

Charge Asymmetry in Low-Energy Lepton-Proton Scattering

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Proton radius puzzle

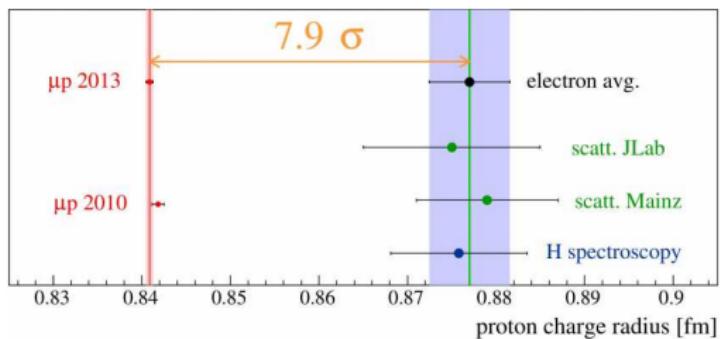
Proton Radius Puzzle in 2013: A series of precise experiments of the proton charge radius produced results which were in strong disagreement.

From hydrogen spectroscopy
and electron scattering

$$r_p(\text{CODATA}) = 0.8775(51)\text{fm}$$

From muon spectroscopy

$$r_p(\text{CREAMA}) = 0.84(39)\text{fm}$$



Many new experiments were planned for proton radius measurement to address this issue using

- Hydrogen spectroscopy
- Electron Scattering of proton
- Muon scattering of proton

Shrinking Proton

Two very recent measurements support a smaller radius

Electron spectroscopy→

$$r_p = 0.833 \pm 0.010 \text{ fm}$$

N. Bezginov, et al., Science 365 (2019)

Electron scattering→

$$r_p = 0.831 \pm 0.014 \text{ fm}$$

W. Xiong, et al., Nature 575 (2019)

Shrinking Proton

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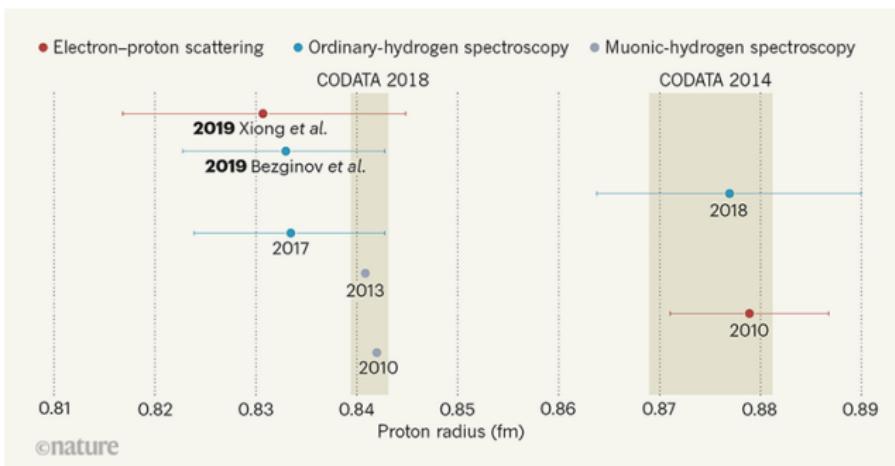
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N. Bezginov, et al., Science 365 (2019)

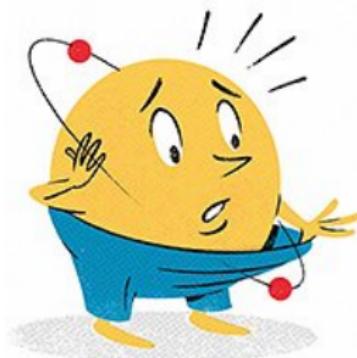
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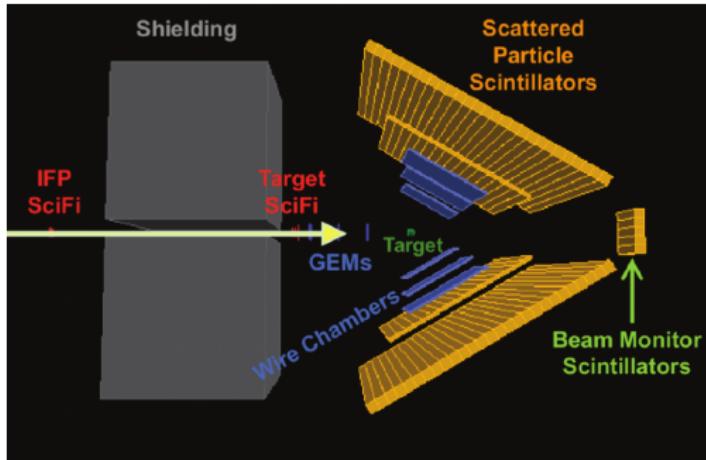
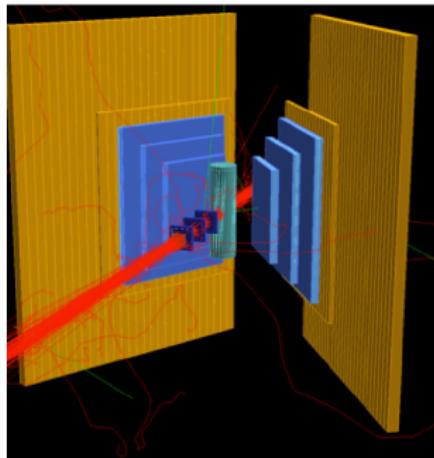
W. Xiong, et al., Nature 575 (2019)



Nature 575, 61-62 (2019)



MUSE Experiment



Intention is to

- measure both $\mu^+ p/\mu^- p$ and $e^+ p/e^- p$ scattering cross section in the low Q^2 region,
- extract form factors and proton radii
- compare ep and μp scattering as test of lepton universality,
- determine **charge asymmetry** to see two-photon exchange effects on the radius extraction.

