

Advances in quark orbital dynamics in the proton from Lattice QCD

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Acknowledgments:
M. Burkardt, S. Liuti, LHPC

Gauge ensemble provided by:
R. Edwards, B. Joó and K. Orginos

Proton spin decompositions

$$\frac{1}{2} = \frac{1}{2} \sum_q \Delta q + \sum_q \textcolor{red}{L}_q + J_g \quad (\text{Ji})$$

$$\frac{1}{2} = \frac{1}{2} \sum_q \Delta q + \sum_q \textcolor{red}{\mathcal{L}}_q + \Delta g + \mathcal{L}_g \quad (\text{Jaffe-Manohar})$$

... and many more (in fact, we will see a continuous interpolation between the two ...)

There isn't one unique way of separating quark and gluon orbital angular momentum – the different decompositions have different, legitimate meanings.

Quark orbital angular momentum

Interpreting terms in the energy-momentum tensor:

$$L_q \sim -i\psi^\dagger(\vec{r} \times \vec{D})_z\psi$$

Can be obtained from $L_q = J_q - S_q$, where S_q and J_q can be related to GPDs (Ji sum rule) – this has been used in Lattice QCD.

$$\mathcal{L}_q \sim -i\psi^\dagger(\vec{r} \times \vec{\partial})_z\psi \quad \text{in light cone gauge}$$

Previously not accessed in Lattice QCD.

Direct evaluation of quark orbital angular momentum

$$\begin{aligned}
 L_3^{\mathcal{U}} &= \int dx \int d^2 k_T \int d^2 r_T (r_T \times k_T)_3 \mathcal{W}^{\mathcal{U}}(x, k_T, r_T) && \text{Wigner distribution} \\
 &= - \int dx \int d^2 k_T \frac{k_T^2}{m^2} F_{14}(x, k_T^2, k_T \cdot \Delta_T, \Delta_T^2) \Big|_{\Delta_T = 0} && \text{Generalized transverse} \\
 &&& \text{momentum-dependent} \\
 &&& \text{parton distribution} \\
 &&& (\text{GTMD}) \\
 &= \frac{1}{2P^+} \epsilon_{ij} \frac{\partial}{\partial z_{T,i}} \frac{\partial}{\partial \Delta_{T,j}} \langle p', + | \bar{\psi}(-z/2) \gamma^+ \mathcal{U}[-z/2, z/2] \psi(z/2) | p, + \rangle \Big|_{z^+ = z^- = 0, \Delta_T = 0, z_T \rightarrow 0}
 \end{aligned}$$

$$p' = P + \Delta_T/2, \quad p = P - \Delta_T/2, \quad P \text{ in 3-direction}, \quad P \rightarrow \infty$$

Same type of operator as used in TMD studies – generalization to off-forward matrix element adds transverse position information

Connection to GTMDs –
A. Metz, M. Schlegel, C. Lorcé,
B. Pasquini ...

Direct evaluation of quark orbital angular momentum

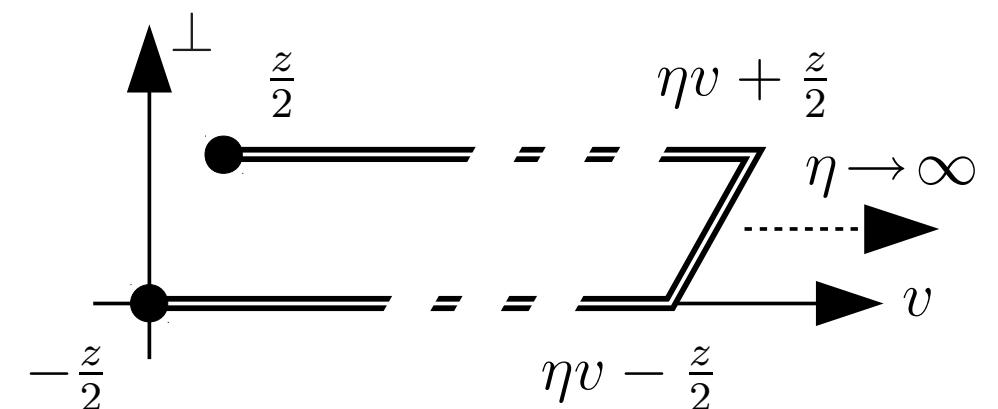
$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij} \frac{\partial}{\partial z_{T,i}} \frac{\partial}{\partial \Delta_{T,j}} \langle p', + | \bar{\psi}(-z/2) \gamma^+ \mathcal{U}[-z/2, z/2] \psi(z/2) | p, + \rangle|_{z^+ = z^- = 0, \Delta_T = 0, z_T \rightarrow 0}}{\langle p', + | \bar{\psi}(-z/2) \gamma^+ \mathcal{U}[-z/2, z/2] \psi(z/2) | p, + \rangle|_{z^+ = z^- = 0, \Delta_T = 0, z_T \rightarrow 0}}$$

Renormalization: Form ratio with number of valence quarks n

Role of the gauge link \mathcal{U} :

Y. Hatta, M. Burkardt:

- Straight $\mathcal{U}[-z/2, z/2] \rightarrow$ Ji OAM
- Staple-shaped $\mathcal{U}[-z/2, z/2] \rightarrow$ Jaffe-Manohar OAM
- Difference is torque accumulated due to final state interaction



Direct evaluation of quark orbital angular momentum

$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij} \frac{\partial}{\partial z_{T,i}} \frac{\partial}{\partial \Delta_{T,j}} \langle p', + | \bar{\psi}(-z/2) \gamma^+ \mathcal{U}[-z/2, z/2] \psi(z/2) | p, + \rangle|_{z^+ = z^- = 0, \Delta_T = 0, z_T \rightarrow 0}}{\langle p', + | \bar{\psi}(-z/2) \gamma^+ \mathcal{U}[-z/2, z/2] \psi(z/2) | p, + \rangle|_{z^+ = z^- = 0, \Delta_T = 0, z_T \rightarrow 0}}$$

Renormalization: Form ratio with number of valence quarks n

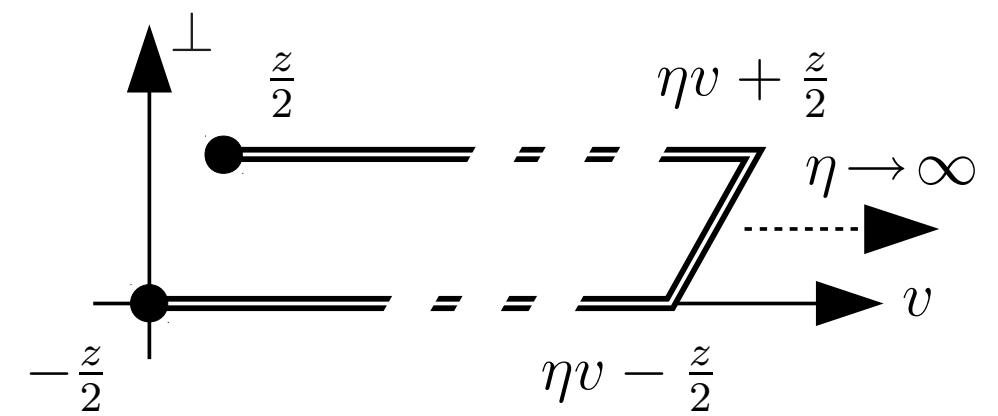
Role of the gauge link \mathcal{U} :

Direction of staple taken off light cone (rapidity divergences)

