Probing gluon TMDs in J/ψ + jet production at EIC

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RK, A. Mukherjee, R. Sangem, arXiv:1908.03698 **QEIC 4-7 Jan 2020, IIT Bombay, India**

Plan of the talk

Gluon TMDs

Gluon Sivers Function (GFS)

Quarkonium Production

Sivers effect in $ep^{\uparrow} \rightarrow J/\psi + jet + \mathrm{X}$

Numerical Results

Gluon TMDs

TMD-PDFs (Transverse Momentum Dependent Parton Distribution Functions): $f(x, k_\perp, Q^2)$
gives the number density of partons , with their intrinsic transverse motion and spin, inside a nucleon, with its spin.

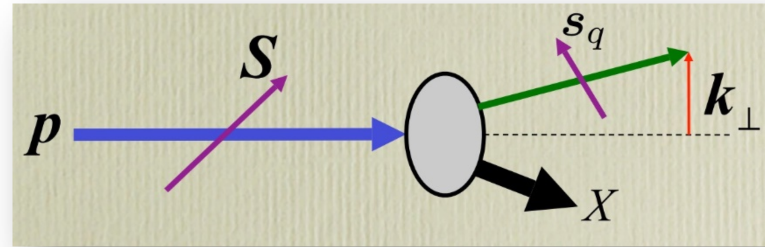
Do not have much information about gluon TMDs, it satisfies positivity bound.

Gluon TMDs are process dependent like the quark TMDs due to presence of gauge link. Each gluon TMD contains two gauge link that makes the process dependence of gluon TMDs more involved than quark TMDs.

The simplest possible gauge link configurations are $++ \; or \; --$ and $-+ \; or \; +-$ where, in literature, in small-x region, they are described as Weizsacker-Williams (WW) type and Dipole distribution respectively.

Gluon Sivers function (GSF)

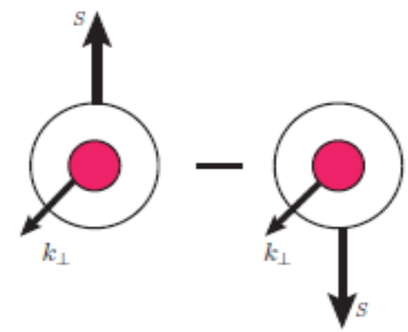
$\boldsymbol{S} \cdot (\boldsymbol{p} \times \boldsymbol{k}_\perp)$: **Sivers effect**



- GSF describes the density of unpolarized gluons inside a transversely polarized nucleon.

$$\Delta^N f_{g/p^\uparrow}(x, \boldsymbol{k}_\perp) = f_{g/p^\uparrow}(x, \boldsymbol{k}_\perp) - f_{g/p^\downarrow}(x, \boldsymbol{k}_\perp)$$

$\Delta^N f =$



$$f_{g/p^\uparrow}(x, k_\perp) = f_{g/p}(x, k_\perp) + \frac{1}{2}\Delta^N_{g/p^\uparrow}(x, k_\perp)\hat{\mathbf{S}}.(\hat{\mathbf{P}} \times \hat{\mathbf{k}}_\perp)$$

Sivers effect

Trento notation

$$\Delta^N f_{g/p^\uparrow} = -2\frac{k_\perp}{M_p} f_{1T}^{\perp g}$$

Gluon Sivers function (GSF)

- The definition of polarized TMD in terms of fields

$$f_{g/p^S}(x,\mathbf{k}_\perp) = \delta_{ij} \int \frac{d\xi^- d^2\xi^\perp}{(2\pi)^3} e^{ixP^+\xi^- - i\mathbf{k}^\perp.\xi^\perp} \langle P,S|\mathrm{Tr}[F^{i+}(\xi)W(\xi,0)F^{j+}(0)W(0,\xi)]|S,P\rangle|_{\xi^+=0}$$

Boer et al, Adv.High Energy Phys. 2015, 371396

- $F^{\mu\nu}$ is the gluon field strength tensor
- Gauge link or Wilson line $W[a,b] = \mathcal{P}\exp\left(-ig\int_a^b dz_\mu A^\mu(z)\right)$

Properties of Sivers function

- Sivers function is Time-reversal odd function.
- Sivers function in DY is equal in magnitude but opposite in sign compared to Sivers function in SIDIS

$$\Delta^N f_{g/p^\uparrow}(x,\mathbf{k}_\perp)|_{\mathrm{DY}} = -\Delta^N f^\perp_{g/p^\uparrow}(x,\mathbf{k}_\perp)|_{\mathrm{SIDIS}}$$

- This kind of non-universal property of Sivers function can be used as tool to test our understanding of QCD

Measure of Sivers function

How to extract the Sivers function? By measuring the single spin asymmetry (SSA)

$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

D. Sivers, PRD 41, 83(1990)

- Non-zero asymmetry indicates the existence of orbital angular momentum

M. Burkardt, Nucl. Phys. A735, 185 (2004)

Bacchetta and Radici, PRL . 107, 212001 (2011)

- SIDIS data come from HERMES, COMPASS and JLab experiments in pion and kaon production.