MUSE continued

lepton charge asymmetry can be defined as

$$\delta_{asym} = \frac{\sigma^{e^- p} - \sigma^{e^+ p}}{\sigma^{e^- p} + \sigma^{e^+ p}}$$

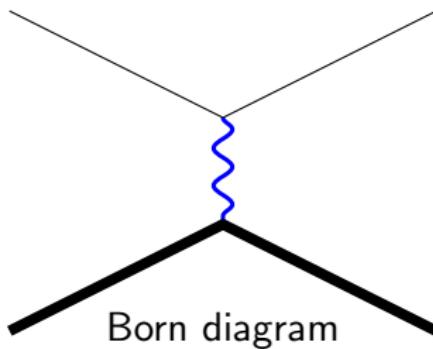
Range of Q^2 for MUSE in GeV^2

	$k = 0.115\text{GeV}$	$k = 0.153\text{GeV}$	$k = 0.210\text{GeV}$
Electron			
$\theta = 20^\circ$	0.0016	0.0028	0.0052
$\theta = 100^\circ$	0.027	0.046	0.082
Muon			
$\theta = 20^\circ$	0.0016	0.0028	0.0052
$\theta = 100^\circ$	0.026	0.045	0.080

R. Gilman, et al., MUSE experiment: R-12-01.1, arXiv:1303.2160 (2013).

Born cross section

Unpolarized elastic e-p cross section in born approximation



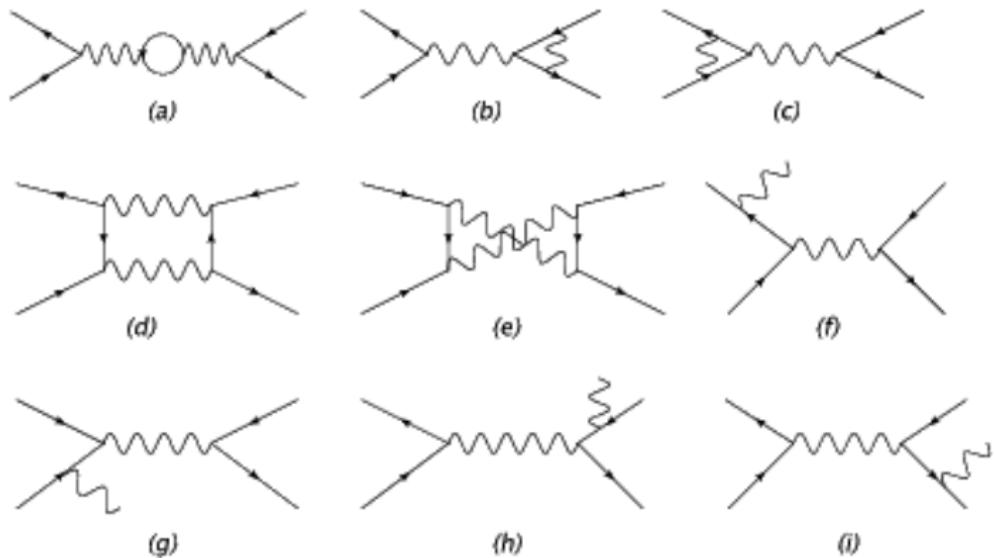
$$\left(\frac{d\sigma}{d\Omega}\right)_{Born} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \frac{\varepsilon G_E^2(Q^2) + \tau G_M^2(Q^2)}{\varepsilon(1+\tau)}$$

$\epsilon = (1 + 2(1 + \tau) \tan^2 \frac{\theta}{2})^{-1}$ → virtual photon polarization parameter

$$\tau = \frac{Q^2}{4M^2}$$

G_E/G_M → Proton electric form factor/Proton magnetic form factor

Going beyond born approximation

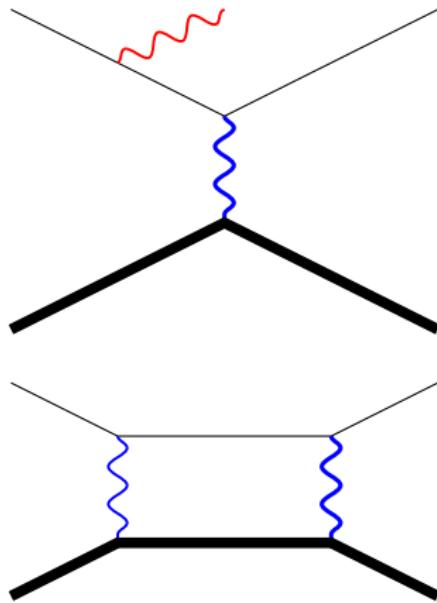


$$d\sigma \propto |\mathcal{M}_{Born}|^2 + 2\text{Re}[\mathcal{M}_{Born}^\dagger (\mathcal{M}_{vac} + \mathcal{M}_{vert}^l + \mathcal{M}_{vert}^p)] + 2\text{Re}[\mathcal{M}_{Born}^\dagger \mathcal{M}_{tpe}] + \\ (\mathcal{M}_{brem}^l + \mathcal{M}_{brem}^p)^2 + \mathcal{O}(\alpha^4)$$

$$\left(\frac{d\sigma}{d\Omega} \right) = (1 + \delta) \left(\frac{d\sigma}{d\Omega} \right)_{Born}$$

Now for $e^- \rightarrow e^+$

- $\mathcal{M}_{born} \rightarrow -\mathcal{M}_{born}$
- $\mathcal{M}_{vac} \rightarrow -\mathcal{M}_{vac}$
- $\mathcal{M}_{vert}^l \rightarrow -\mathcal{M}_{vert}^l$
- $\mathcal{M}_{vert}^p \rightarrow -\mathcal{M}_{vert}^p$
- $\mathcal{M}_{tpe} \rightarrow \mathcal{M}_{tpe}$
- $\mathcal{M}_{brem}^l \rightarrow \mathcal{M}_{brem}^l$
- $\mathcal{M}_{brem}^p \rightarrow -\mathcal{M}_{brem}^p$



$$\delta_{asym} = \delta_{tpe} + \delta_{brem}^{lp}$$

TPE and interference between lepton and proton bremsstrahlung are only the charge odd contributions and will contribute to the charge asymmetry.

Why EFT?

- While lepton photon vertex can be studied with QED, QCD is not applicable to proton photon vertex at low energy.
- It demands that the hadronic vertex be either phenomenologically modeled using *ad hoc* form factors, or derived *ab initio* in the context of an effective field theory (EFT).
- While phenomenological form factor brings in some degree of model dependence, EFT provide a systematic model independent prescription to study low energy dynamics of hadron.

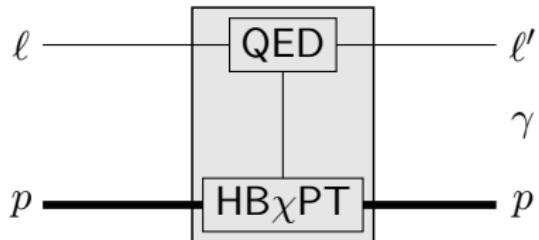


Figure : The ℓp scattering process and bremsstrahlung emission in EFT.

