Extraction of polarized TMDs and the Nucleon Spin Structure

Marco Radici
INFN - Pavia
Transv.-Mom. Dependent Parton Distributions

TMD PDFs ($x, k_T^2; Q$) at leading twist

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<tr>
<th>Nucleon Polarization</th>
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<tr>
<td>U</td>
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<td>f_1 = ⊙</td>
<td></td>
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<td>L</td>
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Transv.-Mom. Dependent Parton Distributions

TMD PDFs \((x, k_T^2; Q)\) at leading twist

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<tr>
<td><strong>U</strong></td>
<td>(f_1 = \bullet)</td>
<td></td>
<td>(*)</td>
<td>(h_1^\perp = \uparrow - \downarrow)</td>
</tr>
<tr>
<td><strong>L</strong></td>
<td>(*)</td>
<td>(g_1 = \rightarrow - \rightarrow)</td>
<td></td>
<td>(h_{1L}^\perp = \rightarrow - \rightarrow)</td>
</tr>
<tr>
<td><strong>T</strong></td>
<td>(f_1^T = \bullet - \bullet)</td>
<td>(g_1^T = \rightarrow - \rightarrow)</td>
<td></td>
<td>(h_{1T}^\perp = \downarrow - \uparrow)</td>
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* forbidden by parity invariance

Mulders & Tangerman, N.P. B461 (96)
Boer & Mulders, P.R. D57 (98)
Bacchetta et al., JHEP 02 (07) 093
**Transv.-Mom. Dependent Parton Distributions**

![Diagram of quark and nucleon with k_T and x vectors]

TMD PDFs \((x,k_T^2; Q)\) at leading twist

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<td>*</td>
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</tr>
<tr>
<td>(L)</td>
<td>*</td>
<td>(g_1 = \bigcirc) - (\bigcirc)</td>
<td>(h_{1L}^\perp = \bigcirc - \bigcirc)</td>
<td></td>
</tr>
<tr>
<td>(T)</td>
<td>(f_{1T}^\perp = \bigcirc) - (\bigcirc) (g_{1T} = \bigcirc) - (\bigcirc)</td>
<td>(h_1^\perp = \bigcirc - \uparrow)</td>
<td>(h_{1T}^\perp = \bigcirc - \uparrow)</td>
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* forbidden by parity invariance

3D maps of
- partonic quantum correlations: spin-spin, spin-momentum (orbit)
- quantum correlations between partonic motion and macroscopic nucleon properties (spin)
- partonic orbital motion (most TMDs vanish with no \(Lq\))

---

* Mulders & Tangerman, N.P. B461 (96)
* Boer & Mulders, P.R. D57 (98)
* Bacchetta et al., JHEP 02 (07) 093
Transv.-Mom. Dependent Parton Distributions

TMD PDFs \((x, k_T^2; Q)\) at leading twist

3D maps of
- partonic quantum correlations: spin-spin, spin-momentum (orbit)
- quantum correlations between partonic motion and macroscopic nucleon properties (spin)
- partonic orbital motion (most TMDs vanish with no \(Lq\))
- color (gauge-inv.) residual interactions and (no) T-reversal symmetry

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Quark polarization

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<td>U (f_1 = )</td>
<td>(\ast)</td>
<td>(h_{1L}^T = )</td>
<td>(h_{1T}^L = )</td>
</tr>
<tr>
<td>L (g_1 = )</td>
<td>(\ast)</td>
<td>(h_{1L}^T = )</td>
<td>(h_{1T}^L = )</td>
</tr>
<tr>
<td>T (f_{1T}^L = )</td>
<td>(g_{1T} = )</td>
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* forbidden by parity invariance

3D maps of

- partonic quantum correlations: spin-spin, spin-momentum (orbit)
- quantum correlations between partonic motion and macroscopic nucleon properties (spin)
- partonic orbital motion (most TMDs vanish with no \(L^q\))
- color (gauge-inv.) residual interactions and T-reversal odd symmetry
- helicity-flipping (chiral-odd) structures; need a chiral-odd partner in the cross section

Mulders & Tangerman, N.P. \textit{B461} (96)
Boer & Mulders, P.R. \textit{D57} (98)
Bacchetta et al., \textit{JHEP 02} (07) 093
Transv.-Mom. Dependent Fragmentations

TMD FFs \((z, p_T^2; Q)\) at leading twist and \(S_h \leq 1/2\)

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<td>(H_{1T}^\perp)</td>
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<td>L</td>
<td></td>
<td>(G_{1L})</td>
<td>(H_{1L}^\perp)</td>
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<td>(D_{1T}^\perp)</td>
<td>(G_{1T})</td>
<td>(H_1^\perp)</td>
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hadron

quark
Transv.-Mom. Dependent Fragmentations

TMD FFs \((z, p_T^2; Q)\) at leading twist and \(S_h \leq 1/2\)

Quark polarization

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<td>(D_1)</td>
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<tr>
<td>(G_{1L})</td>
<td></td>
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<td></td>
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<tr>
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<td></td>
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most of the time, detection of final unpolarized mesons \((\pi, K..)\)
⇒ use only first row of table
Transv.-Mom. Dependent Fragmentations

TMD FFs \((z, p_T^2; Q)\) at leading twist and \(S_h \leq 1/2\)

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<td>(G_{1T})</td>
<td>(H_{1T}^\perp)</td>
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Recent data on \(\Lambda^+\) production \(\rightarrow\) access to \(D_{1T}^\perp\)?

From BELLE: Abdesselam et al. (BELLE), arXiv:1611.06648

(See also old data from Fermilab, Hera-B and CERN-NA48/OPAL + new data from CERN-ATLAS)
Factorization theorems well understood for \( q_T \ll Q \)

**Drell-Yan**

- proton
- proton
- \( q_T \)
- \( Q^2 \)
- TMD PDFs

**e^+e^- annihilation**

- electron
- positron
- \( q_T \)
- \( h \)
- \( Q^2 \)
- TMD FFs

---

Rogers & Aybat, P.R. *D83* (11)  
Collins, “Foundations of Perturbative QCD” (11)  
Echevarria, Idilbi, Scimemi, JHEP *1207* (12)
Factorization theorems well understood for $q_T \ll Q$

**Drell-Yan**
- TMD PDFs

**$e^+e^-$ annihilation**
- TMD FFs

but only very few (recent) data with polarization

Rogers & Aybat, P.R. *D83* (11)
Collins, “Foundations of Perturbative QCD” (11)
Echevarria, Idilbi, Scimemi, JHEP *1207* (12)
TMDs depend on two scales

most data from polarized Semi-Inclusive DIS (SIDIS) under the form of spin asymmetries

\[ q_T = P_{hT}/z = |p_T - k_T| \ll Q \]

