Quarkonia in the QGP

Rishi Sharma

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Plan of the talk

- Introduction
- A model for quarkonia at high p_T *[Aaronson, Borras, Odegard, Sharma, Vitev, PLB 778 (2018)]*
- Quarkonium propagation in the QGP using an open quantum system approach *[Sharma, Tiwari (1912.07036)]*

Overview

System

- A $Q\bar{Q}$ ($c\bar{c}$ or $b\bar{b}$) state moving in the QGP is affected by the thermal medium as it propagates
- The temperature T of the medium decreases with time
- The observable is $R_{AA} = \frac{N_{\mathrm{meson}}(AA)}{N_{\mathrm{bin}} N_{\mathrm{meson}}(pp)}$
- The goal is to learn about the medium (for example its response functions) by comparing our models to experiment

Separation of scales

- Can obtain rough estimates of energy scales by assuming that the states are so small in size that the short distance part of the potential (Coulomb force) dominates for these states
- Then $v \sim \alpha(m_Q v)$ is the relative velocity of Q and $\bar{Q}$
- Inverse size $1/r \sim m_Q v$
- $E_b \sim m_Q v^2$
- Finally, the non-perturbative scale Λ_{QCD}
- If v is small, $m_Q \gg q \gg E_b \gg \Lambda_{QCD}$

Separation of scales

- For the lowest bound states one obtains by solving the Schrödinger equation
- Bottomonia:
  - $m_b \sim 4.5\text{GeV}$
  - $1/r \sim 1\text{GeV}$
  - $E_b \sim 0.5\text{GeV}$
- Charmonia:
  - $m_c \sim 1.34\text{GeV}$
  - $1/r \sim 0.6\text{GeV}$
  - $E_b \sim 0.5\text{GeV}$
- Furthermore, excited states have larger sizes and higher binding energies

Heavy ion collisions and competing scales

- In heavy ion collisions a quark gluon plasma (QGP) is created for a short duration of roughly 10fm$/c$ time
- There are additional scales at finite T
  - In the medium, additional energy scales, temperature T, Debye screening length m_D
  - $T \sim 200 - 400$MeV and m_D is comparable (if not larger)
  - For Bottomonia in particular T is not very different from E_b but T is still sufficiently less than $1/r$
  - Additional time scales: dissociation and formation
  - An estimate for the formation time $t_{\mathrm{form.}} \sim 1/E_b$

Usual paradigm

- Formation is handled in NRQCD *[Bodwin, Braaten, LePage (1994)]*

$$d\sigma(ij \to \mathrm{meson}+X)(p_T) = \sum_n d\sigma(ij \to Q\bar{Q}[n]+X')(p_T)\langle\mathcal{O}[n]\rangle \ ,$$

- $\langle\mathcal{O}[n]\rangle$ are LDMEs
- Assume that production in AA is not affected by the Quark Gluon Plasma (QGP)
- Propagation leads to dissociation

Initial state effects

- Even assuming the LDMEs are the same as in *pp* collisions, production can be affected due to Cold Nuclear Matter (CNM) effects
- Can be constrained using *pA* collisions
- A rigorous framework to analyze these is the paradigm of gluon saturation *[Ma, Venugopalan, Zhang (2015, 2018); ..]*
- Other approaches
  1. Cold Nuclear Energy loss *[Vitev, Goldman, Johnson, Qiu (2006); ..], [Arleo, Peigne (2014);..]*
  2. Saturation and transverse momentum *[Vogt (2015)];..*
  3. Modified nuclear PDFs *[Eskola et. al. (2009), EPS 09; Kovnak et. al. (2016), nCTEQ15; Lansberg, Shao (2017)];..*

Initial state effects

- In this talk I'll be interested in J/ψ and Υ at $y = 0$ and large p_T ($\gtrsim$ 5GeV)
- For these kinematics pPb data is available albeit with substantial errors
- Both ATLAS *[1505.08141, 1709.03089]* and CMS *[1702.01462]* suggest that R_{pA} is above 1 at the level of $5 - 10\%$ but consistent with being 1
- Here we will ignore CNM effects in the production of the quarkonia
- The pp yields, therefore give the initial configuration of the evolution of $Q\bar{Q}$ in the QGP