HERMES Collaboration PRL. 103 (2009) 152002

COMPASS Collaboration PLB. 673 (2009) 127, PLB 744 (2015) 250

JLAB Hall A Collaboration PRL 107 (2011) 072003

- D' Alesio et al have made the 1st attempt to extract the GSF from the RHIC data

D'Alesio et al, JHEP **09** (2015) 119

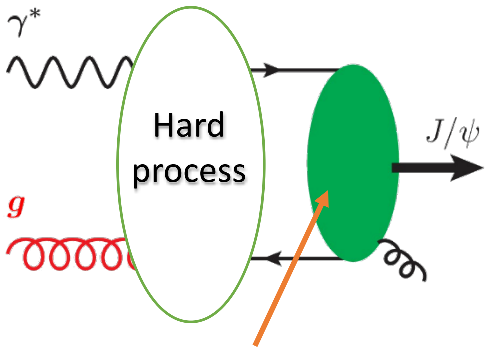
- Anselmino et al, have attempted to study the sign change of Sivers function. Due to the limited data, final conclusion could not be drawn, although W^- production data favors for sign change.

Anselmino et al, JHEP 1704 (2017) 046

Quarkonium Production

- Quarkonium is a bound state of heavy quark and anti-quark ($Q\bar{Q}$)

- In the rest frame of bound state, the relative momenta of two quarks is small compared to their mass, that allows to nonrelativistic approach called NRQCD model.



- The production of $Q\bar{Q}$ pair in a definite quantum state is a hard process and can be calculated perturbatively.

- $Q\bar{Q}$ pair will transform into a quarkonium, by emitting or absorbing soft gluons and it happens at the scale below Λ_{QCD} which is a soft process.

- The probability of states to end up a particular quarkonium state is stored in a non-perturbative quantity called Long Distance Matrix Elements (LDMEs).

NRQCD factorization

LDMEs

$$d\sigma^{ab\to J/\psi} = \sum_n d\hat{\sigma}[ab \to c\bar{c}(n)]\langle 0 \mid \mathcal{O}_n^{J/\psi} \mid 0\rangle$$

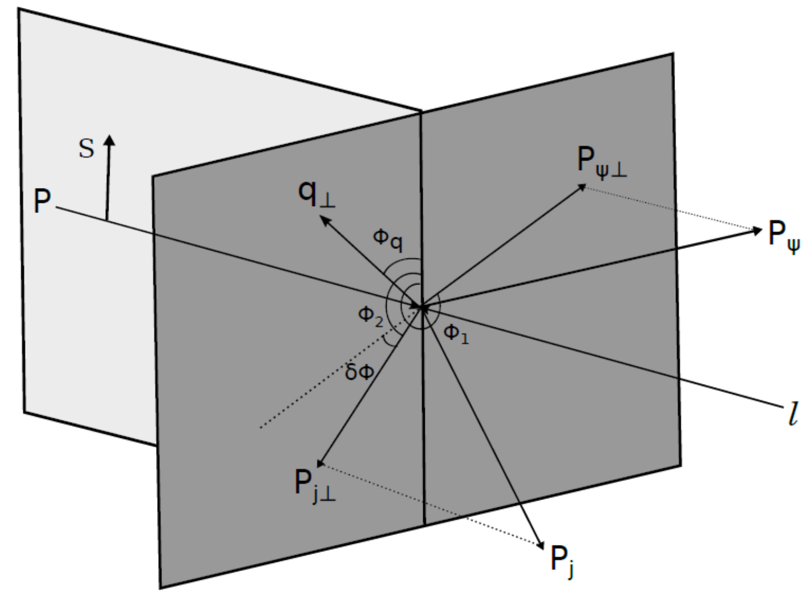
G. T. Bodwin et al, PRD51 (1995)

Sivers effect in J/ψ + jet photoproduction

- Consider the photoproduction process: $e(l) + p^{\uparrow}(P) \to J/\psi(P_\psi) + \text{jet}(P_j) + X,$

- Proton-electron center of mass frame; wherein the proton and electron move along + z and – z directions respectively.

- $P_{\psi\perp}$ and $P_{j\perp}$ are transverse momentum of J/ψ and jet respectively in the plane orthogonal to the momentum of proton P.



- We define sum and difference of transverse momenta

$$\boldsymbol{q}_\perp = \boldsymbol{P}_{\psi\perp} + \boldsymbol{P}_{j\perp} \quad , \quad \boldsymbol{K}_\perp = \boldsymbol{P}_{\psi\perp} - \boldsymbol{P}_{j\perp}$$

ϕ_q denotes azimuthal angle of $\boldsymbol{q}_\perp$

- In the case where $|\boldsymbol{q}_\perp| \ll |\boldsymbol{K}_\perp|$, the J/ψ and jet are almost back to back in the transverse plane.