Heavy Baryon Chiral Perturbation Theory

Wienberg Folk Theorem: *If one writes down the most general possible Lagrangian, including all terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S-matrix.*

Typical features of EFT:

- Has a well defined **Hard Scale** Λ separating long and short distance physics. e.g., in ChPT $\Lambda_\chi \simeq m_n \simeq 1\text{GeV}$
- Heavy/High energy DOFs, e.g., $q, \bar{q}, g, W^\pm, Z, \dots \leftarrow$ "integrated out" (unresolved)
- The Effective low-energy DOFs are hadrons → Mesons, Baryons, Resonances,....
- The Effective interactions reflects the underlying symmetries and its breaking pattern

Chiral Symmetry (Symmetry of the Strong Interaction)

- QCD Lagrangian is invariant under $SU(2)_V \times SU(2)_A$ transformation in the limit of vanishing quark masses.
- $SU(2)_V$ symmetry → Well known Isospin Symmetry
 $SU(2)_A$ symmetry → Meson Spectrum does not reflect it ?
- Axial symmetry is spontaneously broken \Leftrightarrow Pions are the Goldstone-boson
- Low energy hadronic processes are dominated by pions \mapsto All observable can be expressed as an expansion in pion masses and momenta
(origin of Chiral Perturbation Theory)

Heavy-Baryon ChPT

- The heavy-baryon formulation of ChPT consists in an expansion of matrix element in terms of $\frac{p}{m_N}$.
 $p \rightarrow$ nucleon momentum, $m_N \rightarrow$ nucleon mass
- The four momentum of the heavy nucleon is separated into large piece of the order of the nucleon mass and a small residual component.

$$P_\mu = m_N v_\mu + l_\mu, \quad v^2 = 1 \text{ and } v.l = -\frac{l^2}{2m_N} \quad (1)$$

- The Lagrangian is organised in increasing power of $\frac{1}{m_N}$

$$\mathcal{L}_{\pi N} \rightarrow \bar{N}_v (iv.D + g_A S.u) N_v + \sum_{n=1}^{\infty} \frac{1}{(2m_N)^n} \mathcal{L}_{\pi N; 1/m_N}^{(n)}$$

S. Scherer arXiv:hep-ph/0210398

- Where

$$\mathcal{L}_{\pi N}^0 = \bar{N}_v (iv.D + g_A S.u) N_v$$

$$\mathcal{L}_{\pi N}^{(1)} = \bar{N} \left\{ \frac{1}{2M} (v \cdot D)^2 - \frac{1}{2M} (D \cdot D) + \dots \right\} N$$

is the LO HB χ PT Lagrangian and NLO Lagrangian with only the terms relevant to our analysis

$$D_\mu = \partial_\mu + \Gamma_\mu - iv_\mu^{(s)}, \quad \Gamma_\mu = \frac{1}{2}[u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - il_\mu)u^\dagger]$$

$$U(x) = \exp[i\tau.\pi(x)/F_\pi] = u^2$$

$g_A \rightarrow$ axial vector coupling constant, $S_\mu \rightarrow$ spin operator

- $r_\mu, l_\mu, v_\mu^{(s)} \rightarrow$ external fields
- For interaction with external electromagnet field,

$$r_\mu = l_\mu = -e\frac{\tau_3}{2}\mathcal{A}_\mu, \quad v_\mu^{(s)} = -\frac{e}{2}\mathcal{A}_\mu$$

Kinematics and Notations:

$p(E_l, \vec{p})$ → four momentum of the incident lepton

$p'(E'_l, \vec{p}')$ → four momentum of the scattered lepton

$p_p(E'_p, \vec{p}_p)$ / $p'_p(E'_p, \vec{p}'_p)$ → Four momentum of the target/recoiled proton

$k^*(E_\gamma^*, \vec{k}^*)$ → four momentum of real photon

$k(E_\gamma, \vec{k})$ → four momentum of virtual photon

m/M → mass of lepton/proton

θ → scattering angle

The four momentum transfer,

$$Q^2 = (p - p')^2 = 2(m_l^2 - EE' + |\vec{p}| |\vec{p}'| \cos(\theta))$$

$$\text{in rest frame of the target proton, } E' = E + \frac{Q^2}{2M} = E + \mathcal{O}\left(\frac{1}{M}\right)$$

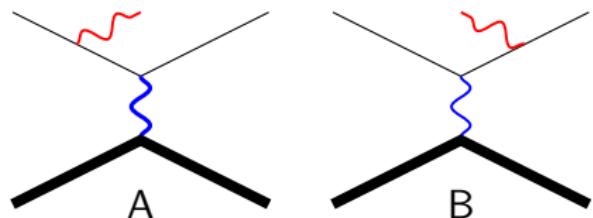
(2)

lepton mass is explicitly included in all our expressions

Bremsstrahlung contribution to asymmetry

At LO in HB χ PT

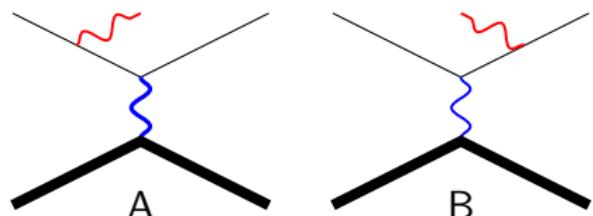
- proton does not radiate at LO
- No asymmetry contribution at LO



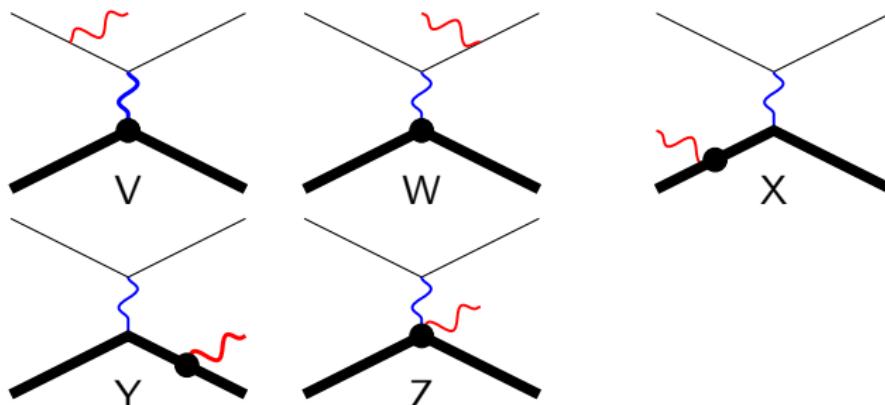
Bremsstrahlung contribution to asymmetry