LDMEs

- The second consideration in the initial state is the quarkonium production mechanism
- We used LDMEs fitted to data from $p_T \sim 5$GeV onwards
- These are famously incompatible with polarization measurements
- Updated by *[Chao et. al. (2012); Bodwin et. al. (2014)]* focussing on high p_T (also see *[Bain et. al. (2017)]*)
- For low p_T see *[Ma, Stebel, Venugopalan (2018); Baccetta et. al. (2018)]*
- In our model, where we start the evolution after the formation of the bound state ($t \sim 1/E_b$), this does not make a (substantial) difference. However, if one wants to start the evolution from the hard process ($t \sim 1/M$), the production mechanism will make a difference

A simple model for high p_T

A simple model for high p_T quarkonia

- We start with the initial state in the vacuum form, assuming the initial formation is not strongly modified
- The formation dynamics can not be handled rigorously: We assume that formation happens on a time scale τ_{form} which we vary from $1-1.5$fm
- The thermal medium screens the interaction between the Q and the $\bar{Q}$. This screening can be obtained from lattice QCD measurements
- Interactions with the thermal gluons in the medium can lead to dissociation which is an added effect to screening

$Q\bar{Q}$ state

- For a boosted object the dynamics are predominantly transverse
- Write the light cone wavefunction

$$
|\vec{P}^+\rangle = \int \frac{d^2\mathbf{k}}{(2\pi)^3} \frac{dx}{2\sqrt{x(1-x)}} \frac{\delta_{c_1c_2}}{\sqrt{3}} \psi(x,\mathbf{k}) \\
\times a_Q^{\dagger\, c_1}(x\vec{P}^+ + \mathbf{k}) b_{\bar{Q}}^{\dagger\, c_2}((1-x)\vec{P}^+ - \mathbf{k})|0\rangle \ ,
$$

- $$
\psi(x,\mathbf{k}) = \mathrm{Norm} \times \exp\left(-\frac{\mathbf{k}^2 + m_Q^2}{2\Lambda^2(T)x(1-x)}\right)
$$

- Λ is related to the width of the wavefunctions in momentum space *[Adil, Vitev (2007)]*

Dissociation rate

- We use a simple model for dissociation: the transverse momentum broadening of high p_T particles *[BDMPS, GLV, Wiedemann, HT...]*
- The Q and $\bar{Q}$ get kicks to the relative transverse momentum $\mathbf{k}$ thus modifying the light cone wavefunction as the $Q\bar{Q}$ propagates in the medium: $\mathbf{k}^2 \to \mathbf{k}^2 + \Delta\mathbf{k}^2$
- The distribution of the transverse kicks is

$$\frac{dP(\Delta k^2)}{d\Delta k^2} \propto e^{-\Delta k^2/(\chi \mu_D^2 \xi)}$$

  where $\chi \mu_D^2 \xi$ is the analog of $\hat{q}L$
- $P_{\mathrm{surv}}(t) = |\langle \Psi_T(t) | \Psi_T(0) \rangle|^2$
- $\frac{1}{t_{\mathrm{diss.}}} = -\frac{1}{P_{\mathrm{surv}}(t)} P_{\mathrm{surv}}(t)$

Rate equations

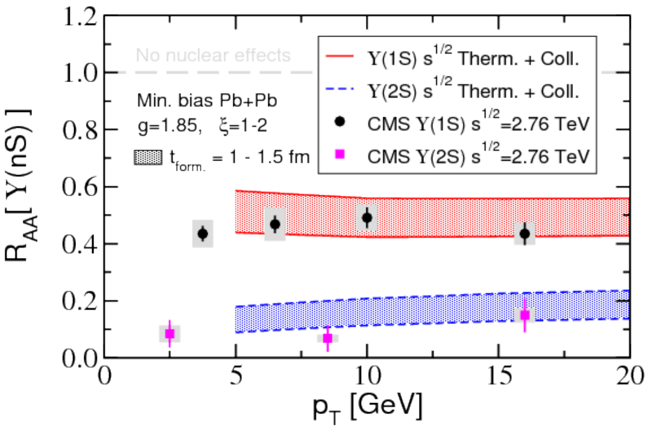
- We have all the ingredients to find the p_T differential yields
- Rate equations

$$\frac{d}{dt}\left(\frac{d\sigma^{\text{meson}}(t;p_T)}{dp_T}\right) = \frac{1}{t_{\text{form.}}}\frac{d\sigma^{Q\bar{Q}}(t;p_T)}{dp_T} - \frac{1}{t_{\text{diss.}}}\frac{d\sigma^{\text{meson}}(t;p_T)}{dp_T}$$