Differential Cross-section

Within the generalized TMD factorization framework

$$\frac{d\sigma}{d^2\boldsymbol{q}_\perp dz d^2\boldsymbol{K}_\perp} = \frac{1}{2(2\pi)^2}\frac{1}{z(1-z)s}\sum_a \int \frac{dx_\gamma}{x_\gamma} f_{\gamma/e}(x_\gamma) f_{a/p}(x_a, q_\perp)\frac{1}{2\hat{s}}|\mathcal{M}_{\gamma a\to J/\psi a}|^2$$

Weizsaker-William distribution function

Weizsaker-William distribution function gives the density of photon inside an unpolarized electron

$$f_{\gamma/e}(x_\gamma) = \frac{\alpha}{2\pi}\left[2m_e^2 x_\gamma\left(\frac{1}{Q^2_{min}} - \frac{1}{Q^2_{max}}\right) + \frac{1+(1-x_\gamma)^2}{x_\gamma}\ln\frac{Q^2_{max}}{Q^2_{min}}\right]$$

$$Q^2_{min} = m_e^2 \frac{x_\gamma^2}{1-x_\gamma}$$

$$Q^2_{max} = 1\ \mathrm{GeV}^2$$

S. Frixione, M. L. Mangano, Phys. Lett. B319, 339 (1993)

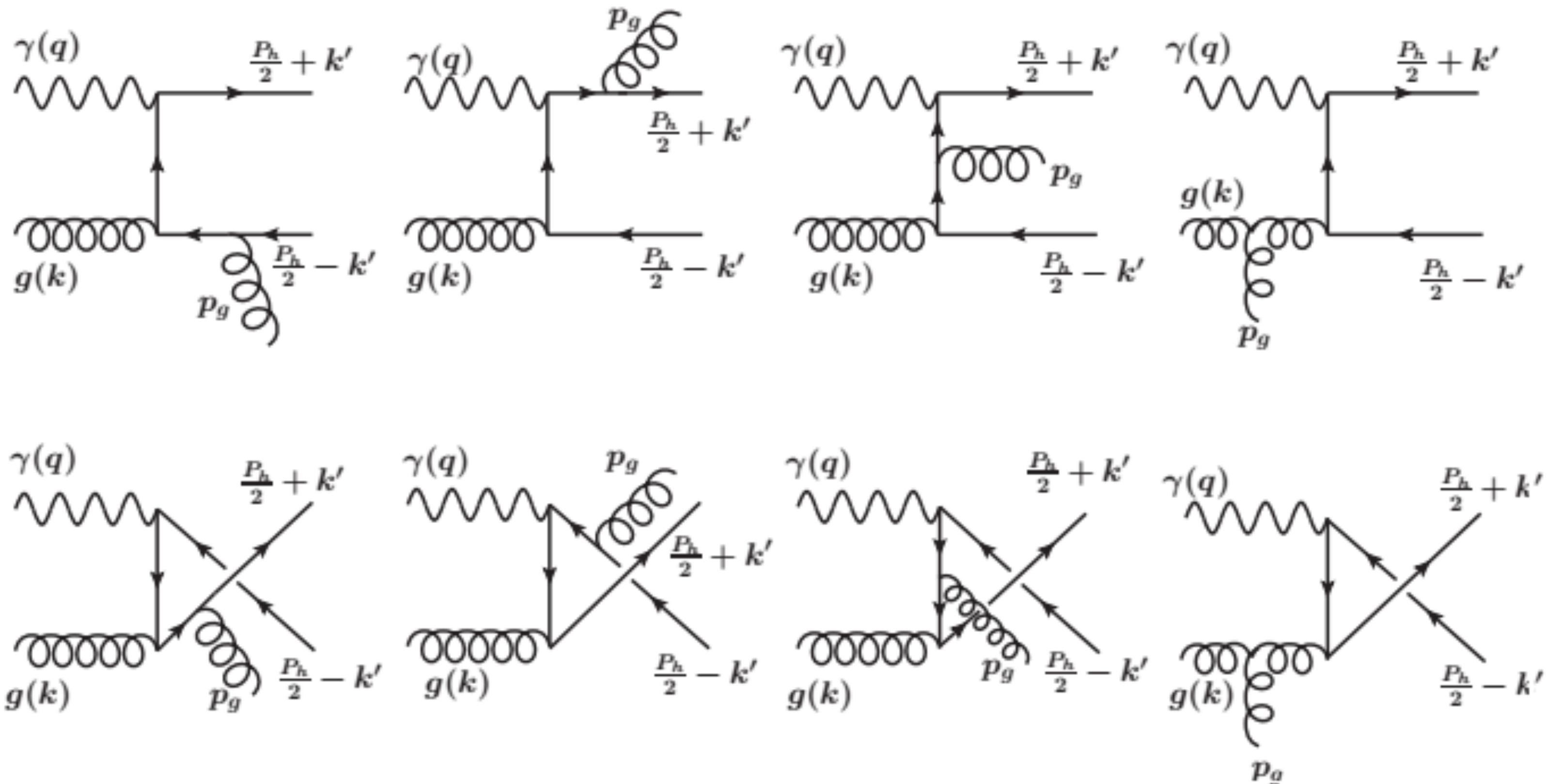
For transversely polarized proton;