At LO in HB χ PT

- proton does not radiate at LO
- No asymmetry contribution at LO



At NLO in HB χ PT



Only the interference between the LO lepton bremsstrahlung and NLO proton bremsstrahlung is going to contribute to charge asymmetry at $\mathcal{O}(\frac{1}{M})$

$$\begin{aligned}
 |\mathcal{M}_{brem}^{lp}|^2 &= 2\text{Re} \sum (\mathcal{M}_A + \mathcal{M}_B)^\dagger (\mathcal{M}_X + \mathcal{M}_Y + \mathcal{M}_Z) \\
 &= \left(\frac{\alpha}{2\pi^2 M} \right) \text{Re} \sum |M_0|^2 \left[\frac{p \cdot p_p}{(v \cdot k^*)(p \cdot k^*)} - \frac{p \cdot p'_p}{(v \cdot k^*)(p \cdot k^*)} \right. \\
 &\quad \left. - \frac{p' \cdot p_p}{(v \cdot k^*)(p' \cdot k^*)} + \frac{p' \cdot p'_p}{(v \cdot k^*)(p' \cdot k^*)} \right] \tag{3}
 \end{aligned}$$

We can read off the bremsstrahlung contribution to charge asymmetry as

$$\delta_{brem}^{lp} = \left(\frac{\alpha}{2\pi^2 M} \right) [p \cdot (p_p - p'_p)L_1 + p' \cdot (p'_p - p_p)L_2]$$

Where,

$$\begin{aligned}
 L_1 &= (2\pi\mu)^{2\epsilon} \int_0^\Delta (k^*)^{d-4} dk^* d^{d-2} \Omega_{k^*} \frac{1}{(p \cdot k^*)} & E_\gamma^* < \Delta \rightarrow \text{SOFT} \\
 L_2 &= 2\pi\mu)^{2\epsilon} \int_0^\Delta (k^*)^{d-4} dk^* d^{d-2} \Omega_{k^*} \frac{1}{(p' \cdot k^*)} & E_\gamma^* > \Delta \rightarrow \text{HARD}
 \end{aligned}$$

We have used phase space slicing method to divide the photons into soft and hard.

$$\begin{aligned}
L_1 &= \frac{2\pi}{E} \left\{ \frac{1}{2\beta} \left[\frac{1}{|\epsilon|} + \gamma_E - \log \frac{4\pi\mu^2}{\Delta^2} \right] \log \frac{1+\beta}{1-\beta} + H(\beta) \right\} \\
L_2 &= \frac{2\pi}{E'} \left\{ \frac{1}{2\beta'} \left[\frac{1}{|\epsilon|} + \gamma_E - \log \frac{4\pi\mu^2}{\Delta^2} \right] \log \frac{1+\beta'}{1-\beta'} + H(\beta') \right\}
\end{aligned} \tag{4}$$

where $H(\beta) = \frac{1}{2} \int_{-1}^1 \frac{\log(1-x^2)}{[1-\beta x]} dx$

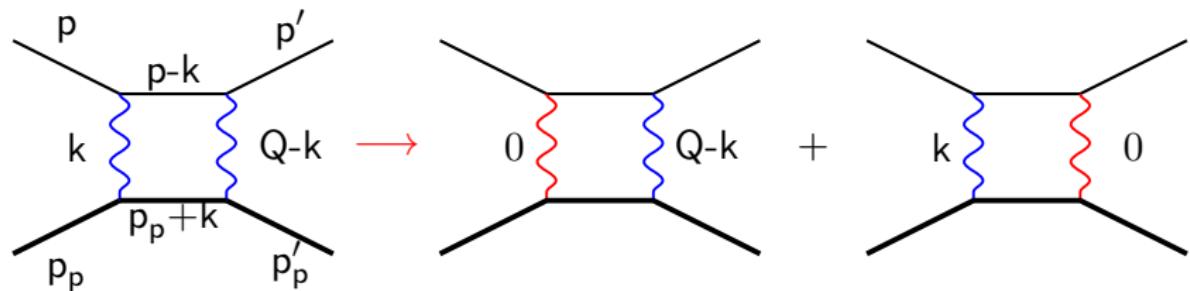
$$\delta_{brem}^{lp}(\Delta) = \text{IR}_{brem}^{lp} + \frac{\alpha}{\pi M} \left[p \cdot (p_p - p'_p) \frac{H(\beta)}{E} + p' \cdot (p'_p - p_p) \frac{H(\beta')}{E'} \right]$$

Where,

$$\text{IR}_{brem}^{lp} = \frac{\alpha}{\pi M} \left\{ \frac{p \cdot (p_p - p'_p)}{|\vec{p}|} \ln \sqrt{\frac{1+\beta}{1-\beta}} + \frac{p' \cdot (p'_p - p_p)}{|\vec{p}'|} \ln \sqrt{\frac{1+\beta'}{1-\beta'}} \right\} \left(\frac{1}{|\epsilon|} + \gamma_E - \ln \left(\frac{4\pi\mu^2}{\Delta^2} \right) \right)$$

Two Photon Exchange Evaluation

Soft Photon Definition

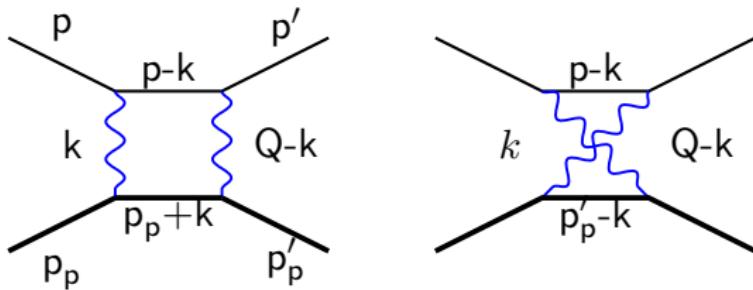


$$i\mathcal{M} = e^4 \int \frac{d^4 k}{(2\pi)^4} \frac{[\bar{u}(p')\gamma^\mu(\not{p} - \not{k} + m)\gamma^\nu u(p)][\chi^\dagger(p'_p)v_\mu v_\nu \chi(p_p)]}{(k^2 + i0)(Q - k)^2(k^2 - 2k \cdot p + i0)(v \cdot k + \frac{\vec{p}_p^2}{2M_p} + i0)}$$