-

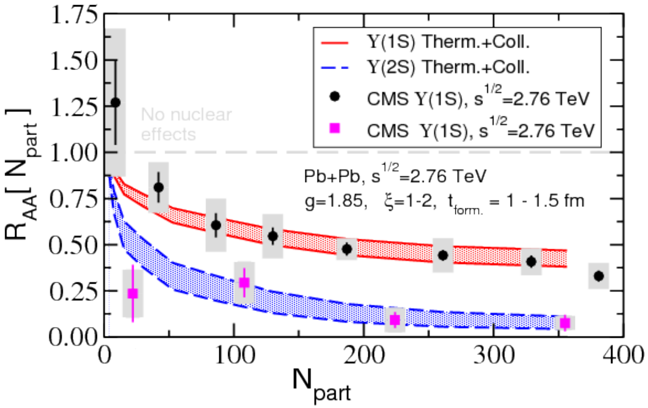
$$\frac{d}{dt}\left(\frac{d\sigma^{Q\bar{Q}}(t;p_T)}{dp_T}\right) = -\frac{1}{t_{\text{form.}}}\frac{d\sigma^{Q\bar{Q}}(p_T)}{dp_T}$$

$R_{AA}(\Upsilon)$



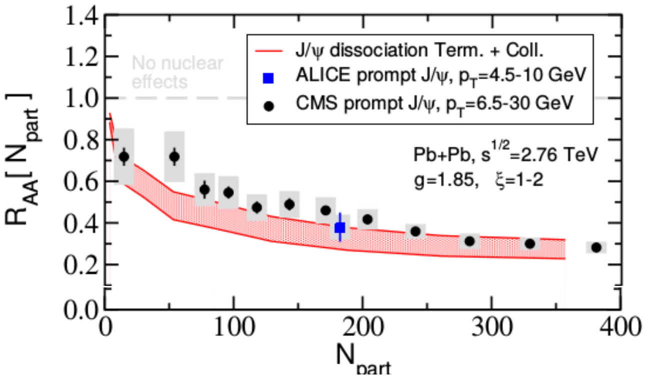
[Aaronson, Borras, Odegard, Sharma, Vitev (2017)] Both screening and dissociation

$R_{AA}(\Upsilon)$



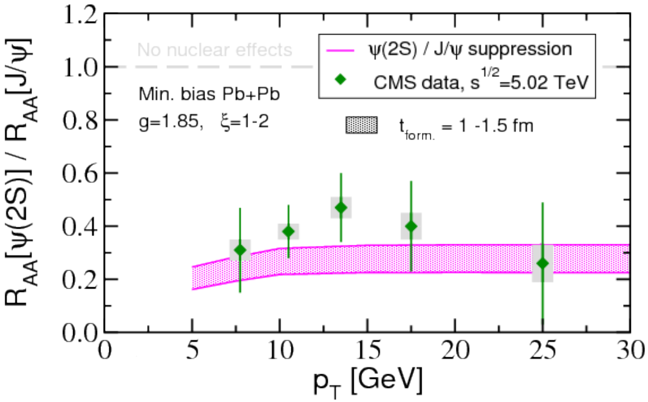
Υ(1S) Therm.+Coll.
Υ(2S) Therm.+Coll.
CMS Υ(1S), s^1/2=2.76 TeV
CMS Υ(1S), s^1/2=2.76 TeV
No nuclear effects
Pb+Pb, s^1/2=2.76 TeV
g=1.85, ξ=1-2, t_form. = 1 - 1.5 fm
R_AA[N_part]
N_part
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
0
100
200
300
400

