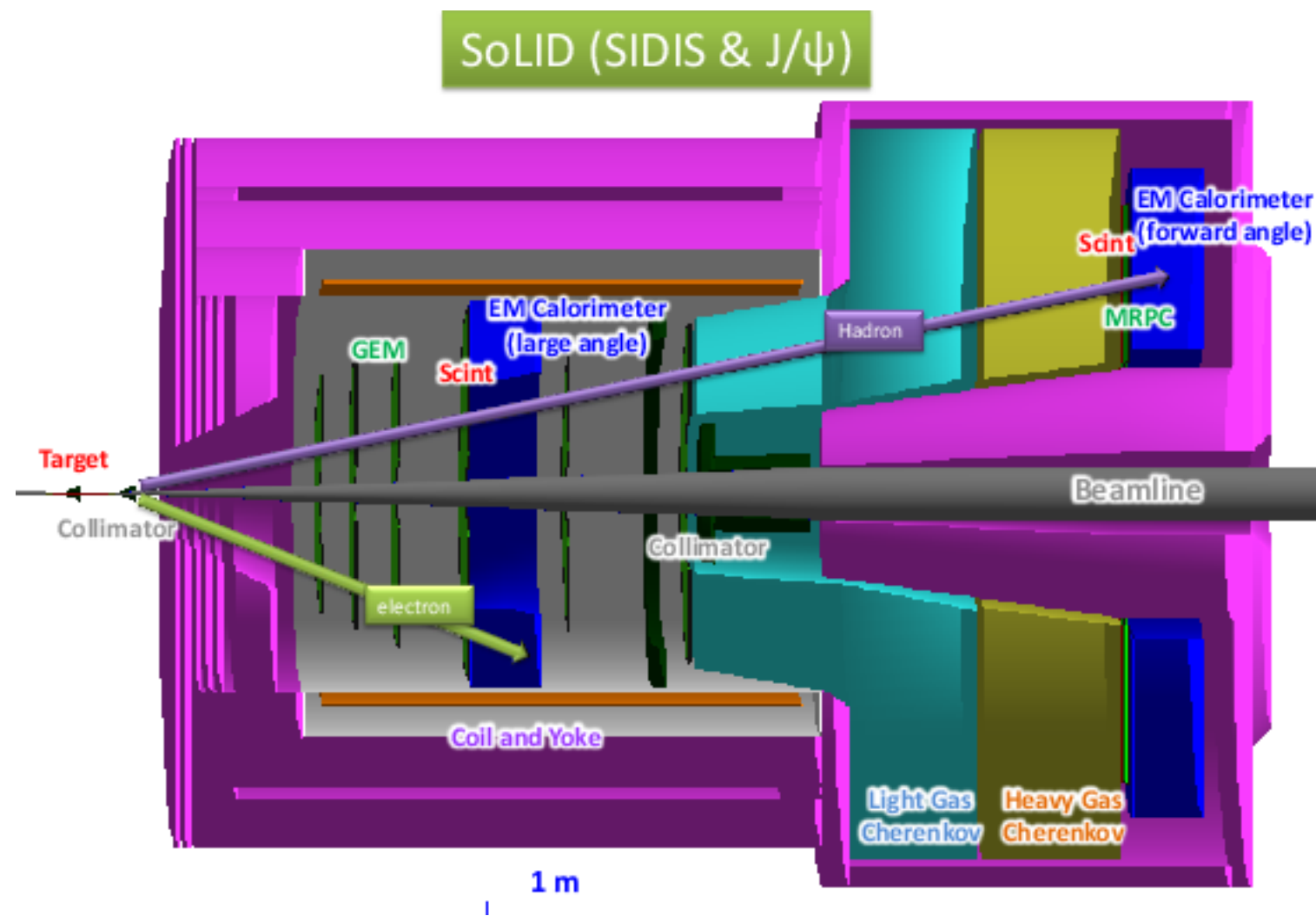


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# AWESOME!

# SOLID @ JLAB

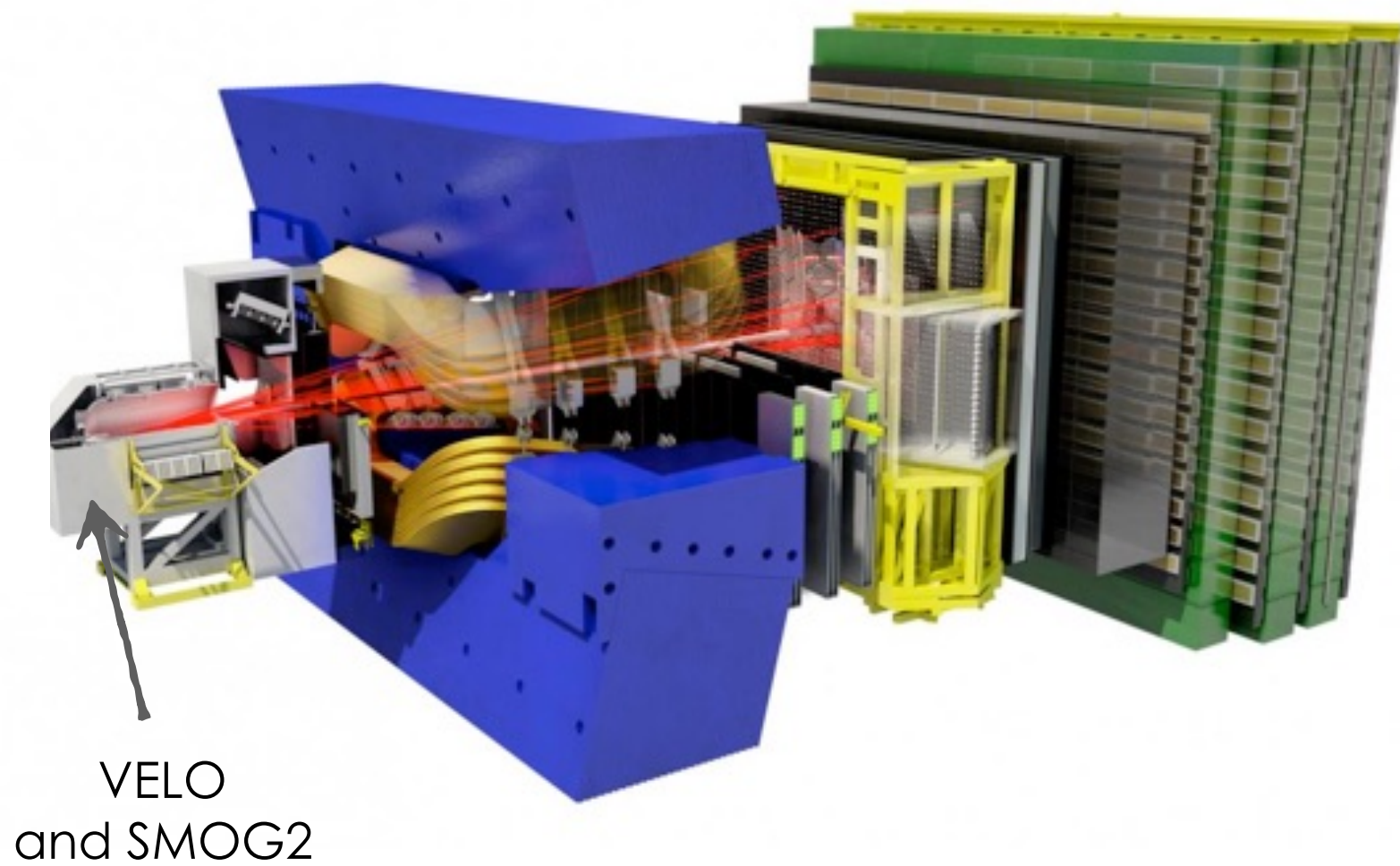
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# LHCb FIXED TARGET, INCLUDING POLARISATION

