

Constraints on the $g \rightarrow \pi^0$ fragmentation function from RHIC data

Abhiram Kaushik

Institute of Mathematical Sciences



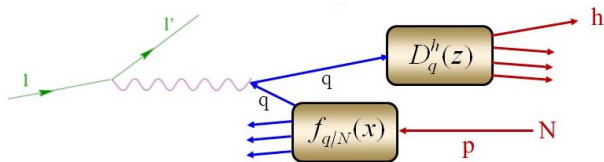
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In collaboration with: Rohini Godbole (IISc), Piet Mulders (NIKHEF) and Emanuele Nocera (NIKHEF)

Fragmentation functions

- Describe the transition of a single parton into a particular colourless bound state i.e., a hadron, in high energy processes D. Sivers, Phys. Rev. D41, 83 (1990); 43, 261 (1991).
- Complementary to PDFs.
- Essential part of a factorised description of QCD processes.

$$d\sigma(ep \rightarrow \pi X) \propto \sum_q f_q(x, Q^2) \times \frac{d\sigma^2(eq \rightarrow e' q')}{dx} \times D_q^h(z, Q^2)$$



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 - ▶ Plsum FFs: $D_i^{\pi^+ + \pi^-}(z, Q)$
 - ▶ Related to FFs for neutral pion through isospin:
$$D_i^{\pi^0}(z, Q) = \frac{1}{2} D_i^{\pi^+ + \pi^-}(z, Q)$$

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 - ▶ Related to FFs for neutral pion through isospin:
$$D_i^{\pi^0}(z, Q) = \frac{1}{2} D_i^{\pi^+ + \pi^-}(z, Q)$$
- ② The need to systematically include theory uncertainties in fits of fragmentation functions.
(In the vein of E. Nocera *et al.*, Eur.Phys.J. C79, 931 (1906.10698))
 - ▶ Brief discussion in the context of the above exercise

Preliminary: brief review of NNPDF methodology

Actually the methodology of Monte Carlo fits in general

- Consider the functional space of PDFs/FFs.
- Limited statistics of experimental \implies PDF cannot be measured without any error, hence consider a probability distribution $\rho[f]$ in the functional space.
- Given this, predictions for observables can be written as,

$$\begin{aligned}\langle O \rangle &= \int d[f] \rho[f] O(f) \\ \sigma_O &= \sqrt{\langle O^2 \rangle - \langle O \rangle^2}\end{aligned}$$

- Perform a Monte Carlo sampling of this functional space:
 - ▶ Generate N_{rep} pseudodata-sets from existing data-set (information on the errors of the data goes into this) and perform a fit of PDF/FF to each pseudodata-set.
 - ▶ Functional form of PDF/FF is parameterised using a neural network:

$$f_i(x) = x^\alpha (1-x)^\beta \text{NN}_i(x)$$

Preliminary: brief review of NNPDF methodology

- Now we perform a Monte Carlo integral over the functional space of PDFs:

$$\begin{aligned}\langle O \rangle_{\text{MC}} &= \frac{1}{N_{\text{rep}}} \sum_{i=1}^{N_{\text{rep}}} O[f_i] \\ \sigma_{O_{\text{MC}}} &= \sqrt{\langle O^2 \rangle_{\text{MC}} - \langle O \rangle_{\text{MC}}^2}\end{aligned}$$

- Propagating errors is straightforward
- Reweighting allows one to *easily and quickly* incorporate new data into the fit without doing a whole new fit (which would be time and resource consuming)

$$\langle O \rangle_{\text{MC}}^{\text{reweighted}} = \frac{1}{N_{\text{rep}}} \sum_{i=1}^{N_{\text{rep}}} w_i O[f_i]$$

Since weights of replicas are different, the reweighted ensemble corresponds to a lower N_{eff} where $0 < N_{\text{eff}} < N_{\text{rep}}$. If $N_{\text{eff}} \ll N_{\text{rep}}$, reweighting is not enough — a full fit is required to incorporate new data.

