Infrared renormalon effects in color dipole TMD PDF at small-x

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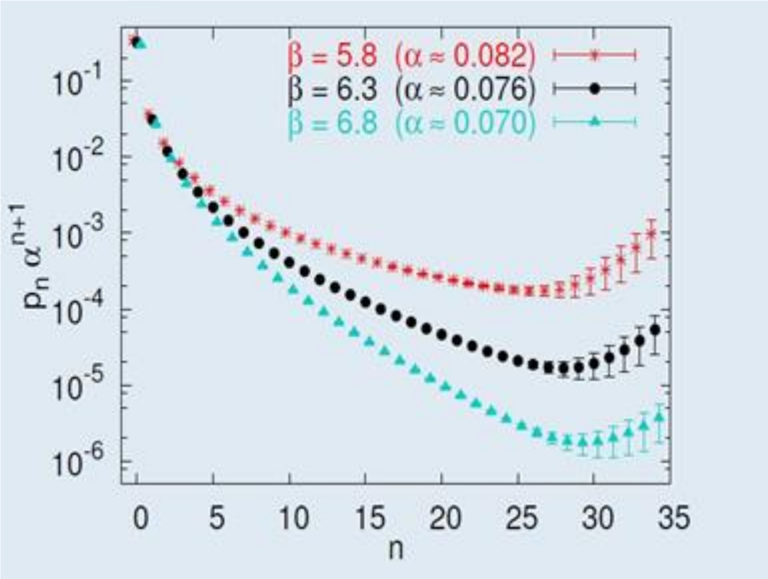
Perturbation Theory

- In Quantum Field Theory the perturbation series is divergent.

$$\sum_n C_n \alpha_s^n$$

- Even after term by term renormalization of mass and charge, perturbation series are divergent.
- This no way restrict predictions from the perturbation series for practical applications.

Each term in the series first decrease and gradually approaches to a minimum and then start to increase without any limit.



Perturbative series

Dyson's argument [Phys. Rev. 85, 631 (1952)]
Perturbation series are typically divergent with zero radius of convergence.

A series is convergent for every real number α such that
$|\alpha| < r$ and $0 < r < \infty$, r is the radius of convergence.
Dyson's idea
For $r \neq 0$ then series also converges for negative value of α.

Combinatorial argument [B. Lautrap] There could be single gauge invariant diagram that contribute to amplitude to grow it like $n!$ in the nth order term of the perturbation series.

Saddle-point method [Lipatov] Showed that in scalar QFT the series coefficients c_n indeed grow factorially with the order $n!$.

Best guess of perturbation theory.

There are ways in which a value can be assign to the sum of a given divergent series.

Borel resummation

For factorially divergent series Borel summation is mostly used.

Borel transform

Corresponding Borel integral

$$\sum_{n=0}^{\infty} r_n \alpha^{n+1} \to B(t) = \sum_{n=0}^{\infty} r_n \frac{t^n}{n!}$$

$$\int_{n=0}^{\infty} dt\, e^{-t/\alpha} B(t)$$

As a generating function for the series coefficient.

Divergent behaviour is encoded in the singularities of Borel transform.

Borel resummation

The analytic structure of the Borel transform $B(t)$ is connected to the large order behavior of the perturbation series $\sum_n r_n \alpha^{n+1}$, with $r_n = n!\, a^n$.

For $a < 0$

- The series alternates in sign.
- The singularity is on the negative real axis of complex Borel plane.
- The series is Borel resummable.

For $a > 0$

- Fixed-sign series.
- The singularity is on the positive real axis of complex Borel plane.
- The series is not Borel resummable.

Singularity on positive real axis spoil Borel summability.

Ambiguity of the Borel integral

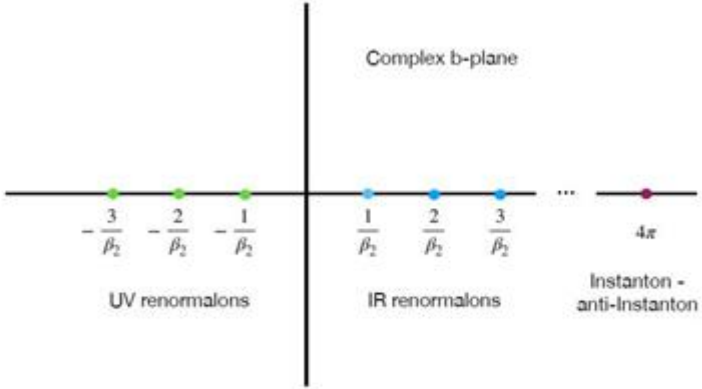
- The Borel integral still be defined by moving the contour above or below the singularities.

- Difference between various regularization prescriptions gives an estimate of the uncertainty due to the Borel singularity.

Divergence of perturbative series in QCD

The divergence of perturbative series in QCD is reflected through the pole singularities in Borel plane

- Infrared (IR) renormalons
- Ultraviolet (UV) renormalons
- Instantons.



Infrared renormalon

Renormalon contribution is found to estimate the non-perturbative QCD contribution to small-x physics.

- The non-perturbative contribution stems from the IR renormalons.
- IR renormalon are the first Borel non-summable singularity of the QCD expansions.
- It lie on the positive real axis of the Borel plane and the integral is not defined.

