

Pion and Kaon structure in light cone quark model

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(under preparation)

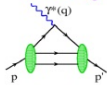


Introduction

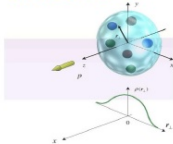
- Probing 3D structure
- Light Cone Quark Model (LCQM)
- $\pi(K)$ Distribution Amplitudes (DAs)
- $\pi(K)$ Parton Distribution Functions (PDFs)
- $\pi(K)$ Generalized Parton Distributions (GPDs)
- $\pi(K)$ Transverse Momentum Distributions (TMDs)

Probing 3D Structure

Elastic Scattering

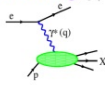


Established extended nature of nucleon

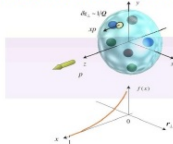


charge and magnetization distribution

Deep Inelastic Scattering

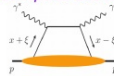


discovered the existence (quarks) inside the nucleon

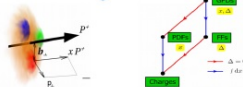
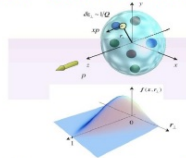


longitudinal momentum distribution

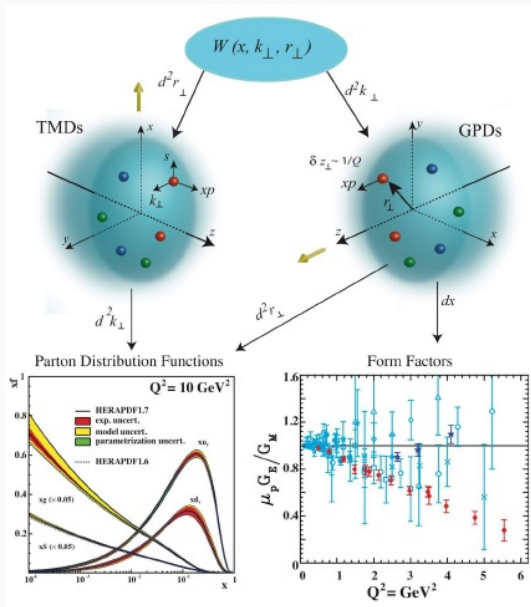
Deeply virtual Compton Scattering



provides 3D spatial structure of the nucleon



- Form factors describe the transverse localization of partons in a fast moving nucleon, irrespective of their longitudinal momenta.
- Parton densities provides the probability to find partons of a given longitudinal momentum fraction x of the parent nucleon with transverse resolution $\frac{1}{Q}$, no information on the transverse position of partons is accessible.



Pion and Kaon structure at EIC

- Understanding the origin and dynamics of hadron structure and in turn that of atomic nuclei is a central goal of nuclear physics.
- An EIC is the ultimate machine like CT scanner for atoms.
- EIC can provide greater insight into the nucleon structure by facilitating multi-dimensional maps of the distributions of partons in space, momentum, spin, and flavor.
- Pions having small mass as compared to other hadrons and can propagate over distances significantly larger than the typical hadronic scale.
- They are critical in generating the force that binds neutrons and protons within nuclei, but also appear to greatly influence the properties of isolated nucleons.

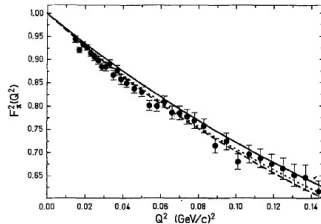
- No understanding of matter is complete without a detailed explanation of the role of pions. It is thus crucial to expose the role played by pions in nucleon structure.
- In addition to this, kaons also play very special role in the mass budget of pion and proton in QCD.

Pion and Kaon Structure at the Electron-Ion Collider

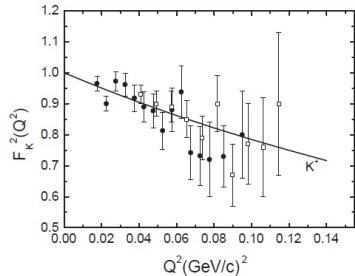
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Light Cone Quark Model (LCQM)

- The LCQM finds application in QCD low-scale regime.
- The LCQM is successful in explaining electromagnetic form factors of the pion and kaon and is consistent with the experimental data.



B-Q Ma, Z. Phys. A 345
(1993) 321.



Bo-Wen Xiao, EPJ A 15 (2002)
523.