We use SPA both in denominator and in the numerator

four point function \Rightarrow three point function

At LO



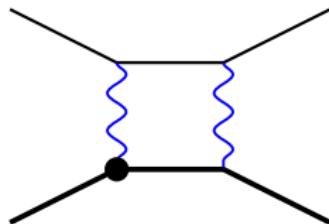
$$i\mathcal{M}_{box}^{LO} = e^4 \int \frac{d^4 k}{(2\pi)^4} \frac{[\bar{u}(p')\gamma^\mu(\not{p} - \not{k} + m)\gamma^\nu u(p)][\chi^\dagger(p'_p)v_\mu v_\nu \chi(p_p)]}{(k^2 + i0)(Q - k)^2(k^2 - 2k \cdot p + i0)(v \cdot k + i0)}$$

$$i\mathcal{M}_{xbox}^{LO} = e^4 \int \frac{d^4 k}{(2\pi)^4} \frac{[\bar{u}(p')\gamma^\mu(\not{p} - \not{k} + m)\gamma^\nu u(p)][\chi^\dagger(p'_p)v_\mu v_\nu \chi(p_p)]}{(k^2 + i0)(Q - k)^2(k^2 - 2k \cdot p + i0)(-v \cdot k + i0)}$$

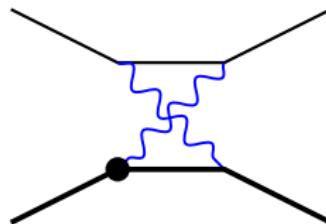
$$Re[\mathcal{M}_{box}^{LO} + \mathcal{M}_{xbox}^{LO}] = 0$$

TPE does not contribute at LO in HB χ PT

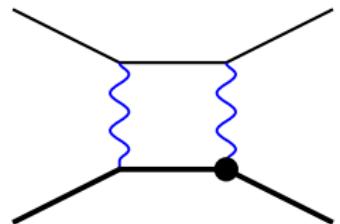
All the TPE diagrams at NLO



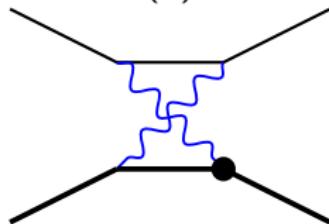
(a)



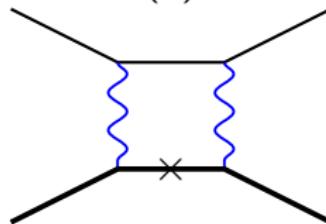
(b)



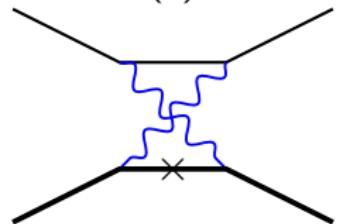
(c)



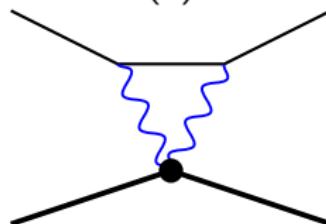
(d)



(e)



(f)



(g)

amplitude of all the diagrams

$$\begin{aligned}
 i\mathcal{M}_a &= \frac{e^4}{2M} \int \frac{d^4 k}{(2\pi)^4} [\bar{u}(p') \gamma^\mu (\not{p} - \not{k} + m) \gamma^\nu u(p)] \frac{1}{(k^2 + i0)(Q - k)^2 (k^2 - 2k \cdot p + i0)(v \cdot k + i0)} \\
 &\quad [\chi^\dagger(p'_p) \{v_\mu(2p_p + k)_\nu - v_\mu v_\nu(v.(2p_p + k))\} \chi(p_p)] \\
 i\mathcal{M}_b &= \frac{e^4}{2M} \int \frac{d^4 k}{(2\pi)^4} [\bar{u}(p') \gamma^\mu (\not{p} - \not{k} + m) \gamma^\nu u(p)] \frac{1}{(k^2 + i0)(Q - k)^2 (k^2 - 2k \cdot p + i0)(-v \cdot k + i0)} \\
 &\quad [\chi^\dagger(p'_p) \{v_\nu(p_p + p'_p - k)_\mu - v_\mu v_\nu(v.(p_p + p'_p - k))\} \chi(p_p)] \\
 i\mathcal{M}_c &= \frac{e^4}{2M} \int \frac{d^4 k}{(2\pi)^4} [\bar{u}(p') \gamma^\mu (\not{p} - \not{k} + m) \gamma^\nu u(p)] \frac{1}{(k^2 + i0)(Q - k)^2 (k^2 - 2k \cdot p + i0)(v \cdot k + i0)} \\
 &\quad [\chi^\dagger(p'_p) \{v_\nu(p_p + p'_p + k)_\mu - v_\mu v_\nu(v.(p_p + p'_p + k))\} \chi(p_p)] \\
 i\mathcal{M}_d &= \frac{e^4}{2M} \int \frac{d^4 k}{(2\pi)^4} [\bar{u}(p') \gamma^\mu (\not{p} - \not{k} + m) \gamma^\nu u(p)] \frac{1}{(k^2 + i0)(Q - k)^2 (k^2 - 2k \cdot p + i0)(-v \cdot k + i0)} \\
 &\quad [\chi^\dagger(p'_p) \{v_\mu(2p'_p - k)_\nu - v_\mu v_\nu(v.(2p'_p - k))\} \chi(p_p)] \\
 i\mathcal{M}_e &= e^4 \int \frac{d^4 k}{(2\pi)^4} [\bar{u}(p') \gamma^\mu (\not{p} - \not{k} + m) \gamma^\nu u(p)] \left[\frac{1}{(k^2 + i0)(Q - k)^2 (k^2 - 2k \cdot p + i0)} \right. \\
 &\quad \left. \left(\frac{1}{2M} + \frac{1}{v.(p_p + k)} - \frac{1}{2M} \frac{(p_p + k)^2}{[v.(p_p + k)]^2} \right) \right] [\chi^\dagger(p'_p) v_\mu v_\nu \chi(p_p)] \\
 i\mathcal{M}_f &= e^4 \int \frac{d^4 k}{(2\pi)^4} [\bar{u}(p') \gamma^\mu (\not{p} - \not{k} + m) \gamma^\nu u(p)] \left[\frac{1}{(k^2 + i0)(Q - k)^2 (k^2 - 2k \cdot p + i0)} \right. \\
 &\quad \left. \left(\frac{1}{2M} + \frac{1}{v.(p'_p - k)} - \frac{1}{2M} \frac{(p'_p - k)^2}{[v.(p'_p - k)]^2} \right) \right] [\chi^\dagger(p'_p) v_\mu v_\nu \chi(p_p)] \\
 i\mathcal{M}_g &= -\frac{2e^4}{2M} \int \frac{d^4 k}{(2\pi)^4} \frac{[\bar{u}(p') \gamma^\mu (\not{p} - \not{k} + m) \gamma^\nu u(p)][\chi^\dagger(p'_p) (g_{\mu\nu} - v_\mu v_\nu) \chi(p_p)]}{(k^2 - 2k \cdot p + i0)(Q - k)^2 (k^2 + i0)}
 \end{aligned}$$

