

peeking into pion using lattice QCD

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role of gluons in hadron mass generation ... the enigma of pion mass

- significant contributions of gluons (trace-anomaly) to proton mass, even in the chiral limit

$$M_p = -\langle T_{\mu\mu} \rangle = \langle H_m \rangle + \langle H_a \rangle$$

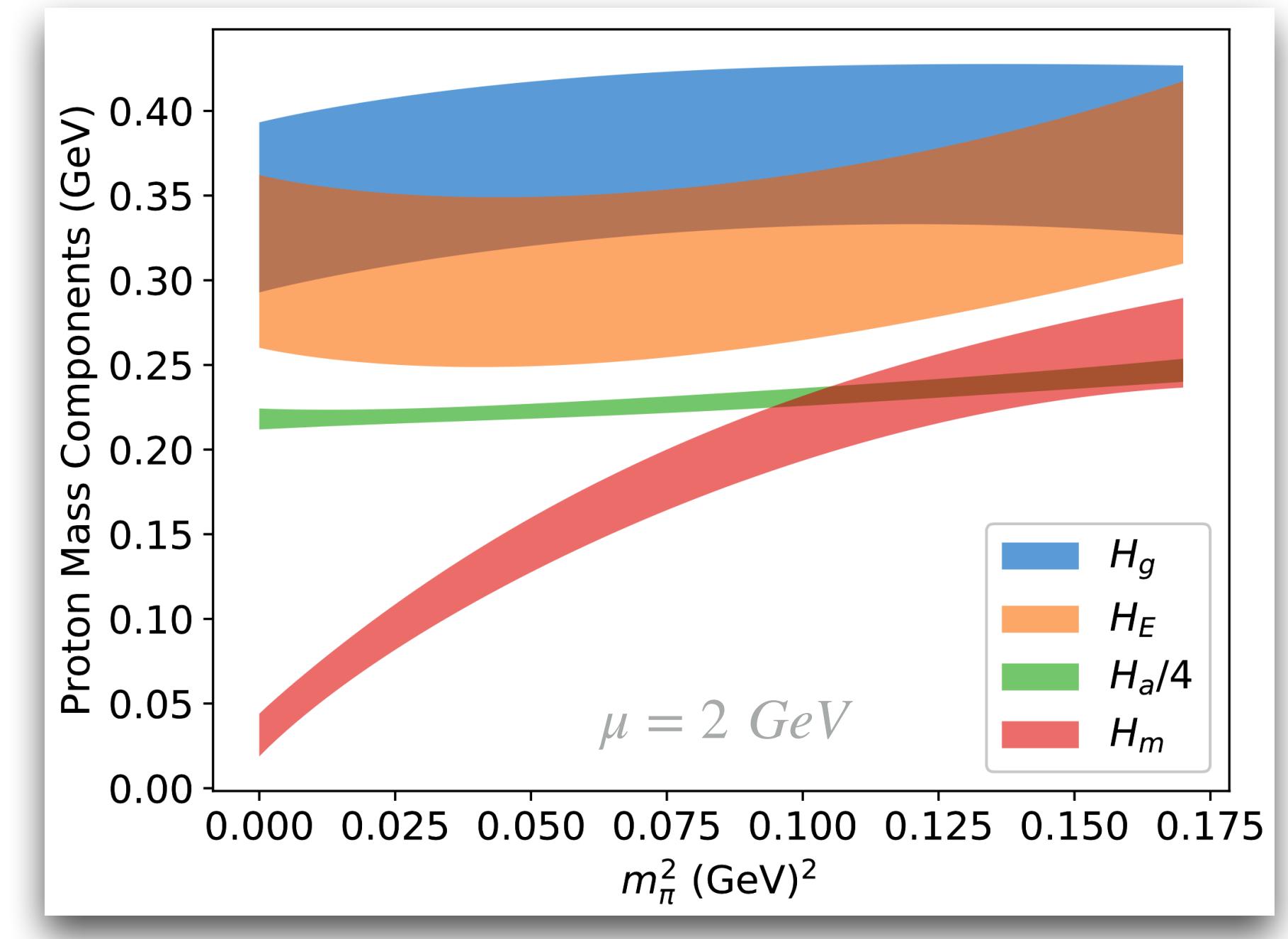
H_a : QCD trace anomaly

H_m : $m_q \times$ quark condensate

X.-D. Ji: Phys. Rev. Lett. 74, 1071 (1995)

$$M_p = -\langle T_{44} \rangle = \langle H_m \rangle + \langle H_E \rangle + \langle H_g \rangle + \langle H_a \rangle / 4$$

H_g : gluon field energy; H_E : quark energy



χ QCD: Phys. Rev. Lett. 121, 212001 (2018)

role of gluons in hadron mass generation ... the enigma of pion mass

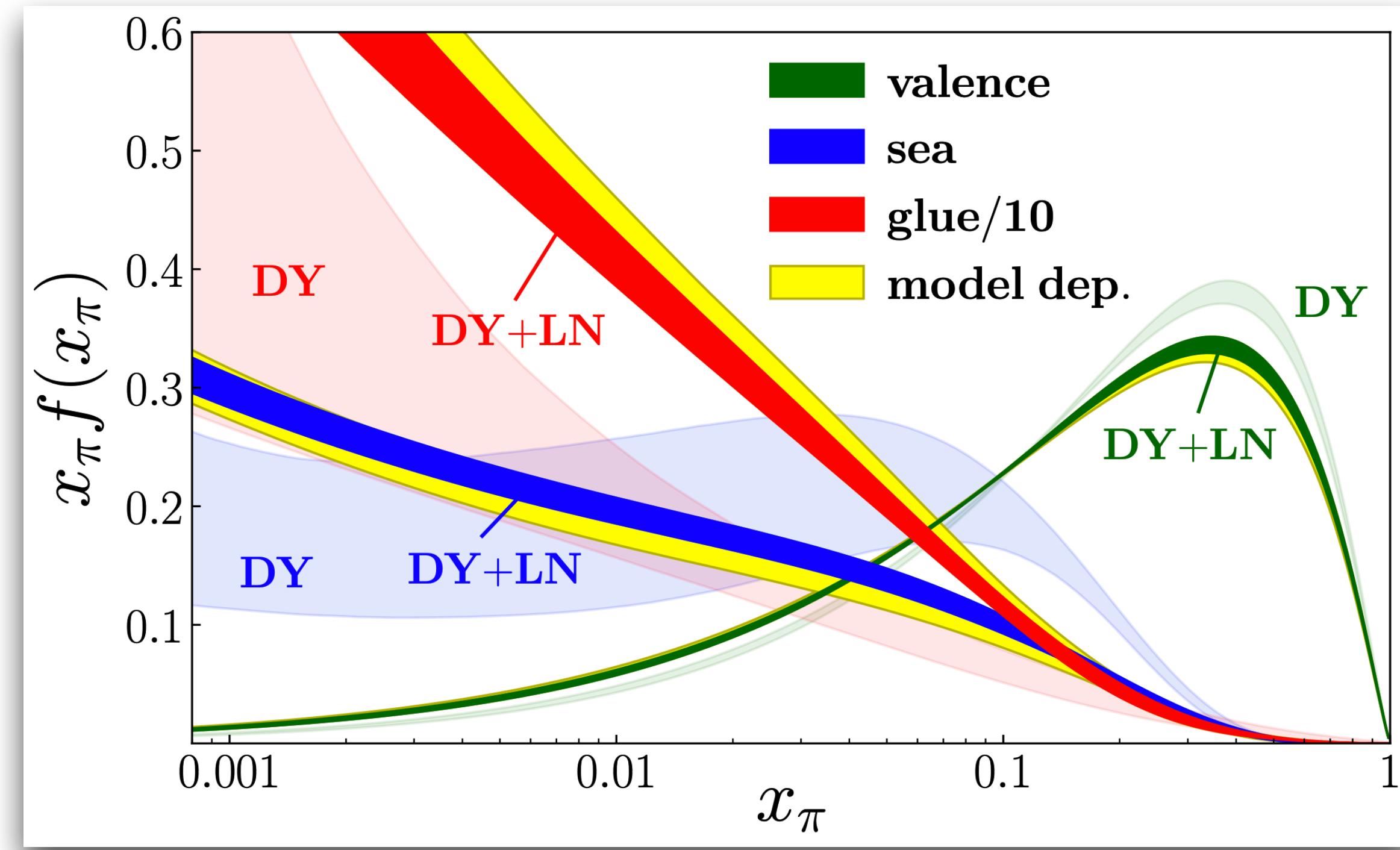
- significant contributions of gluons (trace-anomaly) to proton mass, even in the chiral limit
- contribution of gluons (trace-anomaly) to pion mass ?

$$m_q \rightarrow 0 : -\langle T_{\mu\mu} \rangle = \langle H_m \rangle + \langle H_a \rangle = M_\pi \rightarrow 0$$

a puzzling dichotomy !!

understanding the role of gluons in hadron mass generation will remain incomplete without explaining the *absence* of pions mass

where are the gluons? ... valence quark PDF of pion



JAM collaboration: Phys. Rev. Lett. 121, 152001 (2018)

from fit to:

DY: $\pi A \rightarrow \mu^+ \mu^- X$ (Fermilab E615, CERN NA10)

LN: $e p \rightarrow e' n X$ (HERA ZEUS)

where are the gluons? ... valance quark PDF of pion

different for different models/fits

data: Fermilab E615 Phys. Rev. D39, 92 (1989)

pQCD: $b \simeq 2$ dominated by one-gluon exchange
Berger, Brodsky: Phys. Rev. Lett. 42, 940

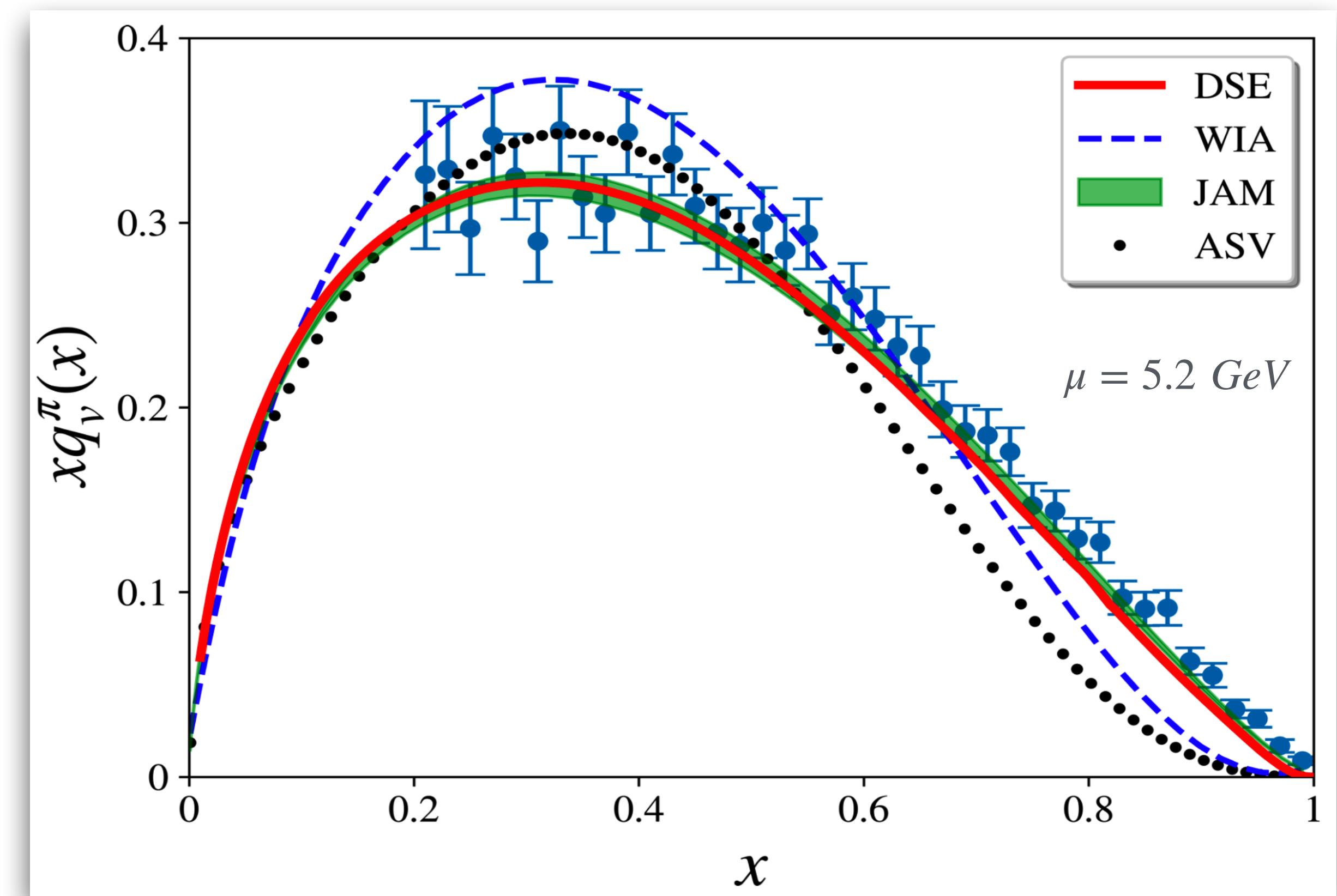
JAM: $b \simeq 1$ Phys. Rev. Lett. 121, 152001 (2018)

