(Quantum) Field Theory and the Electroweak Standard Model

Lecture III

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Outline

- Lecture I
  - What is the Standard Model?
  - Introducing Quantum Fields
  - Interactions and Perturbation Theory
  - Renormalizable or Non-Renormalizable?

- Lecture II
  - An Ode to Symmetry
  - Global Symmetries (and Conserved Quantities)
  - Local Symmetries (and Gauge interactions)
  - From Fermi Model to EW theory

- Lecture III
  - Finalizing the EW SM (a bit of Higgsing)
  - “Features” of the SM
  - Experimental tests of the EW SM
  - Issues and Prospects of the EW SM
Break the symmetry?

Q: How to make \( W \) and \( Z \) massive (keeping the nice features of the gauge theory intact)?

- Explicit breaking via the mass terms

\[
\mathcal{L} \supset m_W^2 W^+_\mu W^-_\mu + \frac{m_Z^2}{2} Z^\mu Z_\mu
\]

leads to inconsistencies...

- We have to do something more clever...

Hidden symmetry?
Hidden Gauge Symmetry

We need to generate masses for $W^{\pm}_\mu$ and $Z_\mu$ but not for $A_\mu$ without explicit breaking of the gauge symmetry.

Let's consider simple $U(1)$ gauge symmetry for scalar electrodynamics:

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial_\mu \phi - V(\phi^\dagger \phi) - \frac{1}{4} F_{\mu\nu}^2 + ie \left( \phi^\dagger \partial_\mu \phi - \phi \partial_\mu \phi^\dagger \right) A_\mu + e^2 A_\mu A_\mu \phi^\dagger \phi,$$

which is invariant under

$$\phi \rightarrow e^{ie\omega(x)} \phi, \quad A_\mu \rightarrow A_\mu + \partial_\mu \omega.$$ 

In polar coordinates ($\phi = \frac{1}{\sqrt{2}} \rho(x)e^{i\theta(x)}$) the Lagrangian

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \rho)^2 + \frac{e^2 \rho^2}{2} \left( A_\mu - \frac{1}{e} \partial_\mu \theta \right) \left( A_\mu - \frac{1}{e} \partial_\mu \theta \right) - V(\rho^2/2) - \frac{1}{4} F_{\mu\nu}^2$$

is still invariant. (Ex: How $\rho$ and $\theta$ change under $U(1)$ transformations?)

NB: The symmetry becomes hidden if we define $B_\mu \equiv A_\mu - \frac{1}{e} \partial_\mu \theta$!
Spontaneous Symmetry Breaking

\[ \mathcal{L} = \frac{1}{2} (\partial_\mu \rho)^2 + \frac{e^2 \rho^2}{2} B_\mu B_\mu - V(\rho^2/2) - \frac{1}{4} F_{\mu\nu}^2(B) \]

Here \( \rho(x) \) is a dynamical field. We get mass term if \( \rho(x) \to \nu = \text{const.} \)

\[ V(\phi) \]

\[ V = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \]

It is invariant under global phase-shift symmetry \( \phi \to e^{i\alpha} \phi \).

The case \( \mu^2 > 0 \) is trivial.

For \( \mu^2 < 0 \) we have a valley of degenerate minima:

\[ \frac{\partial V}{\partial \phi^\dagger} = 0 \Rightarrow \phi_0^\dagger \phi_0 = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2} > 0 \Rightarrow \phi_0 = \frac{v}{\sqrt{2}} e^{i\beta} \]
The Brout-Englert-Higgs mechanism

Non-zero $\phi$ in the minimum of the potential is interpreted as the vacuum expectation value (vev) of the quantum field:

$$\frac{v}{\sqrt{2}} = \langle 0 | \phi(x) | 0 \rangle, \quad \beta = 0 \text{ (Why?)}$$

To introduce particles as excitations we have to shift the field:

$$\phi(x) = \frac{v + h(x)}{\sqrt{2}} e^{i \zeta(x)/v}, \quad \langle 0 | h(x) | 0 \rangle = 0, \quad \langle 0 | \zeta(x) | 0 \rangle = 0$$

$$\mathcal{L} = \frac{1}{2} (\partial_\mu h)^2 + \frac{e^2 v^2}{2} B_\mu B_\mu + e v h B_\mu B_\mu + \frac{e^2}{2} B_\mu B_\mu h^2 - V - \frac{1}{4} F_{\mu \nu}^2 (B)$$

$$V = - \frac{|\mu|^2}{2} (v + h)^2 + \frac{\lambda}{4} (v + h)^4 = \frac{2 \lambda v^2}{2} h^2 + \lambda v h^3 + \frac{\lambda}{4} h^4 - \frac{\lambda}{4} v^4$$

Massive field $B_\mu$ without explicit symmetry breaking! This is the essence of Brout-Englert-Higgs-Hagen-Guralnik-Kibble mechanism. The symmetry is hidden.
The Brout-Englert-Higgs mechanism: Counting DOFs