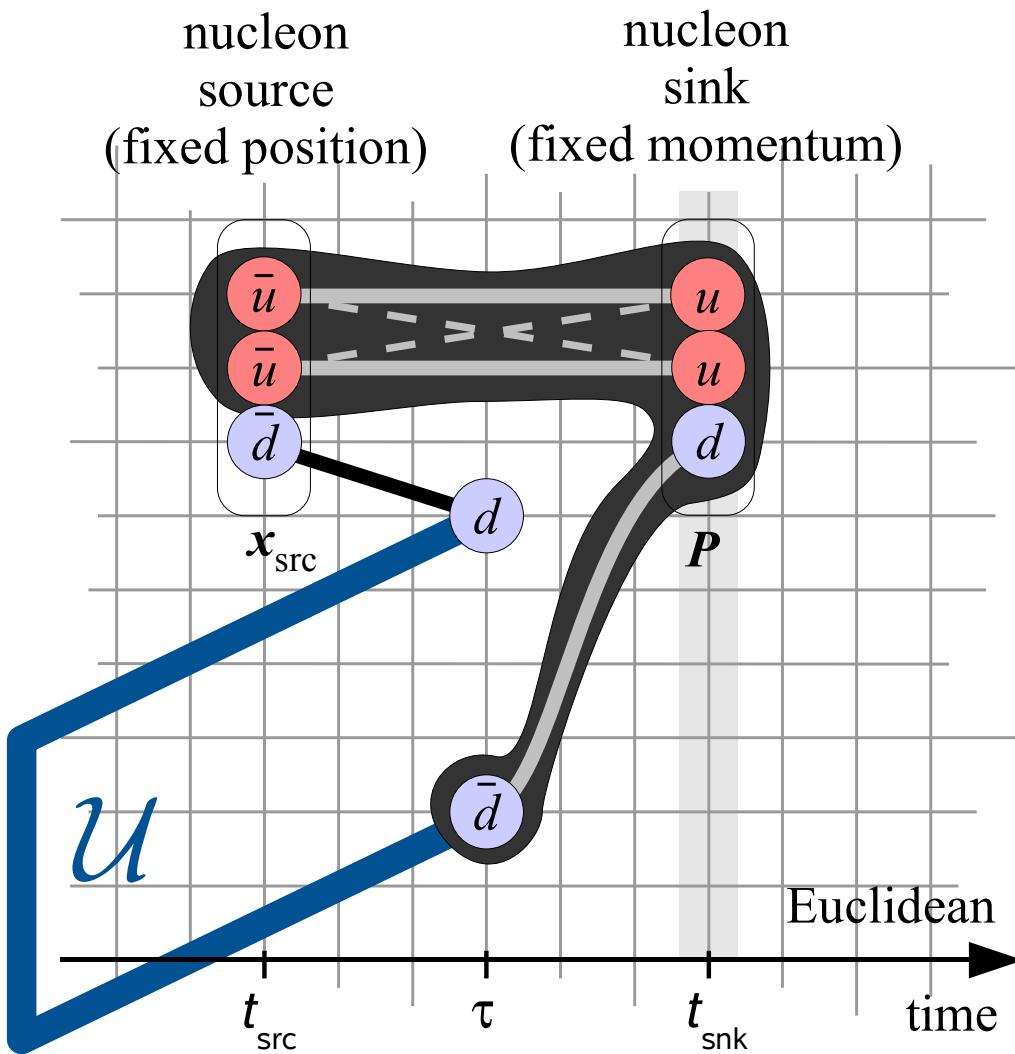
Characterized by Collins-Soper parameter

$$\hat{\zeta} \equiv \frac{P \cdot v}{|P||v|}$$

Are interested in $\hat{\zeta} \rightarrow \infty$; synonymous with $P \rightarrow \infty$ in the frame of the lattice calculation ($v = e_3$)



Lattice setup

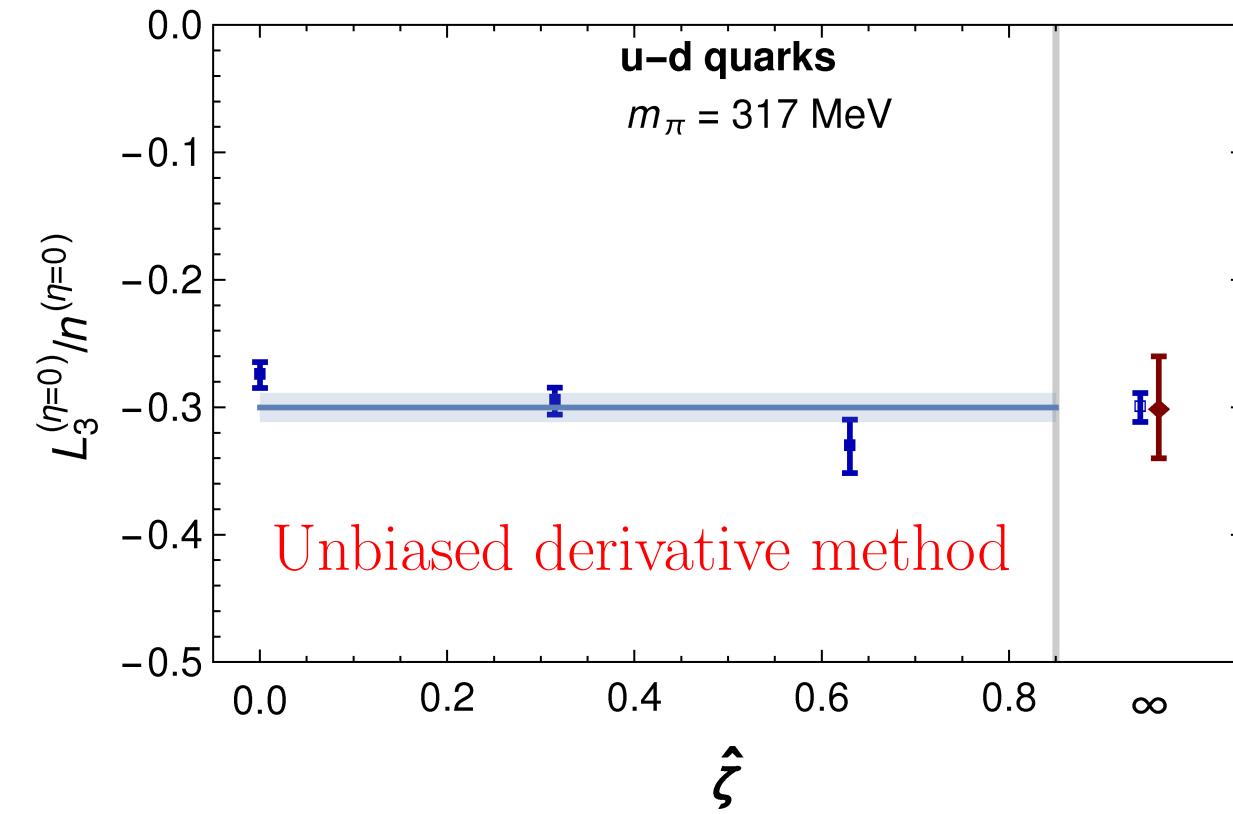
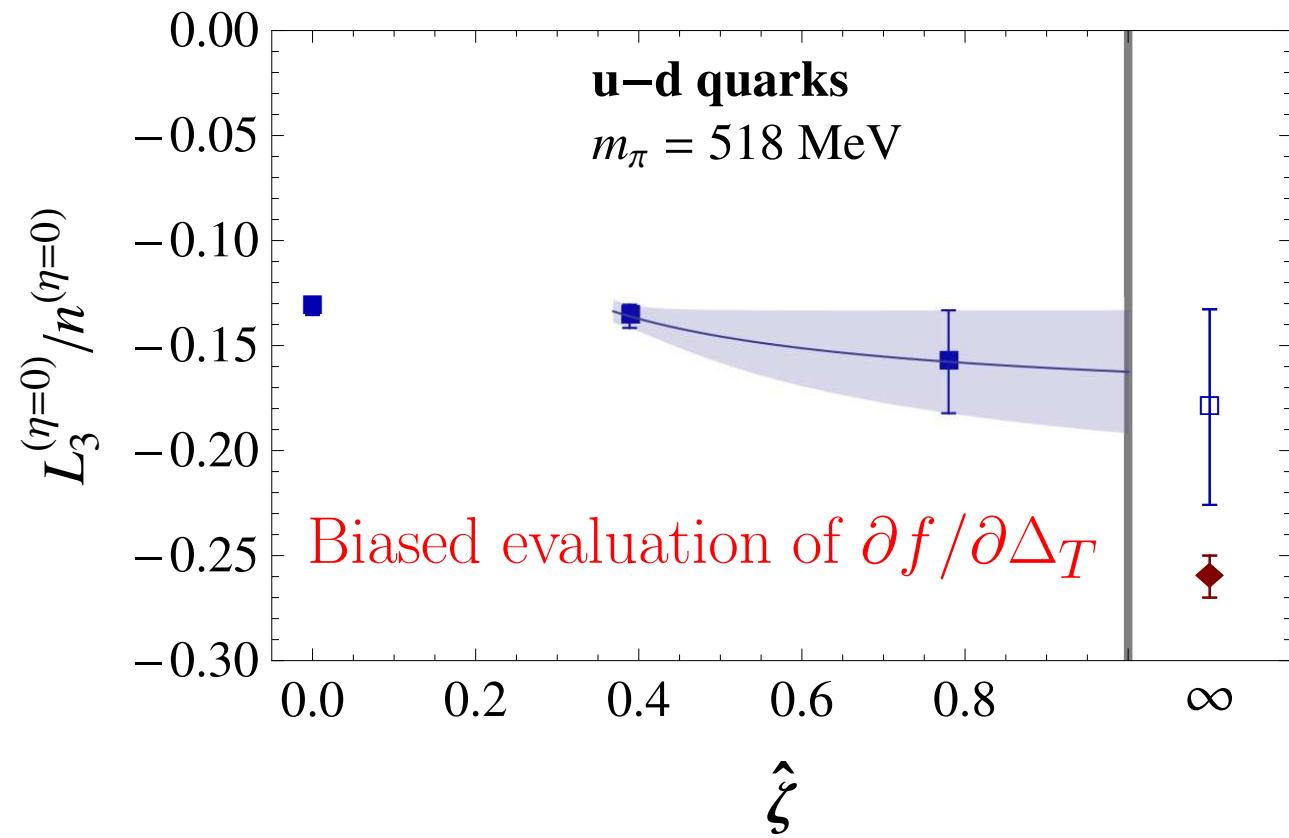


