(Quantum) Field Theory and the Electroweak Standard Model

Lecture III

Alexander Bednyakov

Bogoliubov Laboratory of Theoretical Physics
Joint Institute for Nuclear Research

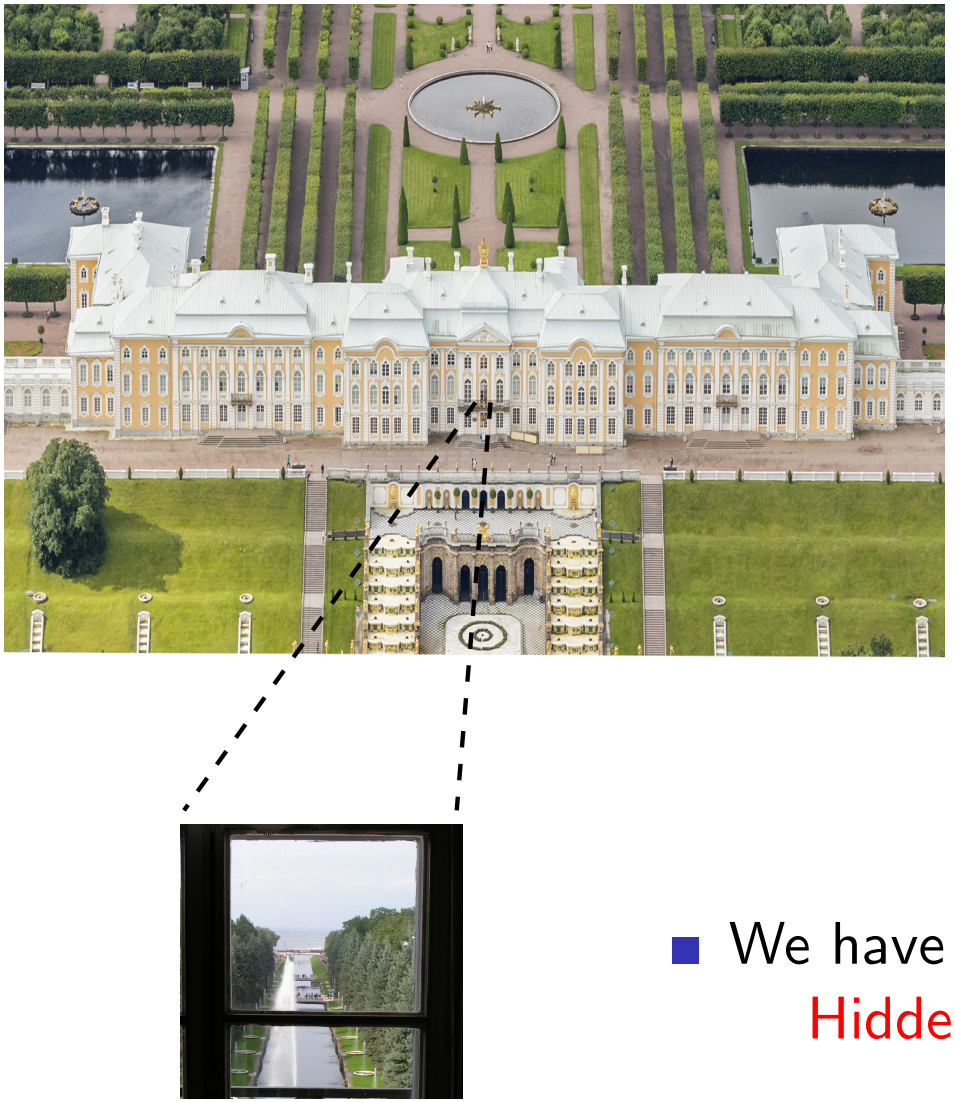
The CERN-JINR European School of High-Energy Physics,
St. Petersburg, Russia, 4 - 17 September 2019

Outline

- Lecture I
  - What is the Standard Model?
  - Introducing Quantum Fields
  - Interactions and Perturbation Theory
  - Renormalizable or Non-Renormalizable?
- Lecture II
  - An Ode to Symmetry
  - Global Symmetries (and Conserved Quantities)
  - Local Symmetries (and Gauge interactions)
  - From Fermi Model to EW theory
- Lecture III
  - Finalizing the EW SM (a bit of Higgsing)
  - "Features" of the SM
  - Experimental tests of the EW SM
  - Issues and Prospects of the EW SM

Break the symmetry?

Q: How to make W and Z massive (keeping the nice features of the gauge theory intact)?



- Explicit breaking via the mass terms

$$\mathcal{L} \ni m_W^2 W_\mu^+ W_-^\mu + \frac{m_Z^2}{2} Z^\mu Z_\mu$$

leads to inconsistencies...

- We have to do something more clever... Hidden symmetry?

Hidden Gauge Symmetry

We need to generate masses for $W_\mu^\pm$ and Z_μ but not for A_μ without explicit breaking of the gauge symmetry.

Let's consider simple $U(1)$ gauge symmetry for scalar electrodynamics:

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial_\mu \phi - V(\phi^\dagger \phi) - \frac{1}{4} F_{\mu\nu}^2 + ie \left(\phi^\dagger \partial_\mu \phi - \phi \partial_\mu \phi^\dagger \right) A_\mu + e^2 A_\mu A_\mu \phi^\dagger \phi,$$

which is invariant under

$$\phi \to e^{ie\omega(x)} \phi, \quad A_\mu \to A_\mu + \partial_\mu \omega.$$

In polar coordinates ($\phi = \frac{1}{\sqrt{2}} \rho(x) e^{i\theta(x)}$) the Lagrangian

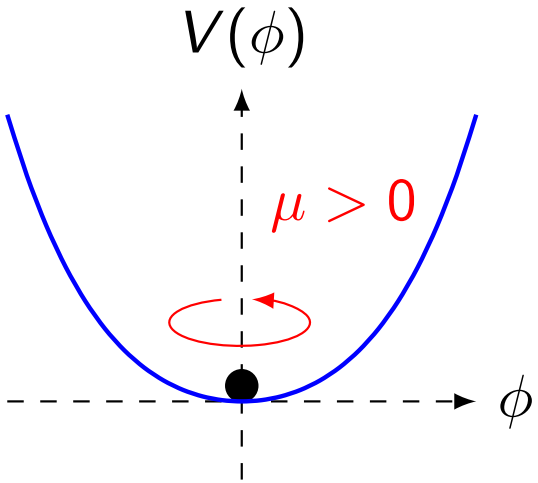
$$\mathcal{L} = \frac{1}{2} (\partial_\mu \rho)^2 + \frac{e^2 \rho^2}{2} \left(A_\mu - \frac{1}{e} \partial_\mu \theta \right) \left(A_\mu - \frac{1}{e} \partial_\mu \theta \right) - V(\rho^2/2) - \frac{1}{4} F_{\mu\nu}^2$$

is still invariant. (Ex: How ρ and θ change under $U(1)$ transformations?)
NB: The symmetry becomes hidden if we define $B_\mu \equiv A_\mu - \frac{1}{e} \partial_\mu \theta$!

Spontaneous Symmetry Breaking

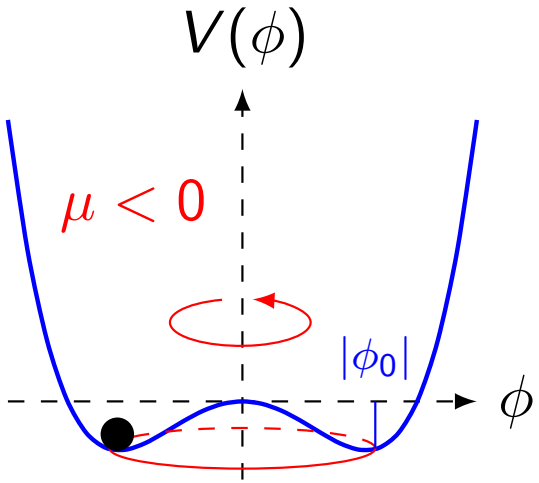
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \rho)^2 + \frac{e^2 \rho^2}{2} B_\mu B_\mu - V(\rho^2/2) - \frac{1}{4} F_{\mu\nu}^2(B)$$

Here $\rho(x)$ is a dynamical field. We get mass term if $\rho(x) \to v = \mathrm{const}$.



$$V = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

It is invariant under global phase-shift symmetry $\phi \to e^{i\alpha}\phi$.



- The case $\mu^2 > 0$ is trivial.
- For $\mu^2 < 0$ we have a valley of degenerate minima:

$$\frac{\partial V}{\partial \phi^\dagger} = 0 \Rightarrow \phi_0^\dagger \phi_0 = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2} > 0 \Rightarrow \phi_0 = \frac{v}{\sqrt{2}} e^{i\beta}$$

The Brout-Englert-Higgs mechanism

Non-zero ϕ in the minimum of the potential is interpreted as the vacuum expectation value (vev) of the quantum field:

$$\frac{v}{\sqrt{2}} = \langle 0|\phi(x)|0\rangle, \qquad \beta = 0 \text{ (Why?)}$$

To introduce particles as excitations we have to shift the field:

$$\phi(x) = \frac{v + h(x)}{\sqrt{2}} e^{i\zeta(x)/v}, \qquad \langle 0|h(x)|0\rangle = 0, \quad \langle 0|\zeta(x)|0\rangle = 0$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu h)^2 + \frac{e^2 v^2}{2} B_\mu B_\mu + evhB_\mu B_\mu + \frac{e^2}{2} B_\mu B_\mu h^2 - V - \frac{1}{4} F_{\mu\nu}^2(B)$$

$$V = -\frac{|\mu|^2}{2}(v+h)^2 + \frac{\lambda}{4}(v+h)^4 = \frac{2\lambda v^2}{2} h^2 + \lambda v h^3 + \frac{\lambda}{4} h^4 - \frac{\lambda}{4} v^4$$

Massive field B_μ without explicit symmetry breaking! This is the essence of Brout-Englert-Higgs-Hagen-Guralnik-Kibble mechanism. The symmetry is hidden..

The Brout-Englert-Higgs mechanism: Counting DOFs

We start with

$$\mathcal{L}_1 = \partial_\mu \phi^\dagger \partial_\mu \phi + ie\left(\phi^\dagger \partial_\mu \phi - \phi \partial_\mu \phi^\dagger\right) A_\mu + e^2 A_\mu A_\mu \phi^\dagger \phi - V - \frac{1}{4} F_{\mu\nu}^2(A),$$

and end with $[\sqrt{2}\phi = (v+h)\exp(i\zeta(x)/v),\ B_\mu = A_\mu - \partial_\mu \zeta/(ev)]$

$$\mathcal{L}_2 = \frac{1}{2}(\partial_\mu h)^2 - \frac{e^2 v^2}{2}\left(1 + \frac{h^2}{v^2}\right) B_\mu B_\mu + evhB_\mu B_\mu - V - \frac{1}{4} F_{\mu\nu}^2(B).$$

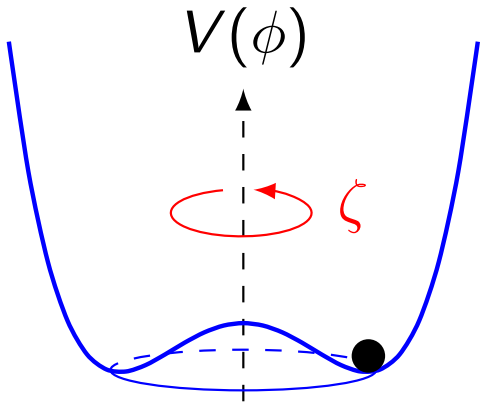
$\mathcal{L}_1$: 2 DOFs (complex scalar ϕ, $\phi^\dagger$) + 2 DOFs (massless vector A_μ).

$\mathcal{L}_2$: 1 DOFs (real scalar h) + 3 DOFs (massive vector field B_μ).

One scalar DOF was "eaten" by the gauge field to become massive.

Q: Which one?

A: The (would-be) Nambu-Goldstone (boson)!