$R_{AA}(J/\psi)$



J/ψ dissociation Term. + Coll.
ALICE prompt J/ψ, p_T=4.5-10 GeV
CMS prompt J/ψ, p_T=6.5-30 GeV
No nuclear effects
Pb+Pb, s^1/2=2.76 TeV
g=1.85, ξ=1-2
R_AA[N_part]
N_part
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
100
200
300
400

$R_{AA}(\psi(2S))/R_{AA}(J/\psi)$



Summary

- Positives
  1. A realistic background medium
  2. Feed-down contributions
  3. Screening as well dissociation included
- Negatives
  1. Main systematic uncertainty due to $t_{\mathrm{form.}}$
  2. Color dynamics not incorporated
  3. Using a rate equation instead of a quantum evolution

Quantum evolution

Competing processes

- Consider a simple setting: a quarkonium state at rest in a thermal medium
- Affected by various processes like
  1. Screening
  2. Gluo-dissociation *[Bhanot, Peskin (1979)]*
  3. Landau damping *[Laine et. al. (2007), Beraudo, Blaizot, Ratti (2007); Brambilla et. al. (2008)]*
- Can all these processes be analyzed in a single framework?
- Screening maintains coherence of the $Q\bar{Q}$ wavefunction while damping and dissociation do not, as energy is lost to the medium

Rate equations

- Several models *Rapp et. al.*, *Aaronson et. al.*, *Strickland et. al.* use rate equations to calculate quarkonium phenomenology

$$\frac{dN_\psi(t)}{dt} = -\Gamma_\psi(t) N_\psi(t)$$
$$R_{AA} = e^{-\int_0^t dt' \Gamma(t')}$$

- The input to the calculation is the decay rate Γ_ψ

Rate equations

- $$\Gamma = \sum_f |\langle f|O|i\rangle|^2$$
- For example, for the process of gluo-dissociation, $|f\rangle$ stand for octet states and O for $\vec{r}\cdot\vec{E}^a$
- The usual set up for perturbation theory: separate the Hamiltonian into H_0 and a perturbation O. H_0 is time independent
- Γ unambiguously gives the decay rate for a state $|i\rangle$ (1) to lowest order in perturbation theory (2) assuming that the un-perturbed Hamiltonian is time independent

Rate equations

- If the un-perturbed Hamiltonian is itself time dependent, the interpretation of Γ unclear
- Hamiltonian itself can lead to transition. Ambiguity in bases
- Choice of the state $|i\rangle$

Two limiting cases

- If $\frac{1}{E_b}|\langle|\frac{dH_0}{dt}|\rangle| \gg 1$
- The change in H_0 is fast: sudden approximation
- $|i\rangle$ can be taken to be the vacuum wavefunction

Two limiting cases

- If $\frac{1}{E_b}|\langle\Psi|\frac{dH_0}{dt}|\Psi\rangle| \gg 1$
- The change in H_0 is fast: sudden approximation
- $|i\rangle$ can be taken to be the vacuum wavefunction
- If $\frac{1}{E_b}|\langle\Psi|\frac{dH_0}{dt}|\Psi\rangle| \ll 1$
- The change in H_0 is slow: adiabatic approximation
- $|i\rangle$ can be taken to be the instantaneous eigenstate
- At early time former is better while at late times the latter is better, and for most of the time neither is very good

Density matrix evolution

Density matrix for the $Q\bar{Q}$ system

- Consider the system $Q\bar{Q}$ interacting with the environment, a thermal medium at temperature T
- The total $H = H_{\rm sys} \otimes I_{\rm med} + I_{\rm sys} \otimes H_{\rm med} + H_{\rm int}$
- The initial condition is $\rho(0) = \rho_{\rm sys}(0) \otimes \rho_{\rm med}(0)$
- $i\frac{d\rho}{dt} = [H, \rho(t)]$
- If the coupling g between the $Q\bar{Q}$ and the gluons in the medium is weak, one can perturbatively trace out the thermal medium

$$\rho_{\rm sys}(t) = \mathrm{Tr}[\rho(t)]$$

- This leads to a density matrix evolution for $\rho_{\rm sys}$, which is not-unitary