IR Renormalons in (color dipole) gluon distribution

Relation between dipole amplitude $\mathcal{N}$ and the unintegrated dipole gluon distribution [Levin and Ryskin;1987]

$$\int d^2b\,\mathcal{N}(r_\perp, b_\perp, x) = \frac{2\pi}{N_c}\int d^2k_\perp\,\left(1 - e^{ik_\perp . r_\perp}\right)\alpha_s(k_\perp^2)\frac{1}{k_\perp^2}\mathcal{F}(x, k_\perp)$$

Unintegrated dipole gluon distribution around saturation region

$$\mathcal{F}(x, k_\perp) \propto \frac{k_\perp^2}{Q_s^2(x)}\,\exp\left[-\frac{k_\perp^2}{Q_s^2(x)}\right] \approx \frac{k_\perp^2}{Q_s^2(x)} \quad (\text{when } k_\perp^2 \ll Q_s^2(x)).$$

Introducing running coupling

Contribution to dipole amplitude around saturation region.

$$\int \frac{d^2k_\perp}{k_\perp^2}\left(1 - e^{ik_\perp . r_\perp}\right)\alpha_s(k_\perp^2)\frac{k_\perp^2}{Q_s^2} \; .$$

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1+\alpha_s(\mu^2)\beta_2 \ln(Q^2/\mu^2)}$$

This gives a divergent perturbative series with coefficients proportional to $n!$.

A running QCD coupling generates renormalons.

$$\frac{r_\perp^2}{2Q_s^2}\mu^4\alpha(\mu)\sum_{n=0}^{\infty}\left(\frac{\alpha(\mu)\beta_2}{2}\right)^n n!\,C \xrightarrow{\text{Resumming}} \sim \int_0^\infty db\, e^{-b/\alpha(\mu^2)}\frac{1}{b-2/\beta_2}$$

This is the effect of Infrared QCD renormalons.

Uncertainty due to renormalon

The size of the IR renormalon uncertainty

$$\sim \frac{1}{\beta_2} \frac{r_\perp^2}{Q_s^2} \Lambda_{QCD}^4$$

Non-perturbative origin of the uncertainty reflects from the fact that the result is proportional to Λ_{QCD} .

IR Renormalons revisited

Unintegrated dipole gluon distribution at small transverse momentum.
[M.Siddiqah, N.Vasim, K.Banu,T.Bhattacharyya and R.Abir; Phys.Rev. D97 (2018) no.5, 054009]

- In leading log accuracy

$$\mathcal{F}(x, k_\perp) \propto \ln\left(\frac{k_\perp^2}{4Q_s^2}\right) \exp\left[-\tau \ln^2\left(\frac{k_\perp^2}{4Q_s^2}\right)\right]$$

- The small-x evolution modifies behaviour of gluon distribution from linear normal to logarithmic log normal.

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On resumming the contribution inside the saturation region , the effect of renormalon in the Borel integral as

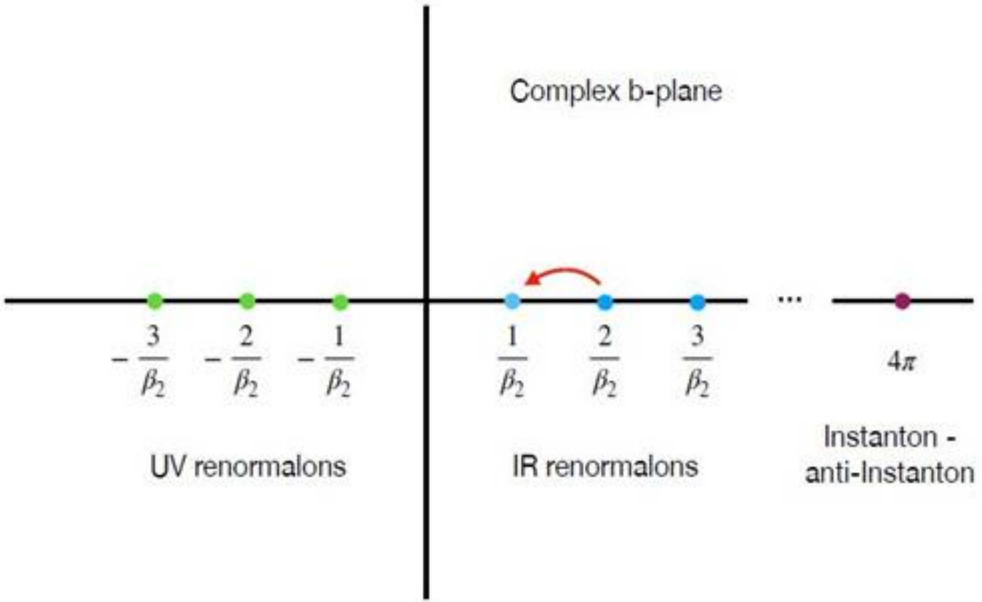
$$\int_0^\infty db \ \exp\left(-\frac{(1+\epsilon)\, b}{\alpha(\mu^2)}\right) \frac{1}{b-1/\beta_2}$$

The first IR pole is at $1/\beta_2$

IR Renormalons revisited

[N.Vasim, R.Abir arxiv.1911.12055]

The non-linear saturation effects at small-x shift the first IR pole at the Borel plane towards zero from $2/\beta_2$ to $1/\beta_2$



Associated uncertainty

$$\sim \frac{r_\perp^2}{\beta_2} \Lambda_{QCD}^2 \ln \frac{\Lambda_{QCD}^2}{4Q_s^2} \exp\left(-\tau \ln^2 \frac{\Lambda_{QCD}^2}{4Q_s^2}\right)$$

An enhanced non-perturbative uncertainty $\longrightarrow$ $\mathcal{O}\left(\Lambda_{QCD}^2\right)$

Presence of the Sudakov type soft factor indicates that the saturation effect tend to suppress the IR renormalon effects at small-x.

Summary

- Non-perturbative effects in QCD actually stems from the diverging nature of the perturbative series.
- The non-linear saturation effects at small-x shift the first IR pole at the Borel plane towards zero.
- The saturation effects suppress the renormalon effect through a Sudakov type of soft factor.

Thank you.