**Factorization theorem** valid for \( q_T \ll Q \)

two scales:
- hard \( Q \) to “see” partons
- soft \( q_T \ll Q \) to be sensitive to \( k_T \) motion of confined partons

\[ J_i, Yuan, Ma, P.R. D71 \ (05) \]
\[ Rogers & Aybat, P.R. D83 \ (11) \]
\[ Collins, “Foundations of Perturbative QCD” (11) \]
\[ Echevarria, Idilbi, Scimemi, JHEP 1207 (12) \]
polarized Semi-Inclusive DIS

dominant diagram
(from diagrammatic approach to OPE)
polarized Semi-Inclusive DIS

Quark polarization

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<td>$g_{1T} = \bigcirc - \bigcirc$</td>
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(From diagrammatic approach to OPE)

dominant diagram

(proton, deuteron, ...

pion, Kaon, ...
polarized Semi-Inclusive DIS

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Quark polarization

- Unpolarized (U)
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Nucleon Polarization

Unpolarized (U)

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Longitudinally Polarized (L)

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Transversely Polarized (T)

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<td>$f_{1T} = \bullet - \bullet$</td>
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dominant diagram

(from diagrammatic approach to OPE)
polarized Semi-Inclusive DIS

\[ \int \frac{d\sigma}{dx\,dy\,dz\,d\phi_h\,dP^2_{hT}} \sim \]

\[ A(y) \, F_U + B(y) \, \cos 2\phi_h \, F_U^{\cos 2\phi_h} \]

\[ + C(y) \, F_{LL} + B(y) \, \sin 2\phi_h \, F_L^{\sin 2\phi_h} \]

\[ + A(y) \, \sin(\phi_h - \phi_S) \, F_T^{\sin(\phi_h - \phi_S)} \]

\[ + B(y) \, \sin(\phi_h + \phi_S) \, F_T^{\sin(\phi_h + \phi_S)} \]

\[ + B(y) \, \sin(3\phi_h - \phi_S) \, F_T^{\sin(3\phi_h - \phi_S)} \]

\[ + C(y) \, \cos(\phi_h - \phi_S) \, F_{LT}^{\cos(\phi_h - \phi_S)} \]

8 structures at leading twist
(more at subleading twist...)

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<td>( h^\perp_1 = )</td>
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<td>( h_{1L} = )</td>
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polarized Semi-Inclusive DIS

\[ \frac{d\sigma}{dx \, dy \, dz \, d\phi_h \, dP_{hT}^2} \sim \]

\[ A(y) \, F_U + B(y) \, \cos 2\phi_h \, F_U^{\cos 2\phi_h} \]

\[ + \, C(y) \, F_{LL} + B(y) \, \sin 2\phi_h \, F_L^{\sin 2\phi_h} \]

\[ + \, A(y) \, \sin(\phi_h - \phi_S) \, F_T^{\sin(\phi_h - \phi_S)} \]

\[ + \, B(y) \, \sin(\phi_h + \phi_S) \, F_T^{\sin(\phi_h + \phi_S)} \]

\[ + \, B(y) \, \sin(3\phi_h - \phi_S) \, F_T^{\sin(3\phi_h - \phi_S)} \]

\[ + \, C(y) \, \cos(\phi_h - \phi_S) \, F_{LT}^{\cos(\phi_h - \phi_S)} \]

Each structure is a convolution in \( k_T, p_T \):

\[ F \sim [\text{TMDPDF}(x, k_T) \otimes \text{TMDFF}(z, p_T)] \]
polarized Semi-Inclusive DIS

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\[
\frac{d\sigma}{dx \, dy \, dz \, d\phi_h \, dP_{hT}} \sim \\
A(y) \, F_U + B(y) \, \cos 2\phi_h \, F_U^{\cos 2\phi_h} + C(y) \, F_{LL} + B(y) \, \sin 2\phi_h \, F_L^{\sin 2\phi_h} \\
+ A(y) \, \sin(\phi_h - \phi_S) \, F_T^{\sin(\phi_h - \phi_S)} + B(y) \, \sin(\phi_h + \phi_S) \, F_T^{\sin(\phi_h + \phi_S)} + B(y) \, \sin(3\phi_h - \phi_S) \, F_T^{\sin(3\phi_h - \phi_S)} \\
+ C(y) \, \cos(\phi_h - \phi_S) \, F_{LT}^{\cos(\phi_h - \phi_S)}
\]

requires polarized electron

each structure is a convolution in \( k_T, p_T \): 
\[
F \sim [\text{TMDPDF}(x, k_T) \otimes \text{TMDFF}(z, p_T)]
\]
Quark polarization

Unpolarized (U) | Longitudinally Polarized (L) | Transversely Polarized (T)
--- | --- | ---
Unpolarized (U) | D₁ | H₁⁺ -

Nucleon Polarization

Unpolarized (U) | Longitudinally Polarized (L) | Transversely Polarized (T)
--- | --- | ---
U | f₁ =.foreach structure is a convolution in kₜ, pₜ: F ≈ [TMDPDF(x, kₜ) ⊗ TMDFF(z, pₜ)]
Azimuthal Spin Asymmetries

SIDIS kinematics & cross section

\[
\frac{d\sigma}{dx \, dy \, dz \, d\phi_h \, dP_{hT}^2} \sim A(y) \, F_U + B(y) \cos 2\phi_h \, F_U^{\cos 2\phi_h}
\]
\[
+ C(y) \, F_{LL} + B(y) \sin 2\phi_h \, F_L^{\sin 2\phi_h}
\]
\[
+ A(y) \sin(\phi_h - \phi_S) \, F_T^{\sin(\phi_h - \phi_S)}
\]
\[
+ B(y) \sin(\phi_h + \phi_S) \, F_T^{\sin(\phi_h + \phi_S)}
\]
\[
+ B(y) \sin(3\phi_h - \phi_S) \, F_T^{\sin(3\phi_h - \phi_S)}
\]
\[
+ C(y) \cos(\phi_h - \phi_S) \, F_{LT}^{\cos(\phi_h - \phi_S)}
\]

- build
\[
\frac{d\sigma^\uparrow(\phi_h, \phi_S) - d\sigma^\downarrow(\phi_h, \phi_S + \pi)}{d\sigma^\uparrow(\phi_h, \phi_S) + d\sigma^\downarrow(\phi_h, \phi_S + \pi)}
\]
(or similar when electron is polarized)