Background and motivations

The NNFF 1.0 FFs are the first fit of fragmentation functions using the NNPDF methodology. [Eur.Phys.J. C77 \(2017\), 516 \(1706.07049\)](#)

- Includes FFs of pions, kaons and protons.
- Fit performed to data in [single-inclusive annihilation \(SIA\)](#):
 $e^+e^- \rightarrow h + X$.
- Monte Carlo framework \implies FF uncertainties are a faithful representation of the uncertainties of the data.

In NNFF 1.0 the [gluon FFs constrained only indirectly](#) (through evolution).

Gluon FFs required in the description of SIDIS ($ep \rightarrow h + X$) and collider processes ($pp \rightarrow h + X$).

We are specifically interested in the $g \rightarrow \pi^0$ FF.

Background and motivations

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- DSS considered e^+e^- , ep and pp data [Phys.Rev. D75 \(2007\) 114010](#), [hep-ph/0703242](#)
- AKK considered e^+e^- and pp data [Nucl. Phys. B 803 \(2008\) 42](#), [0803.2768](#)

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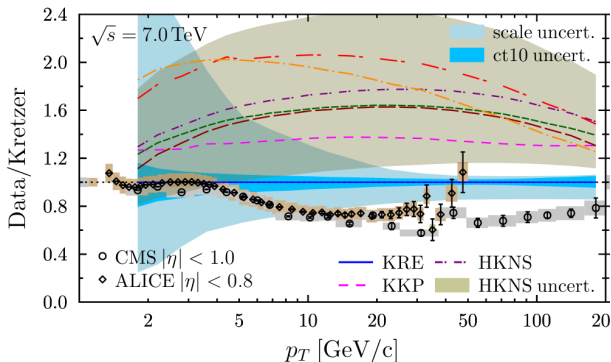
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Motivation I: A fit of π^0 FFs that includes pp data *and also* a systematic treatment of uncertainties (which can be propagated onto the predictions) is still not available.

Background and motivations

Gluon FFs are important in the production of hadrons in colliders.

Issues exist with FF of gluon into unidentified charged hadrons:



D'Enterria, Eskola, Helenius, Pakkunen, Nucl. Phys. B 883 615-628 (1311.1415)

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Motivation 2: Want to study if the same issues exist for the π^0 case...

And do a combined analysis including data from RHIC ($\sqrt{s} = 200$) and ATLAS ($\sqrt{s} = 2.76$ TeV, 7 TeV, 8 TeV)

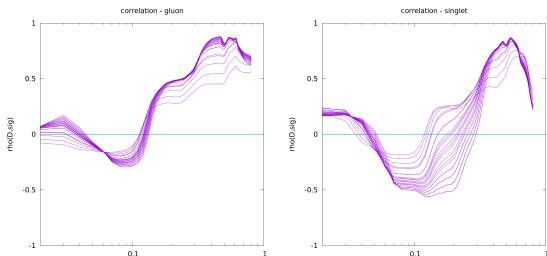
(In this presentation, will be looking at a preliminary fit with only RHIC data)

Reweighting NNFF1.0 with RHIC $pp \rightarrow \pi^0 + X$ data

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Details of the analysis

- Prior (input): NNFF1.0 Plsum FFs at LO and NLO — 100 replicas
- Two datasets:
 - ① PHENIX 2003: $0 < p_T < 14$ GeV [Phys.Rev.Lett. 91 \(2003\) 241803](#)
 - ② PHENIX 2007: $0 < p_T < 18$ GeV [Phys.Rev. D76 \(2007\) 051106](#)
- Two p_T cuts:
 - ① $p_T = 5$ GeV: $N_{\text{dat}} = 24$
“Standard cut” — region where theory uncertainties are under control.
 - ② $p_T = 2$ GeV: $N_{\text{dat}} = 36$
“Experimental cut” — region with significant theory uncertainties.
- Scale choice: $\mu_R = \mu_F = \mu_{\text{frag}} = p_T$



