

# Shear and pressure distributions in the proton from lattice QCD

Will Detmold

work wth Phiala Shanahan  
PRL (2019) arXiv:1810.07589  
PRD (2019) arXiv:1810.04626



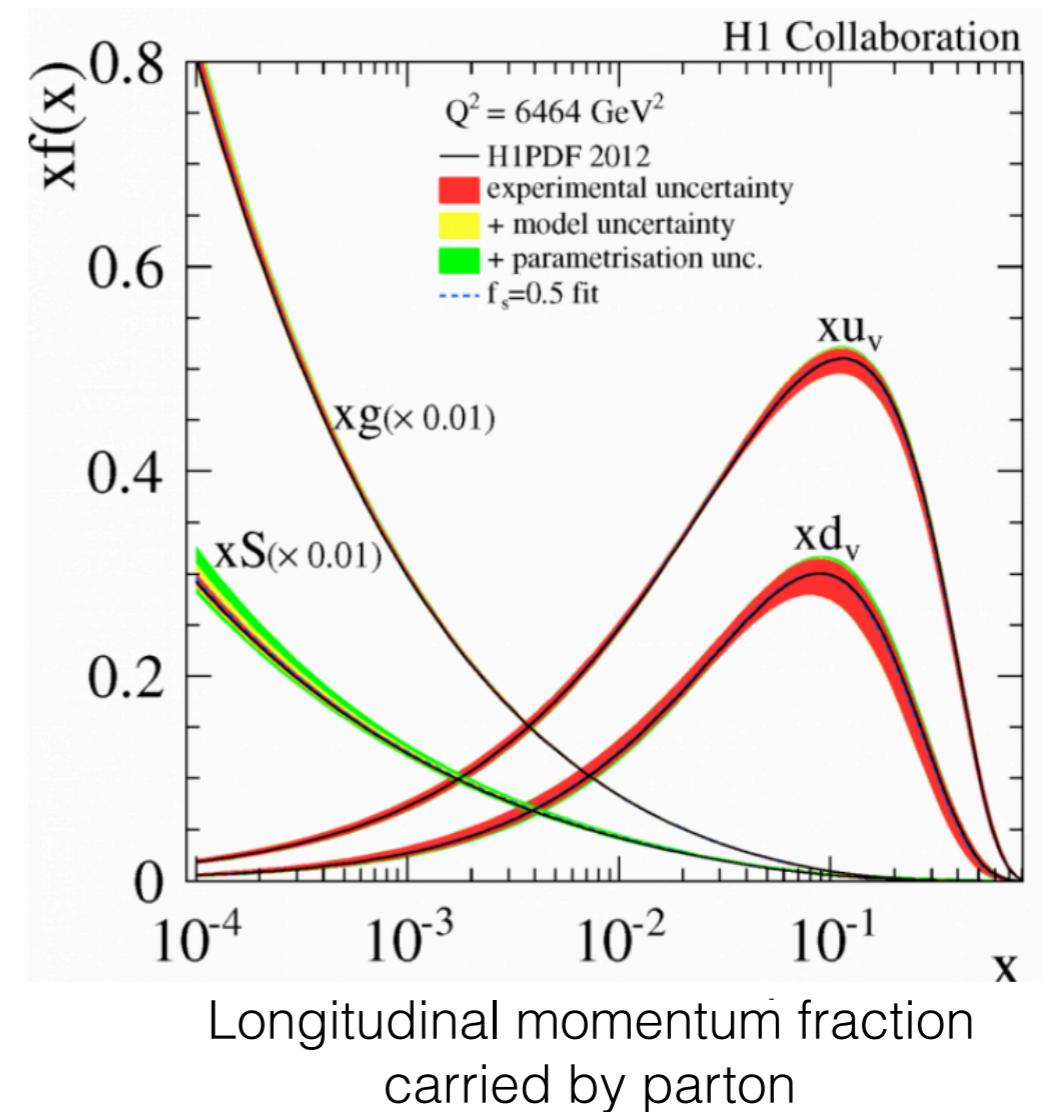
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Institute of  
Technology

# Gluon structure

Gluons offer a new window on nuclear structure

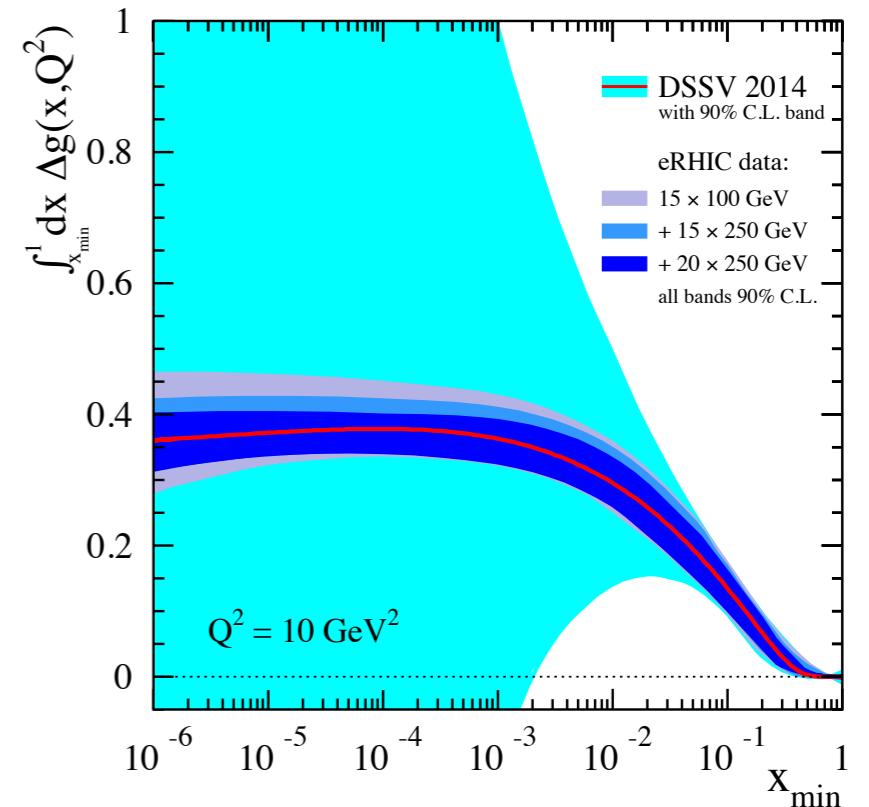
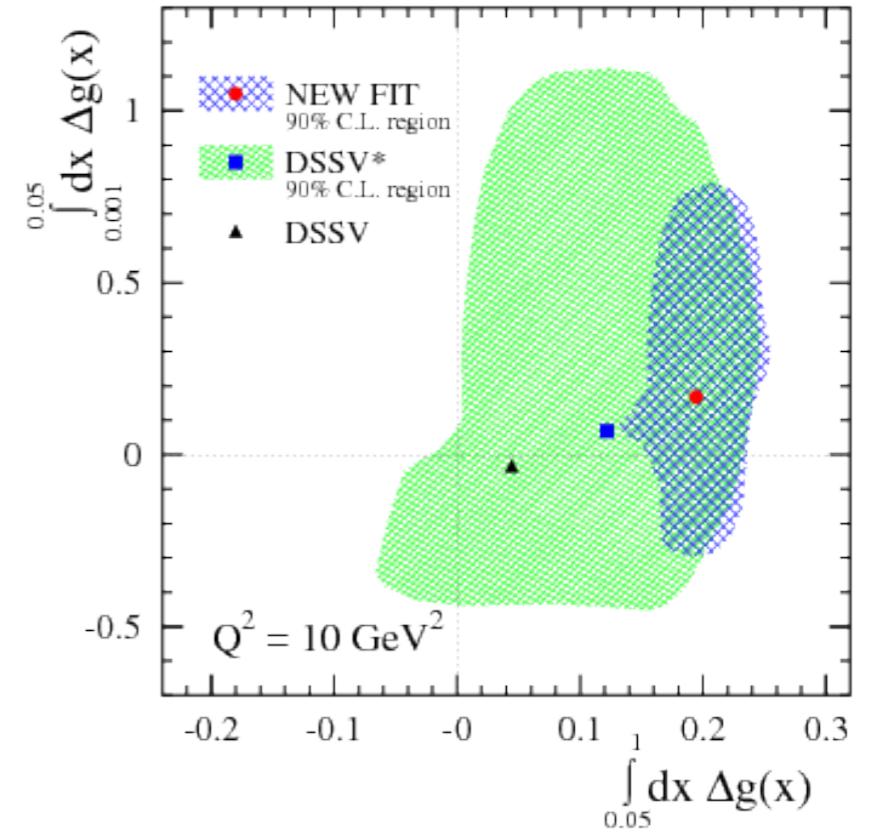
- Past 60+ years: detailed view of quark structure of nucleons
- Gluon structure also important
  - Unpolarised gluon PDF dominant at small longitudinal momentum fraction
- Other aspects of gluon structure relatively unexplored

Parton distributions in the proton



# Gluon angular momentum

- Gluon helicity much less well constrained
  - Major focus of RHIC-spin program
  - Asymmetries in polarised  $p\bar{p} \rightarrow \pi X, D\bar{X}, B\bar{X}, \text{jets}$
- Orbital angular momentum of gluons even less understood
  - Gluon TMDs
- Major motivation for EIC



# Gluon structure

How much do gluons contribute to the proton's

- Momentum
- Spin
- Mass
- D-term

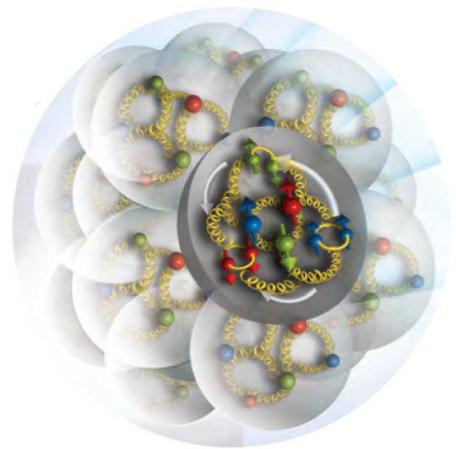


What are the gluon distributions in a proton

- PDFs, GPDs, TMDs
- Pressure, Shear
- Gluon 'radius/radii'

How is the gluon structure of a proton modified in a nucleus

- Gluon 'EMC' effect
- Exotic glue



# Gluon structure

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# Energy-momentum tensor

Many of these properties derived from **Energy-Momentum Tensor**  
(conserved Noether current associated with Lorentz translations)

Matrix elements of traceless gluon EMT for spin-half nucleon:

$$\langle p', s' | G_{\{\mu\alpha}^a G^{a\alpha\}}^{\nu} | p, s \rangle = \bar{U}(p', s') \left( A_g(t) \gamma_{\{\mu} P_{\nu\}} + B_g(t) \frac{i P_{\{\mu} \sigma_{\nu\}} \Delta^{\rho}}{2M_N} + D_g(t) \frac{\Delta_{\{\mu} \Delta_{\nu\}}}{4M_N} \right) U(p, s)$$

Generalised gluon form factors       $\Delta_\mu = p'_\mu - p_\mu$      $P_\mu = (p_\mu + p'_\mu)/2$ ,     $t = -\Delta^2$

- Three generalised gluon form factors  $A_g(t), B_g(t), D_g(t)$
- Sum rules with quark pieces in forward limit
  - Momentum fraction  $A_a(0) = \langle x \rangle_a \rightarrow \sum_{a=q,g} A_a(0) = 1$
  - Spin  $J_a(t) = \frac{1}{2}(A_a(t) + B_a(t)) \rightarrow \sum_{a=q,g} J_a(0) = \frac{1}{2}$
  - D-terms  $D_a(0)$  unknown but equally fundamental!

# D-term

D-term GFF encodes the **pressure** and **shear distributions in the nucleon** (Breit frame)

$$s(r) = -\frac{r}{2} \frac{d}{dr} \frac{1}{r} \frac{d}{dr} \tilde{D}(r), \quad p(r) = \frac{1}{3} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} \tilde{D}(r),$$

$$\tilde{D}(r) = \int \frac{d^3 \vec{p}}{2E(2\pi)^3} e^{-i\vec{p}\cdot\vec{r}} D(-\vec{p}^2)$$

- Quark and gluon shear forces individually well-defined (i.e., scale-dependent partial contributions  $s_{q,g}(r)$ )
- Pressure defined only for the total system (pieces depend also on GFFs related to the trace terms of the EMT that cancel in the sum)

# Generalised parton distributions

GFFs correspond to lowest moments of GPDs:

$$\begin{aligned}\int_0^1 dx H_g(x, \xi, t) &= A_g(t) + \xi^2 D_g(t), & \int_0^1 dx E_g(x, \xi, t) &= B_g(t) - \xi^2 D_g(t) \\ \int_{-1}^1 dx x H_q(x, \xi, t) &= A_q(t) + \xi^2 D_q(t), & \int_{-1}^1 dx x E_q(x, \xi, t) &= B_q(t) - \xi^2 D_q(t)\end{aligned}$$

- **Quark GPDs:** constraints from JLab, HERA, COMPASS, by DVCS, DVMP, future improvements from JLab 12GeV
- **Gluon GPDs:** almost unknown from experiment, future constraints are a central goal of EIC

Leading twist nucleon gluon GPDs:

$$\begin{aligned}& \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p', s' | G_a^{\{\mu\alpha}(-\frac{\lambda}{2}n) \left[ \mathcal{U}_{[-\frac{\lambda}{2}n, \frac{\lambda}{2}n]}^{(A)} \right]_{ab} G_b^{\nu\}}(\frac{\lambda}{2}n) | p, s \rangle \\ &= \frac{1}{2} \left( \textcolor{red}{H_g(x, \xi, t)} \bar{U}(p', s') P^{\{\mu\gamma\nu\}} U(p, s) + \textcolor{red}{E_g(x, \xi, t)} \bar{U}(p', s') \frac{P^{\{\mu i\sigma^\nu\alpha} \Delta_\alpha}}{2M} U(p, s) \right) + \dots ,\end{aligned}$$

Gluon field-strength tensor

$\Delta_\mu = p'_\mu - p_\mu \quad P_\mu = (p_\mu + p'_\mu)/2$   
 $t = \Delta^2 \quad n^2 = 0 \quad \xi = -\frac{1}{2}n \cdot \Delta / n \cdot P$

GPDs(Bjorken x, skewness, mom transfer)

# D-term from JLab DVCS

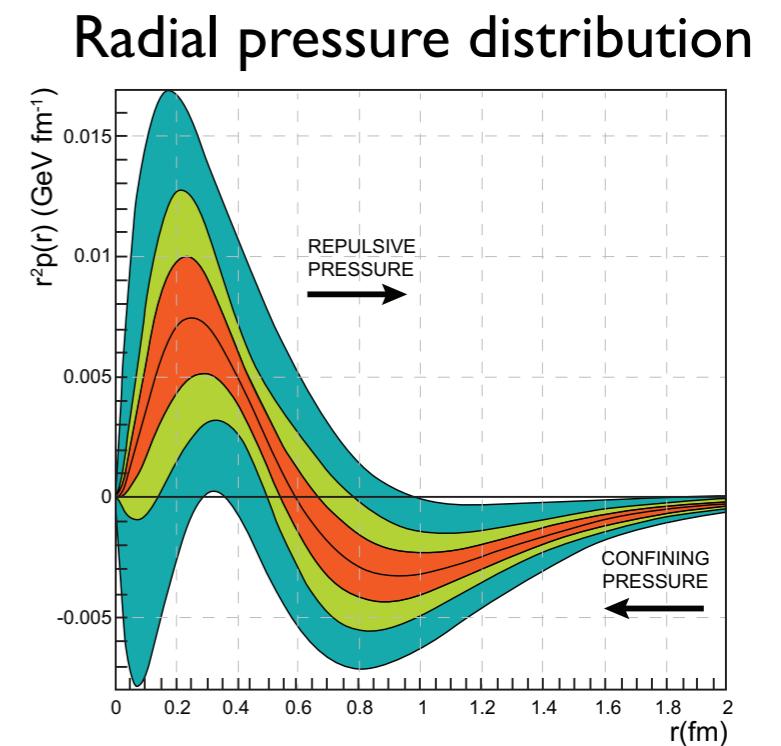
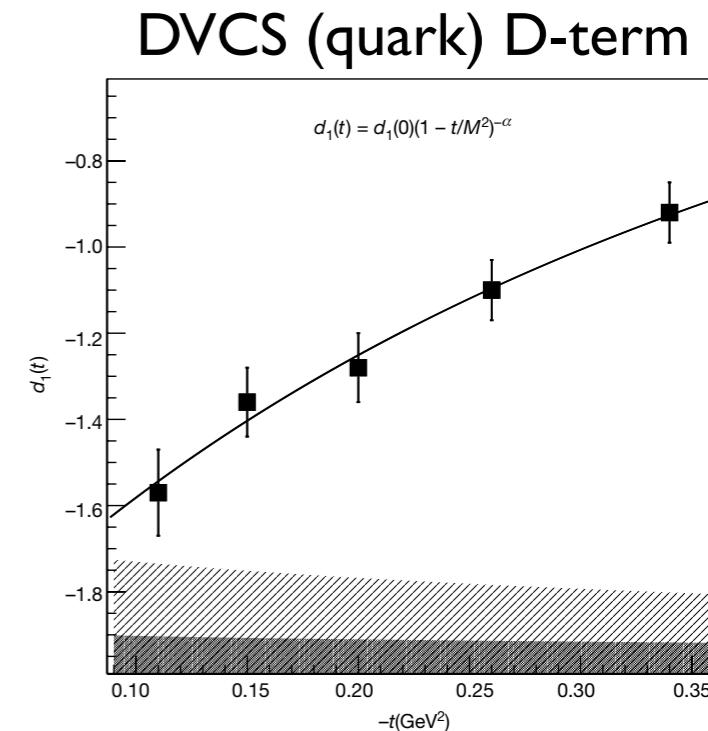
Recent experimental determination of DVCS D-term and extraction of proton pressure distribution

V. D. Burkert, L. Elouadrhiri, and F. X. Girod, Nature 557, 396 (2018)

$$s(r) = -\frac{r}{2} \frac{d}{dr} \frac{1}{r} \frac{d}{dr} \tilde{D}(r), \quad p(r) = \frac{1}{3} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} \tilde{D}(r)$$

- Strong repulsive pressure near the centre of the proton
- Binding pressure at greater distances.
- Peak pressure near the centre  $\sim 1035$  Pascal, greater than pressure estimated for neutron stars
- Key assumptions:** gluon D-term same as quark term, tripole form factor model,  $D_u(t, \mu) = D_d(t, \mu)$

Test assumptions in pressure extraction



# Gluon structure

First-principles QCD calculations

→ QCD benchmarks and predictions ahead of experiment

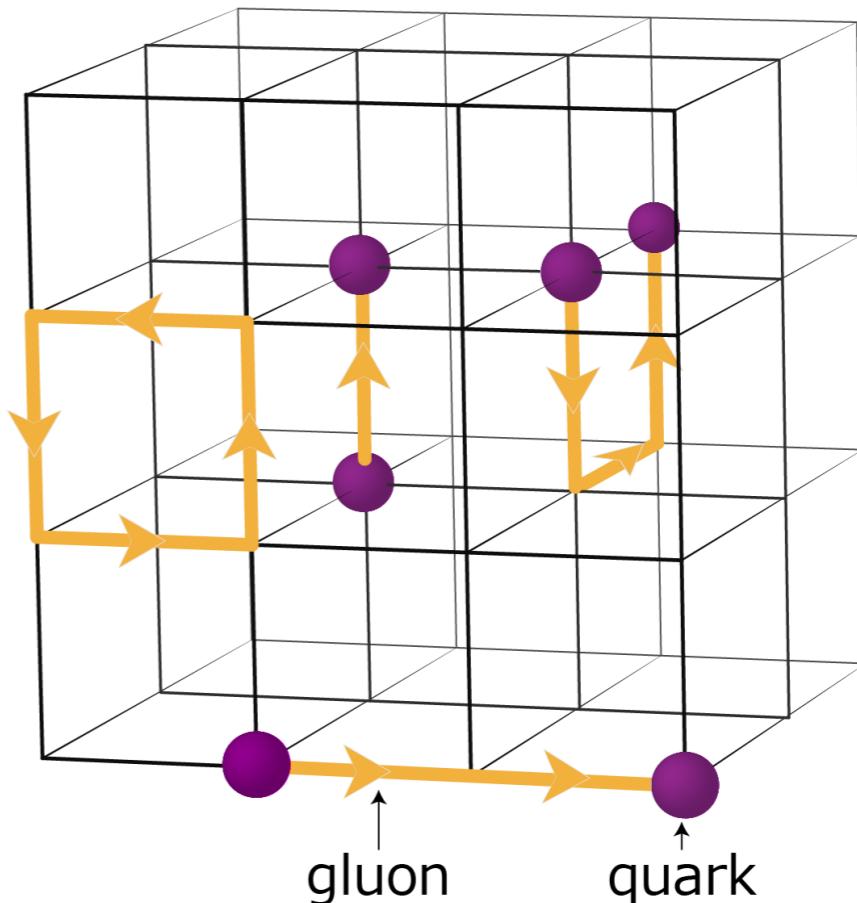


Cover image from EIC whitepaper arXiv:1212.1701

# Lattice QCD

Numerical first-principles approach to non-perturbative QCD

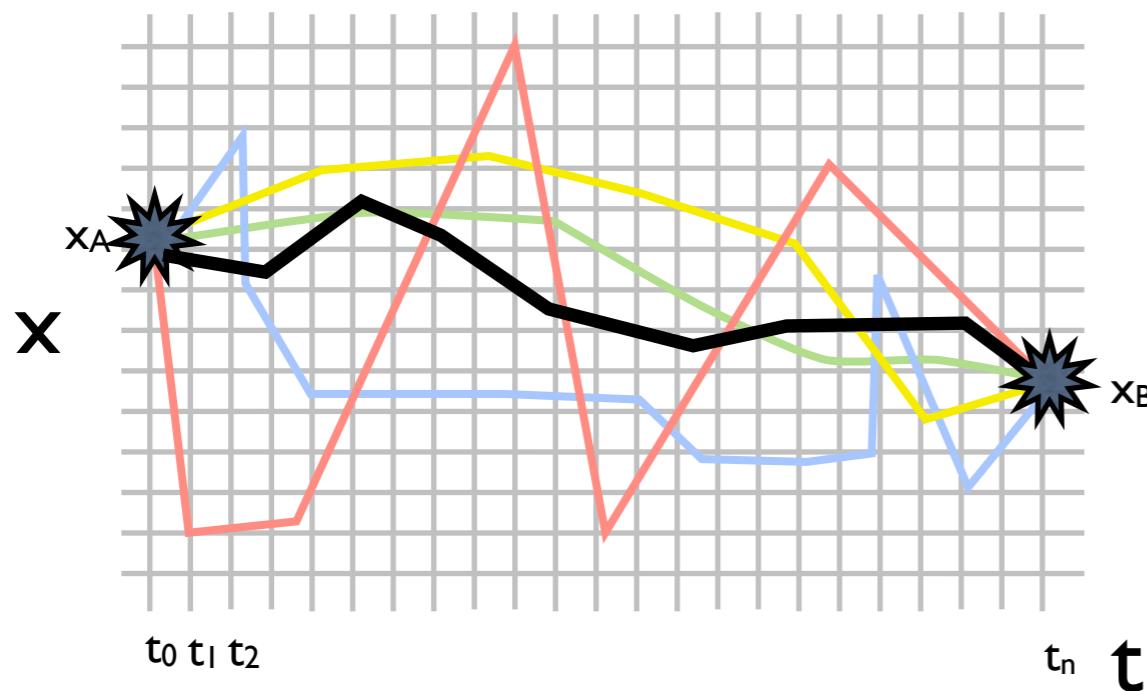
- Discretise equations of QCD onto space-time grid
- Calculate physical quantities
- Take limit of vanishing discretisation afterwards



