Cosmology and Dark Matter

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Lecture 3

Outline of Lecture 3

- Baryon asymmetry of the Universe
  - Generalities.
  - Electroweak baryon number non-conservation
  - What can make electroweak mechanism work?
  - Leptogenesis and neutrino masses
- Before the hot epoch

Baryon asymmetry of the Universe

- There is matter and no antimatter in the present Universe.
- Baryon-to-photon ratio, almost constant in time:

$$\eta_B \equiv \frac{n_B}{n_\gamma} = 6 \cdot 10^{-10}$$

Baryon-to-entropy, constant in time: $n_B/s = 0.9 \cdot 10^{-10}$

What's the problem?

Early Universe ($T > 10^{12}$ K $= 100$ MeV):
creation and annihilation of quark-antiquark pairs $\Rightarrow n_q, n_{\bar{q}} \approx n_\gamma$
Hence

$$\frac{n_q - n_{\bar{q}}}{n_q + n_{\bar{q}}} \sim 10^{-9}$$

Very unlikely that this excess is initial condition. Certainly not, if inflation. How was this excess generated in the course of the cosmological evolution?

Sakharov'67, Kuzmin'70

Sakharov conditions

To generate baryon asymmetry from symmetric initial state, three necessary conditions should be met at the same cosmological epoch:

- B-violation
- C- and CP-violation
- Thermal inequilibrium

NB. Reservation: L-violation with B-conservation at $T \gg 100$ GeV would do as well $\Longrightarrow$ Leptogenesis.

Can baryon asymmetry be due to electroweak physics?

Baryon number is violated in electroweak interactions.
"Sphalerons".

Non-perturbative effect

Hint: triangle anomaly in baryonic current B^μ:

$$\partial_\mu B^\mu = \left(\frac{1}{3}\right)_{B_q} \cdot 3_{colors} \cdot 3_{generations} \cdot \frac{g_W^2}{32\pi^2} \varepsilon^{\mu\nu\lambda\rho} F^a_{\mu\nu} F^a_{\lambda\rho}$$

$F^a_{\mu\nu}$: $SU(2)_W$ field strength; g_W: $SU(2)_W$ coupling

Likewise, each leptonic current ($n = e, \mu, \tau$)

$$\partial_\mu L^\mu_n = \frac{g_W^2}{32\pi^2} \cdot \varepsilon^{\mu\nu\lambda\rho} F^a_{\mu\nu} F^a_{\lambda\rho}$$

B is violated, $B - L \equiv B - L_e - L_\mu - L_\tau$ is not.

Large field fluctuations, $F^a_{\mu\nu} \propto g_W^{-1}$ may have

$$Q \equiv \int d^3x dt \; \frac{g_W^2}{32\pi^2} \cdot \varepsilon^{\mu\nu\lambda\rho} F^a_{\mu\nu} F^a_{\lambda\rho} \neq 0$$

Then

$$B_{fin} - B_{in} = \int d^3x dt \; \partial_\mu B^\mu = 3Q$$

Likewise

$$L_{n,\, fin} - L_{n,\, in} = Q$$

B is violated, $B - L$ is not.

How can baryon number be not conserved without explicit B-violating terms in Lagrangian?

Consider massless fermions in background gauge field $\vec{A}(\mathbf{x},t)$ (gauge $A_0 = 0$). Let $\vec{A}(\mathbf{x},t)$ start from vacuum value and end up in vacuum.

NB: This can be a fluctuation

Dirac equation

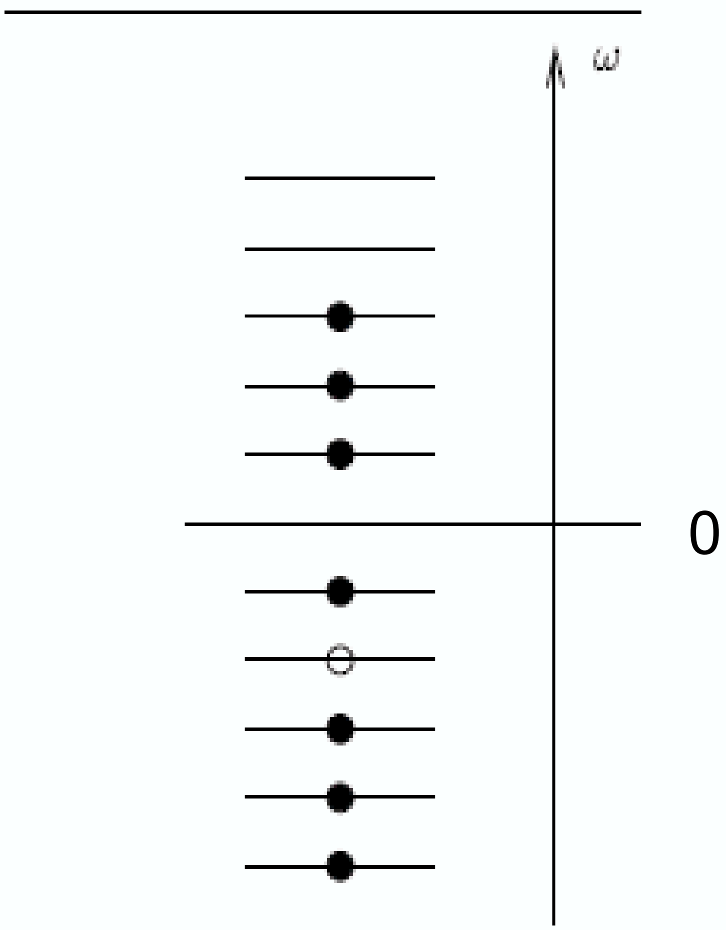
$$i\frac{\partial}{\partial t}\psi = i\gamma^0\vec{\gamma}(\vec{\partial} - ig\vec{A})\psi = H_{Dirac}(t)\psi$$

Suppose for the moment that $\vec{A}$ slowly varies in time. Then fermions sit on levels of instantaneous Hamiltonian,

$$H_{Dirac}(t)\psi_n = \omega_n(t)\psi_n$$

How do eigenvalues behave in time?

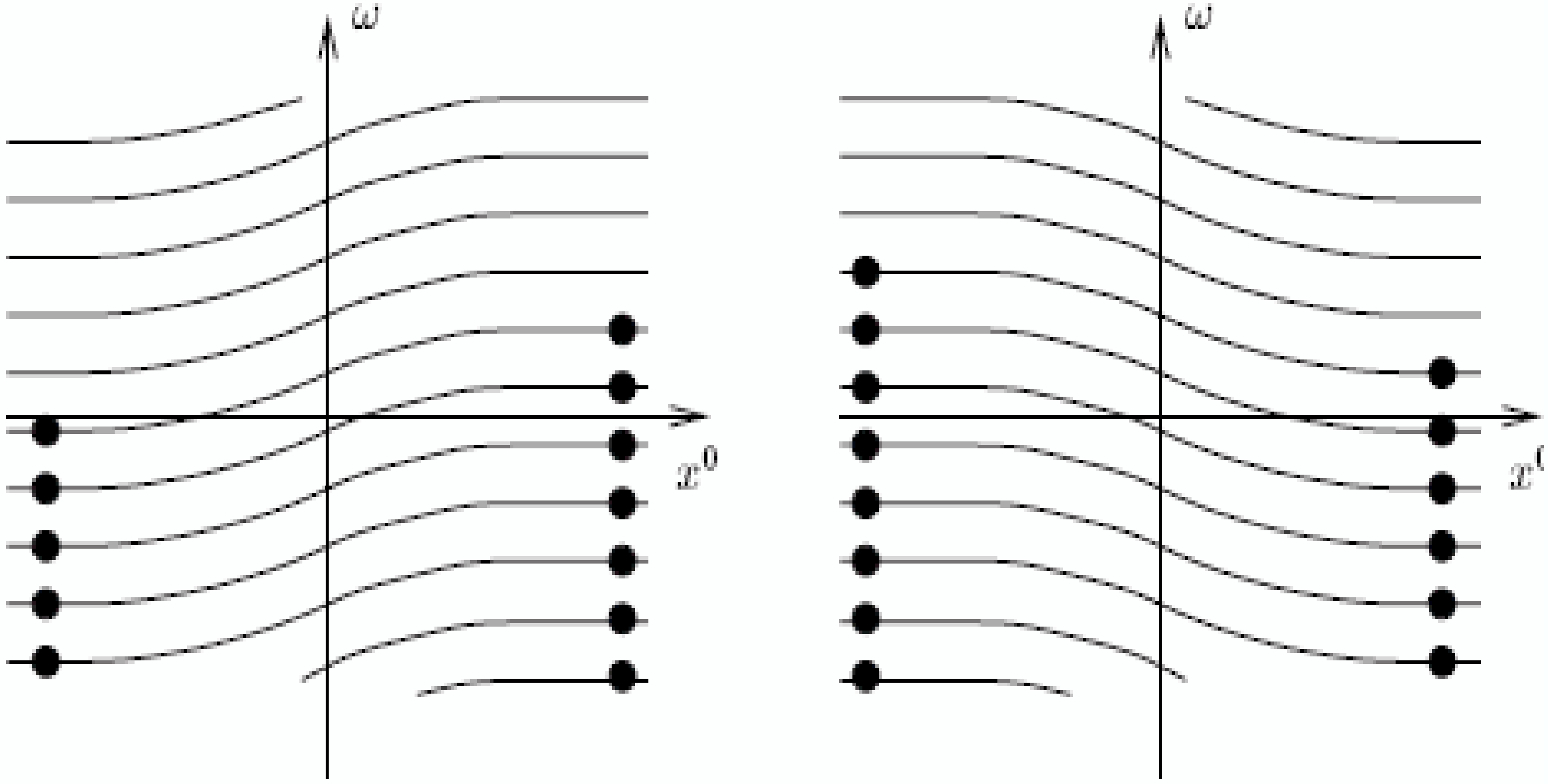
Dirac picture at $\vec{A} = 0$, $t \to \pm\infty$