SSB and Renormalizability

Both $\mathcal{L}_1$ and $\mathcal{L}_2$ has some issues:

$\mathcal{L}_1$: Manifestly gauge-invariant, but not suitable for PT (imaginary mass);

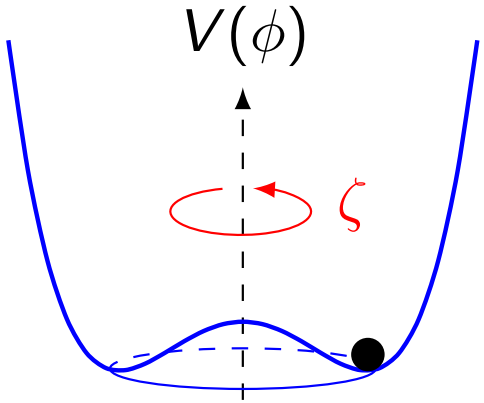
$\mathcal{L}_2$: Hidden gauge symmetry, only physical DOFs, but non-renormalizable by power counting.

There is another, explicitly renormalizable version of the Lagrangian with shifted ϕ written in cartesian coordinates: $\phi = \frac{1}{\sqrt{2}}(v + \eta + i\xi)$

$$\mathcal{L}_3 = \frac{v^4\lambda}{4} - \frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{e^2v^2}{2}A_\mu A_\mu + \frac{1}{2}\partial_\mu\xi\,\partial_\mu\xi - evA_\mu\partial_\mu\xi$$
$$+ \frac{1}{2}\partial_\mu\eta\,\partial_\mu\eta - \frac{2v^2\lambda}{2}\eta^2 + eA_\mu\xi\partial_\mu\eta - eA_\mu\eta\partial_\mu\xi - v\lambda\eta(\eta^2 + \xi^2)$$
$$- \frac{\lambda}{4}(\eta^2 + \xi^2)^2 + \frac{e^2}{2}A_\mu A_\mu(2v\eta + \eta^2 + \xi^2).$$

NB: Now we have massless unphysical field ξ in the spectrum, but it mixes with longitudinal component of A_μ ("partially eaten").

A Remark on the Goldstone Theorem

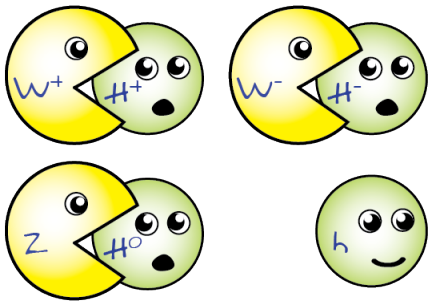


The Goldstone theorem states that if the vacuum breaks a global continuous symmetry there is a massless boson (Nambu-Goldstone) in the spectrum: any non-derivative interactions violates

$$\zeta \to \zeta + ev\omega, \quad \omega = \text{const}$$

Fortunately, we have local symmetry with hungry A_μ...

- We need 3 massive bosons $W^\pm, Z_\mu$.
- 3 symmetries out of $SU(2)_L \times U(1)_Y$ has to be spontaneously broken to get 3 ~~victims~~ (would-be) Nambu-Goldstone bosons.



NB: The Higgs boson (see lect. by J. Ellis) was a by-product of the mass-generation mechanism (the main task was to "exorcise" massless fields).

SSB in the EW theory (GWS)

- To break $SU(2)_L \times U(1)_Y$ we can use a scalar $SU(2)$-doublet:

$$\Phi = \frac{1}{\sqrt{2}} \exp\left(i \frac{\zeta_i \sigma^i}{2v} \right) \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}, \qquad \Phi_0 \equiv \langle 0 | \Phi | 0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

- Let it be also charged under $U(1)_Y$:

$$\Phi \to \exp\left(ig \frac{\sigma^i}{2} \omega_a + ig' \frac{Y_H}{2} \omega' \right) \Phi$$

- We do not want to break $U(1)_{\text{em}}$ and seek for a combination of $SU(2)_L$ and $U(1)_Y$ generators that annihilate the vacuum:

$$Q\Phi_0 = 0, \qquad Q = \frac{\sigma_3}{2} + \frac{Y_H}{2} = \frac{1}{2} \begin{pmatrix} 1 + Y_H & 0 \\ 0 & -1 + Y_H \end{pmatrix}$$

- For $Y_H = 1$, the vacuum has zero electric charge.

SSB in the EW theory (GWS)

- The Lagrangian for Φ

$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi), \quad \text{with} \quad V(\Phi) = m_\Phi^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

leads to $v = \frac{|m_\Phi^2|}{\lambda}$ provided $m_\Phi^2 < 0$.

- Gauge symmetry allows us to get rid of ζ_i (unitary gauge):

$$|D_\mu \Phi|^2 = \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2}{8}(v+h)^2 |W_\mu^1 + iW_\mu^2|^2 + \frac{1}{8}(v+h)^2 (gW_\mu^3 - g' Y_H B_\mu)^2.$$

- We do not want to generate mass for the photon ($Y_H = 1$):

$$0 = g \sin\theta_W - g' \cos\theta_W, \qquad W_\mu^3 = Z_\mu \cos\theta_W + A_\mu \sin\theta_W$$

$$e = g \sin\theta_W = gg'/\sqrt{g^2 + g'^2} \qquad B_\mu = -Z_\mu \sin\theta_W + A_\mu \cos\theta_W$$

SSB in the EW theory (GWS)

- The Lagrangian for Φ

$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi), \quad \text{with} \quad V(\Phi) = m_\Phi^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

leads to $v = \frac{|m_\Phi^2|}{\lambda}$ provided $m_\Phi^2 < 0$.

- Gauge symmetry allows us to get rid of ζ_i (unitary gauge):

$$|D_\mu \Phi|^2 = \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2}{4}(v+h)^2 W_\mu^+ W_\mu^- + \frac{1}{8}(v+h)^2 (g^2 + g'^2) Z_\mu^2.$$

- Masses of W- and Z-bosons are related:

$$M_W = gv/2, \quad M_Z = \sqrt{g^2 + g'^2}\, v/2$$

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1(?)$$

$$W_\mu^3 = \quad Z_\mu \cos\theta_W + A_\mu \sin\theta_W$$

$$B_\mu = -Z_\mu \sin\theta_W + A_\mu \cos\theta_W$$

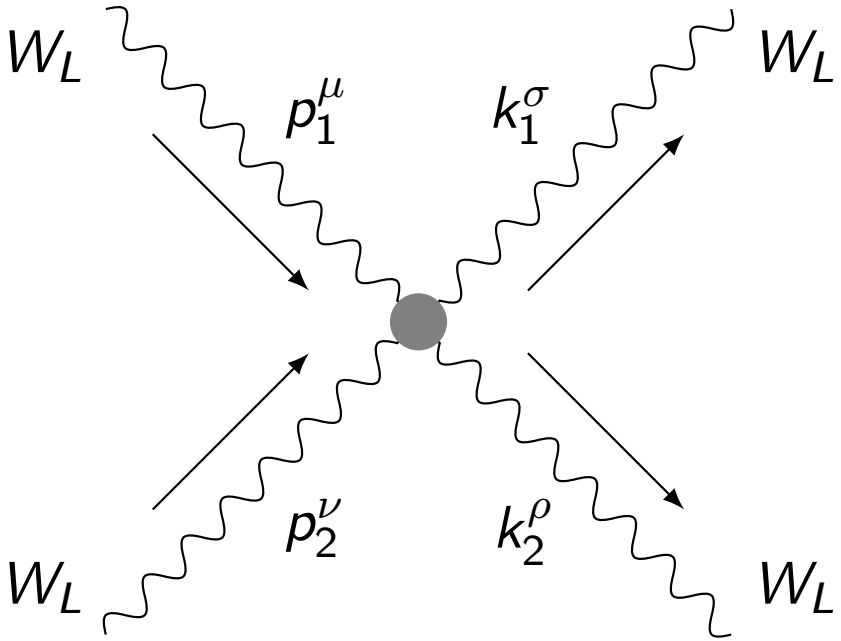
On Scattering of Longitudinal W-bosons and Higgs boson

At high-energies the amplitude of longitudinal W scattering:

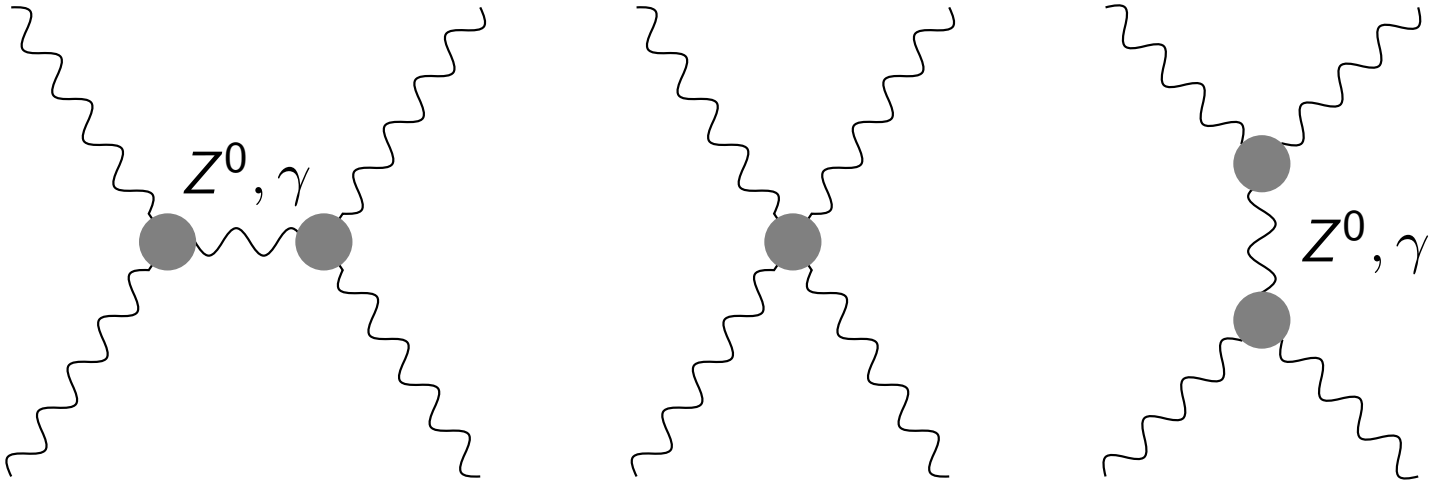
$$\mathcal{M} = g^2 \epsilon_L^\mu(p_1)\epsilon_L^\nu(p_2)\left(2g_{\mu\sigma}g_{\nu\rho} - g_{\mu\rho}g_{\nu\sigma} - g_{\mu\nu}g_{\rho\sigma}\right)\epsilon_L^\rho(k_1)\epsilon_L^\sigma(k_2)$$

$$\mathcal{M} \propto g^2 \frac{E^4}{M_W^4}$$

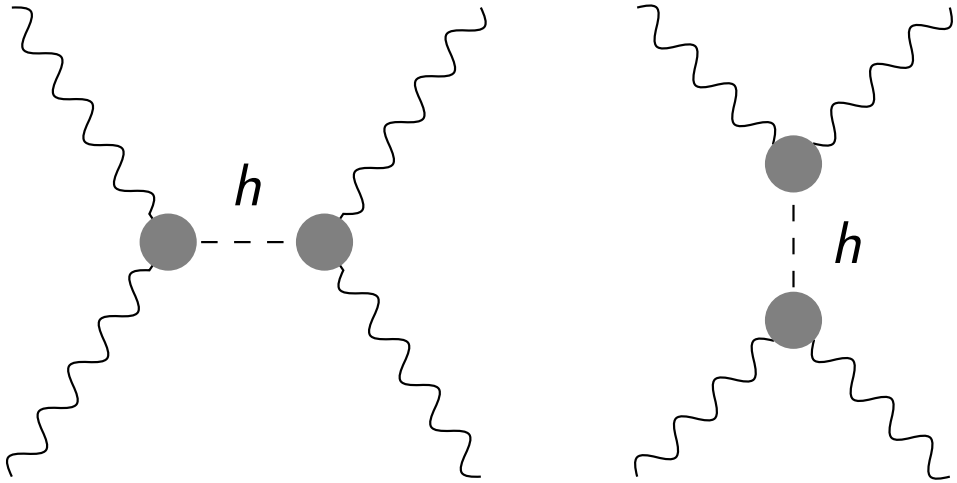
We expect perturbative unitarity violation for $E \sim M_W$.



On Scattering of Longitudinal W-bosons and Higgs boson



$$\mathcal{M} \propto g^2 \frac{E^2}{M_W^2}$$



$$\mathcal{M} \propto -g^2 \frac{E^2}{M_W^2}$$

The Higgs is crucial for unitarization of WW-scattering!