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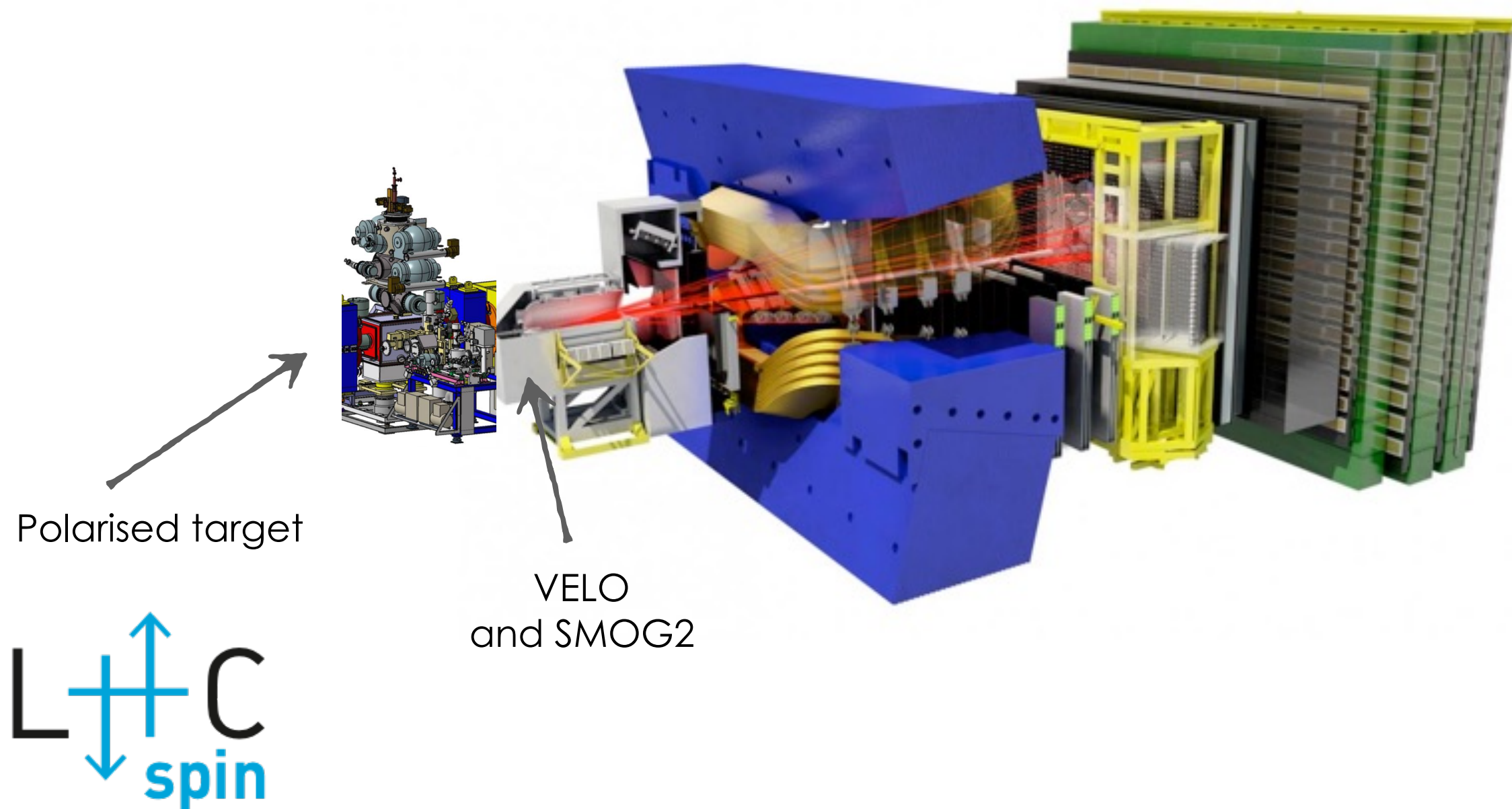
<https://indico.cern.ch/event/755856/>





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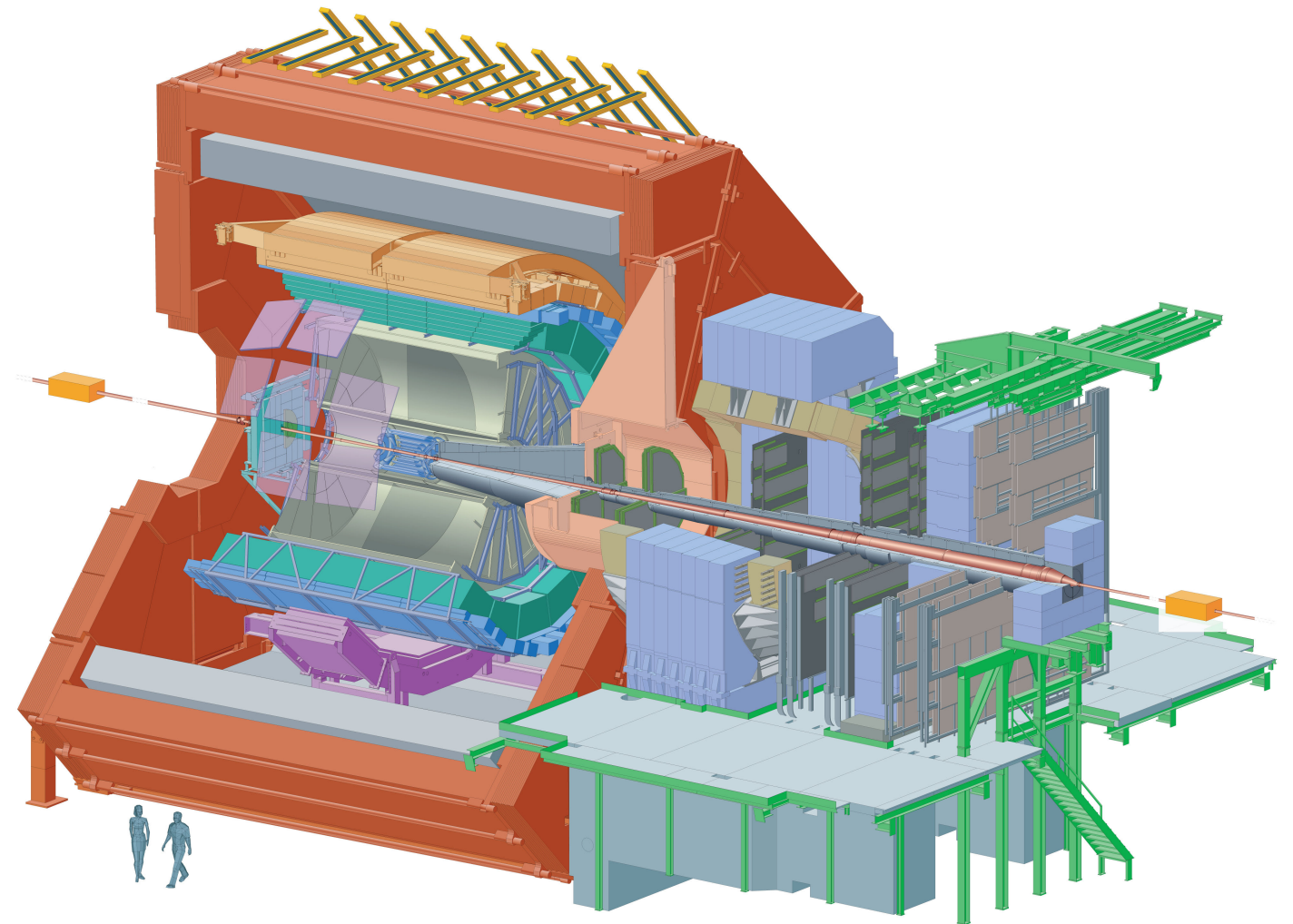
<https://indico.cern.ch/event/755856/>