ASV: $b \simeq 1.3$ NLL threshold resummation
Phys. Rev. Lett. 105, 252003 (2010)

WIA: $b \simeq 2$ DSE, rainbow-ladder truncation
Nguyen et al: Phys. Rev. C83, 062201 (2011)

DSE: $b \simeq 2$ another DSE w/ different quark dressing,
different contributions of gluons
Bednar, Cloet, Tandy: arXiv:1811.12310

$$q_v^\pi(x) = Ax^a(1-x)^b[1 + \text{sub leading}]$$



Bednar, Cloet, Tandy: arXiv:1811.12310

partonic imaging of pion at EIC

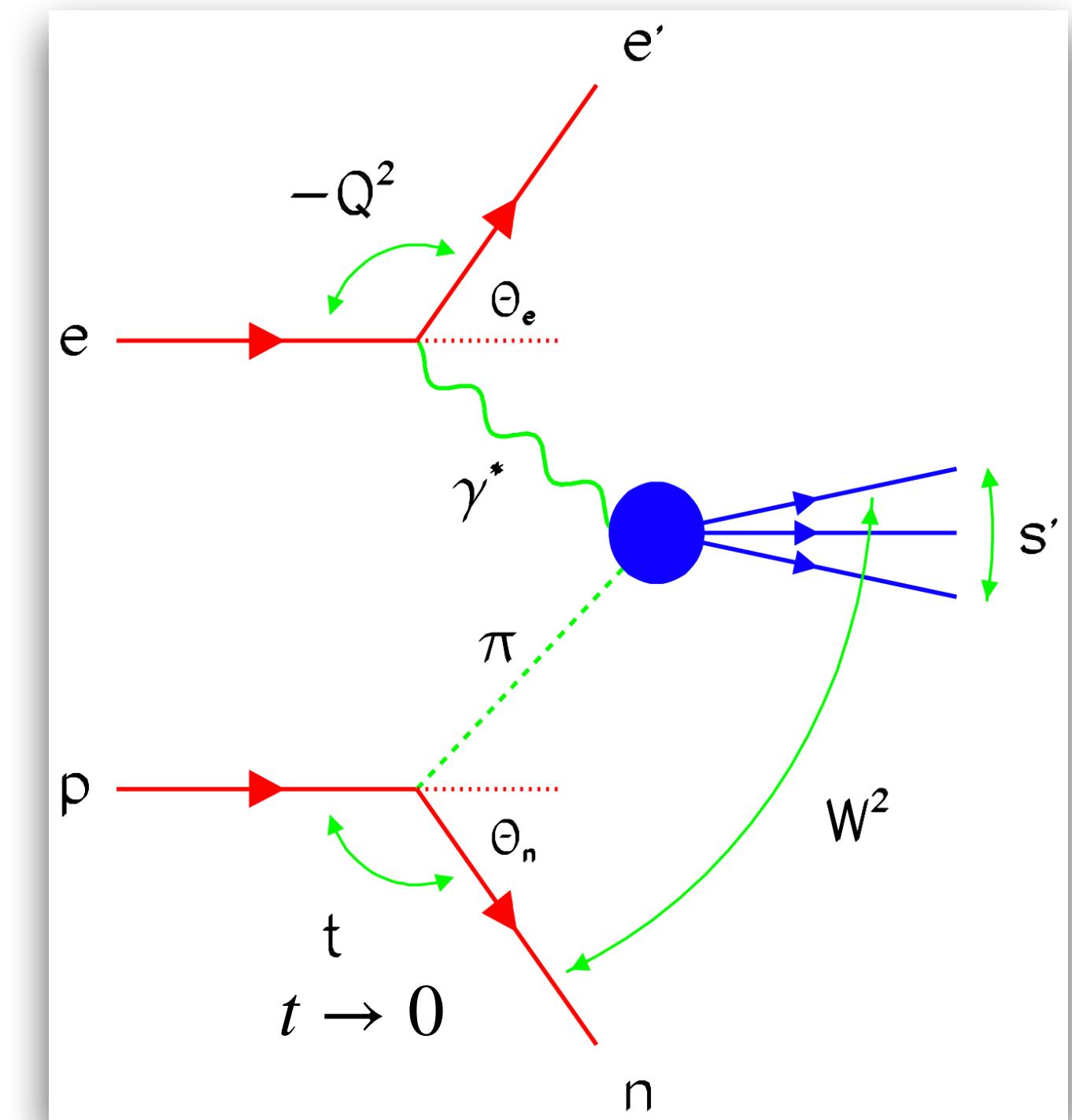
whitepaper: Eur. Phys. J. A55,190 (2019)

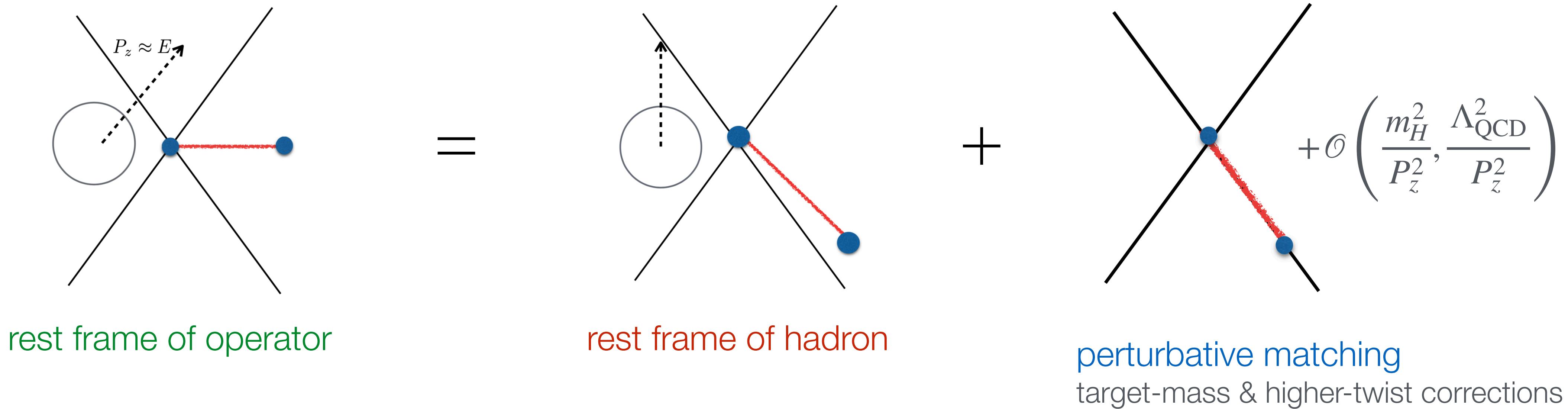
Pion and Kaon Structure at the Electron-Ion Collider

abstract

1 GeV mass-scale that characterizes atomic nuclei appear; why does it have the observed value; and, enigmatically, why are the composite Nambu-Goldstone (NG) bosons in quantum chromodynamics (QCD) abnormally light in comparison? In this perspective, we provide an analysis of the mass budget of the pion and proton in QCD; discuss the special role of the kaon, which lies near the boundary between dominance of strong and Higgs mass-generation mechanisms; and explain the need for a coherent effort in QCD phenomenology and continuum calculations, in exa-scale computing as provided by lattice QCD, and in experiments to make progress in understanding the origins of hadron masses and the distribution of that mass within them. We compare the unique capabilities foreseen at the electron-ion collider (EIC) with those at the hadron-electron ring accelerator (HERA), the