SSB and Fermion Masses

- The $SU(2)_L \times U(1)_Y$ forbids fermion mass terms:

$$m\bar{\psi}\psi = m\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right)$$

- To add masses in the SM we introduce Yukawa interactions:

$$\mathcal{L}_Y = -y_e(\bar{L}\Phi)e_R - y_d(\bar{Q}\Phi)d_R - y_u(\bar{Q}\Phi^c)u_R + \text{h.c.}$$

with $SU(2)$ doublets

$$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, Q = \begin{pmatrix} u \\ d \end{pmatrix}_L, \Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \Phi^c = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$$

- $\Phi \to \langle 0|\Phi|0\rangle$:

$$\mathcal{L}_m^f = -\left[\frac{y_e v}{\sqrt{2}}\bar{e}e + \frac{y_d v}{\sqrt{2}}\bar{d}d + \frac{y_u v}{\sqrt{2}}\bar{u}u\right], \qquad m_f = \frac{y_f v}{\sqrt{2}}$$

Ex: Check that all terms in $\mathcal{L}_Y$ are singlets w.r.t $SU(2)_L \times U(1)_Y$.

SSB and Fermion Masses: Towards Flavour Physics

- The Lagrangian $\mathcal{L}_Y$ can be generalized to account for known three generations (introduce indices $i, j = 1, 2, 3$):

$$\mathcal{L}_Y = -y_l^{ij}(\bar{L}_i\Phi)l_{jR} - y_d^{ij}(\bar{Q}_i\Phi)d_{jR} - y_u^{ij}(\bar{Q}_i\Phi^c)u_{jR} + \mathrm{h.c.}$$

- Yukawa Matrices are non-diagonal (and complex) $\Rightarrow$

$$\mathcal{L}_m^f = -\frac{y_f^{ij}v}{\sqrt{2}}\bar{f}_L^i f_R^j + \mathrm{h.c.} \qquad M_f^{ij} = \frac{y_f^{ij}v}{\sqrt{2}}$$

- Mass Eigenstates: Diagonalizing M_f we obtain masses m_f.

NB1: Couplings of the mass eigenstates f to h are proportional to m_f.
NB2: Three generations are required to have $\mathcal{CP}$ violation in quark sector.
NB3: Dirac mass for neutrinos can also be introduced in a similar way.

(see details in lect. by M. Vysotsky and C. Gonzalez-Garcia)

The SM Lagrangian: Gauge

Let us combine all the ingredients and write down the Full SM Lagrangian

$$\begin{aligned}\mathcal{L}_{SM} = &\ \mathcal{L}_{\text{Gauge}}(g_s, g, g') \\ &+ \mathcal{L}_{\text{Yukawa}}(y_u, y_d, y_l) \\ &+ \mathcal{L}_{\text{Higgs}}(\lambda, m_\Phi^2) \quad \left[-V_{\text{Higgs}}(\lambda, m_\Phi^2)\right] \\ &+ \mathcal{L}_{\text{Gauge-fixing}} + \mathcal{L}_{\text{Ghosts}}\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{\text{Gauge}} = &-\frac{1}{4}\underbrace{G^a_{\mu\nu}G^a_{\mu\nu}}_{SU(3)_c} - \frac{1}{4}\underbrace{W^i_{\mu\nu}W^i_{\mu\nu}}_{SU(2)_L} - \frac{1}{4}\underbrace{B_{\mu\nu}B_{\mu\nu}}_{U(1)_Y} + (D_\mu\Phi)^\dagger(D_\mu\Phi) \\ &+ \underbrace{\bar{L}_i\, i\hat{D}\, L_i + \bar{Q}_i\, i\hat{D}\, Q_i}_{SU(2)_L \text{ doublets}} + \underbrace{\bar{l}_{Ri}\, i\hat{D}\, l_{R_i} + \bar{u}_{Ri}\, i\hat{D} u_{R_i} + \bar{d}_{Ri}\, i\hat{D}\, d_{R_i}}_{SU(2)_L \text{ singlets}}\end{aligned}$$

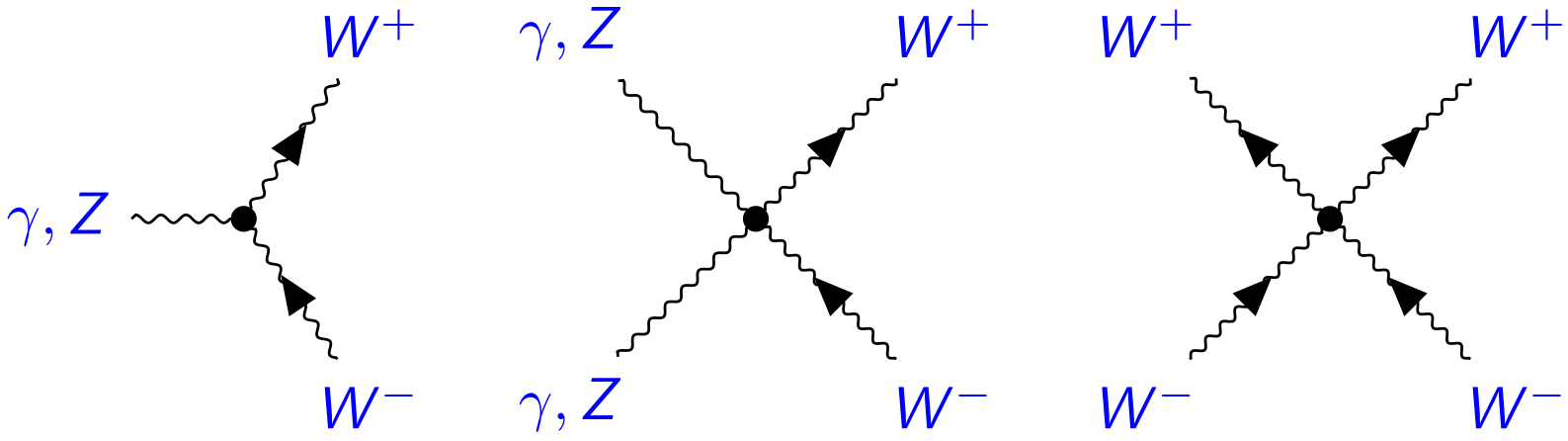
We have three parameters g_s, g, g' in the Gauge sector.

The SM Lagrangian: Gauge Self-Interactions

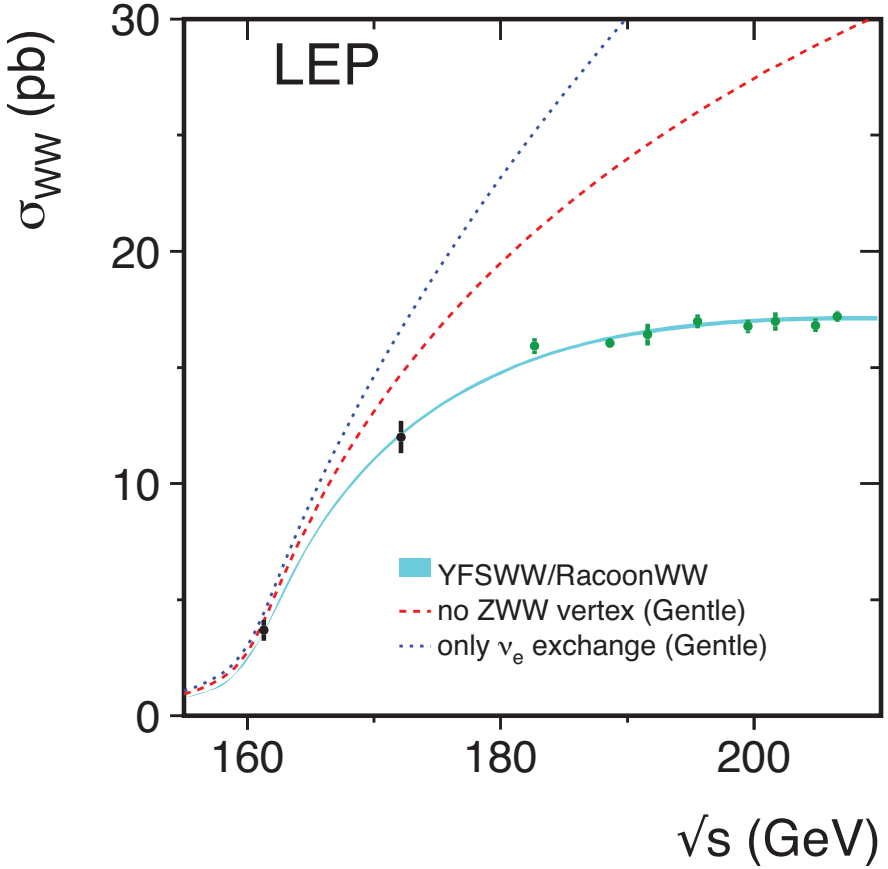
Let us combine all the ingredients and write down the Full SM Lagrangian

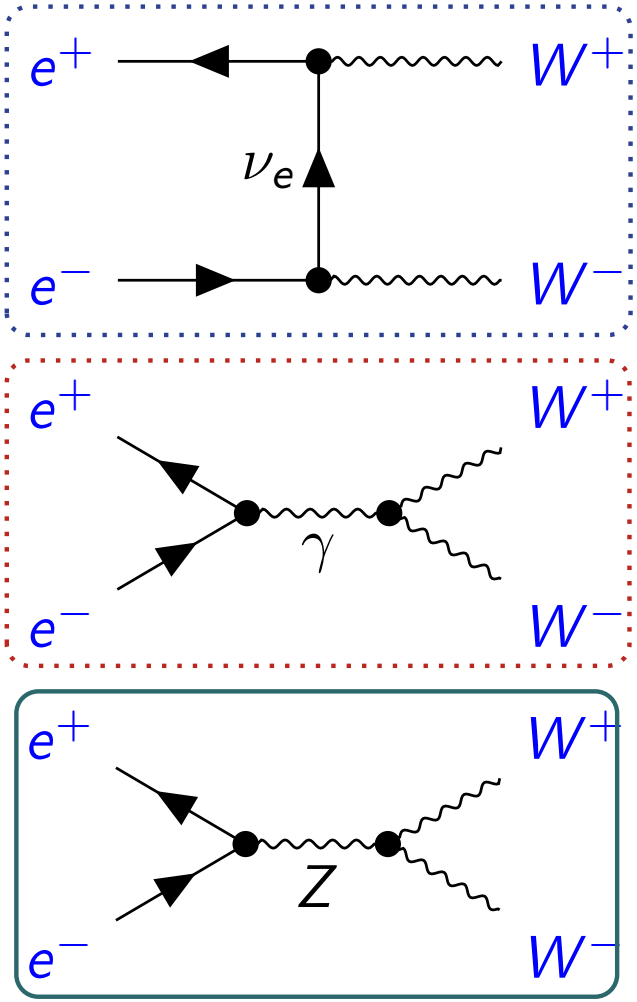
$$\begin{aligned}\mathcal{L}_{SM} = {} & \mathcal{L}_{\text{Gauge}}(g_s, g, g') \\ & + \mathcal{L}_{\text{Yukawa}}(y_u, y_d, y_l) \\ & + \mathcal{L}_{\text{Higgs}}(\lambda, m_\Phi^2) \quad \left[-V_{\text{Higgs}}(\lambda, m_\Phi^2)\right] \\ & + \mathcal{L}_{\text{Gauge-fixing}} + \mathcal{L}_{\text{Ghosts}}\end{aligned}$$

Due to Non-Abelian nature we have Gauge-boson self-interactions:



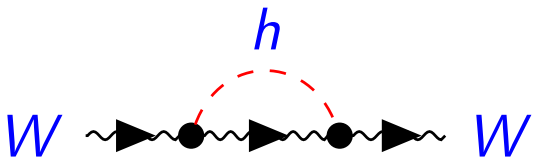
The SM Lagrangian: Gauge Self-Interactions