We start with

\[ \mathcal{L}_1 = \partial_\mu \phi^\dagger \partial^\mu \phi + i e \left( \phi^\dagger \partial_\mu \phi - \phi \partial_\mu \phi^\dagger \right) A_\mu + e^2 A_\mu A_\mu \phi^\dagger \phi - V - \frac{1}{4} F_{\mu \nu}^2 (A), \]

and end with [\sqrt{2}\phi = (v + h) \exp(i\zeta(x)/v), \quad B_\mu = A_\mu - \partial_\mu \zeta / (ev)]

\[ \mathcal{L}_2 = \frac{1}{2} (\partial_\mu h)^2 - \frac{e^2 v^2}{2} \left( 1 + \frac{h^2}{v^2} \right) B_\mu B_\mu + ev h B_\mu B_\mu - V - \frac{1}{4} F_{\mu \nu}^2 (B). \]

\( \mathcal{L}_1 \): 2 DOFs (complex scalar \( \phi, \phi^\dagger \)) + 2 DOFs (massless vector \( A_\mu \)).

\( \mathcal{L}_2 \): 1 DOFs (real scalar \( h \)) + 3 DOFs (massive vector field \( B_\mu \)).

One scalar DOF was “eaten” by the gauge field to become massive.

\textbf{Q:} Which one?

\textbf{A:} The (would-be) Nambu-Goldstone (boson)!
SSB and Renormalizability

Both $\mathcal{L}_1$ and $\mathcal{L}_2$ has some issues:

$\mathcal{L}_1$ : Manifestly gauge-invariant, but not suitable for PT (imaginary mass);

$\mathcal{L}_2$ : Hidden gauge symmetry, only physical DOFs, but non-renormalizable by power counting.

There is another, explicitly renormalizable version of the Lagrangian with shifted $\phi$ written in cartesian coordinates: $\phi = \frac{1}{\sqrt{2}}(v + \eta + i\xi)

\begin{align*}
\mathcal{L}_3 &= \frac{v^4\lambda}{4} - \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{e^2 v^2}{2} A_\mu A_\mu + \frac{1}{2} \partial_\mu \xi \partial_\mu \xi - ev A_\mu \partial_\mu \xi \\
&\quad + \frac{1}{2} \partial_\mu \eta \partial_\mu \eta - \frac{2v^2\lambda}{2} \eta^2 + eA_\mu \xi \partial_\mu \eta - eA_\mu \eta \partial_\mu \xi - v\lambda \eta(\eta^2 + \xi^2) \\
&\quad - \frac{\lambda}{4}(\eta^2 + \xi^2)^2 + \frac{e^2}{2} A_\mu A_\mu(2v\eta + \eta^2 + \xi^2).
\end{align*}

NB: Now we have massless unphysical field $\xi$ in the spectrum, but it mixes with longitudinal component of $A_\mu$ ("partially eaten").
A Remark on the Goldstone Theorem

The Goldstone theorem states that if the vacuum breaks a global continuous symmetry there is a massless boson (Nambu-Goldstone) in the spectrum: any non-derivative interactions violates

\[ \zeta \rightarrow \zeta + ev\omega, \quad \omega = \text{const} \]

Fortunately, we have local symmetry with hungry \( A_\mu \)...

- We need 3 massive bosons \( W^\pm, Z_\mu \).
- 3 symmetries out of \( SU(2)_L \times U(1)_Y \) has to be spontaneously broken to get 3 victims (would-be) Nambu-Goldstone bosons.

**NB:** The Higgs boson (see lect. by J. Ellis) was a by-product of the mass-generation mechanism (the main task was to “exorcise” massless fields).
SSB in the EW theory (GWS)

- To break $SU(2)_L \times U(1)_Y$ we can use a scalar $SU(2)$-doublet:

  $$\Phi = \frac{1}{\sqrt{2}} \exp \left( i \frac{\zeta_i \sigma^i}{2v} \right) \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}, \quad \Phi_0 \equiv \langle 0 | \Phi | 0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

- Let it be also charged under $U(1)_Y$:

  $$\Phi \rightarrow \exp \left( ig \frac{\sigma^i}{2} \omega_a + ig' \frac{Y_H}{2} \omega' \right) \Phi$$

- We do not want to break $U(1)_{em}$ and seek for a combination of $SU(2)_L$ and $U(1)_Y$ generators that annihilate the vacuum:

  $$Q \Phi_0 = 0, \quad Q = \frac{\sigma_3}{2} + \frac{Y_H}{2} = \frac{1}{2} \begin{pmatrix} 1 + Y_H & 0 \\ 0 & -1 + Y_H \end{pmatrix}$$

- For $Y_H = 1$, the vacuum has zero electric charge.
SSB in the EW theory (GWS)

■ The Lagrangian for $\Phi$

$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi), \quad \text{with} \quad V(\Phi) = m_\Phi^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

leads to $v = \frac{|m_\Phi^2|}{\lambda}$ provided $m_\Phi^2 < 0$.