TIME EVOLUTION OF LEVELS

IN SPECIAL (TOPOLOGICAL) GAUGE FIELDS

Left-handed fermions Right-handed

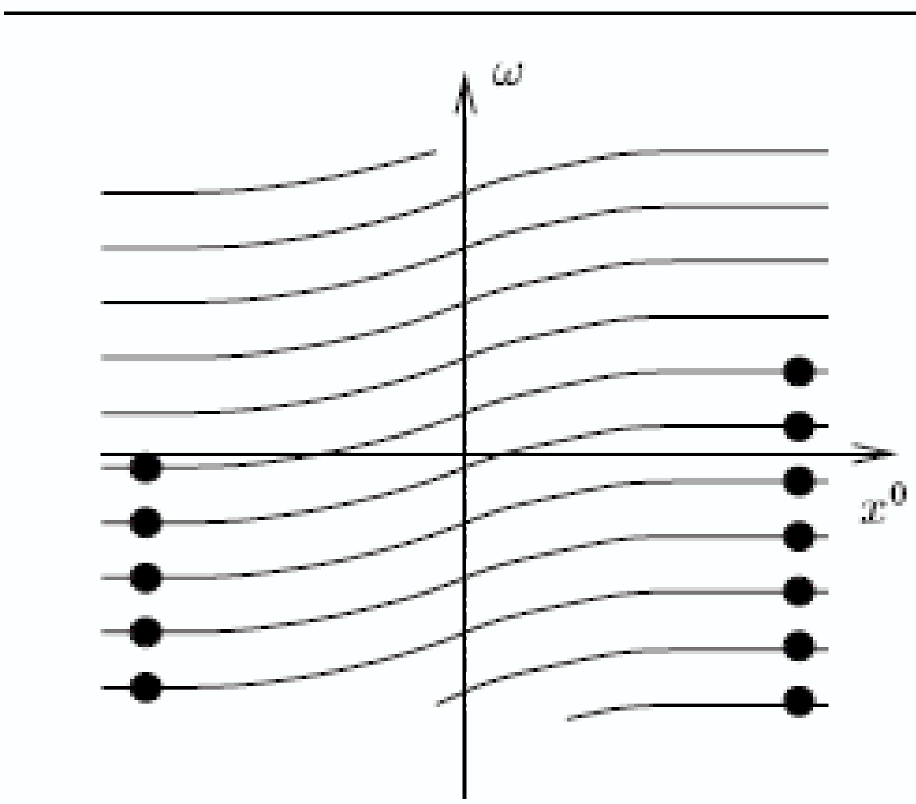


The case for QCD

$B = N_L + N_R$ is conserved, $Q^5 = N_L - N_R$ is not

If only left-handed fermions interact with gauge field,

then number of fermions is not conserved



The case for $SU(2)_W$

Fermion number of every doublet changes by equal amount

Need large field fluctuations. At zero temprature these are instantons; their rate is suppressed by

$$\mathrm{e}^{-\frac{16\pi^2}{g_W^2}} \sim 10^{-165}$$

High temperatures: large thermal fluctuations ("sphalerons").

B-violation rapid as compared to cosmological expansion at

$$\langle\phi\rangle_T < T$$

$\langle\phi\rangle_T$: Higgs expectation value at temperature T.

Possibility to generate baryon asymmetry at electroweak epoch, $T_{EW} \sim 100$ GeV ?

Problem: Universe expands slowly. Expansion time

$$H^{-1} \sim 10^{-10}\ \mathrm{s}$$

Too large to have deviations from thermal equilibrium?

The only chance: 1st order phase transition, highly inequilibrium process

Electroweak symmetry is restored, $\langle\phi\rangle_T = 0$ at high temperatures

Just like superconducting state becomes normal at “high” T

Transition may in principle be 1st order

Fig

1st order phase transition occurs from supercooled state via spontaneous creation of bubbles of new (broken) phase in old (unbroken) phase.

Bubbles then expand at $v \sim 0.1c$

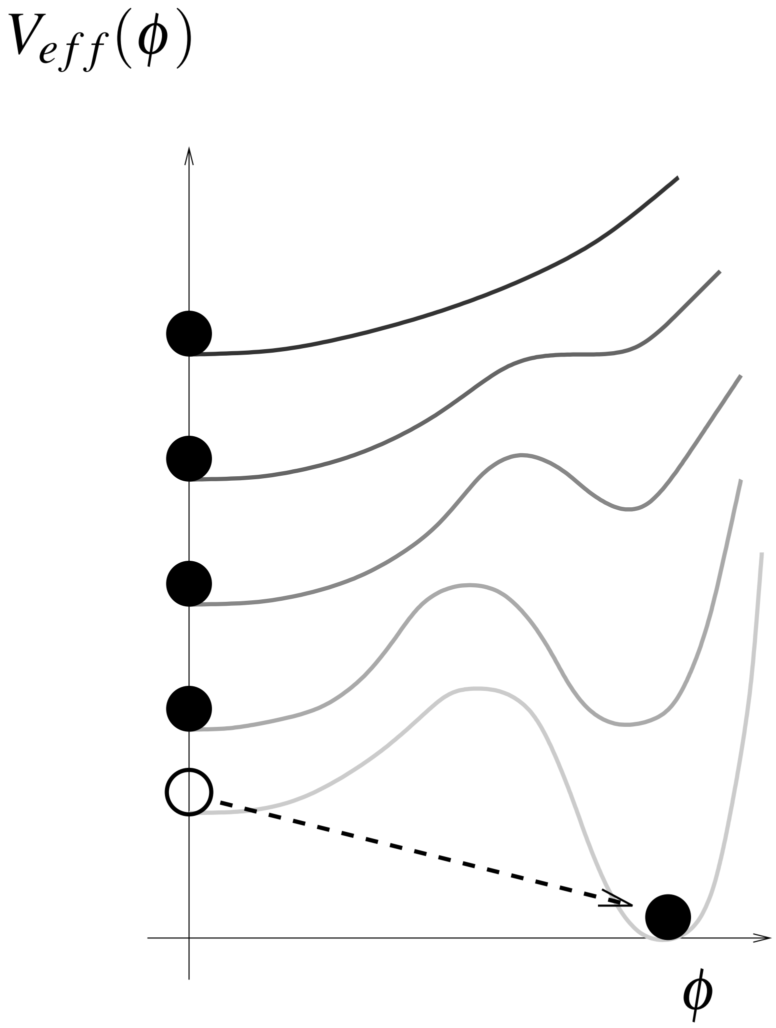
Fig

Beginning of transition: about one bubble per horizon

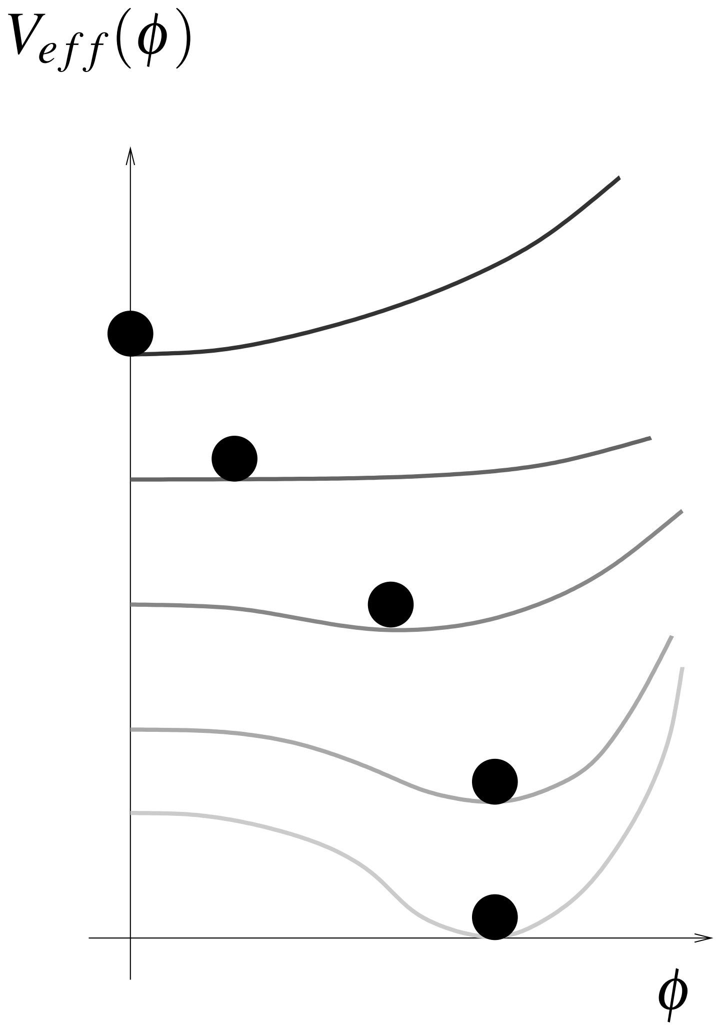
Bubbles born microscopic, $r \sim 10^{-16}$ cm, grow to macroscopic size, $r \sim 0.1H^{-1} \sim 1$ mm, before their walls collide