# Lattice QCD

Numerical first-principles approach to non-perturbative QCD

- QCD equations  $\leftrightarrow$  integrals over the values of quark and gluon fields on each site/link (QCD path integral)
- $\sim 10^{12}$  variables (for state-of-the-art)



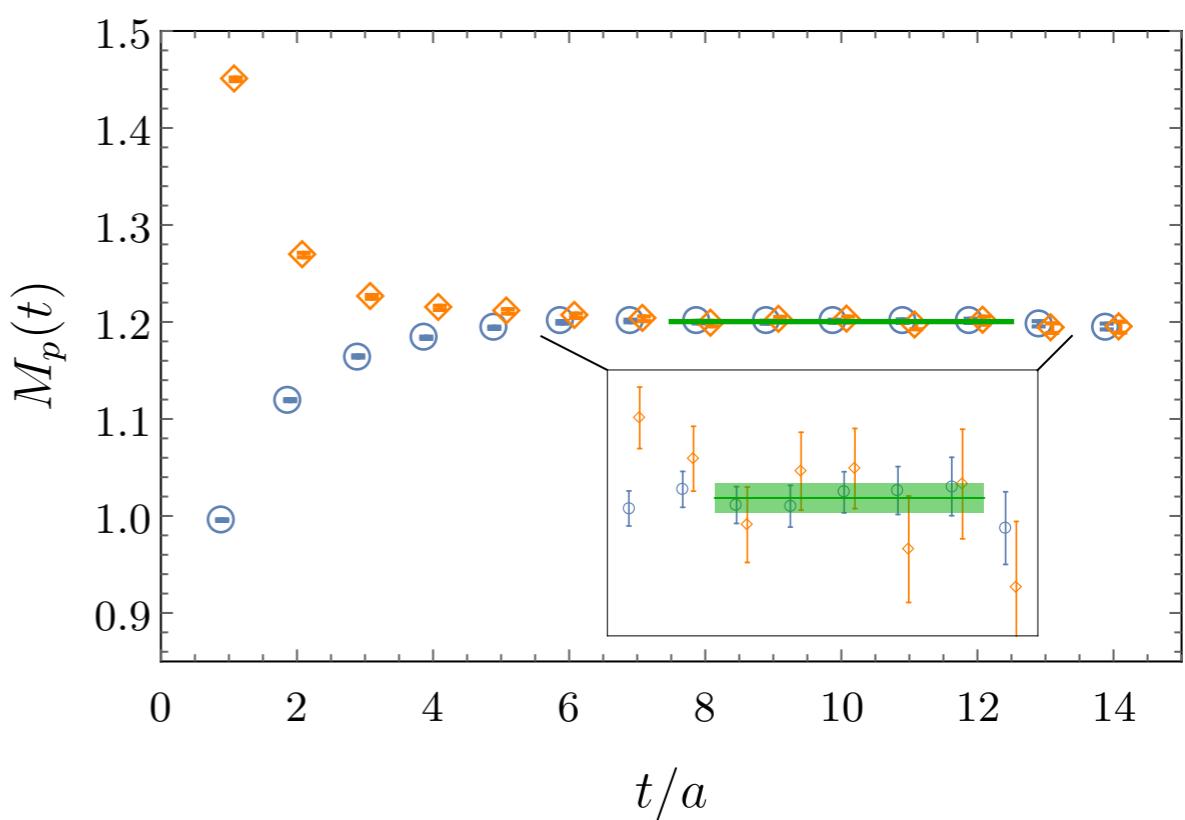
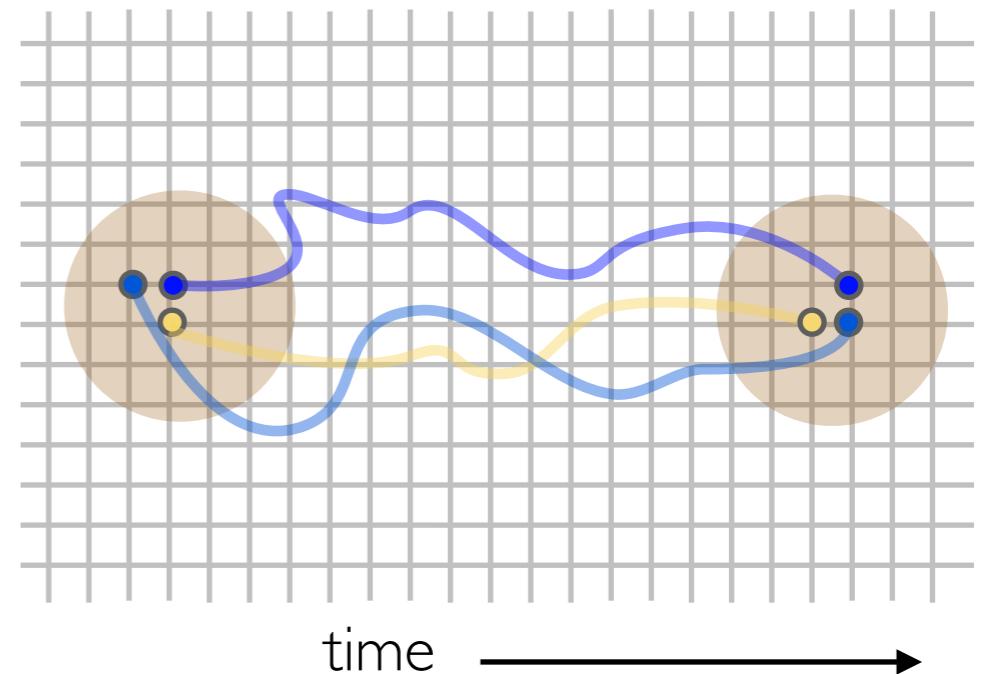
- Evaluate by importance sampling
- Paths near classical action dominate
- Calculate physics on a set (ensemble) of samples of the quark and gluon fields

# Lattice QCD

## Calculate the nucleon mass

- Create three quarks, annihilate them far from source  
Tie together creation and annihilation operators in all possible ways (contractions)
- QCD path integral adds quark anti-quark pairs and gluons automatically
- Measure exponentially decaying correlation to extract mass

$$M(t) = \ln \left[ \frac{C(t)}{C(t+1)} \right] \xrightarrow{t \rightarrow \infty} E_0$$



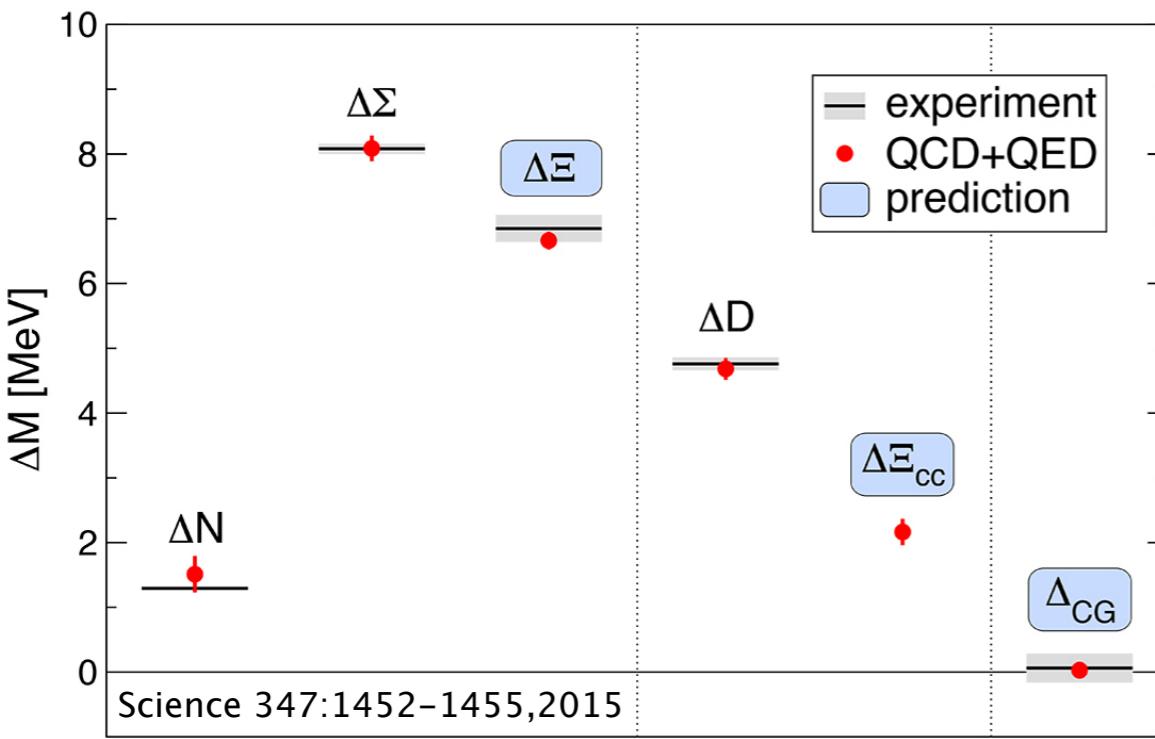
# Lattice QCD

- Calculations use world's largest computers
- Many millions of CPU/GPU hours
- Specifically designed processors for QCD  
(QCDOC precursor of BlueGene computers)

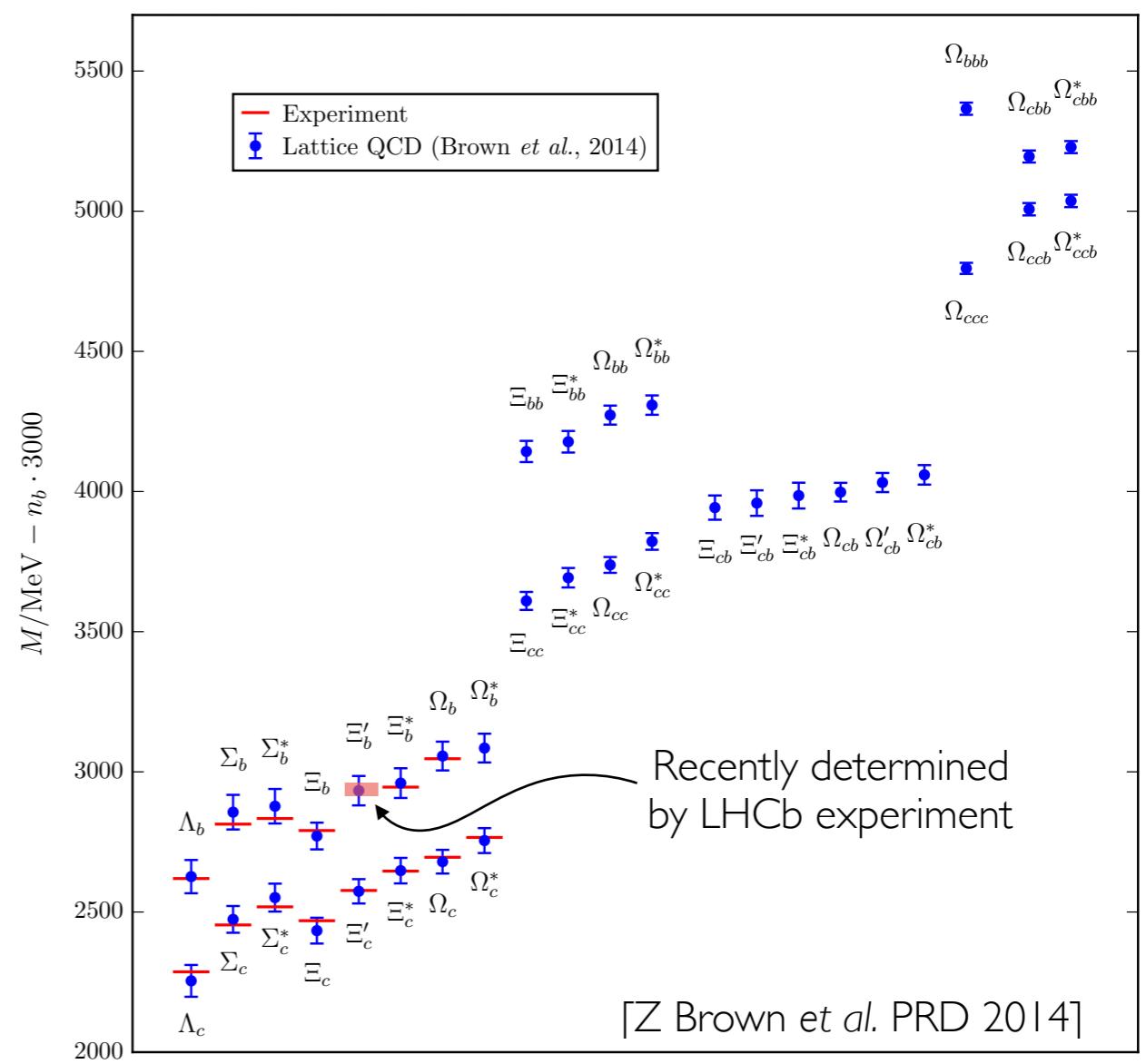


# Lattice QCD

- Ground state hadron spectrum reproduced
- p-n mass splitting reproduced
- ...



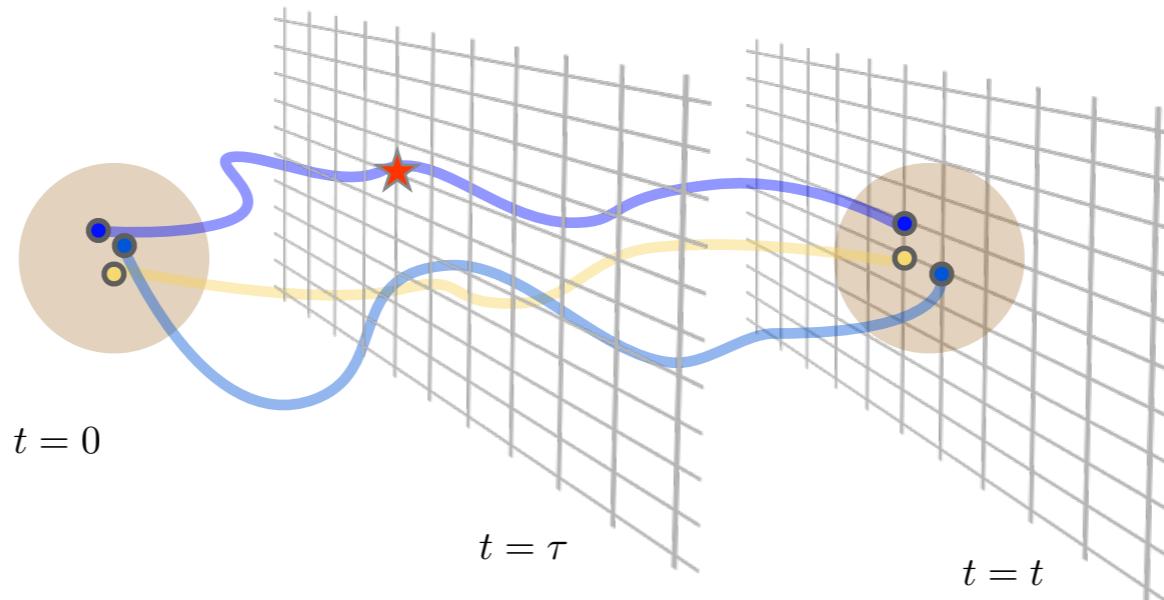
- Predictions for new states with controlled uncertainties



# Lattice QCD Matrix Elements

## Calculate matrix elements

- Create three quarks (correct quantum numbers) at a source and annihilate the three quarks at sink far from source
- Insert operator at intermediate timeslice



- Remove time-dependence by dividing out with two-point correlators:

$$\frac{C_3(t, \tau, \vec{p}', \vec{q})}{C_2(t - \tau, p') C_2(\tau, p)} \xrightarrow{t \rightarrow \infty} \langle N(p') | \mathcal{O}(q) | N(p) \rangle$$

# Gluon GFFs from LQCD

Construct system of equations for **generalised gluon form factors**

Ratios of 3pt and 2pt correlation functions:

$$R_{s;\mathfrak{R},i}(\vec{p}, \vec{p}', t_f, \tau) = \frac{C_{s;\mathfrak{R},i}^{3\text{pt}}(\vec{p}, \vec{p}', t_f, \tau)}{C_s^{2\text{pt}}(\vec{p}', t_f)} \sqrt{\frac{C_s^{2\text{pt}}(\vec{p}, t_f - \tau) C_s^{2\text{pt}}(\vec{p}', t_f) C_s^{2\text{pt}}(\vec{p}', \tau)}{C_s^{2\text{pt}}(\vec{p}', t_f - \tau) C_s^{2\text{pt}}(\vec{p}, t_f) C_s^{2\text{pt}}(\vec{p}, \tau)}} \xrightarrow{t_f \gg \tau \gg 0} \frac{\text{Tr} [\Gamma_s(\not{p}' + M_N) \mathcal{F}_i[A_g, B_g, D_g](\not{p} + M_N)]}{8\sqrt{E_{\vec{p}}^{(N)} E_{\vec{p}'}^{(N)} (E_{\vec{p}}^{(N)} + M_N) (E_{\vec{p}'}^{(N)} + M_N)}}$$

$$\mathcal{F}_{\mu\nu}[A_g, B_g, D_g] = \color{red} A_g(t) \gamma_{\{\mu} P_{\nu\}} + B_g(t) \frac{i P_{\{\mu} \sigma_{\nu\}\rho} \Delta^{\rho}}{2M_N} + D_g(t) \frac{\Delta_{\{\mu} \Delta_{\nu\}}}{4M_N}$$

Generalised gluon form factors

$$\Delta_\mu = p'_\mu - p_\mu \quad P_\mu = (p_\mu + p'_\mu)/2 \quad t = |\Delta|^2$$

- Nucleon spin up/down:  $\Gamma_{s=\pm 1}$
- Sink and operator momenta:

$$|\vec{p}'|^2 \leq 5(2\pi/L)^2$$

$$|\vec{\Delta}|^2 \leq 18(2\pi/L)^2$$

- Operator index choices:  
two different irreducible representations of  $H(4)$

$$\mathcal{O}_{i=\{1,\dots,6\}}^{\tau_3^{(6)}} = \left\{ \frac{(-i)^{\delta_{\nu 0}}}{\sqrt{2}} (\mathcal{O}_{\mu\nu} + \mathcal{O}_{\nu\mu}), \quad 0 \leq \mu < \nu \leq 3 \right\}$$

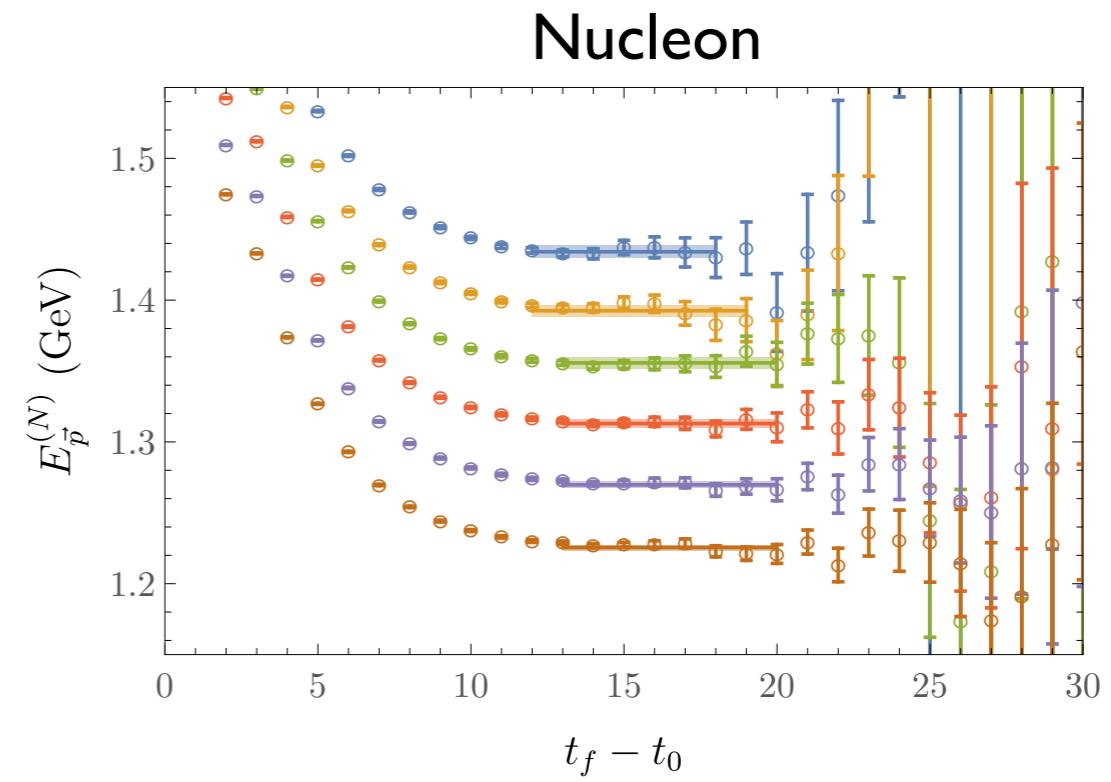
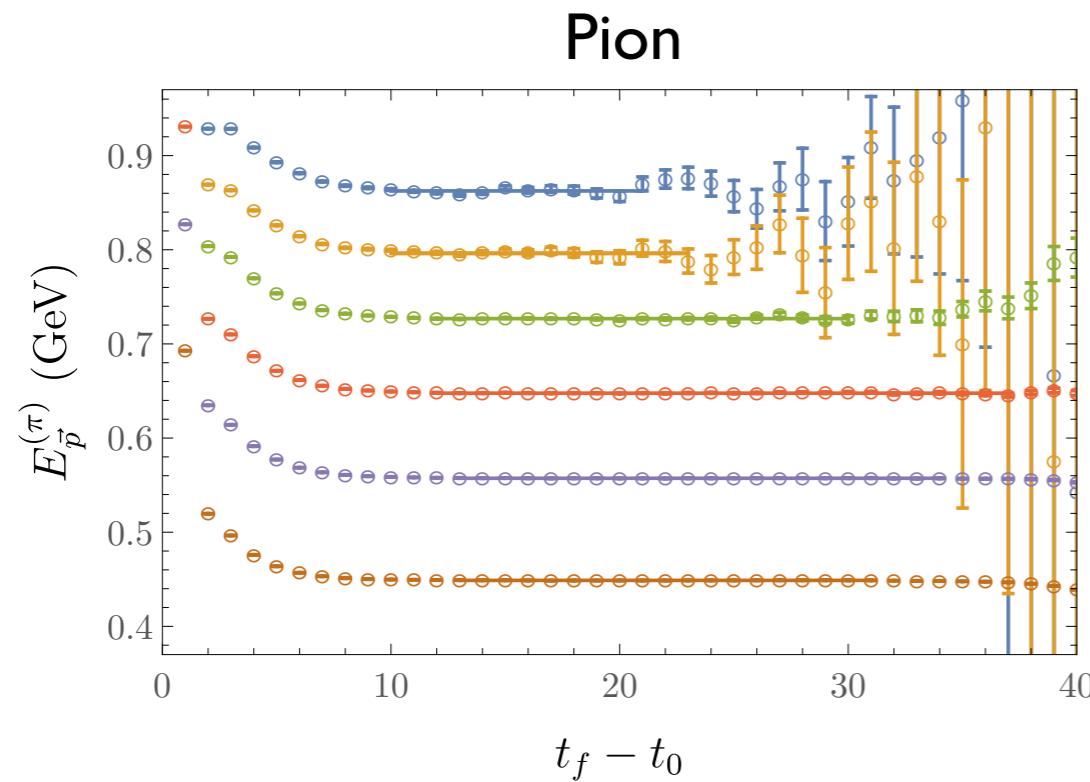
$$\mathcal{O}_1^{\tau_1^{(3)}} = \frac{1}{2} (\mathcal{O}_{11} + \mathcal{O}_{22} - \mathcal{O}_{33} + \mathcal{O}_{00}), \quad \dots,$$