- The charge radii and decay constant are well predicted and consistent with experimental data.
- The hadron eigenstate in connection with multi-particle Fock eigenstates $|n\rangle$ is

$$\begin{aligned}
 |M(P^+, \mathbf{P}_\perp, S_z)\rangle &= \sum_{n, \lambda_i} \int \prod_{i=1}^n \frac{dx_i d^2 \mathbf{k}_{\perp i}}{\sqrt{x_i} 16\pi^3} 16\pi^3 \delta(1 - \sum_{i=1}^n x_i) \delta^{(2)}(\sum_{i=1}^n \mathbf{k}_{\perp i}) \\
 &\times |n : x_i P^+, x_i \mathbf{P}_\perp + \mathbf{k}_{\perp i}, \lambda_i\rangle \psi_{n/M}(x_i, \mathbf{k}_{\perp i}, \lambda_i),
 \end{aligned}$$

- The light-cone wavefunction in LCQM is

$$\begin{aligned}
 \psi_{S_z}^F(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) &= \varphi(x, \mathbf{k}_\perp) \chi_{S_z}^F(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) \\
 \varphi(x, \mathbf{k}_\perp) &\rightarrow \text{momentum space wavefunction} \\
 \chi_{S_z}^F(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) &\rightarrow \text{spin wavefunction}
 \end{aligned}$$

- The light-cone wavefunction of the pion (or kaon) can be obtained through the transformation of the SU(6) instant form wavefunction using Melosh-Wigner rotation.
- For the pseudo-scalar meson, the spin wavefunction is

$$\chi_T = \frac{\chi_1^\uparrow \chi_2^\downarrow - \chi_2^\uparrow \chi_1^\downarrow}{\sqrt{2}}$$

where $\chi_i^{\uparrow,\downarrow}$ are the two component Pauli spinors.

- One can relate the light-cone spin states $|J, \lambda\rangle_F$ and instant-form spin states $|J, s\rangle_T$ as

$$|J, \lambda\rangle_T = \sum U_{s\lambda}^J |J, s\rangle_T$$

where U^J is the Melosh-Wigner rotation operator.

- The final spin wavefunction for the pseudoscalar meson becomes

$$\chi^P(x, \mathbf{k}_\perp) = \sum_{\lambda_1 \lambda_2} \kappa_{S_z}^F \chi_1^{\lambda_1}(F) \chi_2^{\lambda_2}(F)$$

where S_z and λ are the spin projections and quark helicity respectively.

$$\begin{aligned} \kappa_0^F(x, \mathbf{k}_\perp, \uparrow, \downarrow) &= \omega_1 \omega_2 [(q_1^+ + m_1)(q_2^+ + m_2) - q_\perp^2] / \sqrt{2}, \\ \kappa_0^F(x, \mathbf{k}_\perp, \downarrow, \uparrow) &= -\omega_1 \omega_2 [(q_1^+ + m_1)(q_2^+ + m_2) - q_\perp^2] / \sqrt{2}, \\ \kappa_0^F(x, \mathbf{k}_\perp, \uparrow, \uparrow) &= \omega_1 \omega_2 [(q_1^+ + m_1)q_2^L - (q_2^+ + m_2)q_1^L] / \sqrt{2}, \\ \kappa_0^F(x, \mathbf{k}_\perp, \downarrow, \downarrow) &= \omega_1 \omega_2 [(q_1^+ + m_1)q_2^R - (q_2^+ + m_2)q_1^R] / \sqrt{2} \end{aligned}$$

where $q_1^+ = q_1^0 + q_1^3 = x_1 \mathcal{M}$, $q_2^+ = q_2^0 + q_2^3 = x_2 \mathcal{M}$, and $\mathbf{k}_\perp = \mathbf{q}_\perp$, with

$$\mathcal{M}^2 = \frac{m_1^2 + \mathbf{k}_\perp^2}{x_1} + \frac{m_2^2 + \mathbf{k}_\perp^2}{x_2}.$$

- The momentum space wavefunction are adopted using Brodsky-Huang-Lepage (BHL) prescription.
- For pion

$$\varphi^{\pi}(x, \mathbf{k}_{\perp}) = A^{\pi} \exp\left[-\frac{1}{8\beta_{\pi}^2} \frac{\mathbf{k}_{\perp}^2 + m^2}{x(1-x)}\right]$$

- For Kaon

$$\varphi^K(x, \mathbf{k}_{\perp}) = A^K \exp\left[-\frac{\frac{\mathbf{k}_{\perp}^2 + m_1^2}{x} + \frac{\mathbf{k}_{\perp}^2 + m_2^2}{1-x}}{8\beta_K^2} - \frac{(m_1^2 - m_2^2)^2}{8\beta_K^2 \left(\frac{\mathbf{k}_{\perp}^2 + m_1^2}{x} + \frac{\mathbf{k}_{\perp}^2 + m_2^2}{1-x}\right)}\right]$$