$$\mathrm{d}\Delta\sigma \equiv \frac{d\sigma^\uparrow}{d^2\boldsymbol{q}_\perp dz d^2\boldsymbol{K}_\perp} - \frac{d\sigma^\downarrow}{d^2\boldsymbol{q}_\perp dz d^2\boldsymbol{K}_\perp} = \frac{1}{2(2\pi)^2}\frac{1}{z(1-z)s}\sum_a \int \frac{dx_\gamma}{x_\gamma} f_{\gamma/e}(x_\gamma)\Delta\hat{f}_{a/p}(x_a, \boldsymbol{q}_\perp)\frac{1}{2\hat{s}}|\mathcal{M}_{\gamma a\to J/\psi a}|^2,$$

Sivers function

x_γ is momentum fraction of photon.
x_a is momentum fraction of parton.
z is energy fraction of photon carried by J/ψ in proton rest frame.

Feynman diagrams



Feynman diagrams for process: $\gamma + g \rightarrow J/\psi + g$

$$P_h \equiv P_\psi \; and \; p_g \equiv P_j$$

Amplitude calculations

- The amplitude for quarkonium production is given by P. L Cho et al, Phys. Rev. **D53**, 6203 (1996)

$$\mathcal{M}\left(\gamma g \to Q\bar{Q}[^{2S+1}L_J^{(1,8)}](P_h)+g\right) = \sum_{L_z S_z}\int \frac{d^3\boldsymbol{k}'}{(2\pi)^3}\Psi_{LL_z}(\boldsymbol{k}')\langle LL_z;SS_z|JJ_z\rangle$$

$$\times \mathrm{Tr}[O(q,k,P_h,k')\mathcal{P}_{SS_z}(P_h,k')],$$

$\Psi_{LL_z}(\boldsymbol{k}')$ is the eigenfunction of orbital angular momentum *L*

$\langle LL_z;SS_z|JJ_z\rangle$ Clebsch-Gordan coefficient projects the orbital angular momentum

- Spin projection operator $\mathcal{P}_{SS_z}(P_h,k') = \frac{1}{4M^{3/2}}(-\not{P}_h + 2\not{k}' + M)\Pi_{SS_z}(\not{P}_h + 2\not{k}' + M)$ with $\Pi_{SS_z} = \gamma^5$ for singlet (S=0) and $\Pi_{SS_z} = \not{\epsilon}_{s_z}(P_h)$ for triplet state (S=1)

- $O(q,k,P_h,k')$ represents the amplitude of $Q\bar{Q}$ pair without considering the external heavy quark and anti-quark legs.

- $^3S_1^{(1,8)},\ ^1S_0^{(8)},\ ^3P_0^{(8)},\ ^3P_1^{(8)}$ and $^3P_2^{(8)}$ states which contribute in CS and CO models

Amplitude calculations

- For CO state ${}^{3}S_{1}^{(8)}$, amplitude can be written as

$$\mathcal{M}[{}^3S_1^{(8)}](P_h, k) = \frac{1}{4\sqrt{\pi M}} R_0(0) \frac{\sqrt{2}}{2} d_{abc} \mathrm{Tr}\left[\sum_{m=1}^{3} O_m(0)(-\not{P}_h + M)\not{\epsilon}_{s_z}\right],$$

where

$$\sum_{m=1}^{3} O_m(0) = g_s^2 (ee_c)\varepsilon^{\mu}_{\lambda_a}(k)\varepsilon^{\nu}_{\lambda_b}(q)\varepsilon^{\rho*}_{\lambda_g}(p_g)\left[\frac{\gamma_\nu(\not{P}_h - 2\not{q} + M)\gamma_\mu(-\not{P}_h - 2\not{p}_g + M)\gamma_\rho}{(\hat{s} - M^2)(\hat{u} - M^2)} + \frac{\gamma_\rho(\not{P}_h + 2\not{p}_g + M)\gamma_\nu(-\not{P}_h + 2\not{k} + M)\gamma_\mu}{(\hat{s} - M^2)(\hat{t} - M^2)} + \frac{\gamma_\nu(\not{P}_h - 2\not{q} + M)\gamma_\rho(-\not{P}_h + 2\not{k} + M)\gamma_\mu}{(\hat{t} - M^2)(\hat{u} - M^2)}\right] \quad (42$$

Square of the amplitude is

$$|\mathcal{M}[{}^3S_1{}^{(8)}]|^2 = \frac{5\pi^3 e_c^2 \alpha_s^2 \alpha}{36M} \langle 0 \mid \mathcal{O}_8^{J/\psi}({}^3S_1) \mid 0\rangle \frac{512M^2}{s_1^2 t_1^2 u_1^2} \times \left\{s_1^2(s_1 + M^2)^2 + u_1^2(u_1 + M^2)^2 + t_1^2(t_1 + M^2)^2\right\}$$