Progress

- Lindblad equations in weak coupling *[Akamatsu (2013, 2015), Akamatsu et. al. (2017, 2019)]*
- Lindblad equation using NRQCD *[Brambilla et. al (2017, 2018, 2019)]*
- Boltzmann equations from the density matrix formalism. *[Yao, Mehen (2018);Yao, Muller (2018);Yao et. al. (2019) ..]*

Density matrix evolution

- Density matrix equation assuming $g \ll 1$ and $E_b \ll m_D$ derived by *[Akamatsu (2015)]*
-

$$i\frac{\partial}{\partial t}\left[\begin{array}{c}\rho_1\\ \rho_8\end{array}\right]_{(t,\vec{r},\vec{s})} = \left(\frac{-\vec{\nabla}_r^2+\vec{\nabla}_s^2}{M}\right)\left[\begin{array}{c}\rho_1\\ \rho_8\end{array}\right] + (V_r - V_s)\left[\begin{array}{cc}C_{\mathrm{F}} & 0\\ 0 & -1/2N_{\mathrm{c}}\end{array}\right]\left[\begin{array}{c}\rho_1\\ \rho_8\end{array}\right.$$
$$- i\mathcal{D}(\vec{r},\vec{s})\left[\begin{array}{c}\rho_1\\ \rho_8\end{array}\right]$$

-

$$\mathcal{D}(\vec{r},\vec{s}) = 2C_{\mathrm{F}}D(\vec{0}) - (D(\vec{r})+D(\vec{s}))\left[\begin{array}{cc}C_{\mathrm{F}} & 0\\ 0 & -1/2N_{\mathrm{c}}\end{array}\right]$$
$$- 2D\left(\frac{\vec{r}-\vec{s}}{2}\right)\left[\begin{array}{cc}0 & 1/2N_{\mathrm{c}}C_{\mathrm{F}}\,C_{\mathrm{F}} - 1/2N_{\mathrm{c}}\end{array}\right]$$
$$+ 2D\left(\frac{\vec{r}+\vec{s}}{2}\right)\left[\begin{array}{cc}0 & 1/2N_{\mathrm{c}}\\ C_{\mathrm{F}} & -1/N_{\mathrm{c}}\end{array}\right]$$

$D(\vec{r})$

- $D(r)$ appears with an i and leads to a loss of coherence of the $\bar{Q}Q$ state. The trace of the density matrix remains 1 because the heavy quarks do not get lost
- But even if one starts from a pure state, it gets converted to a mixed state
- The function $D(r)$ is related to Landau damping of the exchanged gluons by the thermal medium
- $$D(\vec{r}) = -g^2 T \int \frac{d^3k}{(2\pi)^3} \frac{\pi m_{\mathrm{D}}^2 e^{i\vec{k}\cdot\vec{r}}}{k\left(k^2 + m_{\mathrm{D}}^2\right)^2}$$
- $m_D \sim gT$ is the Debye screening mass
- $D(r)$ decays $\sim e^{-m_D r}$ at large r and hence if $m_D \ll 1/r$ then the quarkonium state is not affected by decoherence. Conversely if $m_D \gtrsim 1/r$, then a significant effect

Markovian approximation

- A strong assumption, $E_b \ll m_D$ needed to obtain the density matrix equation in this form. This ensures on-shell gluons can not be created and the density matrix equation is local in time
- The environment has no memory effects and the density matrix evolution is Markovian
- As mentioned above, the hierarchy between E_b and m_D is not very strong
- Therefore it is worth exploring if we can relax it

Stochastic Schrödinger equation

- The density matrix equation above can be solved using a Stochastic Schrödinger equation
- Introduce noise fields $\theta^a(\vec{r}, t)$ with

$$\langle\langle\theta^a(\vec{r}, t)\rangle\rangle = 0\langle\langle\theta^a(\vec{r}, t)\theta^a(\vec{r}', t)\rangle\rangle = D(\vec{r} - \vec{r}')\delta(t - t')$$