# Gluon GFFs from LQCD

One ensemble,  $m_\pi \sim 450$  MeV (physical masses running now)

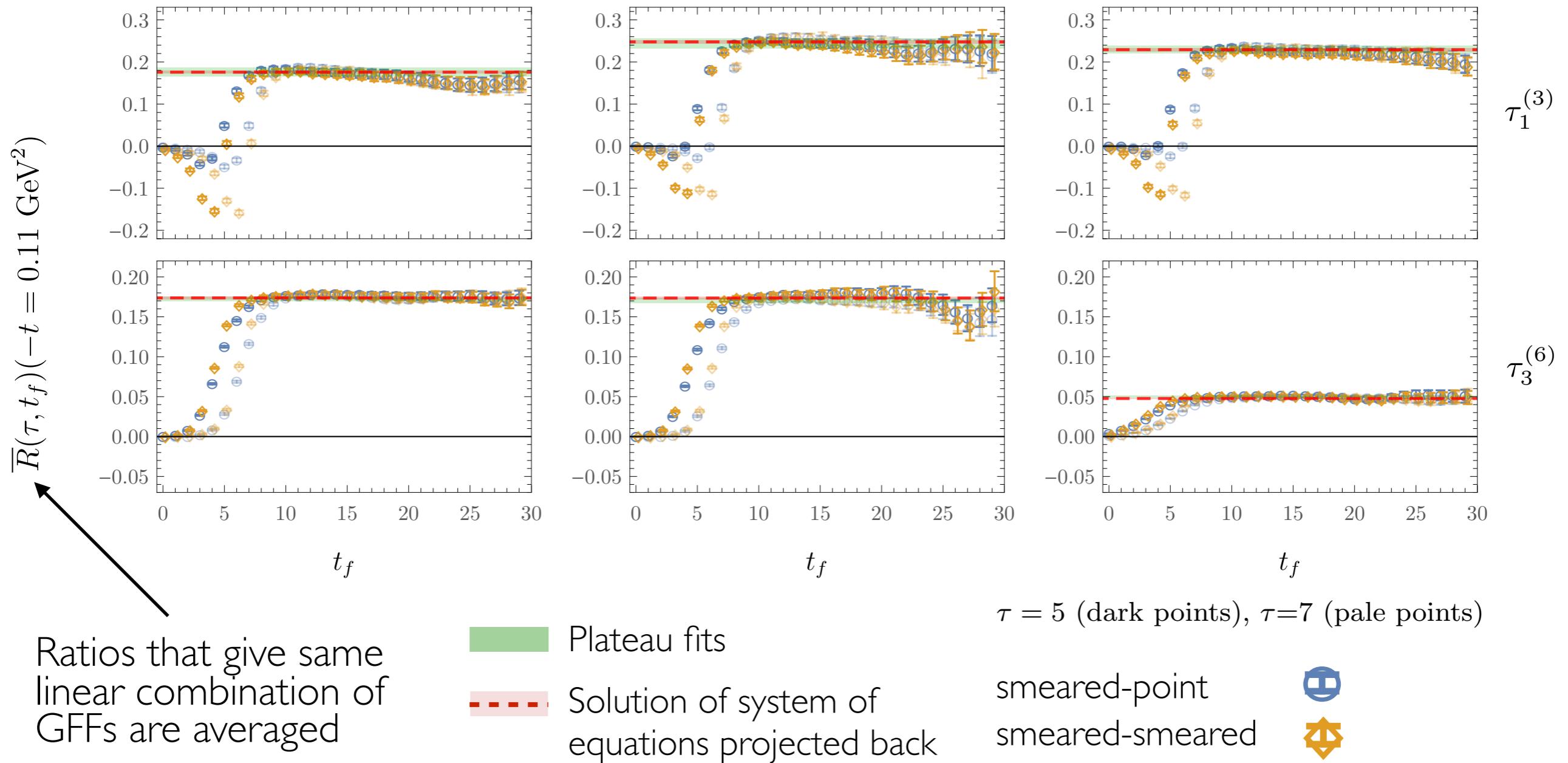
$L/a$	$T/a$	$\beta$	$am_l$	$am_s$	$a$ (fm)	$L$ (fm)	$T$ (fm)	$m_\pi$ (MeV)	$m_K$ (MeV)	$m_\pi L$	$m_\pi T$	$N_{\text{cfg}}$	$N_{\text{meas}}$
32	96	6.1	-0.2800	-0.2450	0.1167(16)	3.7	11.2	450(5)	596(6)	8.5	25.6	2821	203

Clean plateaus in effective masses for  $|\vec{p}'|^2 \leq 5(2\pi/L)^2$



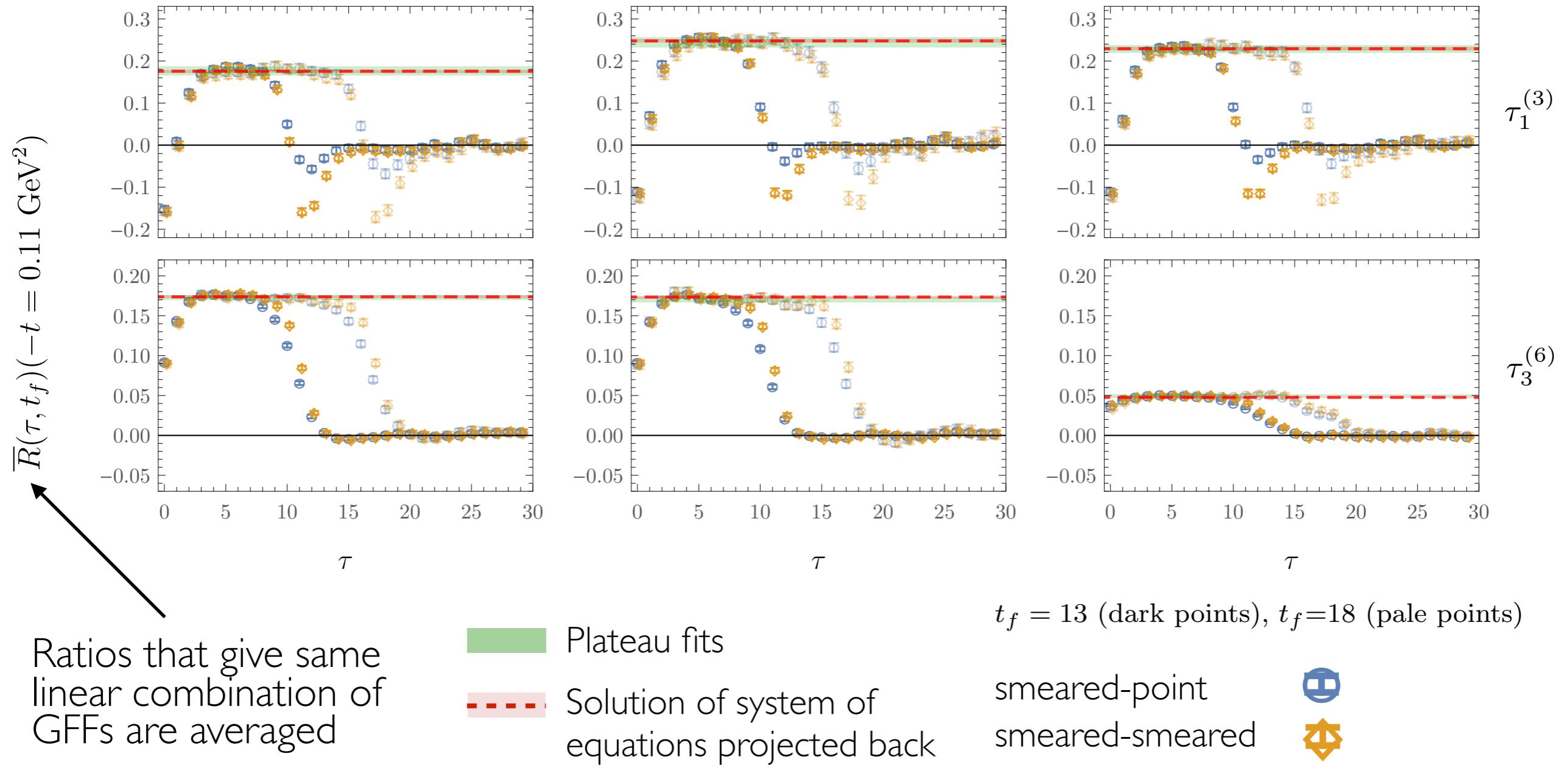
# Gluon GFFs from LQCD

PION: Clean signals in 3pt/2pt ratios (examples)



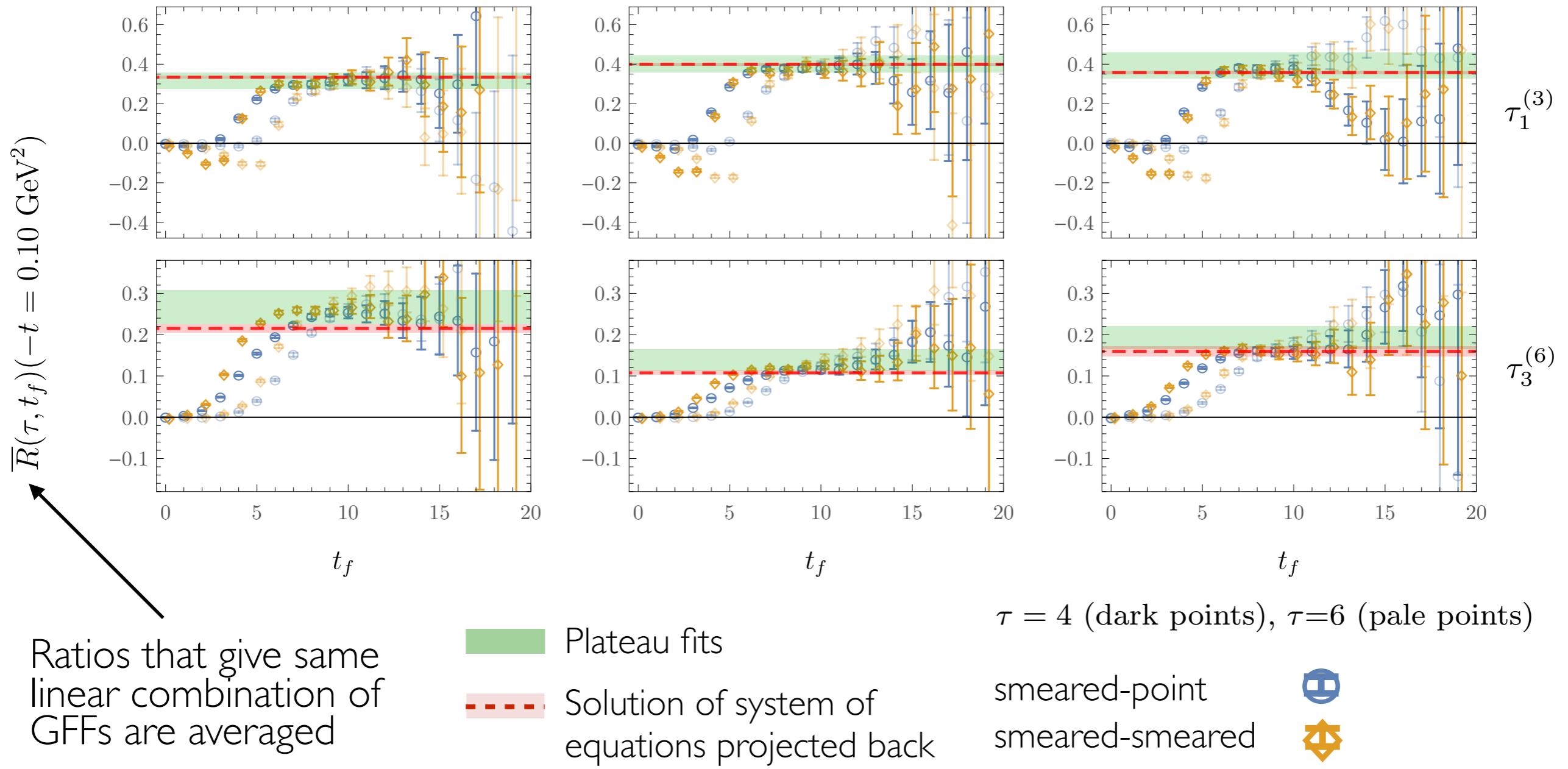
# Gluon GFFs from LQCD

PION: Clean signals in 3pt/2pt ratios (examples)



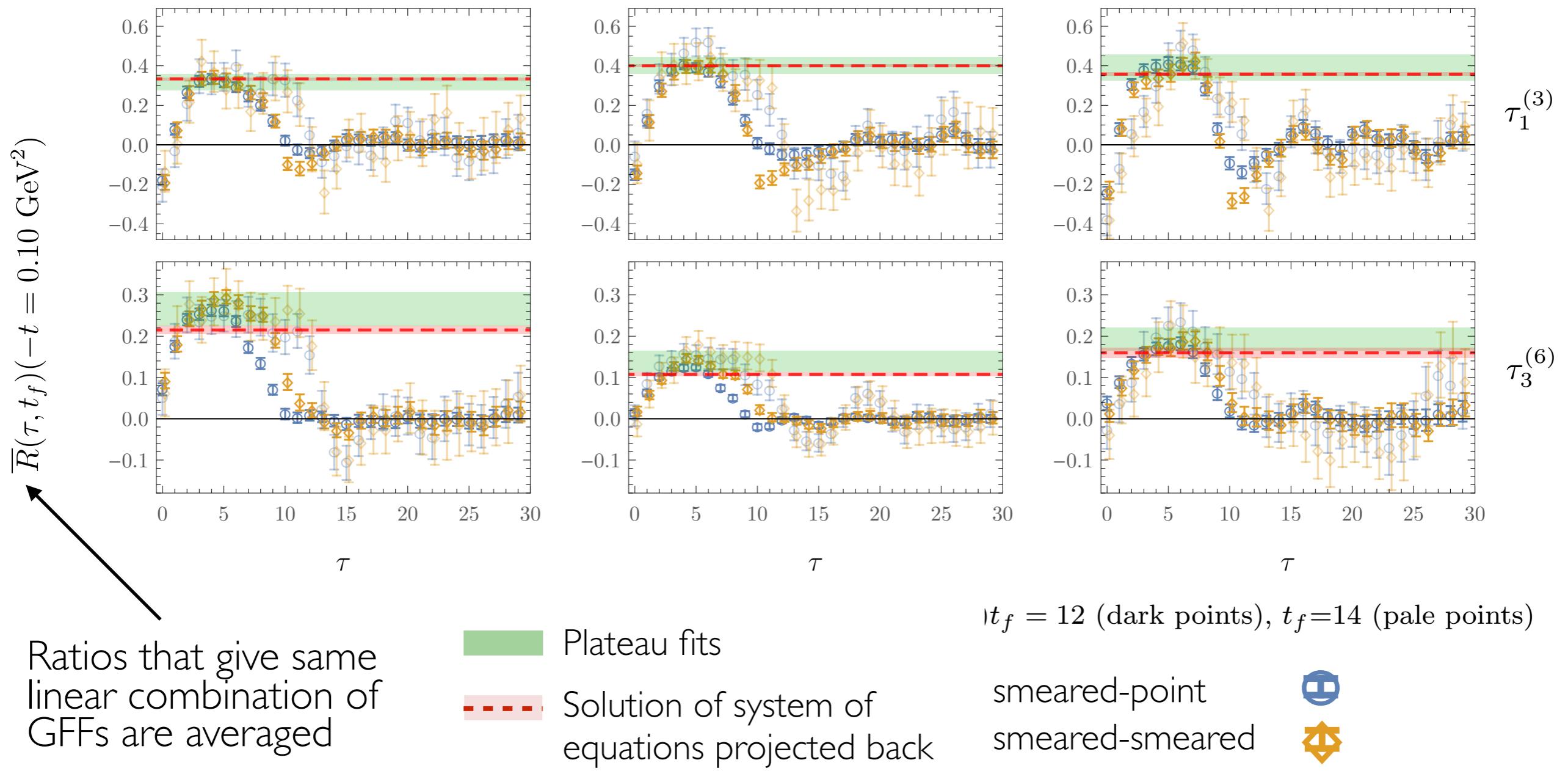
# Gluon GFFs from LQCD

**NUCLEON:** Clean signals in 3pt/2pt ratios (examples)



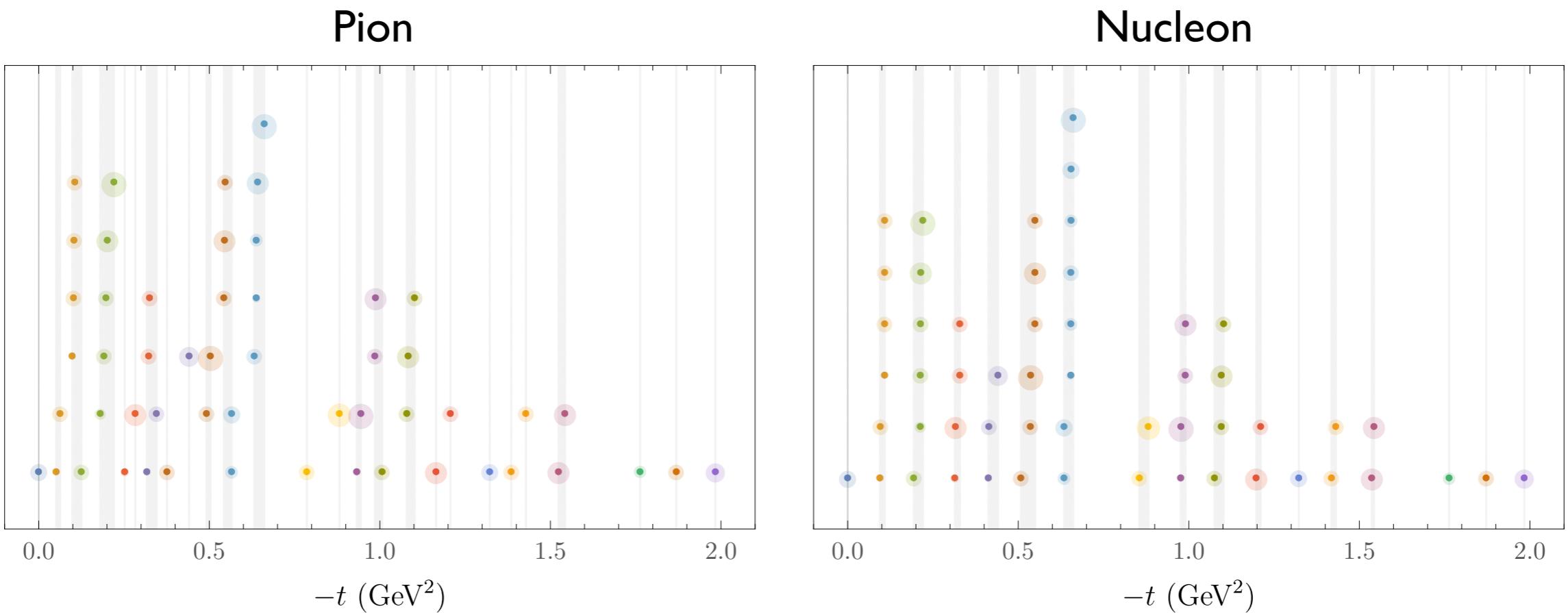
# Gluon GFFs from LQCD

**NUCLEON:** Clean signals in 3pt/2pt ratios (examples)



# Gluon GFFs from LQCD

Solve system of equations for GFFs in bins in  $t = (p' - p)^2$



- Colour coding: three momentum transfer  $\vec{\Delta}^2 = (\vec{p}' - \vec{p})^2$
- Point size  $\propto$  number of three-momenta at that  $\vec{\Delta}^2$
- Grey bands: bins in  $t$