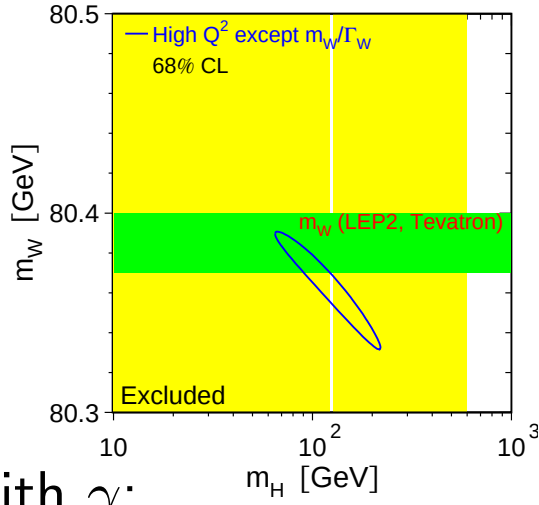


The SM Lagrangian: Gauge-Higgs Interactions

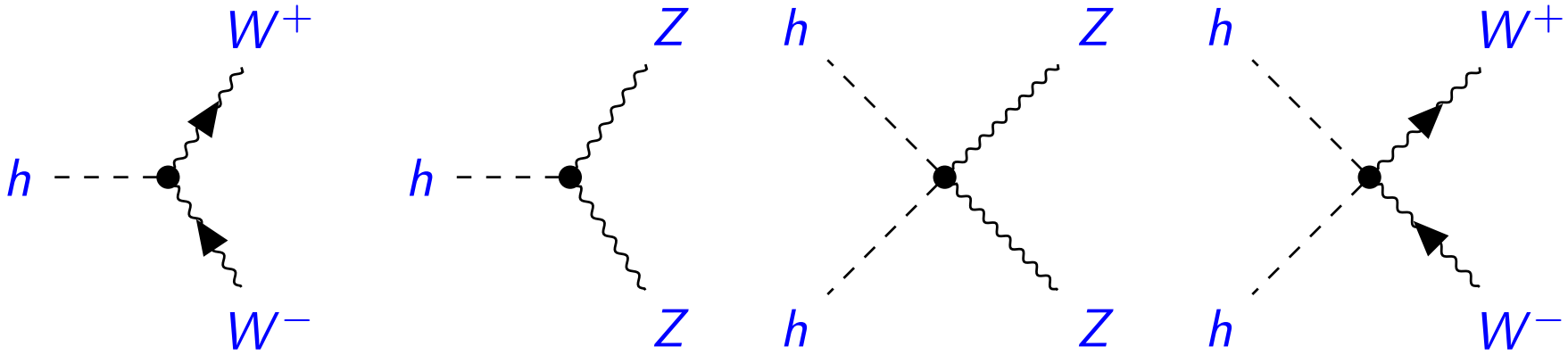
Let us combine all the ingredients and write down the Full SM Lagrangian



$$\mathcal{L}_{SM} = \mathcal{L}_{\text{Gauge}}(g_s, g, g') + \mathcal{L}_{\text{Yukawa}}(y_u, y_d, y_l) + \mathcal{L}_{\text{Higgs}}(\lambda, m_\Phi^2) \quad \left[-V_{\text{Higgs}}(\lambda, m_\Phi^2)\right] + \mathcal{L}_{\text{Gauge-fixing}} + \mathcal{L}_{\text{Ghosts}}$$



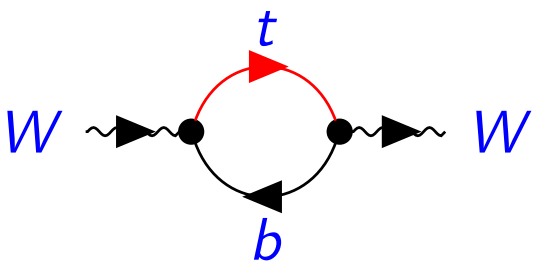
The Higgs boson h interacts with W and Z, but not with γ:



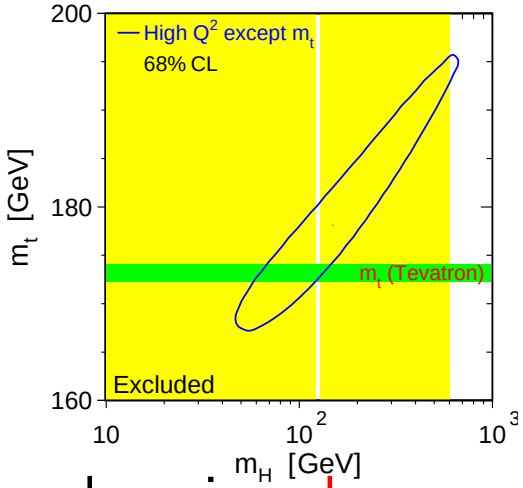
Ex: What are the couplings?

The SM Lagrangian: Gauge-Fermion Interactions

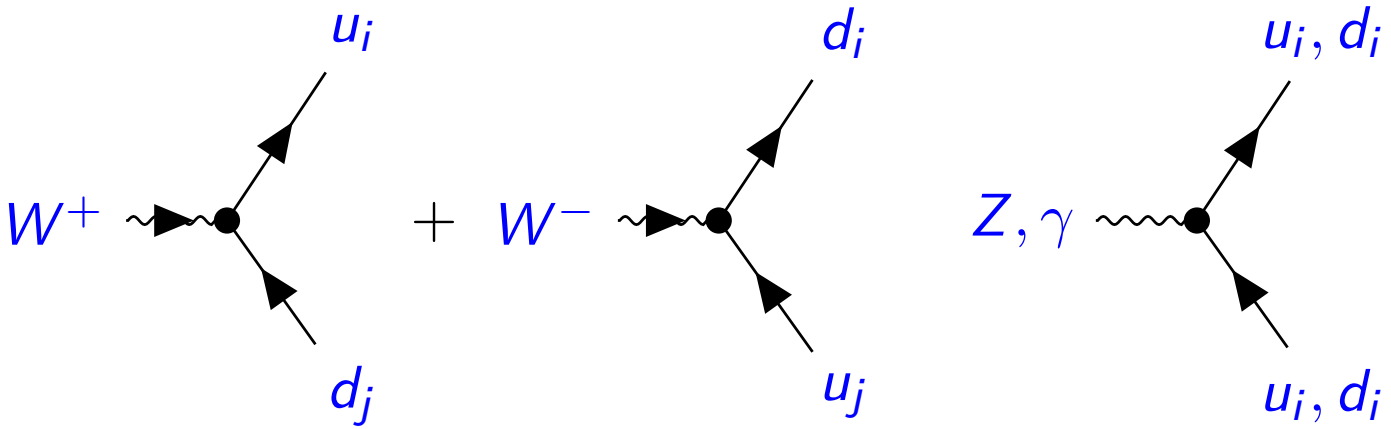
Let us combine all the ingredients and write down the Full SM Lagrangian



$$\mathcal{L}_{SM} = \mathcal{L}_{\text{Gauge}}(g_s, g, g') + \mathcal{L}_{\text{Yukawa}}(y_u, y_d, y_l) + \mathcal{L}_{\text{Higgs}}(\lambda, m_\Phi^2) \quad \left[-V_{\text{Higgs}}(\lambda, m_\Phi^2)\right] + \mathcal{L}_{\text{Gauge-fixing}} + \mathcal{L}_{\text{Ghosts}}$$



After diagonalization of Yukawa matrices, we have flavour-changing charge-current interactions, but flavour-conserving neutral-current transitions:



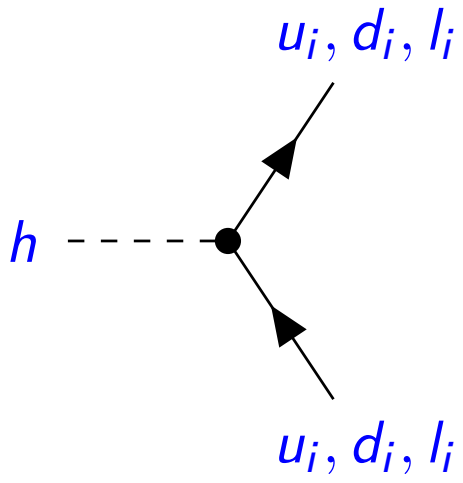
NB: The CKM matrix appears in charged currents (see lect. by M. Vysotsky)

The SM Lagrangian: Yukawa

Let us combine all the ingredients and write down the Full SM Lagrangian

$$\begin{aligned}\mathcal{L}_{SM} = &\ \mathcal{L}_{\text{Gauge}}(g_s, g, g') \\ &+ \mathcal{L}_{\text{Yukawa}}(y_u, y_d, y_l) \\ &+ \mathcal{L}_{\text{Higgs}}(\lambda, m_\Phi^2) \quad \left[-V_{\text{Higgs}}(\lambda, m_\Phi^2)\right] \\ &+ \mathcal{L}_{\text{Gauge-fixing}} + \mathcal{L}_{\text{Ghosts}}\end{aligned}$$

The Higgs boson h interacts with fermions (again, no change of flavour). Not all parameters in complex matrices y_f are physical. The physical ones are $6_q + 3_l$ masses, 3 angles in CKM matrix and 1 CPV phase.



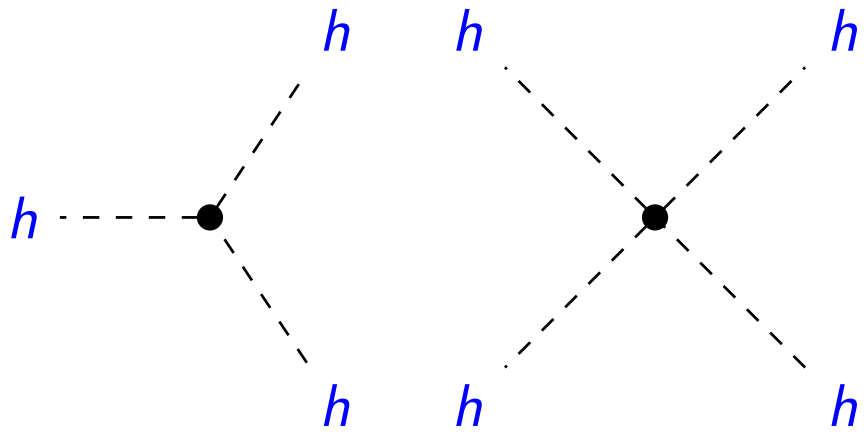
Ex: What are the couplings? Why no Flavour-Changing-Neutral-Current?

The SM Lagrangian: Higgs self-interactions

Let us combine all the ingredients and write down the Full SM Lagrangian

$$\begin{aligned}\mathcal{L}_{SM} = &\ \mathcal{L}_{\text{Gauge}}(g_s, g, g') \\ &+ \mathcal{L}_{\text{Yukawa}}(y_u, y_d, y_l) \\ &+ \mathcal{L}_{\text{Higgs}}(\lambda, m_\Phi^2) \quad \left[-V_{\text{Higgs}}(\lambda, m_\Phi^2)\right] \\ &+ \mathcal{L}_{\text{Gauge-fixing}} + \mathcal{L}_{\text{Ghosts}}\end{aligned}$$

The shift in the Higgs field generates triple self-interaction h:



NB: Two parameters in the Higgs sector: $M_h = 2\lambda v^2$ and $v^2 = -m_\Phi^2/\lambda$.

The SM Input Parameters

- There are 18+1 free parameters in the canonical SM (no ν_R).
- To make predictions they have to be extracted from experiment.
- There are different, yet related, sets of parameters. For example,

18=	1	1	1	1	1	9	4
primary:	g_s	g	g'	λ	m_Φ	y_f	y_{ij}
practical:	α_s	M_Z	α	G_F	M_H	m_f	V_{CKM}

$$\alpha_s = \frac{g_s^2(Q)}{4\pi}, \quad (4\pi)\alpha = g^2 g'^2/(g^2+g'^2), \quad M_Z^2 = \frac{(g^2+g'^2)v^2}{4},$$

$$G_F = \frac{1}{\sqrt{2}v^2}, \qquad M_h = 2\lambda v^2 = 2|m_\Phi^2|, \qquad m_f = y_f v/\sqrt{2}$$

Ex: Where is μ?

NB: Tree-level relations, can be corrected in high orders of perturbation theory (renormalization scheme!).

On the Importance of Radiative Corrections

Let us use our physical parameter set to predict M_W at the tree level:

- Given the relations

$$\frac{G_F}{\sqrt{2}} = \frac{1}{2v^2}, \quad (4\pi)\alpha = \frac{g^2 g'^2}{g^2 + g'^2}, \quad M_Z^2 = \frac{g_z^2 v^2}{4}, \quad M_W^2 = \frac{g^2 v^2}{4}$$

  we derive

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2M_W^2(1 - M_W^2/M_Z^2)}.$$

- Plugging the PDG2019 values

$$\alpha^{-1} = 137.035999139(31),$$
$$M_Z = 91.1876(21) \text{ GeV},$$
$$G_F = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2},$$

  we compare "theory" and exp

$$M_W^{tree} = 80.9387(25) \text{ GeV}$$
$$M_W^{exp} = 80.379(12) \text{ GeV}$$

Our naive prediction is off by about 47σ!

