

Probing initial state fluctuations in heavy-ion collisions with power spectrum of flow coefficients

Ajit M. Srivastava

Institute of Physics
Bhubaneswar, India

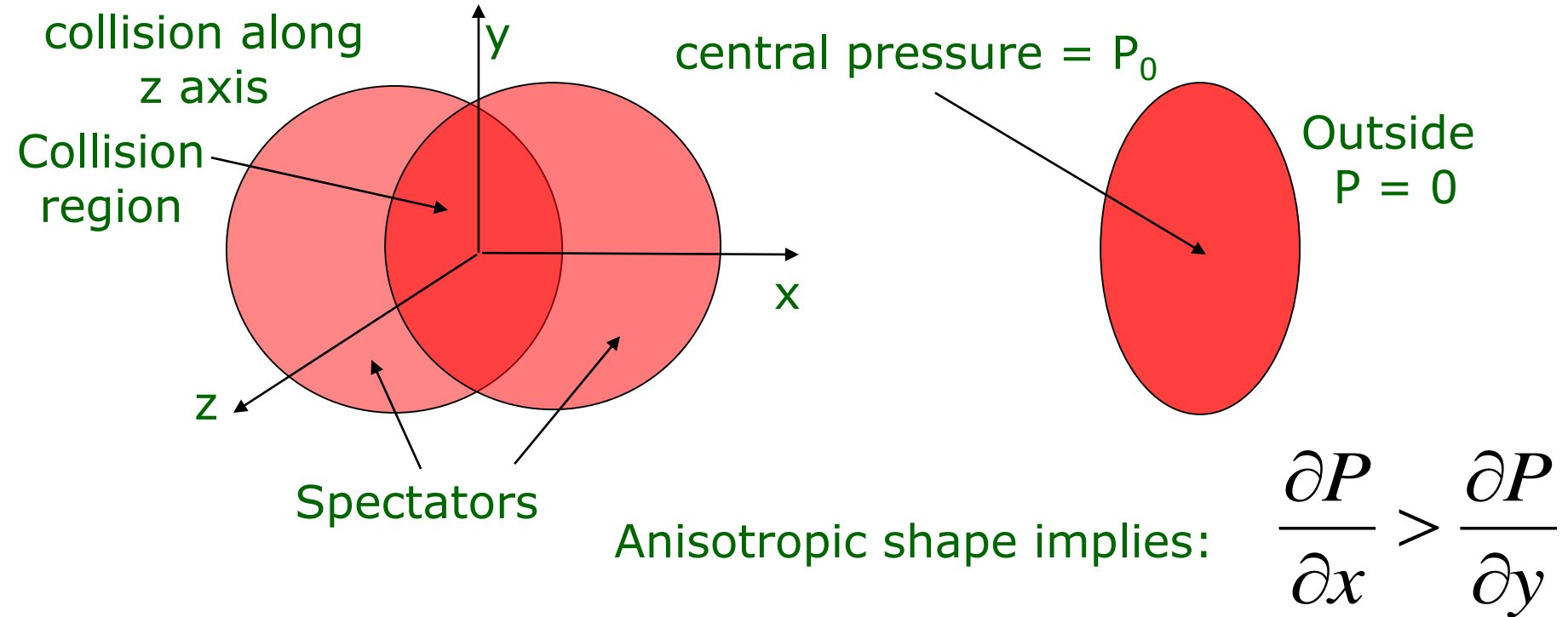
Outline:

1. Relativistic heavy-ion collision experiments (RHICE):
Elliptic Flow and other flow coefficients
2. Initial state fluctuations: Focus for electron-ion collider
Earlier considerations of initial state fluctuations for determination of axes for flow coefficients. Mostly non-central collisions.
3. Our approach: Flow fluctuations important, **especially central collisions**: Detailed information about initial state fluctuations.
Power spectrum of flow fluctuation coefficients v_n :
Valuable probe for initial fluctuations: like CMBR power spectrum.
4. Applying CMBR analysis techniques to RHICE. Deep similarities with CMBR anisotropies: Acoustic peaks and superhorizon fluctuations for heavy-ion collisions as for CMBR power spectrum
5. Hydrodynamics simulations: Large wavelength initial fluctuations
6. Initial stage Magnetic fields and Flow fluctuations:
MHD simulations: power difference in even-odd v_n
Another independent probe for initial state fluctuations.

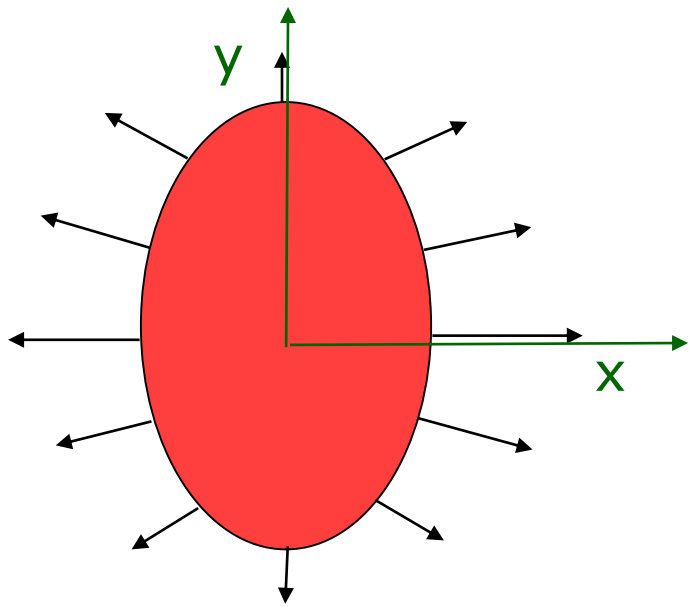
Relativistic heavy-ion collision experiments:

Recall: Elliptic Flow

In non-central collisions: central QGP region is anisotropic



Important: Initially no transverse expansion
Anisotropic pressure gradient implies:
Buildup of plasma flow larger in x direction
than in y direction



Initial particle momentum distribution isotropic : it develops anisotropy due to larger flow in x direction

This momentum anisotropy is characterized by the 2nd Fourier coefficient V_2 (Elliptic flow).

Higher Fourier coefficients denoted as V_n , the higher flow coefficients.

Note: Elliptic flow strong evidence for thermalization. No other way To get anisotropic momentum distribution only from spatial Anisotropy.

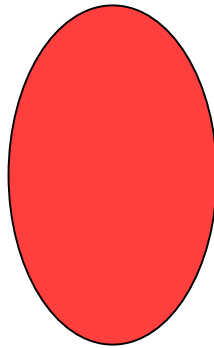
Led to very important results:

Strong constraints on η/s : values determined to be in range 1 -3 times AdS/CFT bound. Lower than any known liquid.

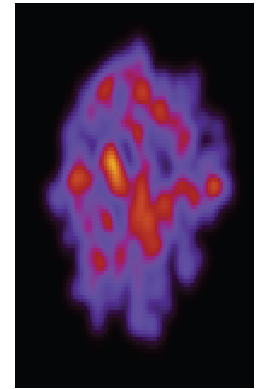
Initial state fluctuations and elliptic flow:

It was known that there are fluctuations present at the initial stage itself.

Thus: situation is
not like this



Rather, it
is like this



However, earlier, these initial state fluctuations were only discussed in the context of determination of the axes of eccentricity for elliptic flow calculations, and the axes for calculations of V_2 , V_4 , and V_6 .

No other flow coefficients were discussed.

In particular, no odd harmonics were discussed.

Primary focus was on non-central collisions to get elliptic flow which gave information about equation of state, viscosity etc.

Initial state fluctuations had to be accounted for the determination of these flow coefficients with proper choice of axes.

Initial state fluctuations and flow coefficients

In a series of papers (2008-2010), we argued that initial state fluctuations are extremely important, originating from initial conditions (parton distributions) inside the colliding nuclei.

We, thus, argued that, in particular, central collisions are very important. (Non-central collisions also important, however then the very large elliptic flow tends to mask the effects of initial state fluctuations.)

So better to focus on central collisions

We argued that due to these initial state fluctuations all flow coefficients will be non-zero in general:

Inhomogeneities of all scales are present, even in central collisions:
Arising from initial state fluctuations

All Fourier coefficients V_n are of interest, say, $n=1$ to 30-40, **including Odd harmonics**, these were never discussed earlier.

Power spectrum of flow fluctuations:

We emphasized: Learn from CMBR power spectrum analysis:

**Calculate root-mean-square values of V_n ,
and NOT their average values.**

That is: Calculate the Power spectrum of flow coefficients

Original proposal:

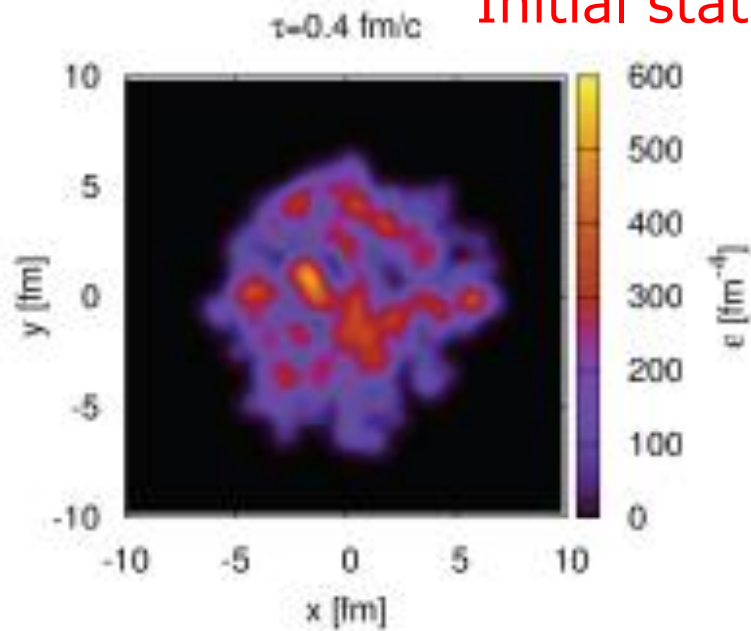
A.P. Mishra, R. Mohapatra, Saumia P.S., AMS:

- 1) Super-horizon fluctuations and acoustic oscillations in relativistic heavy-ion collisions: PRC 77, 064902 (2008)
- 2) Using CMBR analysis tools for flow anisotropies in relativistic heavy-ion collisions: PRC 81, 034903 (2010)

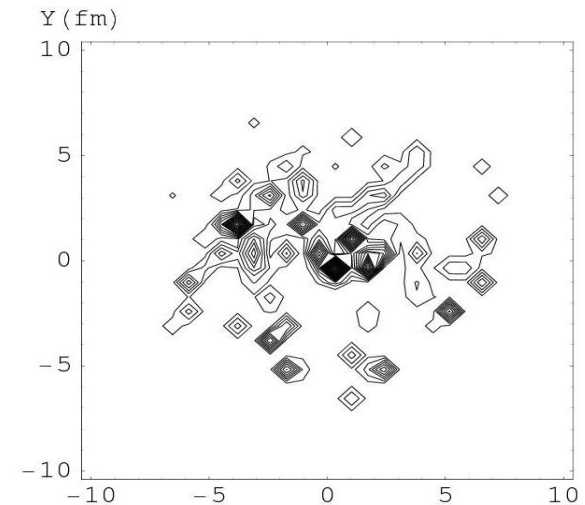
Hydrodynamics simulation:

Saumia P.S., AMS, Mod. Phys. Lett. A31, 1650197 (2016)

Initial state fluctuations



(Blaizot's talk, Matter at extreme conditions, 2014)



transverse energy density:
Au-Au **central collision**
at 200 GeV/A, HIJING

Inhomogeneities of all scales are present, even in central collisions:
Arising from initial state fluctuations

Thus: the equilibrated matter will also have azimuthal anisotropies
(as well as radial fluctuations) of similar level.

We emphasized: Lesson from CMBR power spectrum analysis:
Plot of root-mean-square values: Enormous information about nature
of initial state fluctuations, their evolution, equation of state, etc.

Important lesson for heavy-ion collisions from CMBR analysis

CMBR temperature anisotropies analyzed using Spherical Harmonics

$$\frac{\Delta T}{T}(\theta, \phi) = a_{lm} Y_{lm}(\theta, \phi)$$

Now: Average values of these expansions coefficients are zero due to overall isotropy of the universe

$$\langle a_{lm} \rangle = 0$$

However: their standard deviations are non-zero and contain crucial information.

$$C_l = \langle |a_{lm}|^2 \rangle$$

This gives the celebrated Power Spectrum of CMBR anisotropies

Lesson : Apply same technique for RHICE also

For central events average values of flow coefficients will be zero

$$\langle V_n \rangle = 0$$

(same is true even for non-central events if a coordinate frame with fixed orientation in laboratory system is used).