After SPA

$$\begin{aligned}\mathcal{M}_a &= \frac{4p \cdot p_p}{2M_p} e^2 M_\gamma K^v[p] \\ \mathcal{M}_b &= \frac{4p' \cdot p_p}{2M_p} e^2 M_\gamma K^{-v}[p'] \\ \mathcal{M}_c &= \frac{4p' \cdot p'_p}{2M_p} e^2 M_\gamma K^v[p'] \\ \mathcal{M}_d &= \frac{4p \cdot p'_p}{2M_p} e^2 M_\gamma K^{-v}[p]\end{aligned}$$

where

$$\begin{aligned}K^{-v}[p] &= \frac{1}{i} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 + i0)(k^2 - 2k \cdot p + i0)(-v \cdot k + i0)} \\ K^v[p] &= \frac{1}{i} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 + i0)(k^2 - 2k \cdot p + i0)(v \cdot k + i0)}\end{aligned}$$

$$K^v[p] = -\frac{1}{(4\pi)^2 |\vec{p}|} \left[\left(\frac{1}{|\epsilon|} + \gamma_E - \ln\left(\frac{4\pi\mu^2}{m^2}\right) + \mathcal{O}(\epsilon) \right) \left(\frac{1}{2} \ln\left(\frac{1+\beta}{1-\beta}\right) - i\epsilon \right) - \frac{\pi^2}{2} - \text{Li}_2\left(\frac{2\beta}{1+\beta}\right) \right. \\ \left. + \frac{1}{4} \ln^2\left(\frac{1+\beta}{1-\beta}\right) - 2i\pi \ln\left(\frac{2|\vec{p}|}{m}\right) \right]$$

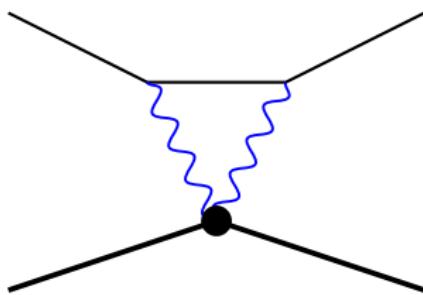
$$K^{-v}[p'] = -\frac{1}{(4\pi)^2 |\vec{p}'|} \left[\left(\frac{1}{|\epsilon|} + \gamma_E - \ln\left(\frac{4\pi\mu^2}{m^2}\right) + \mathcal{O}(\epsilon) \right) \frac{1}{2} \ln\left(\frac{1+\beta}{1-\beta}\right) - \text{Li}_2\left(\frac{2\beta}{1+\beta}\right) \right. \\ \left. + \frac{1}{4} \ln^2\left(\frac{1+\beta}{1-\beta}\right) \right]$$

Where $\beta = \frac{|\vec{p}|}{E}$, $\beta' = \frac{|\vec{p}'|}{E'}$

$$2 \sum_{spin} M_\gamma^\dagger (M_a + M_b + M_c + M_d) \\ = -\frac{\alpha}{\pi M} \sum_{spin} |M_\gamma|^2 \left\{ \frac{p \cdot (p_p - p'_p)}{|\vec{p}|} \left[\left(\frac{1}{|\epsilon|} + \gamma_E - \ln\left(\frac{4\pi\mu^2}{m^2}\right) \right) \frac{1}{2} \ln\left(\frac{1+\beta}{1-\beta}\right) - \frac{\pi^2}{2} - \text{Li}_2\left(\frac{2\beta}{1+\beta}\right) \right. \right. \\ \left. \left. + \frac{1}{4} \ln^2\left(\frac{1+\beta}{1-\beta}\right) \right] + \frac{p' \cdot (p'_p - p_p)}{|\vec{p}'|} \left[\left(\frac{1}{|\epsilon|} + \gamma_E - \ln\left(\frac{4\pi\mu^2}{m^2}\right) \right) \frac{1}{2} \ln\left(\frac{1+\beta'}{1-\beta'}\right) \right. \right. \\ \left. \left. - \text{Li}_2\left(\frac{2\beta'}{1+\beta'}\right) + \frac{1}{4} \ln^2\left(\frac{1+\beta'}{1-\beta'}\right) \right] \right\}$$

The Seagull

Seagull diagram does not have any IR or UV divergence, can be evaluated without any approximation



- Without SPA $\Rightarrow M_g = -\frac{e^4}{M_p} [\bar{u}(p') \gamma^\alpha K^0[p] \gamma_\alpha u(p)] [\chi_p^\dagger(p'_p) \chi_p(p_p)]$

Where,

$$K^0[p] = -i \int \frac{d^4 k}{(2\pi)^4} \frac{\not{p} - \not{k} + m}{(k^2 - 2k \cdot p + i0)(Q - k)^2(k^2 + i0)}$$