Sullivan process w/ off-shell pion



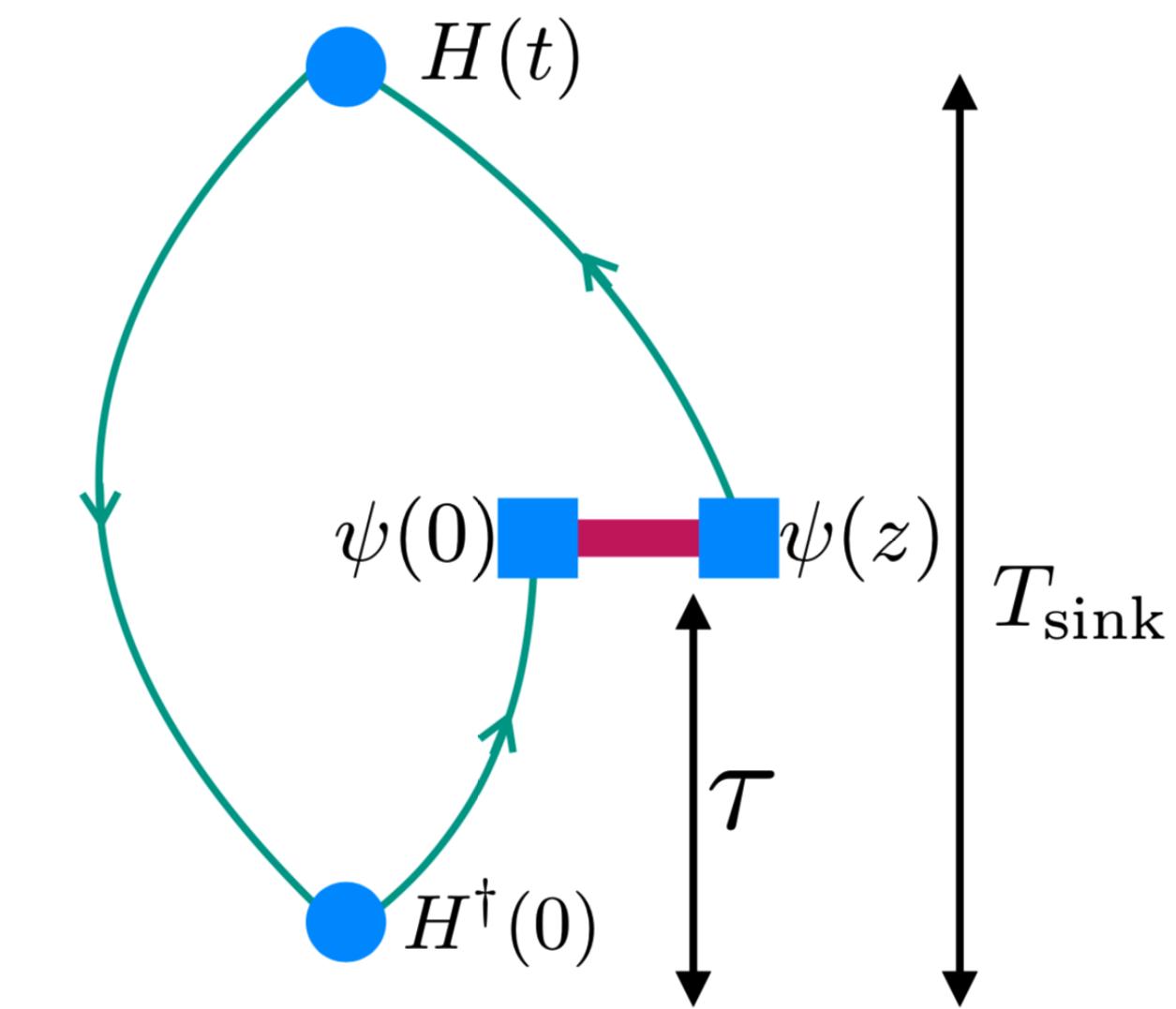


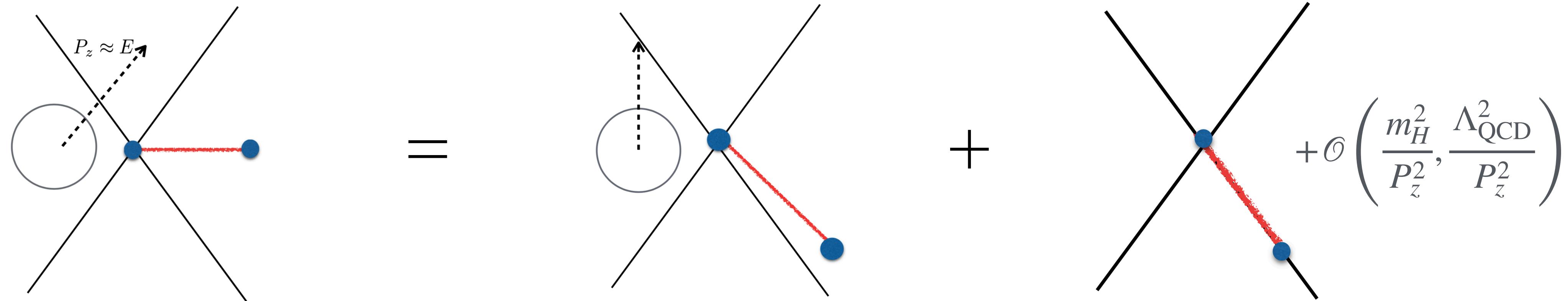
- bare matrix element, extract ground-state aptitude

$$Q_b(z, p_z) = \langle H(p_z, 0) | \bar{\psi}(0, \tau) \gamma_0 W(0, z) \psi(z, \tau) | H(p_z, T_{\text{sink}}) \rangle$$

equal-time, non-local operator within boosted hadron

$W(0, z)$: Wilson line from 0 to z





- renormalized operator:

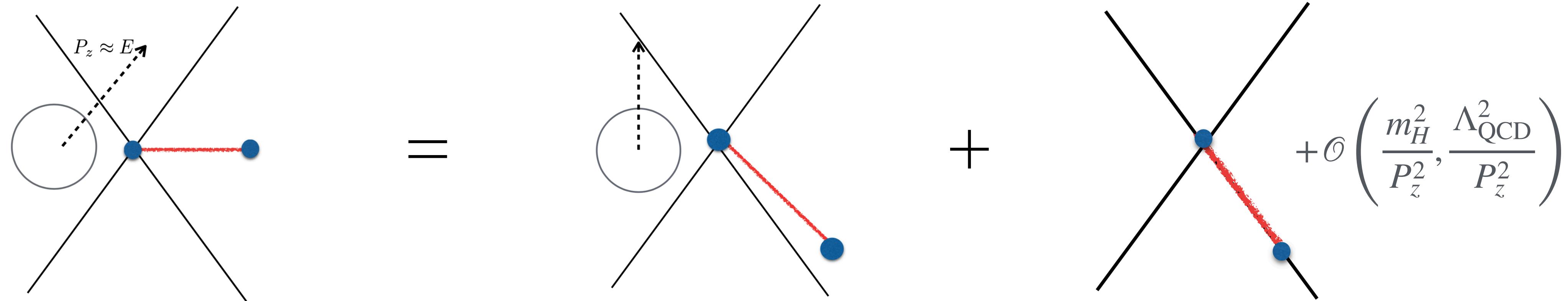
$$Q(z, p_z, p^R) = Z(z, p^R) Q_b(z, p_z)$$

non-perturbative RI-MOM

by computing the same matrix element for Landau-gauge fixed off-shell quarks with $p^2 > 0$

renormalization condition:

$$Z(z, p^R, a) Q_b(z, p = p^R, a) \equiv e^{izp_z^R}$$



rest frame of operator

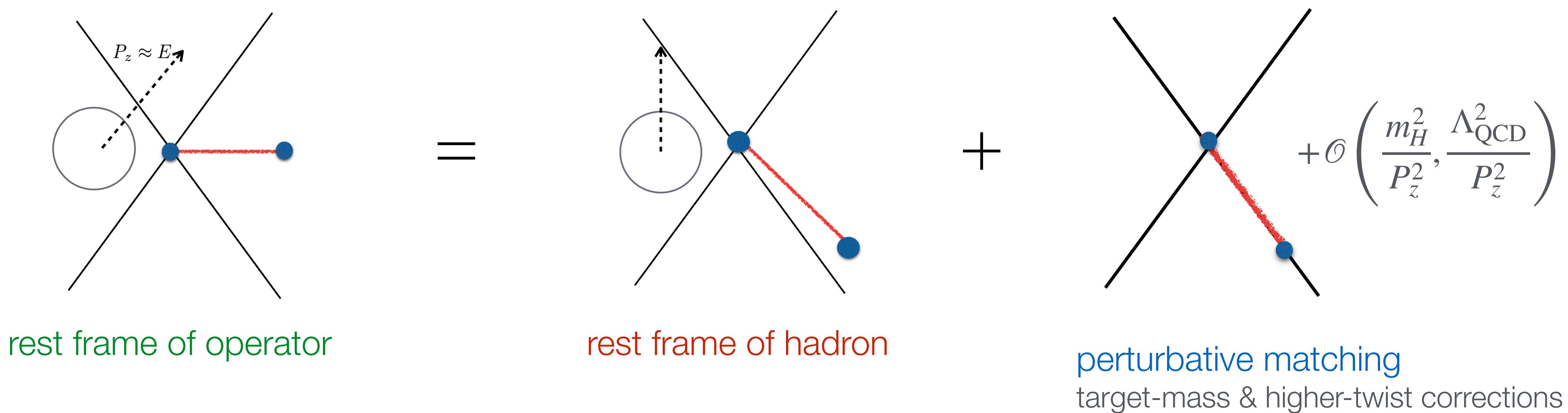
rest frame of hadron

perturbative matching
target-mass & higher-twist corrections

- RI-MOM renormalized quasi-PDF (qPDF):

$$q(x, p_z, p^R) = \frac{1}{4\pi} \int_0^\infty dz e^{-izp_z x} Q(z, p_z, p^R)$$

fixed p_z , small $x \rightarrow$ large $|z| \rightarrow$ contaminations of $\Lambda_{\text{QCD}}, m_H$



- RI-MOM qPDF to $\overline{\text{MS}}$ -PDF:

$$q(x, P_z, P^R) = \int \frac{dy}{|y|} f(x, \mu) C \left(\frac{x}{y}, \frac{\mu}{y P_z}, \frac{P_\perp^R}{P_z^R}, \frac{y P_z}{P_z^R} \right) + \mathcal{O}\left(\frac{m_h^2}{P_z^2}, \frac{\Lambda_{\text{QCD}}^2}{P_z^2}\right)$$

first continuum limit, then $P_z \rightarrow 0$

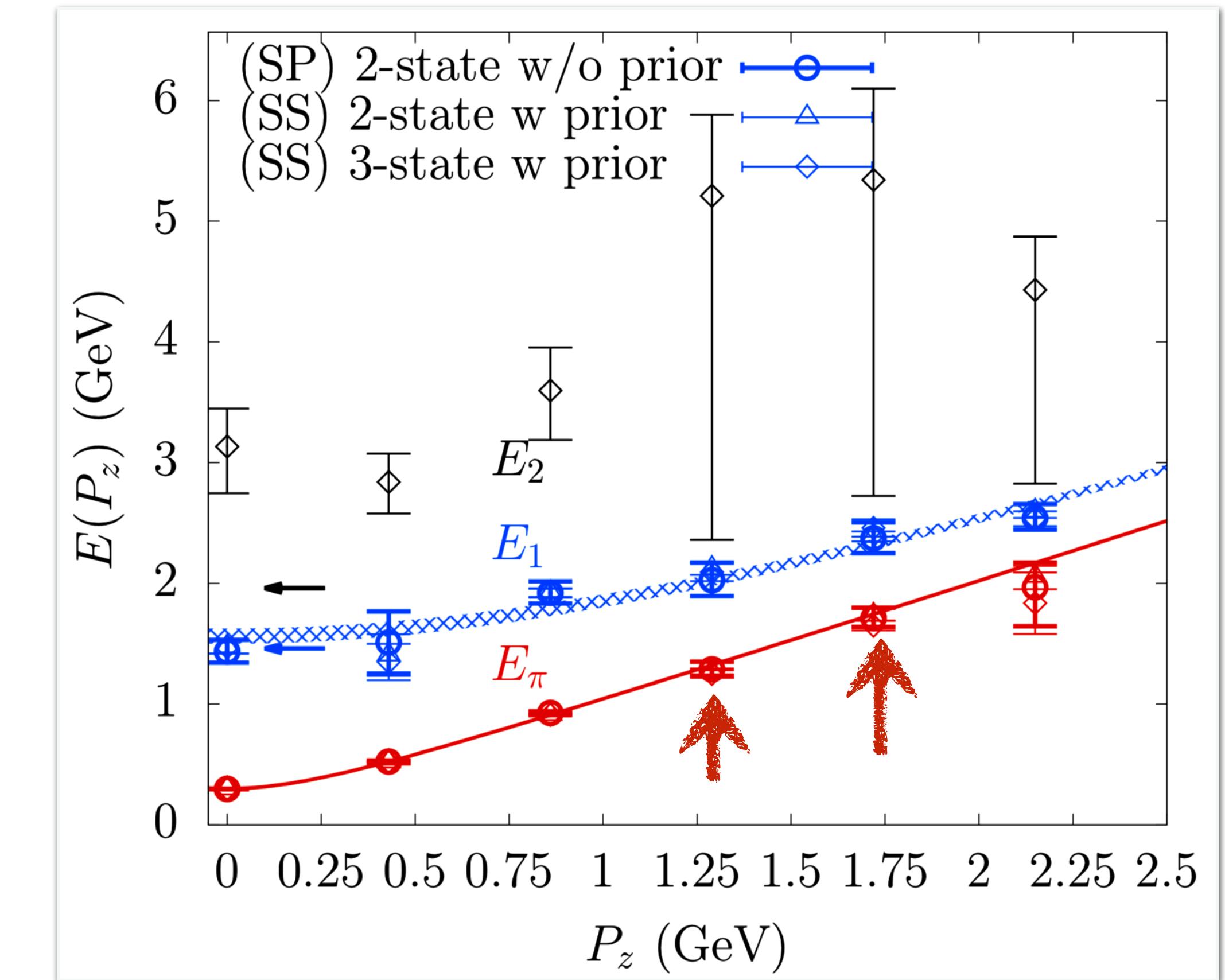
universal for all hadrons, presently only 1-loop