- Evaluate directly

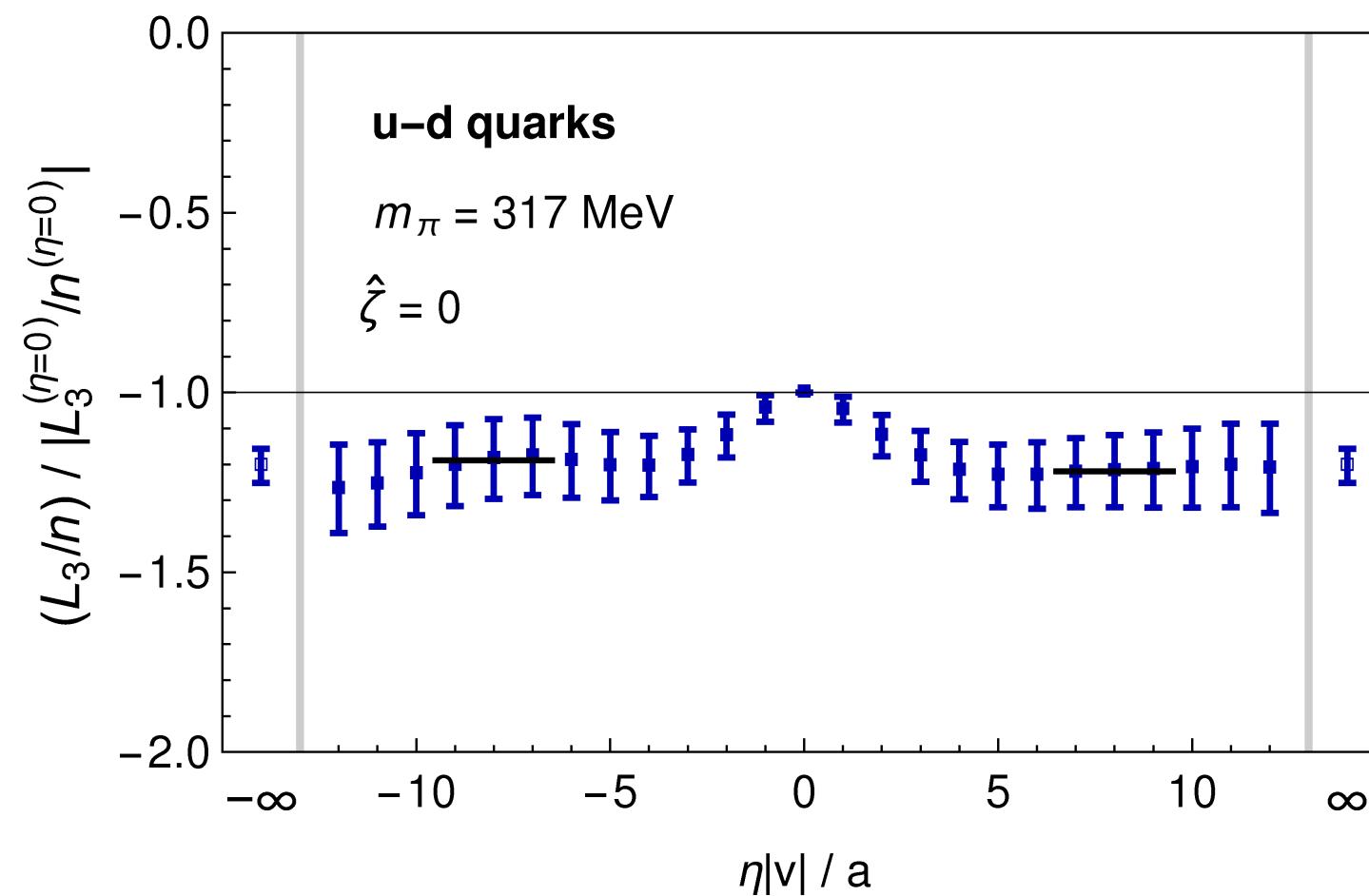
$$\langle p', + | \bar{\psi}(-z/2) \gamma^+ \mathcal{U}[-z/2, z/2] \psi(z/2) | p, + \rangle$$

- $m_\pi = 317 \text{ MeV}$
- $a = 0.114 \text{ fm}$
- Use direct derivative method to achieve unbiased evaluation of Δ_T -derivative

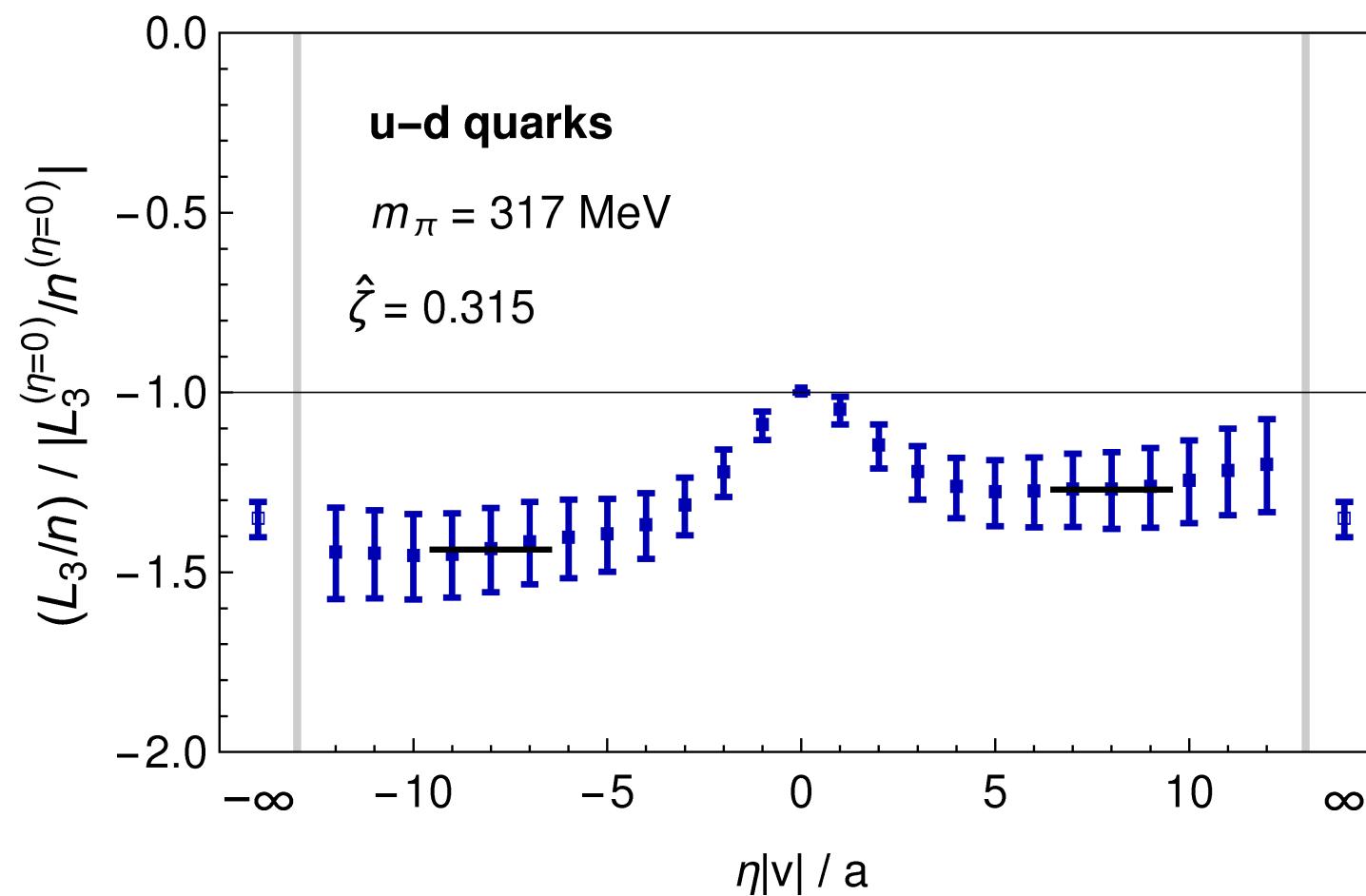
Ji quark orbital angular momentum: $\eta = 0$