■ Gauge symmetry allows us to get rid of $\zeta_i$ (unitary gauge):

$$|D_\mu \Phi|^2 = \frac{1}{2} (\partial_\mu h)^2 + \frac{g^2}{8} (v + h)^2 |W^1_\mu + iW^2_\mu|^2 + \frac{1}{8} (v + h)^2 (gW^3_\mu - g' Y_H B_\mu)^2.$$ 

■ We do not want to generate mass for the photon ($Y_H = 1$): 

\[
0 = g \sin \theta_W - g' \cos \theta_W, \\
e = g \sin \theta_W = \frac{gg'}{\sqrt{g^2 + g'^2}} \\
W^3_\mu = Z_\mu \cos \theta_W + A_\mu \sin \theta_W \\
B_\mu = -Z_\mu \sin \theta_W + A_\mu \cos \theta_W
\]
SSB in the EW theory (GWS)

The Lagrangian for $\Phi$

$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi), \quad \text{with} \quad V(\Phi) = m_\Phi^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

leads to $v = \frac{|m_\Phi^2|}{\lambda}$ provided $m_\Phi^2 < 0$.

Gauge symmetry allows us to get rid of $\zeta_i$ (unitary gauge):

$$|D_\mu \Phi|^2 = \frac{1}{2} (\partial_\mu h)^2 + \frac{g^2}{4} (v + h)^2 W_\mu^+ W_\mu^-$$

$$+ \frac{1}{8} (v + h)^2 (g^2 + g'^2) Z_\mu^2.$$

Masses of $W$- and $Z$-bosons are related:

$$M_W = \frac{gv}{2}, \quad M_Z = \sqrt{g^2 + g'^2} \frac{v}{2}$$

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = 1(?)$$

$$W_\mu^3 = Z_\mu \cos \theta_W + A_\mu \sin \theta_W$$

$$B_\mu = -Z_\mu \sin \theta_W + A_\mu \cos \theta_W$$
On Scattering of Longitudinal $W$-bosons and Higgs boson

At high-energies the amplitude of longitudinal $W$ scattering:

$$\mathcal{M} = g^2 \epsilon^\mu_L(p_1) \epsilon^\nu_L(p_2) (2g_{\mu\sigma}g_{\nu\rho} - g_{\mu\rho}g_{\nu\sigma} - g_{\mu\nu}g_{\rho\sigma}) \epsilon^\rho_L(k_1) \epsilon^\sigma_L(k_2)$$

$$\mathcal{M} \propto g^2 \frac{E^4}{M_W^4}$$

We expect perturbative unitarity violation for $E \sim M_W$. 
On Scattering of Longitudinal W-bosons and Higgs boson

\[
M \propto g^2 \frac{E^2}{M_W^2}
\]

The Higgs is crucial for unitarization of WW-scattering!
SSB and Fermion Masses

- The $SU(2)_L \times U(1)_Y$ forbids fermion mass terms:

$$m \bar{\psi} \psi = m \left( \bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L \right)$$

- To add masses in the SM we introduce Yukawa interactions:

$$\mathcal{L}_Y = -y_e (\bar{L} \Phi) e_R - y_d (\bar{Q} \Phi) d_R - y_u (\bar{Q} \Phi^c) u_R + \text{h.c.}$$

with $SU(2)$ doublets

$$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad Q = \begin{pmatrix} u \\ d \end{pmatrix}_L, \quad \Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \Phi^c = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$$

- $\Phi \rightarrow \langle 0 | \Phi | 0 \rangle$:

$$\mathcal{L}_m^f = -\left[ \frac{y_e v}{\sqrt{2}} \bar{e} e + \frac{y_d v}{\sqrt{2}} \bar{d} d + \frac{y_u v}{\sqrt{2}} \bar{u} u \right], \quad m_f = \frac{y_f v}{\sqrt{2}}$$

**Ex:** Check that all terms in $\mathcal{L}_Y$ are singlets w.r.t $SU(2)_L \times U(1)_Y$. 

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SSB and Fermion Masses: Towards Flavour Physics

- The Lagrangian $\mathcal{L}_Y$ can be generalized to account for known three generations (introduce indices $i, j = 1, 2, 3$):

$$\mathcal{L}_Y = -y_{ij}^l (\bar{L}_i \Phi) l_{jR} - y_{ij}^d (\bar{Q}_i \Phi) d_{jR} - y_{ij}^u (\bar{Q}_i \Phi^c) u_{jR} + \text{h.c.}$$

- Yukawa Matrices are non-diagonal (and complex) ⇒

$$\mathcal{L}_f^m = -\frac{y_{ij}^f v}{\sqrt{2}} \bar{f}_i f_j + \text{h.c.} \quad M_{ij}^f = \frac{y_{ij}^f v}{\sqrt{2}}$$

- Mass Eigenstates: Diagonalizing $M_f$ we obtain masses $m_f$.