Stewart, Zhao: Phys. Rev. D 97, 054512 (2018)

chasing after pions on the light-cone

- Wilson-Clover valance quarks: $m_\pi^{val} = 300$ MeV
- boosts: $p_z = 1.29, 1.72$ GeV
- lattice spacing: $a = 0.06$ fm
- lattice size: 3.8×2.9^3 fm⁴, $m_\pi^{val} L = 4.4$
- 2+1 flavor HISQ HotQCD gauge configurations:
 $m_\pi^{sea} \simeq 160$ MeV, $m_K^{sea} \simeq 500$ MeV

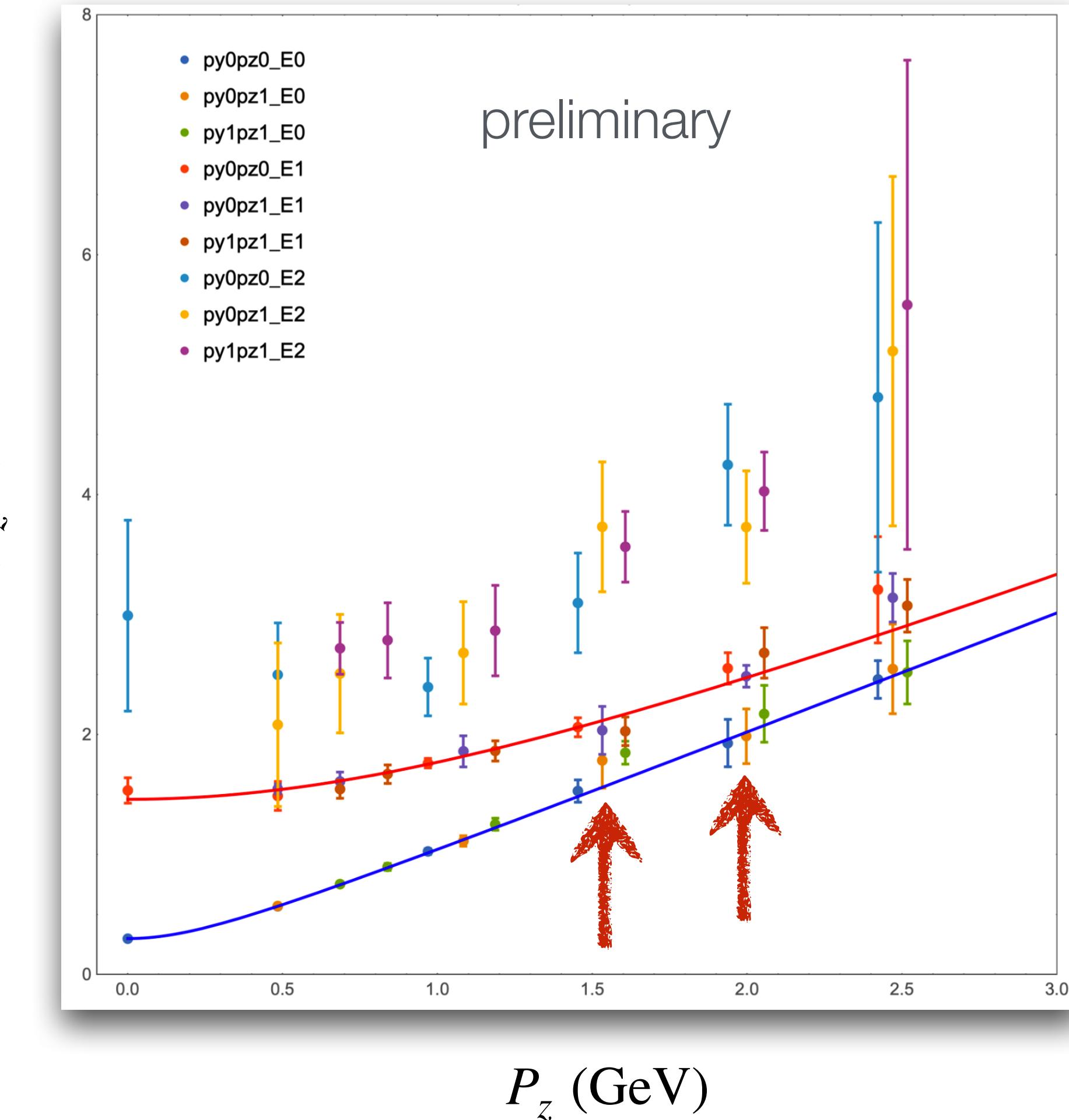
energy levels of boosted pion



BNL-SBU (Nikhil Karthik): Phys.Rev. D100, 034516 (2019)

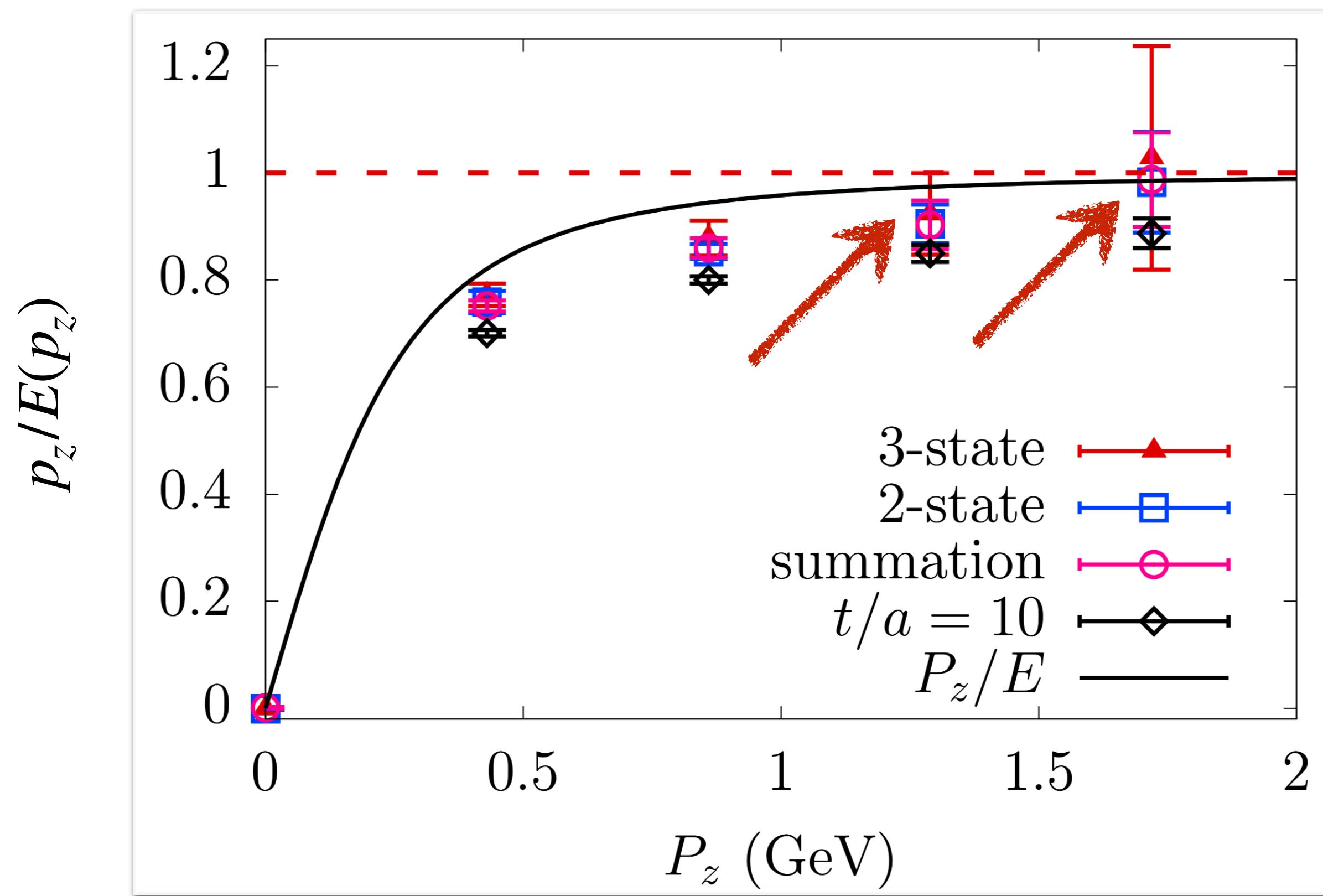
energy levels of boosted pion

- Wilson-Clover valance quarks: $m_\pi^{val} = 300$ MeV
- boosts: $p_z = 1.45, 1.93$ GeV
- lattice spacing: $a = 0.04$ fm
- lattice size: 2.6^4 fm 4 , $m_\pi^{val}L = 4$
- 2+1 flavor HISQ HotQCD gauge configurations:
 $m_\pi^{sea} \simeq 160$ MeV, $m_K^{sea} \simeq 500$ MeV