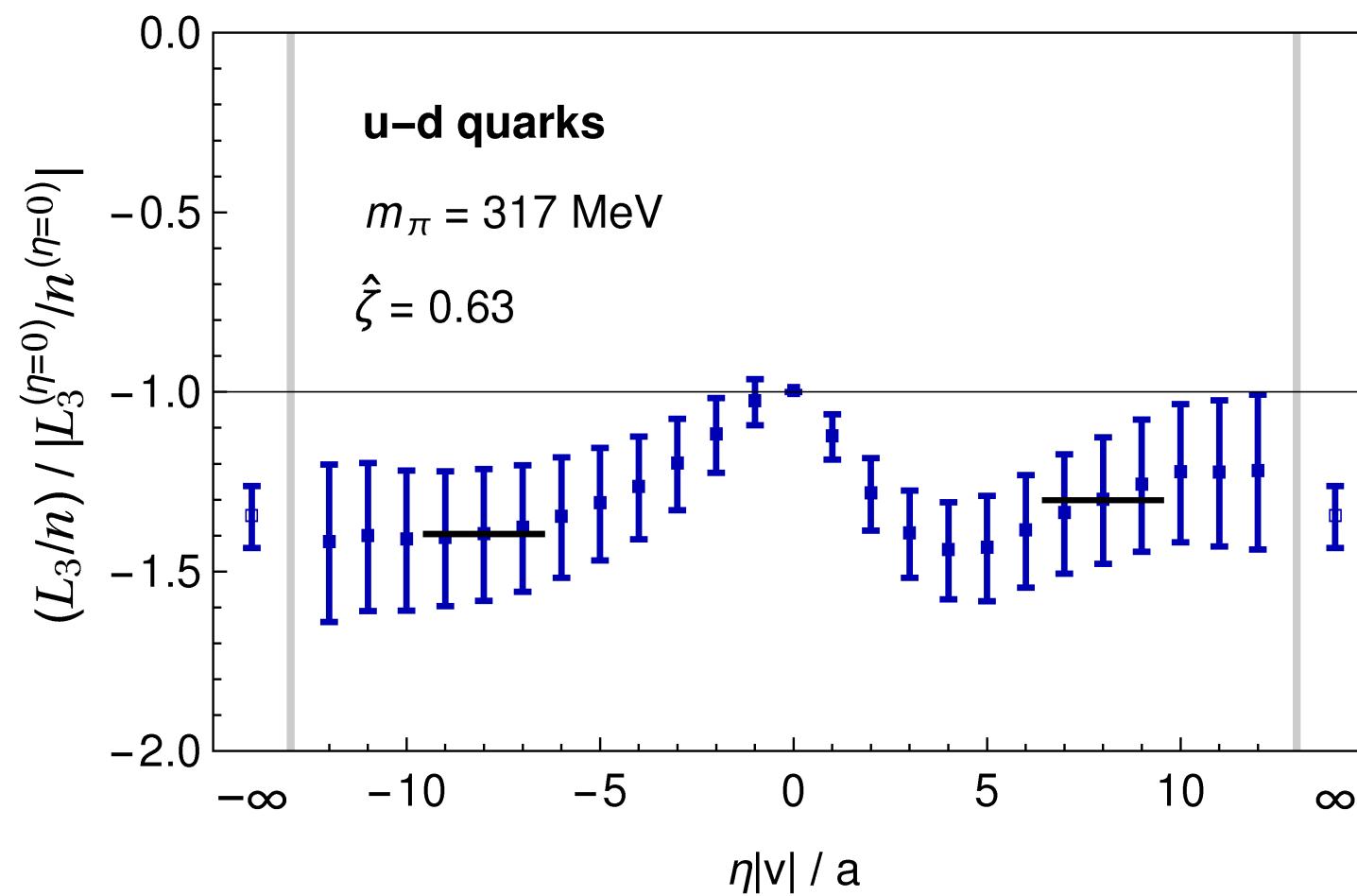
From Ji to Jaffe-Manohar quark orbital angular momentum



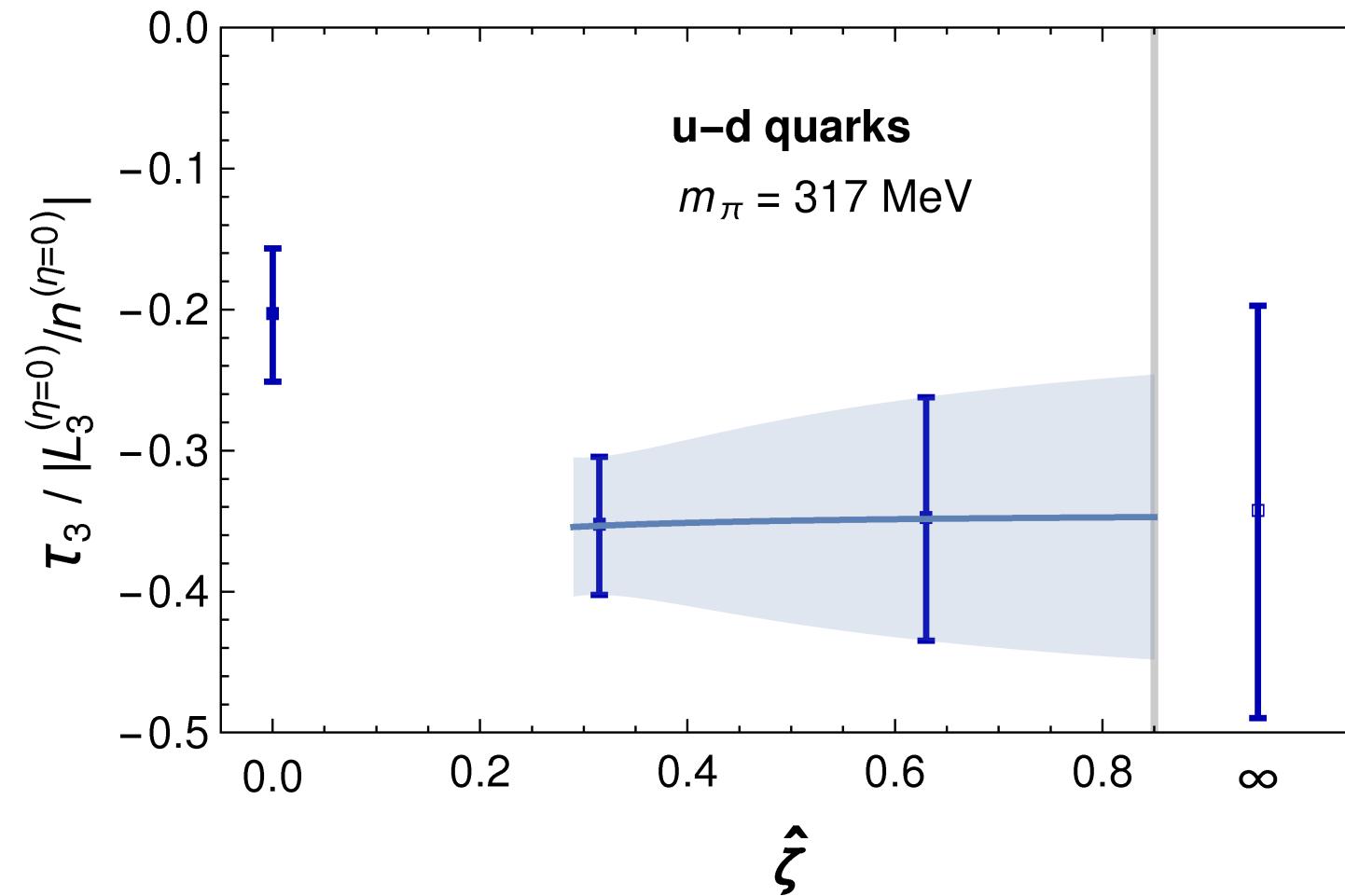
From Ji to Jaffe-Manohar quark orbital angular momentum



From Ji to Jaffe-Manohar quark orbital angular momentum



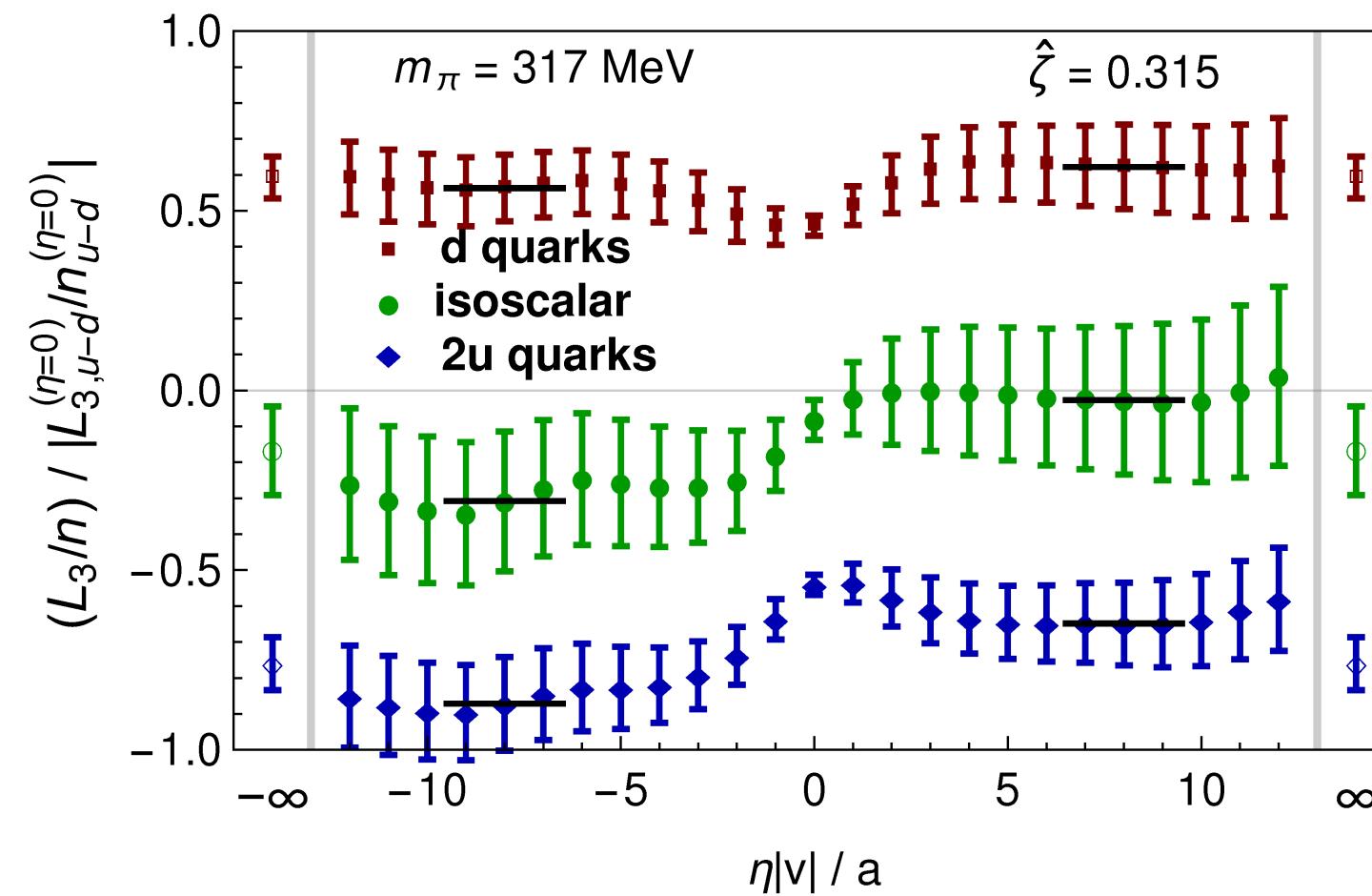
Burkardt's torque – extrapolation in $\hat{\zeta}$