Boiling Universe, strongly out of equilibrium

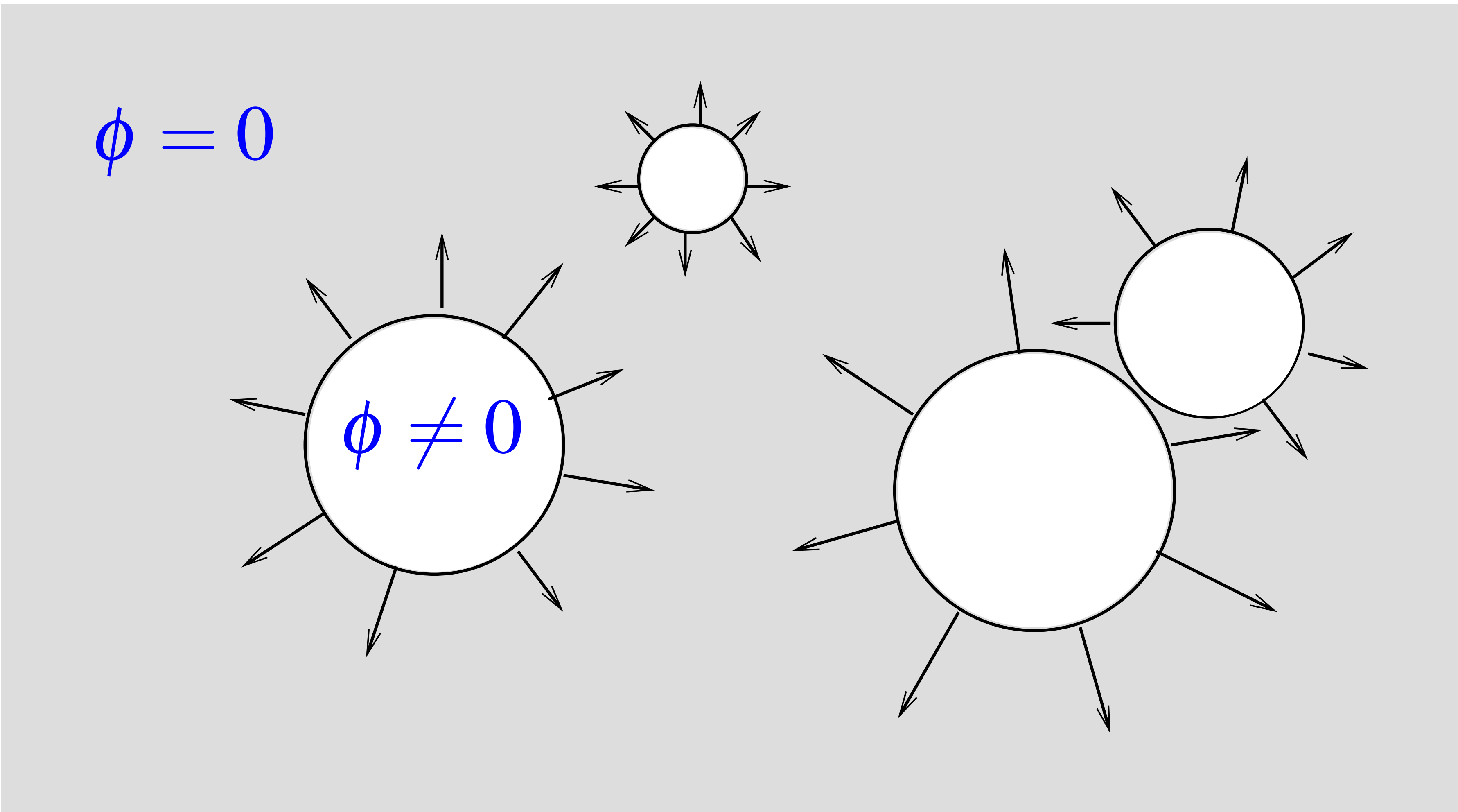
$V_{eff}(\phi)$ = free energy density



1st order



2nd order


$\phi = 0$
$\phi \neq 0$

Baryon asymmetry may be generated in the course of 1st order phase transition, provided there is enough C- and CP-violation.

Does this really happen?

Not in SM

- Given the Higgs boson mass

$$m_H = \sqrt{2\lambda} v = 125 \text{ GeV}$$

No phase transition at all; smooth crossover

- Way too small CP-violation due to CKM phase

What can make EW mechanism work?

- Extra bosons
  - Should interact fairly strongly with Higgs(es)
  - Should be present in plasma at EW epoch
    $\Longrightarrow$ physics at or below TeV scale

- Plus extra source of CP-violation.
  Better in Englert–Brout–Higgs sector $\Longrightarrow$ Several scalar fields
  - Electric dipole moments of neutron and electron.
  - Recent limit $d_e < 1.1 \cdot 10^{-29}$ e cm (ACME) kills many concrete models

More generally, EW baryogenesis requires
complex dynamics in EW symmetry breaking sector
at $E \sim (\text{a few}) \cdot 100$ GeV

LHC's FINAL WORD

Is EW the only appealing scenario?

By no means!

— Leptogenesis

— Something theorists never thought about

Why $\Omega_B \approx \Omega_{DM}$?

Leptogenesis:

Baryon asymmetry and neutrino masses

B is violated in electroweak interactions,

$B-L$ is conserved

But we know that lepton numbers are violated anyway: neutrino oscillations.

Neutrinos have tiny masses. We know two differences of mass squared:

$$m_2^2 - m_1^2 = (0.01 \text{ eV})^2\,, \qquad |m_3^2 - m_1^2| = (0.05 \text{ eV})^2$$

We also know that all masses are small,

$$m_\nu < 2 \text{ eV} \quad \text{(Experiment)}$$

$$m_\nu < 0.1S \text{ eV} \quad \text{(Cosmology)}$$

Leptogenesis: use the physics responsible for neutrino masses to generate lepton asymmetry in the Universe. Electroweak interactions automatically reprocess part of lepton asymmetry into baryon asymmetry.

Standard Model in thermal equilibrium at $T \gg 100$ GeV with $B - L \neq 1$:

$$B = C \cdot (B - L) \, , \qquad L = -(1 - C) \cdot (B - L)$$

$$C = \frac{8N_{gen} + 4N_{Higgs}}{22N_{gen} + 13N_{Higgs}} = \frac{28}{79} \, , \quad T \gg 100 \text{ GeV}$$

Idea: neutrino masses may be generated due to interactions of our neutrinos $\nu_{e,\mu,\tau}$ with heavy sterline neutrinos N_α. Lepton asymmetry generated in decays of N

$$\Gamma(N \to \nu + \text{ Higgs}) \neq \Gamma(N \to \bar{\nu} + \text{ Higgs}) \quad \Longrightarrow \quad n_\nu \neq n_{\bar{\nu}} \, .$$

Need CP violation in lepton sector.

Albeit not directly observable

See-saw in nutshell

See lectures by C. Gonzalez-Garcia

Begin with one lepton doublet $L = (\nu, l)$. To generate neutrino mass, add a new left fermion N, singlet under $SU(2)_W \times U(1)_Y$.

Allowed Majorana mass term and Yukawa interaction

$$L = \frac{M}{2}\bar{N}^c N + y\bar{N}^c \tilde{H}^\dagger L + \text{h.c}$$

In vacuum $\tilde{H}^\dagger = (v/\sqrt{2}, 0)$, so one gets mass terms

$$L = \frac{M}{2}\bar{N}^c N + \frac{yv}{\sqrt{2}}\bar{N}^c \nu + \text{h.c}$$

At energies and momenta small compared to M, equation of motion for N is

$$MN + \frac{yv}{\sqrt{2}}\nu = 0 \quad \Longrightarrow \quad N = -\frac{yv}{M\sqrt{2}}\nu$$

Plug N back into Lagrangian, get Majorana mass of ν:

$$L_{m_\nu} = -\frac{y^2 v^2}{2M}\bar{\nu}^c \nu + \text{h.c.}$$

Small Majorana neutrino mass for large M (see-saw):

$$m_\nu = \frac{y^2 v^2}{2M}\,, \quad M = (10^{15} - 10^{11})\ \text{GeV} \ \text{ for } \ m_\nu = 0.1\ \text{eV}\,, \quad y = 1 - 0.01\,.$$

Three generations:

$$\mathscr{L} = \frac{M_\alpha}{2}\bar{N}^c_\alpha N_\alpha + \left(y_{\alpha\beta}\bar{N}^c_\alpha \tilde{H}^\dagger L_\beta + h.c.\right)$$

$y_{\alpha\beta}$, $\alpha = 1,2,3$: complex Yukawa couplings in basis where M is diagonal and $L_\alpha = (L_e, L_\mu, L_\tau)$.
Once H obtains vev $\tilde{H} = (v/\sqrt{2}, 0)$, SM neutrinos get Majorana masses

$$m_{\alpha\beta} = \frac{v^2}{2} y_{\gamma\alpha}\frac{1}{M_\gamma} y_{\gamma\beta}$$

Lepton asymmetry from N-decays

Complex $y_{\alpha\beta}$ violate CP $\Longrightarrow$

$$\Gamma(N \to lh) \neq \Gamma(N \to \bar{l}h)$$

(do not distinguish h and $\bar{h}$, no conserved number in the Higgs sector).