**NB1:** Couplings of the mass eigenstates $f$ to $h$ are proportional to $m_f$.

**NB2:** Three generations are required to have $\mathcal{CP}$ violation in quark sector.

**NB3:** Dirac mass for neutrinos can also be introduced in a similar way.

(see details in lect. by M. Vysotsky and C. Gonzalez-Garcia)
The SM Lagrangian: Gauge

Let us combine all the ingredients and write down the Full SM Lagrangian

\[ \mathcal{L}_{SM} = \mathcal{L}_{Gauge}(g_s, g, g') + \mathcal{L}_{Yukawa}(y_u, y_d, y_l) + \mathcal{L}_{Higgs}(\lambda, m^2_{\Phi}) + \mathcal{V}_{Higgs}(\lambda, m^2_{\Phi}) + \mathcal{L}_{Gauge-fixing} + \mathcal{L}_{Ghosts} \]

\[ \mathcal{L}_{Gauge} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W^i_{\mu\nu} - \frac{1}{4} B_{\mu\nu} B_{\mu\nu} + (D_\mu \Phi)^\dagger (D_\mu \Phi) \]

\[ + \bar{L}_i i \hat{D} L_i + \bar{Q}_i i \hat{D} Q_i + \bar{I}_{Ri} i \hat{D} I_{Ri} + \bar{u}_{Ri} i \hat{D} u_{Ri} + \bar{d}_{Ri} i \hat{D} d_{Ri} \]

We have three parameters \( g_s, g, g' \) in the Gauge sector.
The SM Lagrangian: Gauge Self-Interactions

Let us combine all the ingredients and write down the Full SM Lagrangian

$$\mathcal{L}_{SM} = \mathcal{L}_{\text{Gauge}}(g_s, g, g') + \mathcal{L}_{\text{Yukawa}}(y_u, y_d, y_l) + \mathcal{L}_{\text{Higgs}}(\lambda, m_\Phi^2) \quad [ - V_{\text{Higgs}}(\lambda, m_\Phi^2) ] + \mathcal{L}_{\text{Gauge-fixing}} + \mathcal{L}_{\text{Ghosts}}$$

Due to Non-Abelian nature we have Gauge-boson self-interactions:
The SM Lagrangian: Gauge Self-Interactions

\begin{align*}
\sigma_{WW} \text{(pb)} \quad \sqrt{s} \text{(GeV)} \\
\text{LEP}
\end{align*}

\[ e^+ \rightarrow W^+ \quad e^- \rightarrow W^- \]

\[ e^+ \rightarrow \gamma \quad e^- \rightarrow W^- \]

\[ e^+ \rightarrow Z \quad e^- \rightarrow W^- \]
The SM Lagrangian: Gauge-Higgs Interactions

Let us combine all the ingredients and write down the Full SM Lagrangian

\[ \mathcal{L}_{SM} = \mathcal{L}_{Gauge}(g_s, g, g') + \mathcal{L}_{Yukawa}(y_u, y_d, y_l) + \mathcal{L}_{Higgs}(\lambda, m^2) \left[ -V_{Higgs}(\lambda, m^2) \right] + \mathcal{L}_{Gauge-fixing} + \mathcal{L}_{Ghosts} \]

The Higgs boson \( h \) interacts with \( W \) and \( Z \), but not with \( \gamma \):

Ex: What are the couplings?
The SM Lagrangian: Gauge-Fermion Interactions

Let us combine all the ingredients and write down the Full SM Lagrangian

\[ \mathcal{L}_{SM} = \mathcal{L}_{Gauge}(g_s, g, g') + \mathcal{L}_{Yukawa}(y_u, y_d, y_l) + \mathcal{L}_{Higgs}(\lambda, m^2_\Phi) \left[ -V_{Higgs}(\lambda, m^2_\Phi) \right] + \mathcal{L}_{Gauge-fixing} + \mathcal{L}_{Ghosts} \]

After diagonalization of Yukawa matrices, we have flavour-changing charge-current interactions, but flavour-conserving neutral-current transitions:

\[ W^+ \sim u_i \rightarrow d_j \quad \text{and} \quad W^- \sim d_i \rightarrow u_j \]

\[ Z, \gamma \sim u_i, d_i \]

NB: The CKM matrix appears in charged currents (see lect. by M. Vysotsky)
The SM Lagrangian: Yukawa

Let us combine all the ingredients and write down the Full SM Lagrangian

\[ \mathcal{L}_{SM} = \mathcal{L}_{\text{Gauge}}(g_s, g, g') \]
\[ + \mathcal{L}_{\text{Yukawa}}(y_u, y_d, y_l) \]
\[ + \mathcal{L}_{\text{Higgs}}(\lambda, m^2_\Phi) \left[ - V_{\text{Higgs}}(\lambda, m^2_\Phi) \right] \]
\[ + \mathcal{L}_{\text{Gauge-fixing}} + \mathcal{L}_{\text{Ghosts}} \]

The Higgs boson \( h \) interacts with fermions (again, no change of flavour). Not all parameters in complex matrices \( y_f \) are physical. The physical ones are \( 6_q + 3_l \) masses, 3 angles in CKM matrix and 1 CPV phase.