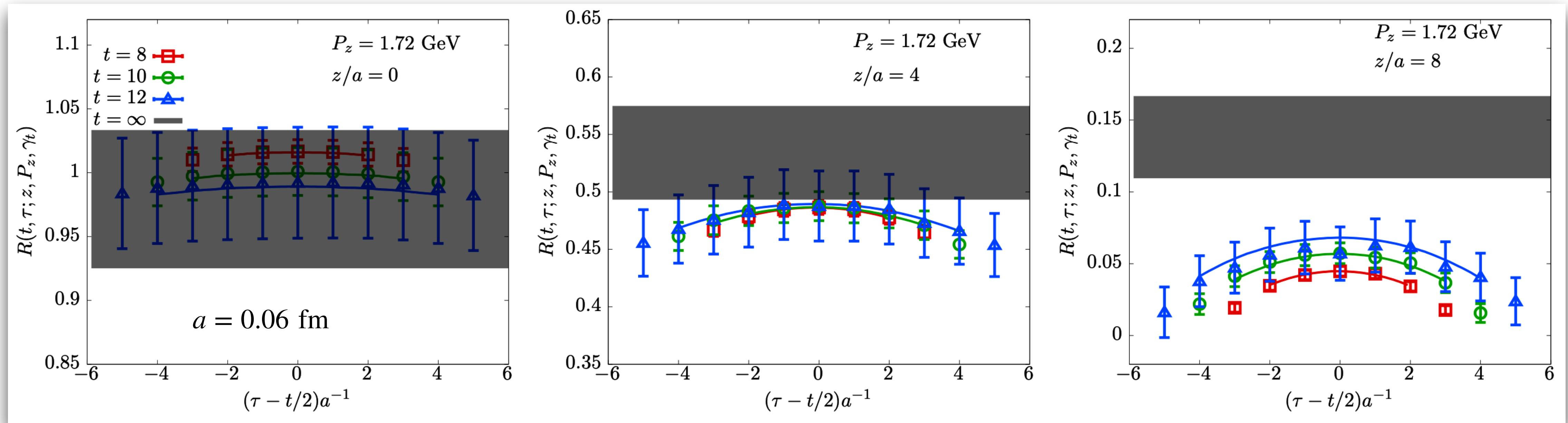


BNL-SBU-Tsinghua (Xiang Gao): on-going

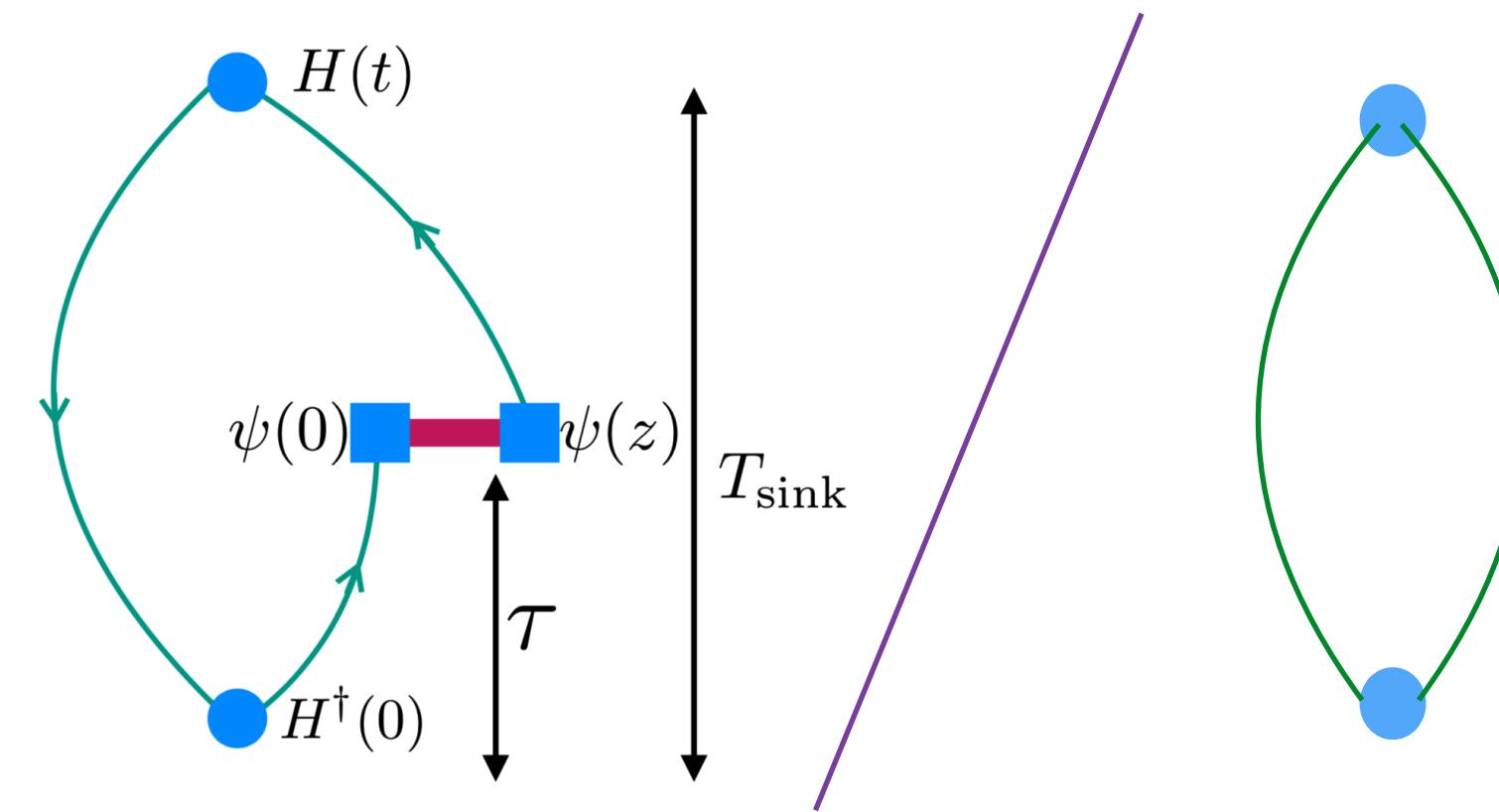
pion almost on the light-cone



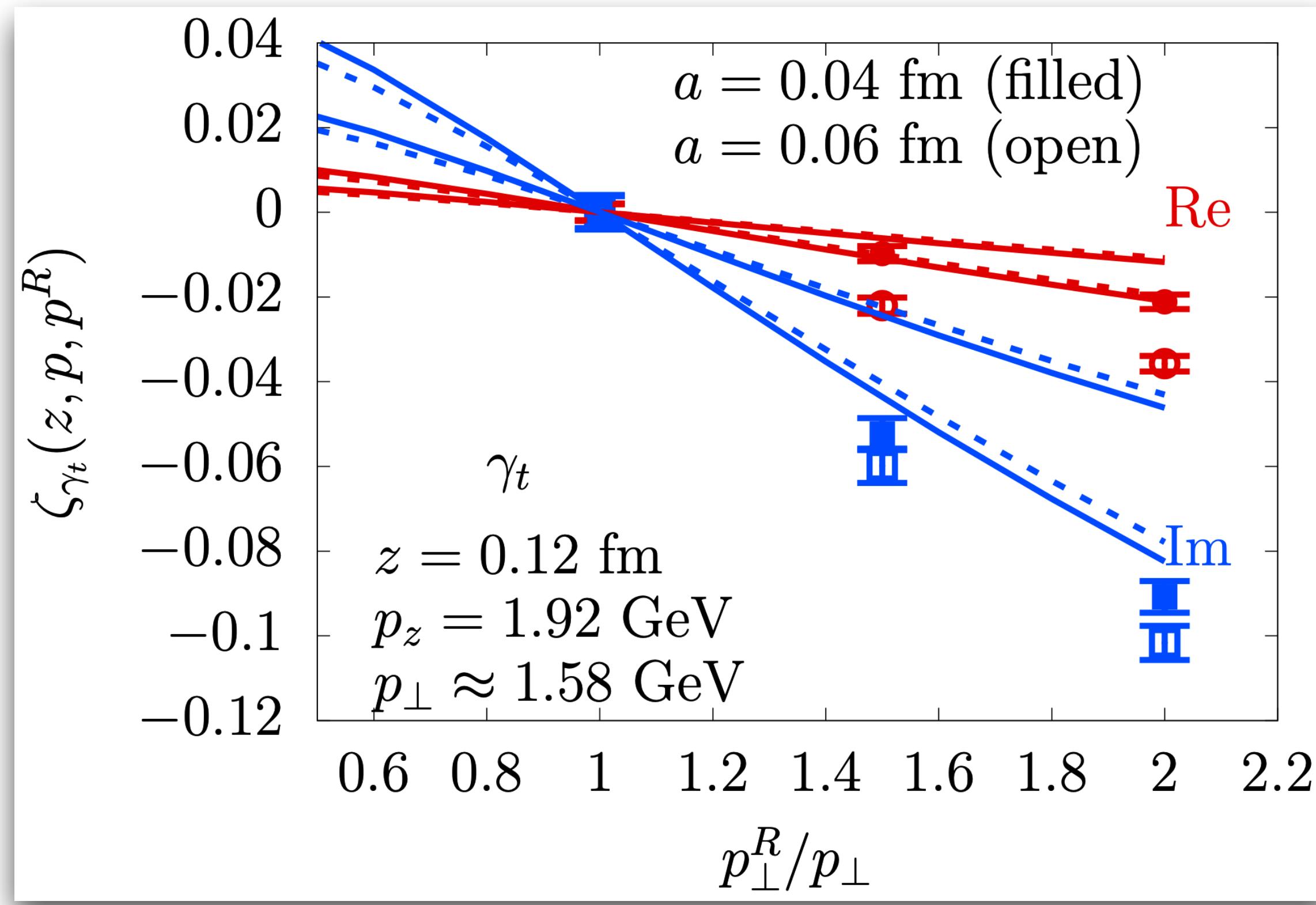
bare 3-pt operators in z-space



ratio of 3-pt to 2-pt corr. function



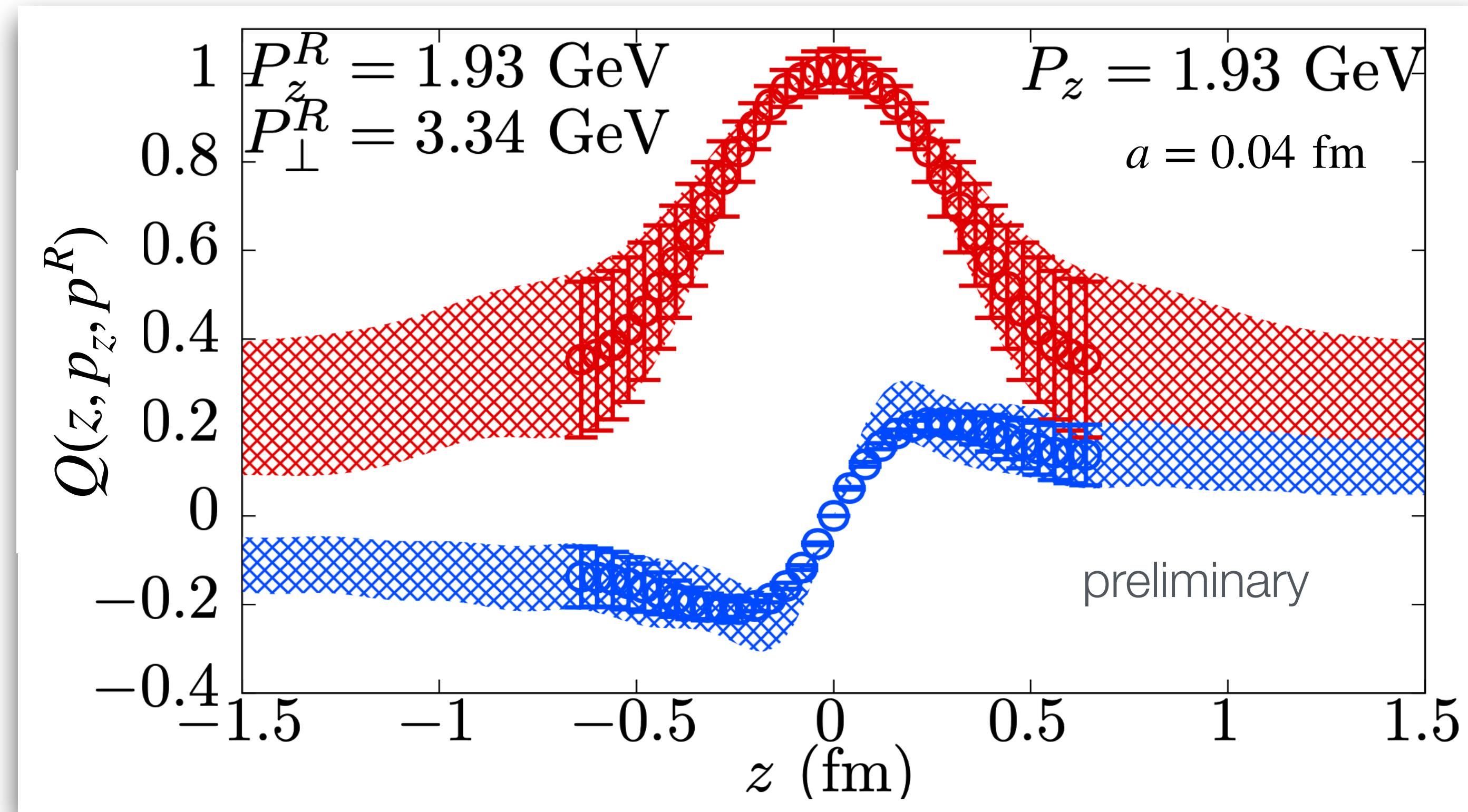
scale dependence of non-perturbative renormalization factor