-G.P. Lepage, S.J. Brodsky, PRD 22, 2157(1980)

- The two-particle Fock state expansion for pion(kaon) can be described in terms of LCWFs as

$$\begin{aligned}
|\pi(K)(P^+, \mathbf{P}_\perp, S_z)\rangle &= \int \frac{d^2\mathbf{k}_\perp d\mathbf{x}}{2(2\pi)^3} [\psi_{S_z}^{\pi(K)}(x, \mathbf{k}_\perp, \uparrow, \uparrow) |xP^+, \mathbf{k}_\perp, \uparrow, \uparrow\rangle \\
&+ \psi_{S_z}^{\pi(K)}(x, \mathbf{k}_\perp, \uparrow, \downarrow) |xP^+, \mathbf{k}_\perp, \uparrow, \downarrow\rangle \\
&+ \psi_{S_z}^{\pi(K)}(x, \mathbf{k}_\perp, \downarrow, \uparrow) |xP^+, \mathbf{k}_\perp, \downarrow, \uparrow\rangle \\
&+ \psi_{S_z}^{\pi(K)}(x, \mathbf{k}_\perp, \downarrow, \downarrow) |xP^+, \mathbf{k}_\perp, \downarrow, \downarrow\rangle]
\end{aligned}$$

- The constituent quark masses and harmonic scale (β) are two input parameters

B. -Q. Ma, Z. Phys. A 345, 321 (1993)

W. Qian, and B.-Q. Ma, Eur. Phys. J. C 65, 457 (2010)

<i>Meson</i>	<i>Mass in GeV</i>	<i>β in GeV</i>	<i>A</i>
$\pi (u\bar{d})$	$m = 0.2$	0.410	44.236
$K (u\bar{s})$	$m_1 = 0.2, m_2 = 0.556$	0.405	74.033

Distribution Amplitudes

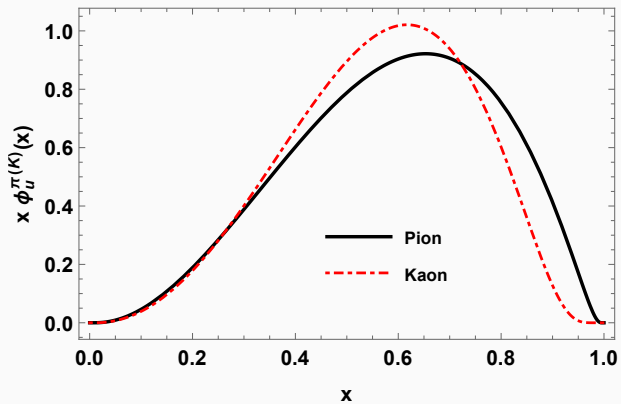
- DAs gives the momentum distribution of meson constituent viz. quark (or anti-quark) in longitudinal direction and represent the coupling of quark-antiquark in the meson.
- The representation of the leading-twist DAs is defined through the correlation

$$\langle 0 | \bar{\Psi}(z) \gamma^+ \gamma_5 \Psi(-z) | \mathcal{P}^+(P) \rangle = i k^+ f_{\mathcal{P}} \int_0^1 dx e^{i(x-1/2)k^+z^-} \phi(x) \Big|_{z^+, \mathbf{z}_{\perp}=0}$$

-Y. Li, P. Maris, and J. P. Vary, PRD 96, 016022 (2017).

- In this work, DAs in terms of LCWFs

$$\frac{f_{\pi(K)}}{2\sqrt{2N_c}}\phi(x) = \frac{1}{\sqrt{2x(1-x)}} \int \frac{d^2\mathbf{k}_{\perp}}{16\pi^3} [\psi_0^{\pi(K)}(x, \mathbf{k}_{\perp}, \uparrow, \downarrow) - \psi_0^{\pi(K)}(x, \mathbf{k}_{\perp}, \downarrow, \uparrow)]$$



- We have also taken into account the evolution of pion DA.
The leading order evolution is

$$\phi^\pi(x, \mu) = 6x(1-x) \sum_{n=0}^{\infty} C_n^{\frac{3}{2}}(2x-1) a_n(\mu),$$

-E. R. Arriola, PRD 66, 094016 (2002).