Ex: What are the couplings? Why no Flavour-Changing-Neutral-Current?
The SM Lagrangian: Higgs self-interactions

Let us combine all the ingredients and write down the Full SM Lagrangian

$$\mathcal{L}_{SM} = \mathcal{L}_{\text{Gauge}}(g_s, g, g') + \mathcal{L}_{\text{Yukawa}}(y_u, y_d, y_l) + \mathcal{L}_{\text{Higgs}}(\lambda, m^2_\Phi) - \mathcal{V}_{\text{Higgs}}(\lambda, m^2_\Phi) + \mathcal{L}_{\text{Gauge-fixing}} + \mathcal{L}_{\text{Ghosts}}$$

The shift in the Higgs field generates triple self-interaction $h$:  

![Diagram of Higgs field interactions](image)

**NB:** Two parameters in the Higgs sector: $M_h = 2\lambda v^2$ and $v^2 = -m^2_\Phi/\lambda$. 
The SM Input Parameters

- There are 18+1 free parameters in the canonical SM (no $\nu_R$).
- To make predictions they have to be extracted from experiment.
- There are different, yet related, sets of parameters. For example,

  \[
  18 = 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 9 \quad 4
  \]

  primary: $g_s$  $g$  $g'$  $\lambda$  $m_\Phi$  $y_f$  $y_{ij}$

  practical: $\alpha_s$  $M_Z$  $\alpha$  $G_F$  $M_H$  $m_f$  $V_{CKM}$

\[
\alpha_s = \frac{g_s^2(Q)}{4\pi}, \quad (4\pi)\alpha = \frac{g^2 g'^2}{(g^2 + g'^2)}, \quad M_Z^2 = \frac{(g^2 + g'^2) v^2}{4},
\]

\[
G_F = \frac{1}{\sqrt{2} v^2}, \quad M_h = 2\lambda v^2 = 2|m_\Phi|^2, \quad m_f = y_f v/\sqrt{2}
\]

**Ex:** Where is $\mu$?

**NB:** Tree-level relations, can be corrected in high orders of perturbation theory (renormalization scheme!).
On the Importance of Radiative Corrections

Let us use our physical parameter set to predict $M_W$ at the tree level:

- Given the relations

$$\frac{G_F}{\sqrt{2}} = \frac{1}{2v^2}, \quad (4\pi)\alpha = \frac{g^2 g'^2}{g^2 + g'^2}, \quad M_Z^2 = \frac{g_Z^2 v^2}{4}, \quad M_W^2 = \frac{g^2 v^2}{4}$$

we derive

$$\frac{G_F}{\sqrt{2}} = \frac{\pi \alpha}{2 M_W^2 (1 - M_W^2 / M_Z^2)}.$$ 

- Plugging the PDG2019 values

$$\alpha^{-1} = 137.035999139(31),$$
$$M_Z = 91.1876(21) \text{ GeV},$$
$$G_F = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2},$$

we compare “theory” and exp

$$M_W^{\text{tree}} = 80.9387(25) \text{ GeV}$$
$$M_W^{\text{exp}} = 80.379(12) \text{ GeV}$$

Our naive prediction is off by about $47\sigma$!

Do we have to go beyond the SM?
On the Importance of Radiative Corrections

Let us use our physical parameter set to predict $M_W$ at the tree level:

- Given the relations

$$\frac{G_F}{\sqrt{2}} = \frac{1}{2v^2}, \quad (4\pi)\alpha = \frac{g^2g'^2}{g^2 + g'^2}, \quad M_Z^2 = \frac{g_Z^2v^2}{4}, \quad M_W^2 = \frac{g^2v^2}{4}$$

we derive

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2M_W^2(1 - M_W^2/M_Z^2)}.$$ 

- Plugging the PDG2019 values

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we compare “theory” and exp

$$M_W^{\text{tree}} = 80.9387(25) \text{ GeV}$$

$$M_W^{\text{exp}} = 80.379(12) \text{ GeV}$$

Nope, have to account for higher orders..
Scale dependence of primary SM parameters

- Grand Unification?

\[
\frac{d\alpha}{d \log \mu^2} = \beta \alpha^2 + \mathcal{O}(\alpha^3), \quad \alpha = \frac{g^2}{4\pi}
\]

\[
\beta = -\frac{1}{4\pi} \left[ \frac{11}{3} C_2 - \frac{2}{3} \sum F T_F - \frac{1}{3} \sum S T_S \right]
\]

\[
C_2 = N \text{ for } SU(N), \quad T_F = T_S = \frac{1}{2}
\]

for Fermions and Scalars

- Vacuum (Meta)Stability?

**Fig. made by A. Pikelner, MR package**
(Anti)Neutrino-(Anti)Quark scattering

Due to masslessness* of (anti)neutrino, it is produced in LH chiral state. Can be used to directly probe the (anti)quark content of the proton or neutron!

\[ d \rightarrow \nu \mu \]

\[ \mu^{-} \rightarrow W^{+} \]

\[ u \rightarrow \bar{u} \]

\[ \bar{d} \rightarrow \mu^{-} \]

\[ \theta^{*} \]

\[ \sigma^{\nu q} = \frac{G_{F} s}{\pi} \]

\[ \sigma^{\nu \bar{q}} = \frac{G_{F} s}{3\pi} \]

\[ \frac{d\sigma_{\nu q}}{d\Omega^{*}} = \frac{G_{F} s}{4\pi^{2}} \]

\[ \frac{d\sigma_{\nu \bar{q}}}{d\Omega^{*}} = \frac{G_{F} s}{4\pi^{2}} \cos^{4} \frac{\theta^{*}}{2} \]

Ex: Compare with the \( \theta^{*} \)-dependence of \( e^{\pm} q \) scattering due to \( \gamma \)-exchange.

\[ A. \text{ Bednyakov (JINR)} \]