Other CO states amplitude expressions are very lengthy

R. Sangem, R. K., A. Mukherjee, PhysRevD.98.014007

Sivers asymmetry

Weighted Sivers asymmetry

$$A_N^{W(\phi_q)} \equiv \frac{\int d\phi_q W(\phi_q)(\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow})}{\int d\phi_q (\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow})} \equiv \frac{\int d\phi_q W(\phi_q)\mathrm{d}\Delta\sigma(\phi_q)}{\int d\phi_q 2\mathrm{d}\sigma},$$

D. Boer, P. J. Mulders, C. Pisano, and J. Zhou, JHEP 08, 001 (2016)

where azimuthal weight factor $W(\phi_q) = -\sin(\phi_q)$, given by

$$-\sin(\phi_q) = \frac{(\boldsymbol{S} \times \hat{\boldsymbol{P}}) \cdot \boldsymbol{q}_\perp}{|\boldsymbol{S} \times \hat{\boldsymbol{P}}||\hat{\boldsymbol{P}} \times \boldsymbol{q}_\perp|}.$$

Sivers function satisfies the positivity bound

$$|\Delta^N f_{a/p^{\uparrow}}(x_a, q_\perp)| \leq 2 f_{a/p}(x_a, q_\perp)$$

Parameterization of TMDs

Gaussian parameterization for GSF within the DGLAP evolution approach

$$\Delta^N f_{a/p^\uparrow}(x_a, q_\perp) = 2\,\frac{\sqrt{2e}}{\pi}\,\mathcal{N}_a(x_a)\, f_{a/p}(x_a)\,\sqrt{\frac{1-\rho}{\rho}}\, q_\perp\,\frac{e^{-q_\perp^2/\rho\langle q_\perp^2\rangle}}{\langle q_\perp^2\rangle^{3/2}}$$

$$\mathcal{N}_a(x_a) = N_a x_a^{\alpha}(1-x_a)^{\beta}\,\frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha \beta^\beta}, \qquad \rho = \frac{M'^2}{\langle q_\perp^2\rangle + M'^2}$$

$\alpha, \beta, N_a\ and\ \langle q_\perp^2\rangle$ are parameters which can be extracted from experiments

Anselmino, D’Alesio, F. Murgia, PhysRevD.67.074010 (2003)

- D’ Alesio et al. have reported three set of parameters “SIDIS1”,“SIDSI2” and “SIDIS3” for GSF

D’Alesio et al, JHEP **09**, 119 (2015)

D’Alesio et al, PhysRevD99,036013 (2019)

TMD Evolution

J. Collins, Foundations of Perturbative QCD, 2011
S. M. Aybat et al. PRD 85 034043 (2012)

- In the TMD evolution formalism,

$$f_{a/p}(x_a, p_{a\perp}, Q_f) = \frac{1}{2\pi}\int_0^\infty db_\perp b_\perp J_0(p_{a\perp}b_\perp) f_{a/p}(x_a, b_\perp, Q_f),$$

$$f_{1T}^{\perp}(x_a, p_{a\perp}, Q_f) = -\frac{1}{2\pi p_{a\perp}}\int_0^\infty db_\perp b_\perp J_1(p_{a\perp}b_\perp) f_{1T}^{\prime\perp}(x_a, b_\perp, Q_f)$$

Derivative of the Sivers function obeys same evolution equation as unpolarized TMD

$$f_{1T}^{\prime\perp}(x_a, b_\perp, Q_f, \zeta) = f_{1T}^{\prime\perp}(x_a, b_\perp, Q_i)\, R_{pert}\left(Q_f, Q_i, b_*\right) R_{NP}\left(Q_f, b_\perp\right)$$

$$R_{pert}\left(Q_f, b_*\right) = \exp\left\{-\int_{c/b_*}^{Q_f}\frac{d\mu}{\mu}\left(A\log\left(\frac{Q_f^2}{\mu^2}\right)+B\right)\right\},\quad R_{NP}(Q_f, b_\perp) = \exp\left\{-\left[g_1^{\mathrm{TMD}}+\frac{g_2}{2}\log\frac{Q_f}{Q_0}\right]b_\perp^2\right\}$$

A and B are perturbative terms

C. Kouvaris, J.-W. Qiu, W. Vogelsang, and F. Yuan, Phys. Rev. D74, 114013 (2006)

At initial scale

$$T_{a,F}(x_a, Q_i) = \mathcal{N}_a(x_a) f_{a/p}(x_a, Q_i)$$

Qiu-Sterman function

$$f_{1T}^{\prime\perp}(x_a, b_\perp, Q_i) \simeq \frac{M_p b_\perp}{2} T_{a,F}(x_a, Q_i),$$

$\mathcal{N}_a(x_a)$ is same as before

$$Q_i = \frac{2}{b_*}e^{-0.577} = \sqrt{\zeta_0},\ Q_f = \sqrt{\zeta} = M$$

$$b_* = \frac{b_\perp}{\sqrt{1+\left(\frac{b_\perp}{b_{\max}}\right)^2}} \begin{array}{l}\approx b_{\max};\ (b_\perp \to \infty)\\ \approx 0;\ (b_\perp \to 0)\end{array}$$

TMD Evolution

- So far the best fit parameters of GSF has not been extracted in the TMD evolution approach.