# ALICE FIXED TARGET

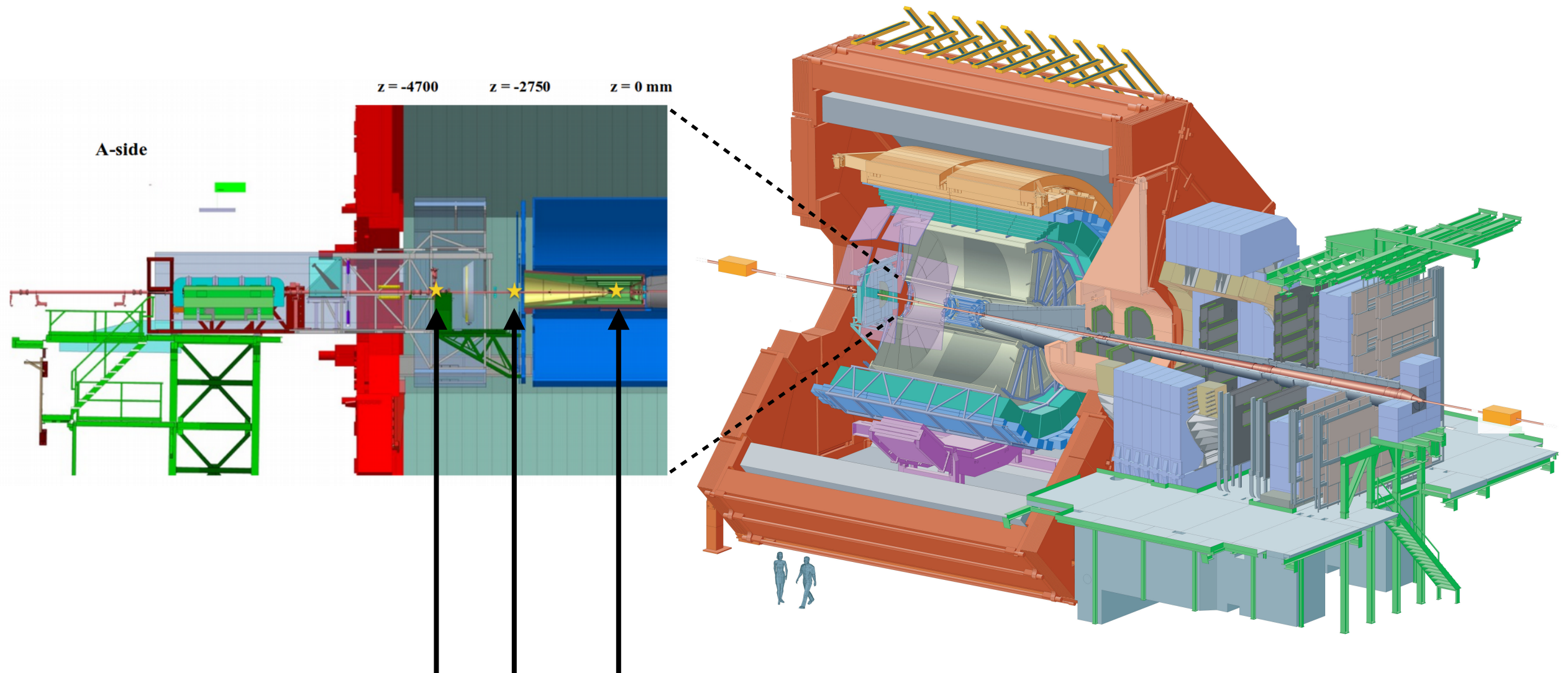
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<https://indico.cern.ch/event/755856/>