with

$$a_n(\mu) = \frac{2(2n+3)}{3(n+1)(n+2)} \left(\frac{\alpha(\mu)}{\alpha(\mu_0)} \right)^{\frac{\gamma_n^{(0)}}{2\beta_0}} \int_0^1 dx C_n^{\frac{3}{2}}(2x-1) \phi^\pi(x, \mu_0),$$

where $C_n^{\frac{3}{2}}(2x-1)$ is a Gegenbauer polynomial and n contains only the even values i.e. $n = 0, 2, 4, \dots$, and the factor $\alpha(\mu)$ defines the strong coupling constant and is defined as

$$\alpha(\mu) = \frac{4\pi}{\beta_0 \ln\left(\frac{\mu^2}{\Lambda_{QCD}^2}\right)}$$

- The factor $\frac{\gamma_n^{(0)}}{2\beta_0}$ defines the anomalous dimensions, we have

$$\gamma_n^{(0)} = -2c_F \left(3 + \frac{2}{(n+1)(n+2)} - 4 \sum_{m=1}^{n+1} \frac{1}{m} \right); \quad \text{with} \quad c_F = \frac{4}{3},$$

and

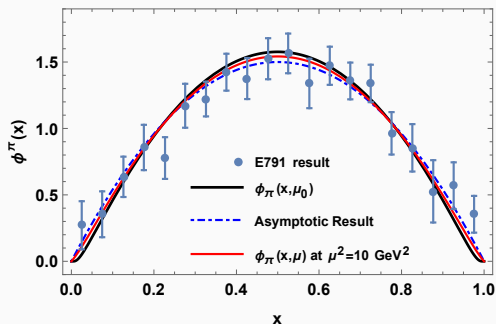
$$\beta_0 = \frac{11}{3}c_A - \frac{2}{3}n_F,$$

where $c_A = 3$ and n_F corresponds to the number of active quarks.

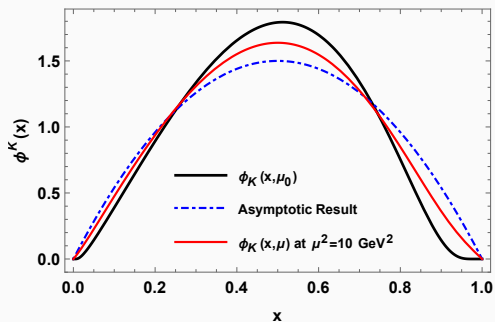
- The relation between moments of DA and its Gegenbauer coefficients are given as

$$\langle z_n \rangle = \int_0^1 dx \, z^n \phi(x, \mu),$$

where z can be $\xi = (2x - 1)$ or x^{-1} .



QCD evolution at energy scale $\mu^2 = 10 \text{ GeV}^2$ with initial scale $\mu_0^2 = 0.246 \text{ GeV}^2$.



QCD evolution at initial scale $\mu_0^2 = 0.194 \text{ GeV}^2$.

- Comparison of first two possible moments and inverse moment in this model with the available theoretical results for pionic DA.

Pion DA	μ [GeV]	$\langle \xi_2 \rangle$	$\langle \xi_4 \rangle$	$\langle x^{-1} \rangle$
Asymptotic	∞	0.200	0.085	3.00
LCQM (This model)	10^4	0.197	0.0836	2.974
LCQM (This model)	1, 2	0.189, 0.192	0.077, 0.079	2.855, 2.894
LF Holographic ($B = 0$)	1, 2	0.180, 0.185	0.067, 0.071	2.81, 2.85
LF Holographic ($B \gg 1$)	1, 2	0.200, 0.200	0.085, 0.085	2.93, 2.95
LF Holographic	~ 1	0.237	0.114	4.0
Platykurtic	2	$0.220^{+0.009}_{-0.006}$	$0.098^{+0.008}_{-0.005}$	$3.13^{+0.14}_{-0.10}$
LF Quark Model	~ 1	0.24[0.22]	0.11[0.09]	
Sum Rules	1	0.24	0.11	-
Renormalon model	1	0.28	0.13	-
Instanton vacuum	1	0.22, 0.21	0.10, 0.09	-
NLC Sum Rules	2	$0.248^{+0.016}_{-0.015}$	$0.108^{+0.05}_{-0.03}$	$3.16^{+0.09}_{-0.09}$
Sum Rules	2	0.343	0.181	4.25
Dyson-Schwinger[RL,DB]	2	0.280, 0.251	0.151, 0.128	5.5, 4.6
Lattice	2	0.28(1)(2)	-	-
Lattice	2	0.2361(41)(39)	-	-
Lattice	2	0.27 ± 0.04	-	-

- Comparison of first four possible moments and inverse moment in this model with the available theoretical results for kaonic DA.