Following CMBR analysis, we proposed to calculate root-mean-square values of these flow coefficients using a lab fixed coordinate system, and plot it for a large range of values of $n = 1, 30-40$

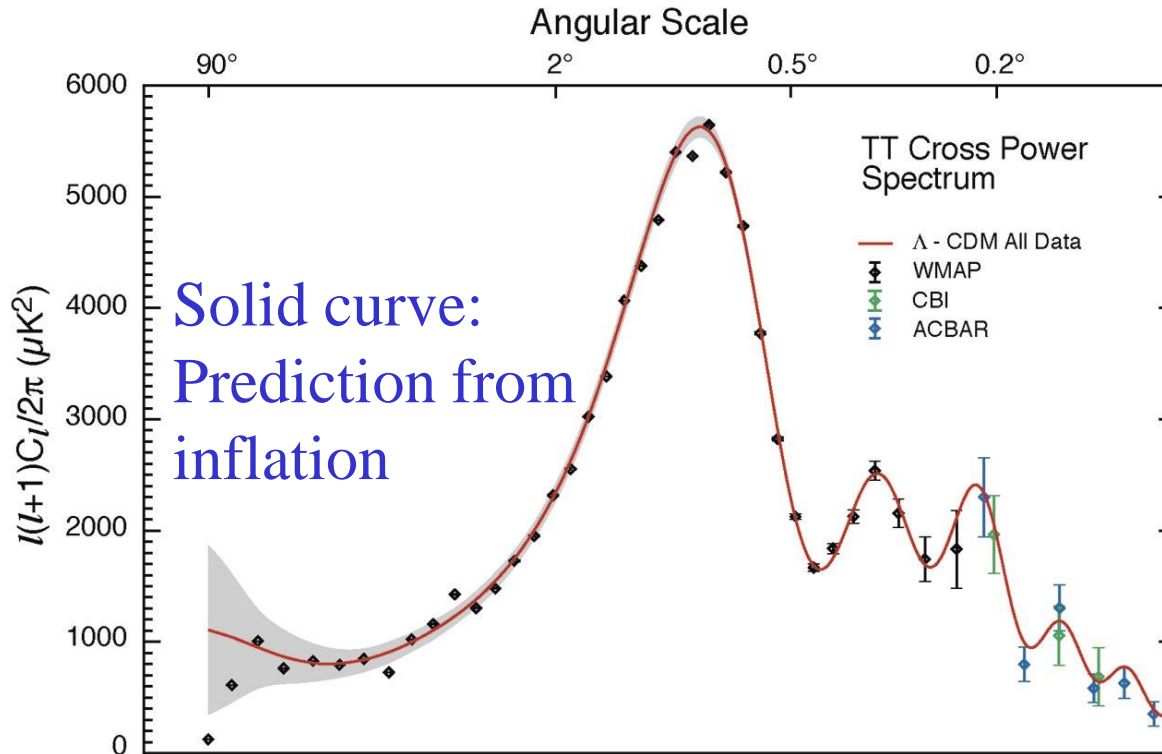
$$V_n^{rms} = \sqrt{\langle V_n^2 \rangle}$$

These values will be generally non-zero for even very large n and will carry important information .

Conclusion: Plot power spectrum for V_n for entire range of values of n . The whole plot will have information about initial state fluctuations and their evolution.

Power Spectrum of flow fluctuations: Further insight

Recall: Acoustic peaks in CMBR anisotropy power spectrum



Can such a power spectrum be expected for heavy-ion collisions ?

So far we discussed: Plot of V_n^{rms} for large values of n will give important information about initial density fluctuations.

We now discuss: Such a plot may also reveal non-trivial structure like acoustic peaks for CMBR, and **suppression in the power of long wavelength initial state fluctuations.**

We have noted that initial state fluctuations of different length scales are present in Relativistic heavy-ion collisions even for central collisions

The process of equilibration will lead to some level of smoothing. However, thermalization happens quickly (for RHIC, within 1 fm)

No homogenization can be expected to occur beyond length scales larger than this at this thermalization stage.

This provides a natural concept of causal Horizon

Thus, inhomogeneities, especially anisotropies with wavelengths larger than the thermalization time scale should be necessarily present at the thermalization stage when the hydrodynamic description is expected to become applicable.

As time increases, the causal horizon (or, more appropriately, the sound horizon) increases with time.

An Important feature of flow power spectrum
Fluctuations with **superhorizon** wavelengths

Meaning of Horizon for the Universe:

Horizon size = speed of light c x age of the universe t
No physical effect possible for distances larger than this

In the universe, density fluctuations with wavelengths of
superhorizon scale have their origin in the inflationary period.
Tiny fluctuations are stretched by superluminal expansion

Meaning of Horizon for Heavy-ion collisions:

System equilibrates in time τ_0 less than 1 fm/c. **Horizon size** = $c \tau_0$
No physical effects possible for distances larger than $c \tau_0 = 1$ fm.
(more precisely the sound horizon given by the speed of sound)

Note: Initial state fluctuations present of all wavelengths even at
time τ_0 . These arise from fluctuations in nucleon/partons phase
space distributions. **Focus for Electron-Ion collider.**

All fluctuations larger than 1 fm are **superhorizon** at time τ_0 .
At any later time τ , any fluctuation larger than $c\tau$ is superhorizon.

Recall: Two crucial aspects of the inflationary density fluctuations leading to the remarkable signatures of acoustic peaks in CMBR:

Coherence and Acoustic oscillations.

Note: Coherence of inflationary density fluctuations essentially results from the fact that the fluctuations initially are stretched to superhorizon sizes and are subsequently frozen out dynamically.

In the context of heavy-ion collisions, this freezing out is similar to absence of initial transverse expansion velocity for QGP.

Initially, fluctuations are only in spatial distribution of energy density, they become dynamical, converting to momentum anisotropies through hydrodynamical evolution.

For all fluctuations of certain size, it happens **ONLY** after a certain time when causal horizon equals the fluctuation size. Until then The fluctuations are almost frozen.

Thus coherence (meaning phase locking) will be expected to hold for RHICE also.

Oscillatory behavior for the fluctuations.

Important: Small perturbations in a fluid will always propagate as acoustic waves, hence oscillations are naturally present.

Note: **The only difference from the universe is the absence of Gravity for RHICE.**

However, in the universe, the only role of attractive Gravity is to compress (collapse) the initial overdensities of cosmic fluid. Acoustic oscillations happen on top of these collapsed fluctuations.

Thus: for RHICE one will get harmonic oscillations (for a given mode) of plasma, while for the Universe one gets oscillations of a forced oscillator (gravity acting as extra force) for the cosmic fluid.

Conclusion: For RHICE also, one should have acoustic oscillations, Which are coherent: just as for CMBR.

We will see that hydrodynamics evolution supports this.

Important: Oscillations occur only for sub-horizon fluctuations.

We argued that sub-horizon fluctuations in heavy-ion collisions should display oscillatory behavior just as fluctuations for CMBR

What about super-horizon fluctuations ?

Recall: For CMBR, the importance of horizon entering is for the growth of fluctuations due to gravity.

This leads to increase in the amplitude of density fluctuations, with subsequent oscillatory evolution, leaving the imprints of these important features in terms of acoustic peaks.

Superhorizon fluctuations for universe do not oscillate (are frozen, as we discussed earlier).

Importantly, they also do not grow, That is: they are suppressed compared to the fluctuations which enter the horizon and grow by Gravitational collapse.

For heavy-ion collisions, such Superhorizon fluctuations will be extremely important: Information about long range correlations in the Initial state.

For heavy-ion collisions, there is a similar (though not the same, due to absence of gravity here) importance of horizon entering.

One can argue that flow anisotropies for superhorizon Fluctuations in heavy-ion collisions should be suppressed by a factor of order $\frac{H_{\text{fr}}^s}{\lambda / 2}$
 H_{fr}^s is the sound horizon at the freezeout time t_{fr} ($\sim 5\text{-}10$ fm for heavy-ion collisions), λ is the wavelength of fluctuation.

This is because in heavy-ion collisions, spatial variations of density are not directly detected.

This is in contrast to the Universe where one directly detects the spatial density fluctuations in terms of angular variations of CMBR.

For heavy-ion collisions, spatial fluctuation of a given scale (i.e. a definite mode) has to convert to fluid momentum anisotropy of the corresponding angular scale. This will get imprinted on the final hadrons and will be experimentally measured.

This conversion of spatial anisotropy to Momentum anisotropy (via pressure gradients) is not effective for Superhorizon modes.

Thus: Superhorizon modes will be suppressed in heavy-ion collisions

Emphasize again: Superhorizon Density Fluctuations:

For Universe: Quantum fluctuations of sub-horizon scale are stretched out to superhorizon scales during the inflationary period.

During subsequent evolution, after the end of the inflation, fluctuations of sequentially increasing wavelengths keep entering the horizon. The largest ones to enter the horizon, and grow, at the stage of decoupling of matter and radiation lead to the first peak in CMBR anisotropy power spectrum.

We have seen that superhorizon fluctuations should be present in RHICE at the initial equilibration stage itself.

Note: sound horizon, $H_s = c_s t$ here, where c_s is the sound speed, is smaller than 1 fm at $t = 1$ fm. At time t from the birth of the plasma, physical effects cannot propagate to distances beyond H_s .

With the nucleon size being about 1.6 fm, the equilibrated matter will necessarily have density inhomogeneities with superhorizon wavelengths at the equilibration stage.

We expect these superhorizon flow fluctuations to be suppressed

Our earlier Results: A.P. Mishra, R. Mohapatra, Saumia P.S., AMS:
Super-horizon fluctuations and acoustic oscillations in relativistic heavy-ion
collisions: PRC 77, 064902 (2008)

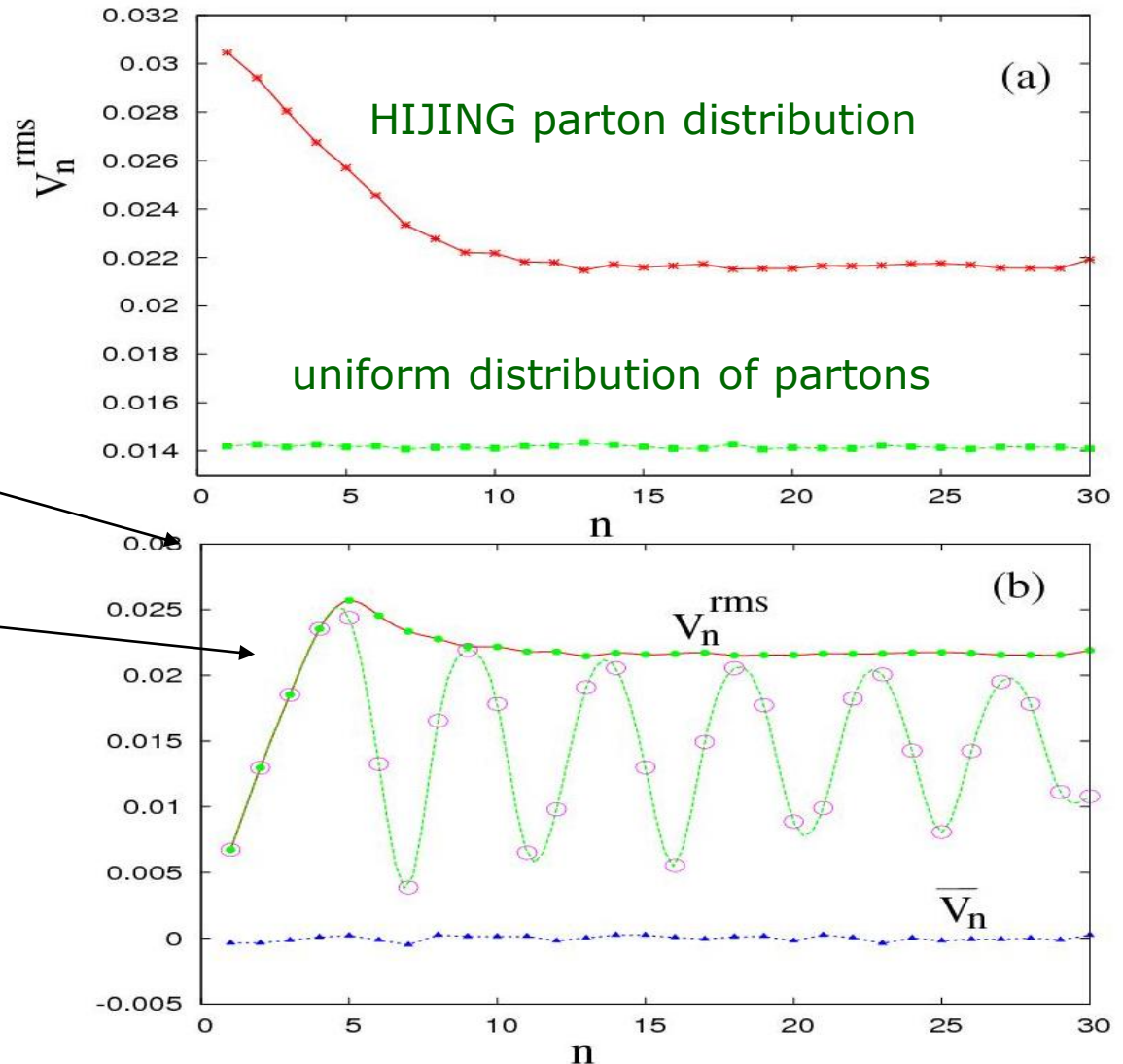
(modeling only, no hydrodynamical simulation in this work)