# Renormalisation

## Non-perturbative RI-MOM renormalisation of gluon operator

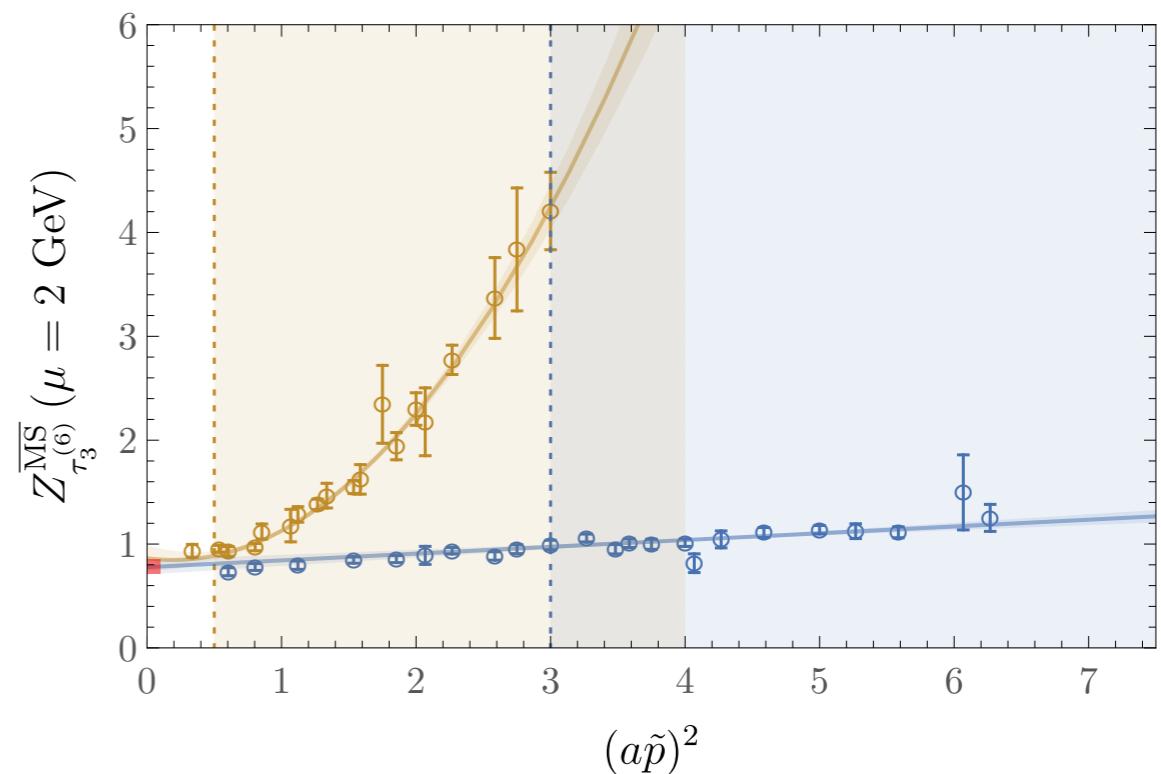
- Mixing with quark operator neglected  
Found to be small in lattice PT e.g., Alexandrou et al., 1611.06901
- One-loop perturbative matching to  $\overline{\text{MS}}$  scheme: Yang et al., 1612.02855

$$\mathcal{O}^{\overline{\text{MS}}}(\mu^2) = Z_{\mathcal{O}}^{\overline{\text{MS}}}(\mu^2) \mathcal{O}^{\text{latt}} = \mathcal{R}^{\overline{\text{MS}}}(\mu^2, \mu_R^2) Z_{\mathcal{O}}^{\text{RI-MOM}}(\mu_R^2) \mathcal{O}^{\text{latt}}$$

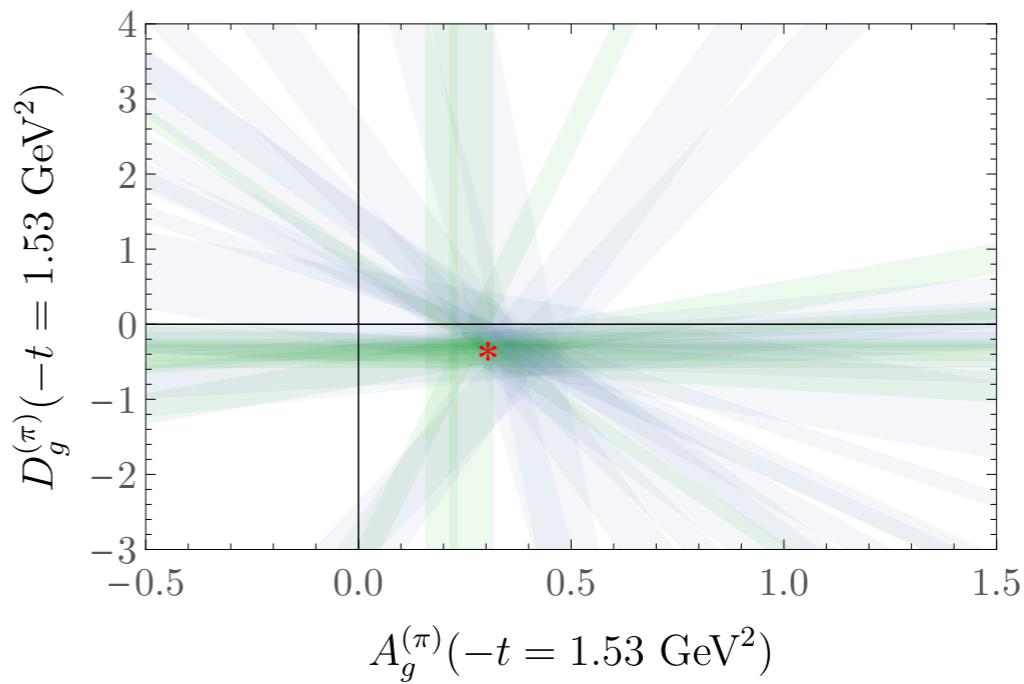
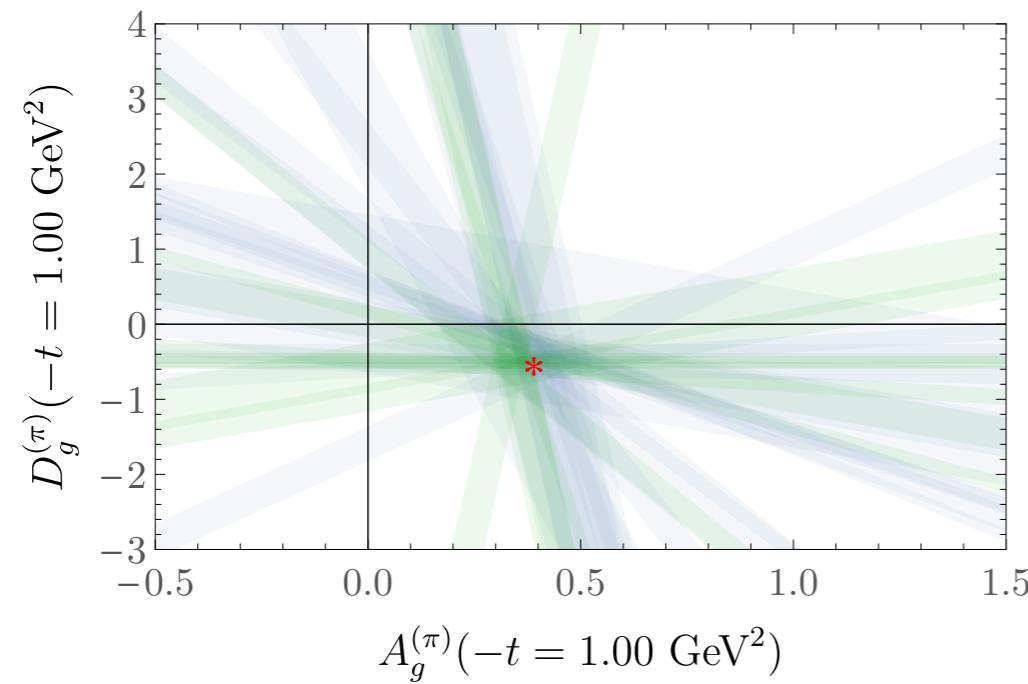
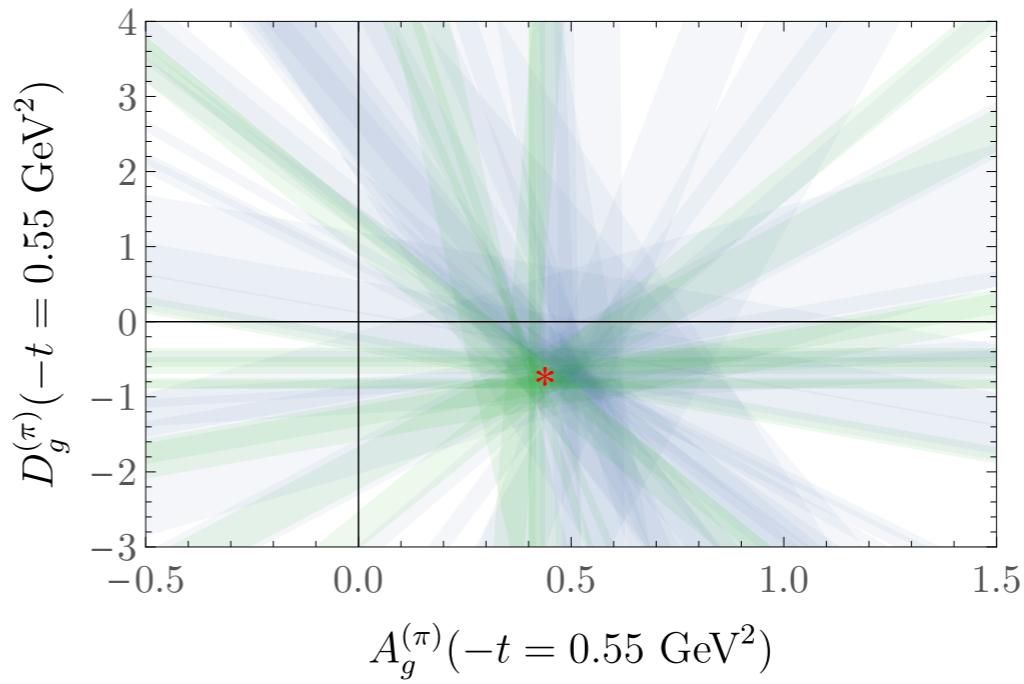
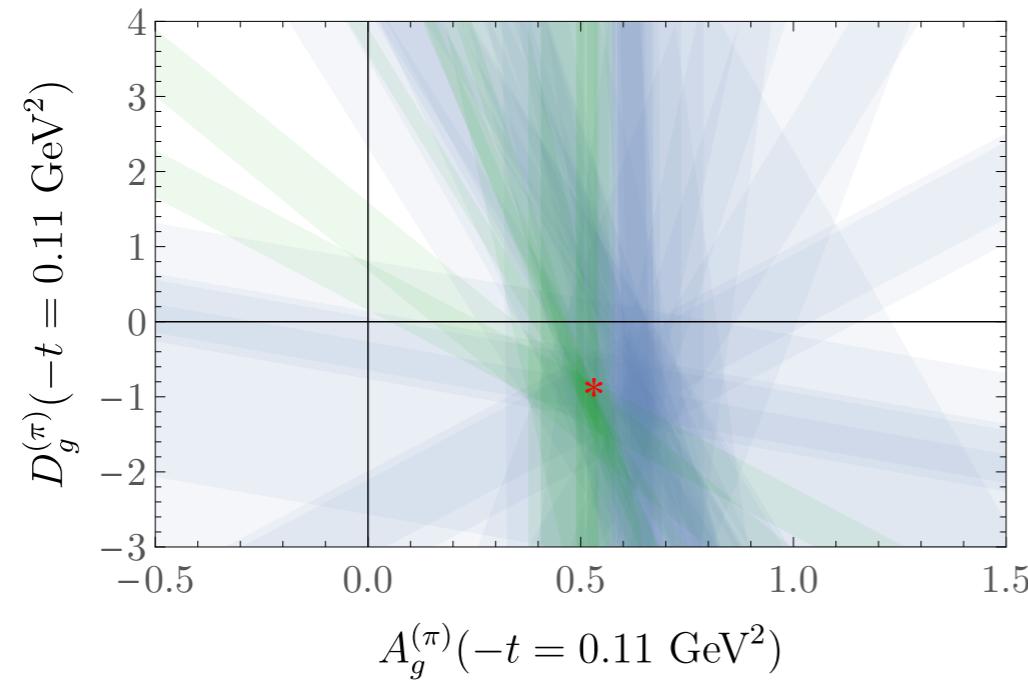
Calculate RI-MOM coefficient  
using Landau-gauge fixed gluon  
2pt function

$$(Z_{\hat{\mathcal{O}}}^{\text{RI-MOM}}(\mu_R^2))^{-1} = \frac{4p^2 \langle \hat{\mathcal{O}}_{\alpha\beta} \text{Tr}[A_\tau(p)A_\tau(-p)] \rangle}{\Lambda_{\hat{\mathcal{O}}}^{\text{tree}}(p) \langle \text{Tr}[A_\tau(p)A_\tau(-p)] \rangle} \Big|_{\substack{p^2=\mu_R^2 \\ \tau \neq \alpha \neq \beta \\ p_\tau=0}}$$
$$\Lambda_{\hat{\mathcal{O}}}^{\text{tree}}(p) = \langle \hat{\mathcal{O}}_{\alpha\beta}^{\mathfrak{R}} \text{Tr}[A_\tau(p)A_\tau(-p)] \rangle_{\text{amp}}^{\text{tree}}$$

- Wilson-flowed gluon 2pts
- No flow in 2pts



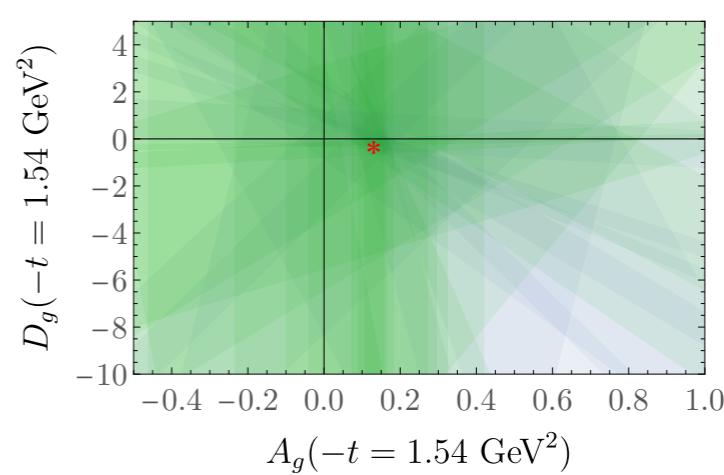
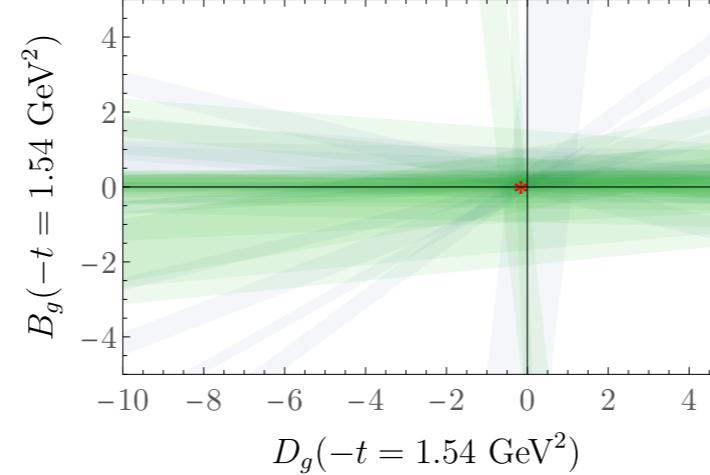
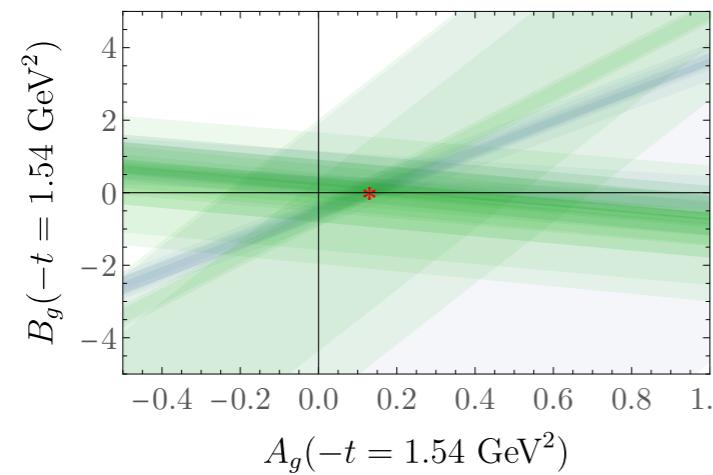
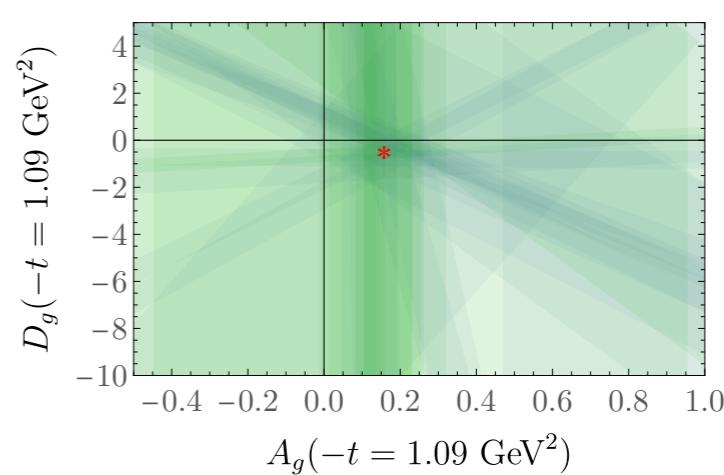
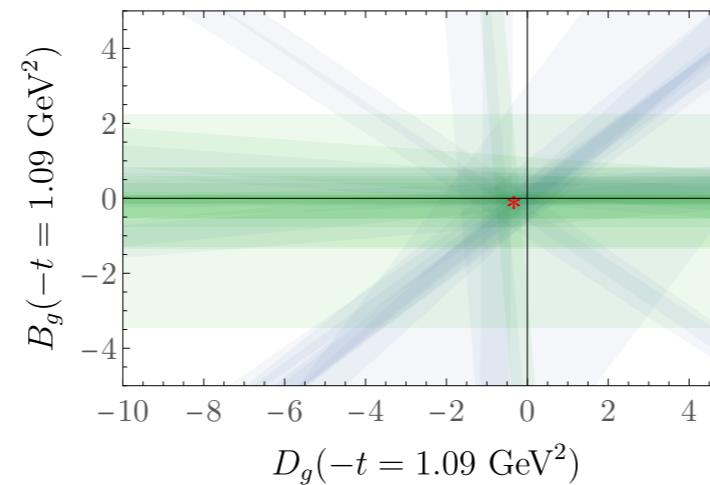
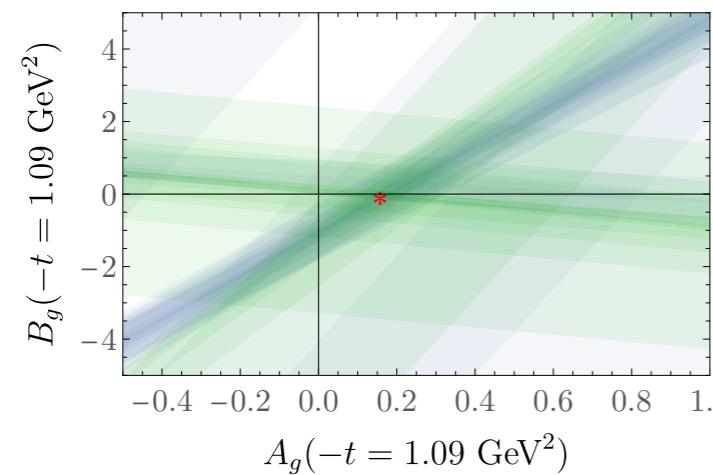
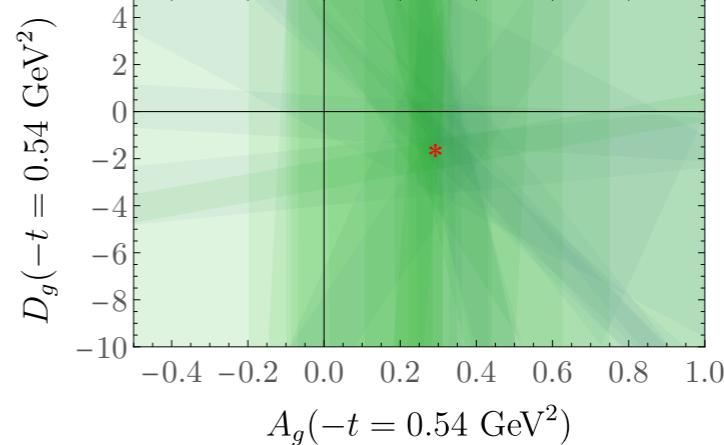
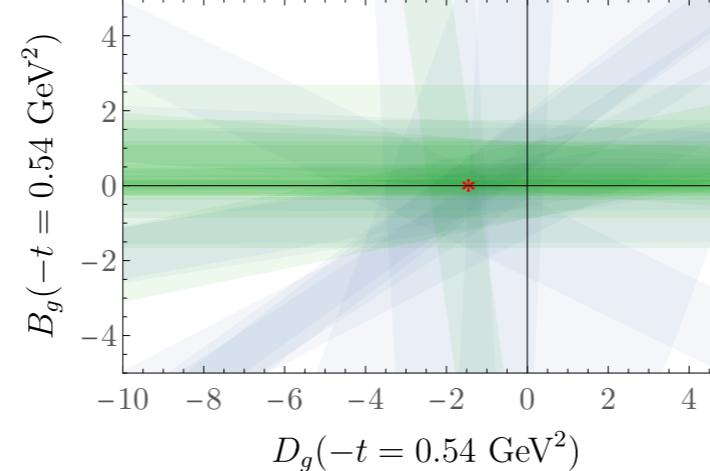
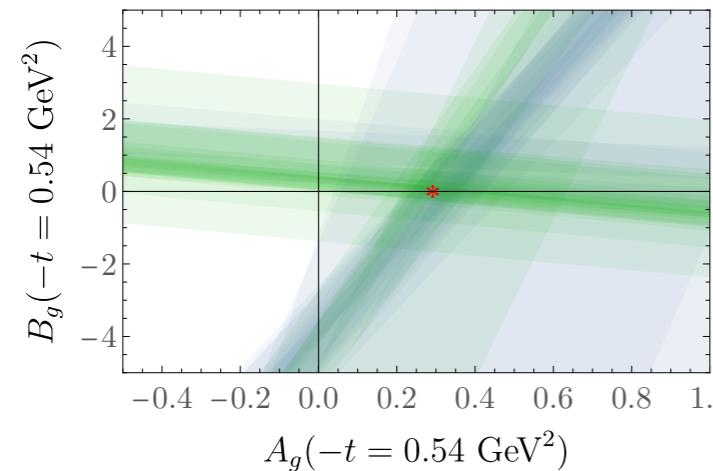
# Gluon GFFs from LQCD