Kaon DA	μ [GeV]	$\langle \xi_1 \rangle$	$\langle \xi_2 \rangle$	$\langle \xi_3 \rangle$	$\langle \xi_4 \rangle$	$\langle x^{-1} \rangle$
Asymptotic	∞	0	0.200	0	0.085	3.00
LCQM (This model)	10^4	0.0093	0.191	0.005	0.0796	2.971
LCQM (This model)	1, 2	0.02, 0.0175	0.169, 0.176	0.011, 0.009	0.065, 0.069	2.838, 2.881
Holographic ($B = 0$)	1, 2	0.055, 0.047	0.175, 0.180	0.021, 0.018	0.062, 0.067	2.55, 2.62
Holographic ($B \gg 1$)	1, 2	0.094, 0.081	0.194, 0.195	0.039, 0.034	0.080, 0.081	2.60, 2.66
Lattice	2	0.036(2)	0.26(2)	-	-	-
LF Quark Model	~ 1	0.06[0.08]	0.21[0.19]	0.03[0.04]	0.09[0.08]	-
Sum Rules	1	0.036	0.286	0.015	0.143	3.57
Dyson-Schwinger[RL,DB]	2	0.11, 0.040	0.24, 0.23	0.064, 0.021	0.12, 0.11	-
Instanton vacuum	1	0.057	0.182	0.023	0.070	-

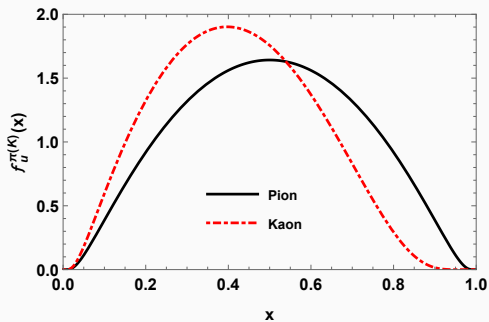
Parton Distribution Functions

- PDFs were introduced by Feynman in 1969.
- PDFs $f(x)$ imparts an information about the probability of finding a parton carrying a longitudinal momentum fraction x inside the hadron. At fixed light-front time, the PDF can be expressed as

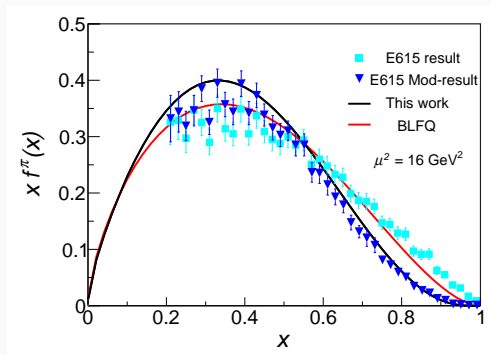
$$f^{\mathcal{P}}(x) = \frac{1}{2} \int \frac{dz^-}{4\pi} e^{ik^+ z^- / 2} \langle \mathcal{P}^+(P); S | \bar{\Psi}(0) \gamma^+ \Psi(z^-) | \mathcal{P}^+(P); S \rangle |_{z^+ = z_{\perp} = 0},$$

- The overlap form of PDF is defined by putting the pion (kaon) states

$$f^{\pi(K)}(x) = \int \frac{d^2 \mathbf{k}_{\perp}}{16\pi^3} \left[|\psi_0^{\pi(K)}(x, \mathbf{k}_{\perp}, \uparrow, \uparrow)|^2 + |\psi_0^{\pi(K)}(x, \mathbf{k}_{\perp}, \uparrow, \downarrow)|^2 + |\psi_0^{\pi(K)}(x, \mathbf{k}_{\perp}, \downarrow, \uparrow)|^2 + |\psi_0^{\pi(K)}(x, \mathbf{k}_{\perp}, \downarrow, \downarrow)|^2 \right].$$



Parton distribution function $f(x)$ for pion and kaon.



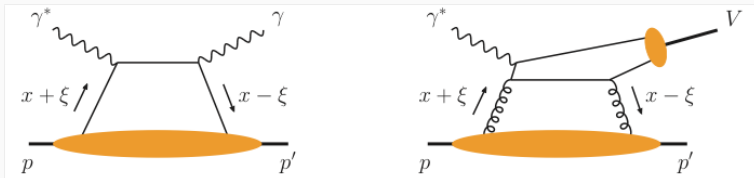
QCD evolution for the pion PDF in LCQM. The analysis is done with using the Higher Order Perturbative Parton Evolution Tool (HOPPET) kit to numerically solve the NNLO DGLAP equations.

Generalized Parton Distributions

- PDFs $f(x)$ imparts an information about the probability of finding a parton carrying a longitudinal momentum fraction x inside the hadron.
- But how partons are distributed in the plane transverse to the motion of hadron?**

This missing information was then compensated in generalized parton distributions (GPDs).