Errors less
than $\sim 2\%$

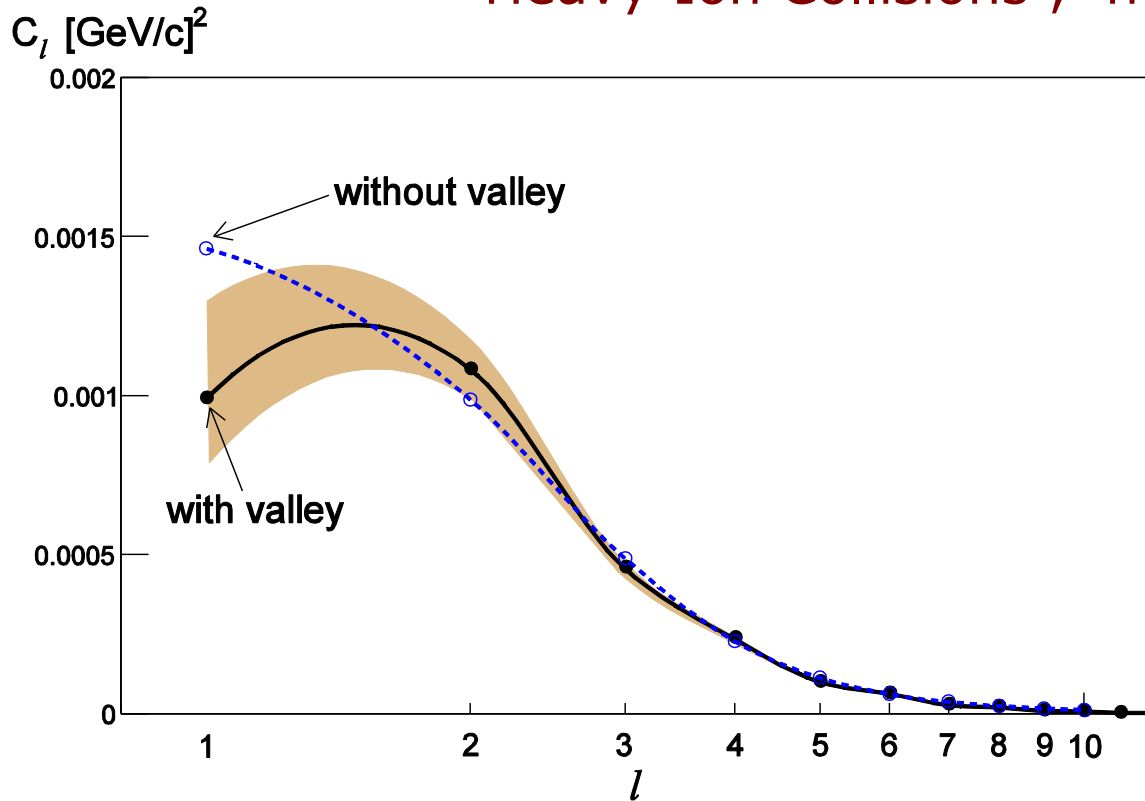
Include superhorizon
suppression

Include oscillatory
factor also

Note: Dissipation, e.g.
from viscosity, diffusion,
will damp higher n modes



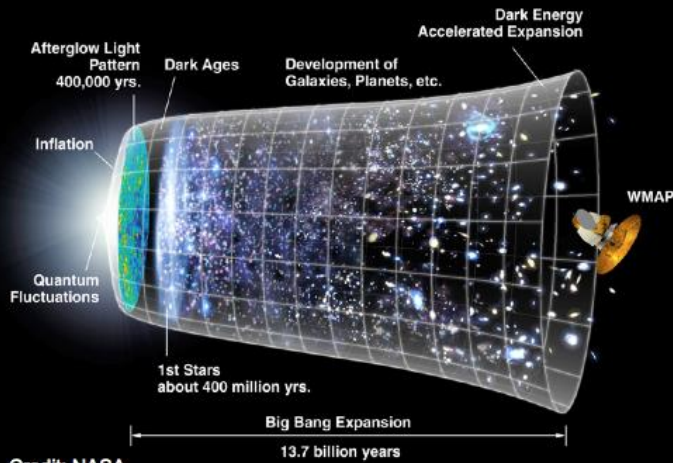
Paul Sorensen "Searching for Superhorizon Fluctuations in Heavy-Ion Collisions", nucl-ex/0808.0503



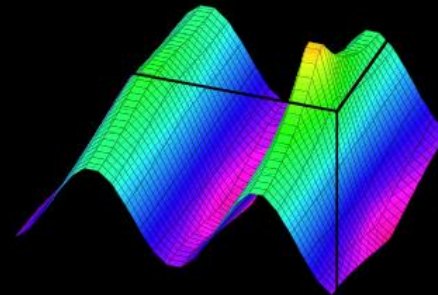
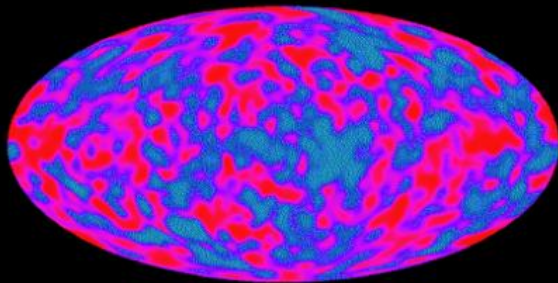
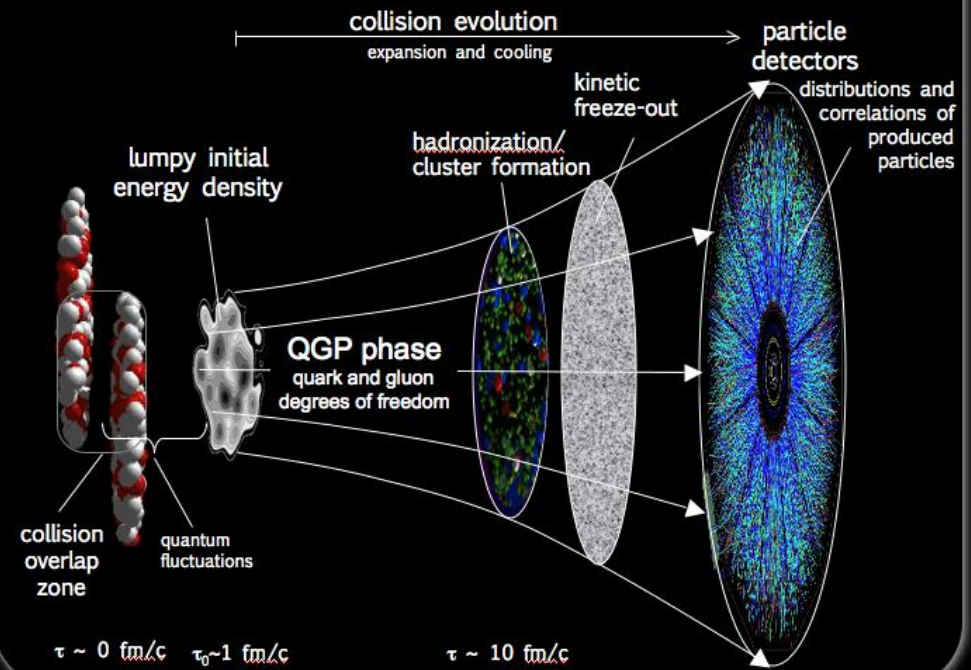
See, also, youtube video by Sorensen from STAR:
<http://www.youtube.com/watch?v=jF8QO3Cou-Q>

Analogies with the early universe

The Universe: Slow Expansion

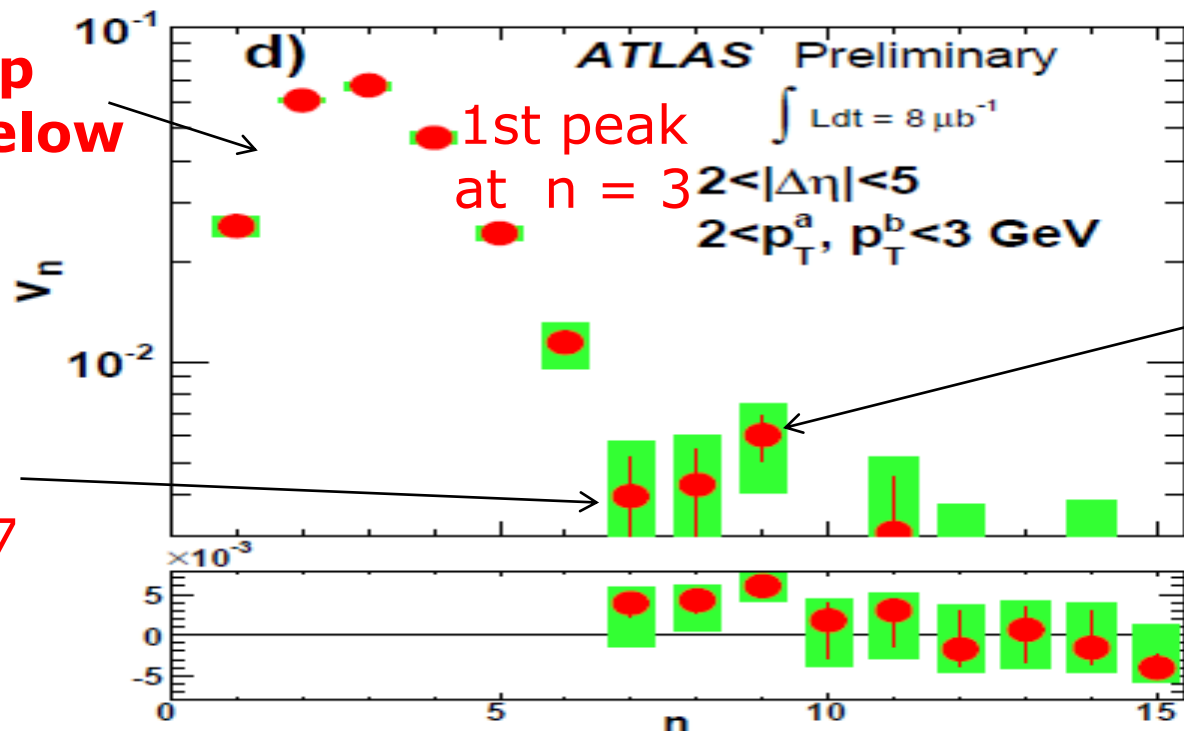


Heavy-ion Collisions: Rapid Expansion



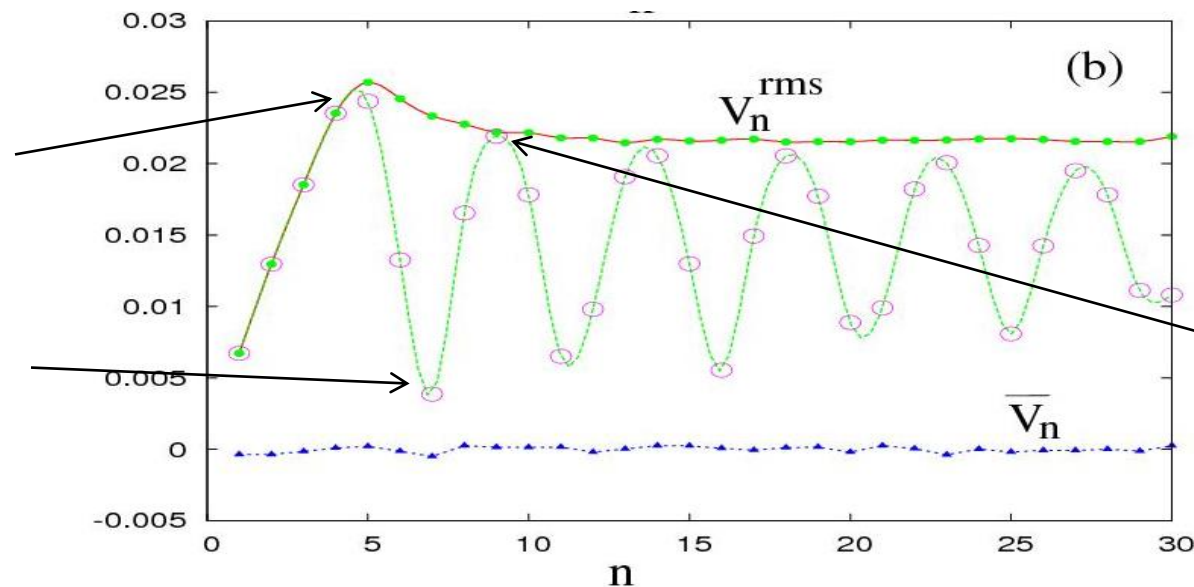
Paul Sorensen "Searching for Superhorizon Fluctuations in Heavy-Ion Collisions", nucl-ex/0808.0503