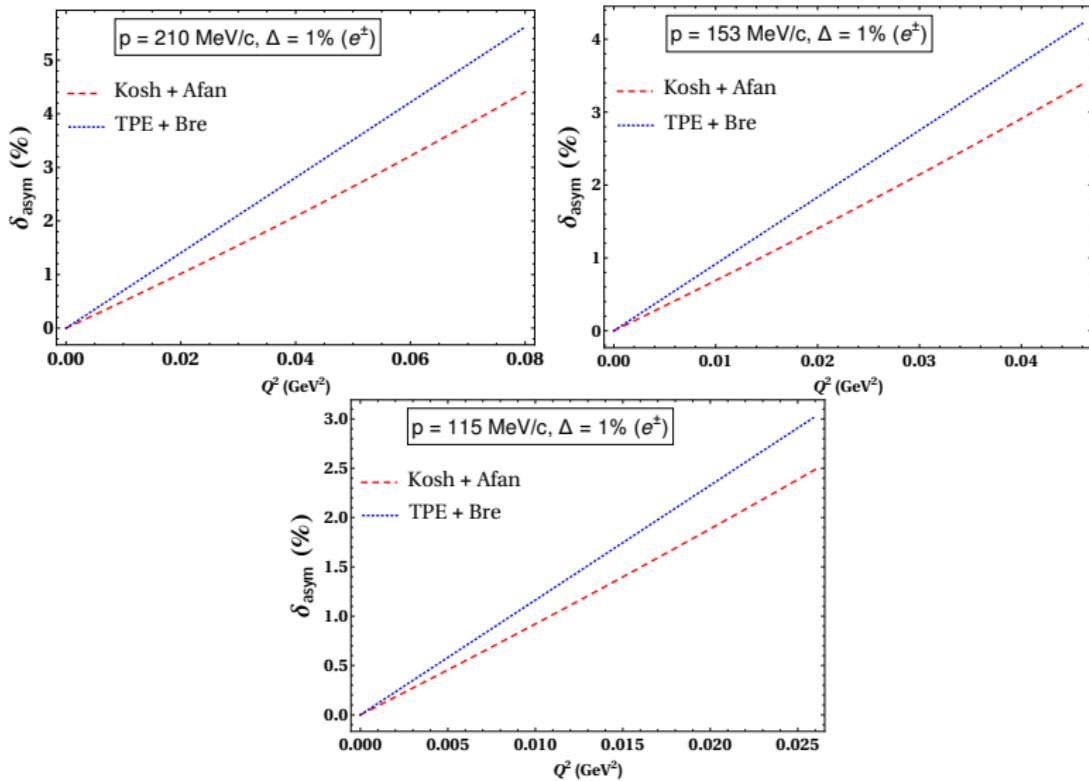
$$\begin{aligned}
\delta_{tpe} &= \frac{2 \sum_{spin} M_\gamma^\dagger (\mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c + \mathcal{M}_d)}{\sum_{spin} |M_\gamma|^2} \\
&= -\frac{\alpha Q^2}{2\pi M E} \left[\left\{ \frac{1}{\epsilon} - \gamma_E + \ln \left(\frac{4\pi\mu^2}{m_l^2} \right) \right\} \left\{ \frac{1}{\beta} \ln \sqrt{\frac{1+\beta}{1-\beta}} + \frac{E}{E'\beta'} \ln \sqrt{\frac{1+\beta'}{1-\beta'}} \right\} + \right. \\
&\quad \left. + \frac{1}{\beta} \left\{ \frac{\pi^2}{2} - \ln^2 \sqrt{\frac{1+\beta}{1-\beta}} - \text{Sp} \left(\frac{2\beta}{1+\beta} \right) \right\} + \frac{E}{E'\beta'} \left\{ \frac{\pi^2}{2} - \ln^2 \sqrt{\frac{1+\beta'}{1-\beta'}} - \text{Sp} \left(\frac{2\beta'}{1+\beta'} \right) \right\} \right] \\
&= \text{IR}_{tpe} - \frac{\alpha Q^2}{\pi M E \beta} \left[\frac{\pi^2}{2} + \ln \left(\frac{-Q^2}{m_l^2} \right) \ln \sqrt{\frac{1+\beta}{1-\beta}} - \ln^2 \sqrt{\frac{1+\beta}{1-\beta}} - \text{Sp} \left(\frac{2\beta}{1+\beta} \right) \right] + \mathcal{O} \left(\frac{1}{M^2} \right)
\end{aligned}$$

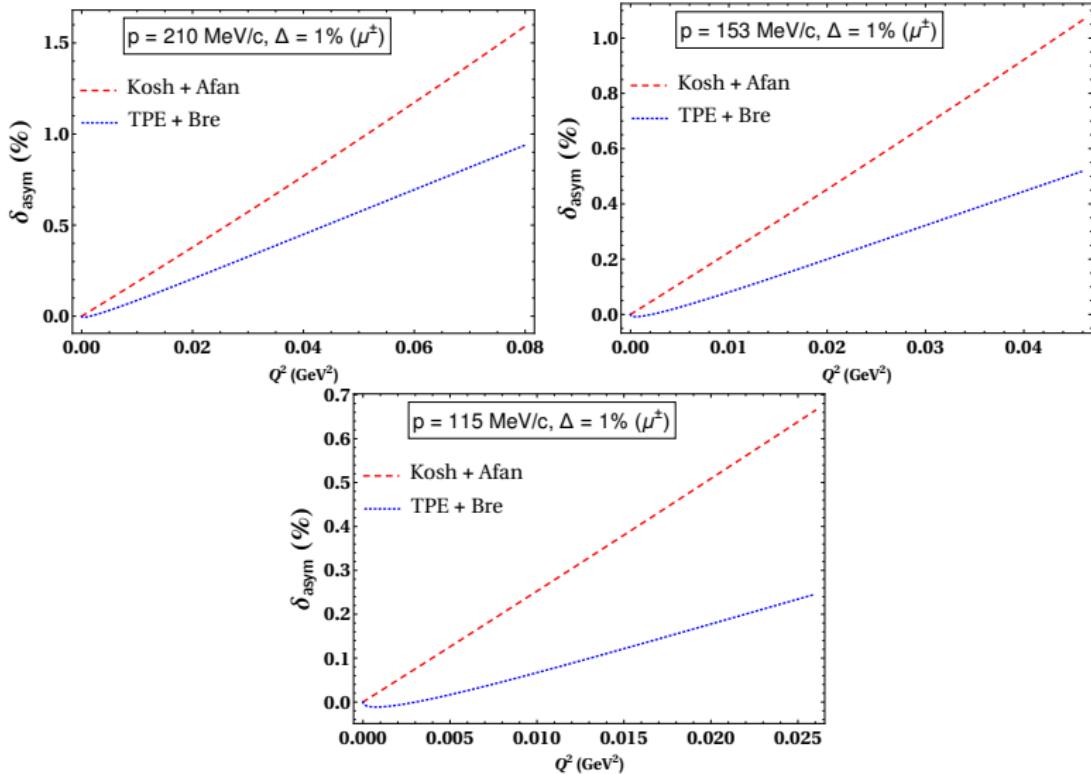
Where, $\text{IR}_{tpe} = -\frac{\alpha}{\pi M} \left\{ \frac{p \cdot (p_p - p'_p)}{|\vec{p}|} \ln \sqrt{\frac{1+\beta}{1-\beta}} + \frac{p' \cdot (p'_p - p_p)}{|vec{p}'|} \ln \sqrt{\frac{1+\beta'}{1-\beta'}} \right\} \left(\frac{1}{|\epsilon|} + \gamma_E - \ln \left(\frac{4\pi\mu^2}{m^2} \right) \right)$

$$\begin{aligned}
\delta_{tpe}^{(\text{seagull})}(Q^2) &= \frac{2\mathcal{R}e \sum_{spin} \left(\mathcal{M}_\gamma^* \widetilde{\mathcal{M}}_{\text{seagull}}^{(i)} \right)}{\sum_{spin} |\mathcal{M}_\gamma|^2} \\
&= -\frac{2\alpha Q^2}{\pi M E} \left[\frac{E^2 + EE'}{Q^2 + 4EE'} \right] \left(\mathcal{I}_1(Q^2) + \mathcal{I}_2(Q^2) + \frac{Q^2}{m_l^2} [\mathcal{I}_3(Q^2) - \mathcal{I}_4(Q^2)] \right) \\
&= -\frac{4\alpha Q^2}{\pi M E} \left[\frac{E^2}{Q^2 + 4E^2} \right] \left(\mathcal{I}_1(Q^2) + \mathcal{I}_2(Q^2) + \frac{Q^2}{m_l^2} [\mathcal{I}_3(Q^2) - \mathcal{I}_4(Q^2)] \right) + \mathcal{O} \left(\frac{1}{M^2} \right)
\end{aligned}$$