- An essential tool to investigate hadron structure is the study of DIS, where individual quarks and gluons are resolved.



- GPDs have three support regions: $-1 \leq x \leq -\xi$, $-\xi \leq x \leq \xi$ and $\xi \leq x \leq 1$.
- GPDs for the pion and kaon are studied in the LCQM.
- However, we restrict ourself to only DGLAP region i.e. $\xi < x < 1$.
- One can define the correlation to evaluate the unpolarized GPD

$$\begin{aligned}
 H^{\mathcal{P}}(x, 0, t) &= \frac{1}{2} \int \frac{dz^-}{4\pi} e^{ixP^+z^-/2} \\
 &\times \langle \mathcal{P}^+(P') | \bar{\Psi}(0) \gamma^+ \Psi(z) | \mathcal{P}^+(P) \rangle |_{z^+=z_{\perp}=0}.
 \end{aligned}$$

- Unpolarized quark GPD of the pion in terms of overlap of light-cone wavefunctions

$$H^\pi(x, 0, t) = \int \frac{d^2 \mathbf{k}_\perp}{16\pi^3} \left[\left((x\mathcal{M}'^\pi + m)((1-x)\mathcal{M}'^\pi + m) - \mathbf{k}_\perp'^2 \right) \left((x\mathcal{M}^\pi + m)((1-x)\mathcal{M}^\pi + m) - \mathbf{k}_\perp^2 \right) + (\mathcal{M}'^\pi + 2m)(\mathcal{M}^\pi + 2m) \right] \frac{\varphi^{\pi*}(x, \mathbf{k}_\perp') \varphi^\pi(x, \mathbf{k}_\perp)}{\omega_1' \omega_2' \omega_1 \omega_2}$$

with

$$\mathcal{M}^\pi = \sqrt{\frac{m^2 + \mathbf{k}_\perp^2}{x(1-x)}}, \quad \mathcal{M}'^\pi = \sqrt{\frac{m^2 + \mathbf{k}_\perp'^2}{x(1-x)}},$$

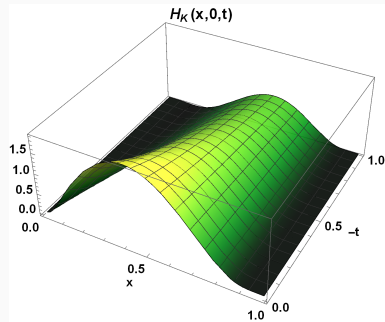
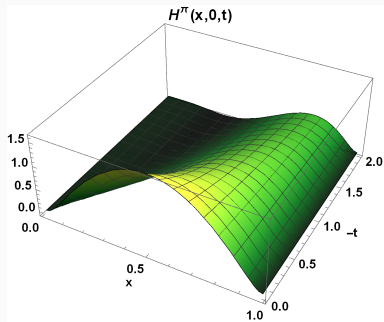
- For the case of kaon, we have

$$H^K(x, 0, t) = \int \frac{d^2 \mathbf{k}_\perp}{16\pi^3} \left[\left((x\mathcal{M}'^K + m_1)((1-x)\mathcal{M}'^K + m_2) - \mathbf{k}_\perp'^2 \right) \left((x\mathcal{M}^K + m_1)((1-x)\mathcal{M}^K + m_2) - \mathbf{k}_\perp^2 \right) + (\mathcal{M}'^K + m_1 + m_2)(\mathcal{M}^K + m_1 + m_2) \right] \frac{\varphi^{K*}(x, \mathbf{k}_\perp') \varphi^K(x, \mathbf{k}_\perp)}{\omega_1' \omega_2' \omega_1 \omega_2}$$

with

$$\mathcal{M}^K = \sqrt{\frac{m_1^2 + \mathbf{k}_\perp^2}{x} + \frac{m_2^2 + \mathbf{k}_\perp^2}{1-x}},$$

$$\mathcal{M}'^K = \sqrt{\frac{m_1^2 + \mathbf{k}_\perp'^2}{x} + \frac{m_2^2 + \mathbf{k}_\perp'^2}{1-x}},$$



Pion and Kaon GPD $H^{\pi,k}(x, 0, t)$.