Focus on dip
for low n below
First peak

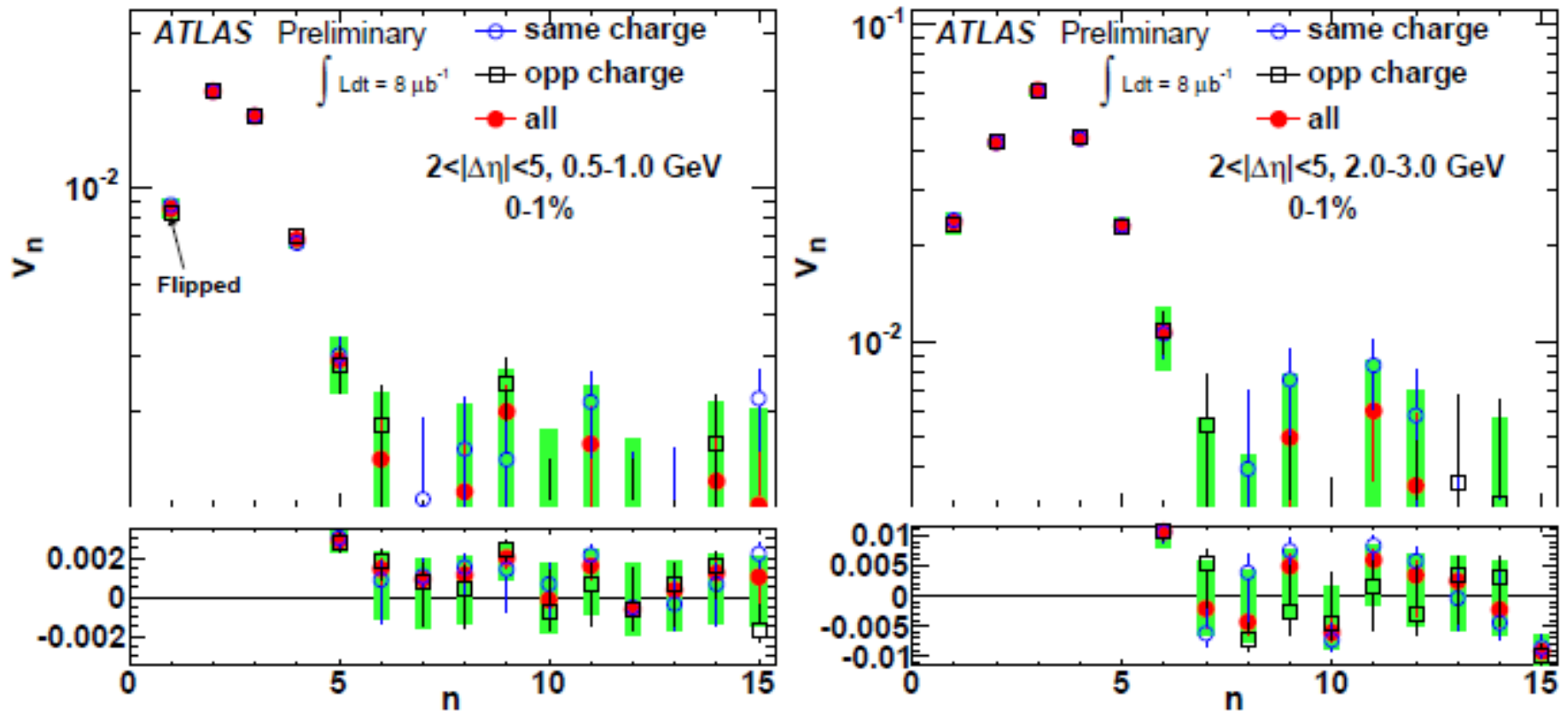


1st peak
at $n = 5$

1st dip
at $n = 7$



arXiv: 1107.1468, LHC Pb-Pb at 2.76 TeV



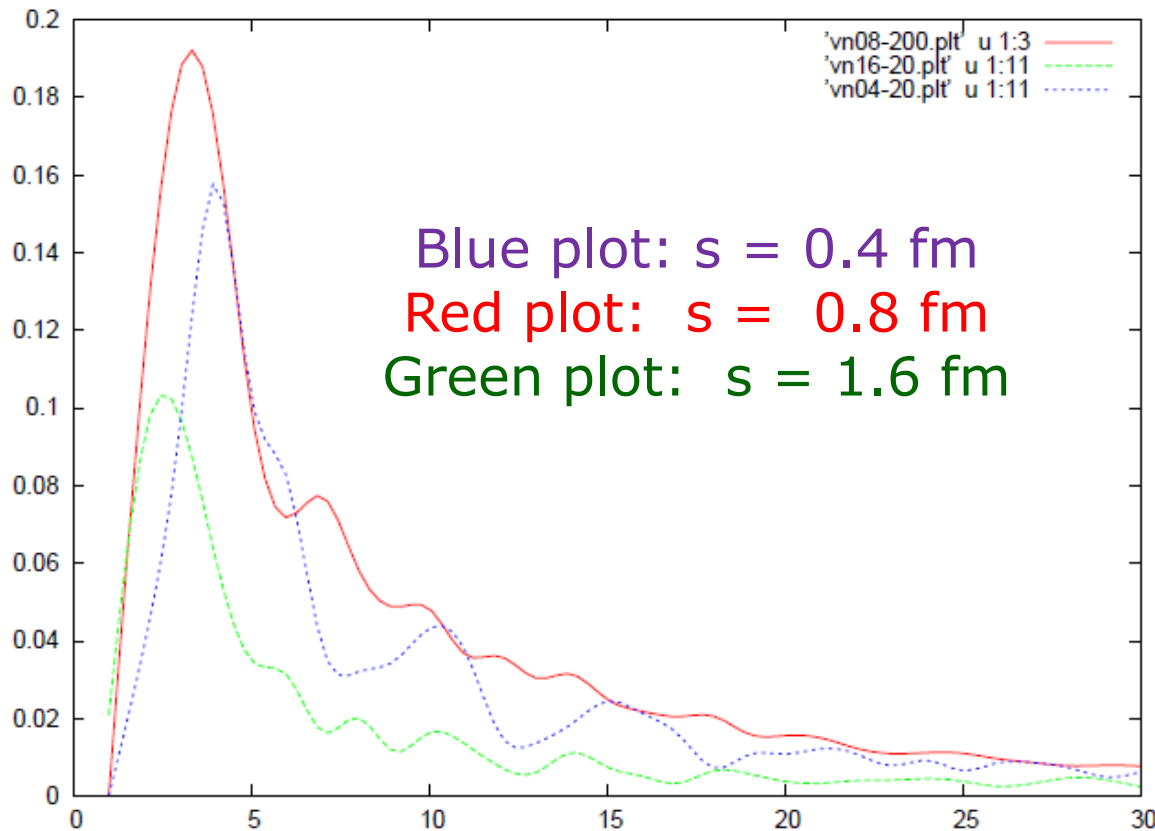
Note: Very important to understand suppression of low n harmonics
It contains the information about freezeout horizon size

And about long wavelength fluctuations at the initial stage.

Relativistic Hydrodynamics Simulations:

Saumia P.S., AMS, Mod. Phys. Lett. A31, 1650197 (2016)

Plots of v_n^{rms} vs. n for Gaussian fluctuations of width s



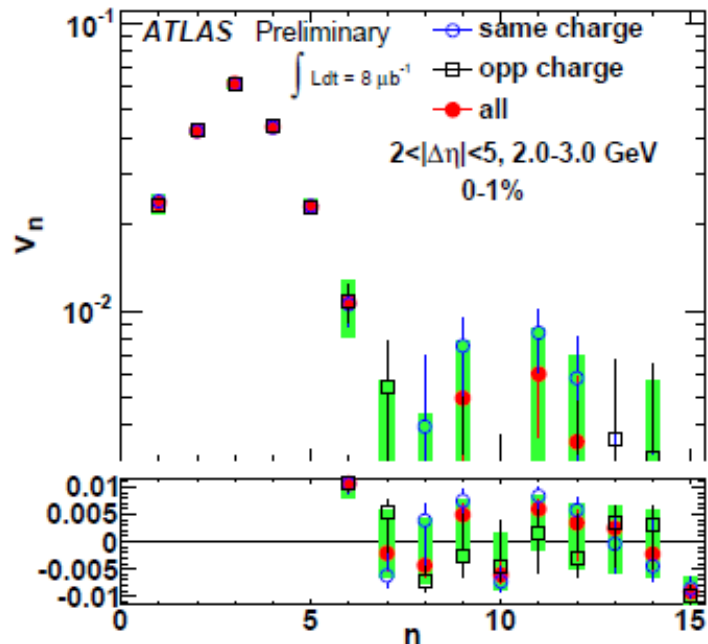
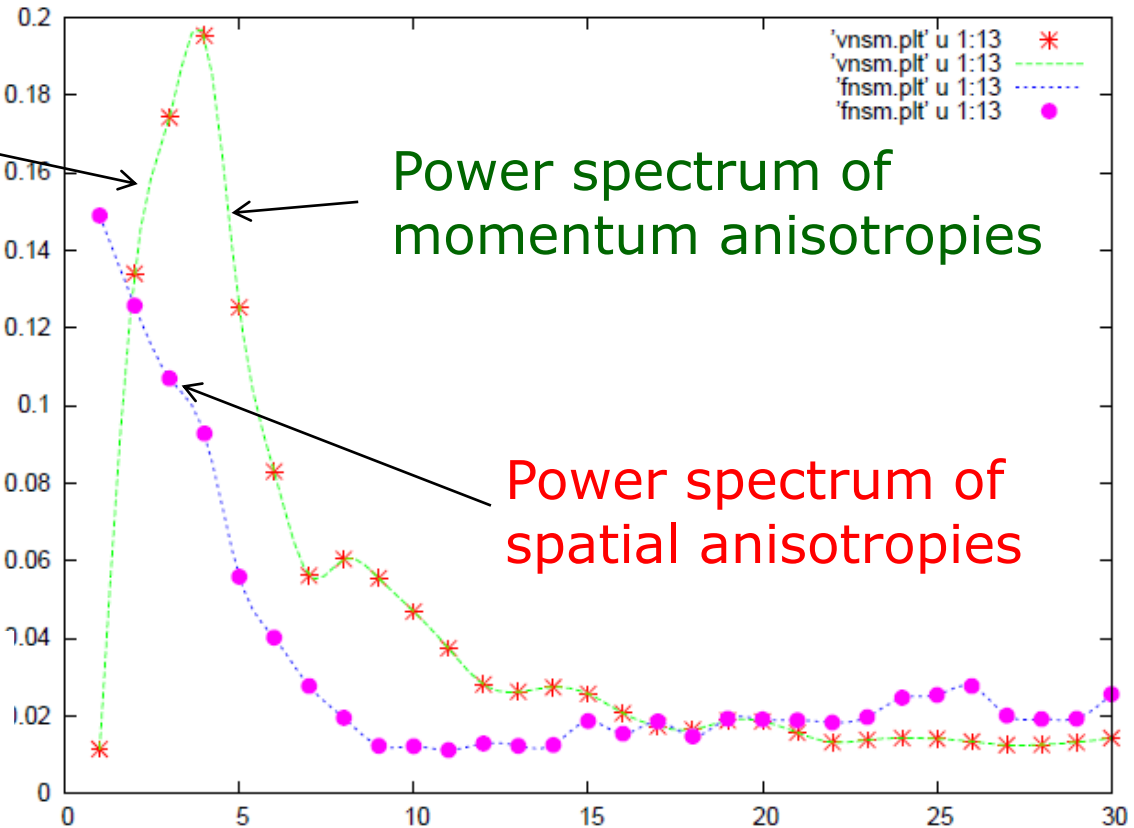
Note: Here peak position gives information about length scale of initial fluctuations

Just as for CMBR, where first peak location directly gives size of largest fluctuation at last scattering surface

Woods-Saxon density profile, 2 fm radius with 10 Gaussian fluctuations, $T_0 = 500$ MeV

Evidence for Superhorizon suppression from hydro simulations

Superhorizon suppression



Data shows this suppression:
Very Important to understand this.

These relate to long range correlations
at the initial stage.

Magnetic field in heavy-ion collisions:

Important physics motivations: Chiral magnetic effect, chiral vortical effect,.....

Problem: Magnetic field strong only for very early times

We use this limitation to isolate the initial distribution of fluctuations, separate from their later evolution.

Magnetic field affects every aspect of flow: enhancement of v_2
Effect on all flow coefficients: On the power spectrum

We start with discussion of effect of Magnetic Field on v_2 :
We earlier pointed out that initial magnetic field can enhance elliptic flow (Mohapatra, Saumia, AMS, MPLA 26, 2477 (2011)).

Similar enhancement confirmed by Tuchin: J.Phys.G39, 025010(2012)

Later simulations: no effect found: Inghirami et al: arXiv:1609.03042
Important to understand these discrepancies
Will show results of our present simulations

Complex factors affect elliptic flow (in fact, every aspect of flow) in the presence of magnetic field:

Briefly recall basic physics of our original argument:

In presence of magnetic field, there are different types of waves in the plasma. Fast magnetosonic waves: Generalized sound waves with significant contributions from the magnetic pressure.

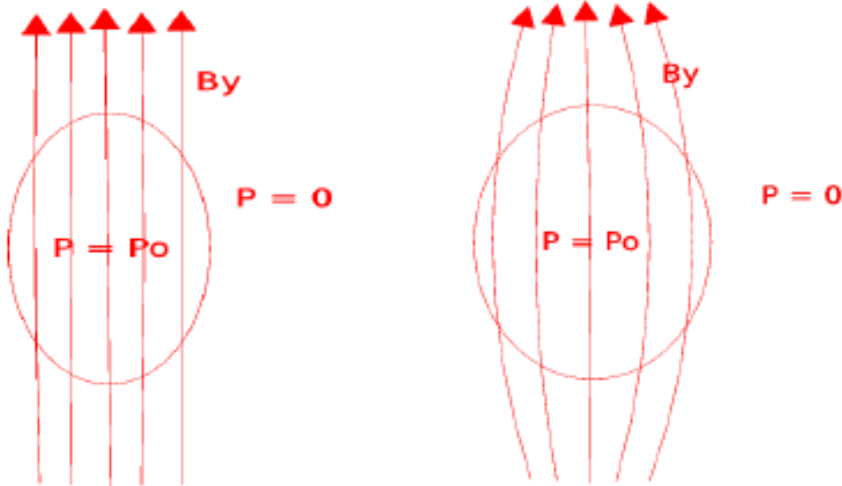
Basically, distortions of magnetic field in transverse direction costs energy, equation of state stiffer in that direction

Expect larger sound speed in transverse direction.

Flow velocity proportional to c_s^2

So we argued: Flow in x direction will be enhanced, while in y direction will not change:

Conclusion was: B increases v_2 .



However, the physics of this is not that simple.
Other factors can be present.

For example, it is known that under certain situations, expansion of a conducting plasma into regions of magnetic field gets hindered.

One can expect it from Lenz's law: expanding conductor squeezes magnetic flux, which should oppose expansion of plasma (cause of squeezing). Such an argument will imply suppression of v_2 due to B . This will be expected when magnetic field extends well beyond plasma region.

However, this is also not correct, as this completely misses the factor of distortion of magnetic field costing energy (which was the argument we used in our paper arguing for increase of v_2 .) We can expect that to hold true when magnetic field is entirely contained inside plasma region.

In general, all such factors are present. As we will see later, in some situation one factor will dominate, while in another, the other factor. Along with these two factors, fluctuations also play important role. Final effect is a combination of all these factors.

First we note: very complex flow patterns can develop due to magnetic field when fluctuations are also present.