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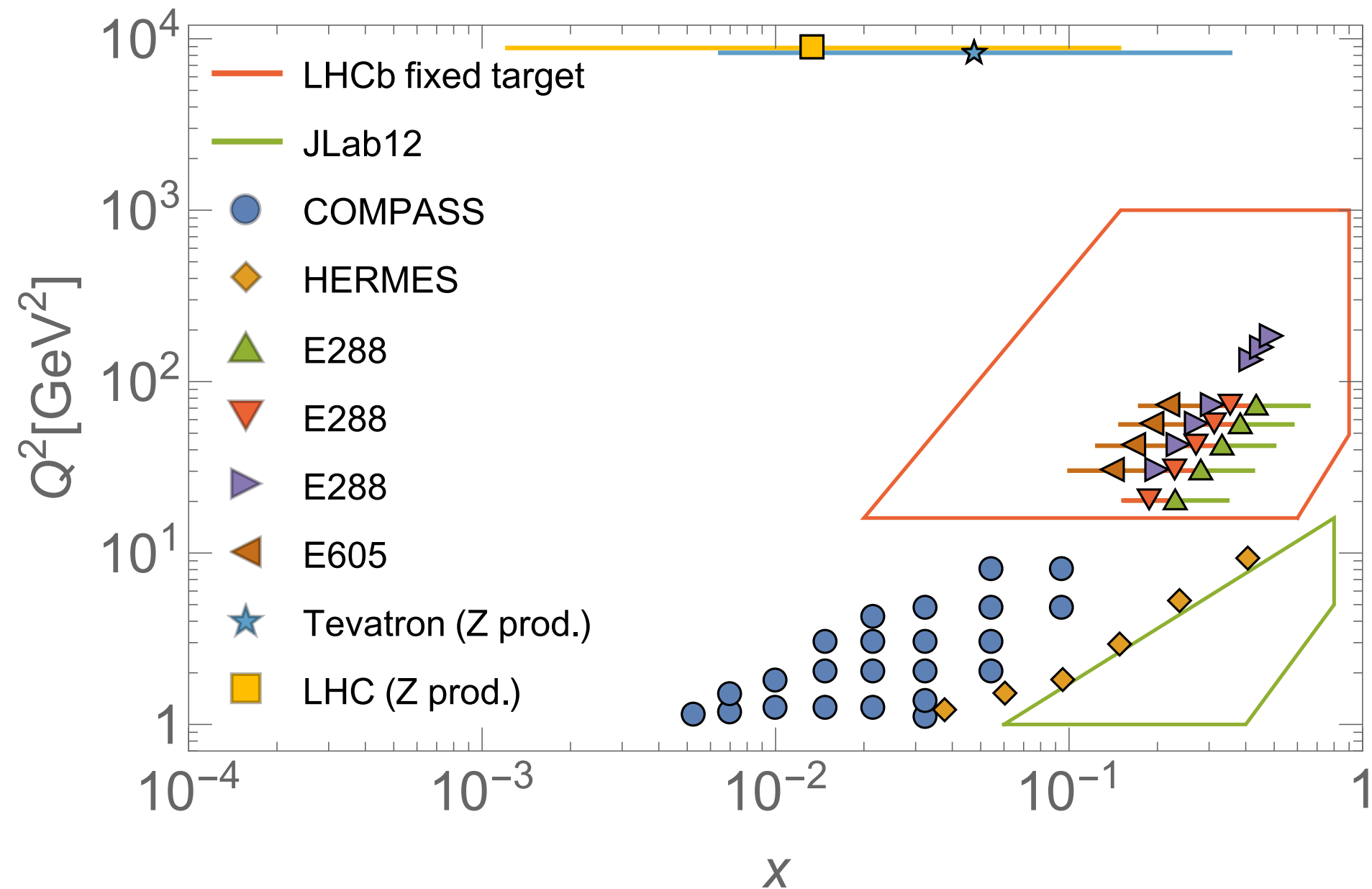


Possible fixed-target positioning



# EXPECTED EXTENSION OF DATA RANGE

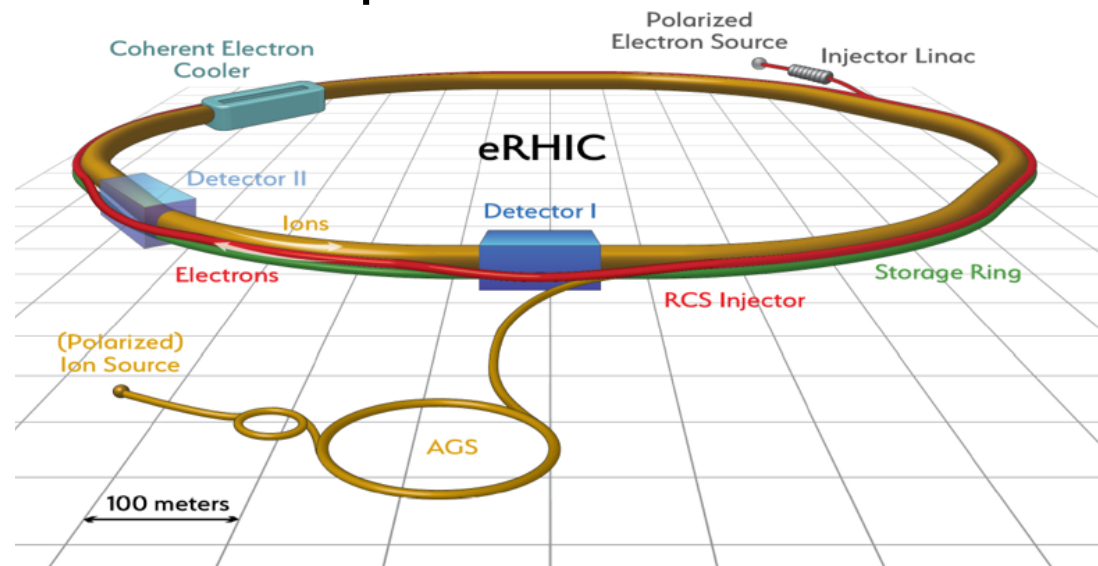
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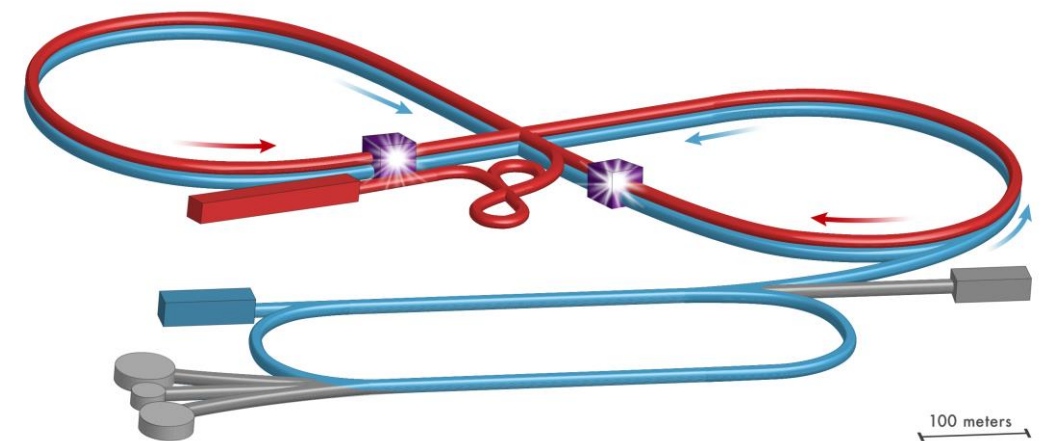
# THE ELECTRON-ION COLLIDER PROJECT

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## BNL concept



## JLab concept



- High luminosity: ( $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ )
- Variable CM energy: 20-100 GeV
- Highly polarized beams
- Protons and other nuclei

# Transversity 2020



25-29 May 2020  
Almo Collegio Borromeo, Pavia, Italy  
Europe/Rome timezone

## Overview

[Committees](#)

[Timetable](#)

[Registration](#)

[Participant List](#)

[Accommodation](#)

## Contacts

✉ [transversity2020@unipv.it](mailto:transversity2020@unipv.it)

✉ [info@pragmacongressi.it](mailto:info@pragmacongressi.it)

☎ +39 0382 309579

Transversity 2020 is the 6th international workshop on transverse polarization phenomena in hard processes, following those held in 2005 on Lake Como (Italy), 2008 in Ferrara (Italy), 2011 in Lošinj (Croatia), 2014 in Cagliari (Italy), and 2017 in Frascati (Italy)

The aim of the workshop is to provide an environment in which present theoretical and experimental knowledge in the field of transversity, transverse-momentum dependent distribution and fragmentation functions as well as generalised parton distribution functions will be presented and discussed in depth, together with new theoretical ideas and experimental perspectives. The workshop represents a valuable opportunity to gather the spin physics community, with a broad participation of theorists, as well as of experimentalists working in international collaborations at BEPC-II, BNL, CERN, DESY, KEK and Jefferson Lab (JLab), all deeply involved in this area of research. The workshop will also be a unique occasion for young researchers to form a detailed and up-to-date perspective on this fast-developing research field, and to present and discuss their own work and projects in a highly stimulating and reactive context.