$$\tau_3 = (L_3^{(\eta=\infty)} / n^{(\eta=\infty)}) - (L_3^{(\eta=0)} / n^{(\eta=0)})$$

Integrated torque accumulated by struck quark leaving proton

Flavor separation – from Ji to Jaffe-Manohar quark orbital angular momentum



Lorentz invariance and equation of motion relations (straight link)

$$\frac{d}{dx} \int d^2 k_T \frac{k_T^2}{m^2} F_{14} = \tilde{E}_{2T} + H + E$$

$$-x \tilde{E}_{2T} = \tilde{H} - \int d^2 k_T \frac{k_T^2}{m^2} F_{14} + \mathcal{M}$$

\mathcal{M} : quark-gluon-quark correlator

$$L = - \int dx \int d^2 k_T \frac{k_T^2}{m^2} F_{14} = \frac{1}{2} \int dx x (H + E) - \frac{1}{2} \int dx \tilde{H} = \int dx x (\tilde{E}_{2T} + H + E)$$

- A. Rajan, A. Courtoy, M.E., S. Liuti, PRD 94 (2016) 034041
A. Rajan, M.E., S. Liuti, PRD 98 (2018) 074022

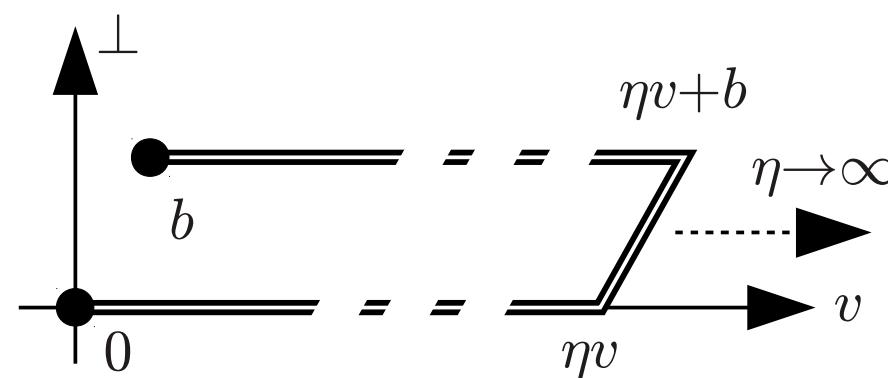
Preliminary sketch: x -dependence of Sivers shift

Sivers shift: Average transverse momentum of unpolarized quarks in a nucleon polarized in the other transverse direction

$$\frac{1}{2} \langle P, S | \bar{q}(0) \gamma^+ \mathcal{U}[0, \dots, b] q(b) | P, S \rangle = 2P^+ (\bar{A}_{2B} + i m_N \epsilon_{ij} b_i S_j \bar{A}_{12B})$$

$$\langle k_T \rangle_{TU} (b_T^2, x, \dots) = m_N \frac{\bar{f}_1^{\perp(1)}(b_T^2, x, \dots)}{\bar{f}_1^{(0)}(b_T^2, x, \dots)} = -m_N \frac{\int d(b \cdot P) \exp(ixb \cdot P) \bar{A}_{12B}(-b_T^2, b \cdot P, \hat{\zeta}, \eta v \cdot P)}{\int d(b \cdot P) \exp(ixb \cdot P) \bar{A}_{2B}(-b_T^2, b \cdot P, \hat{\zeta}, \eta v \cdot P)}$$

Note soft factors do not depend on $b \cdot P$ – can be factored outside the Fourier transform



Preliminary sketch: x -dependence of Sivers shift

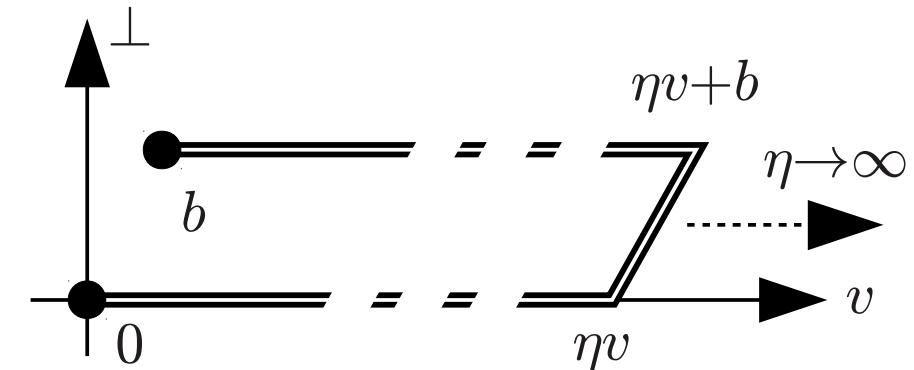
Phenomenological frame: $P_T = v_T = 0, b^+ = 0$

Expressed in Lorentz-invariant fashion: $\frac{v \cdot b}{v \cdot P} = \frac{b \cdot P}{m_N^2} \left(1 - \sqrt{1 + 1/\hat{\zeta}^2} \right)$