The expression for group velocity in relativistic MHD

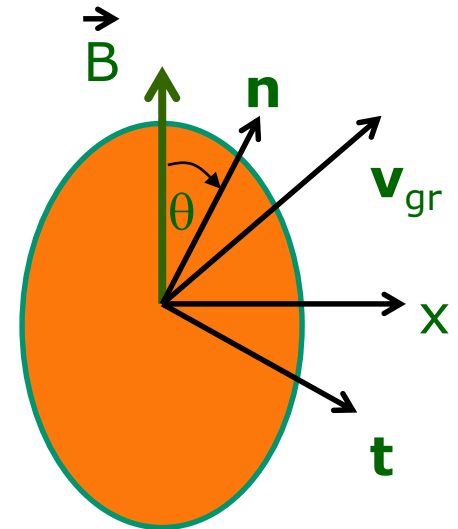
$$\mathbf{v}_{gr} = v p_h \left[\mathbf{n} + \mathbf{t} \frac{\sigma \pm 2\delta(a \mp (1 + \delta \cos^2 \theta)) \sin \theta \cos \theta}{2(1 + \delta \cos^2 \theta \pm a)a} \right]$$

$$v_{ph} = \mathbf{n} \frac{\left[\left(\frac{4\rho}{3\omega} \right) c_s^2 + v A^2 \right]^{1/2}}{2} (1 + \delta \cos^2 \theta \pm a)^{1/2}$$

$$\mathbf{n} = \mathbf{k}/k, \mathbf{t} = \left[\left(\frac{\mathbf{B}}{B} \right) \times \mathbf{n} \right] \times \mathbf{n}$$

$$a^2 = (1 + \delta \cos^2 \theta) - \sigma \cos^2 \theta,$$

$$\delta = \frac{c_s^2 v_A^2}{\left(\frac{4\rho}{3\omega} \right) c_s^2 + v A^2}, \sigma = \frac{4 c_s^2 v_A^2}{\left[\left(\frac{4\rho}{3\omega} \right) c_s^2 + v A^2 \right]^2}, \omega = \left(\frac{4\rho}{3} \right) + B^2$$



Note: Direction of group velocity depends on the coefficient of \mathbf{t} above, which depends on local pressure (for a given \mathbf{B}). Thus: with pressure variations, direction keeps changing: **Very complex flow pattern (May lead to generation of vortices due to strong fluctuations).**

Relativistic Ideal Magneto-Hydrodynamics Simulations:

Arpan Das, Shreyansh S. Dave, Saumia P.S., AMS, PRC 96, 034902 (2017)

We do not discuss the issue of survival of magnetic field in the plasma. Due to conductivity (Tuchin) magnetic field does not decay very rapidly in the plasma, field diffusion time at least several fm.

We take an initial value of the field, at a given time after the collision, calculated by taking uniformly charged nuclei (spherical or ellipsoidal for deformed case), and Lorentz transforming for oppositely moving nuclei with required impact parameter.

We carry out 3+1 dimensional simulation using Glauber-like initial conditions for QGP, with profile in z-direction being Woods-Saxon with appropriate size. We work in the limit of infinite conductivity: so use equations of Ideal Relativistic MHD:

We follow formalism from:

Mignone and Bodo, Mon. Not. R. Astron. Soc. (2005)

We first give a brief summary of the formalism:

Conservation of total energy-momentum tensor (perfect fluid QGP + magnetic field):

$$\partial_\alpha [(\rho + p_g + |b|^2)u^\alpha u^\beta - b^\alpha b^\beta + (p_g + \frac{|b|^2}{2})\eta^{\alpha\beta}] = 0$$

Maxwell's equations:

$$\partial_\alpha (u^\alpha b^\beta - b^\alpha u^\beta) = 0$$

Where:

$$b^\alpha = \gamma[\vec{v} \cdot \vec{B}, \frac{\vec{B}}{\gamma^2} + \vec{v}(\vec{v} \cdot \vec{B})]$$

and:

$$u^\alpha b_\alpha = 0, \text{ and } |b|^2 \equiv b^\alpha b_\alpha = \frac{|\vec{B}|^2}{\gamma^2} + (\vec{v} \cdot \vec{B})^2$$

For simulation, these equations are cast in the following form

$$\frac{\partial U}{\partial t} + \sum_k \frac{\partial F^k}{\partial x^k} = 0$$

Where different quantities are defined as:

$$U = (m_x, m_y, m_z, B_x, B_y, B_z, E)$$

$$m_k = [\rho h \gamma^2 + |\vec{B}|^2] v_k - (\vec{v} \cdot \vec{B}) B_k$$

$$E = \rho h \gamma^2 - p_g + \frac{|\vec{B}|^2}{2} + \frac{v^2 |\vec{B}|^2 - (\vec{v} \cdot \vec{B})^2}{2}$$

$$F^x = \begin{pmatrix} m_x v_x - B_x \frac{b_x}{\gamma} + p \\ m_y v_x - B_x \frac{b_y}{\gamma} \\ m_z v_x - B_x \frac{b_z}{\gamma} \\ 0 \\ B_y v_x - B_x v_y \\ B_z v_x - B_x v_z \\ m_x \end{pmatrix}$$

$$p = p_g + \frac{|b|^2}{2}$$

($F^{y,z}$ are similarly defined by appropriate change of indices)

Note: From U at each stage, independent variables (p_g, \vec{v}, \vec{B}) have to be extracted.

(p_g, \vec{v}, \vec{B}) Are extracted by defining:

$$S = \vec{m} \cdot \vec{B} \quad W = \rho h \gamma^2$$

And writing

$$E = W - p_g + \left(1 - \frac{1}{2\gamma^2}\right) |\vec{B}|^2 - \frac{S^2}{2W^2}$$

$$|m|^2 = (W + |\vec{B}|^2)^2 \left(1 - \frac{1}{\gamma^2}\right) - \frac{S^2}{W^2} (2W + |\vec{B}|^2)$$

These equations are written eventually as a single equation for one unknown W (by rewriting equation for $|m|^2$:

$$\gamma = \left(1 - \frac{S^2(2W + |\vec{B}|^2) + |m|^2 W^2}{(W + |\vec{B}|^2)^2 W^2}\right)^{-1/2}$$

$$f(W) \equiv W - p_g + \left(1 - \frac{1}{2\gamma^2}\right) |\vec{B}|^2 - \frac{S^2}{2W^2} - E = 0$$

This equation is solved using Newton-Raphson method to get W, from which other independent variables are obtained using above equations.

Limitations of the simulation:

Due to computer limitations, we use small lattice (200x200x200) so small nuclei used (copper), also for small times only up to maximum of 3 fm time, sometime much shorter time.

We extract initial values of flow coefficients.

We use smaller energy CMS energy of 20 GeV, for very large energies magnetic field becomes very large near receding nuclei (it is 3+1 dimensional simulation), causing problem With fluctuations difficult to run for long times.

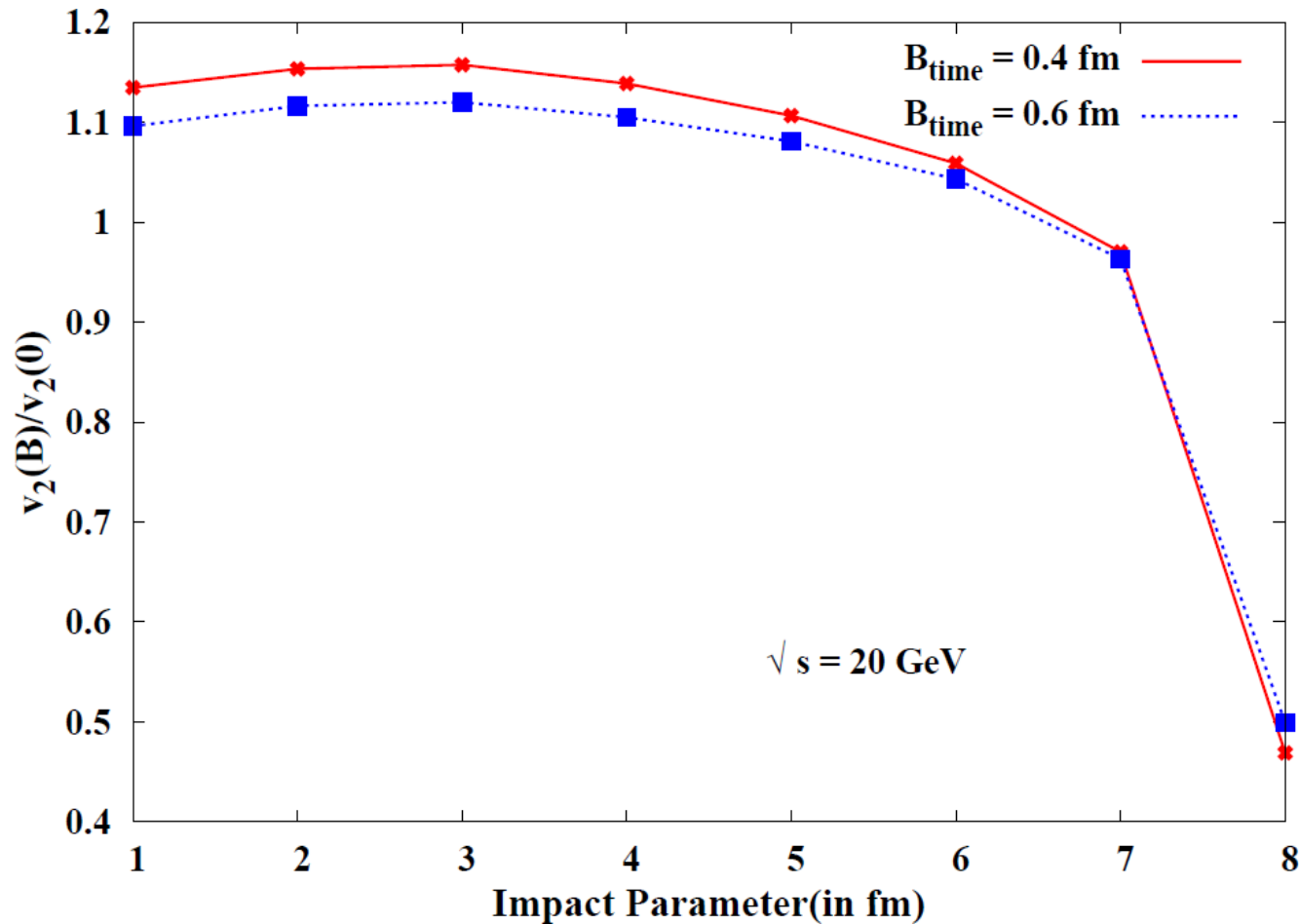
General problem when magnetic field energy density becomes much larger than the plasma density.

Same problem was found in other simulation also (Inghrami et al. where no effect on v_2 was found)

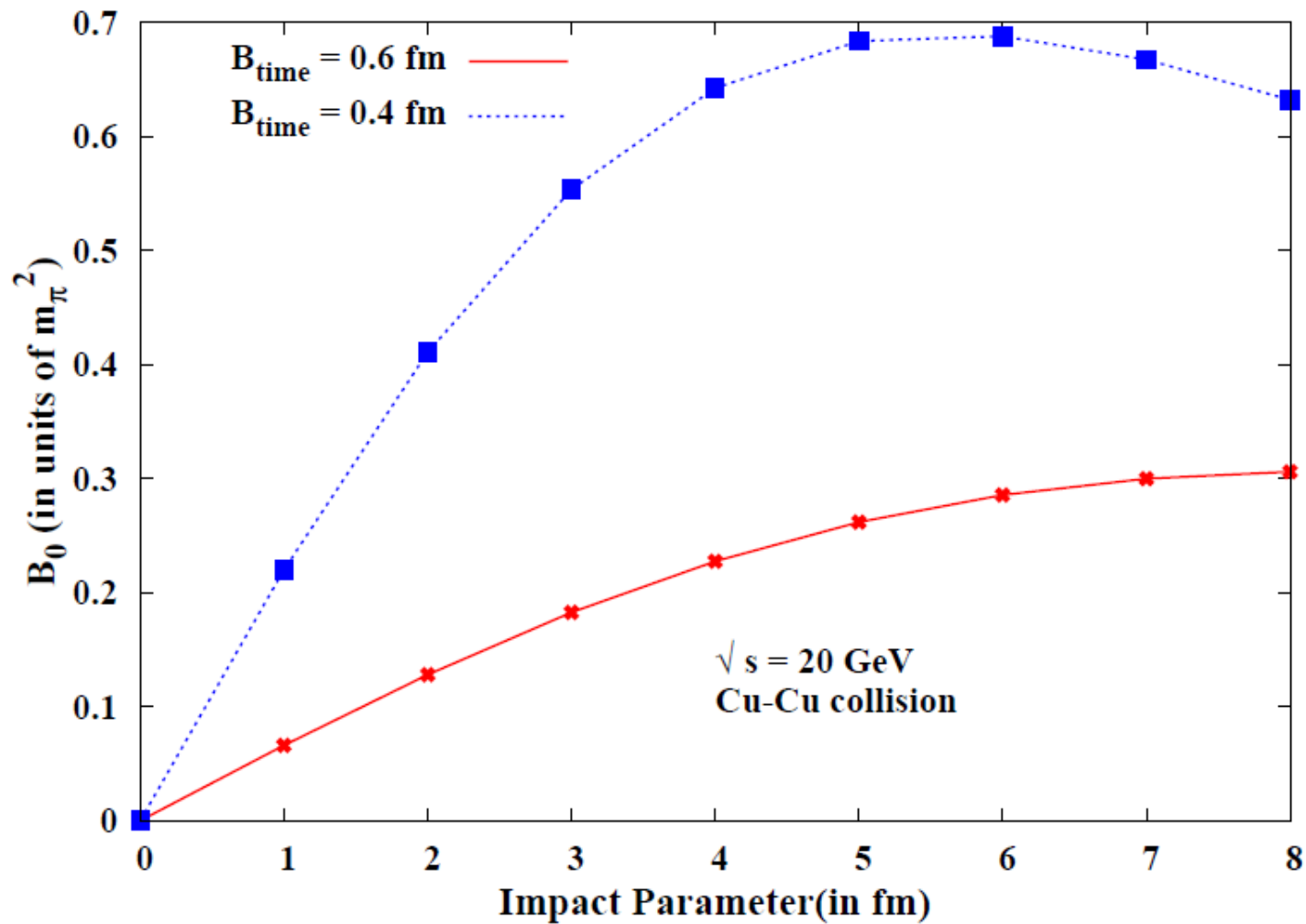
For elliptic flow, we have studied details of the dependence of elliptic flow on magnetic field, and it seems to crucially depend on the relative profiles of B and plasma density.