Uncertainties from renormalisation not shown

$\tau_1^{(3)}$   $\tau_3^{(6)}$

# Gluon GFFs from LQCD



**Cross-sections:** GFF not shown in each projection taken to its central value

$\tau_1^{(3)}$   
 $\tau_3^{(6)}$

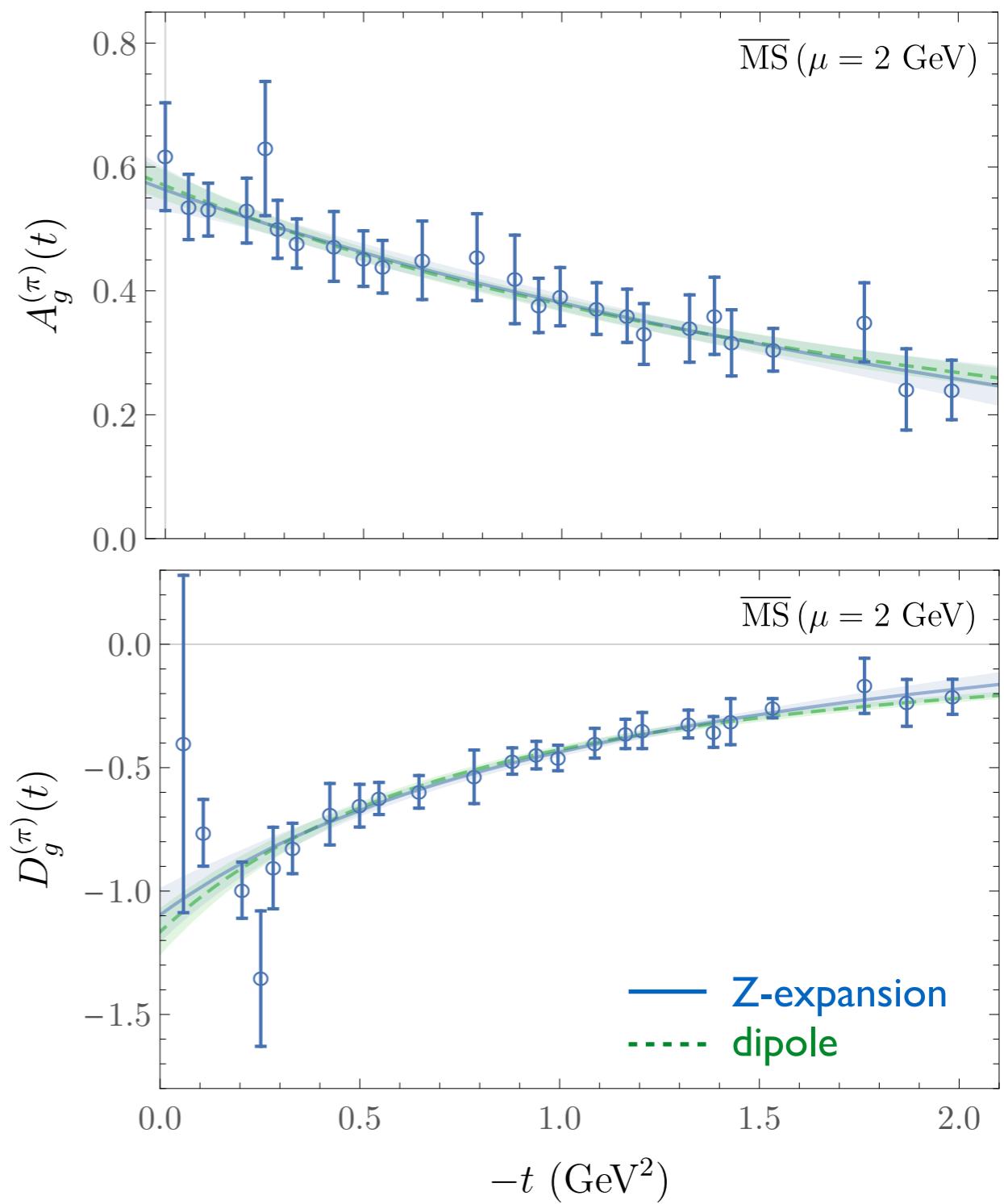
# LQCD Pion GFFs

Pion gluon GFFs  $m_\pi \sim 450$  MeV

Solve system of equations simultaneously for both hypercubic irreps for each binned four-momentum transfer

Dipole-like fall-off with momentum transfer

- Momentum fraction  $A_a(0) = \langle x \rangle_a$ 
 $\rightarrow \sum_{a=q,g} A_a(0) = 1$
- D-terms  $D_a(0)$  related to pressure and shear distributions



# LQCD Pion GFFs

Pion gluon GFFs  $m_\pi \sim 450$  MeV

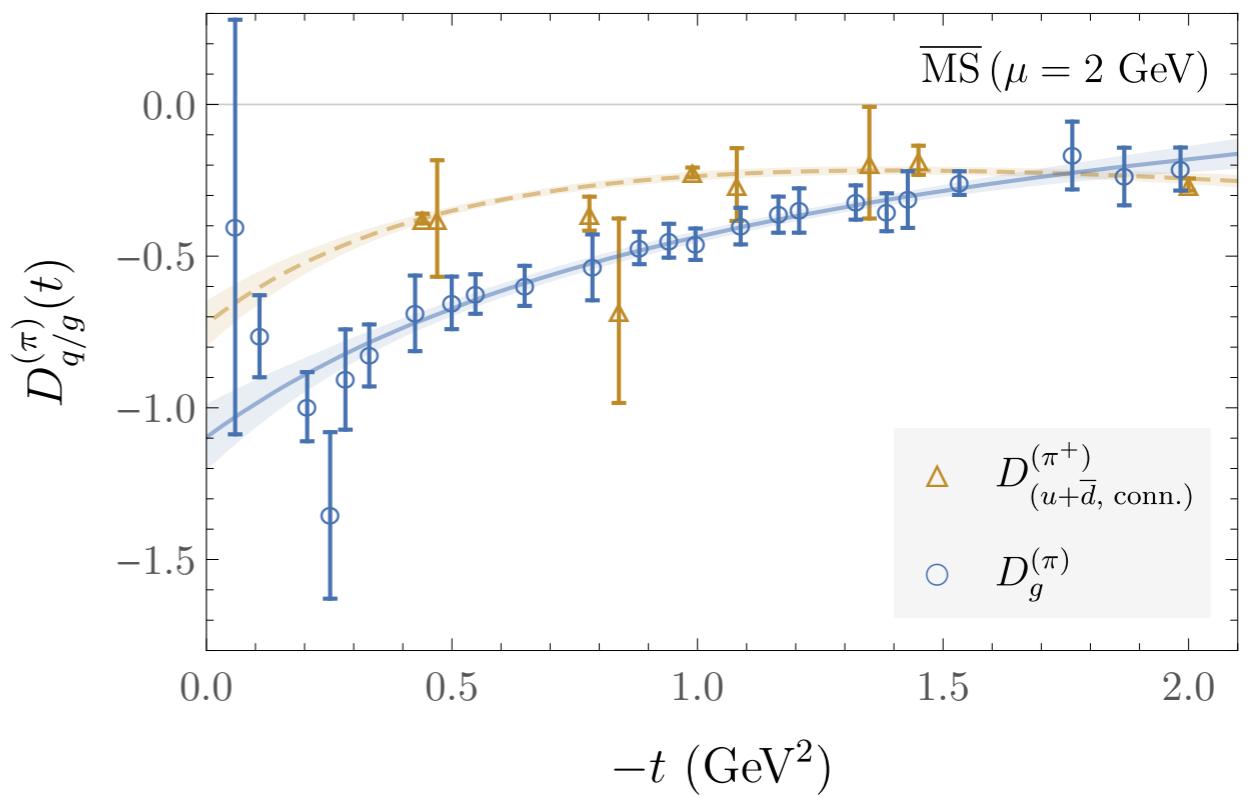
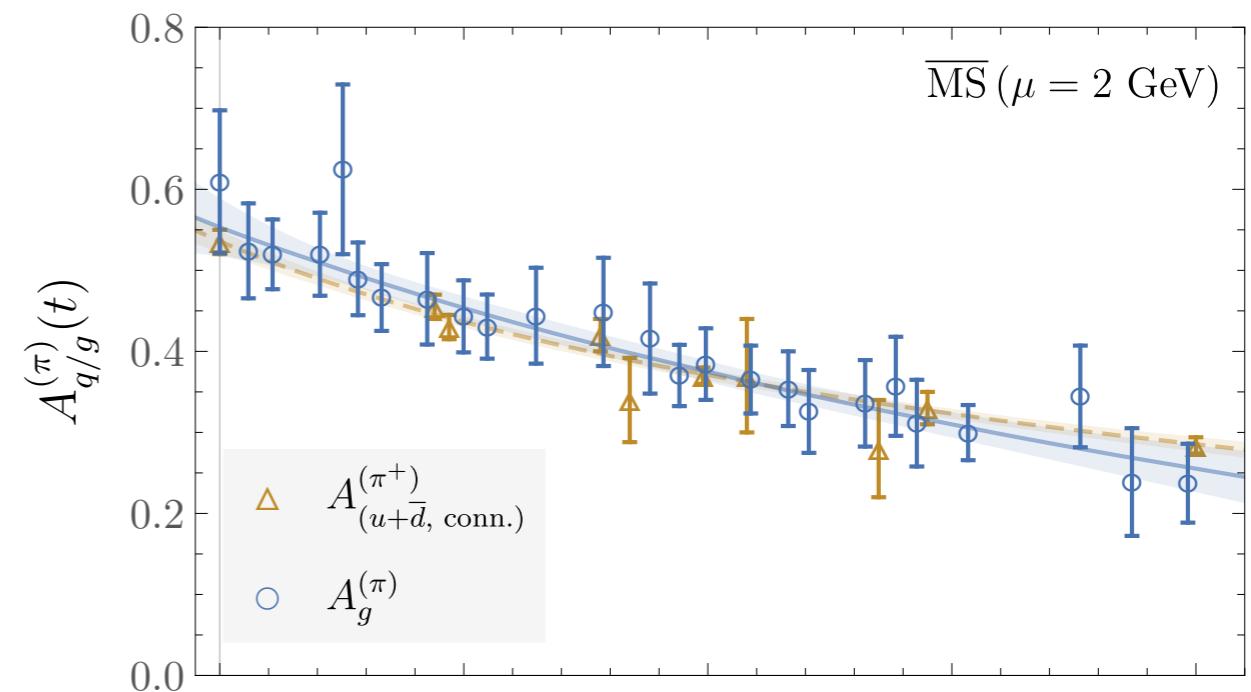
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Dipole-like fall-off with momentum transfer

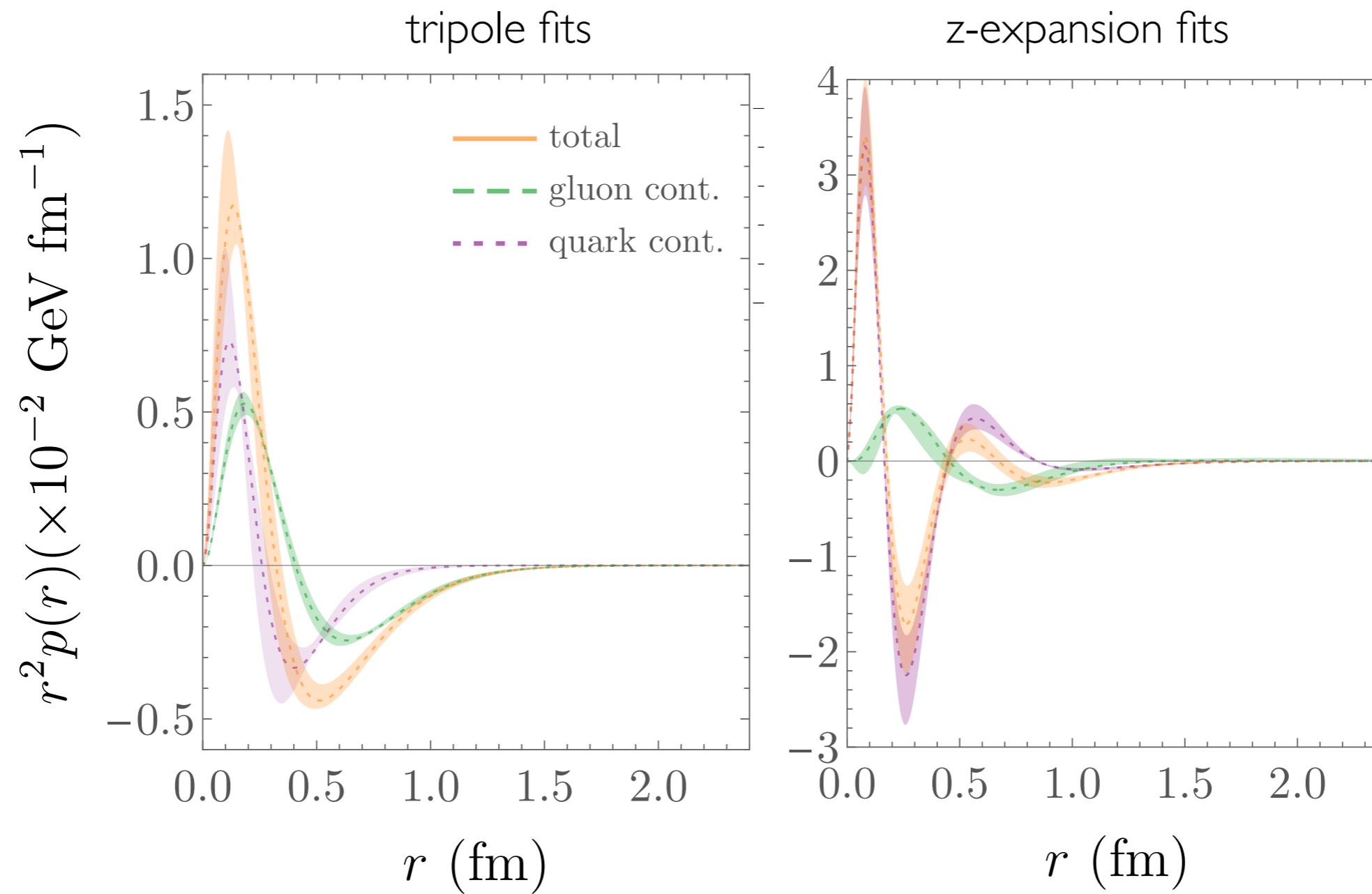
- Momentum fraction  $A_a(0) = \langle x \rangle_a$
- D-terms  $D_a(0)$  related to pressure and shear distributions

gluon: Shanahan, Detmold, PRD (2019)

quark: Brommel Ph.D. thesis (2007)  $m_\pi \sim 840$  MeV



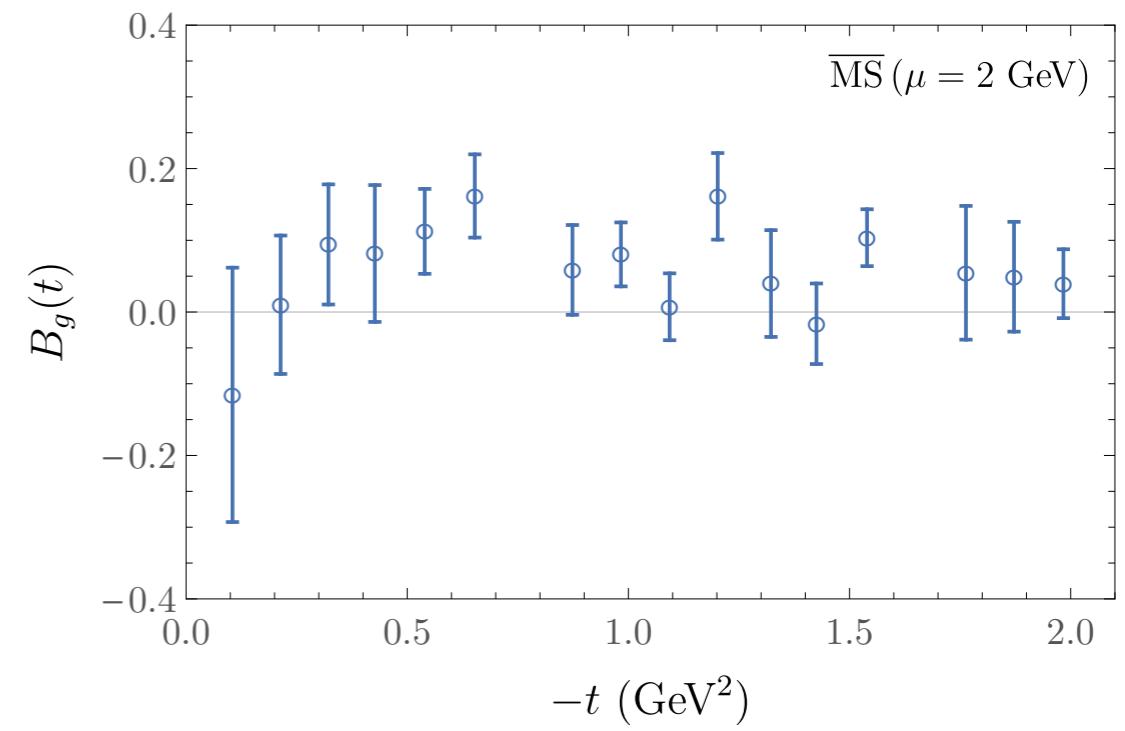
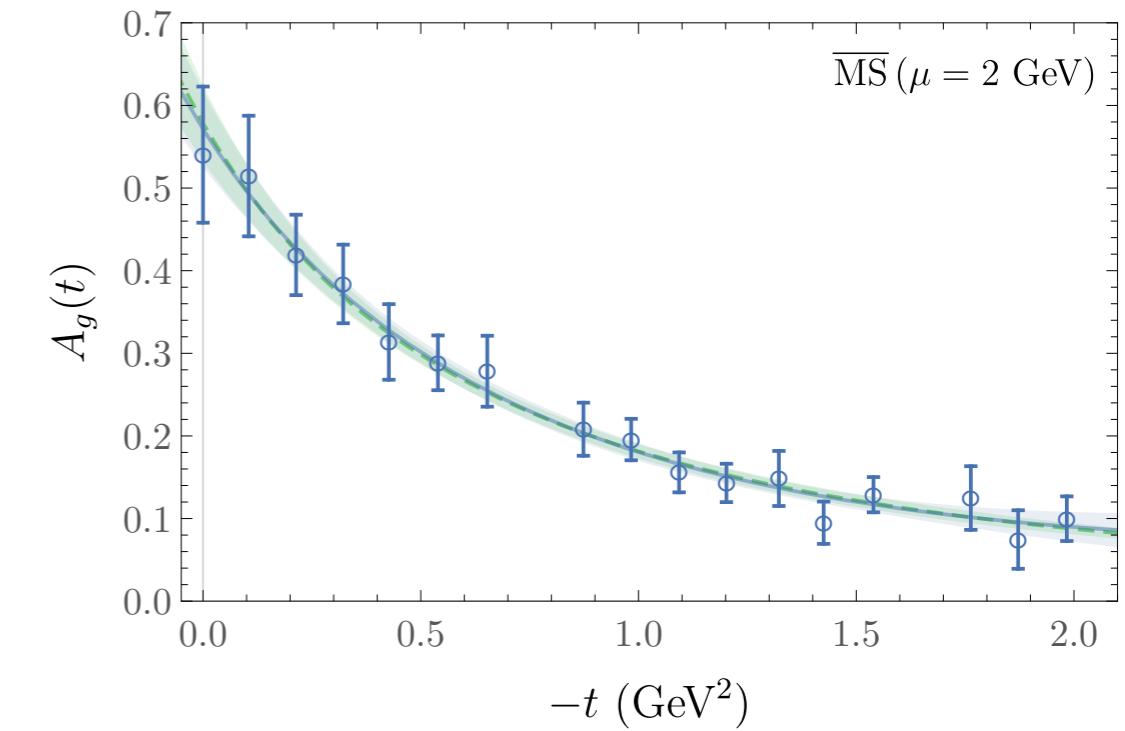
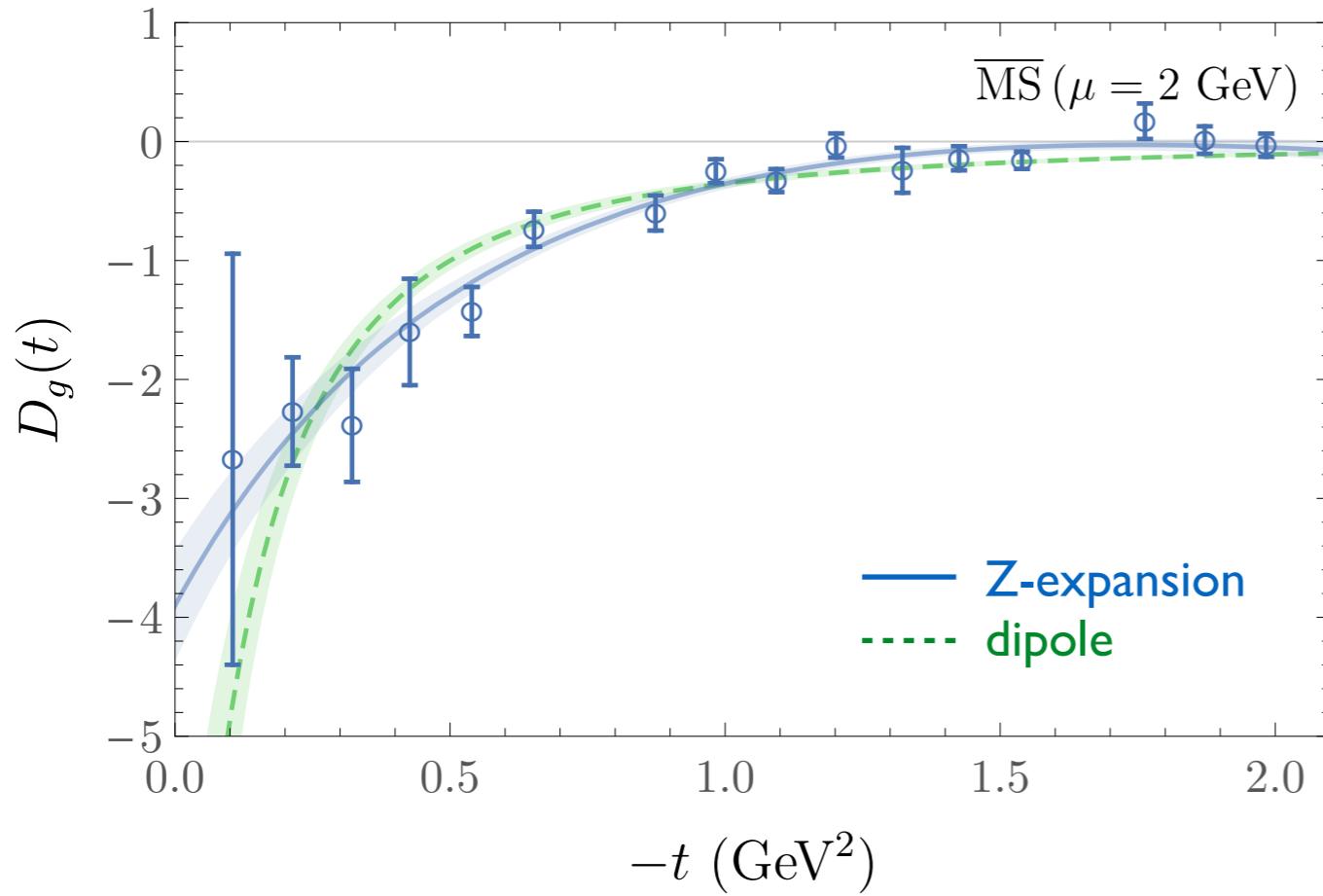
# LQCD pion pressure