real part, imaginary part; lines: 1-loop pQCD

$$\xi_{\gamma_t}(z, p, p^R) = Z_{\gamma_t}(z, p^R)/Z_{\gamma_t}(z, p) - 1 \simeq \partial \ln[Z_{\gamma_t}(z, p)]/\partial p|_{p^R}$$

renormalized 3-pt operator in z-space



real part, imaginary part

Fourier transform w.r.t. z



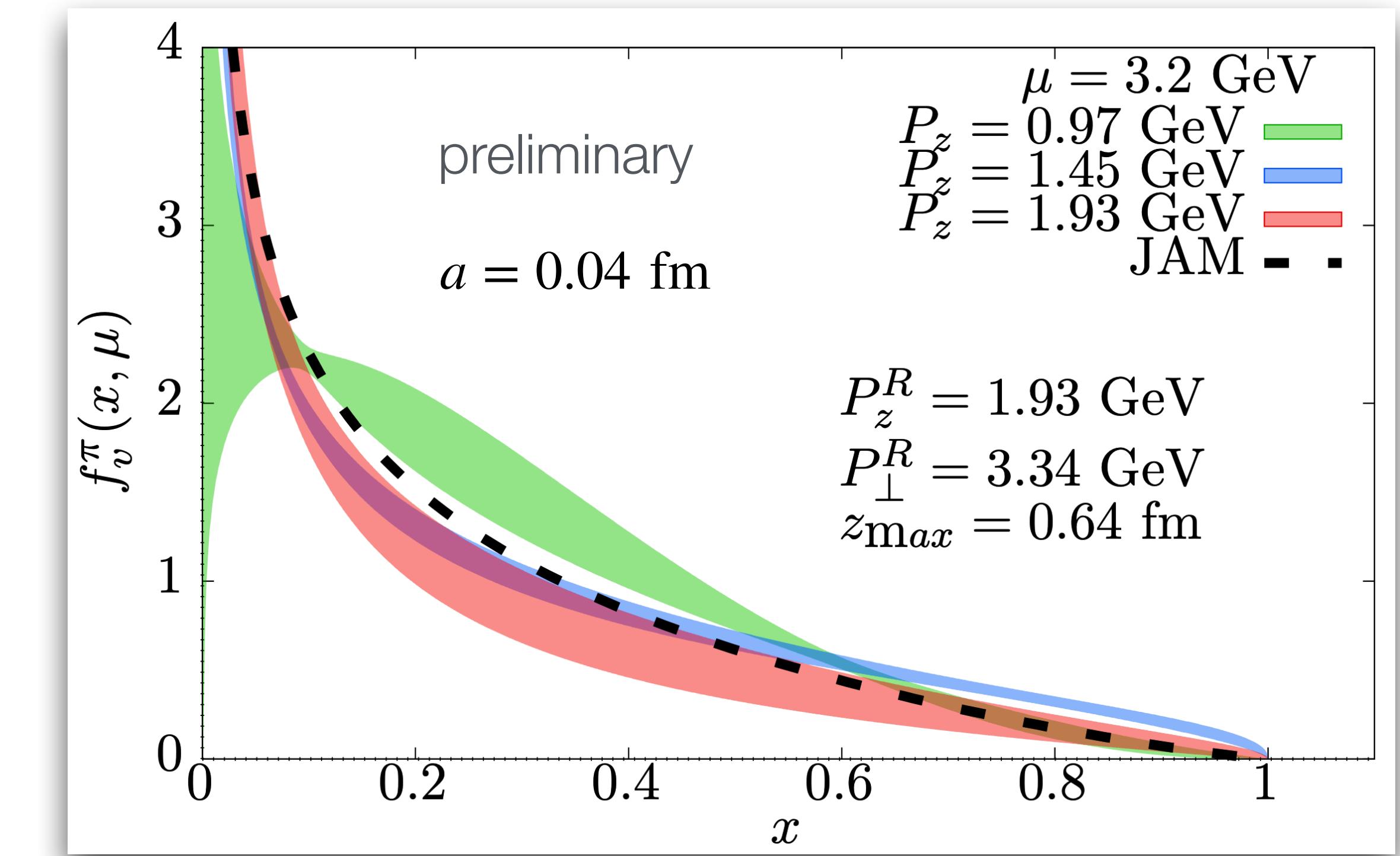
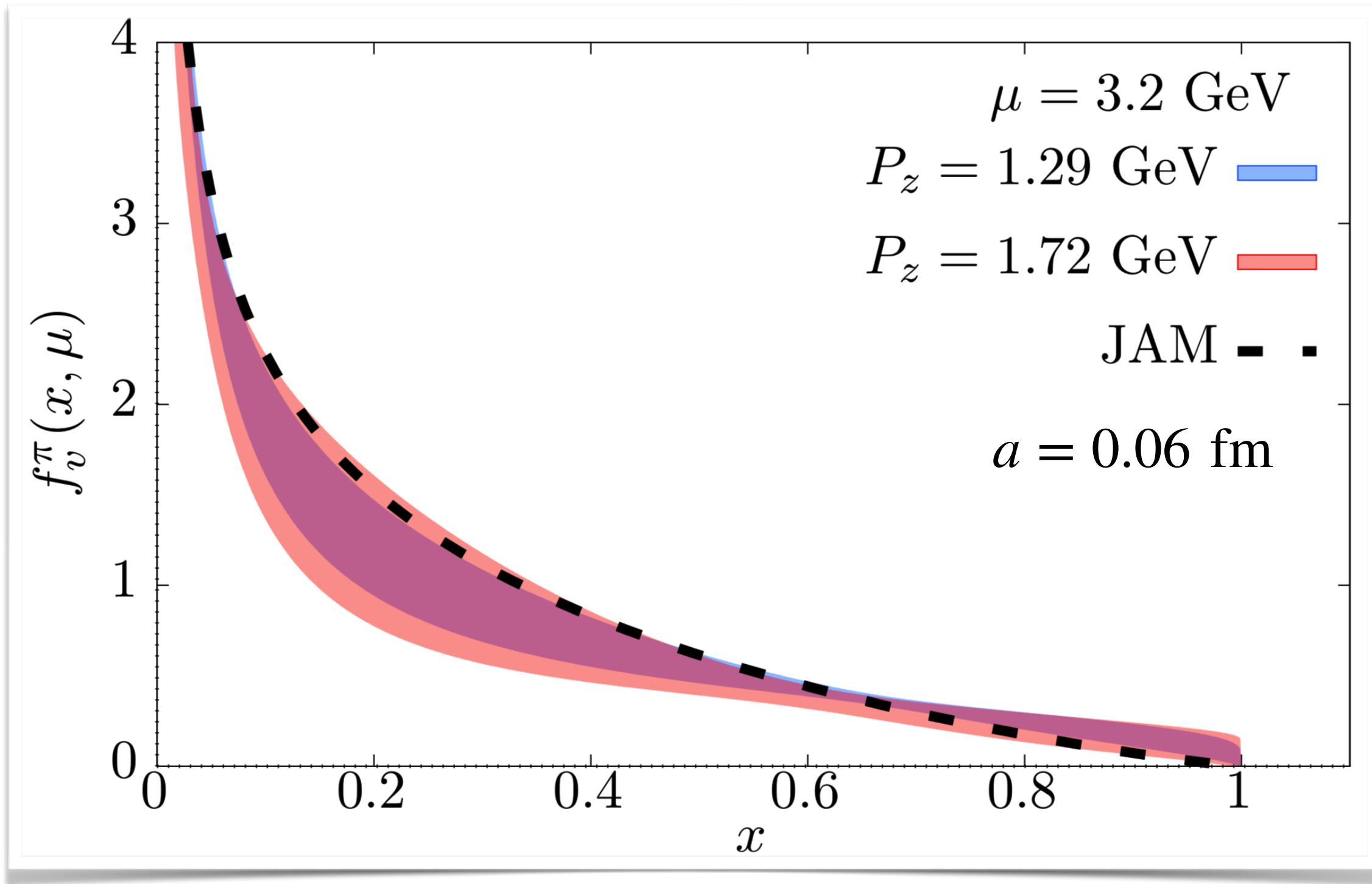
RI-MOM qPDF

pQCD matching



$\overline{\text{MS}}$ PDF

valance quark PDF of pion



BNL-SBU: Phys.Rev. D100, 034516 (2019)

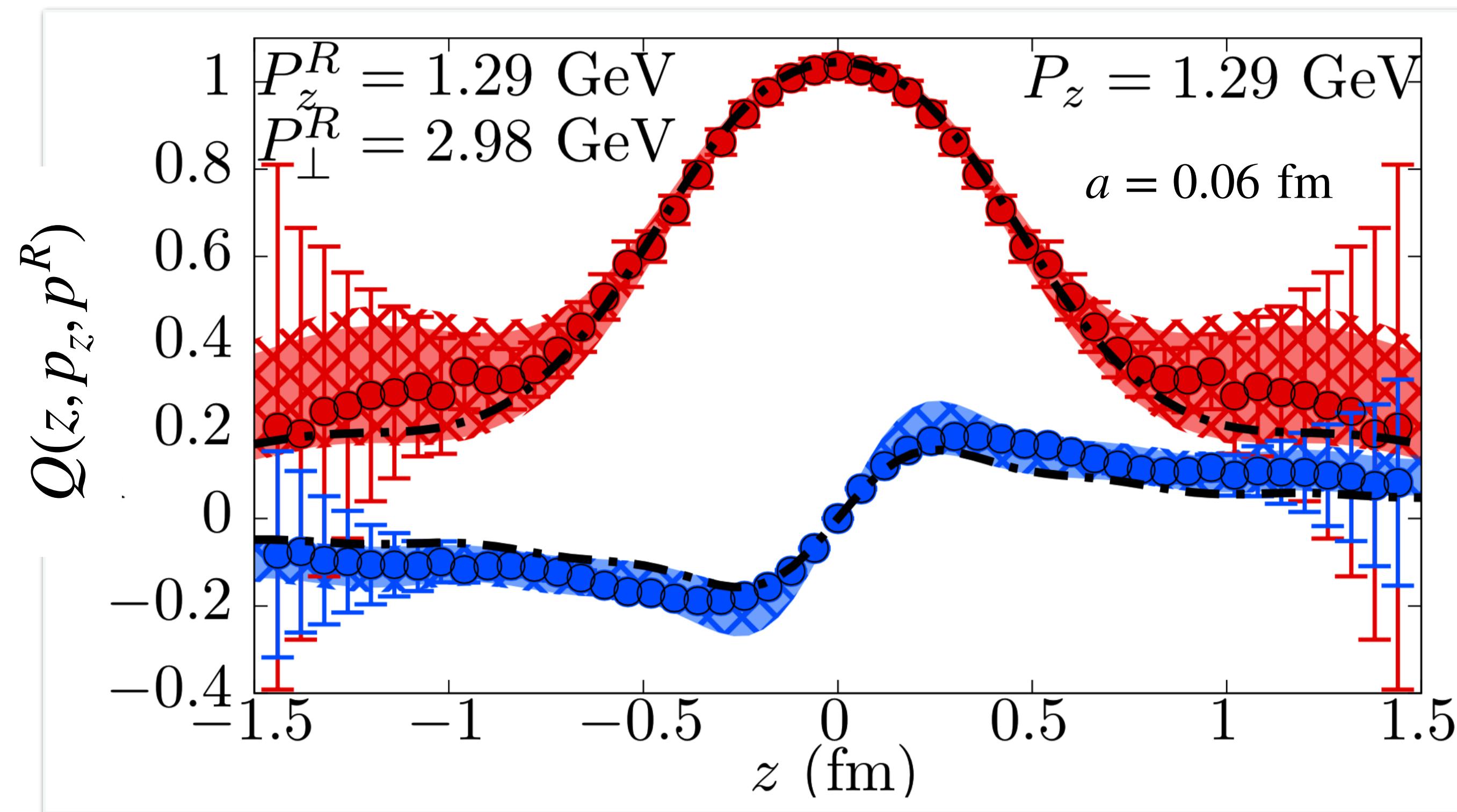
$$f_v^\pi(x) = f_u^\pi(x) - f_d^\pi(x) \dots 0 < x < 1$$