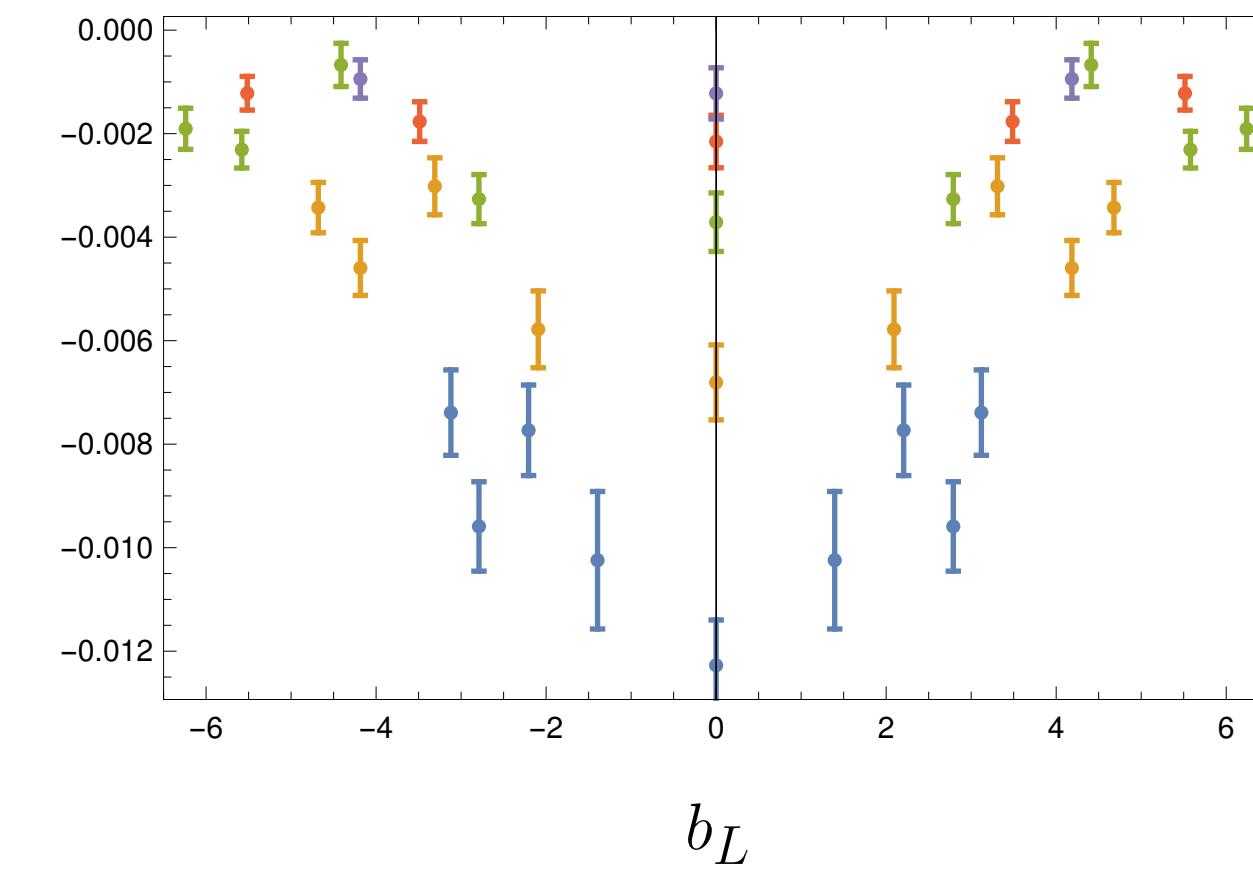
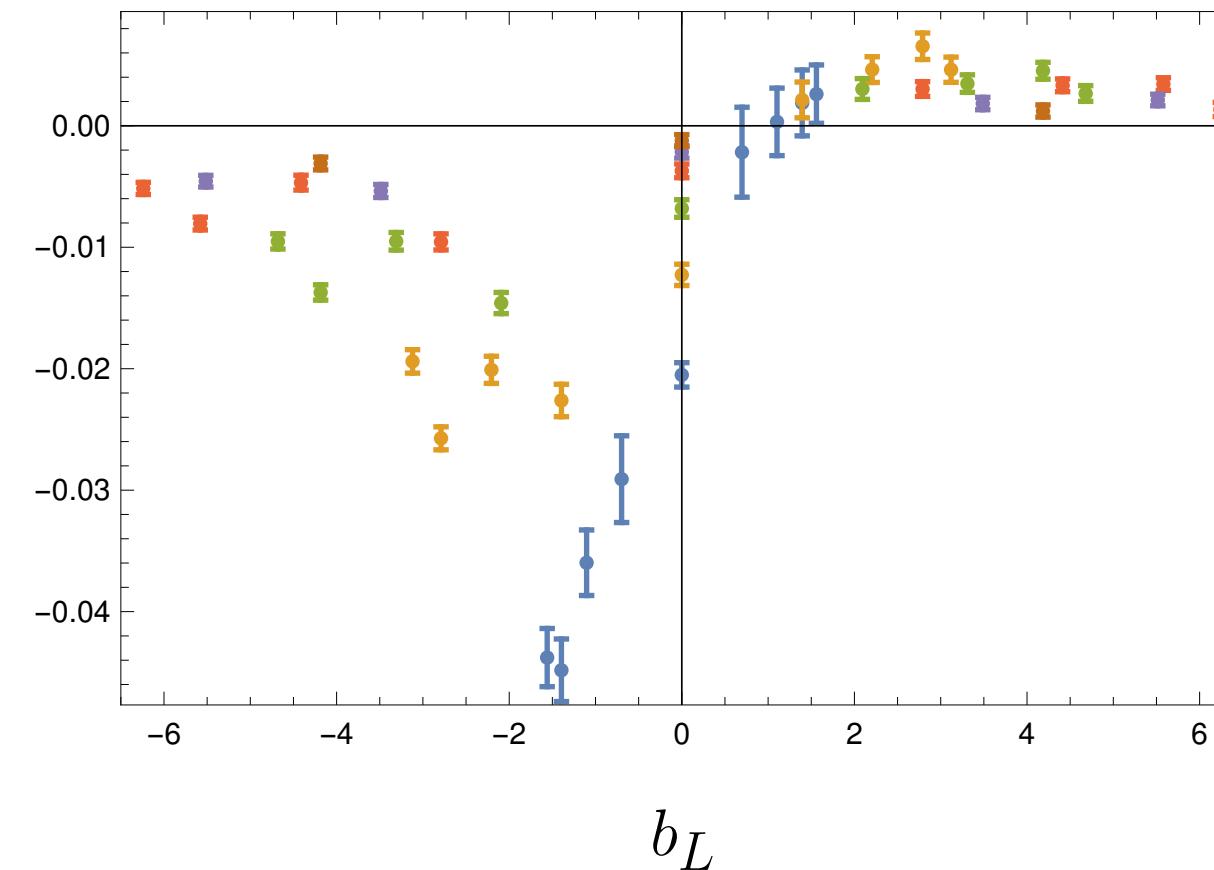
Lattice frame: b, v purely spatial