- Echevarria et al, have extracted u-quark and d-quark Sivers function from latest SIDIS data within TMD evolution scheme. We use u-quark and d-quark Sivers function best fit parameters for GSF as the following ("TMD-a" and "TMD-b")

M. G. Echevarria, A. Idilbi, Z.-B. Kang, and I. Vitev, PhysRevD89,074013 (2014)

$$\begin{aligned} (a) \quad \mathcal{N}_g(x_g) &= (\mathcal{N}_u(x_g) + \mathcal{N}_d(x_g))/2 \\ (b) \quad \mathcal{N}_g(x_g) &= \mathcal{N}_d(x_g). \end{aligned}$$

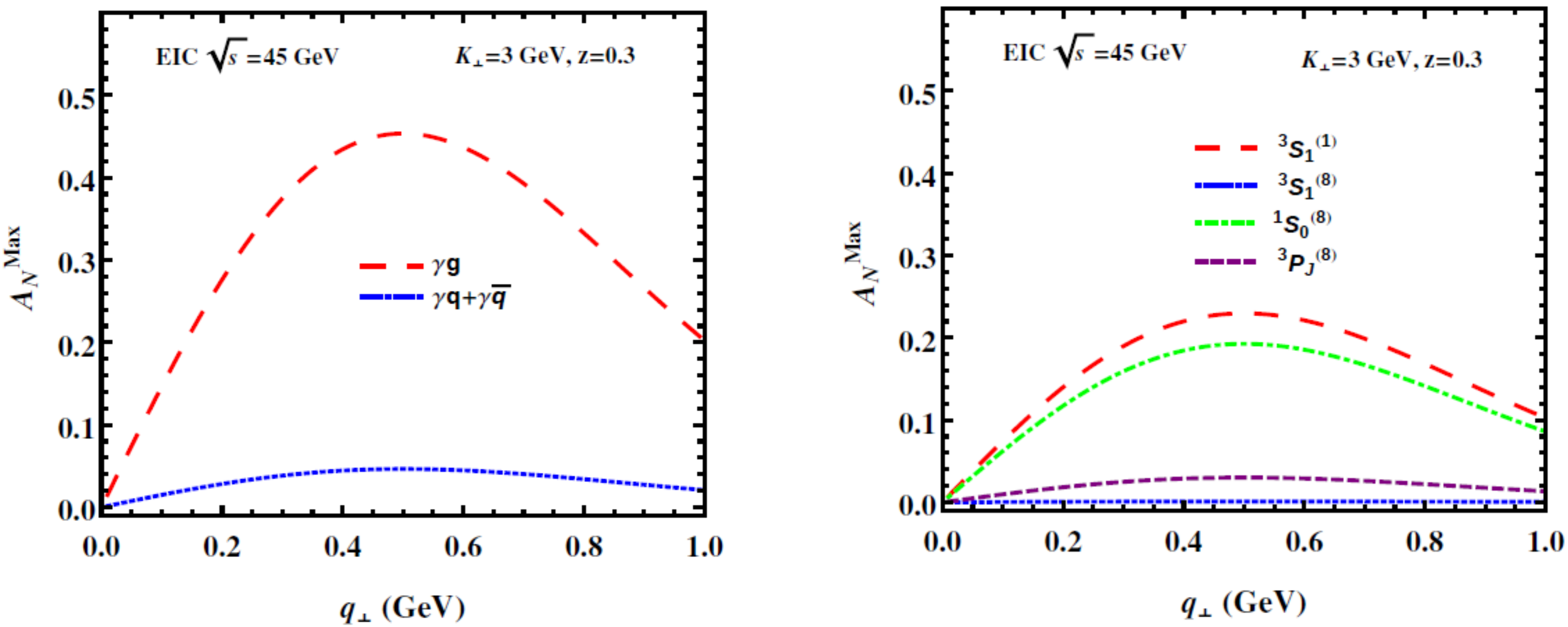
D. Boer and W. Vogelsang, PhysRevD69, 094025 (2004)