Reweighting NNFF1.0 with RHIC $pp \rightarrow \pi^0 + X$ data

Results

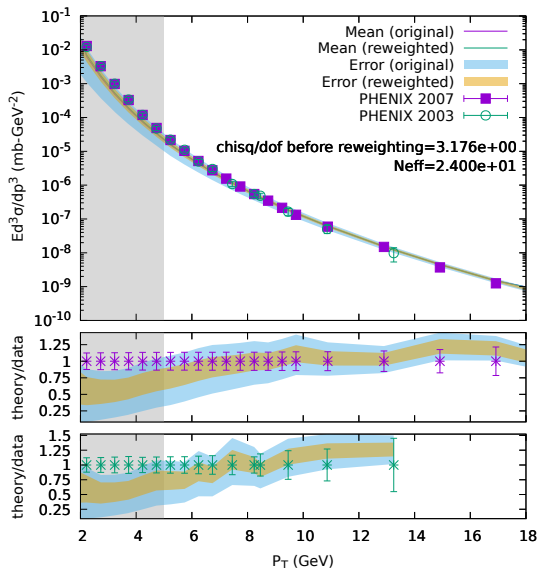


Figure: NLO results with standard p_T cut.

Reweighting NNFF1.0 with RHIC $pp \rightarrow \pi^0 + X$ data

Results — Discussion

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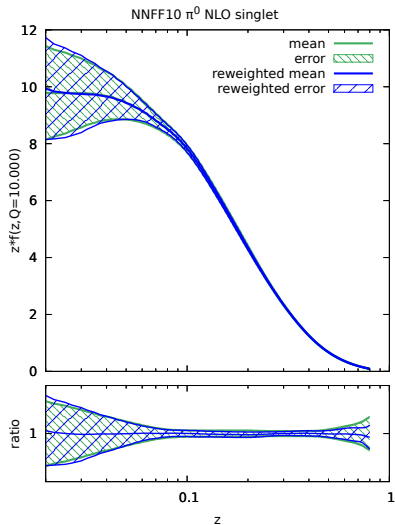
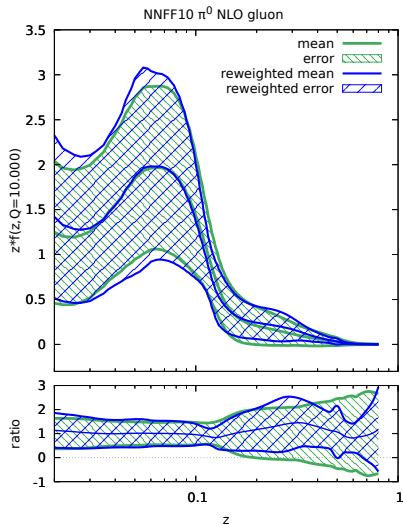
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All the relevant numbers:

p_T cut	FF	$\chi^2/\text{d.o.f}$ before reweighting	$\chi^2/\text{d.o.f}$ after reweighting	N_{eff}
5 GeV	NLO	3.2	1.0	24
	LO	5.3	1.0	1
2 GeV	NLO	9.0	1.0	8
	LO	14.4	1.1	2

Reweighting NNFF1.0 with RHIC $pp \rightarrow \pi^0 + X$ data

Results — Discussion



Gluon FF in $z > 0.1$ significantly affected. Singlet FF remains unaffected.

Including theory uncertainties in the analysis

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So far in our analysis — and in most analyses of PDF/FFs — we have used the χ^2 function as defined below as the standard metric of the *goodness of fit*:

$$\chi^2 = \sum_{i=1}^{N_{\text{dat}}} \frac{(D_i - T_i)^2}{\sigma_i^2} \quad (\text{assuming uncorrelated errors})$$

The use of this metric is based on — at least to a certain extent— two assumptions:

- 1 That σ_i is a faithful representation (in a statistical sense of course) of the deviation between the *true value* T_i of the observable and the value of the data-point D_i .
- 2 That it is possible to find the right theoretical input (values for the fit parameters) such that the difference between the theory prediction T_i and the true value T_i is minimal.