Constraint forces the use of general off-axis directions

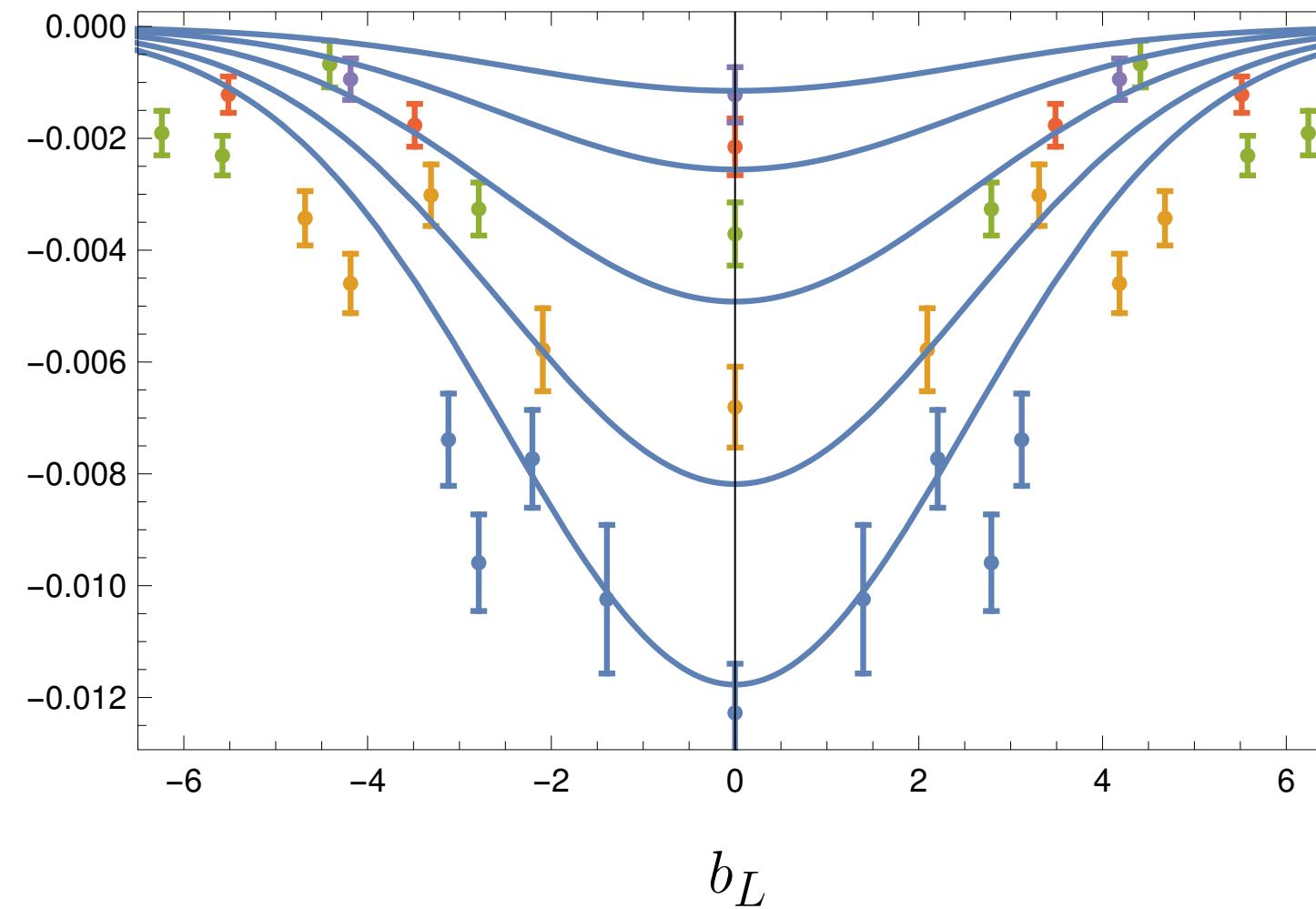
Perform analysis at large staple length



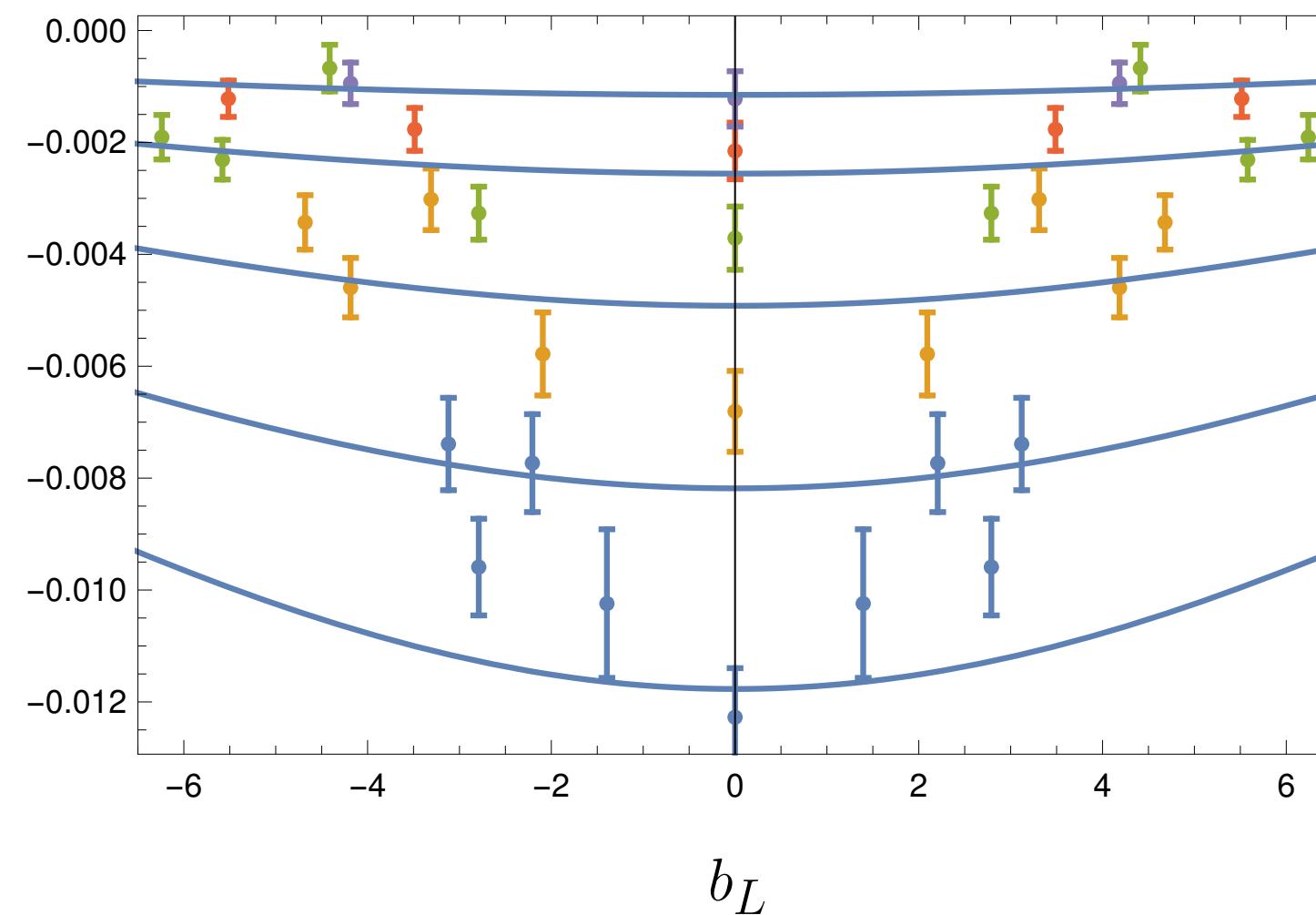
Extract b_L -even component of imaginary part of γ^+ correlator