- isolate specific azimuthal component, coefficient \( F_{XX}(x, z, P_{hT}^2; Q^2) \)

\[
A_{XX}^x(x, z, P_{hT}^2, Q^2) \equiv \frac{F_{XX}(x, z, P_{hT}^2; Q^2)}{F_U(x, z, P_{hT}^2; Q^2)}
\]
All 8 asymmetries have been measured (plus more at subleading twist..)

\[ A_U^{\cos 2\phi_h} \text{ on p & D targets} \quad \text{arXiv:1204.4161} \]
\[ A_L^{\sin 2\phi_h} \text{ on p} \rightarrow \text{ hep-ph/0608048} \]
\[ A_T^{\sin(\phi_h-\phi_S)} \text{ on p}^{\uparrow} \quad \text{arXiv:0906.3918} \]
\[ A_T^{\sin(\phi_h+\phi_S)} \text{ on p}^{\uparrow} \quad \text{arXiv:1006.4221} \]

\[ A_U^{\cos 2\phi_h} \text{ on D targets} \quad \text{arXiv:1401.6284} \]
\[ A_{LL} \text{ on p} \rightarrow \text{ arXiv:1509.03526} \]
\[ A_{LL} \text{ & } A_L^{\sin 2\phi_h} \text{ on p} \rightarrow \text{ arXiv:1509.03526 \ & \ d} \rightarrow \text{ arXiv:1609.06062} \]
\[ A_T^{\sin(\phi_h-\phi_S)} \text{ on p}^{\uparrow} \{ \text{arXiv:1205.5122} \quad \text{arXiv:1609.07374} \} \quad \text{arXiv:1005.5609} \quad \text{arXiv:1408.4405} \]
\[ A_T^{\sin(\phi_h+\phi_S)} \text{ on p}^{\uparrow} \quad \text{arXiv:1205.5121} \]
\[ A_T^{\sin(\phi_h\pm\phi_S)} \text{ on D}^{\uparrow} \quad \text{arXiv:0802.2160} \]
\[ A_T^{\sin(\phi_h\pm\phi_S)} \text{ & } A_{LT}^{\cos(\phi_h-\phi_S)} \text{ & } A_T^{\sin(3\phi_h-\phi_S)} \text{ on p}^{\uparrow} \quad \text{arXiv:1512.06590} \]

**Jefferson Lab**

**Hall A**
\[ A_T^{\sin(\phi_h\pm\phi_S)} \text{ on } {^3}\text{He}^{\uparrow} \text{ with } \pi \quad \text{arXiv:1106.0363} \]
\[ A_T^{\sin(\phi_h-\phi_S)} \text{ on } {^3}\text{He}^{\uparrow} \text{ with } \pi \quad \text{arXiv:1311.1866} \]

**Hall B**
\[ A_{LL} \text{ & } A_L^{\sin 2\phi_h} \text{ on p} \rightarrow \text{ arXiv:1003.4549} \]

**Hall B**
\[ A_{LT}^{\cos(\phi_h-\phi_S)} \text{ on } {^3}\text{He}^{\uparrow} \text{ arXiv:1108.0489} \]
\[ A_T^{\sin(3\phi_h-\phi_S)} \text{ on } {^3}\text{He}^{\uparrow} \text{ arXiv:1312.3047} \]
Two Most Relevant Cases

The EIC White Paper

Accardi et al., E.P.J. A52 (16) 268, arXiv:1212.1701, see also Boer et al., arXiv:1108.1713

Table 2.2: Science Matrix for TMD: 3D structure in transverse momentum space: (upper) the golden measurements; (lower) the silver measurements.
The Sivers effect

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- Unpolarized
- Longitudinally Polarized
- Transversely Polarized

The Sivers function

(spin-orbit) correlation between \( k_T \) of parton and \( S_T \) of proton
distortion of quark distribution in transv. polarized proton \( P^\uparrow \)

Sivers, P.R. \textit{D41} (90) 83
The Sivers effect

Bacchetta & Contalbrigo, Il Nuovo Saggiatore 28 (12) n. 1,2

no polarization

polarization $S_y$

distortion of quark distribution in transversely polarized proton $P^\uparrow$

$$f_{q/p^\uparrow}(x, k_T) = f_1^q(x, k_T^2) - f_{1T}^q(x, k_T^2) S \cdot \left( \frac{\hat{P}}{M} \times k_T \right)$$

density of $q$ in (proton)$^\uparrow$

Sivers, P.R. D41 (90) 83

deflection along $x$ depends on flavor

$f_{1T}^u < 0$

$f_{1T}^d > 0$
Non-universality of Sivers function

Sivers effect apparently forbidden by T-rev. invariance but Sivers function entirely given by residual color interactions, that restore color-gauge invariance

“final-state” color interactions
Non-universality of Sivers function

Sivers effect apparently forbidden by T-rev. invariance but Sivers function entirely given by residual color interactions, that restore color-gauge invariance

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the Sivers function can be extracted also from transv. polarized Drell-Yan $p^\uparrow + p \rightarrow \ell^+ + \ell^- + X$

“initial-state” color interactions
Non-universality of Sivers function

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"final-state" color interactions

the Sivers function can be extracted also from transv. polarized Drell-Yan \( p^\uparrow + p \rightarrow \ell^+ + \ell^- + X \)

"initial-state" color interactions

QCD prediction to be tested:

\[
\text{Sivers}_{\text{SIDIS}} = -\text{Sivers}_{\text{D-Y}}
\]

Collins, P.L. B536 (02)
Non-universality of Sivers on lattice

quark density

\[ f_{q/p^+}(x, k_T) = f_1^q(x, k_T^2) - f_{1T}^{+q}(x, k_T^2) S \cdot \left( \frac{\hat{P}}{M} \times k_T \right) \]

Fourier transform

\[ \tilde{f}_{q/p^+}(b^-, b_T) \sim \tilde{f}_1^{[1](0)} - i \tilde{f}_{1T}^{[1](1)} S \cdot \left( \frac{\hat{P}}{M} \times k_T \right) \]

Sivers shift

\[ \propto \langle P, S | q(0) \gamma^+ U[0, \eta \nu, \eta \nu + b, b] | q(b) \rangle |P, S\rangle \]

see Michael Engelhardt’s talk

Yoon et al., P.R. D96 (17) 094508 arXiv:1706.03406
Exp. test of Sivers sign change

first data on transversely polarized Drell-Yan

Adamczyk et al., PRL 116 (16) 132301
arXiv:1511.06003

\[ p^\uparrow + p \rightarrow W^{\pm}/Z^0 + X \]