<https://agenda.infn.it/e/transversity2020>

# CONCLUSIONS



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- Fragmentation functions need independent data
- Flavor dependence of TMDs still not well constrained
- We expect a steady flow of data coming up in the next years

**BACKUP SLIDES**

.....

# LOW- $b_T$ MODIFICATIONS

---

$$\log(Q^2 b_T^2) \rightarrow \log(Q^2 b_T^2 + 1)$$

*see, e.g., Bozzi, Catani, De Florian, Grazzini*  
[hep-ph/0302104](#)

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- The justification is to recover the integrated result (“unitarity constraint”)
- Modification at low  $b_T$  is allowed because resummed calculation is anyway unreliable there

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These are all choices that should be at some point checked/challenged

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$$\hat{f}_1^q(x, b_T; \mu^2) = \sum_i (C_{qi} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\text{NP}}^q(x, b_T)$$

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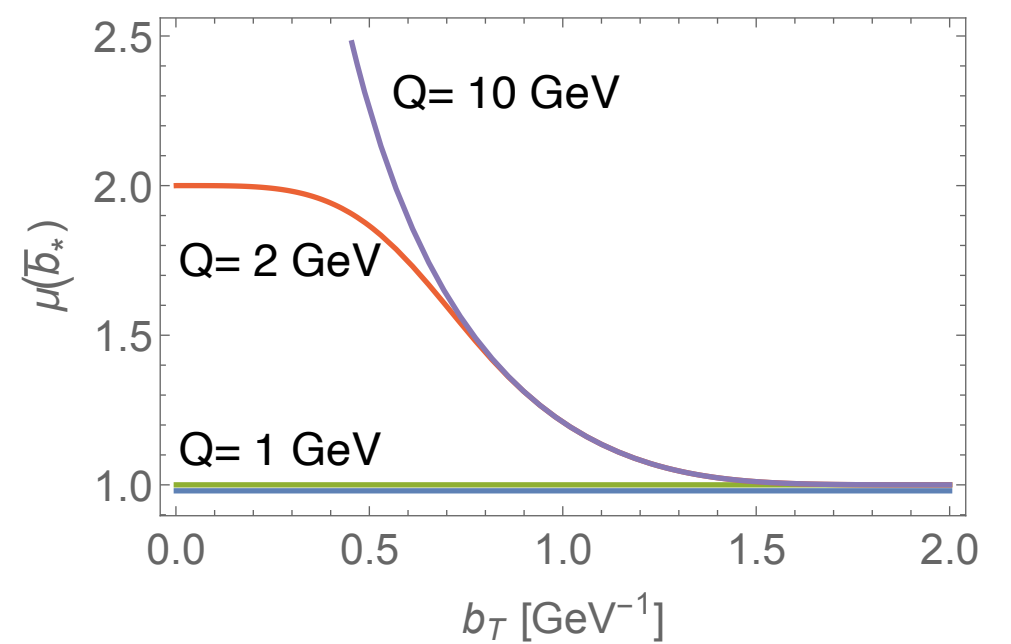
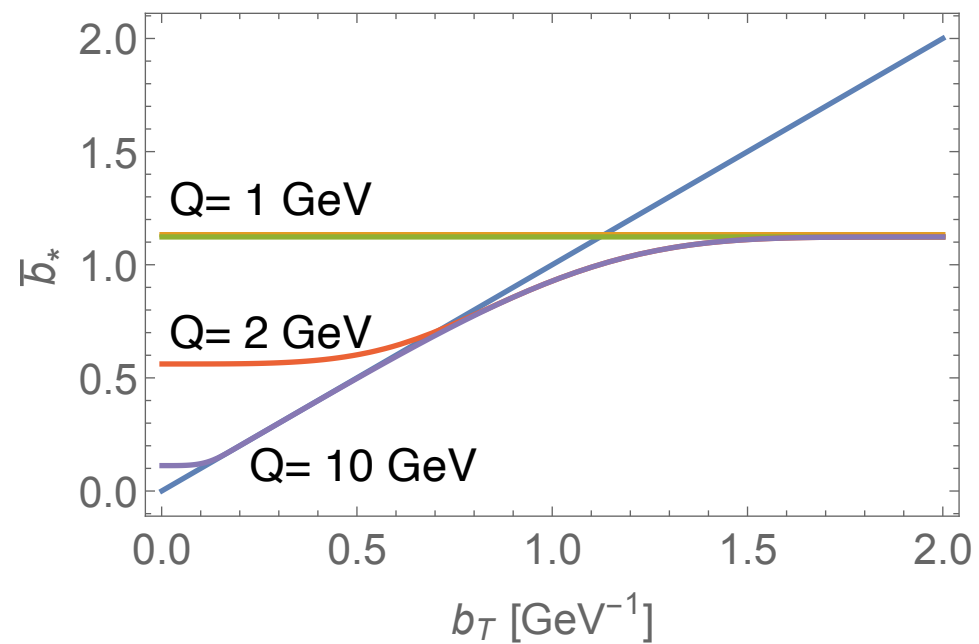
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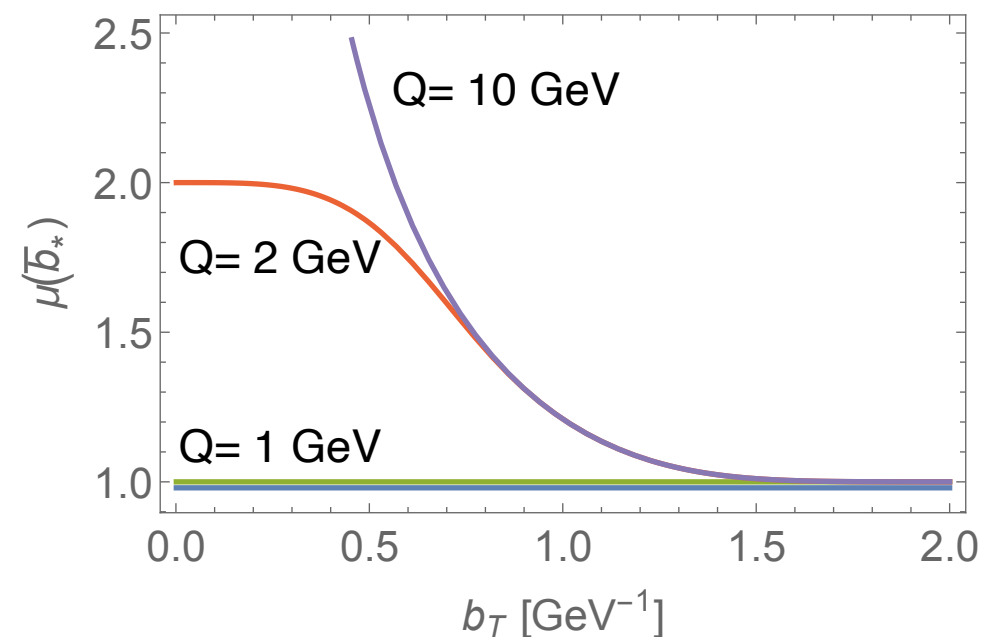
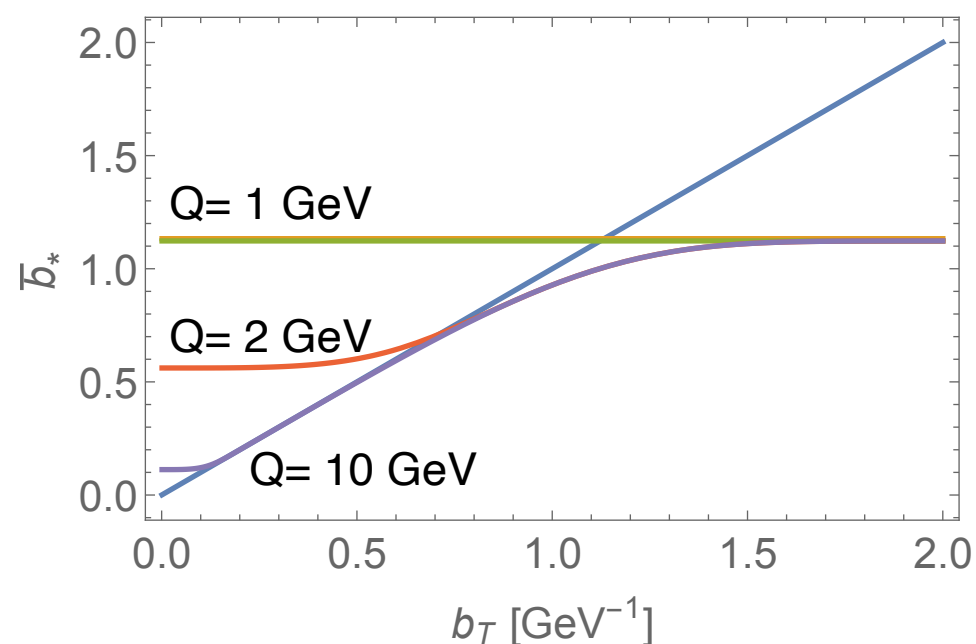
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No significant effect at high  $Q$ , but large effect at low  $Q$   
(inhibits perturbative contribution)