BNL-SBU-Tsinghua: on-going

$$2\langle x \rangle = 0.43(6)$$

$$\text{JAM: } 2\langle x \rangle = 0.437$$

comparison with JAM in z-space



real part, imaginary part

dashed black lines: reconstructions from JAM global fits by reversing the procedure

an alternative angle: Ioffe-time distribution

- pseudo Ioffe-time distribution: Lorentz invariant

Radyushkin: Phys. Rev. D96, 034025 (2017)

$$M_b(z^2, \nu) = \langle H(p_z, 0) | \bar{\psi}(0, \tau) \gamma_0 W(0, z) \psi(z, \tau) | H(p_z, T_{\text{sink}}) \rangle$$

$$\nu \equiv z p_z$$

- approach to light-cone: $z^2 \rightarrow 0$, $\nu = \text{fixed}$

$$\lim_{z^2 \rightarrow 0} M(z^2, \nu) \rightarrow M(x, \mu) = \int_{-1}^1 C(\mu, z) f(x, \mu) e^{ix\mu} d\nu$$

- reduced Ioffe-time distribution:

$$rITD(z^2, \nu) = \frac{M_b(z^2, \nu)}{M_b(z^2, 0)} = \sum_{n=0}^{\infty} \frac{(-i\nu)^n}{n!} \frac{C_n(\mu^2 z^2)}{C_0(\mu^2 z^2)}$$

↔

moments of $\overline{\text{MS}}$ PDF

renormalized

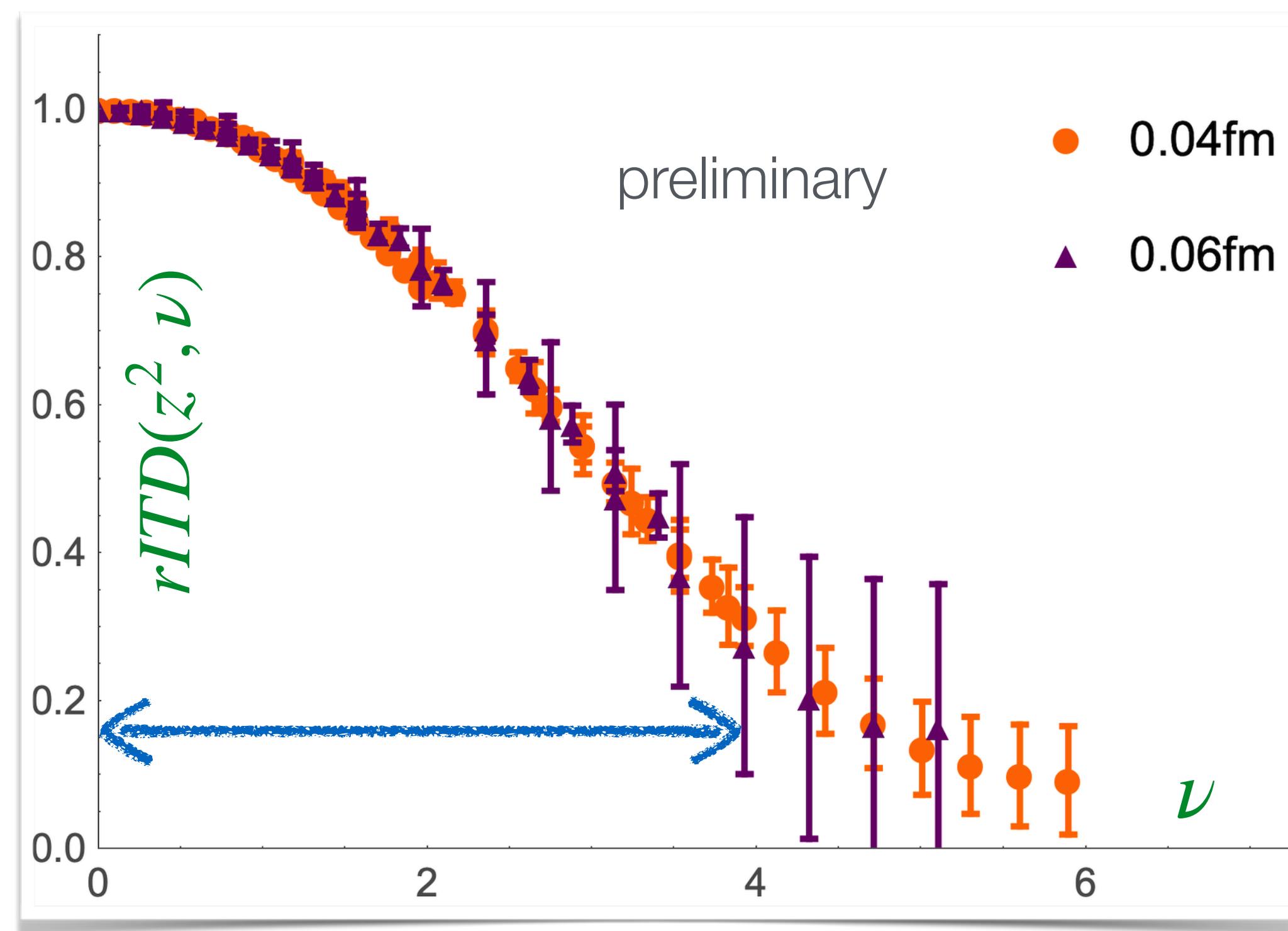
perturbative, 1-loop

target-mass, higher-twist

$\langle x^n \rangle(\mu) + \mathcal{O}(z^2 m_H^2, z^2 \Lambda_{\text{QCD}})$

Izubuchi et. al.: Phys. Rev. D98, 056004 (2018)

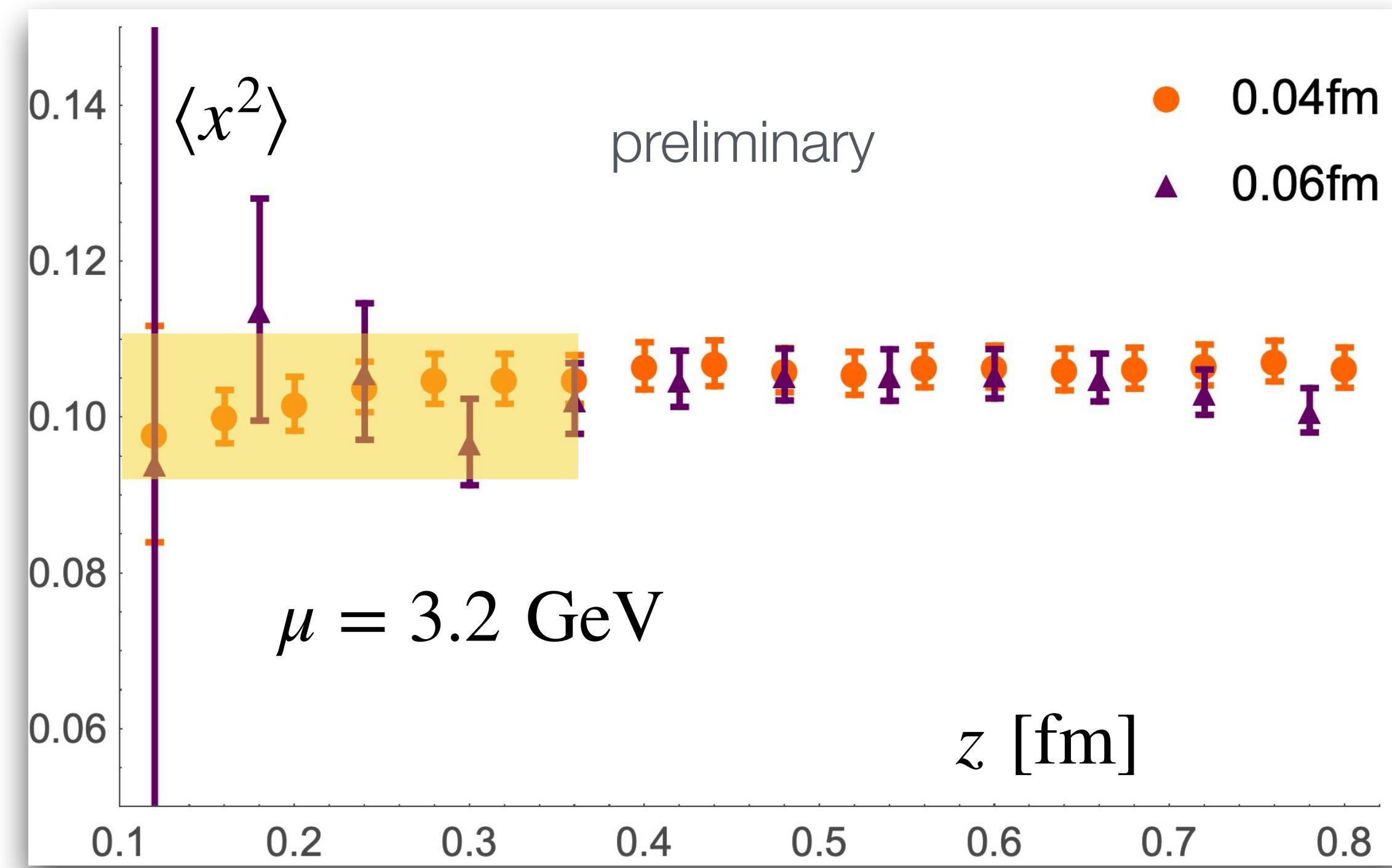
reduced Ioffe-time distribution



joint fit to all z, p_z $\nu = zP_z = 0 - 4$

$$rITD(zp_z, z^2) = \sum_{n=0}^{\infty} \frac{(-izp_z)^n}{n!} \frac{C_n(\mu^2 z^2)}{C_0(\mu^2 z^2)} \langle x^n \rangle(\mu)$$