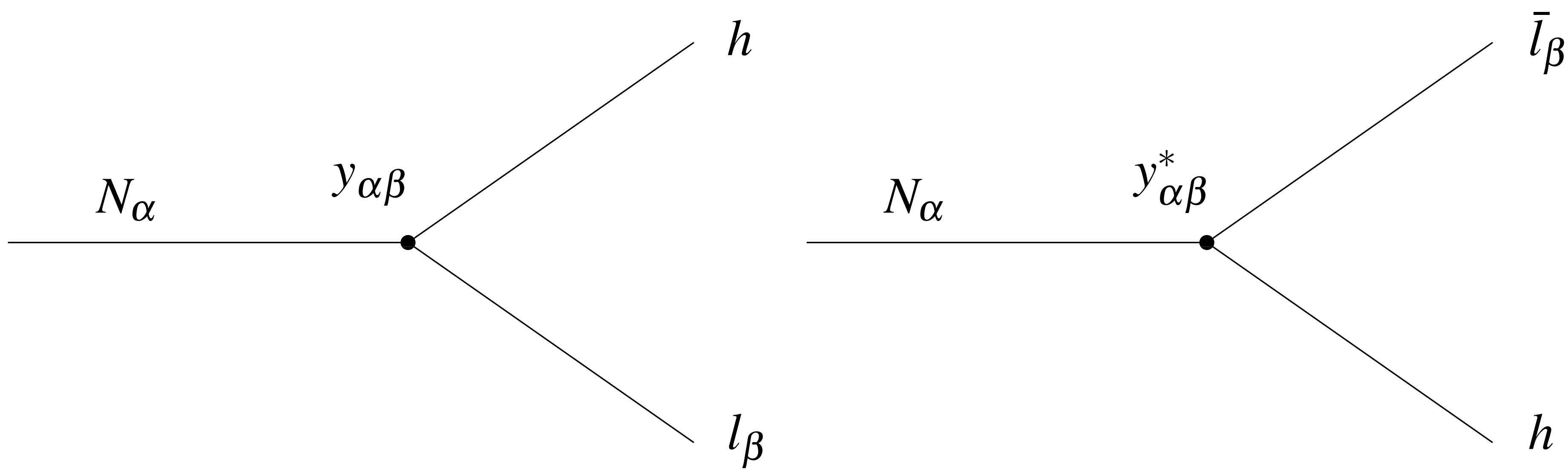
Use to generate lepton asymmetry: as the Universe expands, temperature falls below M_N, heavy N existing in plasma decay and produce lepton asymmetry.

Net asymmetry generated in decays of lightest N, call it N_1.

Equilibrium: decays compensated by inverse decays; decay and inverse decay rates equal to $\Gamma_1 \equiv \Gamma_{N_1}$.

Condition for strong deviation from thermal equilibrium:

$$\Gamma_1 \lesssim H(T = M_1)$$



$$\Gamma_1 = \frac{M_1}{8\pi} \sum_\alpha |y_{1\alpha}|^2 \lesssim H(T = M_1) = \frac{M_1^2}{M_{Pl}^*}$$

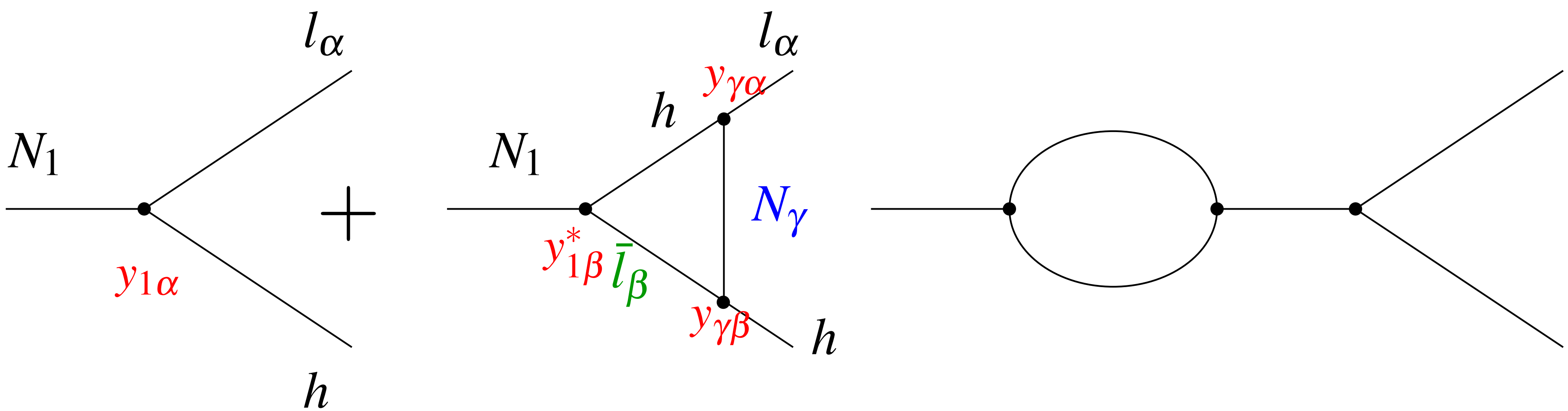
$$\Longrightarrow \quad \tilde{m}_1 = \sum_\alpha \frac{|y_{1\alpha}|^2}{2M_1} \cdot v^2 \lesssim \frac{4\pi}{M_{Pl}^*} \cdot v^2 \sim 10^{-3} \text{ eV}$$

$\tilde{m}_1$: contribution of lightest N to neutrino mass matrix $\Longrightarrow$ need small neutrino masses, not many orders of magnitude larger than 10^{-3} eV.

Microscopic asymmetry

$$\delta = \frac{\Gamma(N_1 \to lh) - \Gamma(N_1 \to \bar{l}h)}{\Gamma(N_1 \to lh) + \Gamma(N_1 \to \bar{l}h)}.$$

Appears due to interferenece of tree and loop



$$\Gamma(N_1 \to lh) = \text{const} \cdot \sum_\alpha \left| y_{1\alpha} + \sum_{\beta,\gamma} D\left(\frac{M_1}{M_\gamma}\right) \cdot y^*_{1\beta} y_{\gamma\alpha} y_{\gamma\beta} \right|^2$$

$D(M_1/M_\gamma)$ = loop factor,

$$\text{Im}D = \frac{1}{24\pi} \frac{M_1}{M_\gamma}, \qquad M_\gamma \gg M_1$$

$$\Gamma(N \to \bar{l}h) = \Gamma(N \to lh; y \to y^*)$$

Asymmetry

$$\delta = \frac{\Gamma(N \to lh) - \Gamma(N \to \bar{l}h)}{\Gamma(N \to lh) + \Gamma(N \to \bar{l}h)} = \frac{M_1}{12\pi} \frac{1}{\sum_\alpha |y_{1\alpha}|^2} \sum_{\alpha\beta\gamma} \mathrm{Im} \left[y_{1\alpha} y_{1\beta} \left(y^*_{\gamma\alpha} \frac{1}{M_\gamma} y^*_{\gamma\beta} \right) \right]$$

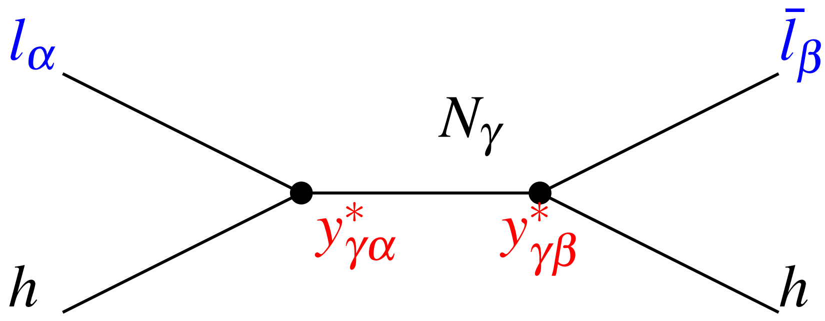
Not particularly small if Yukawas are not very small.

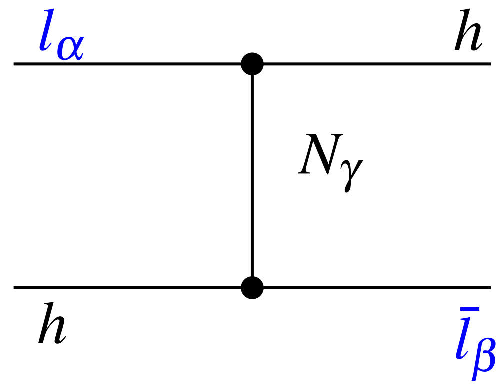
Comment:

- Relevant phases here are not related to phases of neutrino mass matrix: unitary rotations of $(\nu_e, \nu_\mu, \nu_\tau)$ do not affect δ, but eliminate PMNS mixing.

Washout in scattering

Non-resonant scattering with L-violation





Fast processes $\Longrightarrow$ washout, if

$$\langle \sigma v \rangle n_h \gtrsim H(T)$$

At $T \ll M_\alpha$ one has for light h, l

$$\langle \sigma v \rangle \sim \sum_{\alpha\beta\gamma} \left| \frac{y_{\gamma\alpha} y_{\gamma\beta}}{M_\gamma} \right|^2 \sim \frac{\mathrm{Tr}\, m m^\dagger}{v^4} = \frac{\sum m_\nu^2}{v^4}$$

With $n_h \sim T^3$, requirement of absence of washout

$$n_h \langle \sigma v \rangle \sim T^3 \frac{\sum m_\nu^2}{v^4} \ll H(T) = \frac{T^2}{M_{Pl}^*}$$

NB: washout switches off at low T.