Do we have to go beyond the SM?

On the Importance of Radiative Corrections

Let us use our physical parameter set to predict M_W at the tree level:

- Given the relations

$$\frac{G_F}{\sqrt{2}} = \frac{1}{2v^2}, \quad (4\pi)\alpha = \frac{g^2 g'^2}{g^2 + g'^2}, \quad M_Z^2 = \frac{g_z^2 v^2}{4}, \quad M_W^2 = \frac{g^2 v^2}{4}$$

  we derive

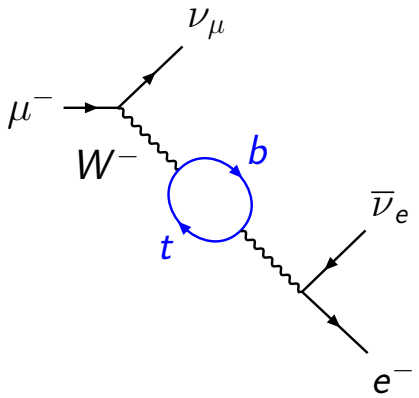
$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2M_W^2(1 - M_W^2/M_Z^2)}.$$

- Plugging the PDG2019 values

$$\alpha^{-1} = 137.035999139(31),$$
$$M_Z = 91.1876(21) \text{ GeV},$$
$$G_F = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2},$$

  we compare "theory" and exp

$$M_W^{tree} = 80.9387(25) \text{ GeV}$$
$$M_W^{exp} = 80.379(12) \text{ GeV}$$



Nope, have to account for higher orders..

Scale dependence of primary SM parameters

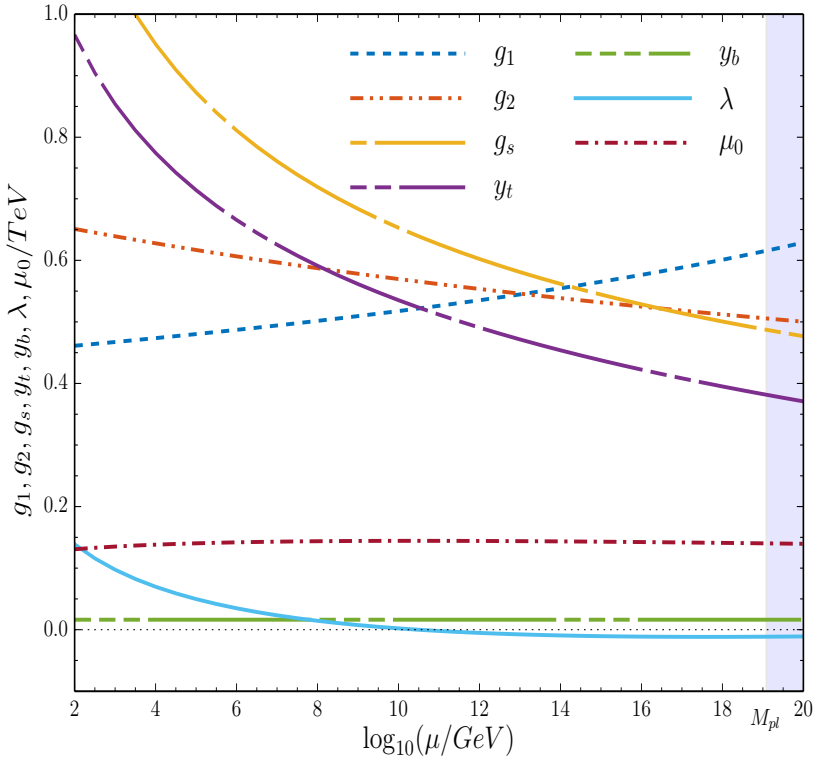
- Grand Unification?

$$\frac{d\alpha}{d\log\mu^2} = \beta\alpha^2 + \mathcal{O}(\alpha^3), \quad \alpha = \frac{g^2}{4\pi}$$

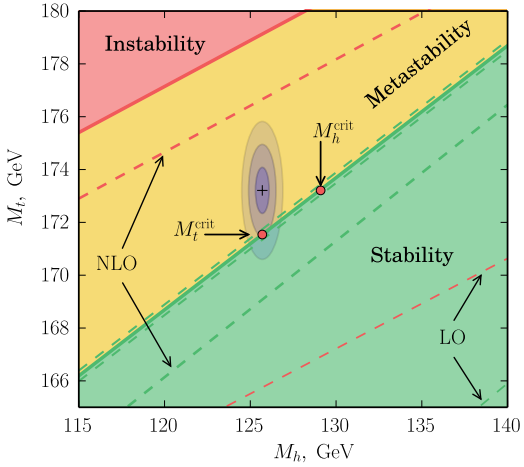
$$\beta = -\frac{1}{4\pi}\left[\frac{11}{3}C_2 - \frac{2}{3}\sum_F T_F - \frac{1}{3}\sum_S T_S\right]$$

$$C_2 = N \text{ for } SU(N), \quad T_F = T_S = \frac{1}{2}$$

for Fermions and Scalars*



- Vacuum (Meta)Stability?



$$V_{eff}(\phi \gg v) \simeq \frac{\lambda(\mu=\phi)}{4}\phi^4$$

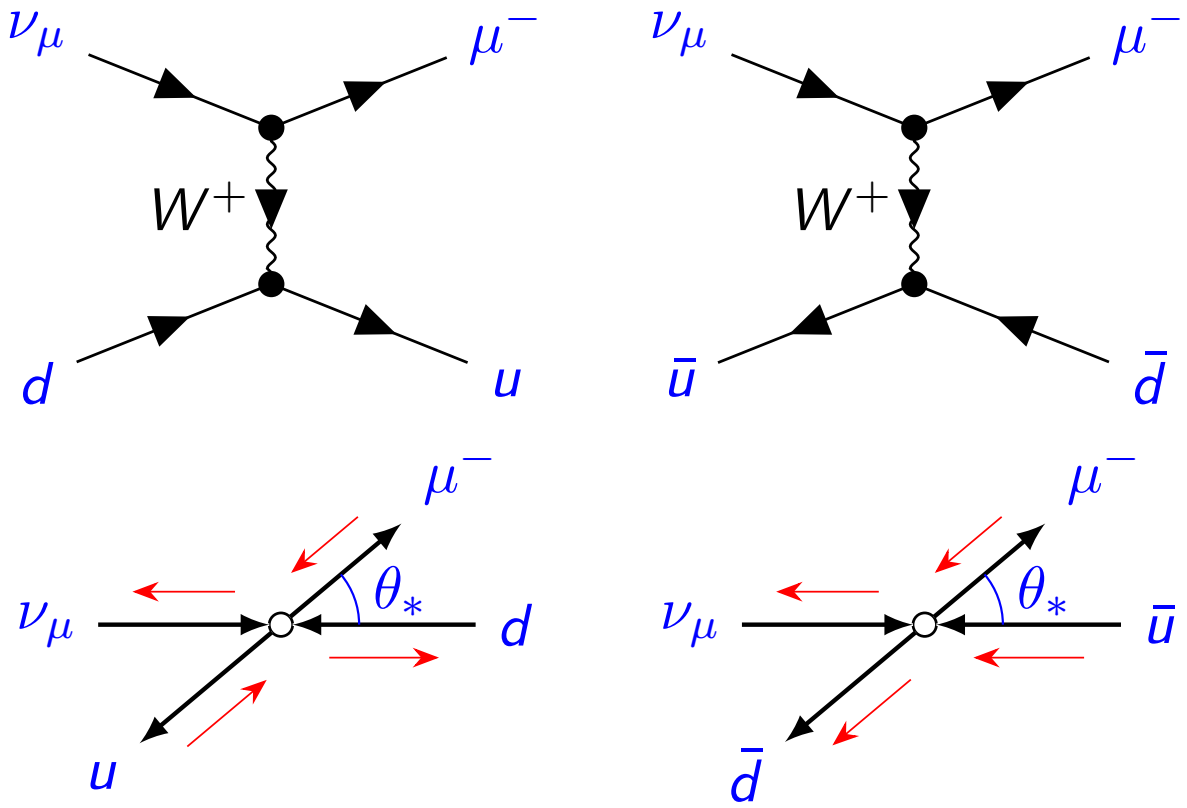
$$(4\pi)^2\frac{d\lambda}{d\log\mu^2} = 12\lambda - 3{y_t}^4 + \ldots$$

Fig. made by A. Pikelner, MR package

(Anti)Neutrino-(Anti)Quark scattering

Due to masslessness* of (anti)neutrino, it is produced in LH chiral state.

Can be used to directly probe the (anti)quark content of the proton or neutron!



$$\frac{d\sigma_{\nu q}}{d\Omega^*} = \frac{G_F s}{4\pi^2} \qquad \frac{d\sigma_{\nu \bar{q}}}{d\Omega^*} = \frac{G_F s}{4\pi^2} \cos^4 \frac{\theta_*}{2}$$

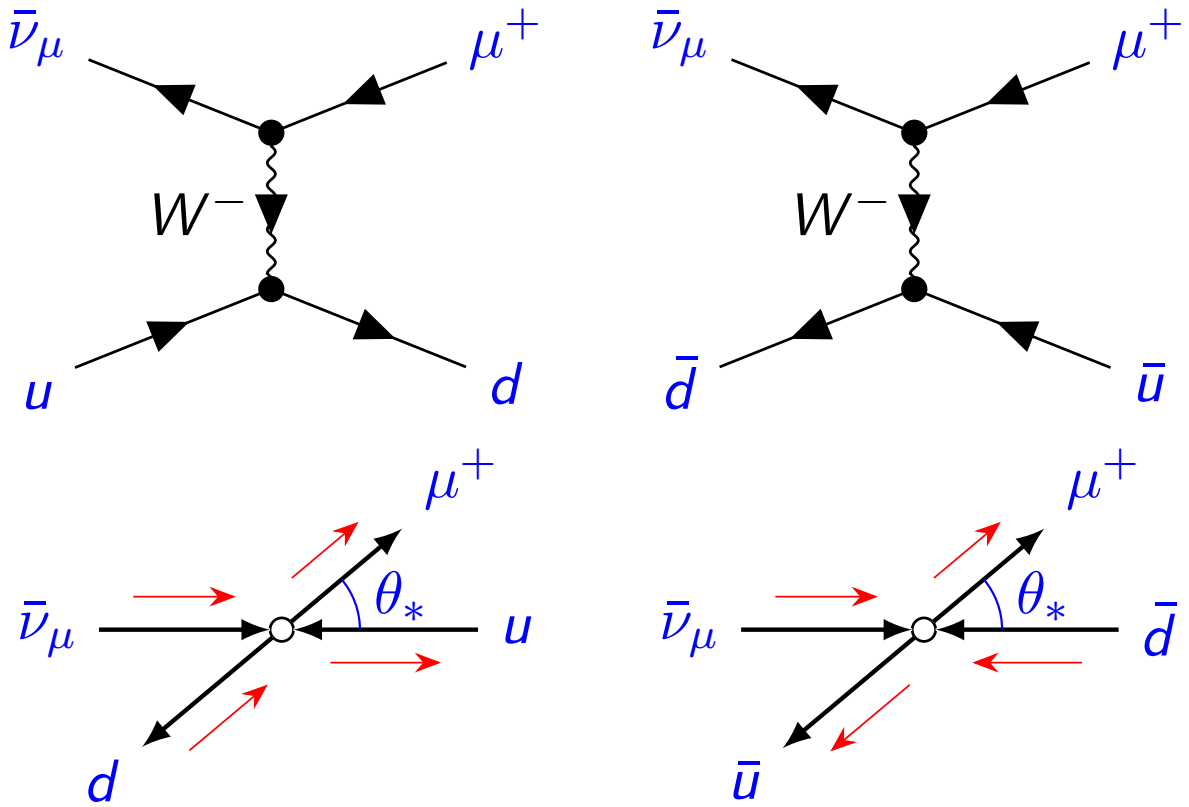
$$\sigma^{\nu q} = \frac{G_F s}{\pi} \qquad \sigma^{\nu \bar{q}} = \frac{G_F s}{3\pi}$$

Ex: Compare with the θ_*-dependence of $e^\pm q$ scattering due to γ-exchange.

(Anti)Neutrino-(Anti)Quark scattering

Due to masslessness* of (anti)neutrino, it is produced in LH chiral state.