Transverse Momentum Distributions

- TMDs provide the distribution of partons in momentum space and are functions of longitudinal momentum fraction and transverse momentum.
- TMDs are sensitive to correlations between the motion of partons and their spin, as well as the spin of the parent hadron.
- These correlations can arise from spin-orbit coupling among the partons, about which very little is known to date.
- To evaluate the pion and kaon TMDs, the unintegrated quark-quark correlator is

$$\Phi^{\mathcal{P}}(x, \mathbf{k}_{\perp}; S) = \frac{1}{2} \int \frac{dz^-}{2\pi} \frac{d^2 \mathbf{z}_{\perp}}{(2\pi)^2} e^{ik \cdot z/2} \langle \mathcal{P}^+(P), S | \bar{\Psi}(0) \gamma^+ \Psi(z) | \mathcal{P}^+(P), S \rangle |_{z^+=0}$$

-S. Meissner et. al., PRD 76 034002 (2007).

- In the present calculations, unpolarized pion and kaon TMDs can be obtained by the overlap of LCWFs

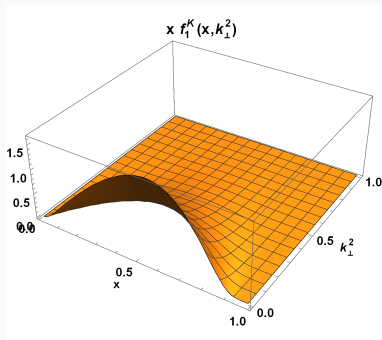
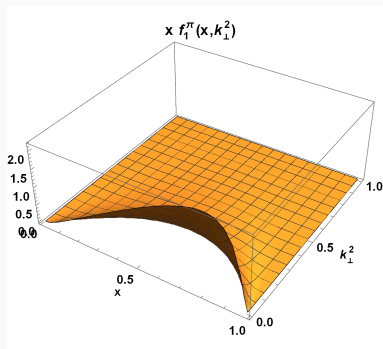
$$f_1^{\pi(K)}(x, \mathbf{k}_\perp^2) = \frac{1}{16\pi^3} [|\psi_0^{\pi(K)}(x, \mathbf{k}_\perp, \uparrow, \uparrow)|^2 + |\psi_0^{\pi(K)}(x, \mathbf{k}_\perp, \uparrow, \downarrow)|^2 + |\psi_0^{\pi(K)}(x, \mathbf{k}_\perp, \downarrow, \uparrow)|^2 + |\psi_0^{\pi(K)}(x, \mathbf{k}_\perp, \downarrow, \downarrow)|^2].$$

- For pion, the TMD evaluated is

$$f_1^\pi(x, \mathbf{k}_\perp^2) = \frac{1}{16\pi^3} \left[((x\mathcal{M}^\pi + m)((1-x)\mathcal{M}^\pi + m) - \mathbf{k}_\perp^2)^2 + (\mathcal{M}^\pi + 2m)^2 \right] \frac{|\varphi^\pi(x, \mathbf{k}_\perp)|^2}{\omega_1^2 \omega_2^2},$$

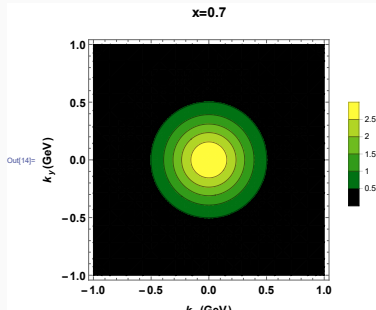
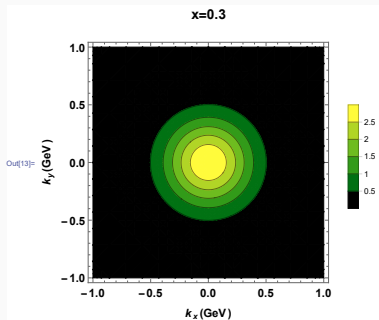
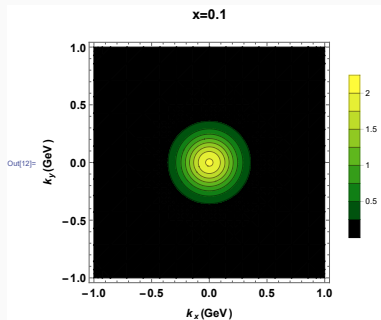
- For kaon, we have

$$f_1^K(x, \mathbf{k}_\perp^2) = \frac{1}{16\pi^3} \left[((x\mathcal{M}^K + m_1)((1-x)\mathcal{M}^K + m_2) - \mathbf{k}_\perp^2)^2 + (\mathcal{M}^K + m_1 + m_2)^2 \right] \frac{|\varphi^K(x, \mathbf{k}_\perp)|^2}{\omega_1^2 \omega_2^2}$$