Most dangerous at generation of lepton asymmetry, $T \sim M_1$.

$$\sum m_\nu^2 \ll \frac{v^4}{M_{Pl}^* M_1}$$

$M_1 \sim 10^{12}$ GeV $\Longrightarrow$

$$m_\nu = \sqrt{\frac{1}{3} \sum m_\nu^2} < 0.1 \text{ eV}$$

In fact, this bound, when combined with successfull leptogenesis, is valid for virtually all M_1.

Conclusions on leptogenesis

- It is intriguing that the mechanism can work for light neutrinos only. Furthermore, neutrino masses suggested by oscillation data are in right ballpark.

- Needs Majorana neutrino masses

- Highly degenerate neutrino masses $m_\nu \sim 0.3 - 1$ eV would be inconsistent with (simple and appealing versions of) leptogenesis. Watch out Katrin (but already ruled out by cosmology).

- Knowning mass matrix of “our” neutrinos is, generally speaking, insufficient for calculating baryon/lepton asymmetry. Even its sign.

- Reversing the argument, models that relate $y_{\alpha\beta}$ to “our” neutrino mass matrix can be ruled out — or have great success — once the mass matrix is completely known.

Before the hot epoch

With Big Bang nucleosynthesis theory and observations
we are confident of the theory of the early Universe
at temperatures up to $T \simeq 1$ MeV, age $t \simeq 1$ second

With the LHC, we hope to be able to go
up to temperatures $T \sim 100$ GeV, age $t \sim 10^{-10}$ second

Are we going to have a handle on even earlier epoch?

Key: cosmological perturbations

Our Universe is not exactly homogeneous.

Inhomogeneities: ⊙ density perturbations and associated gravitational potentials (3d scalar), observed;

⊙ gravitational waves (3d tensor), not observed (yet?).

$$ds^2 = (1+2\Phi(x))dt^2 - a^2(t)[\delta_{ij}(1+2\Psi(x)) + h_{ij}]dx^i dx^j$$

Φ, Ψ: scalar perturbations, generated by perturbations in energy density; h_{ij} transverse–traceless, $\partial_i h_{ij} = h_i^i = 0$: tensor perturbations (gravity waves).

Today: inhomogeneities strong and non-linear

In the past: amplitudes small, linear analysis appropriate,

$$\frac{\delta\rho}{\rho}, \Phi, \Psi = 10^{-4} - 10^{-5}$$

How are they measured?

- Cosmic microwave background: photographic picture of the Universe at age 380 000 yrs, $T = 3000$ K
  - Temperature anisotropy
  - Polarization
- Deep surveys of galaxies and quasars, cover good part of entire visible Universe
- Gravitational lensing, etc.

We have already learned a number of fundamental things

Extrapolation back in time with known laws of physics and known elementary particles and fields $\Longrightarrow$ hot Universe, starts from Big Bang singularity (infinite temperature, infinite expansion rate)

We know that this is not the whole story!

Properties of perturbations in conventional (“hot”) Universe.

Reminder:

Friedmann–Lemaître–Robertson–Walker metric:

$$ds^2 = dt^2 - a^2(t) d\vec{x}^{\,2}$$

$a(t) \propto t^{1/2}$ at radiation domination stage (before $T \simeq 1$ eV, $t \simeq 60$ thousand years)
$a(t) \propto t^{2/3}$ at matter domination stage (until recently).

Cosmological horizon at time t (assuming that nothing preceded hot epoch): distance that light travels from Big Bang moment,

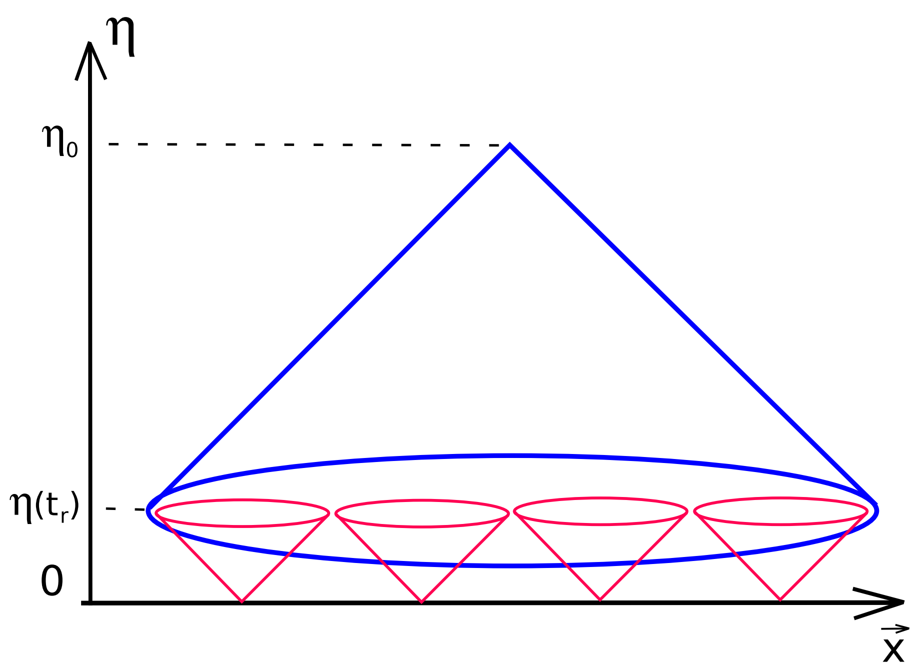
$$l_{H,t} \sim H^{-1}(t) \sim t$$

Wavelength of perturbation grows as $a(t)$.
E.g., at radiation domination

$$\lambda(t) \propto t^{1/2} \quad \text{while} \quad l_{H,t} \propto t$$

Today $\lambda < l_H$, subhorizon regime

Early on $\lambda(t) > l_H$, superhorizon regime.



Angular size of horizon at recombination
(last scattering of CMB photons) $\approx 2^\circ$

Major issue: origin of perturbations

Causality $\Longrightarrow$ perturbations can be generated only when they are subhorizon.

Off-hand possibilities:

- Perturbations were never superhorizon, they were generated at the hot cosmological epoch by some causal mechanism.

  E.g., seeded by topological defects (cosmic strings, etc.)

  The only possibility, if expansion started from hot Big Bang.

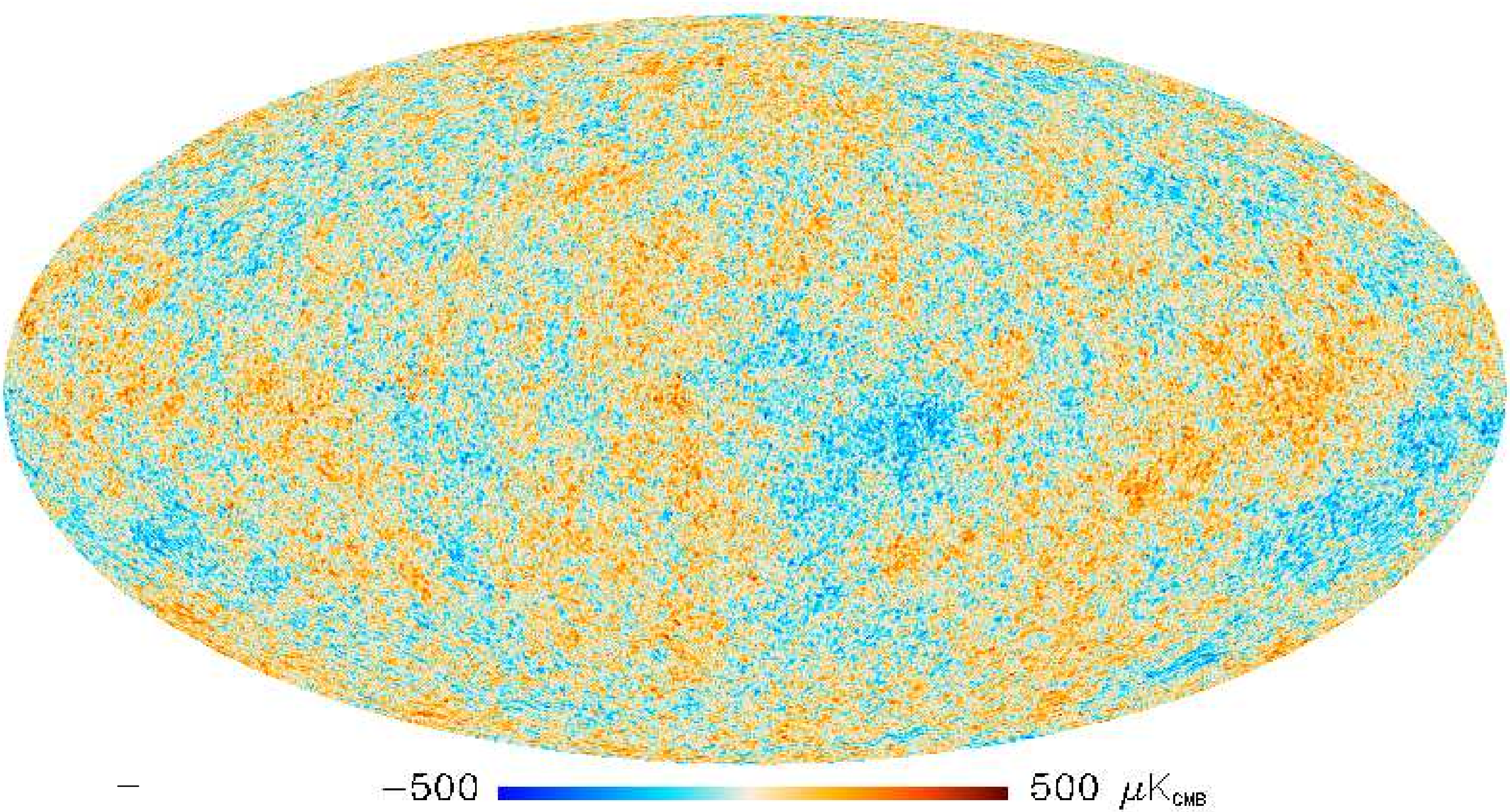
No longer an option!