Can be used to directly probe the (anti)quark content of the proton or neutron!



$$\frac{d\sigma_{\bar{\nu}q}}{d\Omega^*} = \frac{G_F s}{4\pi^2} \cos^4 \frac{\theta_*}{2}$$

$$\sigma^{\bar{\nu}q} = \frac{G_F s}{3\pi}$$

$$\frac{d\sigma_{\bar{\nu}\bar{q}}}{d\Omega^*} = \frac{G_F s}{4\pi^2}$$

$$\sigma^{\bar{\nu}\bar{q}} = \frac{G_F s}{\pi}$$

Ex: Compare with the θ_*-dependence of $e^\pm q$ scattering due to γ-exchange.

Neutrino DIS $\nu N \to \mu X$ and anti-quark content of p

- For neutrino - proton νp and neutrino - neutron νn scattering:

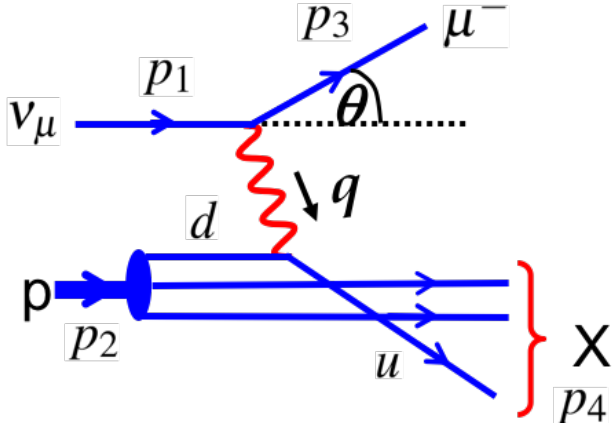
$$\sigma_{\nu p} = \frac{G_F^2 s}{\pi}\left[f_d + \frac{1}{3}f_{\bar{u}}\right], \quad \sigma_{\nu n} = \frac{G_F^2 s}{\pi}\left[f_u + \frac{1}{3}f_{\bar{d}}\right],$$

where $f_q = \int\limits_0^1 xq(x)dx$ - the fraction of proton momentum carried by parton q, and we use $f_u(proton) = f_d(neutron) \equiv f_u$, etc. (why?)

- For an isoscalar target (equal number of protons and neutrons)

$$\sigma_{\nu N} = \frac{1}{2}\left[\sigma_{\nu p} + \sigma_{\nu n}\right] = \frac{G_F^2 s}{2\pi}\left[f_q + \frac{1}{3}f_{\bar{q}}\right], f_u = f_d + f_u,$$

$$\sigma_{\bar{\nu} N} = \frac{G_F^2 s}{2\pi}\left[f_{\bar{q}} + \frac{1}{3}f_q\right]$$

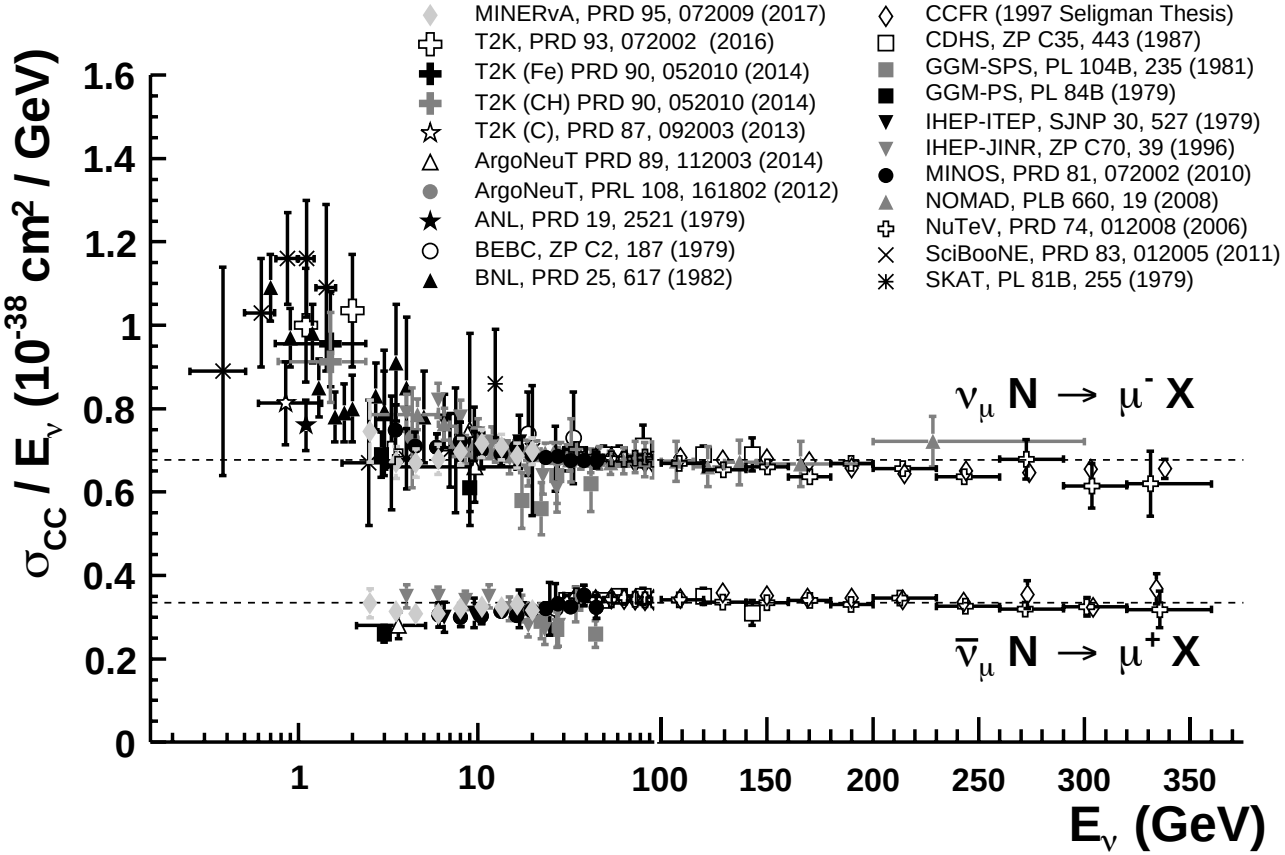


- The ratio probes antiquark content:

$$\frac{\sigma_{\nu N}}{\sigma_{\bar{\nu} N}} = \frac{3f_q + f_{\bar{q}}}{f_q + 3f_{\bar{q}}}$$

Neutrino Total Cross Sections Measurements

In the LAB frame $s \simeq 2m_N E_\nu$ and $\sigma_{\nu N}/E_\nu = \frac{G_F^2 m_N}{2\pi}\left[f_q + \frac{1}{3} f_{\bar{q}}\right]$



$$\sigma^{\nu N}/E_\nu \simeq 0.67 \cdot 10^{-38}\ \text{cm}^2\ \text{GeV}^{-1}, \quad \sigma^{\bar{\nu} N}/E_{\bar{\nu}} \simeq 0.33 \cdot 10^{-38}\ \text{cm}^2\ \text{GeV}^{-1}$$

$$\frac{\sigma_{\nu N}}{\sigma_{\bar{\nu} N}} = 1.984 \pm 0.012 \neq 3(f_{\bar{q}} = 0), \qquad f_q \simeq 0.41 \quad \text{and} \quad f_{\tilde{q}} \simeq 0.08$$

NB: By the way, where is 50% of proton momentum?

Figure from PDG2019

Z-width and branching ratios

$$\mathcal{L}_I \ni g_z Z^\mu J^\mu_Z, \quad J^\mu_Z = \frac{1}{4}\sum_f \bar{f}\left(v_f - a_f\gamma_5\right) f = \sum_f \bar{f}\left(c_L P_L + c_R P_R\right) f$$

$$\Gamma(Z \to f\bar{f}) = \frac{g_Z^2 m_Z}{24\pi}\left(c_L^2 + c_R^2\right)$$

Assuming $\sin^2\theta_W \simeq 0.23$:

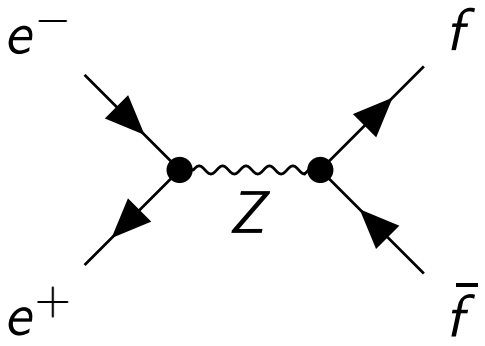
fermion	Q_f	T_3^f	Y_L	Y_R	c_L	c_R
ν_l	0	$+\frac{1}{2}$	-1	0	$+\frac{1}{2}$	0
l^-	-1	$-\frac{1}{2}$	-1	-2	-0.27	$+0.23$
u	$+\frac{2}{3}$	$+\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{4}{3}$	$+0.35$	-0.15
d	$-\frac{1}{3}$	$-\frac{1}{2}$	$+\frac{1}{3}$	$-\frac{2}{3}$	-0.42	$+0.08$

Ex1: Find the relations between $a_f(v_f)$ and $c_L(c_R)$.

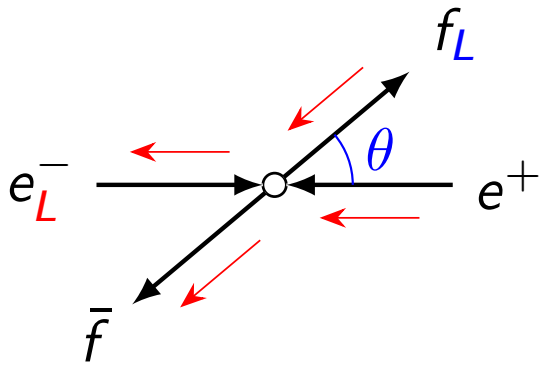
Ex2: Evaluate $\Gamma_{\nu\nu}$ given $g_Z \simeq 0.55$.

Precision tests: Z-resonance

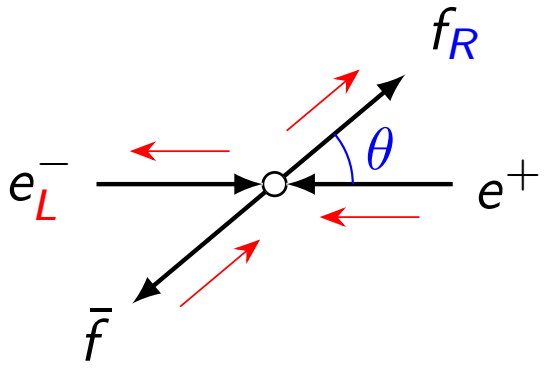
Consider the process
$e^+e^- \to Z \to f\bar{f}$
for $\sqrt{s} \simeq m_Z \gg m_f$



$|P(z)|^2 = 1/[(s - m_Z)^2 + m_Z^2\Gamma_Z^2]$:



$$|M_{LL}|^2 = g_Z^4 |P(z)|^2 [c_L^e]^2 [c_L^f]^2 \, (1 + \cos\theta)^2$$



$$|M_{LR}|^2 = g_Z^4 |P(z)|^2 [c_L^e]^2 [c_R^f]^2 \, (1 - \cos\theta)^2$$

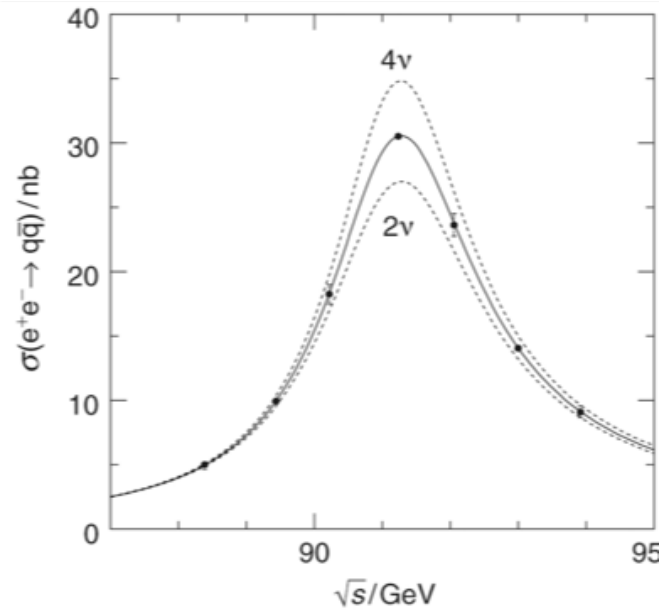
Ex: Evaluate $|M_{RR}|^2$ and $|M_{RL}|^2$.