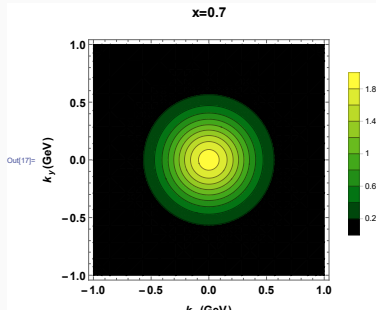
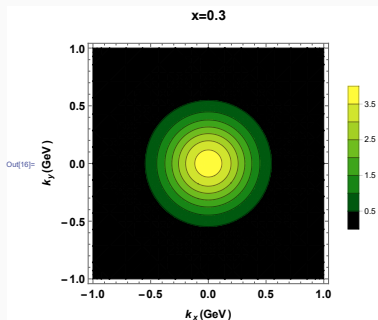
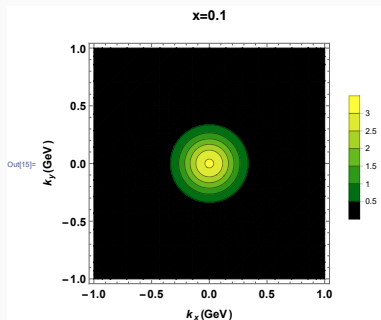


Unpolarized TMD $x f_1(x, \mathbf{k}_\perp^2)$ as a function of x and \mathbf{k}_\perp^2 for pion, and kaon.

Pion TMDs in momentum space



Kaon TMDs in momentum space



TMD Evolution

- Unpolarized TMD evolution is factorized in the framework of Collins-Soper-Sterman (CSS) formalism
J. C. Collins et. al., NPB 250, 199 (1985).
- Includes the perturbative effects in the larger energies and momentum transfer regimes.
- TMD evolution is executed in \mathbf{b}_\perp space by taking the Fourier transformation of $f_1(x, \mathbf{k}_\perp^2)$

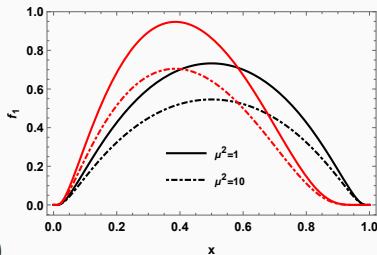
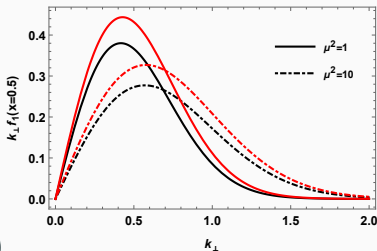
$$\tilde{f}_1(x, \mathbf{b}_\perp^2) = \int_0^\infty d\mathbf{k}_\perp \mathbf{k}_\perp J_0(\mathbf{k}_\perp \mathbf{b}_\perp) f_1(x, \mathbf{k}_\perp^2),$$

$$\tilde{f}_1(x, \mathbf{b}_\perp^2; \mu) = \tilde{f}_1(x, \mathbf{b}_\perp^2) R(\mu, \mu_0, \mathbf{b}_\perp) e^{-g_k(\mathbf{b}_\perp) \ln \frac{\mu}{\mu_0}},$$

where $g_k(\mathbf{b}_\perp)$ is the Sudakov factor and TMD evolution factor $R(\mu, \mu_0, \mathbf{b}_\perp)$ is

$$R(\mu, \mu_0, \mathbf{b}_\perp) = \exp\left(\ln \frac{\mu}{\mu_0} \int_\mu^{\mu_b} \frac{d\mu'}{\mu'} \gamma_K(\mu') + \int_{\mu_0}^\mu \frac{d\mu'}{\mu'} \gamma_F\left(\mu', \frac{\mu^2}{\mu'^2}\right)\right),$$

- Pion TMD is evolved from the model scale $\mu_0^2 = 0.246\text{GeV}^2$ whereas kaon TMD is from $\mu_0^2 = 0.194\text{GeV}^2$.



(a) Unpolarized TMD evolution. Red line is for kaon and black is for pion.

Conclusions

- Quark distributions in pion and kaon is studied from the overlap of LCWFs
- LCWFs are obtained from instant-form wavefunctions through the Melosh-Wigner rotation.
- DAs for pion and kaon under QCD evolution are in good agreement with asymptotic DA result.
- NNLO DGLAP evolution has been done for pion PDF. Results are in good agreement with FNAL-E-615 experimental results.
- Pion and kaon GPDs obtained from the overlap of LCWFs in DGLAP region (providing the spatial distribution of partons).
- Pion and kaon TMDs are also obtained from the overlap of LCWFs (providing the momentum tomography).
- TMDs evolution has been done to includes the perturbative effects.

Thank you!