# LQCD Nucleon GFFs

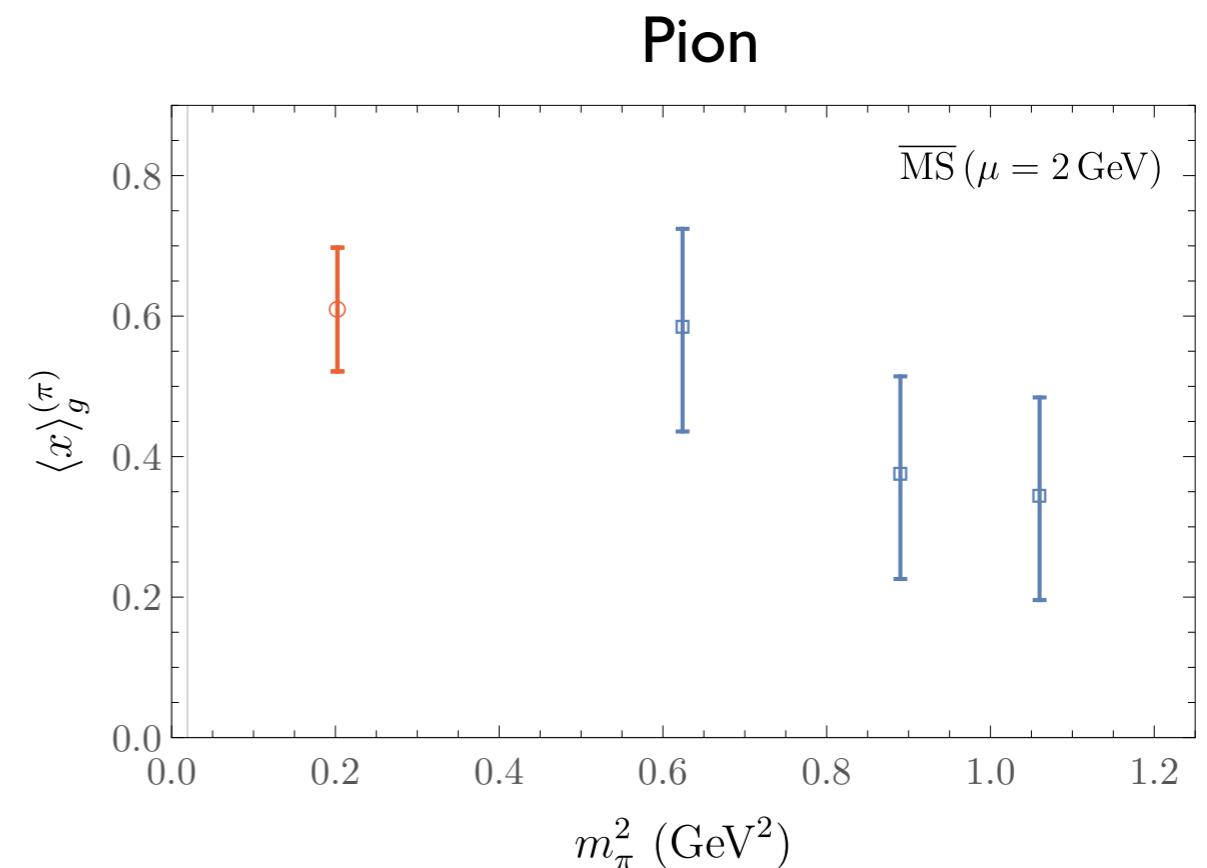
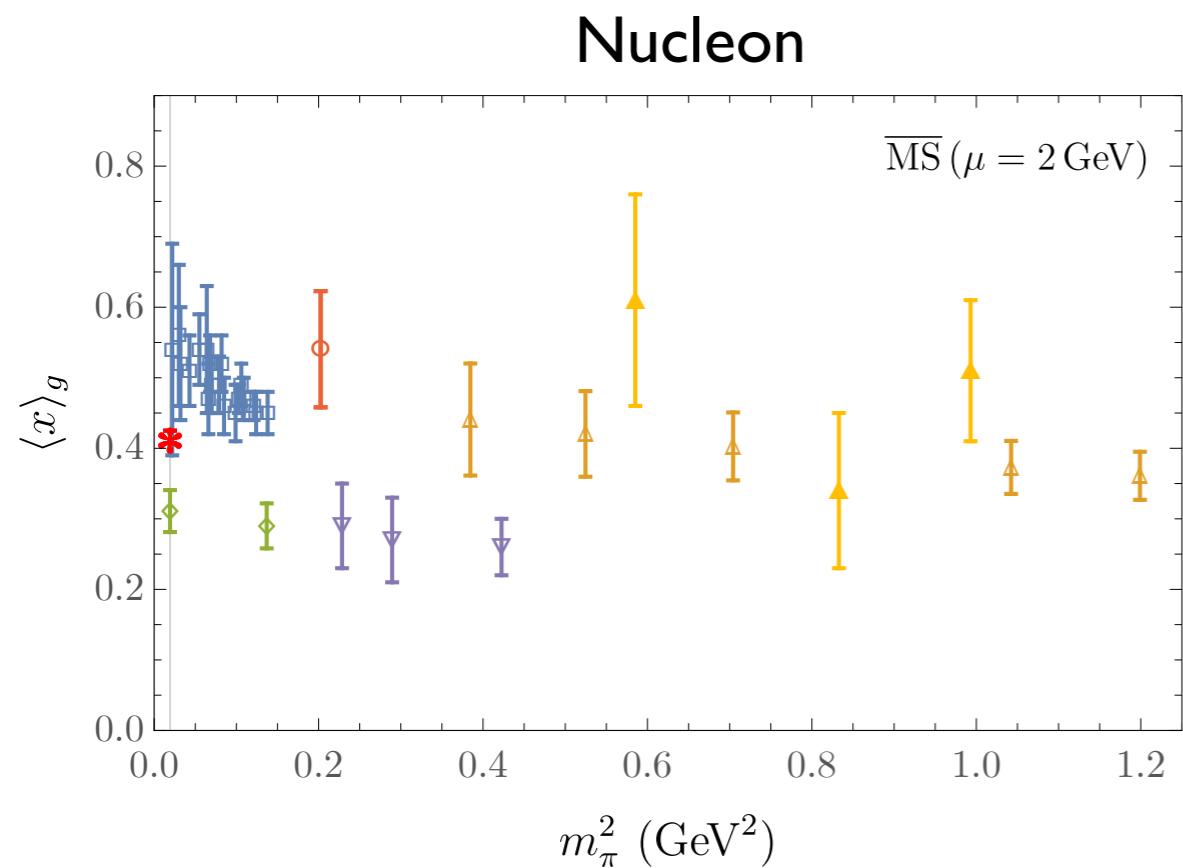
Nucleon gluon GFFs,  $m_\pi \sim 450$  MeV

Dipole-like fall-off with momentum transfer



# Gluon momentum fraction

Gluon momentum fraction  $A_a(0) = \langle x \rangle_a$



xQCD  
 ETM

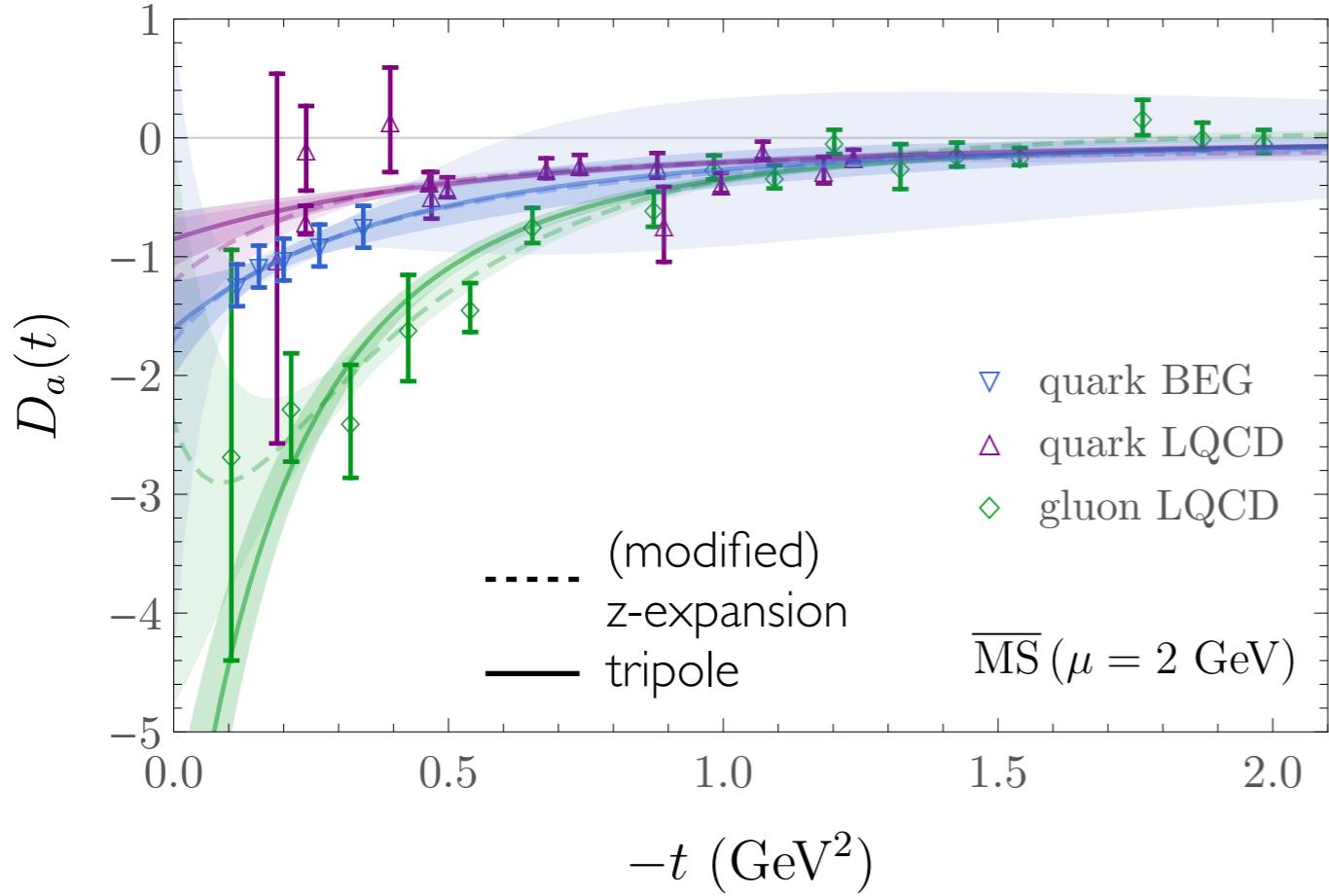
xQCD quenched  
 QCDSF quenched

Meyer/Negele  
quenched

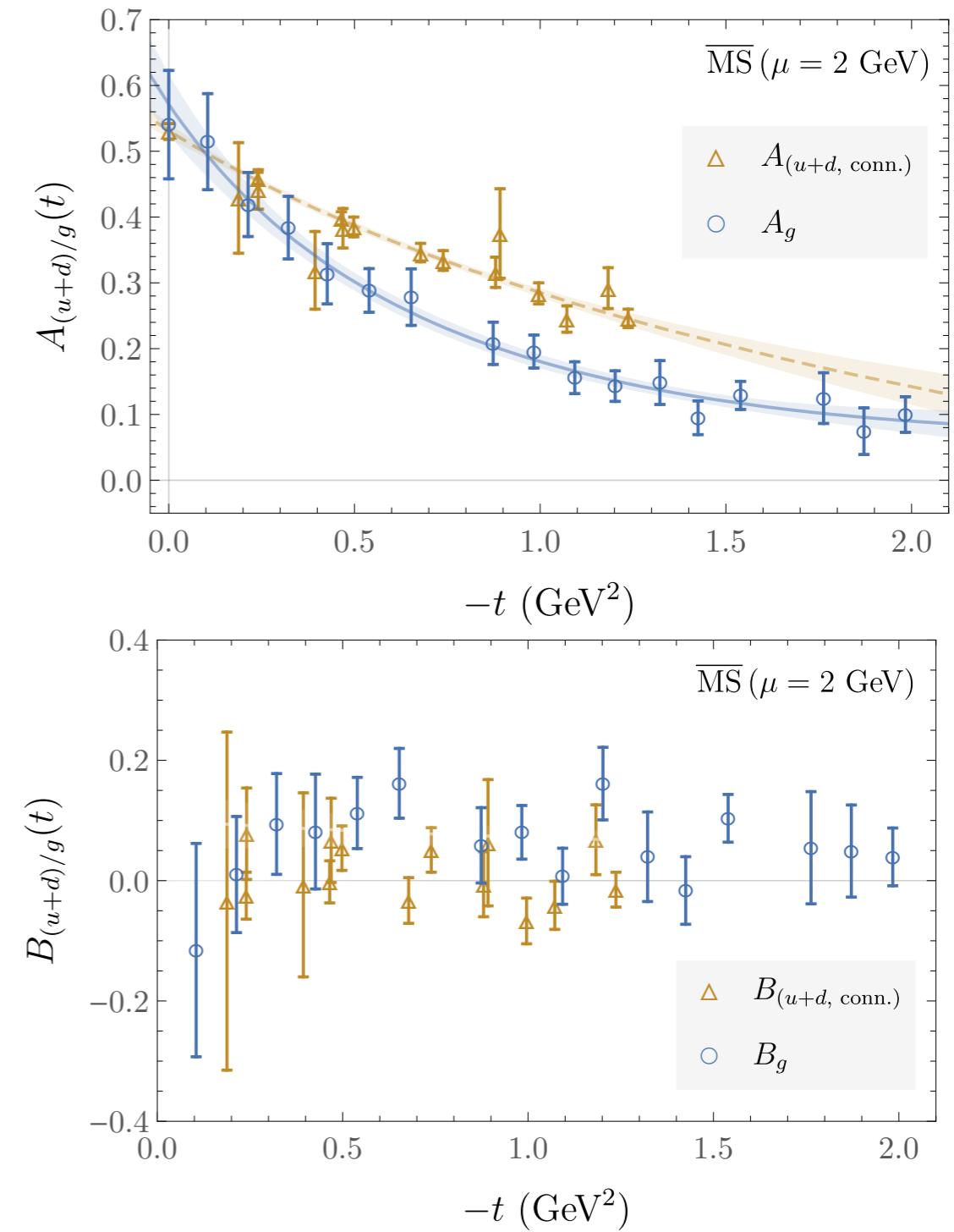
# LQCD Nucleon GFFs

Nucleon gluon GFFs,  $m_\pi \sim 450$  MeV

Tripole-like fall-off with momentum transfer



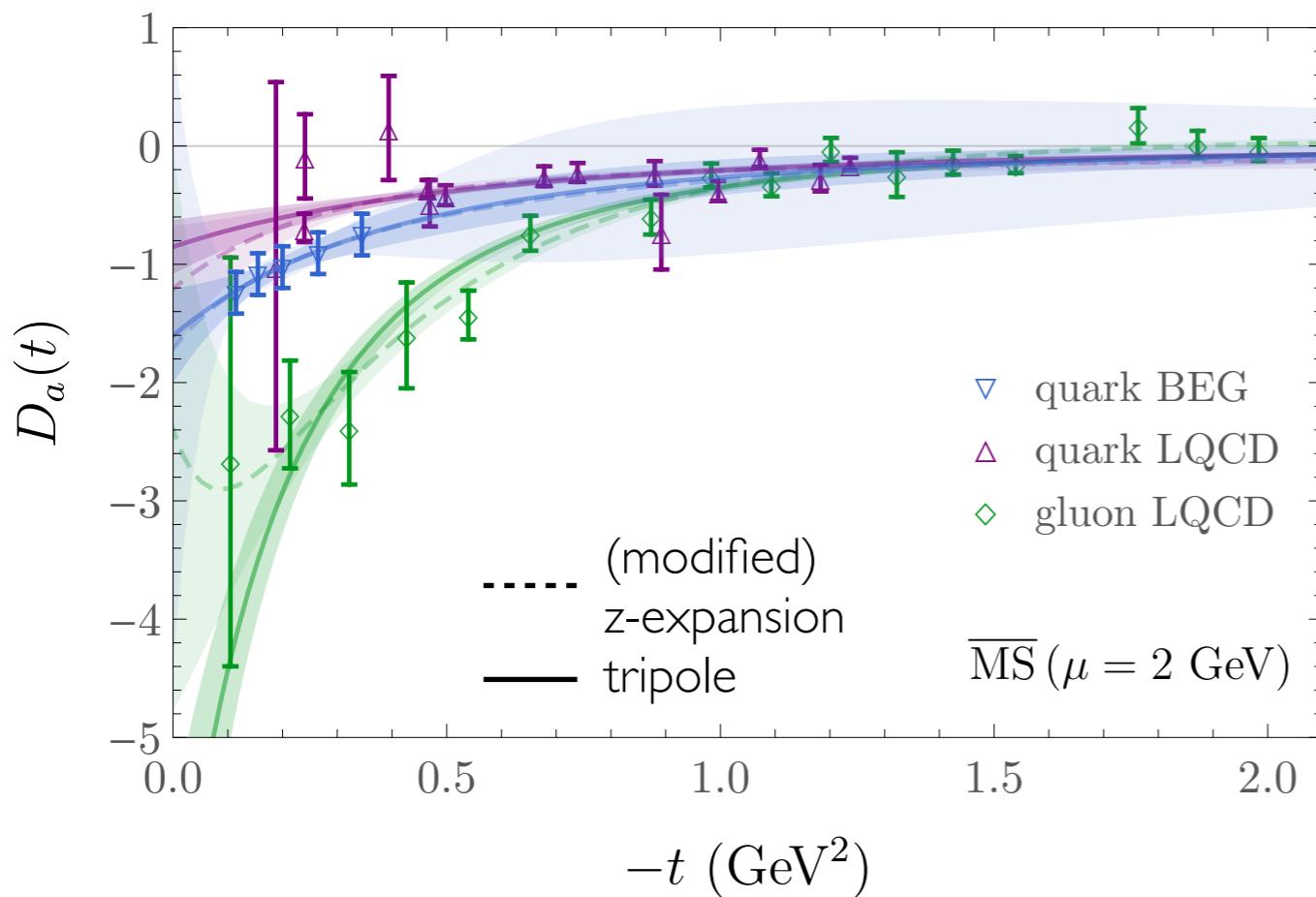
Gluon GFFs: Shanahan, Detmold, PRD (2019) PRL (2019)  
 Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008)  
 Expt quark GFFs (BEG): Burkert et al, Nature 557, 396 (2018)



# Nucleon D-term GFFs

Nucleon gluon GFFs,  $m_\pi \sim 450$  MeV

Tripole-like fall-off with momentum transfer



Gluon GFFs: Shanahan, Detmold, PRD (2019), PRL (2019)

Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008)

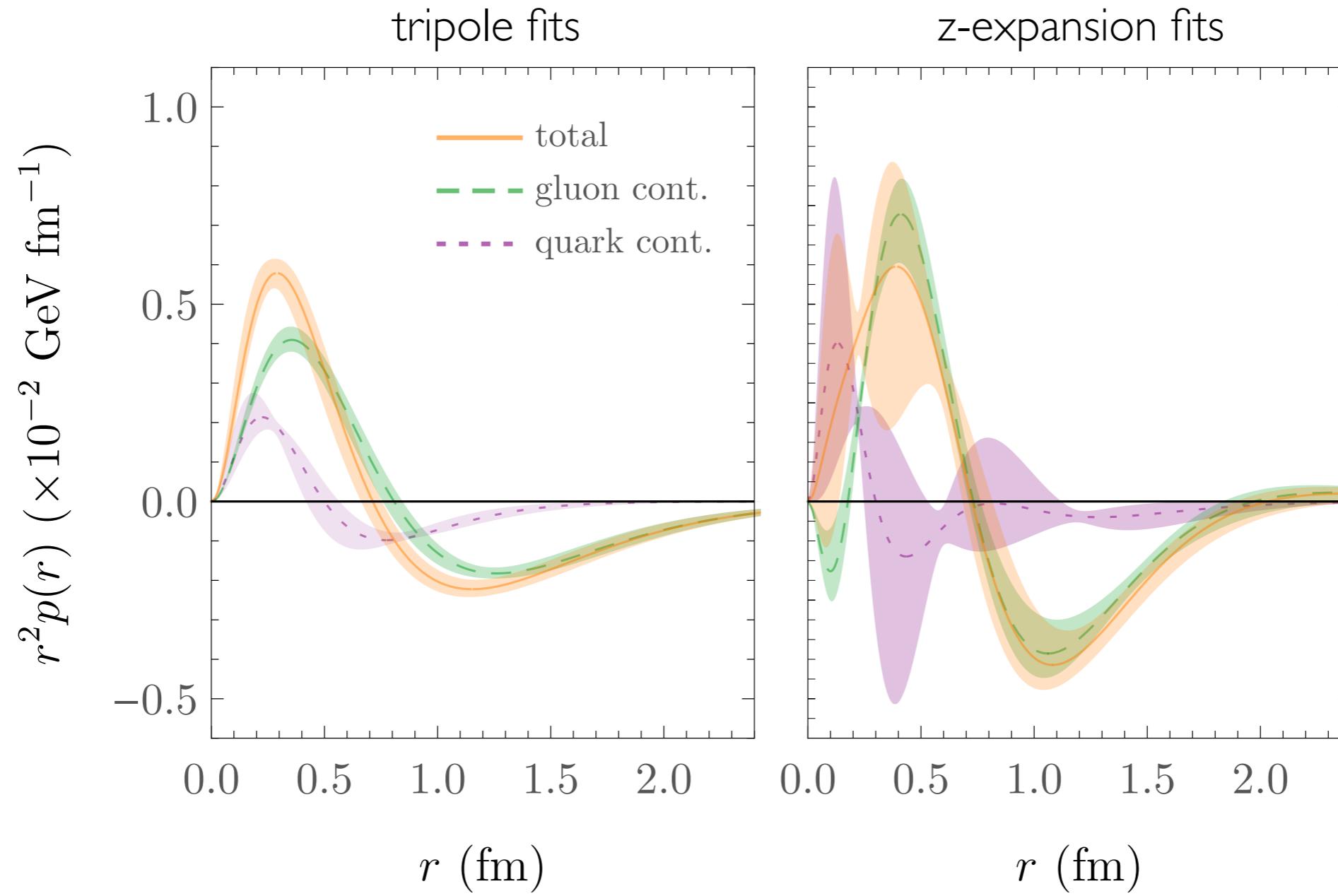
Expt quark GFFs (BEG): Burkert et al, Nature 557, 396 (2018)