We first present these results

Result-1: Elliptic flow in the presence of magnetic field:

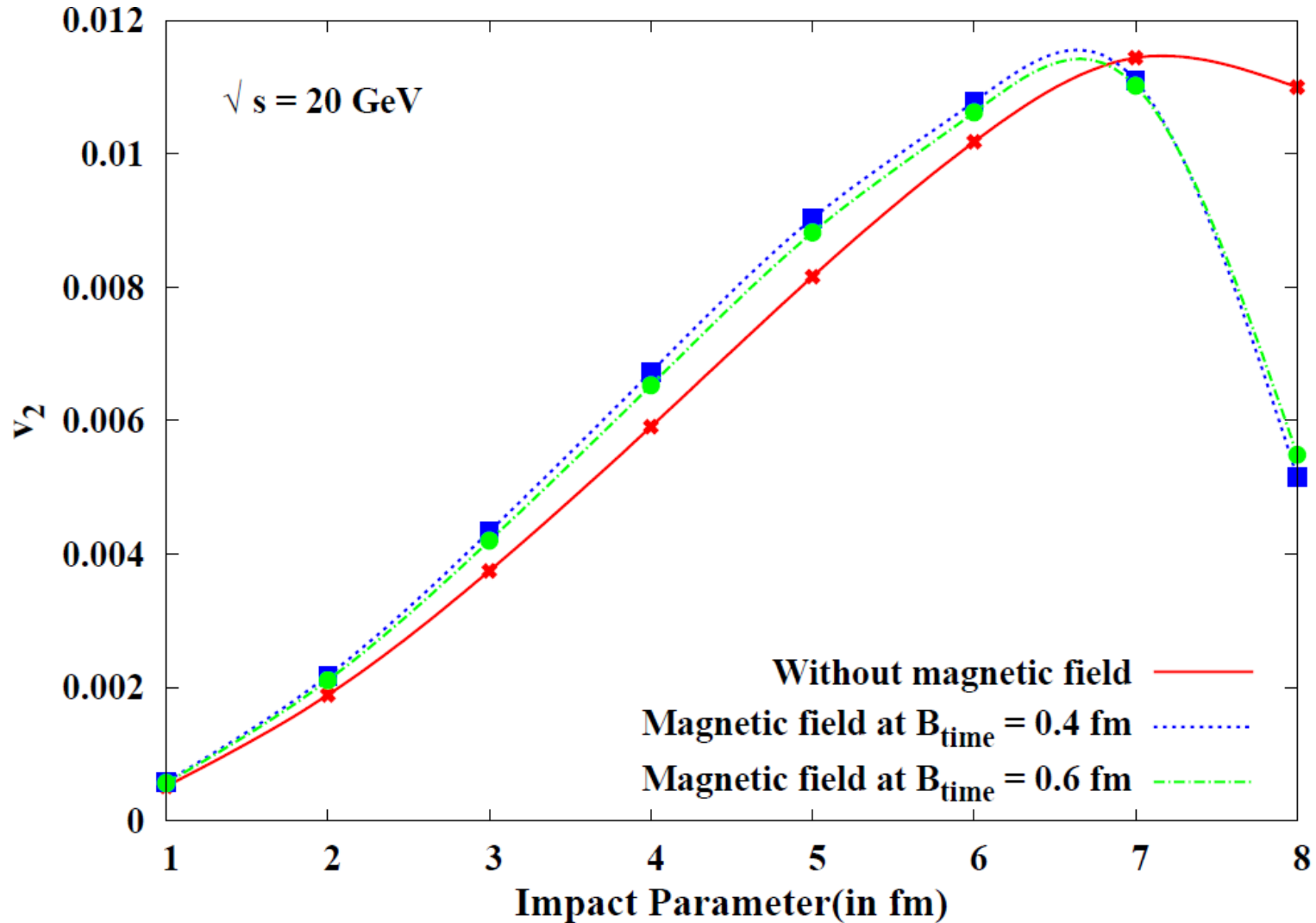


We see that magnetic field enhances v_2 , but only up to impact parameter of about 6 fm, after that B suppresses it. Also Enhancement increases for small impact parameter. Why?



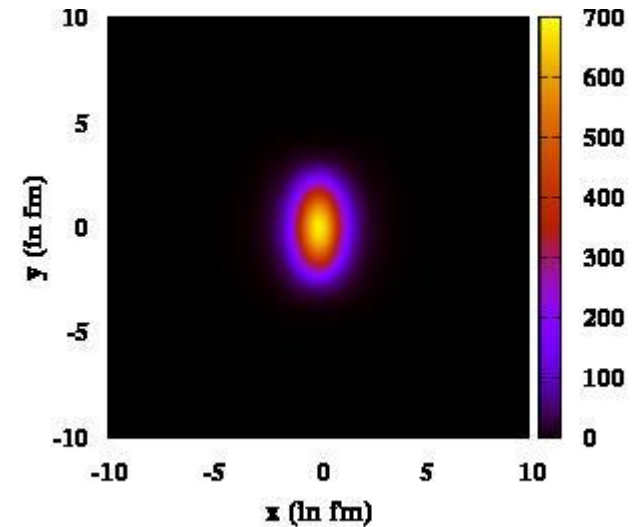
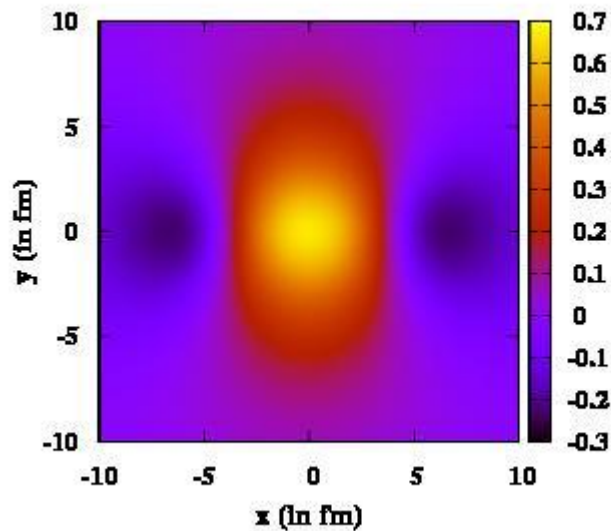
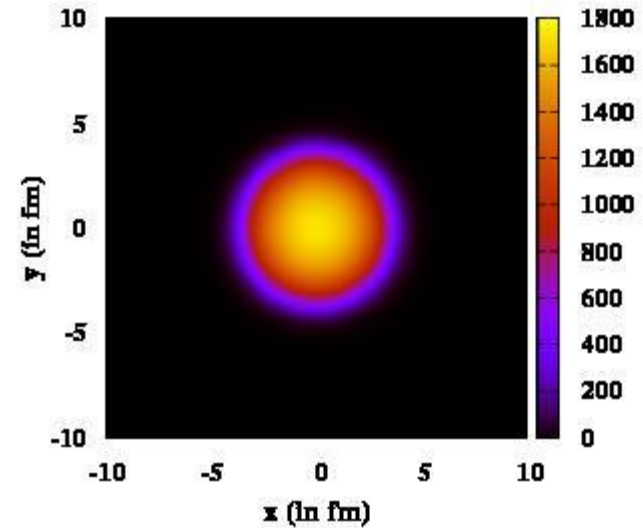
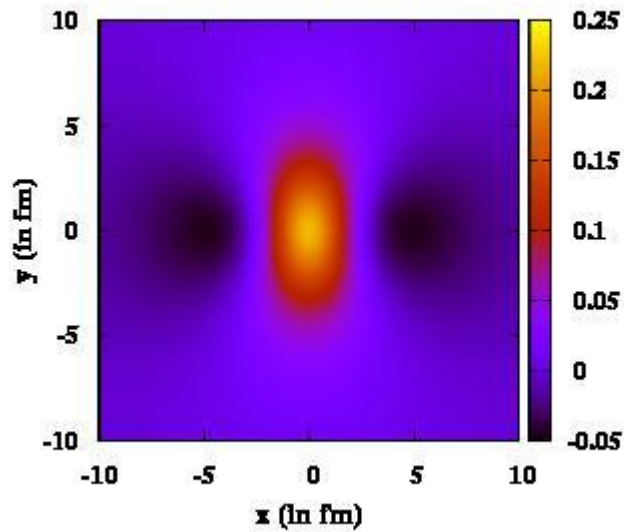
Note: Magnetic field almost monotonically increases with impact parameter.

Elliptic flow with and without magnetic field:



We see that magnetic field enhances v_2 for small impact Parameters, then enhancement decrease. Eventually, magnetic field suppresses elliptic flow.

Magnetic field plots (top) and plasma density plots (bottom) for small 1 fm (left) and large 7 fm (right) impact parameter



Left: B contained entirely within plasma region, expect enhanced v_2 from anisotropic sound speed

Right: B extends well outside plasma region, expect v_2 suppression from flux squeezing (Lenz's law)

Summary of results for effect of magnetic field on elliptic flow:

If magnetic field is contained almost entirely within the plasma region, then elliptic flow is enhanced with increasing magnetic field. This happens for small values of the impact parameter.

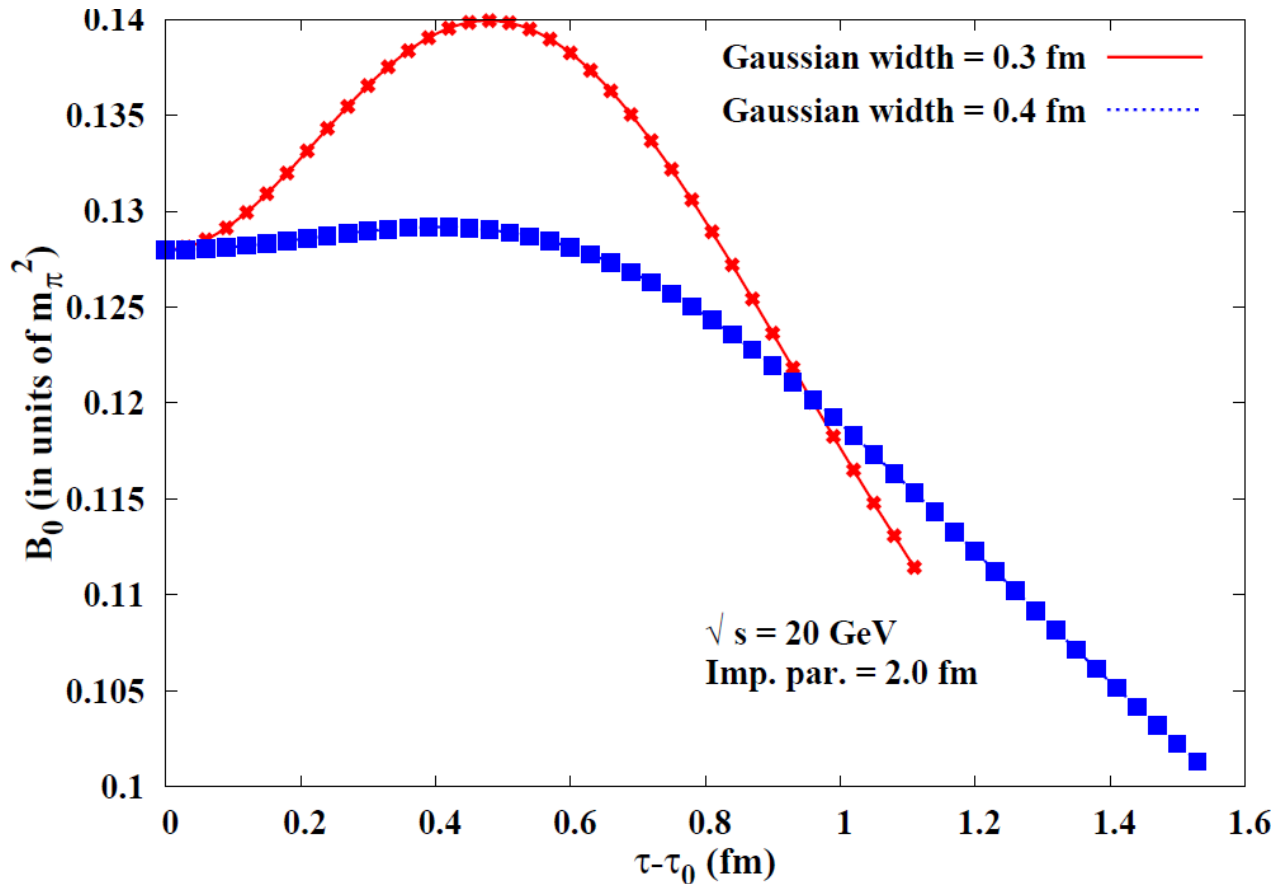
This is in accordance with the original argument of having a stiffer equation of state transverse to magnetic field direction.

However, if the magnetic field extends well beyond the plasma region, then elliptic flow is suppressed by the magnetic field. This will be in accordance with the Lenz's law. This happens when impact parameter is large.