Total cross-section $ee \to Z \to ff$

For unpolarised $e^{\pm}$ beams total cross section (narrow-width):

$$\sigma(ee \to Z \to ff) = \frac{12\pi s}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{ff}}{(s - m_Z)^2 + m_Z^2 \Gamma_Z^2}$$

- Position of the peak: $\sigma(\sqrt{s} = m_Z) = \sigma^0_{ff}$
- Full Width at Half Maximum $\sigma(\sqrt{s} = m_Z \pm \Gamma_Z/2) = \sigma^0_{ff}/2$
- $(12\pi)\Gamma_{ee}\Gamma_{ff} = \sigma^0_{ff}\Gamma_Z^2 m_Z^2$
  - $\to$ $12\pi\Gamma_{ee}^2 = \sigma^0_{\mu\mu}\Gamma_Z^2 m_Z^2$ (Lepton Universality)
  - $\to$ $12\pi\Gamma_{hadrons} = \sigma^0_{hadrons}\Gamma_Z^2 m_Z^2/\Gamma_{ee}$
- $N_\nu = (\Gamma_Z - 3\Gamma_{ee} - \Gamma_{hadrons})/\Gamma^{SM}_{\nu\nu} \simeq 2.98$



Z-peak observables: Forward-Backward Asymmetry

- For charged leptons:

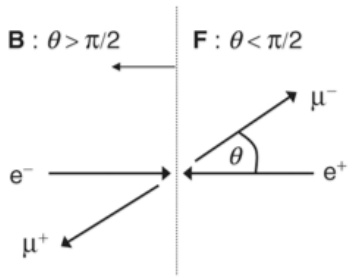
$$\frac{v_l}{a_l} = 1 - 4\sin\theta_W^2$$

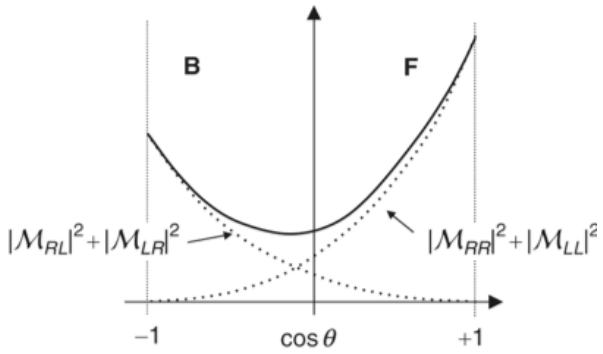
- Forward-Backward Asymmetry

$$A_{FB}^{\mu} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_\mu, \quad \mathcal{A}_f = \frac{(c_L^f)^2 - (c_R^f)^2}{(c_L^f)^2 + (c_R^f)^2} = \frac{v_f/a_f}{1 + (v_f/a_f)^2}$$

$$\sigma_F = 2\pi \int_0^1 \frac{d\sigma}{d\Omega} d(\cos\theta)$$

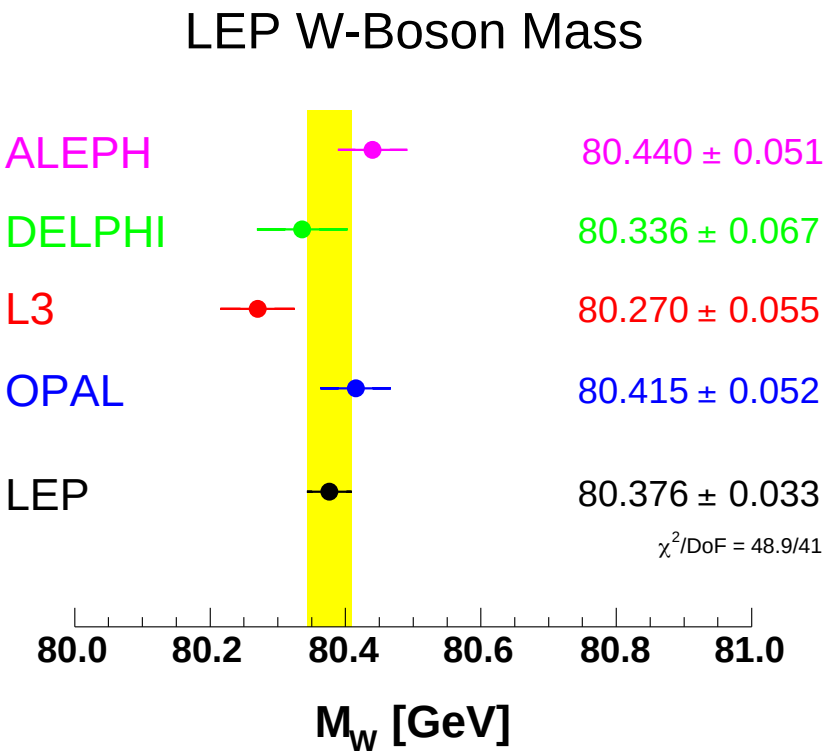
$$\sigma_B = 2\pi \int_{-1}^0 \frac{d\sigma}{d\Omega} d(\cos\theta)$$

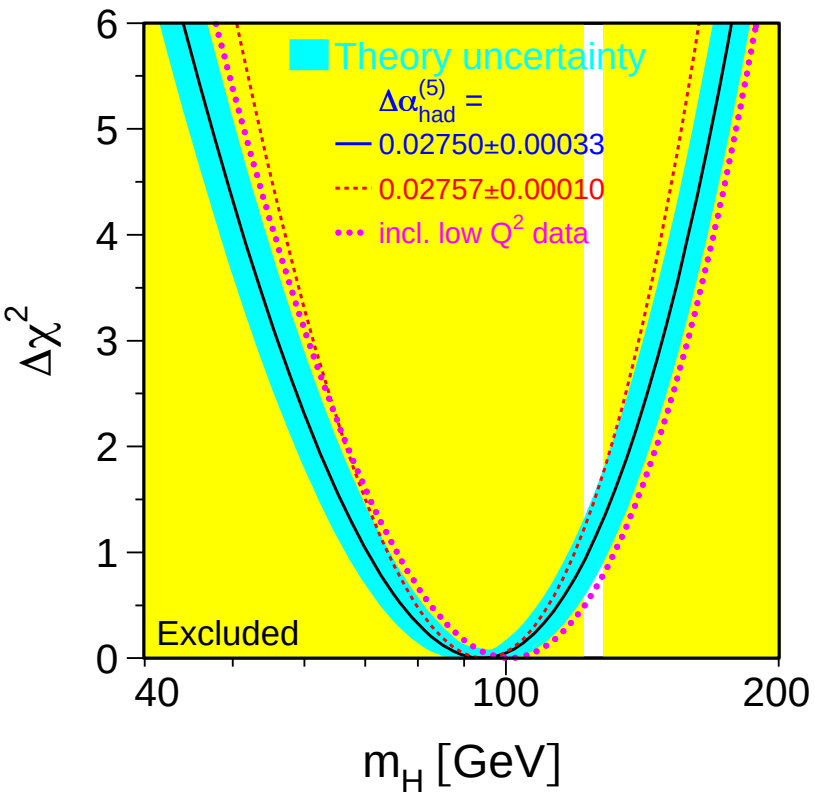




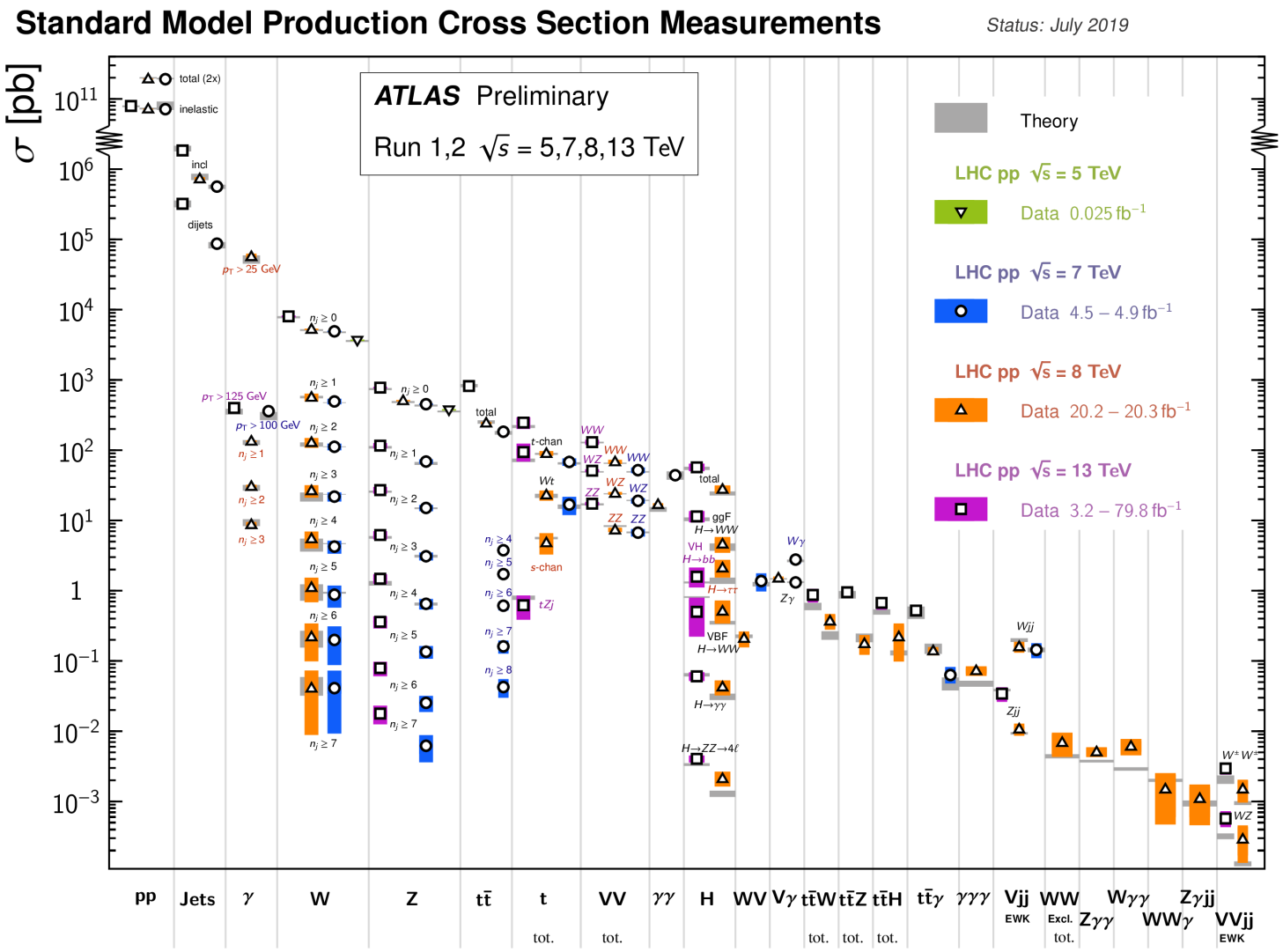
- Studied at LEP: $\mathcal{A}_e \simeq \mathcal{A}_\mu \simeq \mathcal{A}_\tau \simeq 0.15 \Rightarrow \sin^2\theta_W \simeq 0.23 \simeq \frac{1}{4}$