2nd moment of $f_\nu^\pi(x)$

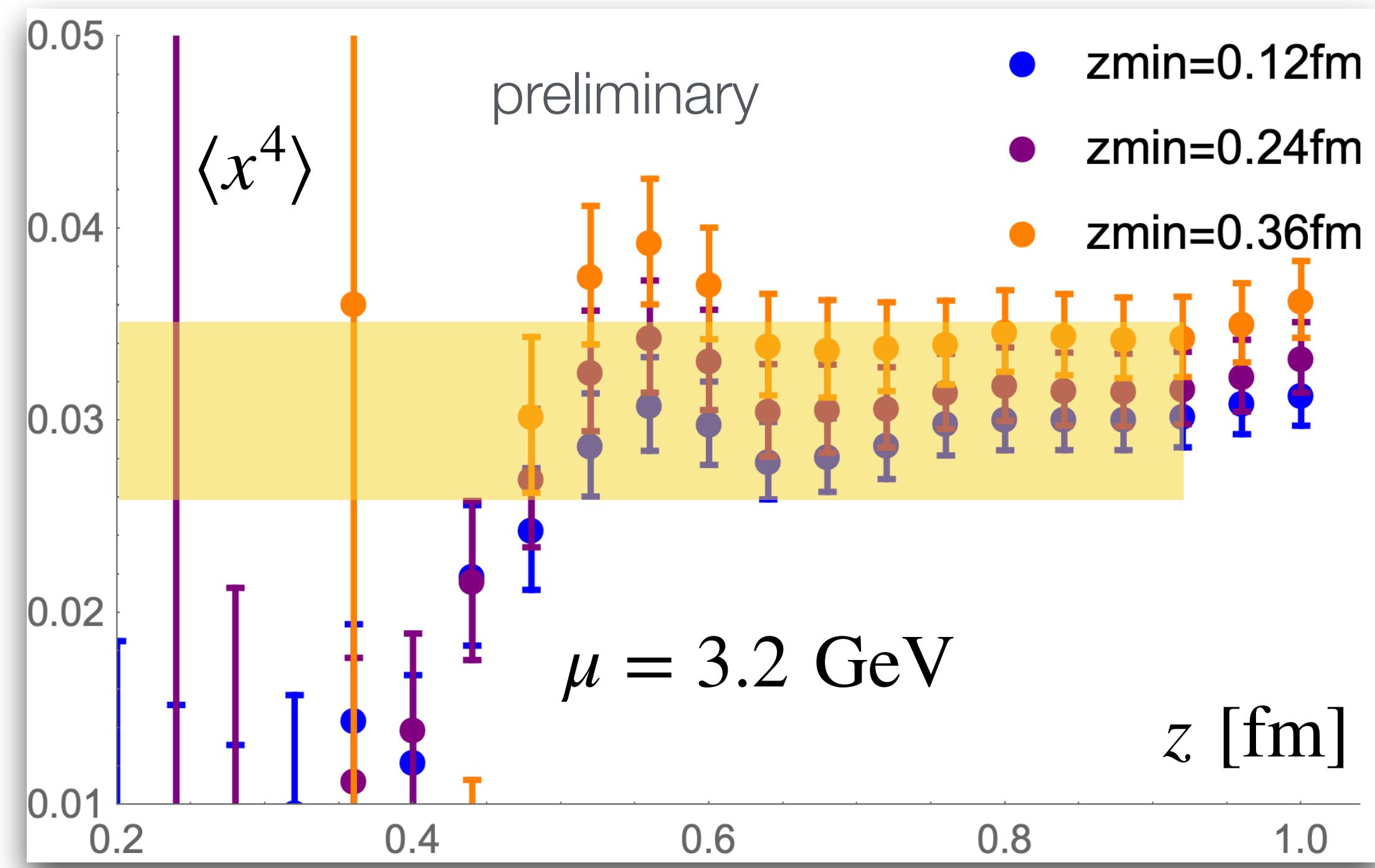


BNL-SBU-Tsinghua, on-going

$$\langle x^2 \rangle \simeq 0.10(1)$$

JAM: $\langle x^2 \rangle = 0.095$

4th moment of $f_\nu^\pi(x)$



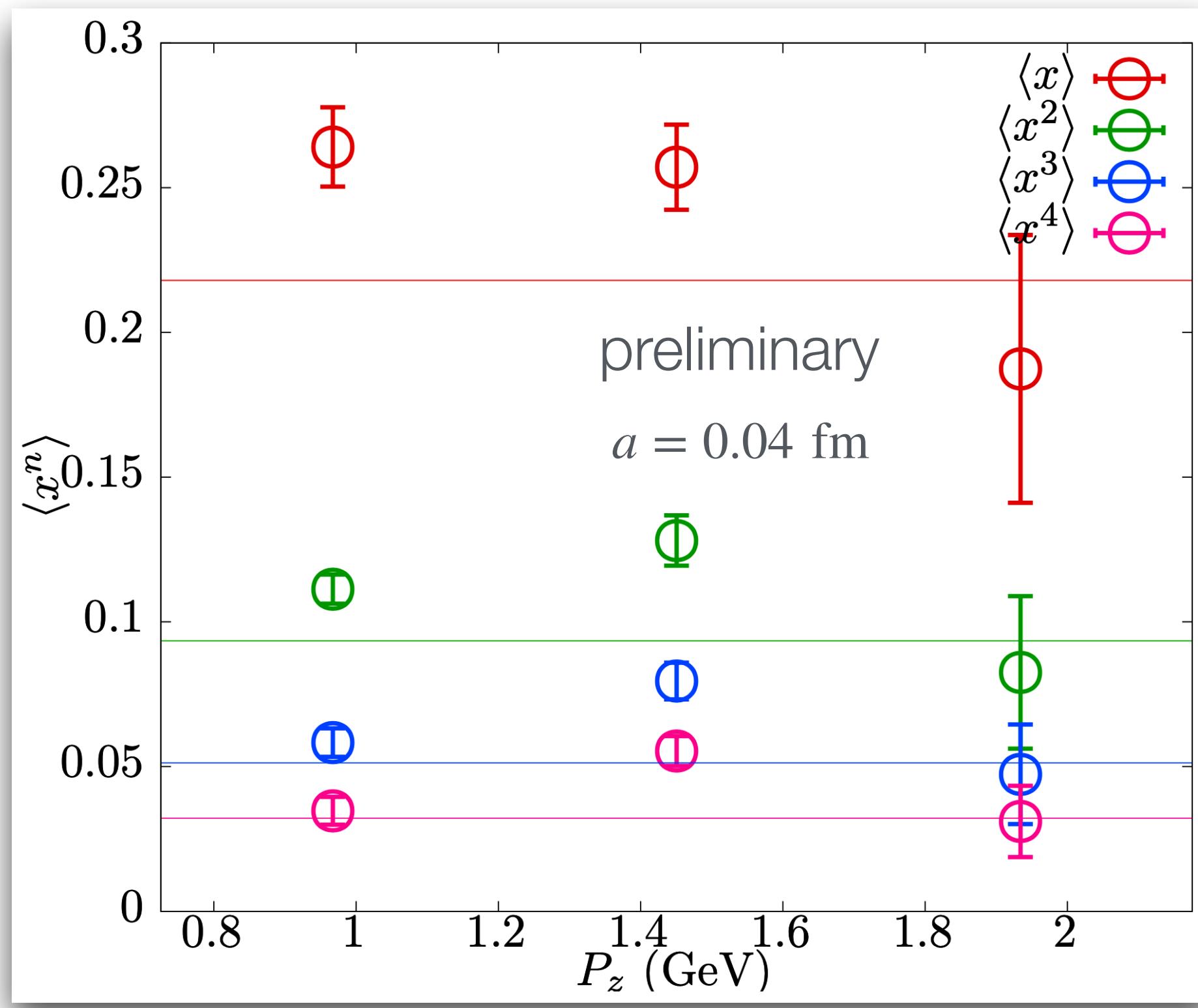
BNL-SBU-Tsinghua, on-going

$$\langle x^4 \rangle \simeq 0.030(5)$$

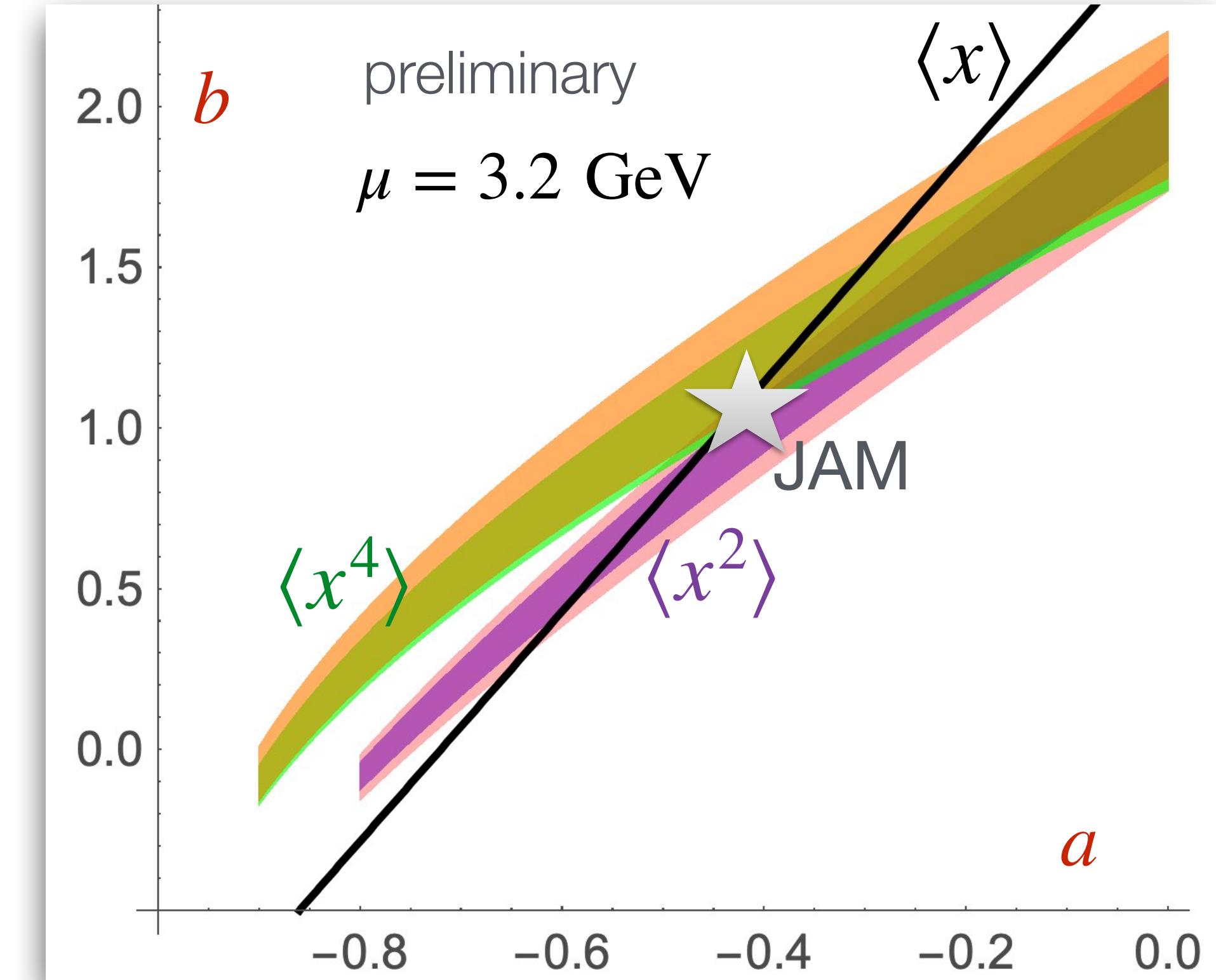
$$\text{JAM: } \langle x^4 \rangle = 0.032$$

instead of a summary ... the shape of $f_\nu^\pi(x)$

$$f_\nu^\pi(x) = Ax^a(1-x)^b$$



from quasi-PDF



from pseudo-ITD

BNL-SBU-Tsinghua, on-going