Aghasyan et al., PRL 119 (17) 112002
arXiv:1704.00488

\[ \pi^- + p^\uparrow \rightarrow \ell^+ \ell^- X \]

hints of sign change statistically favored

Kang & Qiu,
PRL 103 (09) 172001

Echevarria et al., PR D89 (14) 074013
arXiv:1401.5078

Sun & Yuan, PR D88 (13) 114012
arXiv:1308.5003

DGLAP Anselmino et al., JHEP 1704 (17) 046
arXiv:1612.06413
Exp. test of Sivers sign change

first data on transversely polarized Drell-Yan

\[ p^+p \rightarrow W^\pm/Z^0 + X \]

\[ A_N(\sqrt{s} = 500\text{ GeV}) \]

prediction with TMD evolution (??)

hints of sign change statistically favored

Kang & Qiu,
\textit{PRL} 103 (09) 172001

Echevarria et al.,
\textit{PR} D89 (14) 074013
arXiv:1401.5078

Aghasyan et al., \textit{PRL} 119 (17) 112002
arXiv:1704.00488

\[ \pi^- + p^+ \rightarrow \ell^+ \ell^- + X \]

\[ A_{\sin\phi}(\sqrt{s} = 500\text{ GeV}) \]

With sign change

Without sign change

TMD-1

\textit{Echevarria et al., PR} D89 (14) 074013
arXiv:1401.5078

TMD-2

\textit{Sun & Yuan, PR} D88 (13) 114012
arXiv:1308.5003

DGLAP

\textit{Anselmino et al., JHEP} 1704 (17) 046
arXiv:1612.06413
N↑ polarized along y (≈ spin-orbit effect)

charge distribution

distorted along x:

up mostly at x>0
donw at x<0

distortion described by function $E^q$ (GPD)

see Hatta’s talk
Sivers ↔ Nucleon spin?

\[ 1/2 = \sum_q J_y^q (Q^2) \]

charge distribution
distorted along \( x \):
up mostly at \( x > 0 \)
down at \( x < 0 \)

\[ J^q (Q^2) = \text{total angular momentum from parton } q \]

\[ = \frac{1}{2} \int_0^1 dx \ x \ [f_1^q(x, Q^2) + E^q(x, Q^2)] \]

forward limit of \( E^q \)

see Hatta’s talk

distortion described by function \( E^q \) (GPD)

\[ N^\uparrow \text{ polarized along } y \]

(\( \sim \text{spin-orbit effect} \))

charge distribution

distorted along \( x \):
up mostly at \( x > 0 \)
down at \( x < 0 \)

\[ f_{1T}^{-q} \]

distortion in position space

Sivers

\[ E^q \]

distortion in momentum space
The color lensing effect

$N^\uparrow$ polarized along $y$

- distortion along $x$: 
  - **up** mostly at $x>0$
  - **down** at $x<0$

- quarks hit by virtual photon
- try to escape color FSI

- **up** bended to $x<0$
- **down** bended to $x>0$

**described by E**

![Diagram of quark behavior](image)

- $\pi^+$ ($\pi^-$)
  - $\phi = \pi$  $\phi_S = \pi/2$
  - $\sin(\phi_h - \phi_S) > 0$

- $\pi^-$ ($\pi^+$)
  - $\phi = 0$  $\phi_S = \pi/2$
  - $\sin(\phi_h - \phi_S) < 0$

**Sivers $- f_{1T}^\perp$**

Burkardt, P.R. *D66* (02) 114005
The color lensing effect

N↑ polarized along y
- distortion along x:
  - **up** mostly at x>0
  - **down** at x<0

- quarks hit by virtual photon try to escape color FSI
- **up** bended to x<0
- **down** bended to x>0

- described by E
- color lensing $L(x)$
- described by Sivers $-f_{1T}^{\perp}$

- Burkardt, P.R. *D66* (02) 114005

- Bacchetta & Radici, *P.R.L.* 107 (11) 212001

- $\int d\mathbf{k}_T f_{1T}^{\perp q}(x, \mathbf{k}_T; Q_L^2) = -L(x) E^q(x, 0, 0; Q_L^2)$
Sivers function ↔ quark total $J$

- extract Sivers $f_{1T \perp}$ from data +
- model lensing function $L(x)$ +
- Ji's sum rule

models of GPD $E$

Bacchetta & Radici, PRL 107 (11) 212001

color lensing

Pasquini, Rodini, Bacchetta, PR D100 (19) 054039, arXiv:1907.06960

( applicable only to 2-body systems )

Goloskokov & Kroll, EPJ C59 (09) 809
Diehl & Kroll, E.P.J. C73 (13) 2397
Diehl et al., EPJ C39 (05) 1
Guidal et al., PR D72 (05) 054013
Liuti et al., PRD 84 (11) 034007

Pasquini, Rodini, Bacchetta, PR D100 (19) 054039, arXiv:1907.06960

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Liuti et al., PRD 84 (11) 034007

Bacchetta & Radici, PRL 107 (11) 212001

models of GPD $E$

color lensing

lattice
the Sivers Spin Asymmetry in SIDIS

several extractions of Sivers \( f_{1T} \downarrow \):

\[ A_T^{\sin(\phi_h - \phi_S)} \propto \frac{f_{1T} \otimes D_1}{f_1 \otimes D_1} \]

from global fit PV17 of data on SIDIS, DY and Z-boson

\[ \text{Bacchetta et al., JHEP 1706 (17) 081; E 1906 (19) 051} \]
\[ \text{arXiv:1703.10157} \]
(see previous talk)

First extraction of Sivers function using in denominator unpolarized TMDs from global fit consistently in the same TMD framework

\[ \text{Bacchetta, Delcarro, Pisano, Radici, in preparation} \]
The PV19 fit of Sivers $f_{1T\perp}$ (preliminary)

**Bacchetta, Delcarro, Pisano, Radici, in preparation**

**data coverage**

- **proton [H]**
  - HERMES: 95 data points
  - Jefferson Lab: 6 data points
  - *COMPASS* 2009: 88 data points
  - *COMPASS* 2017: 111 data points
  - Qian et al., *P.R.L.* 107 (11) 072003
  - Airapetian et al., *P.R.L.* 103 (09) 152002
- **deuteron [$^6$LiD]**
  - Alekseev et al., *P.L.* B673 (09) 127
- **neutron [$^3$He]**
  - Adolph et al., *P.L.* B770 (17) 138
  - *COMPASS* hadron in the final state. We observe that our parametrization is able to describe very well the predictions, with an overall value of $x^2/\text{d.o.f.} = 1.06 \pm 0.10$.