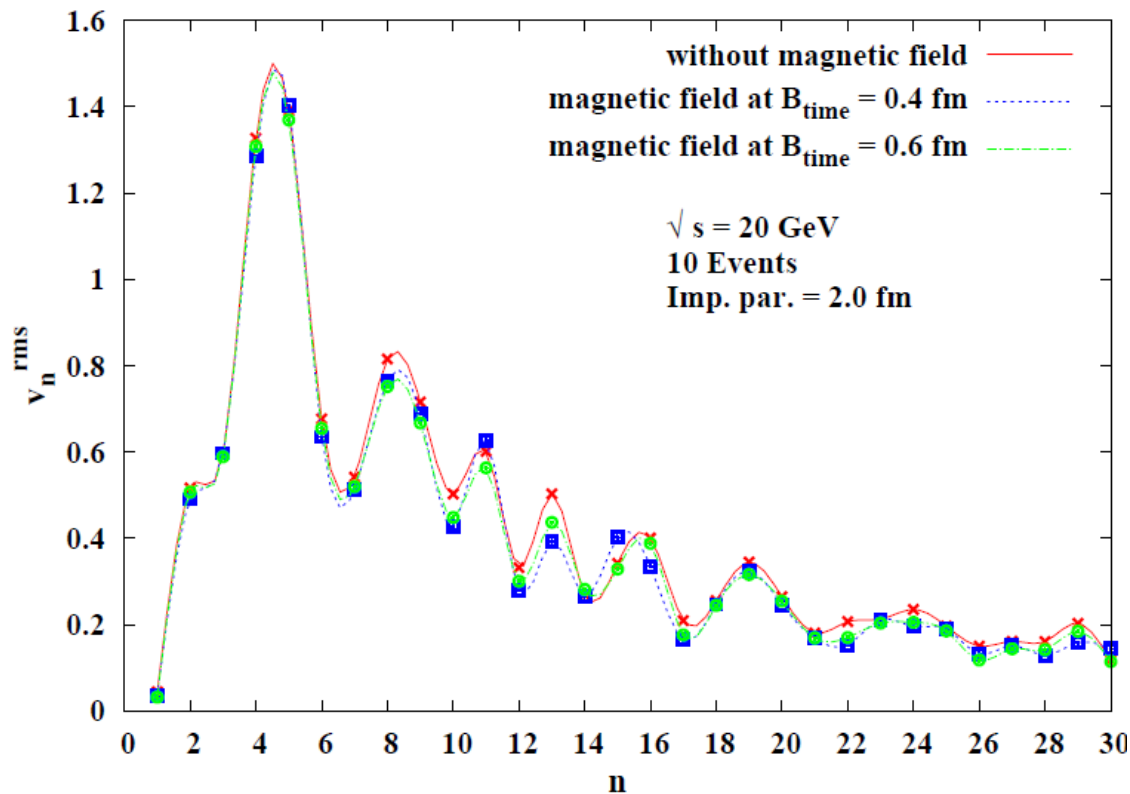
Note: The simulations of Inghirami et al. (arXiv:1609.03042) was for large impact parameter, hence may have been affected by this Lenz's law suppression.

Important to check that simulation for small values of impact parameters.

Result-2: Temporary increase of magnetic field due to flux-rearrangement by evolving initial state density fluctuations
Evolving fluctuations can push around flux lines, leading to temporary, localized concentration of flux.

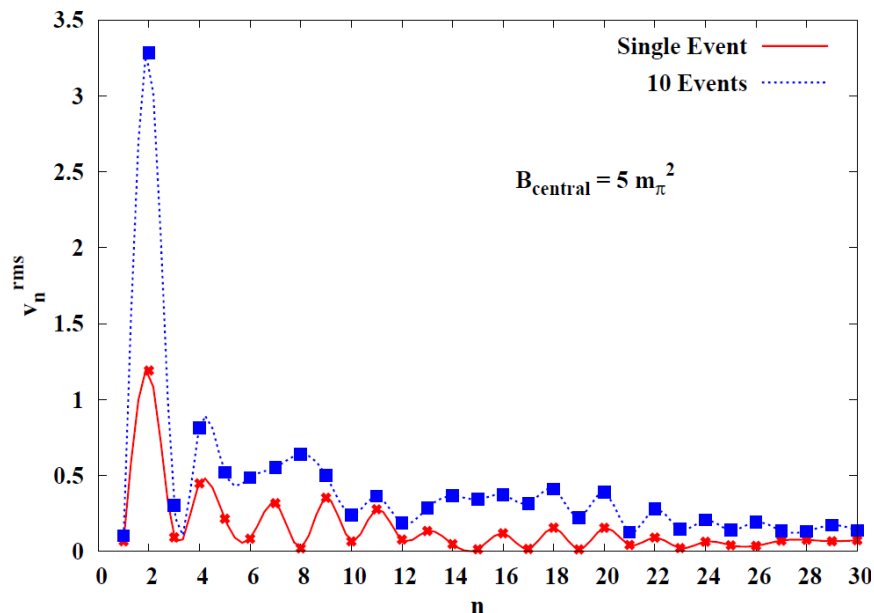


Important for chiral magnetic effect **which is sensitive to local magnetic field (instanton size regions)**. We could only study small fluctuations, for large fluctuations effect can be stronger



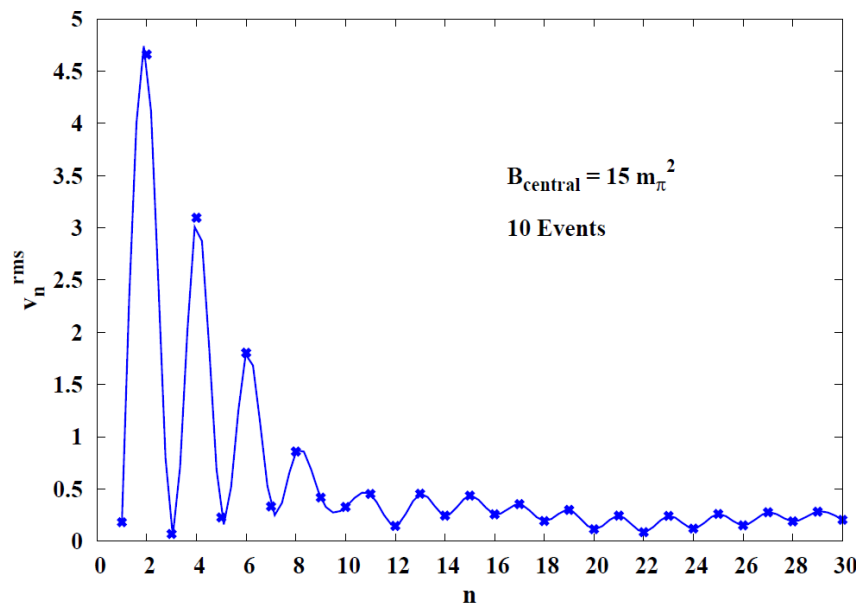
Result-3: Effect of Magnetic field on power spectrum of flow Coefficients.

Top: small magnetic field: $0.1 - 0.4 \text{ m}_{\pi}^2$.
 Very tiny effect

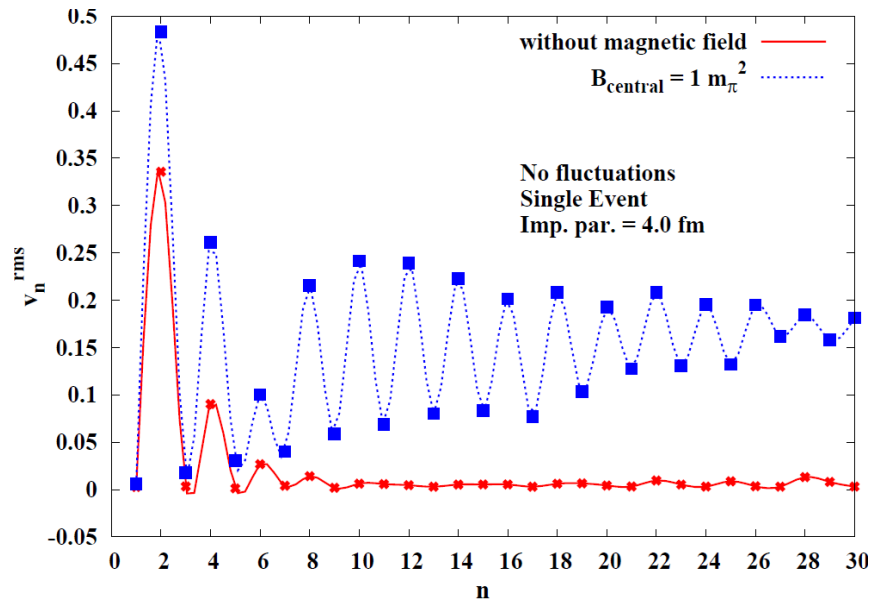


Strong magnetic field 5 m_{π}^2 .
 Significant effect, note first few flow coefficients show some even-odd power difference.

Here magnetic field put in by hand (also taken constant along Y direction for stable simulation, for Gauss' law)



Much stronger magnetic field: 15 m_{π}^2 . Very clear even-odd power difference. Qualitative in nature. Arises from reflection symmetry about the axis of magnetic field, so clear effect only when B dominates over random fluctuations.



To show that fluctuations mask this signal we show power spectrum in the presence of small magnetic field (1 m_{π}^2), but in the absence of any initial state fluctuations. Note: even-odd effect still very strong.

Suppression of even-odd effect occurs due to presence of Initial fluctuations. Independent probe of initial state fluctuations

Conclusions:

1. Power spectrum of flow fluctuations an excellent probe for initial state fluctuations in heavy-ion collisions.

This is exactly like CMBR Power spectrum which is a unique probe of primordial inflationary fluctuations in the Universe.

2. Clear data on suppression of long wavelength fluctuations (lower flow coefficients):
Focus on these to probe long range Correlations at initial stage.
3. Strong initial stage magnetic field will lead to difference in power of even-odd flow coefficients. Initial fluctuations suppress these.

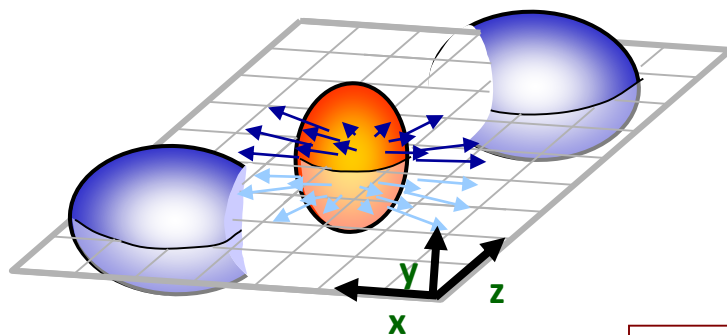
Thus: Suppression of this effect is an independent probe of initial state fluctuations (more sensitive to initial stages due to rapid decay of magnetic field in time).

Thank You

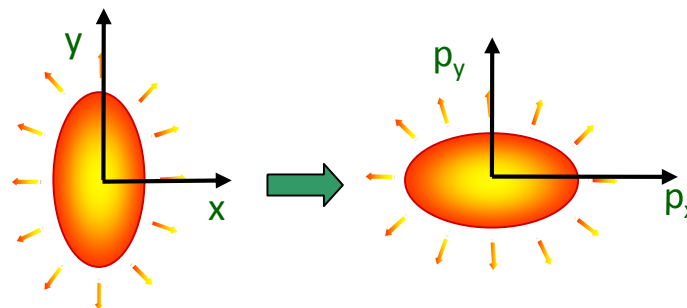
Flow is a phenomenon seen in nucleus-nucleus collisions, which correlates the momentum distributions of the produced particles with the spatial eccentricity of the overlap region.



azimuthal dependence of the pressure gradient.



Reaction plane: z-x plane

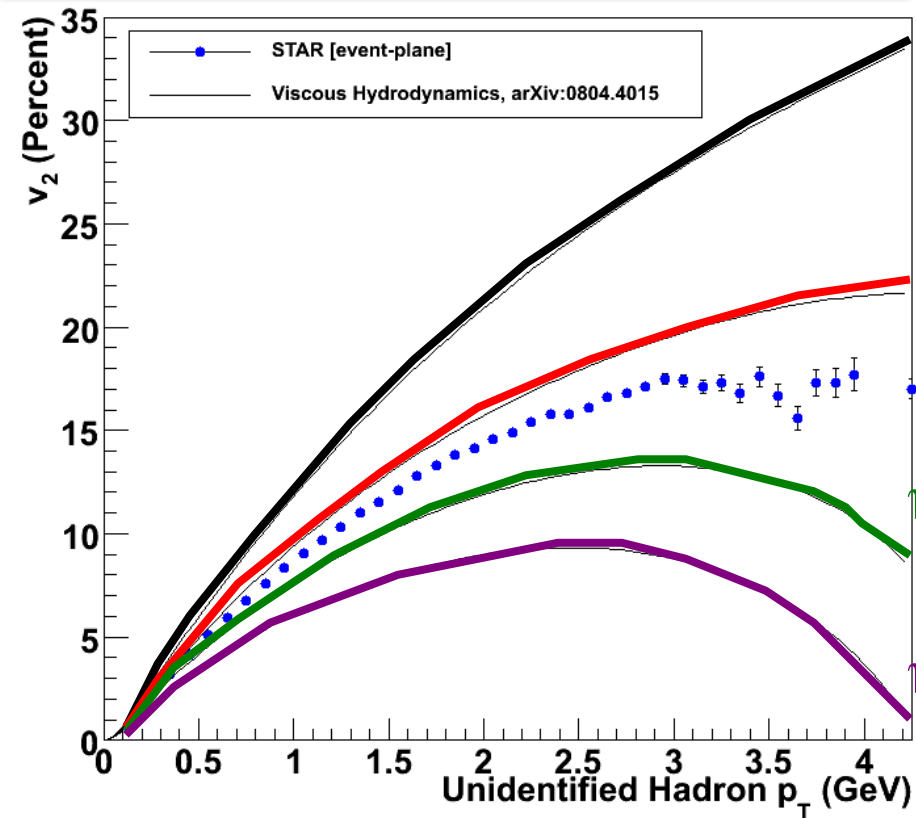


$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$

$$v_2 = \langle \cos 2\varphi \rangle, \quad \varphi = \tan^{-1}\left(\frac{p_y}{p_x}\right)$$

$$E \frac{d^3 N}{d^3 p} = \frac{1}{2\pi} \frac{dN}{p_t dp_t dy} \left(1 + \sum_{n=1} 2v_n \cos(n[\varphi - \Psi_{RP}]) \right) \quad v_2 = \left\langle \cos(2[\varphi - \Psi_2]) \right\rangle$$

Viscosity

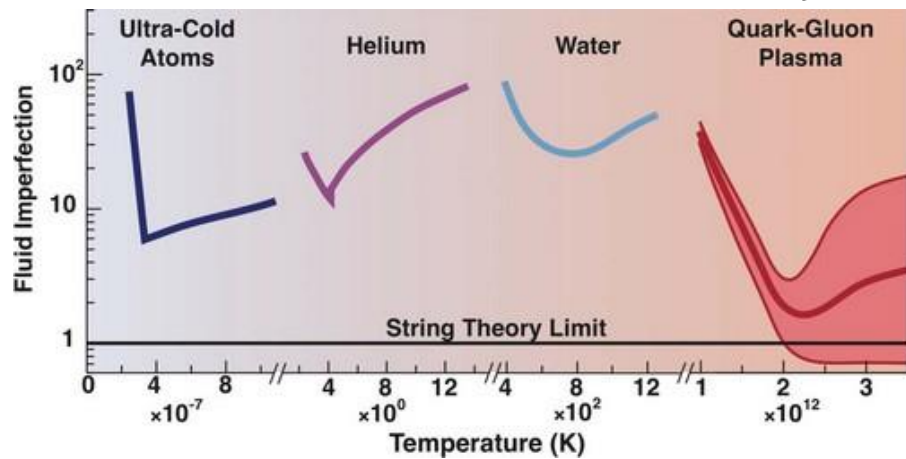


$$\eta/s \sim 0$$

$$\eta/s = 1/4\pi$$

$$\eta/s = 2 \times 1/4\pi$$

$$\eta/s = 3 \times 1/4\pi$$



Almost
Perfect Fluid

Two crucial aspects of the inflationary density fluctuations leading to the remarkable signatures of acoustic peaks in CMBR:
Coherence and Acoustic oscillations.