- Assuming that the center of mass is at $\vec{0}$,

$$\begin{aligned} \psi_{t+dt} &= e^{-iH_\theta(t)dt}\psi(t) \\ H_\theta(\vec{r}, t) = & -\frac{\vec{\nabla}_r^2}{M} + V(\vec{r})(t^t \otimes t^{a*}) \\ & +\theta^a(t, \vec{r}/2)\,(t^a \otimes 1) - \theta^a(t, -\vec{r}/2)\,(1 \otimes t^{a*}) \end{aligned}$$

- $\rho(t) = \langle\langle\ |\psi_\theta\rangle\langle\psi_\theta|\ \rangle\rangle$
- Solved for Abelian dynamics in 1 dimension by *[Akamatsu et. al. (2017)]*
- We extend it to 3 dimensions with full color structure *See also [Brambilla et. al. (2018)]*
- We also use a modified equation to include on-shell processes
- In order to do this we first introduce a simplification

Small $\vec{r}$ expansion

- Comparing the energy scales $1/r \ll m_D$ may be a good approximation
- Therefore we expand the noise terms in the equation in $\vec{r}$
- $$H_\theta = \frac{-\nabla^2}{M}(1 \otimes 1) + V(r)(t^a \otimes t^{*,a})$$
  $$+(t^a \otimes 1 - 1 \otimes t^{a*})\frac{\vec{r}}{2} \cdot \vec{\nabla}\vec{\theta}^a(t)$$
  $$+(t^a \otimes 1 + 1 \otimes t^{a*})\theta^a(t)$$
  $$+\mathcal{O}(\vec{r}^2)$$
- $$\langle \theta^a(t)\theta^b(t') \rangle = \delta^{ab}\delta(t-t')D(\vec{0}),$$
  $$\langle \nabla_i\theta^a(t)\nabla_j\theta^b(t') \rangle = \delta^{ab}\delta(t-t')\delta_{ij}\frac{-\nabla^2}{3}D(\vec{0})$$

The chromo-electric field

- $$\nabla^2 D(\vec{0}) = g^2 \int_{-\infty}^{\infty} dt\, \mathrm{Tr}_H \Big\langle W(t;-\infty)^\dagger E_i^a(t) t_H^a W(t;0) \times E_i^b(0) t_H^b W(0;-\infty) \Big\rangle \tag{1}$$
- W's are Wilson lines put to make the definition gauge invariant
- Therefore, the noise field $\nabla\theta$ can be intuitively understood as the chromo-electric field
- A nice way to see it is by rewriting the lagrangian in terms of the singlet (S) and the octet (O) wavefunctions in potential non-relativistic QCD (pNRQCD) *[Brambilla et. al. 2000]*
- $$\mathcal{L}_{pNRQCD} = + \int d^3 r Tr\{S^\dagger [i\partial_0 - h_s] S + O^\dagger [iD_0 - h_o] O + (O^\dagger \mathbf{r} \cdot g\mathbf{E} S) + \frac{1}{2} O^\dagger \{\mathbf{r} \cdot g\mathbf{E}, O\} + \mathcal{O}(\vec{r}^2)\}$$

-

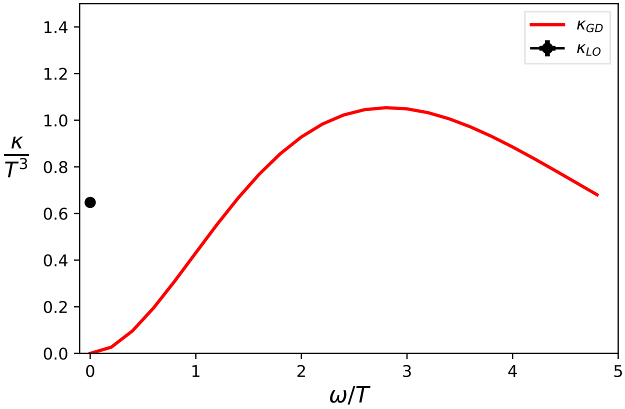
$$\psi(\mathbf{r}, t) = \begin{pmatrix} \psi^S(\mathbf{r}, t) \\ \psi^O(\mathbf{r}, t) \end{pmatrix}$$