- Numerator and denominator of the weighted asymmetry equation in TMD evolution approach;

$$\mathrm{d}\Delta\sigma = -\frac{1}{\pi M_p}\frac{1}{2(2\pi)^2}\frac{1}{z(1-z)s}\sum_a \int \frac{dx_\gamma}{x_\gamma} db_\perp b_\perp J_1(q_\perp b_\perp) f_{1T}^{\prime\perp}(x_a, b_\perp, Q_f) f_{\gamma/e}(x_\gamma)\frac{1}{2\hat{s}}|\mathcal{M}_{\gamma a\to J/\psi a}|^2 \sin(\phi_q),$$

$$2d\sigma = \frac{1}{2(2\pi)^2\pi}\frac{1}{z(1-z)s}\sum_a \int \frac{dx_\gamma}{x_\gamma} db_\perp b_\perp J_0(q_\perp b_\perp) f_{a/p}(x_a, b_\perp, Q_f) f_{\gamma/e}(x_\gamma)\frac{1}{2\hat{s}}|\mathcal{M}_{\gamma a\to J/\psi a}|^2.$$

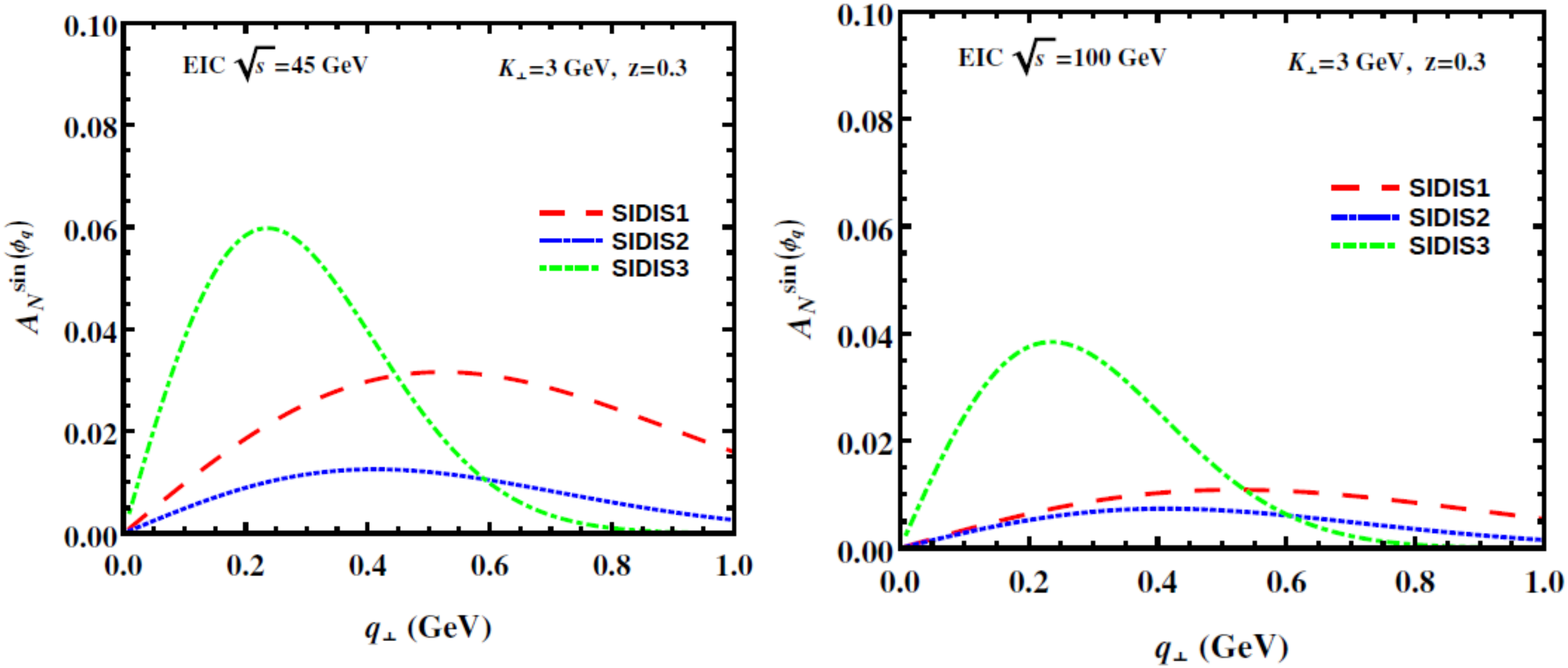
Sivers asymmetry at EIC



- Maximized Sivers asymmetry in $e + p^{\uparrow} \rightarrow J/\psi + jet + X$ process using DGLAP evolution approach.
- Here, we maximize the Sivers function by adopting $\rho = 2/3$ and $\mathcal{N}_a = 1$ in the parametrization of Sivers function.
- Major contributions are coming from $^3S_1^{(1)}$ and $^1S_0^{(8)}$ states.