Note: Coherence of inflationary density fluctuations essentially results from the fact that the fluctuations initially are stretched to superhorizon sizes and are subsequently frozen out dynamically.

Thus, at the stage of re-entering the horizon, when these fluctuations start growing due to gravity, and subsequently start oscillating due to radiation pressure, the fluctuations start with zero velocity.

$$X(t) = A \cos(\omega t) + B \sin(\omega t) = C \cos(\omega t + \phi)$$

where ϕ is the phase of oscillation. Now, the velocity is:

$$dX(t)/dt = -A\omega \sin(\omega t) + B\omega \cos(\omega t) = 0 \text{ at } t = 0$$

→ $B = 0$, So only $\cos(\omega t)$ term survives in oscillations or, phase $\phi = 0$ for all oscillations, irrespective of amplitude. So: all fluctuations of a given wavelength (ω) are **phase locked**. This leads to **clear peaks** in CMBR anisotropy power spectrum

Phase locked oscillations, with varying initial amplitudes
Strong (acoustic) peaks in the power spectrum analysis
(strength of different ℓ modes of spherical harmonics)

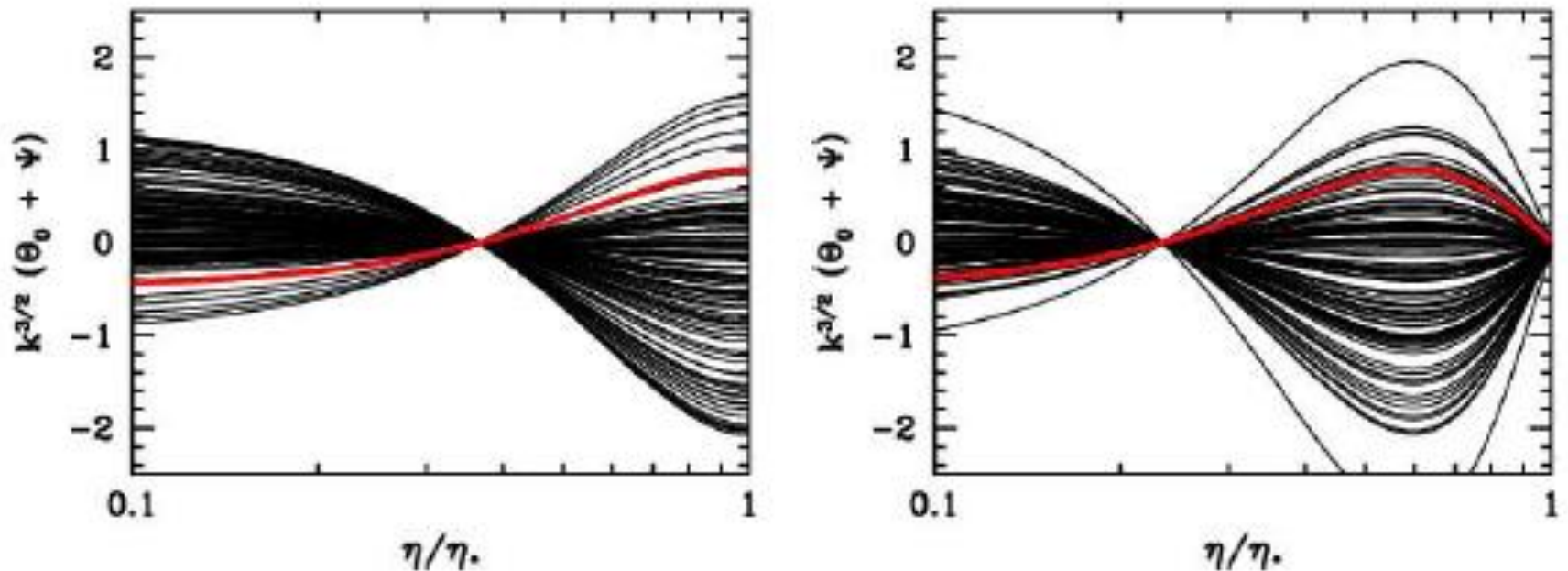


FIGURE 4. The evolution of an infinite number of modes all with the same wavelength. Left panel shows the wavelength corresponding to the first peak, right to the first trough. Although the amplitudes of all these different modes differ from one another, since they start with the same phase, the ones on the left all reach maximum amplitude at recombination; the ones on the right all go to zero at recombination.

Oscillations with varying phases, and initial amplitudes
Peaks get washed out in the power spectrum

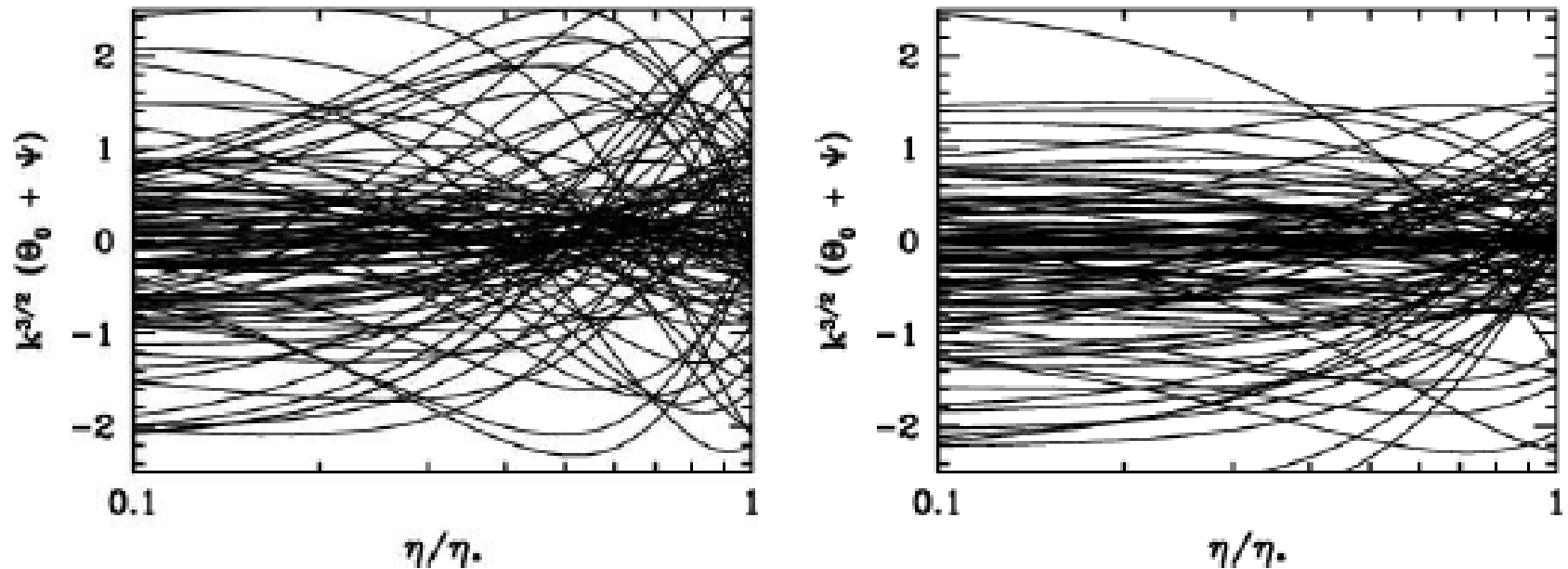


FIGURE 5. Modes corresponding to the same two wavelengths (*First Peak* and *First Trough*) as in Fig. 4, but this time with initial phases scrambled. The anisotropies at the angular scales corresponding to these wavelengths would have identical rms's if the phases were random.

In summary: Crucial requirement for coherence (acoustic peaks):
fluctuations are essentially frozen out until they re-enter the horizon

This should be reasonably true for RHICE 

Note: for RHICE we are considering transverse fluctuations.

Main point: Transverse velocity of fluid to begin with is zero.

Transverse velocity (anisotropic part for us) arises from pressure gradients. However, for a given mode of length scale λ , pressure gradient is not effective for times $t < \lambda/c_s$. In other words, until this time, the mode is essentially frozen, just as in the universe.

(Note: This is just the condition $\lambda > \text{acoustic horizon size } c_s t$)

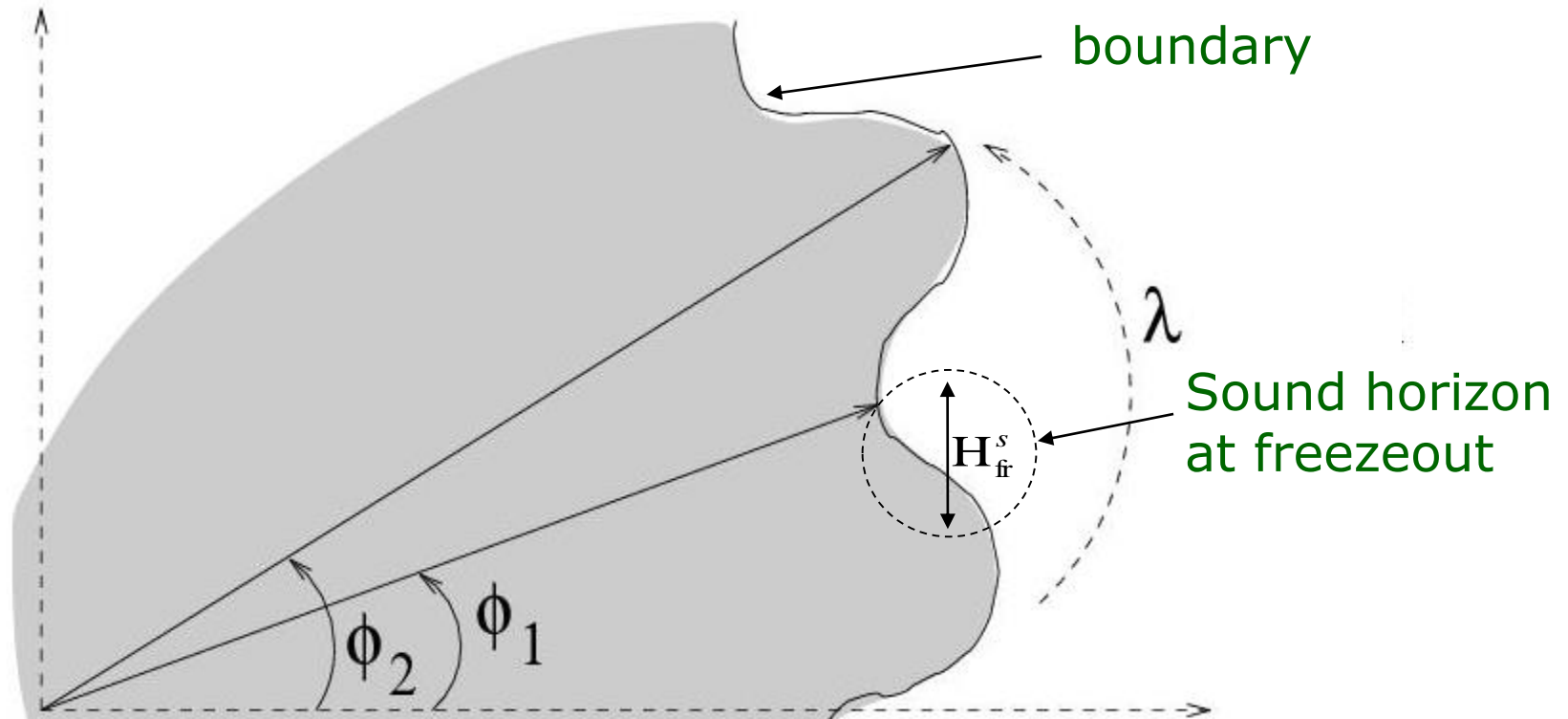
For large wavelengths, those which enter (sound) horizon at times much larger than equilibration time, build up of the radial expansion will not be negligible.

However, our interest is in oscillatory modes.

For oscillatory time dependence even for such large wavelength modes, there is no reason to expect the presence of $\sin(\omega t)$ term at the stage when the fluctuation is entering the sound horizon.

In summary: For RHICE also all fluctuations with scales larger than 1 fm should be reasonably coherent

Presence of such a suppression factor can also be seen for the case when the build up of the flow anisotropies is dominated by the surface fluctuations of the boundary of the QGP region.



When $\lambda \gg H_{\text{fr}}^s$, then by the freezeout time full reversal of spatial anisotropy is not possible: The relevant amplitude for oscillation is only a factor of order $H_{\text{fr}}^s / (\lambda/2)$ of the full amplitude.

CMBR: Changes in the location of peaks with energy-matter density of the Universe, (apparent horizon size changes)