QFT & EW SM
(Anti)Neutrino-(Anti)Quark scattering

Due to masslessness* of (anti)neutrino, it is produced in LH chiral state. Can be used to directly probe the (anti)quark content of the proton or neutron!

\[ d \sigma_{\bar{\nu} q} \over d\Omega^* = \frac{G_F s}{4\pi^2} \cos^4 \frac{\theta^*_s}{2} \]

\[ \sigma_{\bar{\nu} q} = \frac{G_F s}{3\pi} \]

**Ex:** Compare with the \( \theta^*_s \)-dependence of \( e^{\pm} q \) scattering due to \( \gamma \)-exchange.
**Neutrino DIS $\nu N \rightarrow \mu X$ and anti-quark content of $p$**

- For neutrino - proton $\nu p$ and neutrino - neutron $\nu n$ scattering:

  $$\sigma_{\nu p} = \frac{G_F^2 s}{\pi} \left[ f_d + \frac{1}{3} f_{\bar{u}} \right], \quad \sigma_{\nu n} = \frac{G_F^2 s}{\pi} \left[ f_u + \frac{1}{3} f_{\bar{d}} \right],$$

  where $f_q = \int_0^1 xq(x)dx$ - the fraction of proton momentum carried by parton $q$, and we use $f_u(\text{proton}) = f_d(\text{neutron}) \equiv f_u$, etc. (*why?*)

- For an isoscalar target (equal number of protons and neutrons)

  $$\sigma_{\nu N} = \frac{1}{2} \left[ \sigma_{\nu p} + \sigma_{\nu n} \right] = \frac{G_F^2 s}{2\pi} \left[ f_q + \frac{1}{3} f_{\bar{q}} \right], \quad f_u = f_d + f_u,$$

  $$\sigma_{\bar{\nu} N} = \frac{G_F^2 s}{2\pi} \left[ f_{\bar{q}} + \frac{1}{3} f_q \right]$$

- The ratio probes antiquark content:

  $$\frac{\sigma_{\nu N}}{\sigma_{\bar{\nu} N}} = \frac{3f_q + f_{\bar{q}}}{f_q + 3f_{\bar{q}}}$$
Neutrino Total Cross Sections Measurements

In the LAB frame \(s \simeq 2m_N E_\nu\) and \(\sigma_{\nu N}/E_\nu = \frac{G_F^2 m_N}{2\pi} \left[ f_q + \frac{1}{3} f_{\bar{q}} \right]\)

\[
\sigma_{\nu N}/E_\nu \simeq 0.67 \cdot 10^{-38} \text{ cm}^2 \text{ GeV}^{-1}, \quad \sigma_{\bar{\nu} N}/E_{\bar{\nu}} \simeq 0.33 \cdot 10^{-38} \text{ cm}^2 \text{ GeV}^{-1}
\]

\[
\frac{\sigma_{\nu N}}{\sigma_{\bar{\nu} N}} = 1.984 \pm 0.012 \neq 3(f_{\bar{q}} = 0), \quad f_q \simeq 0.41 \quad \text{and} \quad f_{\bar{q}} \simeq 0.08
\]

NB: By the way, where is 50% of proton momentum?

Figure from PDG2019
Z-width and branching ratios

\[ \mathcal{L}_I \ni g_Z Z^\mu J_Z^{\mu}, \quad J_Z^{\mu} = \frac{1}{4} \sum_f \bar{f} (v_f - a_f \gamma_5) f = \sum_f \bar{f} (c_L P_L + c_R P_R) f \]

\[ \Gamma(Z \to f \bar{f}) = \frac{g_Z^2 m_Z}{24\pi} (c_L^2 + c_R^2) \]

Assuming \( \sin^2 \theta_W \simeq 0.23 \):

<table>
<thead>
<tr>
<th>fermion</th>
<th>( Q_f )</th>
<th>( T^f_3 )</th>
<th>( Y_L )</th>
<th>( Y_R )</th>
<th>( c_L )</th>
<th>( c_R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \nu_L )</td>
<td>0</td>
<td>( +\frac{1}{2} )</td>
<td>-1</td>
<td>0</td>
<td>( +\frac{1}{2} )</td>
<td>0</td>
</tr>
<tr>
<td>( l^- )</td>
<td>-1</td>
<td>-( \frac{1}{2} )</td>
<td>-1</td>
<td>-2</td>
<td>-0.27</td>
<td>+0.23</td>
</tr>
<tr>
<td>( u )</td>
<td>( +\frac{2}{3} )</td>
<td>( +\frac{1}{2} )</td>
<td>( +\frac{1}{3} )</td>
<td>( +\frac{4}{3} )</td>
<td>+0.35</td>
<td>-0.15</td>
</tr>
<tr>
<td>( d )</td>
<td>-( \frac{1}{3} )</td>
<td>-( \frac{1}{2} )</td>
<td>( +\frac{1}{3} )</td>
<td>-( \frac{2}{3} )</td>
<td>-0.42</td>
<td>+0.08</td>
</tr>
</tbody>
</table>

Ex1: Find the relations between \( a_f(v_f) \) and \( c_L(c_R) \).