RK, A.Mukherjee, R. Sangem, arXiv:1908.03698

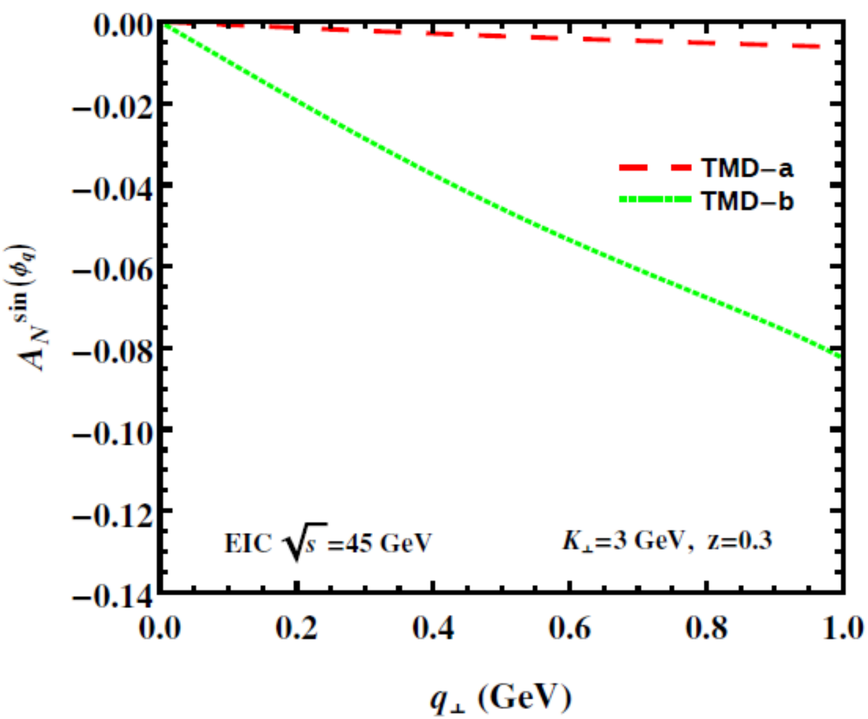
Sivers asymmetry at EIC

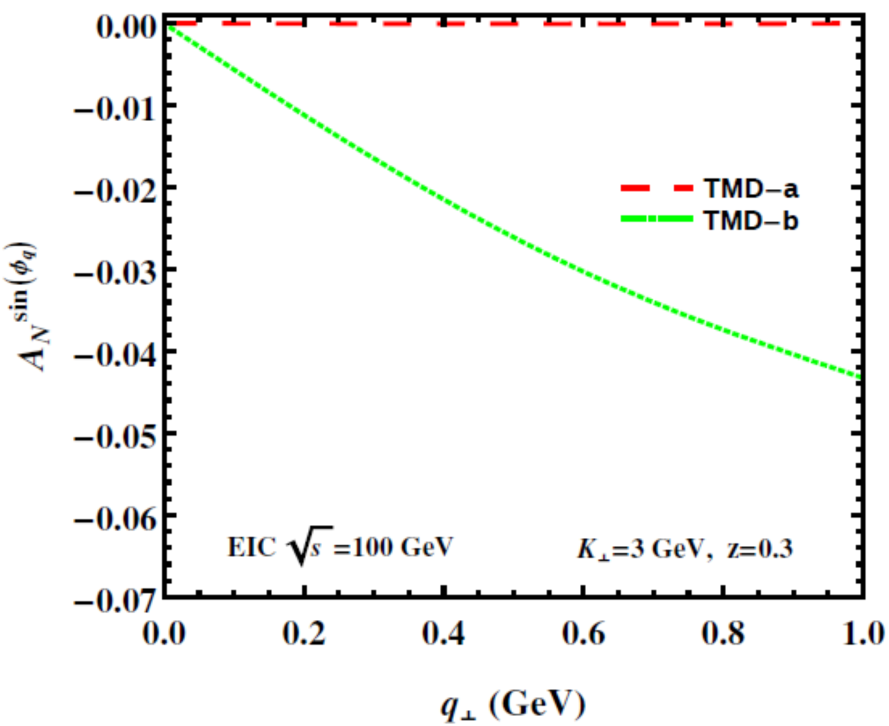


Weighted Sivers asymmetry in $e + p^{\uparrow} \rightarrow J/\psi + jet + X$ process using Gaussian parametrization for TMDs in DGLAP evolution approach.

RK, A.Mukherjee, R. Sangem, arXiv:1908.03698

Sivers asymmetry at EIC





Weighted Sivers asymmetry in $e + p^{\uparrow} \rightarrow J/\psi + jet + X$ process using TMD evolution approach.

Here, asymmetry comes negative which is mainly due to the parametrizations, TMD-a and TMD-b.

RK, A.Mukherjee, R. Sangem, arXiv:1908.03698

Conclusion

- We estimate the Sivers asymmetry in almost back-to-back J/ψ and jet photoproduction at the future EIC.
- The quasi-real photoproduction takes place through the Weizsaker-Williams photon distribution.
- We used NRQCD to calculate the J/ψ production in both the CS and CO model.
- We estimate the asymmetry using DGLAP evolution approach where we used Gaussian parametrization for TMDs and also using TMD evolution approach.
- We have obtained sizable Sivers asymmetry where the major contributions comes from gluon initiated process and quark contribution is small.
- Back-to-back production of J/ψ and jet at EIC can be a promising channel to access the gluon Sivers function.

Thanks for attention