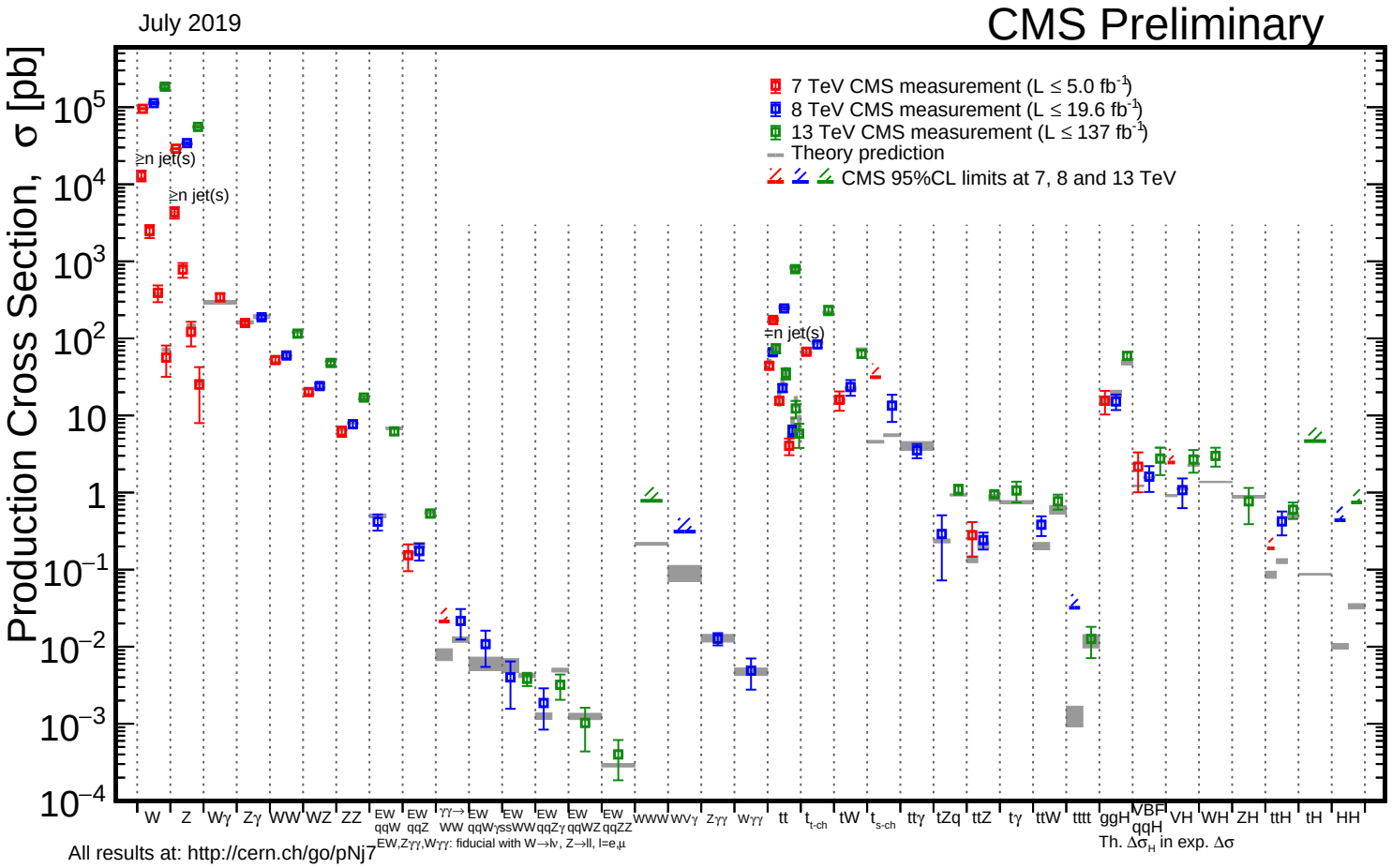
Some of the LEP EWWG Results





From arXiv:1302.3415


Standard Model Production Cross Section Measurements
Status: July 2019
ATLAS Preliminary
Run 1,2 √s = 5,7,8,13 TeV
σ [pb]
Theory
LHC pp √s = 5 TeV
Data 0.025 fb⁻¹
LHC pp √s = 7 TeV
Data 4.5 – 4.9 fb⁻¹
LHC pp √s = 8 TeV
Data 20.2 – 20.3 fb⁻¹
LHC pp √s = 13 TeV
Data 3.2 – 79.8 fb⁻¹
pp
Jets
γ
W
Z
tt̄
t
VV
γγ
H
WV
Vγ
tt̄W
tt̄Z
tt̄H
tt̄γ
γγγ
Vjj
WW
Wγγ
Zγjj
Zγγ
WWγ
VVjj


July 2019
CMS Preliminary
Production Cross Section, σ [pb]
7 TeV CMS measurement (L ≤ 5.0 fb-1)
8 TeV CMS measurement (L ≤ 19.6 fb-1)
13 TeV CMS measurement (L ≤ 137 fb-1)
Theory prediction
CMS 95%CL limits at 7, 8 and 13 TeV
≥n jet(s)
≥n jet(s)
±n jet(s)
W Z Wγ Zγ WW WZ ZZ EW qqW EW qqZ γγ→WW EW qqW EW γsWW EW qqZγ EW qqWZ EW qqZZ WWW WVγ Zγγ Wγγ tt t-ch tW t s-ch ttγ tZq ttZ tγ ttW tttt ggH VBF qqH VH WH ZH ttH tH HH
EW,Zγγ,Wγγ: fiducial with W→lν, Z→ll, l=e,μ
Th. ΔσH in exp. Δσ
All results at: http://cern.ch/go/pNj7

The Absolutely Amazing Theory of Almost Everything: Summary

- Based on Symmetry principles: Lorentz + $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry
- Renormalizable and unitary
- All predicted particles are discovered experimentally
- The structure of all interactions is fixed (but not all couplings are tested experimentally)
- Anomaly free theory
- Can account for rich Flavour Physics (lect. by M. Vysotsky).
- Three generations allow $\mathcal{CP}$-violation (lect. by M. Vysotsky).
- Can be extended to incorporate neutrino masses and mixing (lect by C. Gonzalez-Garcia)
- Survives stringent experimental tests

Issues of the SM

Features to be clarified:

- Unification of all interactions? (Gravity?)
- Pattern behind Flavour Physics (hierarchy in masses and mixing, 3 generations)?
- Symmetry behind SM charge assignment?
- Origin of the Higgs potential?
- Origin of accidental Baryon and Lepton number symmetries*?
- No CP-volation in strong interactions**
- Why the Higgs mass is so low? (Hierarchy/Naturalness problem)
- ...

Phenomenological problems:

- Origin of neutrino masses (see lect. by C. Gonzalez-Garcia)
- Baryon asymmetry (see lect. by V. Rubakov)
- Dark matter, Dark energy, Inflation (see lect. by V. Rubakov)
- Tension in $(g-2)_\mu$, $b \to s\mu\mu$, $b \to cl\nu$.
- ...

Concluding remarks

- The SM is enormously successful theory.
- Any New Physics theory should reproduce the SM as the low-energy limit.
- The values of the SM parameters do not give hints for New Physics scale.
- We still believe that the SM is not an ultimate theory (see lect. by V. Sanz) and ~~we~~ you eagerly search for any deviations ...
- In the absence of direct signal a key role is played by precision measurements

FCC-ee prospects

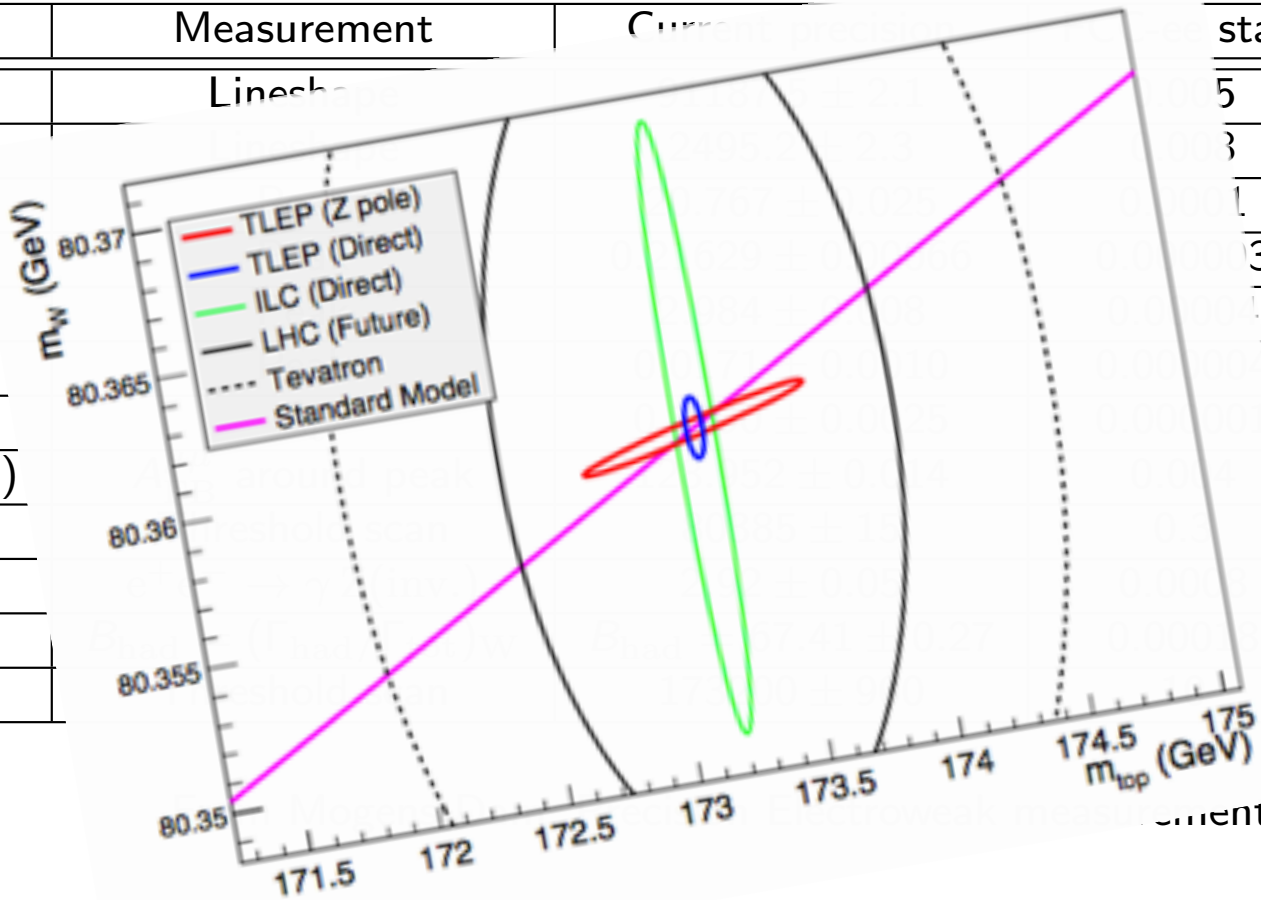
Observable	Measurement	Current precision	FCC-ee stat.	Possible syst.
M_{Z} [MeV]	Lineshape	91187.5 ± 2.1	0.005	< 0.1
Γ_{Z} [MeV]	Lineshape	2495.2 ± 2.3	0.008	< 0.1
R_ℓ	Peak	20.767 ± 0.025	0.0001	< 0.001
R_{b}	Peak	0.21629 ± 0.00066	0.000003	< 0.00006
N_ν	Peak	2.984 ± 0.008	0.00004	0.004
$A_{\mathrm{FB}}^{\mu\mu}$	Peak	0.0171 ± 0.0010	0.000004	< 0.00001
$\alpha_{\mathrm{s}}(M_{\mathrm{Z}})$	R_ℓ	0.1190 ± 0.0025	0.000001	0.00015
$1/\alpha_{\mathrm{QED}}(M_{\mathrm{Z}})$	$A_{\mathrm{FB}}^{\mu\mu}$ around peak	128.952 ± 0.014	0.004	0.002
M_{W} [MeV]	Threshold scan	80385 ± 15	0.3	< 1
N_ν	$\mathrm{e^+e^-} \to \gamma\,\mathrm{Z(inv.)}$	2.92 ± 0.05	0.0008	< 0.001
$\alpha_{\mathrm{s}}(M_{\mathrm{W}})$	$B_{\mathrm{had}} = (\Gamma_{\mathrm{had}}/\Gamma_{\mathrm{tot}})_{\mathrm{W}}$	$B_{\mathrm{had}} = 67.41 \pm 0.27$	0.00018	0.00015
m_{top} [MeV]	Threshold scan	173200 ± 900	10	10

The five orders of magnitude increase of statistics with respect to LEP will yield to considerably more precise measurements of observables such as the Z partial widths and asymmetries, where systematic uncertainties largely cancel in the ratio of cross sections. [FCC Design Study Group]

Table from Mogens Dam, Precision Electroweak measurements at the FCC-ee

FCC-ee prospects

Observable	Measurement	Current	stat.	Possible syst.
M_Z [MeV]	Lineshape		5	< 0.1
Γ_Z [MeV]			3	< 0.1
R_ℓ				< 0.001
R_b			3	< 0.00006
N_ν				0.004
$A_{FB}^{\mu\mu}$				< 0.00001
$\alpha_s(M_Z)$				0.00015
$1/\alpha_{QED}(M_Z)$				0.002
M_W [MeV]				< 1
N_ν				< 0.001
$\alpha_s(M_W)$				0.00015
m_{top} [MeV]				10



ements at the FCC-ee