**Statistical error with replica method (200)**

**Table II: Number of included data points, of free parameters and values of global $x^2/\text{d.o.f.}$**

<table>
<thead>
<tr>
<th>Points</th>
<th>$#$ fit parameters</th>
<th>$x^2/\text{d.o.f.}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>117</td>
<td>14</td>
<td>1.06 ± 0.10</td>
</tr>
</tbody>
</table>

**Q^2 \geq 1.4 \text{ GeV}^2 \quad 0.2 \leq z \leq 0.7 \quad P_{HT} < \min[0.2 \, Q, 0.7 \, Qz] + 0.5 \text{ GeV}**

<table>
<thead>
<tr>
<th>$x$, $z$, $P_{HT}$ data projections</th>
<th>$x^2/\text{d.o.f.}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.0$</td>
<td>$60$</td>
</tr>
<tr>
<td>$1.1$</td>
<td>$50$</td>
</tr>
<tr>
<td>$1.2$</td>
<td>$40$</td>
</tr>
<tr>
<td>$1.3$</td>
<td>$30$</td>
</tr>
<tr>
<td>$1.4$</td>
<td>$20$</td>
</tr>
<tr>
<td>$1.5$</td>
<td>$10$</td>
</tr>
</tbody>
</table>
The PV19 fit of Sivers $f_{1T\perp}$ (preliminary)

\[ Q_0^2[\text{GeV}^2] = \{\text{EIKV} = 2.4, \text{TC18} = 1.2, \text{PV} = 1.0\} \]

\[ f_{1T\perp}^{(1)}(x) = \int dk_T \frac{k_T^2}{2M^2} f_{1T}(x, k_T) \]

- **PV11**: Bacchetta & Radici, P.R.L. 107 (11)
- **EIKV**: Echevarria et al., P.R. D89 (14)
- **TC**: Boglione et al., JHEP 1807 (18)
- **PV19**: Bacchetta, Delcarro, Pisano, Radici, in preparation
The PV19 fit of Sivers $f_{1T}^\perp$ (preliminary)

\[ f_{q/p}^\uparrow(x, k_T) = f_1^q(x, k_T^2) \]

\[ f_{q/p}^\uparrow(x, k_T) = f_1^q(x, k_T^2) - f_{1T}^q(x, k_T^2) S \cdot \left( \frac{\hat{P}}{M} \times k_T \right) \]

(distorted) plots entirely based on real data!
tomography of transversely polarized proton

x = 0.04
x = 0.12
x = 0.2

(distorted) plots entirely based on real data!
The Future

Mid-Term JLab12

Hall B claso

Hall A SoLID

28 Q^2 = 2.4 GeV^2

x = 0.1

95% C.L.

projection with $^3$He↑ and p↑ data

Anselmino et al., E.P.J. A39 (09)
The Future

Mid-Term
JLab12

Hall B

CLAS12 projected results

Hermes
Compass 2010

Hall A
SoLID

projection with $^3$He$^\uparrow$ and p$^\uparrow$ data

$Q^2 = 2.4$ GeV$^2$

95% C.L.

Long-Term
EIC

$\int dk_{\perp}^2 \frac{k_{\perp}^2}{2M^2} f_{1T} u - \bar{u}$

extend range in $x$
explore Sivers effect for sea quarks

Anselmino et al., E.P.J. A39 (09)

Accardi et al.,
E.P.J. A52 (16) 268
arXiv:1212.1701

extend range in $x$
explore Sivers effect for sea quarks

Anselmino et al.,
arXiv:1012.3565

$EIC \sqrt{s} = 45$ GeV

Accardi et al.,
E.P.J. A52 (16) 268
arXiv:1212.1701

$\gamma p \Rightarrow J/\psi p$
The Future

gluon Sivers function basically unknown!

explore:

e p↑ → e+J/ψ+X

Godbole et al.,
arXiv:1201.1066

Mukherjee & Rajesh,
arXiv:1609.05596

Bacchetta et al.,
arXiv:1809.02056

Boer et al.,
arXiv:1605.07934

Rajesh et al.,
arXiv:1802.10359
hadron or jet pair production in hadron-hadron scattering. Besides the terms with higher powers in the O(\alpha_s^2) terms, the hadroproduction of two jets discussed in [18–21] could be described by the distribution function $A$. This asymmetry arises in QED, in the 'tri-gluon' process: $e^+e^- \rightarrow g+g$. Asymmetries in quark gluon operators with the color structure $[22, 23]$ are reconstructed at Fermilab's Tevatron. Since the description involves two TMDs, breaking ISI or FSI can considerably modify the result.$^{23}$

Depending on the process, for example, in HQ production of a hadron, ISI or FSI can considerably modify the result.$^{23}$

The cross section for the process $e+p^\uparrow \rightarrow e+\bar{h}_1+h_2+X$ is better than the magnitude of the result.$^{23}$

Besides the latter process requires helicity flip in quark propagators.$^{23}$

Electron-Ion Collider (EIC) or the Large Hadron electron and hadron collider (LHeC) proposed at CERN. It is essential that the single-spin asymmetries in $e+p^\uparrow \rightarrow e+\text{jet+jet+X}$ are reconstructed (the sum of the transverse momenta $p_T$ of the final state particles being the polar angles of the final state particles). This could be described by the distribution function $A$.$^{23}$

$^{23}$

**The Future**

**gluon Sivers function basically unknown!**

explore:

- $e+p^\uparrow \rightarrow e+\bar{h}_1+h_2+X$
- $e+p^\uparrow \rightarrow e+\text{jet+jet+X}$

Zheng et al., arXiv:1805.05290
Boer et al., arXiv:1605.07934
The Future

gluon Sivers function basically unknown!

e p↑ → e+J/ψ+jet+X

D’Alesio et al., arXiv:1908.00446

see also next talk
gluon Sivers function basically unknown!