At recombination, there were superhorizon perturbations, their angular size today exceeds 2°

Fig.

- Hot epoch was preceeded by some other epoch. Perturbations were generated then.

$$T = 2.726^{\circ}K, \quad \frac{\delta T}{T} \sim 10^{-4} - 10^{-5}$$



Planck

In more detail

Perturbations in baryon-photon plasma = sound waves.

If they were superhorizon, they started off with one and the same phase. Why?

Subhorion regime (late times): acoustic oscillations

$$\frac{\delta\rho}{\rho}(\vec{k},t) = A(\vec{k})\cos\left(\int_0^t v_s\,\frac{k}{a(t)}\,dt + \psi\right), \qquad \psi = \text{ some phase}$$

Recall: Physical distance $dl = adx \Longleftrightarrow$ physical momentum k/a, gets redshifted.

Sound velocity $v_s \approx 1/\sqrt{3}$.

Solutions to wave equation in superhorizon regime in expanding Universe

$$\frac{\delta\rho}{\rho} = \text{const} \qquad \text{and} \qquad \frac{\delta\rho}{\rho} = \frac{\text{const}}{t^{3/2}}$$

Assume that modes were superhorizon. Consistency of the picture: the Universe was not very inhomogeneous at early times, the initial condition is (up to amplitude),

$$\frac{\delta\rho}{\rho} = \text{const} \implies \frac{d}{dt}\frac{\delta\rho}{\rho} = 0$$

Acoustic oscillations start after entering the horizon at zero velocity of medium $\implies$ phase of oscillations well defined.

$$\frac{\delta\rho}{\rho}(\vec{k},t) = A(\vec{k})\cos\left(\int_0^t v_s \frac{k}{a(t)}\, dt\right),$$

no arbitrary phase

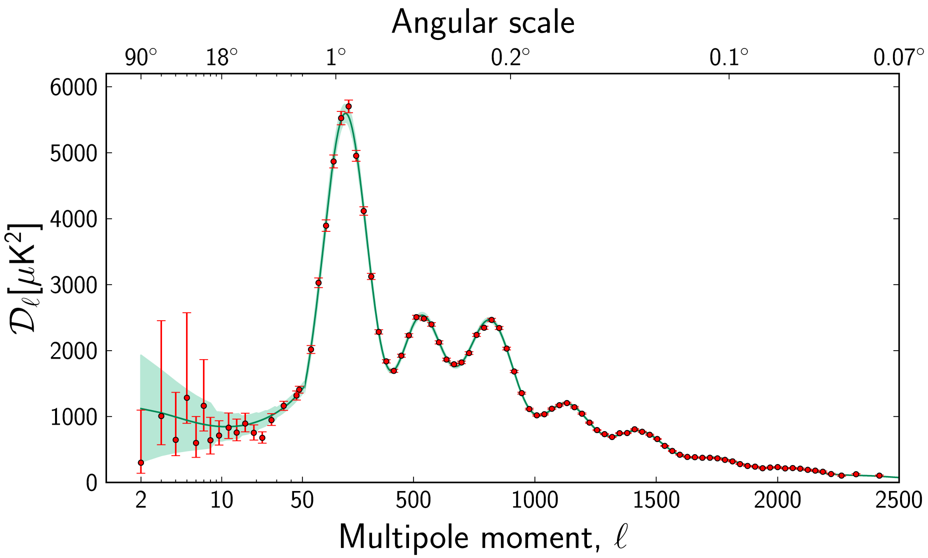
Perturbations come to the time of photon last scattering (= recombination) at different phases, depending on wave vector:

$$\delta(t_r) \equiv \frac{\delta\rho}{\rho}(t_r) \propto \cos\left(k\int_0^{t_r} dt\ \frac{v_s}{a(t)}\right) = \cos(kr_s)$$

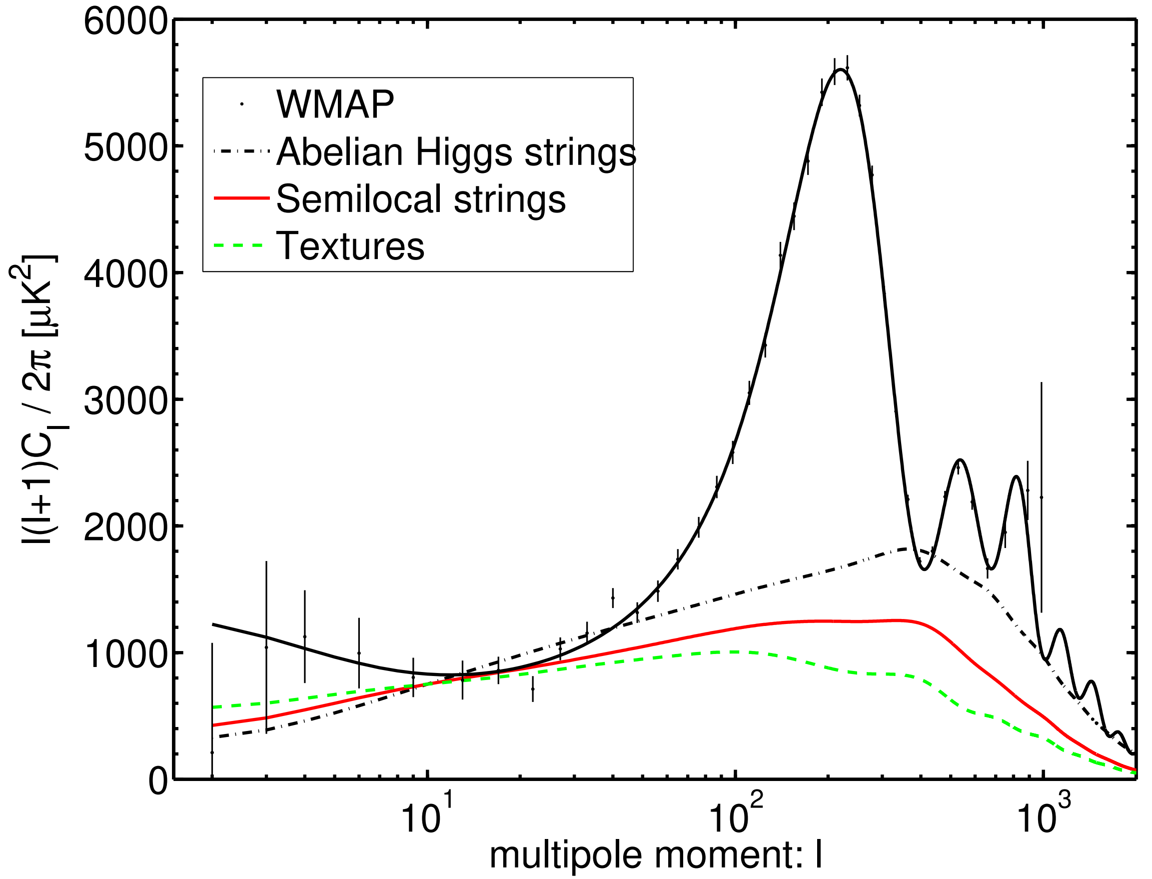
r_s: sound horizon at recombination, $a_0 r_s = 150$ Mpc.

Waves with $k = \pi n/r_s$ have large $|\delta\rho|$, while waves with $k = (\pi n + 1/2)/r_s$ have $|\delta\rho| = 0$ in baryon-photon component.