That is the deviation between the data and theory prediction should be on average the same size as the error on the data point, implying that $\chi^2/N_{\text{dat}} \approx 1$.

Including theory uncertainties in the analysis

The second assumption is obviously not tenable at all times — the theory model may have limitations — may be uncertainties in the theory prediction.

We use **fixed-order perturbative QCD** to describe the process:

- Missing Higher Order Uncertainties (MHOUs) — dependence on unphysical scales: μ_R , μ_F , μ_{frag}
- misses out higher twist effects (negligible in this case)

These theory uncertainties will have to be taken into account for a rigorous analysis — lots of literature on this subject.

Including theory uncertainties in the analysis

In particular a recent paper by the NNPDF collaboration contains a systematic treatment of this issue in the context of PDF fits.

E. Nocera *et al.*, Eur.Phys.J. C79, 931 (1906.10698)

They propose a *theory covariance matrix* in the same spirit as the experimental covariance matrix:

- Assuming that the true values of the observable \mathcal{T} are distributed Gaussianly around the theory predictions T , with the width of the Gaussian being the theory uncertainty.
- This matrix would give the conditional probability for the true values given the theory predictions.
- To include theory uncertainties in the analysis define an improved χ^2 function:

$$\chi^2 = \sum_{i,j=1}^{N_{\text{dat}}} (D_i - T_i)(C_{ij} + S_{ij})(D_j - T_j)$$

- C_{ij} : experimental covariance matrix, contains information on the statistical and systematic errors on the data
- S_{ij} : theory covariance matrix, contains information on theory uncertainties

Including theory uncertainties in the analysis

Estimating the theory covariance matrix S_{ij} :

- the estimation of missing higher order uncertainties in a fixed-order calculation — and thereby the elements of S_{ij} — is a subject of much discussion.

M. Cacciari and N. Houdeau, JHEP 09, 039 (2011), 1105.5152

A. David and G. Passarino, Phys. Lett. B726, 266 (2013), 1307.1843

E. Bagnaschi *et al.*, JHEP 02, 133 (2015), 1409.5036

- We use a simple three-point scale variation method:
 - ▶ Set all scales equal $\mu = \mu_R = \mu_F = \mu_{\text{frag}}$
 - ▶ Vary μ in $\{Q/2, Q, 2Q\}$ where $Q = p_T$.
 - ▶ Then S_{ij} is give by

$$S_{ij} = \frac{1}{2}(\Delta_i^+ \Delta_j^+ + \Delta_i^- \Delta_j^-)$$

where $\Delta_i^+ = T_i(2Q) - T_i(Q)$ and $\Delta_i^- = T_i(Q/2) - T_i(Q)$.

We now take into account the theory uncertainties in the χ^2 metric and see how well the data agrees with the prior \longrightarrow

Including theory uncertainties in the analysis

p_T cut	FF	$\chi^2/\text{d.o.f}$ without theory uncertainties	$\chi^2/\text{d.o.f}$ with theory uncertainties	$\Delta \frac{\chi^2}{\text{d.o.f}}$
5 GeV	NLO	3.2	1.8	1.4
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- With LO theory: tensions between prior SIA dataset and PHENIX dataset. Inclusion of theory uncertainties in analysis could help resolve this.

Note: not discussing results of reweighting for this case since prior fit did not consider theory uncertainties.

Summary

- Presented preliminary results of a reweighting of NNFF1.0 π^0 FFs with PHENIX data on midrapidity π^0 production at $\sqrt{s} = 200$ GeV.
- PHENIX data can significantly reduce the uncertainties on the gluon component of the NLO NNFF1.0 π^0 FFs.
- Including theory uncertainties in the fit can potentially improve the description of data. Could resolve tensions between the SIA and PHENIX datasets when using LO theory.
- A more comprehensive fit incorporating further data from ALICE π^0 data ($\sqrt{s} = 2.76, 7, 8$ TeV) and a prior with 2000 replicas (and hopefully theory uncertainties) in progress.

Thank you!