Ex2: Evaluate \( \Gamma_{\nu\nu} \) given \( g_Z \simeq 0.55 \).
Precision tests: Z-resonance

Consider the process
\[ e^+ e^- \rightarrow Z \rightarrow f \bar{f} \]
for \( \sqrt{s} \approx m_Z \gg m_f \)

\[ |P(z)|^2 = \frac{1}{(s - m_Z)^2 + m_Z^2 \Gamma_Z^2} : \]

\[ |M_{LL}|^2 = g_Z^4 |P(z)|^2 [c_L^e]^2 [c_L^f]^2 (1 + \cos \theta)^2 \]

\[ |M_{LR}|^2 = g_Z^4 |P(z)|^2 [c_L^e]^2 [c_R^f]^2 (1 - \cos \theta)^2 \]

Ex: Evaluate \( |M_{RR}|^2 \) and \( |M_{RL}|^2 \).
Total cross-section $ee \rightarrow Z \rightarrow ff$

For unpolarised $e^\pm$ beams total cross section (narrow-width):

$$\sigma(\text{ee} \rightarrow Z \rightarrow \text{ff}) = \frac{12\pi s}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{ff}}{(s - m_Z)^2 + m_Z^2\Gamma_Z^2}$$

- Position of the peak: $\sigma(\sqrt{s} = m_Z) = \sigma_{ff}^0$
- Full Width at Half Maximum $\sigma(\sqrt{s} = m_Z \pm \Gamma_Z/2) = \sigma_{ff}^0/2$
- $(12\pi)\Gamma_{ee}\Gamma_{ff} = \sigma_{ff}^0\Gamma_Z^2 m_Z^2$
  \[ \rightarrow 12\pi\Gamma_{ee}^2 = \sigma_{\mu\mu}^0 \Gamma_Z^2 m_Z^2 \text{ (Lepton Universality)} \]
  \[ \rightarrow 12\pi\Gamma_{\text{hadrons}}^2 = \sigma_{\text{hadrons}}^0 \Gamma_Z^2 m_Z^2 / \Gamma_{ee} \]
- $N_\nu = (\Gamma_Z - 3\Gamma_{ee} - \Gamma_{\text{hadrons}}) / \Gamma_{\nu\nu}^{\text{SM}} \simeq 2.98$
Z-peak observables: Forward-Backward Asymmetry

- For charged leptons:
  \[ \frac{v_l}{a_l} = 1 - 4 \sin^2 \theta_W \]

- Forward-Backward Asymmetry

  \[ A_{FB}^\mu = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4} A_e A_\mu, \quad A_f = \frac{(c^f_L)^2 - (c^f_R)^2}{(c^f_L)^2 + (c^f_R)^2} = \frac{v_f/a_f}{1 + (v_f/a_f)^2} \]

  \[ \sigma_F = 2\pi \int_0^1 \frac{d\sigma}{d\Omega} d(\cos \theta) \]

  \[ \sigma_B = 2\pi \int_{-1}^0 \frac{d\sigma}{d\Omega} d(\cos \theta) \]

- Studied at LEP: \( A_e \simeq A_\mu \simeq A_\tau \simeq 0.15 \Rightarrow \sin^2 \theta_W \simeq 0.23 \simeq \frac{1}{4} \)
Some of the LEP EWWG Results

**LEP W-Boson Mass**

- **ALEPH**: $80.440 \pm 0.051$
- **DELPHI**: $80.336 \pm 0.067$
- **L3**: $80.270 \pm 0.055$
- **OPAL**: $80.415 \pm 0.052$
- **LEP**: $80.376 \pm 0.033$

$\chi^2$/DoF = 48.9/412

From arXiv:1302.3415
Standard Model Production Cross Section Measurements

ATLAS Preliminary
Run 1,2 $\sqrt{s} = 5,7,8,13$ TeV

Theory

LHC pp $\sqrt{s} = 5$ TeV
- Data: 0.025 fb$^{-1}$

LHC pp $\sqrt{s} = 7$ TeV
- Data: 4.5 – 4.9 fb$^{-1}$

LHC pp $\sqrt{s} = 8$ TeV
- Data: 20.2 – 20.3 fb$^{-1}$

LHC pp $\sqrt{s} = 13$ TeV
- Data: 3.2 – 79.8 fb$^{-1}$
Production Cross Section, $\sigma$, in fb

CMS Preliminary

July 2019

- 7 TeV CMS measurement ($L \leq 5.0$ fb$^{-1}$)
- 8 TeV CMS measurement ($L \leq 19.6$ fb$^{-1}$)
- 13 TeV CMS measurement ($L \leq 137$ fb$^{-1}$)
- Theory prediction
- CMS 95% CL limits at 7, 8 and 13 TeV

All results at: http://cern.ch/go/pNj7
The Absolutely Amazing Theory of Almost Everything: Summary

- Based on Symmetry principles: Lorentz + $SU(3)_C \times SU(2)_L \times U(1)_Y$
gauge symmetry
- Renormalizable and unitary
- All predicted particles are discovered experimentally
- The structure of all interactions is fixed (but not all couplings are tested experimentally)
- Anomaly free theory
- Can account for rich Flavour Physics (lect. by M. Vysotsky).
- Three generations allow $CP$-violation (lect. by M. Vysotsky).
- Can be extended to incorporate neutrino masses and mixing (lect by C. Gonzalez-Garcia)
- Survives stringent experimental tests
Issues of the SM

Features to be clarified:

- Unification of all interactions? (Gravity?)
- Pattern behind Flavour Physics (hierarchy in masses and mixing, 3 generations)?
- Symmetry behind SM charge assignment?
- Origin of the Higgs potential?
- Origin of accidental Baryon and Lepton number symmetries*?
- No CP-volation in strong interactions**
- Why the Higgs mass is so low? (Hierarchy/Naturalness problem)
- ...