This translates into oscillations in CMB angular spectrum

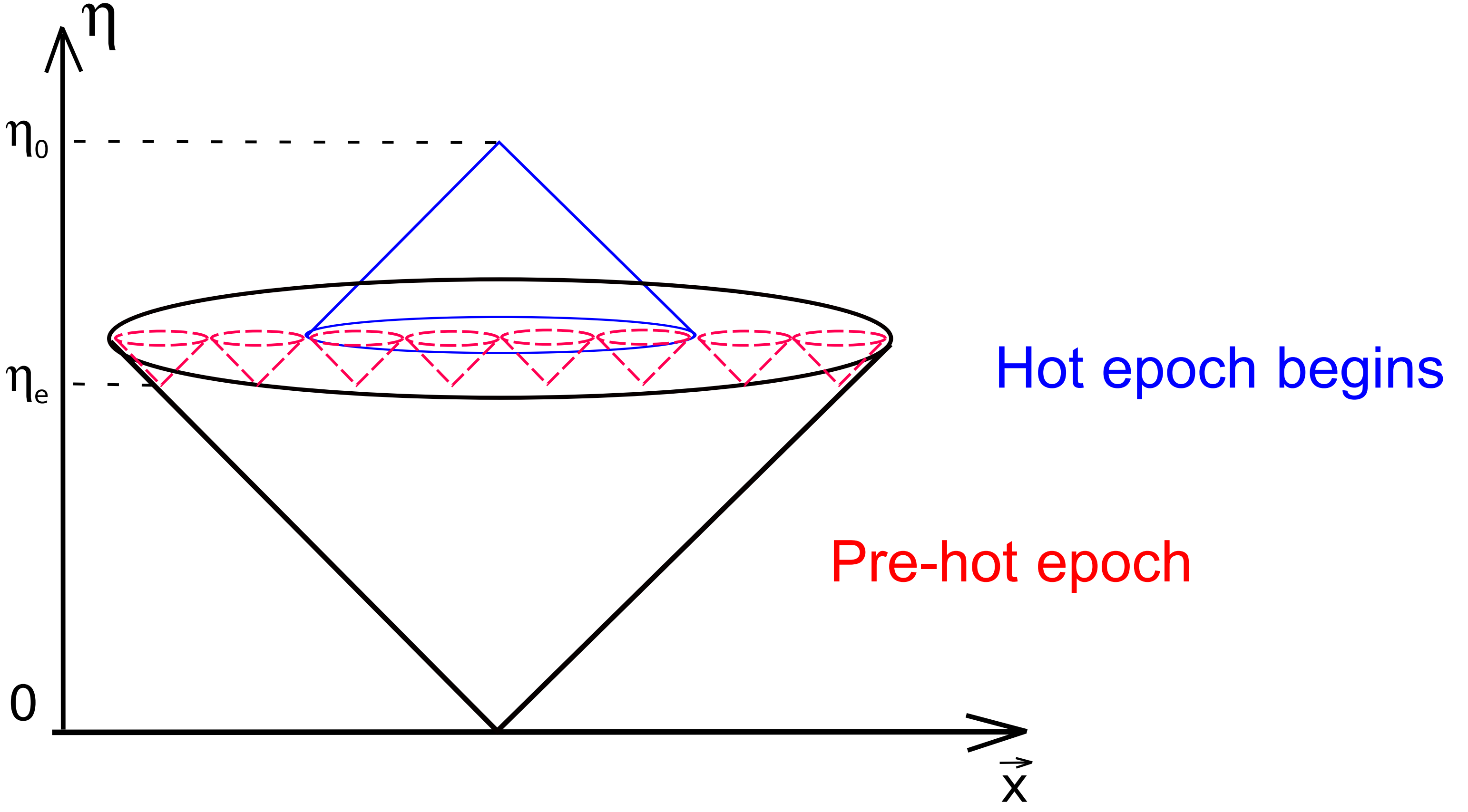


These properties would not be present if perturbations were generated at hot epoch in causal manner: phase ψ would be random function of k, no oscillations in CMB angular spectrum.



Primordial perturbations were generated at some yet unknown epoch before the hot expansion stage.

That epoch must have been long (in a certain sense) and unusual: perturbations were subhorizon early at that epoch, our visible part of the Universe was in a causally connected region.



Excellent guess: inflation

Starobinsky'79; Guth'81; Linde'82; Albrecht and Steinhardt'82

Exponential expansion with almost constant Hubble rate,

$$a(t) = \mathrm{e}^{\int H dt} \,, \qquad H \approx \text{const}$$

- Initially Planck-size region expands to entire visible Universe in $t \sim 100\, H^{-1}$ $\Longrightarrow$ for $t \gg 100\, H^{-1}$ the Universe is VERY large

- Perturbations subhorizon early at inflation:

$$\lambda(t) = 2\pi \frac{a(t)}{k} \ll H^{-1}$$

since $a(t) \propto \mathrm{e}^{Ht}$ and $H \approx$ const;
wavelengths get redshifted, Hubble parameter stays constant

Alternatives to inflation:

Contraction — Bounce — Expansion,

Start up from static state ("Genesis")

Difficult, but not impossible.

Other suggestive observational facts about density perturbations

(valid within certain error bars!)

- Perturbations in overall density, not in composition (jargon: “adiabatic”)

$$\frac{\text{baryon density}}{\text{entropy density}} = \frac{\text{dark matter density}}{\text{entropy density}} = \text{const in space}$$

Consistent with generation of baryon asymmetry and dark matter at hot stage.

Perturbation in chemical composition (jargon: “isocurvature” or “entropy”) $\Longrightarrow$ wrong prediction for CMB angular spectrum $\Longleftrightarrow$ strong constraints from Planck.

NB: even weak variation of composition over space would mean exotic mechanism of baryon asymmetry and/or dark matter generation.

- Primordial perturbations are Gaussian.

Gaussian random field $\delta(\mathbf{k})$: correlators obey Wick's theorem,

$$\begin{aligned}\langle\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle &= 0\\ \langle\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\delta(\mathbf{k}_4)\rangle &= \langle\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\rangle\cdot\langle\delta(\mathbf{k}_3)\delta(\mathbf{k}_4)\rangle\\ &+ \text{permutations of momenta}\end{aligned}$$

- $\langle\delta(\mathbf{k})\delta^*(\mathbf{k}')\rangle$ means averaging over ensemble of Universes. Realization in our Universe is intrinsically unpredictable.
- strong hint on the origin:
  enhanced vacuum fluctuations of free quantum field
  Free quantum field

$$\phi(\mathbf{x},t) = \int d^3k e^{-i\mathbf{kx}}\left(f_{\mathbf{k}}^{(+)}(t)a_{\mathbf{k}}^{\dagger} + e^{i\mathbf{kx}}f_{\mathbf{k}}^{(-)}(t)a_{\mathbf{k}}\right)$$

In vacuo $f_{\mathbf{k}}^{(\pm)}(t) = \mathrm{e}^{\pm i\omega_k t}$

Enhanced perturbations: large $f_{\mathbf{k}}^{(\pm)}$. But in any case, Wick's theorem valid

- Inflation does the job very well: vacuum fluctuations of all light fields get enhanced greatly due to fast expansion of the Universe.

  Including the field that dominates energy density (inflaton) $\Longrightarrow$ perturbations in energy density.

  Mukhanov, Chibisov'81; Hawking'82; Starobinsky'82; Guth, Pi'82; Bardeen et.al.'83

- Enhancement of vacuum fluctuations is less automatic in alternative scenarios

- Non-Gaussianity: big issue
  - Very small in the simplest inflationary theories
  - Sizeable in more contrived inflationary models and in alternatives to inflation. Often begins with bispectrum (3-point function; vanishes for Gaussian field)

$$\langle\delta(\vec{k}_1)\delta(\vec{k}_2)\delta(\vec{k}_3)\rangle = \delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)\; G(k_i^2;\; \vec{k}_1\cdot\vec{k}_2;\; \vec{k}_1\cdot\vec{k}_3)$$

    Shape of $G(k_i^2;\; \vec{k}_1\cdot\vec{k}_2;\; \vec{k}_1\cdot\vec{k}_3)$ different in different models $\Longrightarrow$ potential discriminator.

  - In some models bispectrum vanishes, e.g., due to some symmetries. But trispectrum (connected 4-point function) may be measurable.

Non-Gaussianity has not been detected yet
strong constraints from Planck

Primordial power spectrum is nearly flat

Homogeneity and anisotropy of Gaussian random field:

$$\langle \frac{\delta\rho}{\rho}(\vec{k}) \frac{\delta\rho}{\rho}(\vec{k}') \rangle = \frac{1}{4\pi k^3} \mathscr{P}(k) \delta(\vec{k}+\vec{k}')$$

$\mathscr{P}(k) =$ power spectrum, gives fluctuation in logarithmic interval of momenta,

$$\langle \left(\frac{\delta\rho}{\rho}(\vec{x}) \right)^2 \rangle = \int_0^\infty \frac{dk}{k} \mathscr{P}(k)$$

Flat spectrum: $\mathscr{P}$ is independent of k

Harrison' 70; Zeldovich' 72, Peebles,Yu' 70

Parametrization

$$\mathscr{P}(k) = A \left(\frac{k}{k_*} \right)^{n_s - 1}$$

$A =$ amplitude, $(n_s - 1) =$ tilt, $k_* =$ fiducial momentum (matter of convention). Flat spectrum $\Longleftrightarrow n_s = 1$.
Experiment: $n_s = 0.966 \pm 0.004$ (WMAP, Planck, ...)

There must be some symmetry behind flatness of spectrum

- Inflation: symmetry of de Sitter space-time

$$ds^2 = dt^2 - \mathrm{e}^{2Ht} d\vec{x}^{\,2}$$

Relevant symmetry: spatial dilatations supplemented by time translations

$$\vec{x} \to \lambda \vec{x}\,, \quad t \to t - \frac{1}{2H} \log \lambda$$

- Alternative: conformal symmetry

Conformal group includes dilatations, $x^\mu \to \lambda x^\mu$.
$\Longrightarrow$ No scale, good chance for flatness of spectrum

Exploratory stage: toy models + general arguments so far.

- Tensor modes = primordial gravitational waves

Sizeable amplitude, (almost) flat power spectrum predicted by simplest (and hence most plausible) inflationary models but not alternatives to inflation

Smoking gun for inflation

Tensor perturbations = gravity waves

Metric perturbations

$$ds^2 = dt^2 - a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j$$

$h_{ij} = h_{ij}(\vec{x}, t)$, $h^i_i = \partial_i h^i_j = 0$, spin 2.

Gravity waves: effects on CMB

- Temperature anisotropy (in addition to effect of scalar perturbations)
  WMAP, Planck

NB: gravity wave amplitudes are time-independent when superhorizon and decay as $h_{ij} \propto a^{-1}(t)$ in subhorizon regime.