# DATA SELECTION IN PAVIA 2017

---

$$Q^2 > 1.4 \text{ GeV}^2$$

$$0.2 < z < 0.7$$

$$P_{hT}, q_T < \text{Min}[0.2 Q, 0.7 Qz] + 0.5 \text{ GeV}$$

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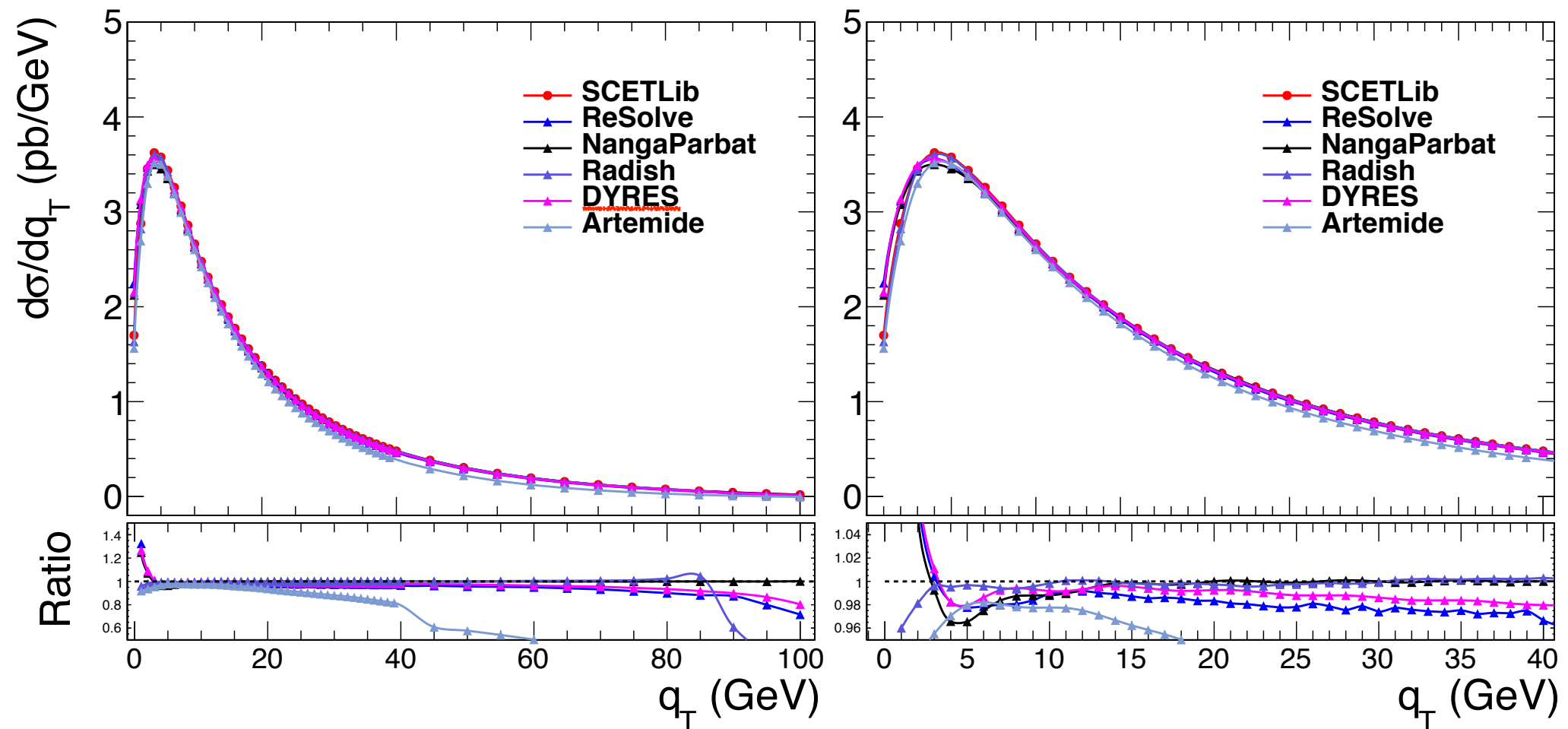
Total number of data points: 477