**RHIC & LHC**

see also:

\[ p^\uparrow p \rightarrow J/\psi + X \]

\[ p^\uparrow p \rightarrow D + X \]

see also next talk

\[ e^+ e^- \rightarrow e^+ J/\psi + j + X \]

\[ D'\text{Alesio et al., arXiv:1908.00446} \]

\[ \text{Godbole et al., arXiv:1703.01991} \]

\[ D'\text{Alesio et al., arXiv:1705.04169} \]

\[ \text{arXiv:1910.09640} \]
Transversity and the Collins effect

- chiral-odd structure
  in spin-1/2 hadron no gluon transversity $\rightarrow h_1$ is a non-singlet object

- related to tensor operator $\bar{q} \sigma^\mu\nu q$ not included in $\mathcal{L}_{SM}$ (at tree level)
  $\rightarrow$ low-energy footprint of BSM physics at higher scale?
Examples of BSM connections

- neutron EDM: estimate CPV induced by quark chromo-EDM $d_q$

\[ \mathcal{L}_{CPV} \supset ie \sum_{f=u,d,s,e} d_f \bar{f} \sigma_{\mu\nu} \gamma_5 f F^{\mu\nu} \]

\[ F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \]

\[ d_n = \delta u_d + \delta d_d + \delta s_d \]

exp. bounds + tensor charge \[ \delta q(Q^2) = \int_0^1 dx \left[ h^q_1(x, Q^2) - h^\bar{q}_1(x, Q^2) \right] \]
Examples of BSM connections

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$$L_{CPV} \supset ie \sum_{f=u,d,s,e} d_f \bar{f} \sigma_{\mu\nu} \gamma_5 f F^{\mu\nu}$$

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exp. bounds + tensor charge

$$\delta q(Q^2) = \int_0^1 dx \ [h^q_1(x, Q^2) - h^q_1(x, Q^2)]$$

constraints on CPV q EDM
Examples of BSM connections

- **neutron EDM**: estimate CPV induced by quark chromo-EDM $d_q$

  \[ \mathcal{L}_{\text{CPV}} \supset i e \sum_{f=u,d,s} d_f \bar{f} \sigma_{\mu\nu} \gamma_5 f F^{\mu\nu} \]

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  \[ \delta q(Q^2) = \int_0^1 dx \left[ h_1^q(x, Q^2) - h_1^q(x, Q^2) \right] \]

  + tensor charge

- **nuclear $\beta$-decay**: effective field theory including, e.g., tensor operator

  hadron level: $n \rightarrow p \ e^- \ \bar{\nu}_e$

  \[ C_T \ \bar{p} \sigma^{\mu\nu} n \ \bar{e} \sigma_{\mu\nu}(1 - \gamma_5) \nu_e \]

  \[ C_T \leftrightarrow g_T \epsilon_T \]

  quark level: $d \rightarrow u \ e^- \ \nu_e$

  \[ \langle p | \bar{u} \sigma^{\mu\nu} d | n \rangle \epsilon_T \bar{e} \sigma_{\mu\nu}(1 - \gamma_5) \nu_e \]

  \[ g_T = \delta u - \delta d \]

  + isovector tensor charge
**Examples of BSM connections**

- **neutron EDM**: estimate CPV induced by quark chromo-EDM \( d_q \)

\[
\mathcal{L}_{\text{CPV}} \supset ie \sum_{f=u,d,s,e} d_f \bar{f} \sigma_{\mu\nu} \gamma_5 f \, F^{\mu\nu}
\]

\[
d_n = \delta u d_u + \delta d d_d + \delta s d_s
\]

**exp. bounds** + **tensor charge** \( \delta q(Q^2) = \int_0^1 dx \left[ h_1^q(x, Q^2) - h_1^\bar{q}(x, Q^2) \right] \)

- **nuclear \( \beta \)-decay**: effective field theory including, e.g., tensor operator

**hadron level** : \( n \rightarrow p \ e^- \bar{\nu}_e \)

\[
C_T \bar{p} \sigma^{\mu\nu} n \quad \bar{e} \sigma_{\mu\nu} (1 - \gamma_5) \nu_e
\]

**exp. data**

\( g_T = \delta u - \delta d \)

**exp. bounds**

**quark level** : \( d \rightarrow u \ e^- \nu_e \)

\[
\langle p | \bar{u} \sigma^{\mu\nu} d | n \rangle \quad \epsilon_T \bar{e} \sigma_{\mu\nu} (1 - \gamma_5) \nu_e
\]

\[
g_T \leftrightarrow \epsilon_T
\]

**constraints on unknown \( \epsilon_T \)**
Transversity and the Collins effect

<table>
<thead>
<tr>
<th>Quark polarization</th>
<th>Unpolarized ((U))</th>
<th>Longitudinally Polarized ((L))</th>
<th>Transversely Polarized ((T))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>U</strong></td>
<td>(f_1 = \bullet)</td>
<td></td>
<td>(h_{1T}^+ = \uparrow) - (\Box)</td>
</tr>
<tr>
<td><strong>L</strong></td>
<td>(g_1 = \downarrow) - (\bullet)</td>
<td>(h_{1L}^{-} = \downarrow) - (\Box)</td>
<td></td>
</tr>
<tr>
<td><strong>T</strong></td>
<td>(f_{1T}^{+} = \circ) - (\circ)</td>
<td>(g_{1T}^{-} = \circ) - (\circ)</td>
<td>(h_{1}^{-} = \uparrow) - (\Box)</td>
</tr>
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</table>

Collins function

\[ s_T \cdot (k \times P_h) \]

- chiral-odd structure in spin-1/2 hadron no gluon transversity \(\rightarrow h_1\) is a non-singlet object
- related to tensor operator \(\bar{q} \sigma^{\mu \nu} q\) not included in \(\mathcal{L}_{SM}\) (at tree level)
  \(\rightarrow\) low-energy footprint of BSM physics at higher scale?
Transversity and the Collins effect

combined fit of azimuthal asymmetries in SIDIS

\[ A_T^{\sin(\phi_h + \phi_S)} \propto \frac{h_1 \otimes H_1^\perp}{f_1 \otimes D_1} \]

\[ A_0^{\cos 2\phi} \propto \frac{H_1^\perp \otimes \overline{H}_1^\perp}{D_1 \otimes \overline{D}_1} \]

**“fixed-hadron” frame**

alternative requires defining thrust axis, which poses problems with TMD factorization th.
Transversity and the Collins effect

combined fit of azimuthal asymmetries in SIDIS

\[ A_T^{\sin(\phi_h+\phi_S)} \propto \frac{h_1 \otimes H_1^\perp}{f_1 \otimes D_1} \]

\[ A_0^{\cos 2\phi} \propto \frac{H_1^\perp \otimes \overline{H}_1^\perp}{D_1 \otimes \overline{D}_1} \]

e^+e^− exp. data (for \(\pi\pi\) pairs)