- The Hamiltonian can be written in the intuitive form

$$H = \begin{pmatrix} -\frac{\nabla_{\vec{r}}^2}{M} + V_s(r) & g\vec{r} \cdot \vec{E}(t) \\ g\vec{r} \cdot \vec{E}(t) & -\frac{\nabla_{\vec{r}}^2}{M} + V_O(r) + g\vec{r} \cdot \vec{E} \end{pmatrix}$$

- The $\vec{E}$ is calculated at the centre of mass

Noise correlator



- $\kappa = -\frac{g^2}{3}\nabla^2 D(0)$

Final upshot

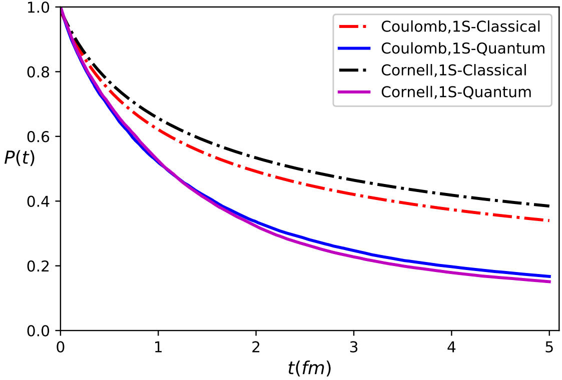
- We start from the stochastic equation above
- In the $m_D \gg E_b$ limit (decoherence), use $\langle\langle g^2 E^a(t) E^a(t') \rangle\rangle = -\delta(t-t') D(\vec{0})$
- To include onshell gluons, i.e. gluo-dissociation $\langle\langle g^2 E^a(t) E^a(t') \rangle\rangle = \frac{g^2 T^4}{\pi^2} \int_0^\infty d\xi \xi^3 \cos(\xi T(t-t')) \frac{1}{e^\xi - 1}$
- With the on-shell gluons, the Markovian nature is lost and noise is correlated over time
- We explore both cases below

Results

Background medium

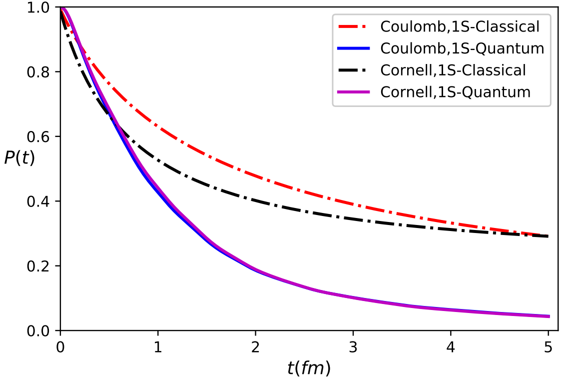
- Bjorken expanding medium
- For only the ground state of Bottomonia for which the Coulombic description might be valid
- Not for a quantitative comparison with data

Suppression for Υ (decoherence)



- *[Sharma, Tiwari; (2019)]*

Suppression for Υ (gluo-dissociation)



- *[Sharma, Tiwari (2019)]*

Some lessons

- Quantum evoution gives larger suppression than the rate equation for both decoherence and gluo-dissociation
- Contribution of gluo-dissociation is as important as the decoherence contribution. Use of the full frequency dependent spectral function to include both effects is underway

Some lessons

- In the calculations we used $g \sim 2$ (motivated by estimates for m_D) *[Kaczmareck et. al. (2004)]*
- Clearly not a perturbative medium
- The pNRQCD formalism provides the framework to go beyond the perturbative framewrok. If we know $\Re e[V_s]$, $\Im m[V_s]$, $\Re e[V_r]$, $\Im m[V_r]$, and $\langle E^a(0) E^b(t, \vec{r}) \rangle$, we will have all ingredients to solve the density matrix evolution equation *[Brambilla et. al. (2019)]*
- In the last year significant progress on the non-perturbative calculation of these quantities using lattice QCD *[Bala, Datta (2019); Larsen et. al. (2019); Burnier, Rothkopf (2016);.....]*

Some lessons

- Connection with phenomenology will also require better control on the initial state: pdfs for pA, AA, and better understanding of the quarkonium production mechanism. EIC promises to provide both