$$\text{Total } \chi^2/\text{dof} = 0.96$$

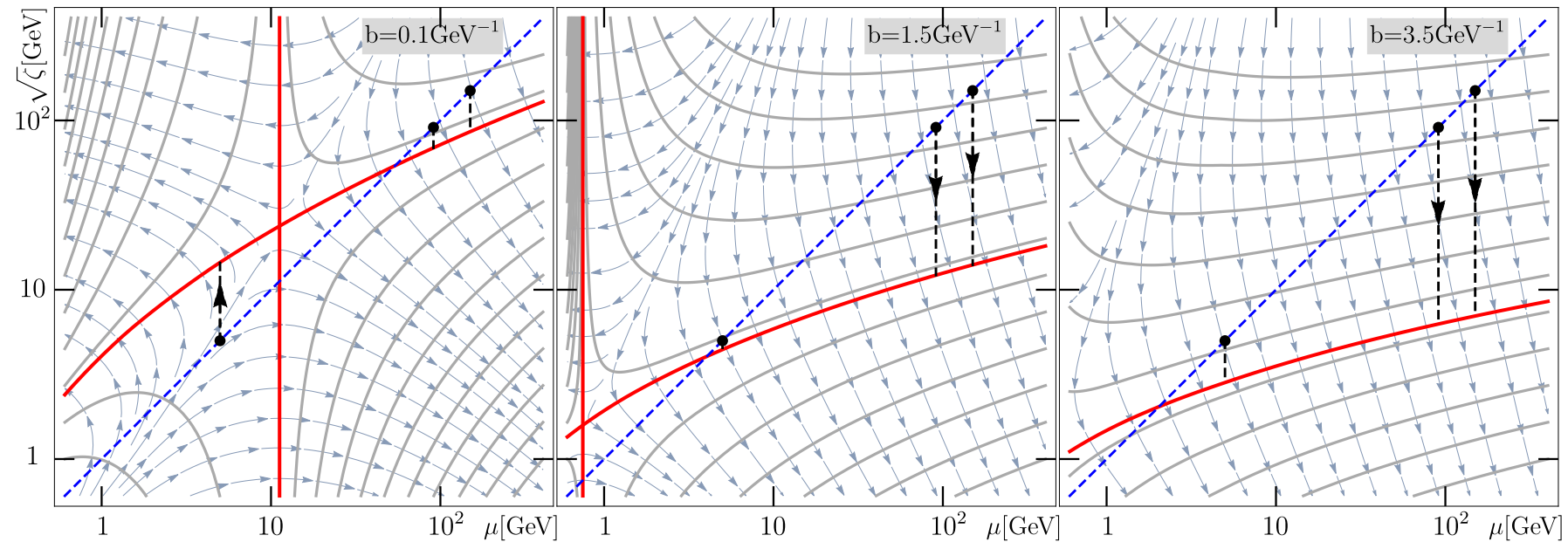
$$\text{Total } \chi^2/\text{dof} = 1.02$$

# BENCHMARKING OF DIFFERENT CODES

V. Bertone's talk at LHC EW WG General Meeting, Dec 2019  
<https://indico.cern.ch/event/849342/>



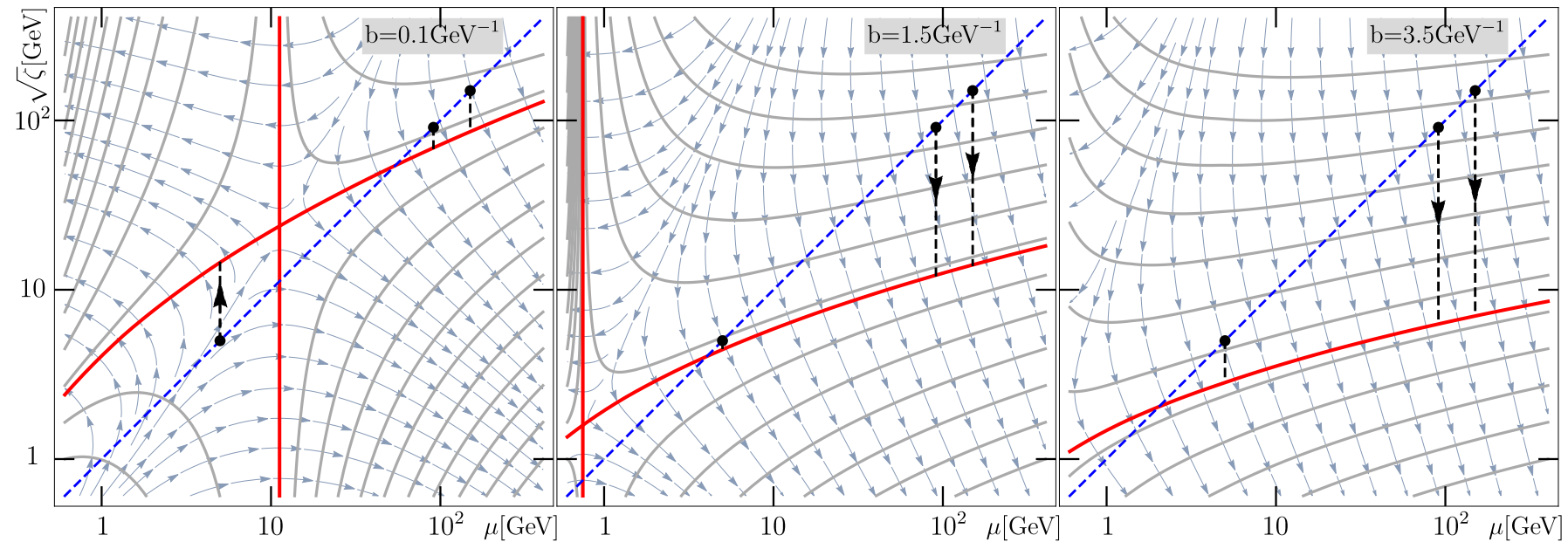
# TMDS AND TWO-SCALE EVOLUTION



The  $\zeta$ -prescription is equivalent to the popular CSS-scheme since it satisfies the same set of differential equations. Nonetheless, this equivalence is strict only within an all-order perturbation theory and it is numerically violated for any truncated series.