Abe et al., P.R.L. 96 (06) 232002
Seidl et al., P.R. D78 (08) 032011
D86 (12) 039905(E)

\[ \sqrt{s} = 10.58 \text{ GeV} \]

Lees et al., P.R. D90 (14) 052003
Lees et al., P.R. D92 (15) 111101

includes also \(P_{hT1}\) & \(P_{hT2}\) dependence and Kaons

“fixed-hadron” frame

alternative requires defining thrust axis, which poses problems with TMD factorization th.

\[ \sqrt{s} = 3.60 \text{ GeV} \]

Ablikim et al., P.R.L. 116 (16) 042001

predicted, not fitted
Transversity and the Collins function

### Transversity

\( x h_1(x, Q_0^2=2.4 \text{ GeV}^2) \)

- **Kang et al. (2015)**
- **Anselmino et al. (2013)**

**Collins**

\[
\propto H_1^{\perp(1)}(z) = \int dp_T \frac{P_T^2}{2M_h^2} H_1^{\perp}(z, p_T)
\]

- **Kang et al. (2015)**
- **Anselmino et al. (2013)**

TMD framework with proper evolution equations

Generalized Parton Model (GPM) with no evolution for \( k_T \) dependence

update that smoothes differences in Collins funct.
**The Future**

**Hall B**

**Mid-Term**

**JLab12**

**Hall A  SoLID**

Single hadron channel: C12-11-111 Hall-B

- **π**
- **K**
- **K**

CLAS12 projected results

- **HERMES**
- **COMPASS 07**

Transversity

\[ \sin(q + q_S) \]

\[ h_{UT} \]

**Parton TMDs at large x** – ECT*16, 12th April 2016, Trento

**Distributions:**

**Di-hadron channel:**

\[ \sigma \]

\[ U_T \]

\[ s_i n (\phi + \phi_S) \]

\[ \propto h_{1} \otimes H_{1} \]

\[ \perp D \]

- **x**
- **z**
- **P_T [GeV/c]**

**Q^2=2.41 GeV^2**

- **projection with ^3He↑ and p↑ data**

- **Anselmino et al., P.R. D92 (15) 114023**
TMD factorization breaking

Factorization breaking in $p+p \rightarrow$ hadrons; is it large?

Rogers & Mulders, P.R. D81 (10)
Buffing, Kang, Lee, Liu, arXiv:1812.07549

no global fit is possible in TMD framework
Transv.-Mom. Dependent Parton Distributions

TMD PDFs \((x, k_T^2; Q)\) at leading twist

<table>
<thead>
<tr>
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<td>(*))</td>
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<td></td>
</tr>
<tr>
<td>L</td>
<td>(*))</td>
<td>(g_1 = \rightarrow - \bigcirc)</td>
<td>(h_{1L}^\perp = \bigcirc - \rightarrow)</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>(f_{1T}^\perp = \bigcirc - \uparrow)</td>
<td>(g_{1T} = \bigcirc - \rightarrow)</td>
<td>(h_1 = \downarrow - \bigcirc)</td>
<td></td>
</tr>
</tbody>
</table>

* forbidden by parity invariance

Mulders & Tangerman, N.P. **B461** (96)
Boer & Mulders, P.R. **D57** (98)
Bacchetta et al., JHEP **02** (07) 093
Transv. Mom. Dependent Parton Distributions

TMD PDFs \((x, x^2; Q)\) at leading twist

Quark polarization

<table>
<thead>
<tr>
<th>Nucleon Polarization</th>
<th>Unpolarized ((U))</th>
<th>Longitudinally Polarized ((L))</th>
<th>Transversely Polarized ((T))</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>(f_1 = \circ)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>(g_1 = \circ)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td></td>
<td>(h_1 = \circ)</td>
<td></td>
</tr>
</tbody>
</table>
Alternative: hadron-in-jet framework

**SIDIS**

**hybrid scheme:**
- TMD framework for TMD fragmentation
- collinear framework for PDF

**Factorization theorem** for $j_T \ll Q$
**universality** for TMD fragmentation

Kang, Liu, Ringer, Xing, JHEP **1711** (17), arXiv:1705.08443
Kang, Prokudin, Ringer, Yuan, P.L. **B774** (17), arXiv:1707.00913

see Felix Ringer’s talks
Collins effect for hadron-in-jet

\[ A^{\sin(\phi_S - \phi_H)}_T \propto \frac{h_1^q \otimes f_1^{q'} \otimes H_{1}^{\perp q}}{f_1^{q'} \otimes f_1^{q} \otimes D_{1}^{q}} \]

Adamczyk et al. (STAR), P.R. D97 (18) 032004

PDF & TMDFF from SIDIS + e+e− analysis

DMP+2013 (no evolution)
Anselmino et al., P.R. D87 (13) 094019
D’Alesio et al., P.L. B773 (17) 300

KPRY (no evolution)
KPRY-NLL (TMD evolution)
Kang et al., P.L. B774 (17) 635
Alternative: the di-hadron mechanism

Collins effect
\[ s_T \cdot (k \times P_h) \]

di-hadron mechanism
\[ s_T \cdot (P_2 \times P_1) \]

Correlation \( s_T \) and \( R_T \) → azimuthal asymmetry in \( \Phi_R \)

\[ \text{hadron} \]

\[ \text{quark} \]

\[ \begin{align*}
P_h &= P_1 + P_2 \\
2R &= P_1 - P_2
\end{align*} \]
Alternative: the di-hadron mechanism

Collins effect
\[ s_T \cdot (k \times P_h) \]

di-hadron mechanism
\[ s_T \cdot (P_2 \times P_1) \]

correlation \( s_T \) and \( R_T \) \(
\rightarrow \) azimuthal asymmetry in \( \Phi_R \)

survives to \( \int dP_{hT} \rightarrow P_h \parallel k \)

can be studied in collinear framework

if \( R_T \ll Q \) correlation encoded in di-hadron FF (DiFF)

\[ H_1^A(z = z_1 + z_2, R_T^2 \propto M_h^2 ; Q^2) \]

pair invariant mass
Global fit for di-hadron production

\[ A_T^\sin(\phi_R + \phi_S) \propto \frac{h_1}{f_1} \frac{H_1^\perp}{D_1} \]

\[ A^\cos(\phi_R + \bar{\phi}_R) \propto \frac{H_1^\perp}{D_1} \]

Airapetian et al., JHEP 0806 (08) 017
Adolph et al., P.L. B713 (12)
Braun et al., E.P.J. Web Conf. 85 (15) 02018

\[ A^\sin(\phi_S - \phi_R) \propto \frac{f_1 \otimes h_1 \otimes H_1^\perp}{f_1 \otimes f_1 \otimes D_1} \]

Adamczyk et al. (STAR), P.R.L. 115 (2015) 242501
Adamczyk et al. (STAR), P.L. B780 (18) 332
Our global fit

Radici and Bacchetta, P.R.L. **120** (18) 192001
arXiv:1802.05212

**SIDIS**

- 18 data points
- [Image of COMPASS and hermes]

**pp collisions**

- 4 data points
- [Image of STAR]

- Run 2006 (s=200 GeV²)

10 independent data points

Probability density function of \( \chi^2 \) distribution for 22 d.o.f.