Key assumptions in pressure extraction from DVCS

- Gluon D-term same as quark term in magnitude and shape  
Factor of  $\sim 2$  difference in magnitude, somewhat different t-dependence
- Tripole form factor model  
LQCD results consistent with ansatz, but more general form is less well constrained
- Isovector quark D-term vanishes  
 $D_{u-d}(t) \sim 0$  from other LQCD studies

# LQCD proton pressure

Nucleon pressure using LQCD results for quark and gluon GFFs,  $m_\pi \sim 450$  MeV

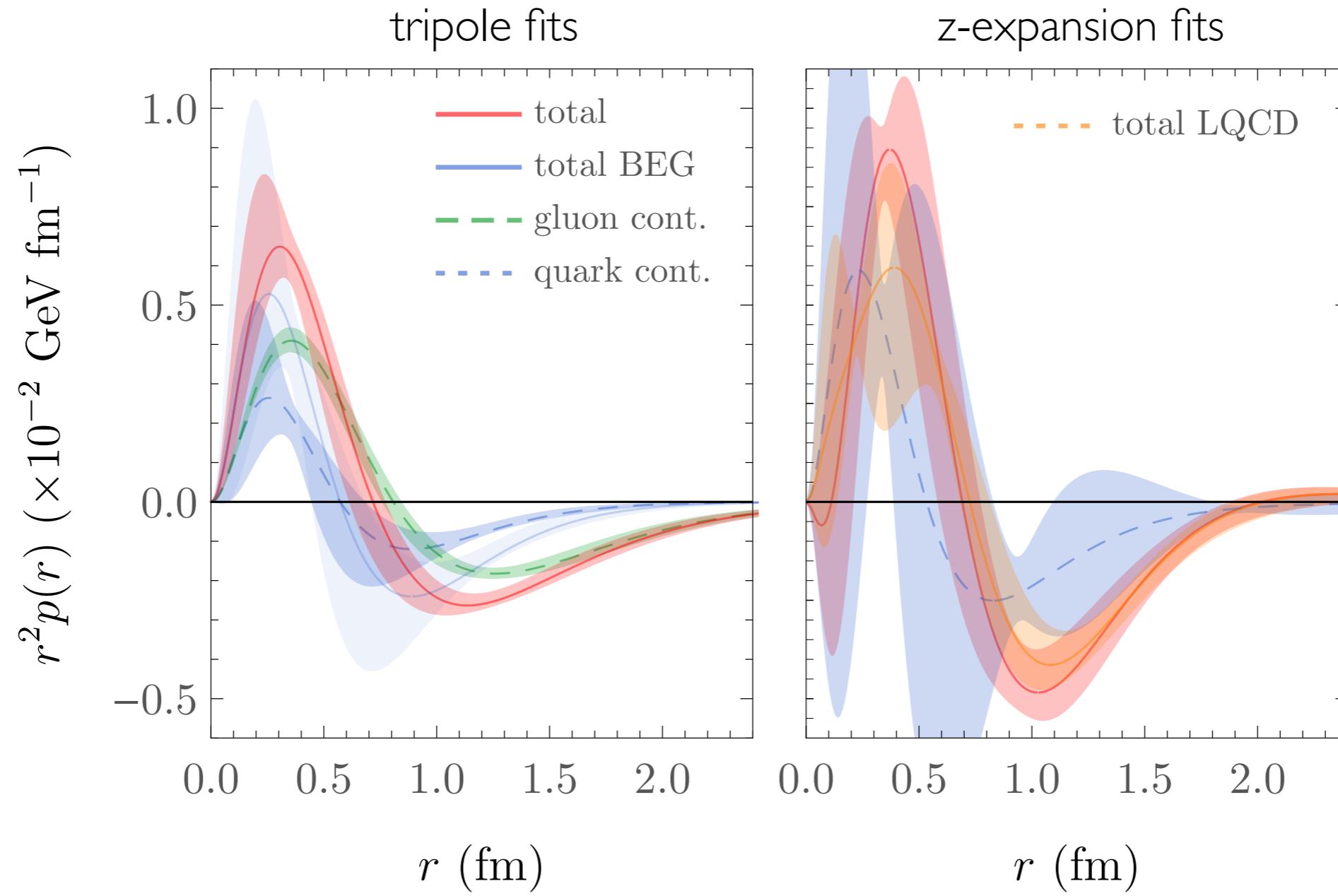


Gluon GFFs: Shanahan, Detmold, PRD (2019), PRL (2019)

Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008)

# LQCD + Expt proton pressure

Nucleon pressure using LQCD results for gluon GFF, JLab results for quark GFF



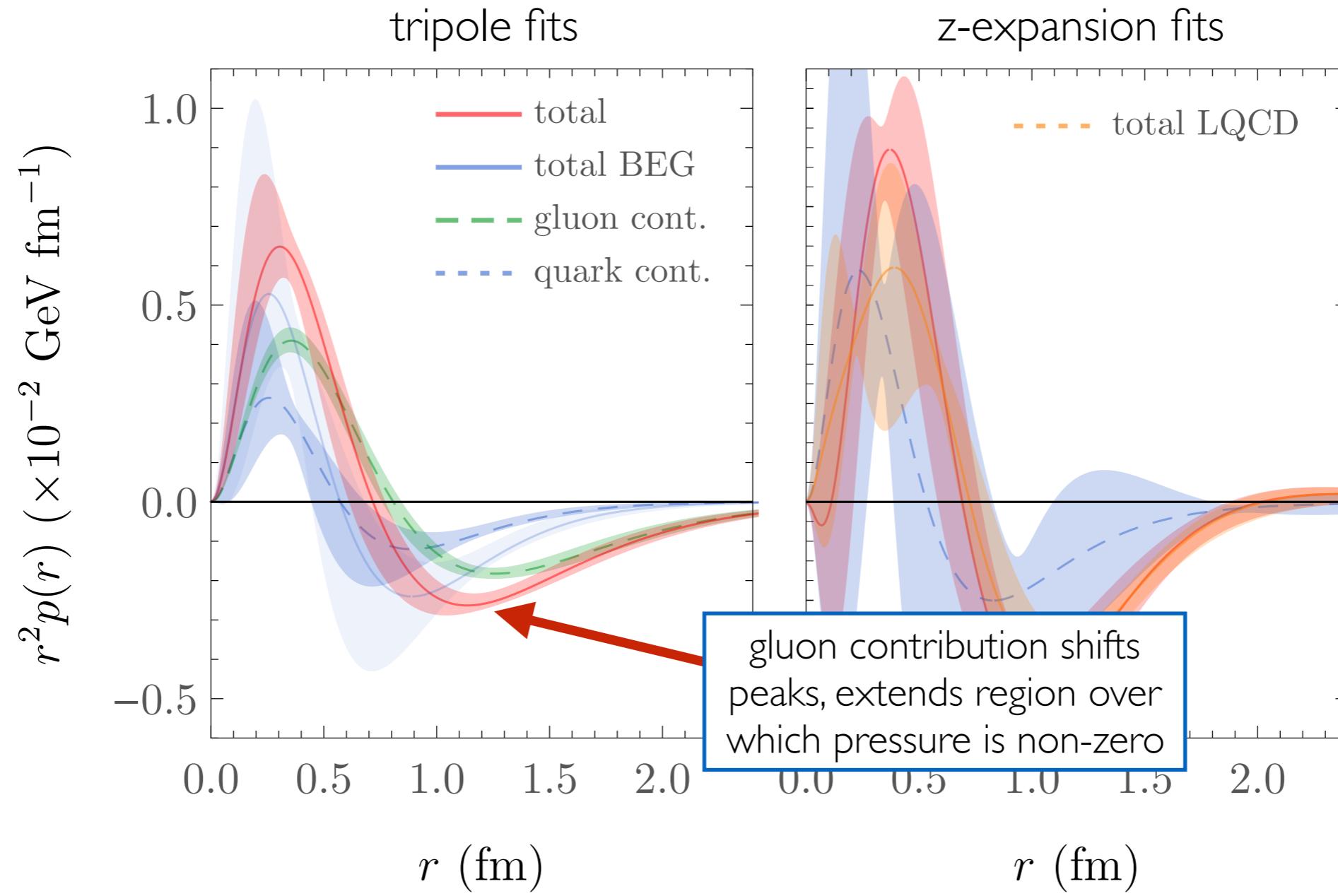
Gluon GFFs: Shanahan, Detmold, PRD (2019), PRL (2019)

Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008)

Expt quark GFFs (BEG): Burkert et al, Nature 557, 396 (2018)

# LQCD + Expt proton pressure

Nucleon pressure using LQCD results for gluon GFF, JLab results for quark GFF

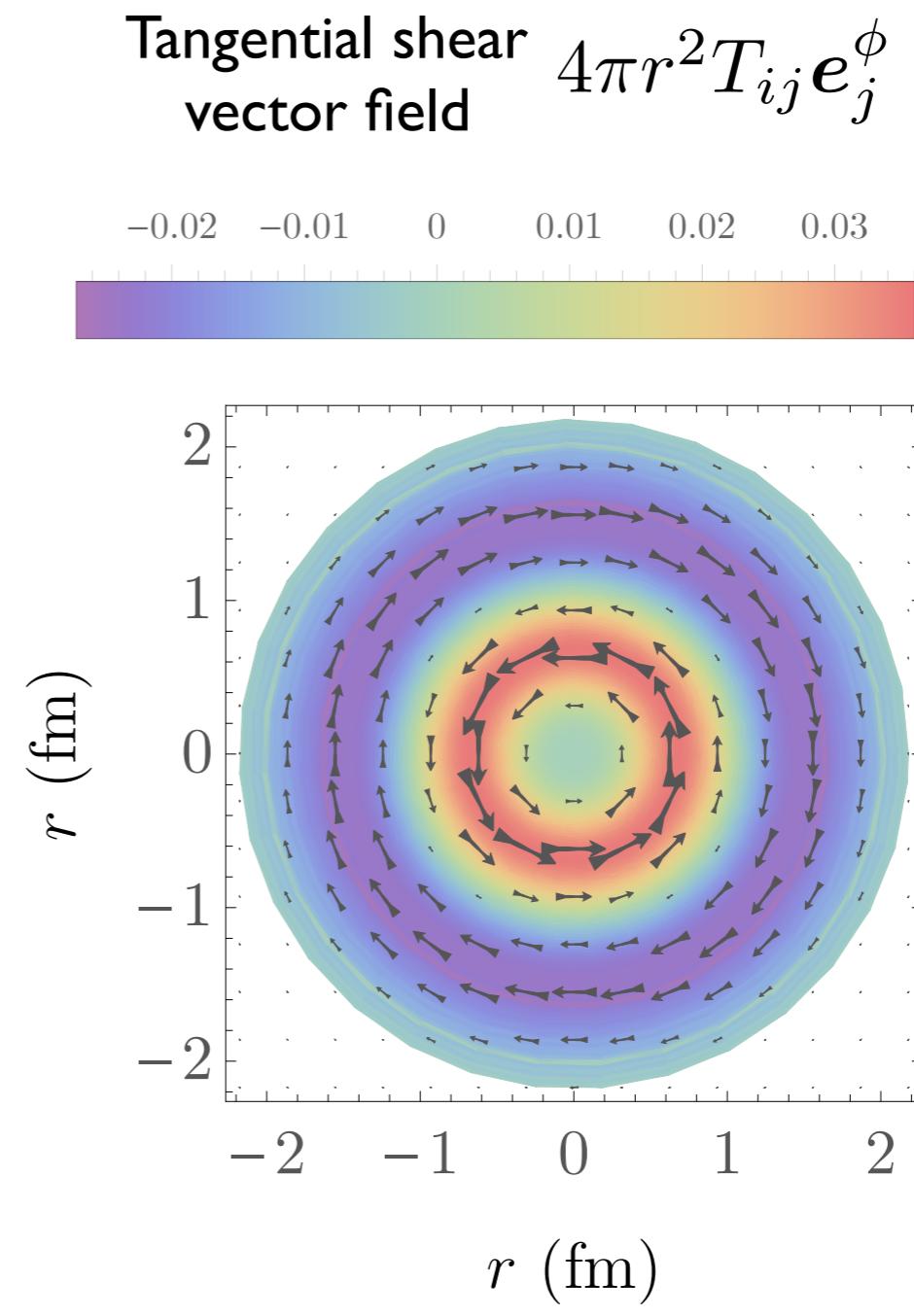
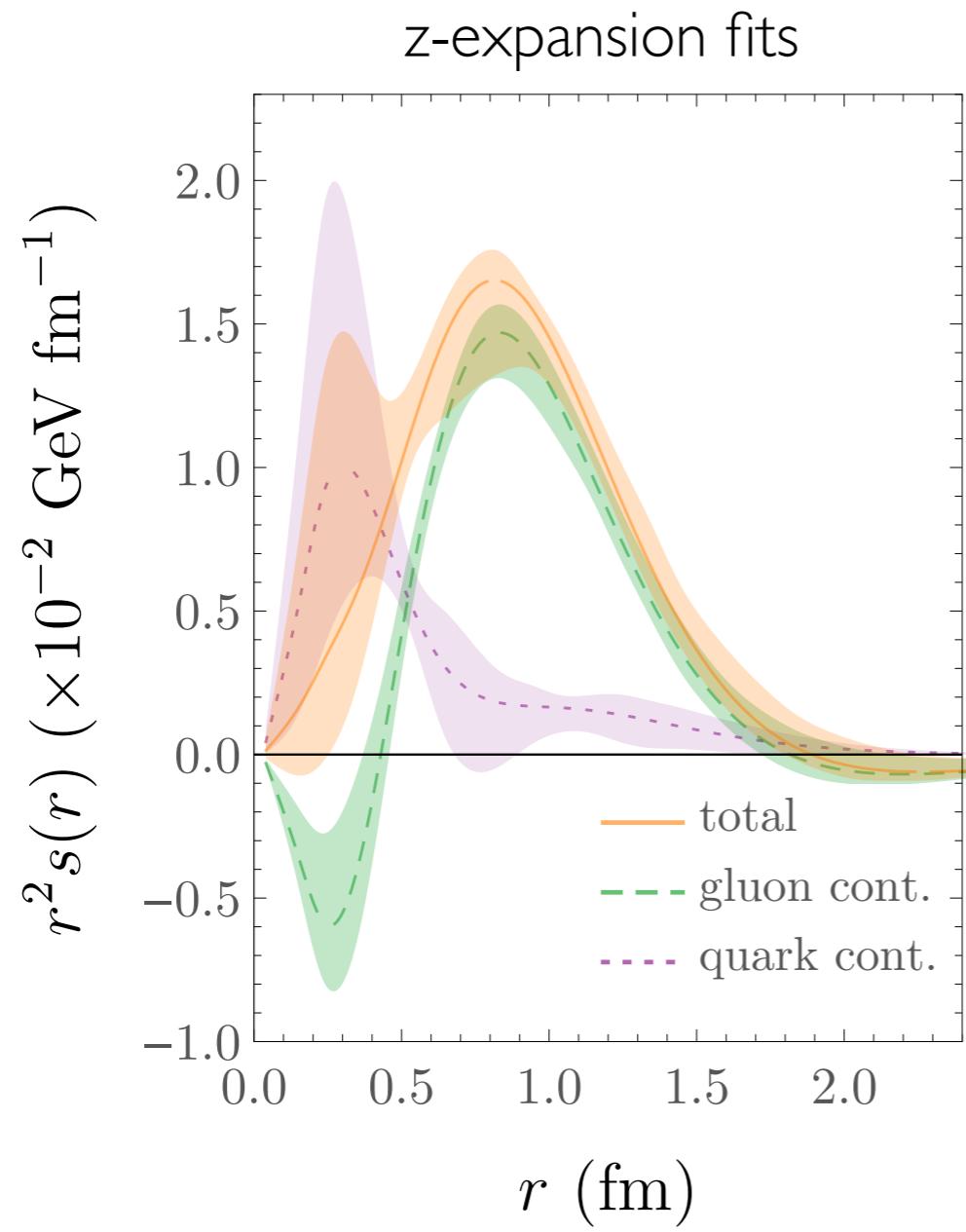


Gluon GFFs: Shanahan, Detmold, PRD (2019), PRL (2019)

Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008)

Expt quark GFFs (BEG): Burkert et al, Nature 557, 396 (2018)

# LQCD proton shear



Gluon GFFs: Shanahan, Detmold, PRD (2019), PRL (2019)

Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008)

Expt quark GFFs (BEG): Burkert et al, Nature 557, 396 (2018)

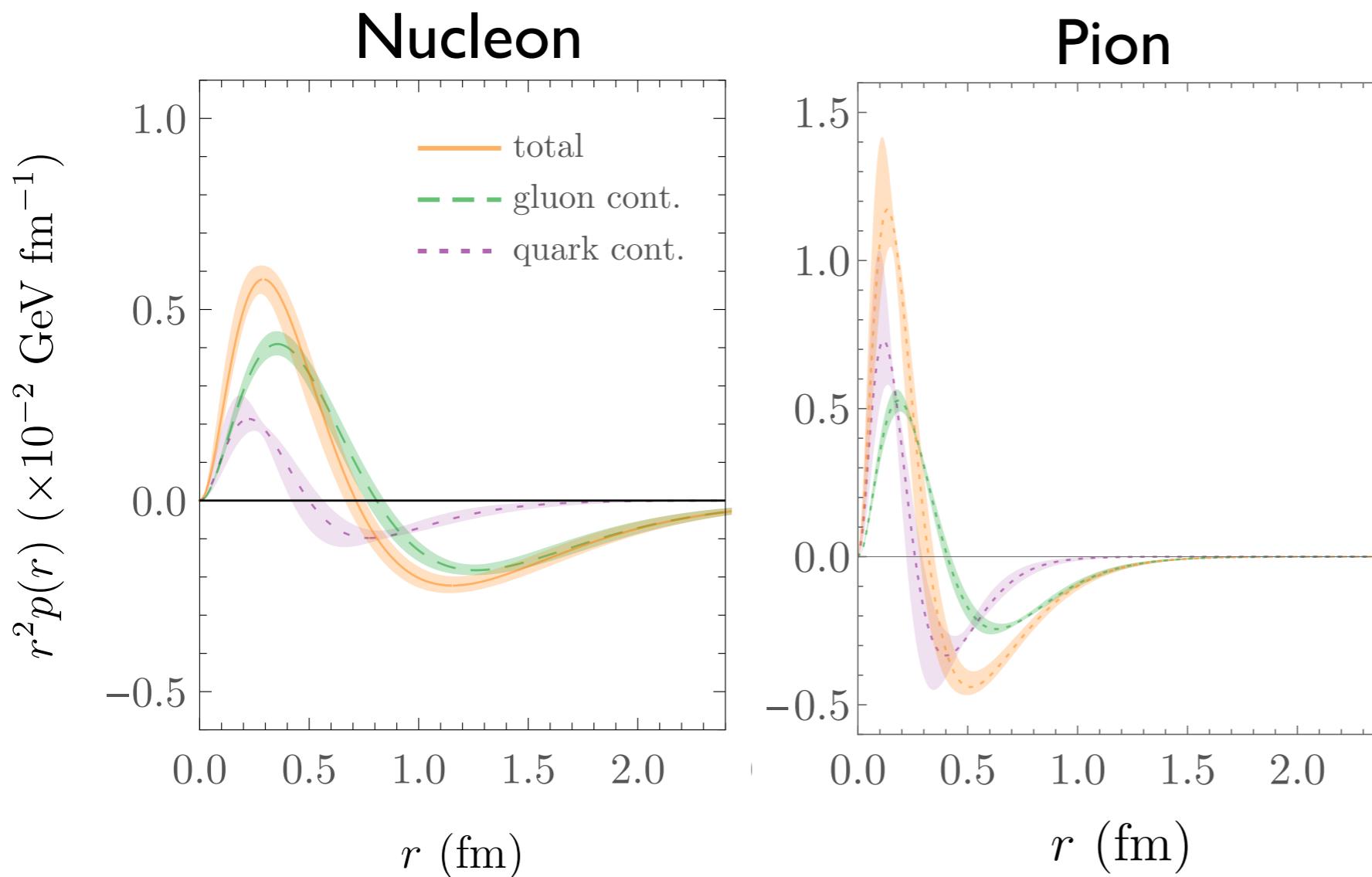
# Gluon structure from LQCD

## LQCD calculations of proton and pion energy momentum tensor

- Gluon and quark gravitational form factors
- Shear and pressure distributions
- New physical mass calculations are ongoing
- Complements recent experimental studies
- Support analysis assumptions
- Suggest target kinematics for future model independent extractions at JLab12 and EIC

# Next: pressure in nuclei

Pressure in light nuclei  
c.f. pressure in the nucleon?