# TMDS AND TWO-SCALE EVOLUTION

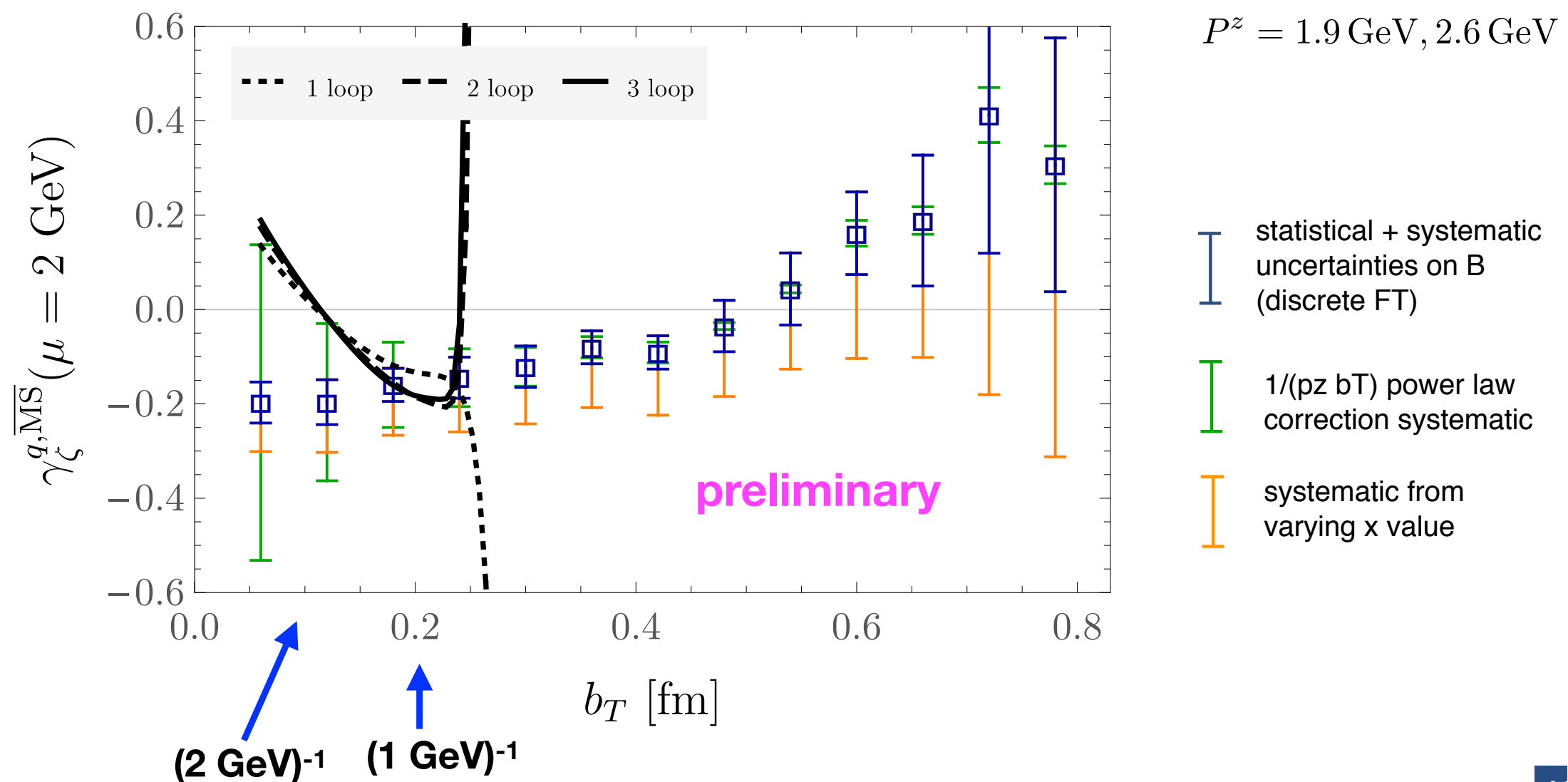
Scimemi, Vladimirov, arXiv:1912.06532



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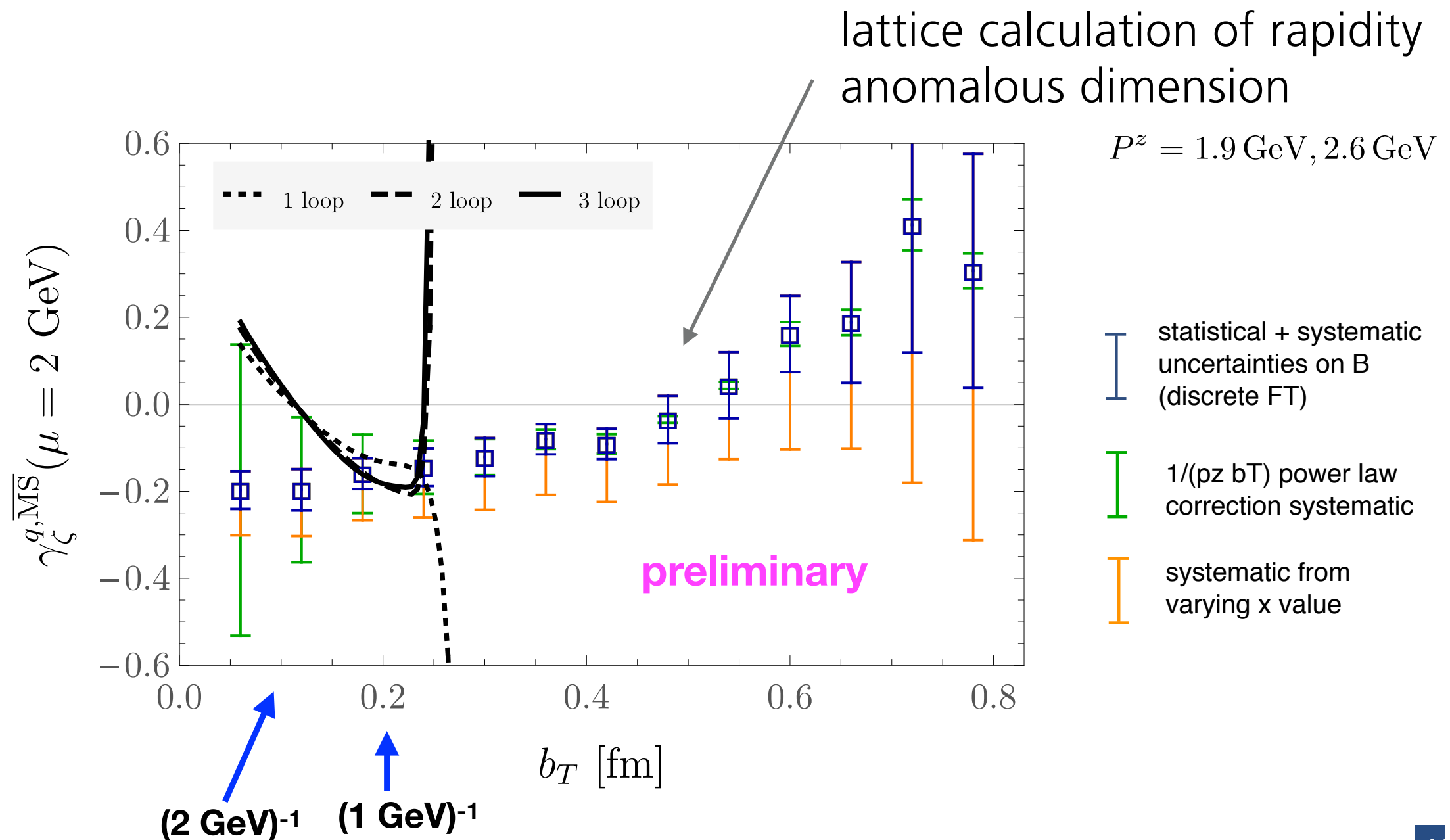
# NONPERTURBATIVE TMD EVOLUTION FROM LATTICE

talk by I. Stewart at REF2019, work in progress with P. Shanahan, M. Wagman, Y. Zhao



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