(for \( \chi^2/\text{dof} = 1 \) perfect overlap)

\[
\chi^2/\text{dof} = 1.76 \pm 0.11
\]
The transversity from first ever global fit

uncertainty band from
90% of 600 replicas =
max uncertainty on $D_{1g}(Q_0)$

diverges less than $1/x$
Comparison with other extractions

Collins effect, only SIDIS data

Anselmino et al., 
P.R. D 87 (13) 094019

Torino

Radici and Bacchetta, 
P.R.L. 120 (18) 192001

global fit

Kang et al., 
P.R. D 93 (16) 014009

up

down
Comparison with lattice

**our global fit**
Radici and Bacchetta, *P.R.L.* **120** (18) 192001

**JAM Collab.**
Lin et al., *P.R.L.* **120** (18) 152502
Collins effect
only SIDIS data
constrained to $g_{T}^{u-d}$ from lattice

**ETMC quasi-$h_{1}$**
Alexandrou et al., *P.R. D99* (19) 114504

courtesy of F. Steffens

Graph showing $h_{1}^{u}-h_{1}^{d}$ vs. $Q^{2}=4$ GeV$^{2}$ with various calculations and data points.
The tensor “charge” of the proton

1\textsuperscript{st} Mellin moment of transversity PDF $\Rightarrow$ tensor “charge”

$$\delta q(Q^2) = \int_0^1 dx \left[ h_1^q(x, Q^2) - h_1^\bar{q}(x, Q^2) \right]$$

tensor charge connected to tensor operator

$$\langle P, S_p \mid \bar{q}\sigma^{\mu\nu} q \mid P, S_p \rangle = (P^\mu S_{p}^{\nu} - P^\nu S_{p}^{\mu}) \delta q$$

$$= (P^\mu S_{p}^{\nu} - P^\nu S_{p}^{\mu}) \int dx h_1^q(x, Q^2)$$

compute on lattice

preferably the isovector $g_T = \delta u - \delta d$

(cancellation of “disconnected” diagrams)

extract transversity from data with transversely polarized protons
Comparison on tensor charge

\[ g_T = \delta u - \delta d \]

Collins effect
no p-p data

\{ JAM \ (Q^2=2) \\
Torino \ (Q^2=1) \\
TMD \ (Q^2=10) \}

\textbf{JAM} includes constraint from “lattice g_T”

1) ETMC '19
   Alexandrou et al., arXiv:1909.00485

2) Mainz '19
   Harris et al., P.R. D100 (19) 034513

3) LHPC '19
   Hasan et al., P.R. D99 (19) 114505

4) JLQCD '18
   Yamanaka et al., P.R. D98 (18) 054516

5) PNDME '18
   Gupta et al., P.R. D98 (18) 034503

6) ETMC '17
   Alexandrou et al., P.R. D95 (17) 114514;
   E. P.R. D96 (17) 099906

7) RQCD '14
   Bali et al., P.R. D91 (15) 054501

8) LHPC '12
   Green et al., P.R. D86 (12) 114509
Comparison on tensor charge

\[ g_T = \delta u - \delta d \]

JAM

global fit

lattice \( Q^2 = 4 \text{ GeV}^2 \)

Torino

TMD

Collins effect

no p-p data

\( \{ \)

JAM \( (Q^2=2) \)

Torino \( (Q^2=1) \)

TMD \( (Q^2=10) \)

\( \} \)

JAM includes constraint from “lattice \( g_T \)”

90% uncertainty band

1) ETMC ’19  
2) Mainz ’19  
3) LHPC ’19  
4) JLQCD ’18  
5) PNDME ’18  
6) ETMC ’17  
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Alexandrou et al., arXiv:1909.00485
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Collins effect
no p-p data

JAM (Q^2=2)
Torino (Q^2=1)
TMD (Q^2=10)

JAM includes constraint from “lattice g_T”

no simultaneous compatibility between “pheno δq” and “lattice δq”
Tension "pheno" - "lattice"

main problem of “pheno δq”
is extrapolating outside data..

if we constrain our global fit with lattice results for all components of tensor charge (up, down, isovector) the $\chi^2$ clearly deteriorate

$$\chi^2 = \int_{x_{\min}}^{x_{\max}} dx \ h_1^{q-\bar{q}} + \int_{x_{\min}}^{x_{\max}} dx \ h_1^{q-\bar{q}} + \int_{1}^{1} dx \ h_1^{q-\bar{q}}$$

$\bar{g}^{\text{latt}} = 1.004 \pm 0.057$

$\delta_u^{\text{latt}} = 0.782 \pm 0.031$

$\delta_d^{\text{latt}} = -0.218 \pm 0.026$

probability density function of $\chi^2$ distribution for … d.o.f.

statistically very unlikely …. 
Mid-Term
JLab12
also:
- need to include data at $s=500$ GeV$^2$
- upcoming new data on deuteron from...
- need data on $pp \rightarrow (\pi\pi)X$ from...

to constrain $D_{1g}$ gluon
The Future

Hall A  SoLID

Mid-Term
JLab12

also:
- need to include data at \( s = 500 \text{ GeV}^2 \)
- upcoming new data on deuteron from
- need data on \( pp \rightarrow (\pi\pi)X \) from \( \star \) to constrain \( D_1^{gluon} \)
- need EIC to extend \((x,Q^2)\) coverage to have better handle on tensor charge
Conclusions

• with polarized TMDs many possible combinations allow to explore various aspects of motion of partons inside hadrons
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• Sivers effect tests QCD at its fundamental level and gives indirect information on partonic contribution to proton spin

• first “tomography” of Sivers effect available from data in a consistent TMD framework

Bacchetta, Delcarro, Pisano, Radici, in preparation
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\[ \text{Radici and Bacchetta, P.R.L. 120 (18) 192001} \]
\[ \text{arXiv:1802.05212} \]

Much more to come with