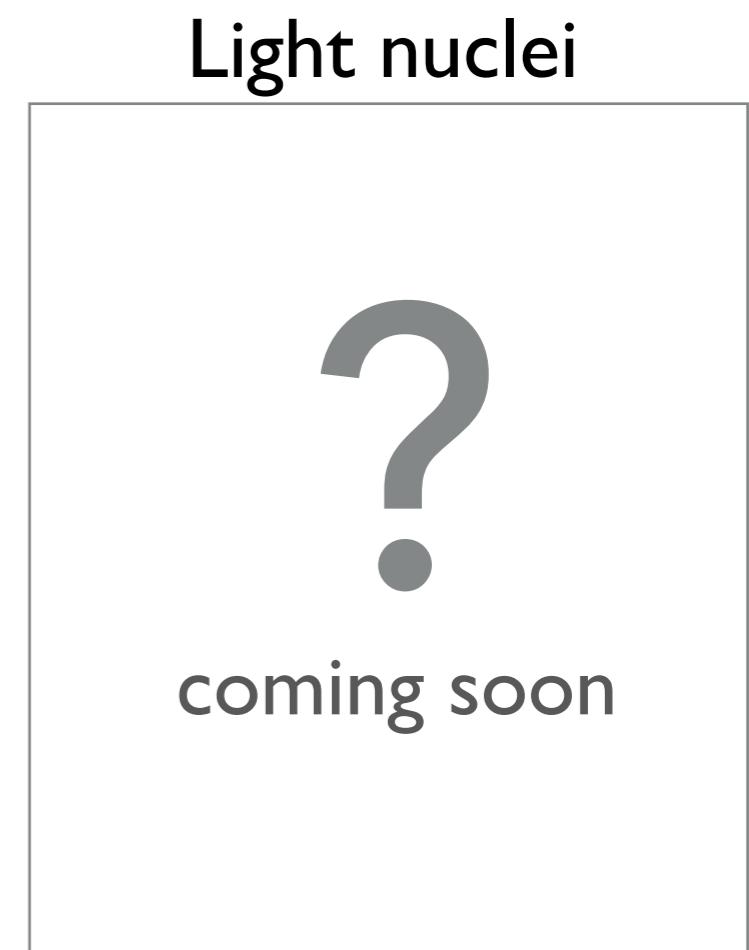
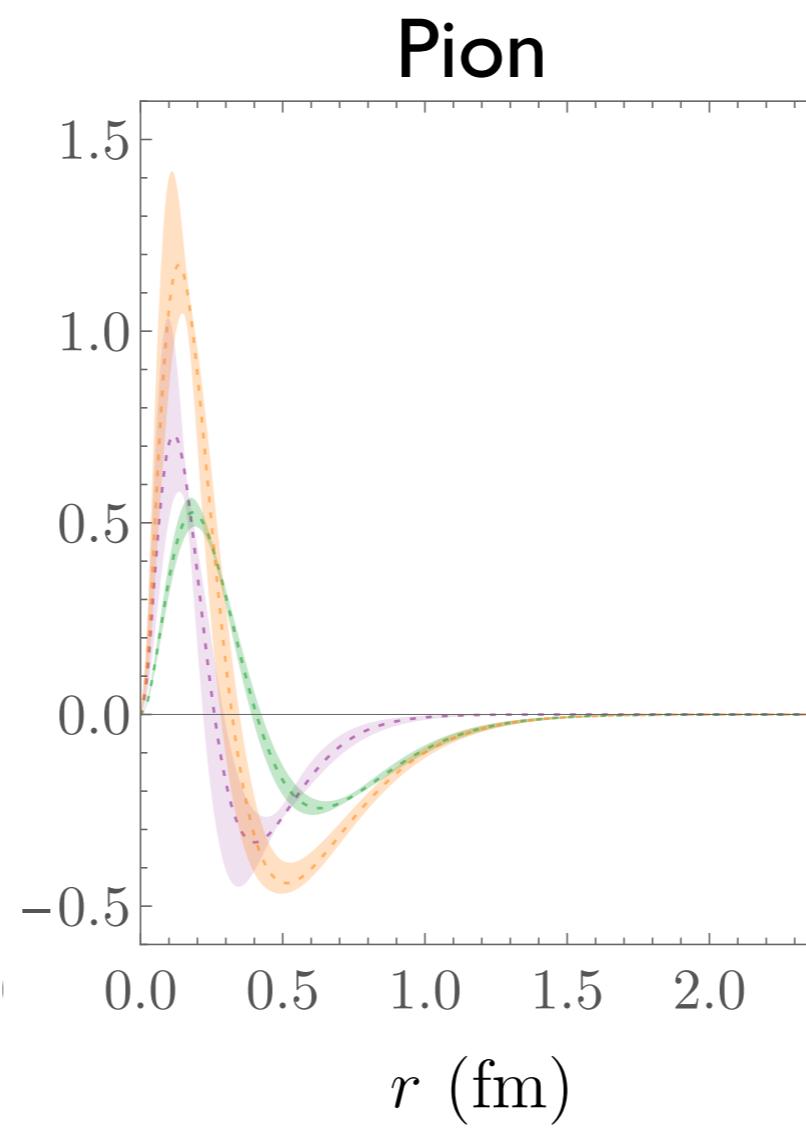
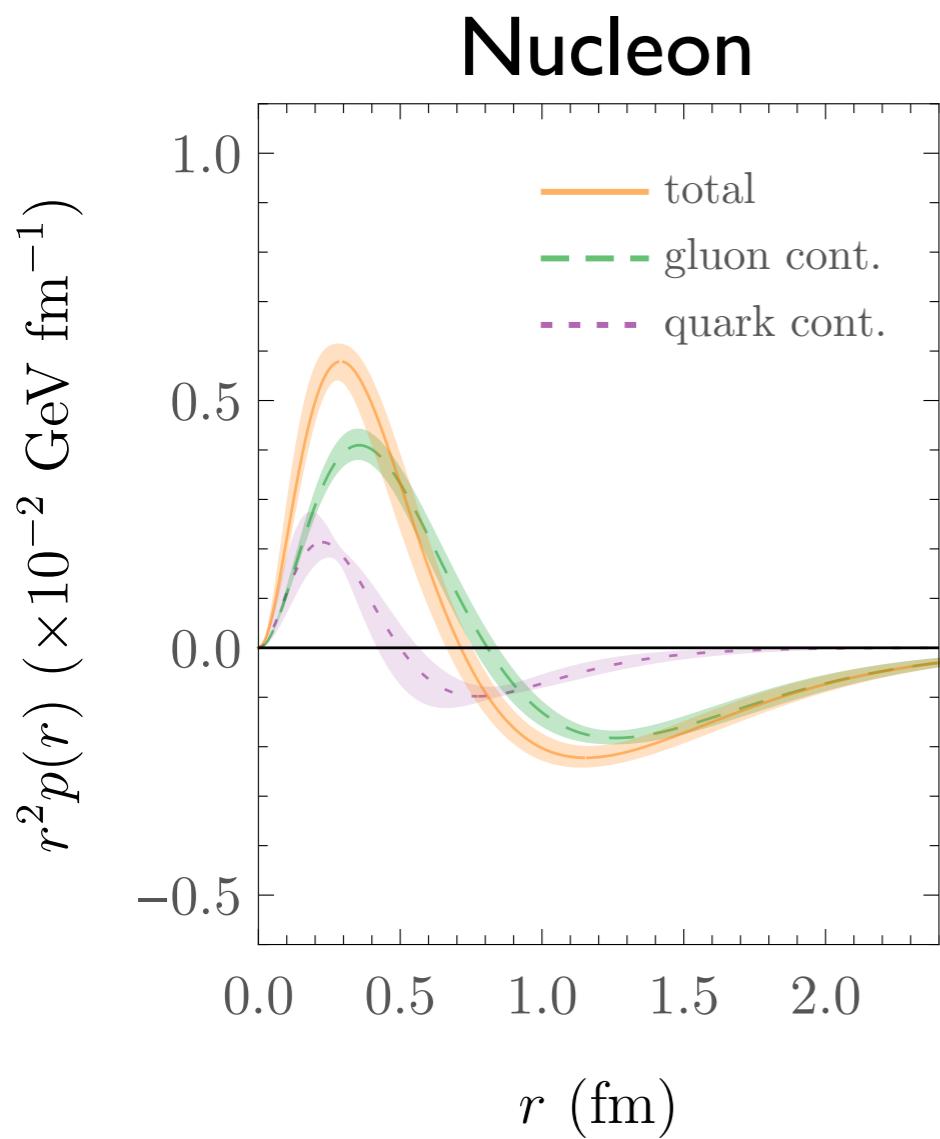
Pion & Nucleon  
quark and gluon  
momentum  
fractions consistent  
within uncertainties,  
but very different  
pressure  
distributions!

Gluon GFFs: Shanahan, Detmold, PRD (2019), PRL (2019)

Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008)

# Next: pressure in nuclei

Pressure in light nuclei  
c.f. pressure in the nucleon?



Gluon GFFs: Shanahan, Detmold, PRD (2019), PRL (2019)

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# Gluon structure of nuclei

How does the gluon structure of a nucleon change in a nucleus?

European Muon Collaboration (1983):  
“EMC effect”

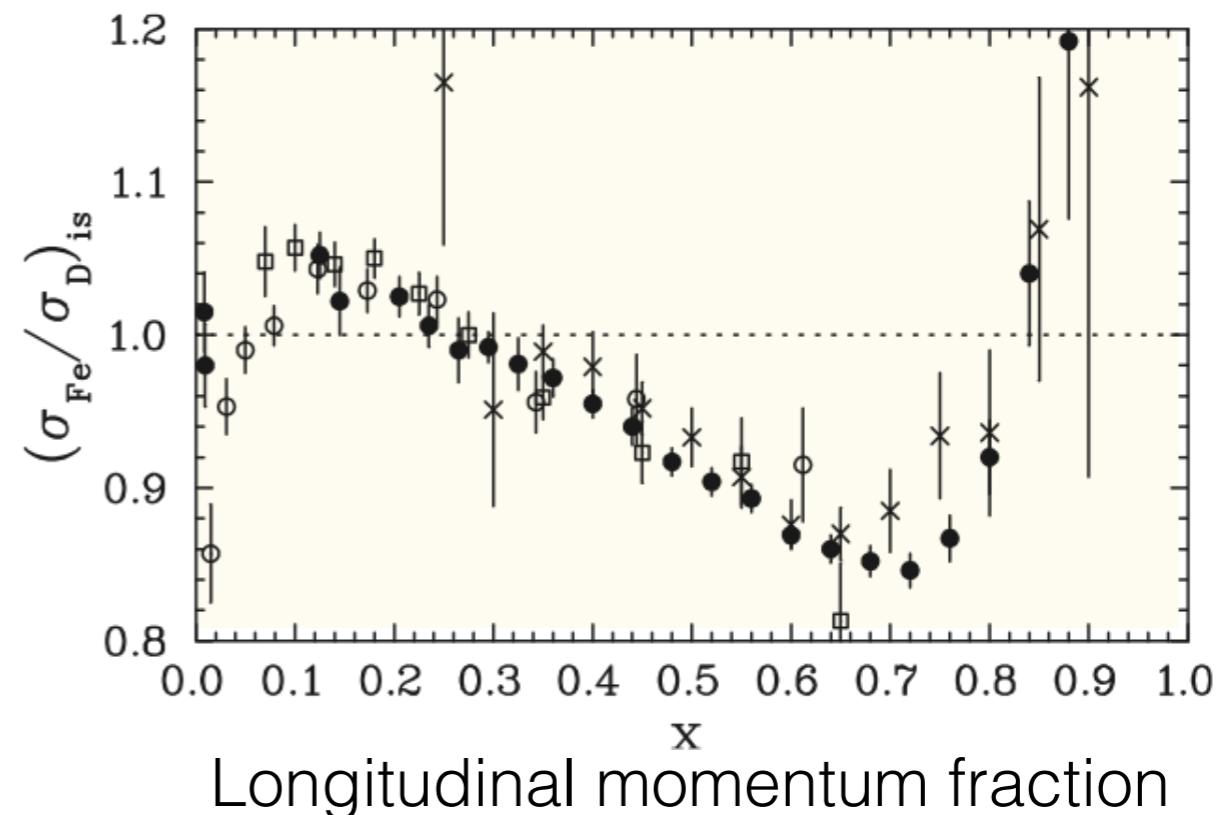
Modification of per-nucleon cross section of nucleons bound in nuclei

Gluon analogue?

Ratio of structure function  $F_2$  per nucleon for iron and deuterium

$$F_2(x, Q^2) = \sum_{q=u,d,s\dots} x e_q^2 [q(x, Q^2) + \bar{q}(x, Q^2)]$$

Number density of partons of flavour q



# Nuclear glue, $m_\pi \sim 450$ MeV

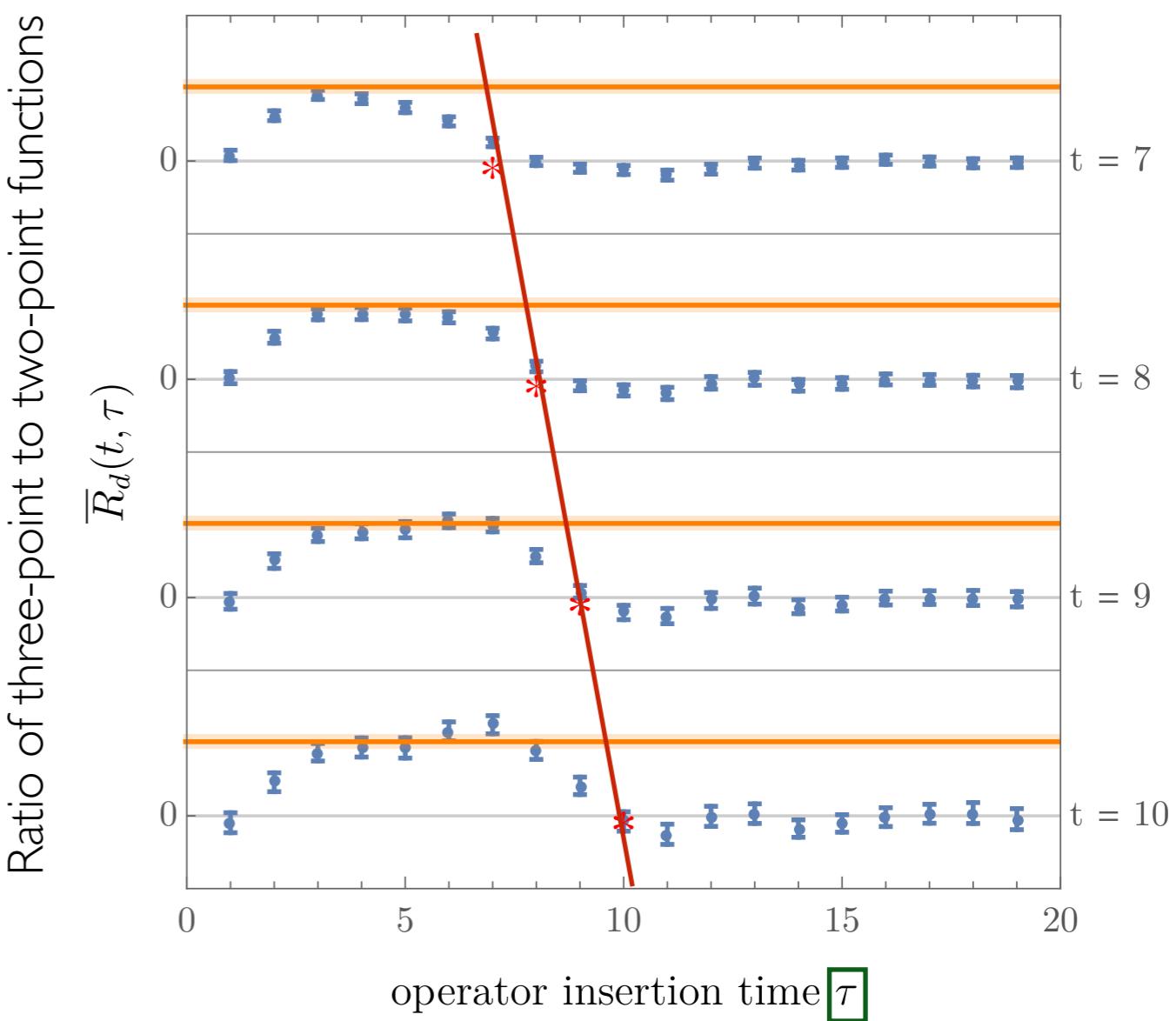
Look for **nuclear (EMC-type) effects** in the first moments of the spin-independent gluon structure function

## Doubly challenging

- Nuclear matrix element
- Gluon observable (suffer from poor signal-to-noise)

## Deuteron gluon momentum fraction

Ratio  $\propto$  matrix element  
for  $0 \ll \tau \ll t$

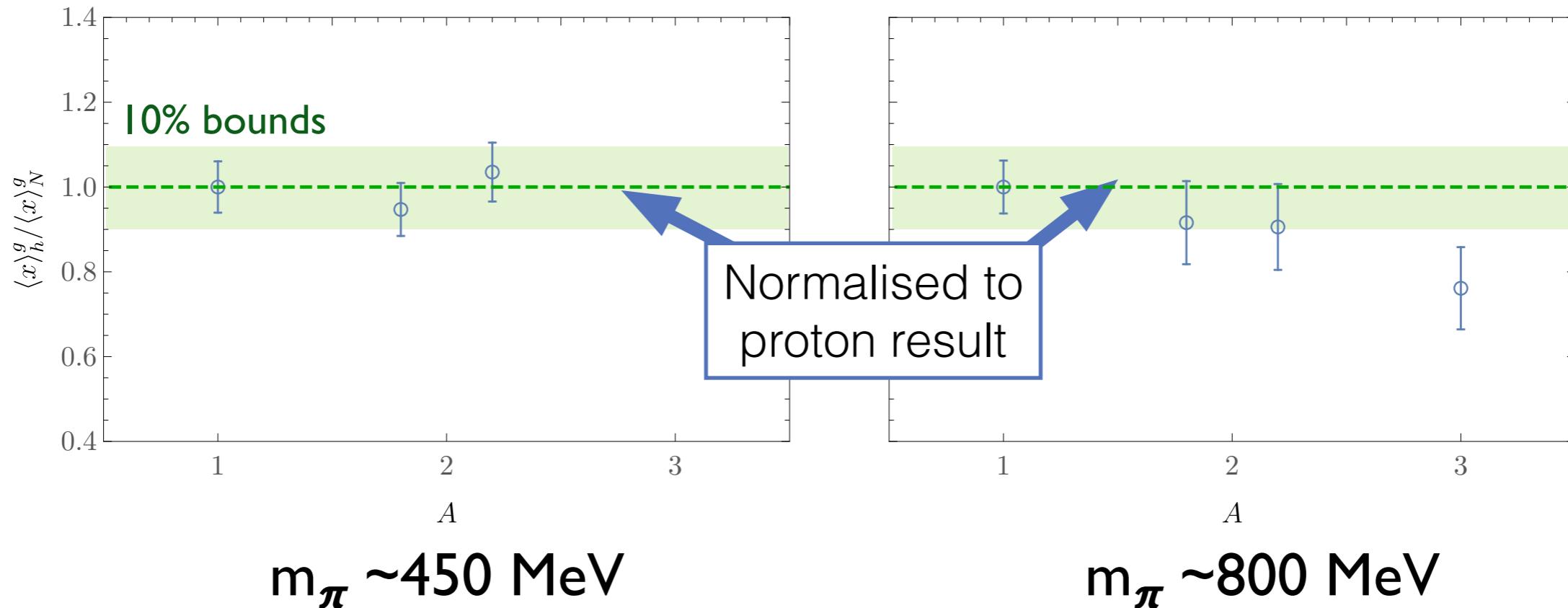


# Gluon momentum fraction

NPLQCD Collaboration PRD96 094512 (2017)

- Matrix elements of the **spin-independent gluon operator** in nucleon and light nuclei
- Present statistics: can't distinguish from no-EMC effect scenario
- Small additional uncertainty from mixing with quark operators

Ratio of gluon momentum fraction in nucleus to nucleon



# Gluon structure of nuclei

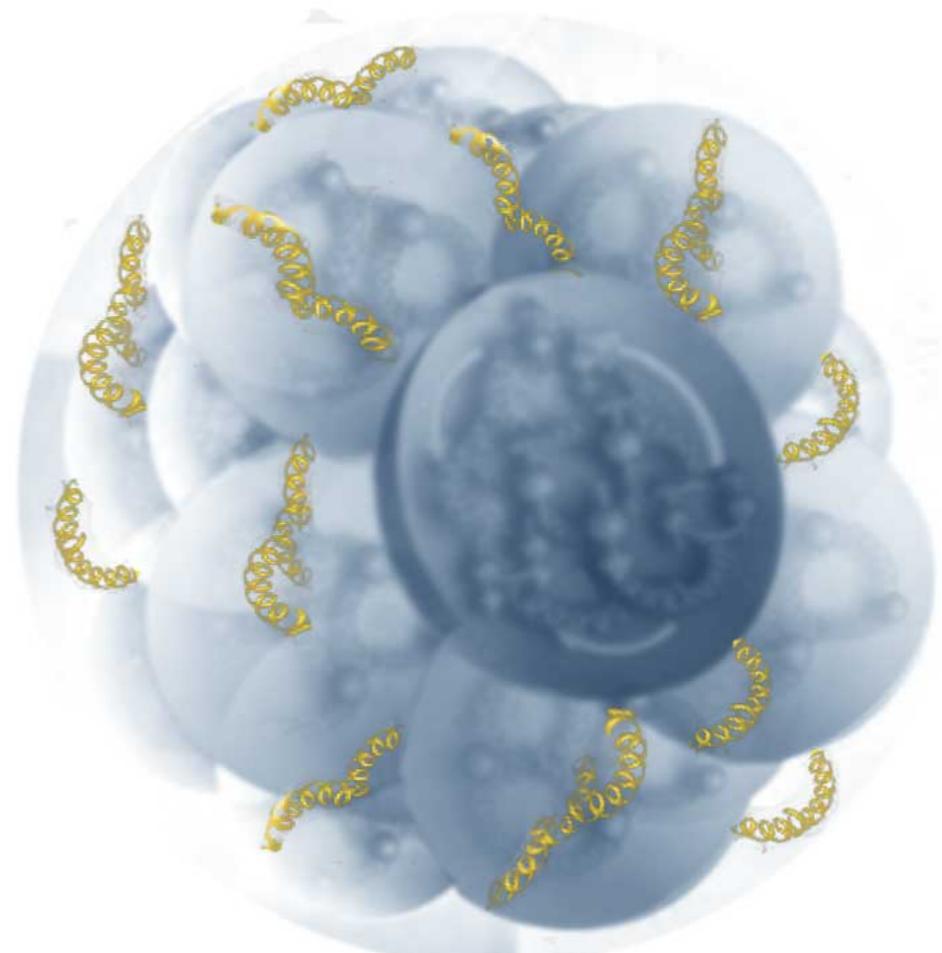
## Exotic Glue

Contributions to nuclear structure from gluons not associated with individual nucleons in nucleus

Exotic glue operator:

$$\text{nucleon } \langle p | \mathcal{O} | p \rangle = 0$$

$$\text{nucleus } \langle N, Z | \mathcal{O} | N, Z \rangle \neq 0$$



Jaffe and Manohar, “Nuclear Gluonometry”  
Phys. Lett. B223 (1989) 218

# Non-nucleonic glue in deuteron

NPLQCD Collaboration PRD96 094512 (2017)

First moment of gluon transversity distribution in the deuteron,  
 $m_\pi \sim 800$  MeV

- First evidence for non-nucleonic gluon contributions to nuclear structure
- Hypothesis of no signal ruled out to better than one part in  $10^7$
- Magnitude relative to momentum fraction as expected from large- $N_c$



Ratio  $\propto$  matrix element  
for  $0 \ll \tau \ll t$

Ratio of 3pt and 2pt functions