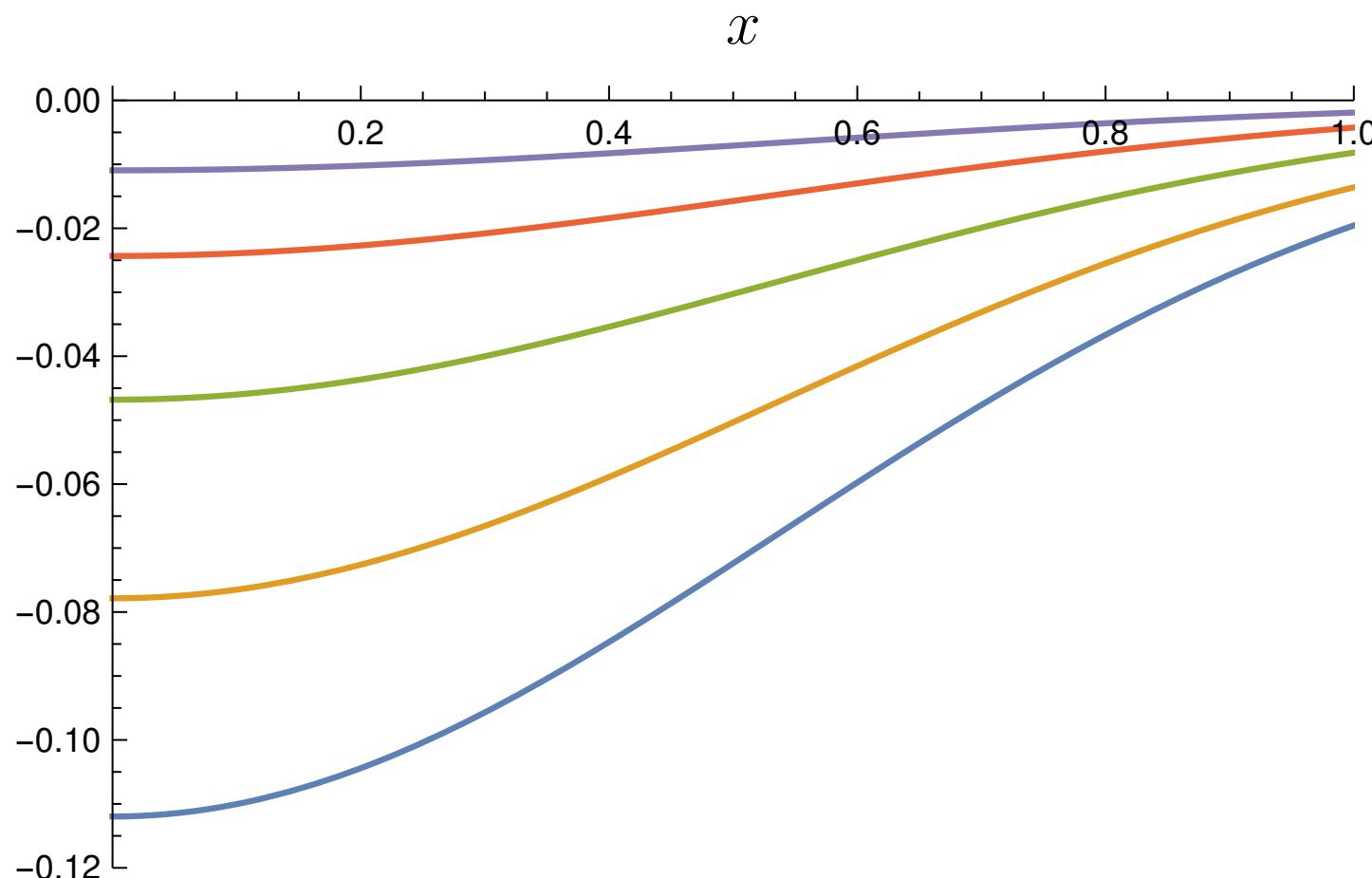
Fit dependence in b_L , $|b_T|$ space



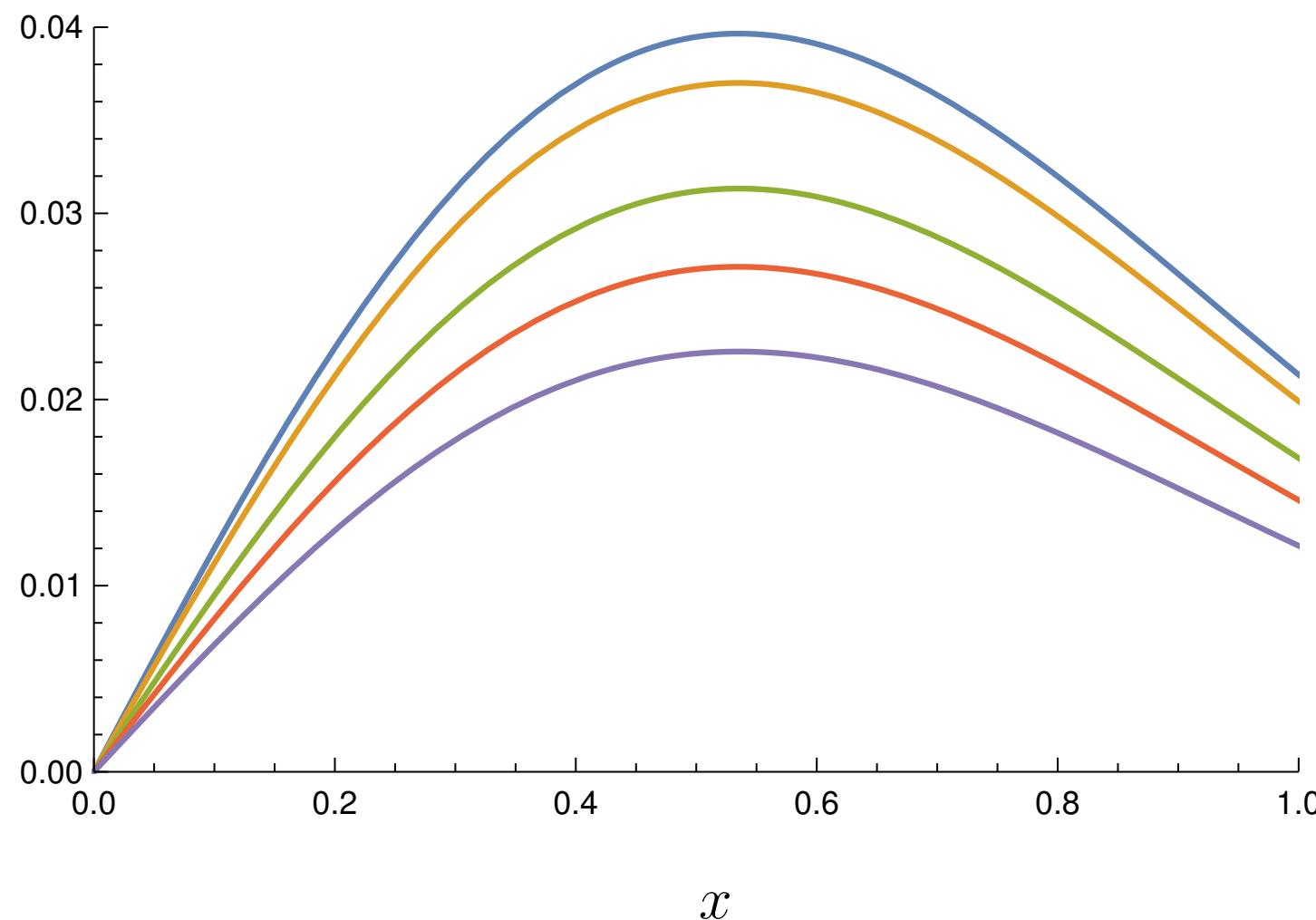
Cast in $b \cdot P, b^2$ space



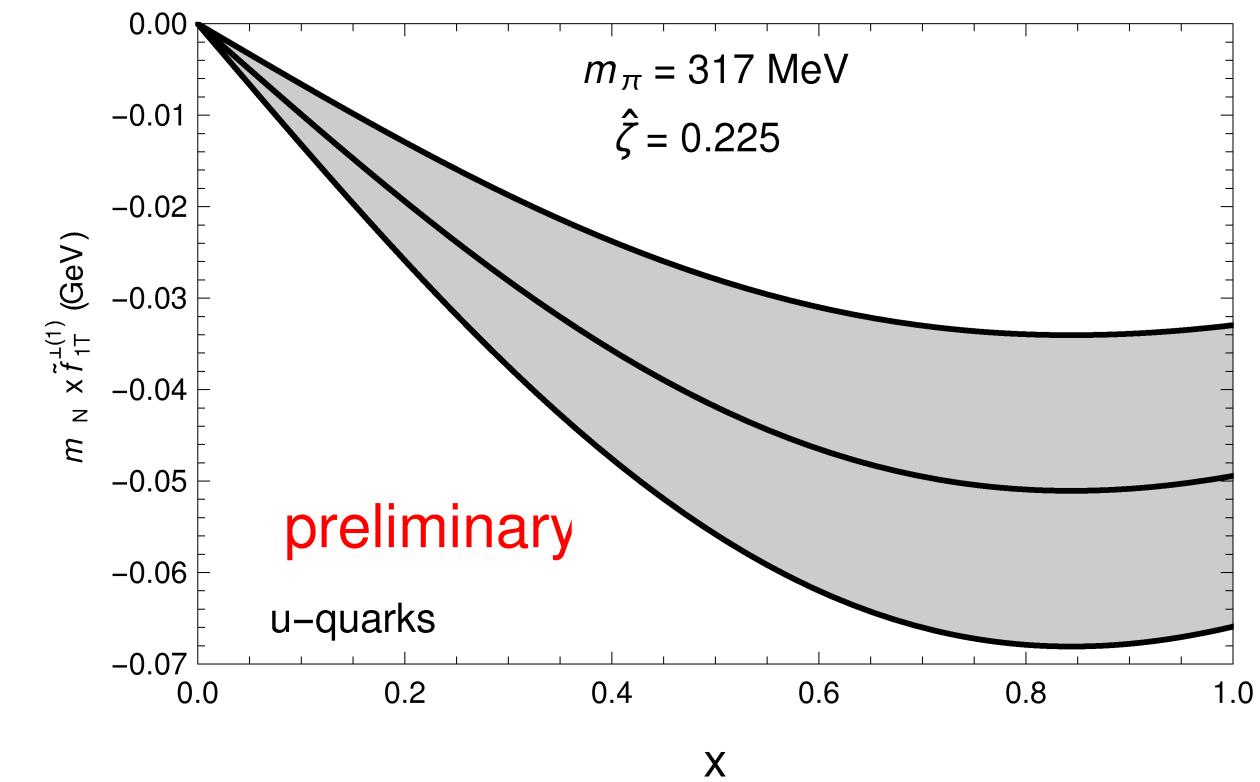
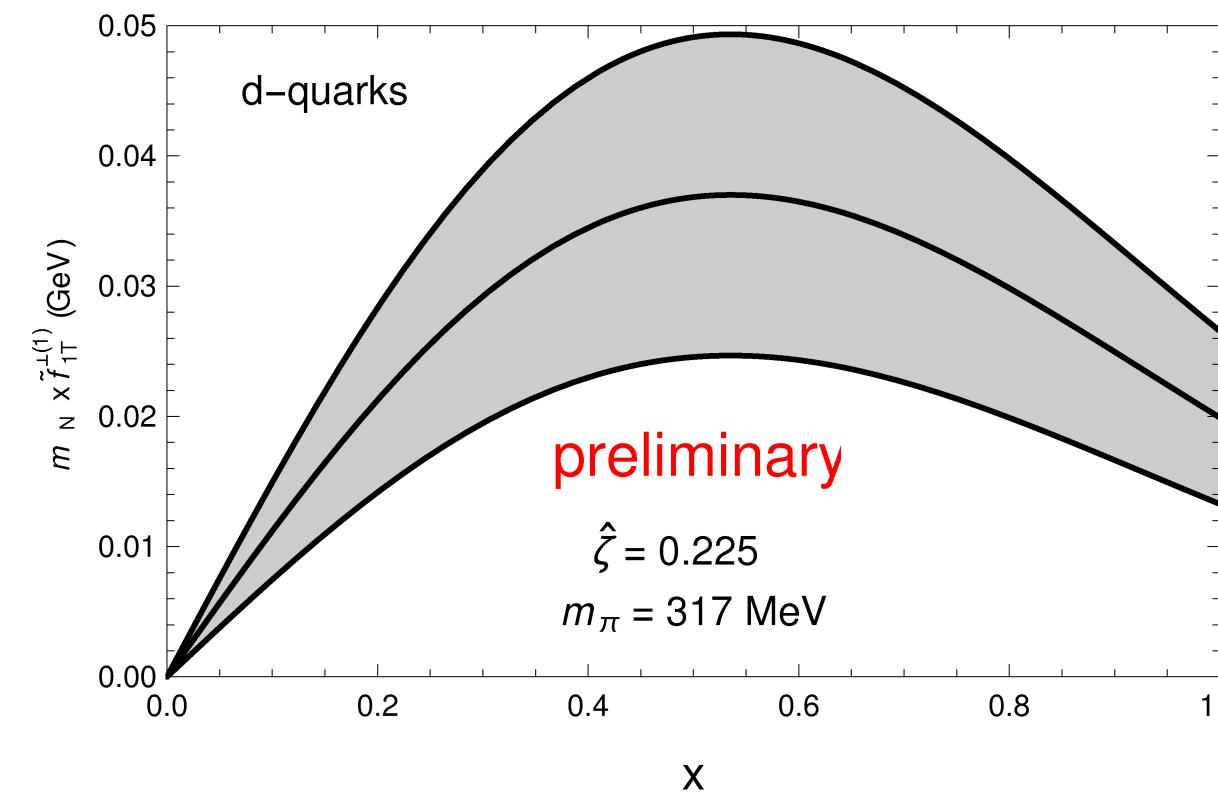
Fourier transform $b \cdot P \longrightarrow x$



Normalize to x -integrated Sivers shift, multiply by x



Eyeball error



Conclusions and Outlook

- Quark orbital angular momentum can be accessed directly in Lattice QCD, continuously interpolating between the Ji and Jaffe-Manohar definitions.
- Agreement with result obtained using Ji sum rule verified
- Difference between the Ji and Jaffe-Manohar definitions (torque accumulated by struck quark leaving a proton) is clearly resolvable, sizeable ($\sim 1/3$ of the original Ji OAM, at $m_\pi = 317 \text{ MeV}$), and leads to an enhancement of Jaffe-Manohar OAM relative to Ji OAM.
- Generalization to x -dependent TMD observables explored – currently producing large data set to permit control of systematics in Sivers shift
- First lattice TMD observables have become available directly at the physical pion mass