Phenomenological problems:

- Origin of neutrino masses (see lect. by C. Gonzalez-Garcia)
- Baryon asymmetry (see lect. by V. Rubakov)
- Dark matter, Dark energy, Inflation (see lect. by V. Rubakov)
- Tension in \((g - 2)_\mu\), \(b \rightarrow s\mu\mu\), \(b \rightarrow c\nu\).
- ...

* accidental

** No CP-volation in strong interactions
Concluding remarks

- The SM is enormously successful theory.
- Any New Physics theory should reproduce the SM as the low-energy limit.
- The values of the SM parameters do not give hints for New Physics scale.
- We still believe that the SM is not an ultimate theory (see lect. by V. Sanz) and we you eagerly search for any deviations ...
- In the absence of direct signal a key role is played by precision measurements.
The five orders of magnitude increase of statistics with respect to LEP will yield to considerably more precise measurements of observables such as the $Z$ partial widths and asymmetries, where systematic uncertainties largely cancel in the ratio of cross sections. [FCC Design Study Group]

Table from Mogens Dam, Precision Electroweak measurements at the FCC-ee

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>$M_Z$ [MeV]</td>
<td>Lineshape</td>
<td>91187.5 ± 2.1</td>
<td>0.005</td>
<td>&lt; 0.1</td>
</tr>
<tr>
<td>$\Gamma_Z$ [MeV]</td>
<td>Lineshape</td>
<td>2495.2 ± 2.3</td>
<td>0.008</td>
<td>&lt; 0.1</td>
</tr>
<tr>
<td>$R_\ell$</td>
<td>Peak</td>
<td>20.767 ± 0.025</td>
<td>0.0001</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td>$R_b$</td>
<td>Peak</td>
<td>0.21629 ± 0.00066</td>
<td>0.000003</td>
<td>&lt; 0.00006</td>
</tr>
<tr>
<td>$N_\nu$</td>
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<td>2.984 ± 0.008</td>
<td>0.00004</td>
<td>0.004</td>
</tr>
<tr>
<td>$A_{FB}^{\mu\mu}$</td>
<td>Peak</td>
<td>0.0171 ± 0.0010</td>
<td>0.000004</td>
<td>&lt; 0.00001</td>
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<tr>
<td>$\alpha_s(M_Z)$</td>
<td>$R_\ell$</td>
<td>0.1190 ± 0.0025</td>
<td>0.000001</td>
<td>0.00015</td>
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<tr>
<td>$1/\alpha_{QED}(M_Z)$</td>
<td>$A_{FB}^{\mu\mu}$ around peak</td>
<td>128.952 ± 0.014</td>
<td>0.004</td>
<td>0.002</td>
</tr>
<tr>
<td>$M_W$ [MeV]</td>
<td>Threshold scan</td>
<td>80385 ± 15</td>
<td>0.3</td>
<td>&lt; 1</td>
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<tr>
<td>$N_\nu$</td>
<td>$e^+e^- \rightarrow \gamma Z$(inv.)</td>
<td>2.92 ± 0.05</td>
<td>0.0008</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td>$\alpha_s(M_W)$</td>
<td>$B_{had} = (\Gamma_{had}/\Gamma_{tot})W$</td>
<td>$B_{had} = 67.41 \pm 0.27$</td>
<td>0.00018</td>
<td>0.00015</td>
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<tr>
<td>$m_{top}$ [MeV]</td>
<td>Threshold scan</td>
<td>173200 ± 900</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

The five orders of magnitude increase of statistics with respect to LEP will yield to considerably more precise measurements of observables such as the $Z$ partial widths and asymmetries, where systematic uncertainties largely cancel in the ratio of cross sections. [FCC Design Study Group]
### Observables and Measurements

<table>
<thead>
<tr>
<th>Observable</th>
<th>Measurement</th>
<th>Current precision</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_Z$ [MeV]</td>
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</tbody>
</table>

**Statistical and Possible Systematic Errors**

<table>
<thead>
<tr>
<th>Observable</th>
<th>Stat.</th>
<th>Possible syst.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z$</td>
<td>&lt; 0.1</td>
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<tr>
<td>$\Gamma_Z$</td>
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<td></td>
</tr>
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<td>&lt; 0.00006</td>
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<td>0.004</td>
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<tr>
<td>$1/\alpha_{QED}(M_Z)$</td>
<td>0.00015</td>
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<tr>
<td>$M_W$</td>
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<tr>
<td>$\alpha_s(M_W)$</td>
<td>&lt; 1</td>
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</tr>
<tr>
<td>$m_{top}$</td>
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<tr>
<td></td>
<td>0.00015</td>
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*Comments at the FCC-ee*