$$\begin{aligned}
\delta_{asym}(\Delta) &= \delta_{tpe} + \delta_{brem}^{lp}(\Delta) \\
&= \frac{\alpha}{\pi M} \left\{ \frac{p \cdot (p_p - p'_p)}{|\vec{p}|} \left[\ln \frac{\Delta^2}{m^2} \ln \sqrt{\frac{1+\beta}{1-\beta}} + \frac{\pi^2}{2} + \text{Li}_2 \left(\frac{2\beta}{1+\beta} \right) - \right. \right. \\
&\quad \left. \frac{1}{4} \ln^2 \left(\frac{1+\beta}{1-\beta} \right) \right] + \frac{p' \cdot (p'_p - p_p)}{|\vec{p}'|} \left[\ln \frac{\Delta^2}{m^2} \ln \sqrt{\frac{1+\beta'}{1-\beta'}} + \text{Li}_2 \left(\frac{2\beta'}{1+\beta'} \right) \right. \\
&\quad \left. \left. - \frac{1}{4} \ln^2 \left(\frac{1+\beta'}{1-\beta'} \right) \right] \right\} + \delta_{seagull} + \frac{\alpha}{\pi M} \left[\frac{p \cdot (p_p - p'_p)}{|\vec{p}|} \right. \\
&\quad \left\{ \ln 16 \ln \sqrt{\frac{1+\beta}{1-\beta}} + \text{Li}_2 \left(\frac{2\beta}{\beta-1} \right) - \text{Li}_2 \left(\frac{2\beta}{1+\beta} \right) \right\} + \frac{p' \cdot (p'_p - p_p)}{|\vec{p}'|} \\
&\quad \left. \left\{ \ln 16 \ln \sqrt{\frac{1+\beta'}{1-\beta'}} + \text{Li}_2 \left(\frac{2\beta'}{\beta'-1} \right) - \text{Li}_2 \left(\frac{2\beta'}{1+\beta'} \right) \right\} \right]
\end{aligned}$$

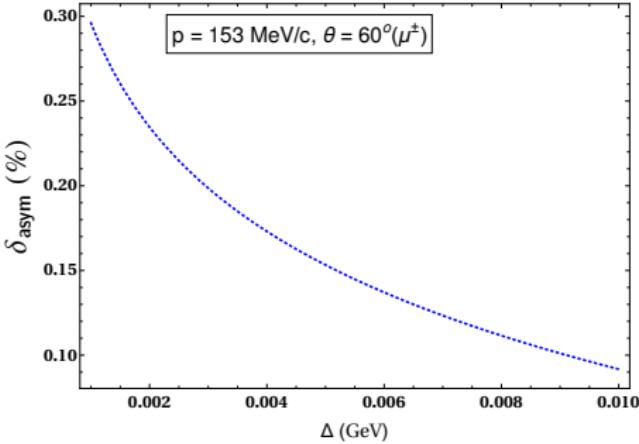
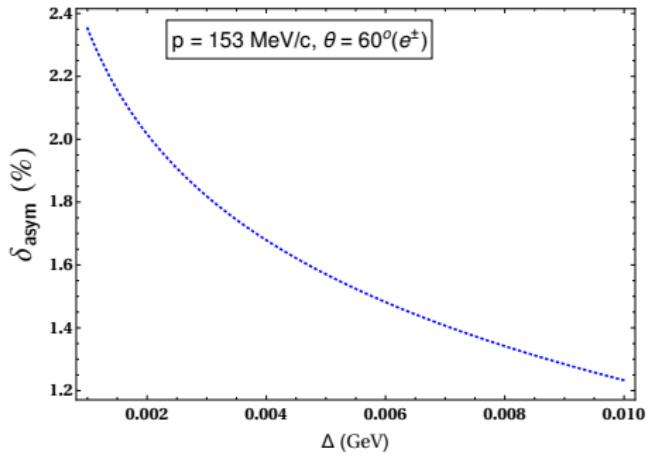
Results and Discussions





P. Talukdar, F. Myhrer, V. Chastry and U. Raha, (PRD 2020) arXiv:1911.06843v1

O. Koshchii and A. Afansev, Phys. Rev. D 96 (2017) 016005



Summary And Conclusion

- We have given an estimate of lepton charge of asymmetry for the ongoing MUSE experiment.
- Our results more or less agrees with the existing calculation.
- In this work we have only considered elastic contribution i.e proton as an intermediate state which can be extended to incorporate Δ particle in HB χ PT.
- TPE does not contribute at LO in HB χ PT
- The contributions from the seagull diagram is very small for e-p scattering but considerable for μ -p scattering.
- As a next step, our aim is to go for full TPE evaluation without any SPA.

THANK YOU

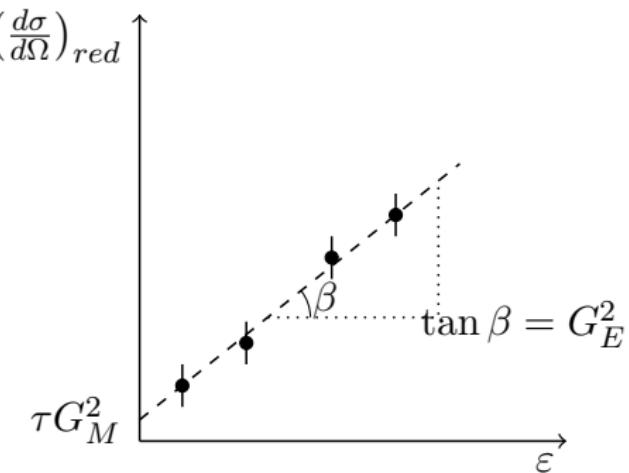
Proton radius from electric form factor

$$\left(\frac{d\sigma}{d\Omega}\right)_{red} = \varepsilon G_E^2(Q^2) + \tau G_M^2(Q^2)$$

At fixed Q^2

from the slope $\rightarrow G_E^2$

from the intercept $\rightarrow G_M^2$



$$r_p^2 = -6 \left(\frac{dG_E(Q^2)}{dQ^2} \right)_{Q^2=0}$$