Strongest contribution to δT at large angles

$$\Delta\theta \gtrsim 2^o, \quad l \lesssim 50, \quad \text{Present wavelengths } \sim 1 \text{ Gpc}$$

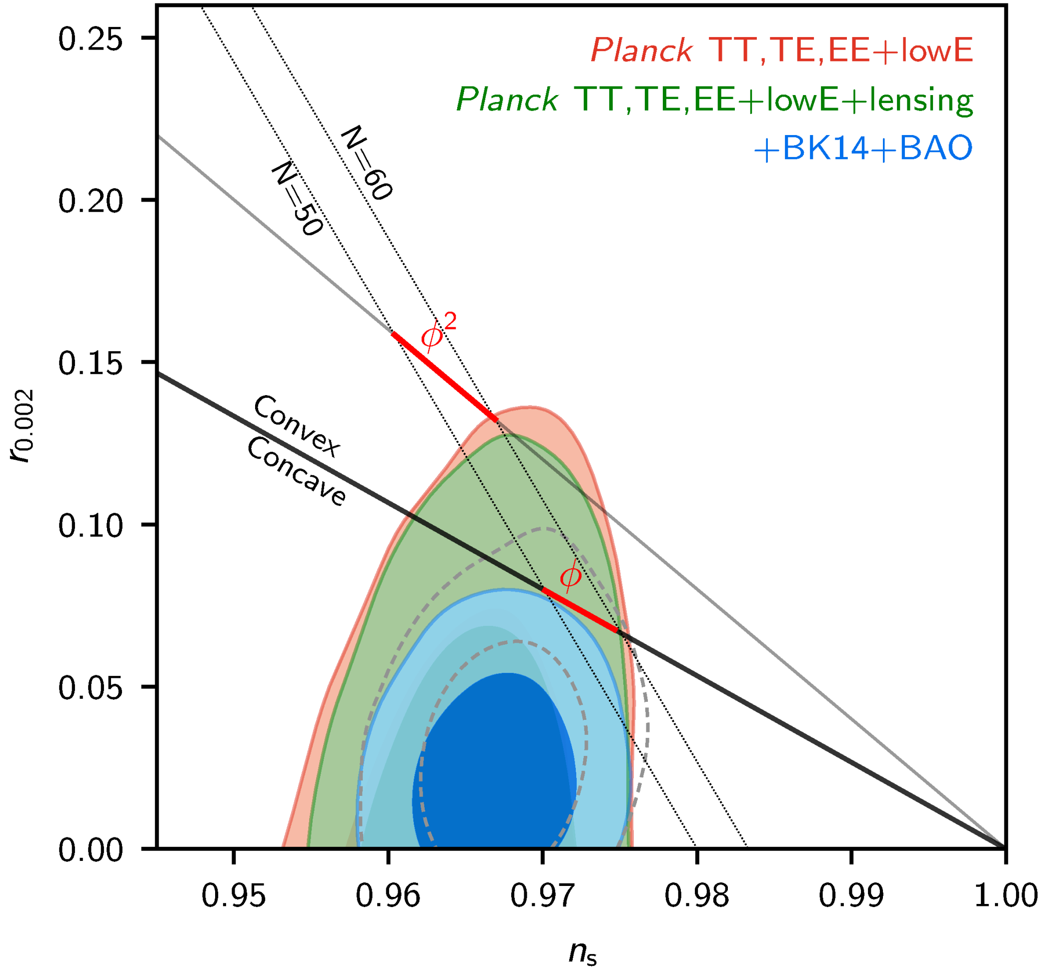
- Polarization
  especially B-mode
  Weak signal, degree of polarization $P(l) \propto l$ at $l \lesssim 50$ and decays with l at $l > 50$.

  Amplitude at $r = 0.2$:

  $P(l \sim 30) \sim 3 \cdot 10^{-8} \Longrightarrow P \cdot T \sim 0.1\ \mu\text{K}$

Planck and everybody else

Scalar spectral index vs. power of tensors



To summarize:

- Available data on cosmological perturbations (notably, CMB anisotropies) give confidence that the hot stage of the cosmological evolution was preceded by some other epoch, at which these perturbations were generated.

- Inflation is consistent with all data. But there are competitors: the data may rather be viewed as pointing towards early conformal epoch of the cosmological evolution.

More options:

Matter bounce,

Negative exponential potential,

Lifshitz scalar, . . .

- Only very basic things are known for the time being.

Good chance for future

- Detection of B-mode of CMB polarization generated by primordial gravity waves $\Longrightarrow$ simple inflation
  - Together with scalar and tensor tilts $\Longrightarrow$ properties of inflaton
- Non-trivial correlation properties of density perturbations (non-Gaussianity) $\Longrightarrow$ contrived inflation, or something entirely different.
  - Shape of non-Gaussianity $\Longrightarrow$ choice between various alternatives
- Statistical anisotropy $\Longrightarrow$ anisotropic pre-hot epoch.
  - Shape of statistical anisotropy $\Longrightarrow$ specific anisotropic model
- Admixture of entropy (isocurvature) perturbations $\Longrightarrow$ generation of dark matter and/or baryon asymmetry before the hot epoch

At the eve of new physics

LHC $\Longleftrightarrow$ Planck,
dedicated CMB polarization experiments,
data and theoretical understanding
of structure formation ...

Good chance to learn
what preceded the hot Big Bang epoch

Barring the possibility that Nature is dull

Backup slides

Dark energy

- Homogeneously distributed over the Universe, does not clump into galaxies, galaxy clusters.
- Determines the expansion rate at late times $\Longrightarrow$ Relation between distance and redshift. Expansion of the Universe accelerates.

Fig.

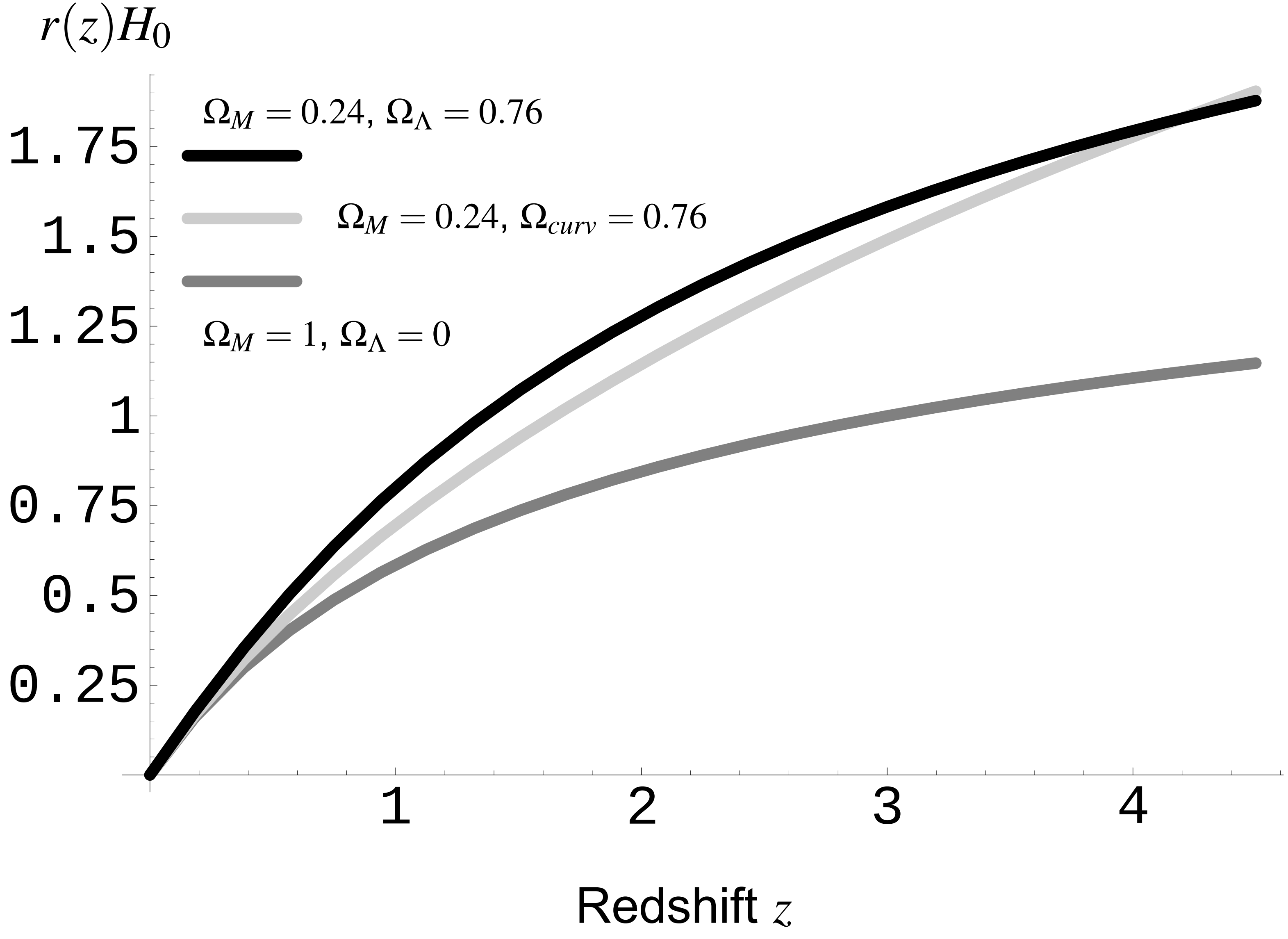
- Measure redshifts ("easy") and distances by using standard candles, objects whose absolute luminocity is known.

Supernovae 1a

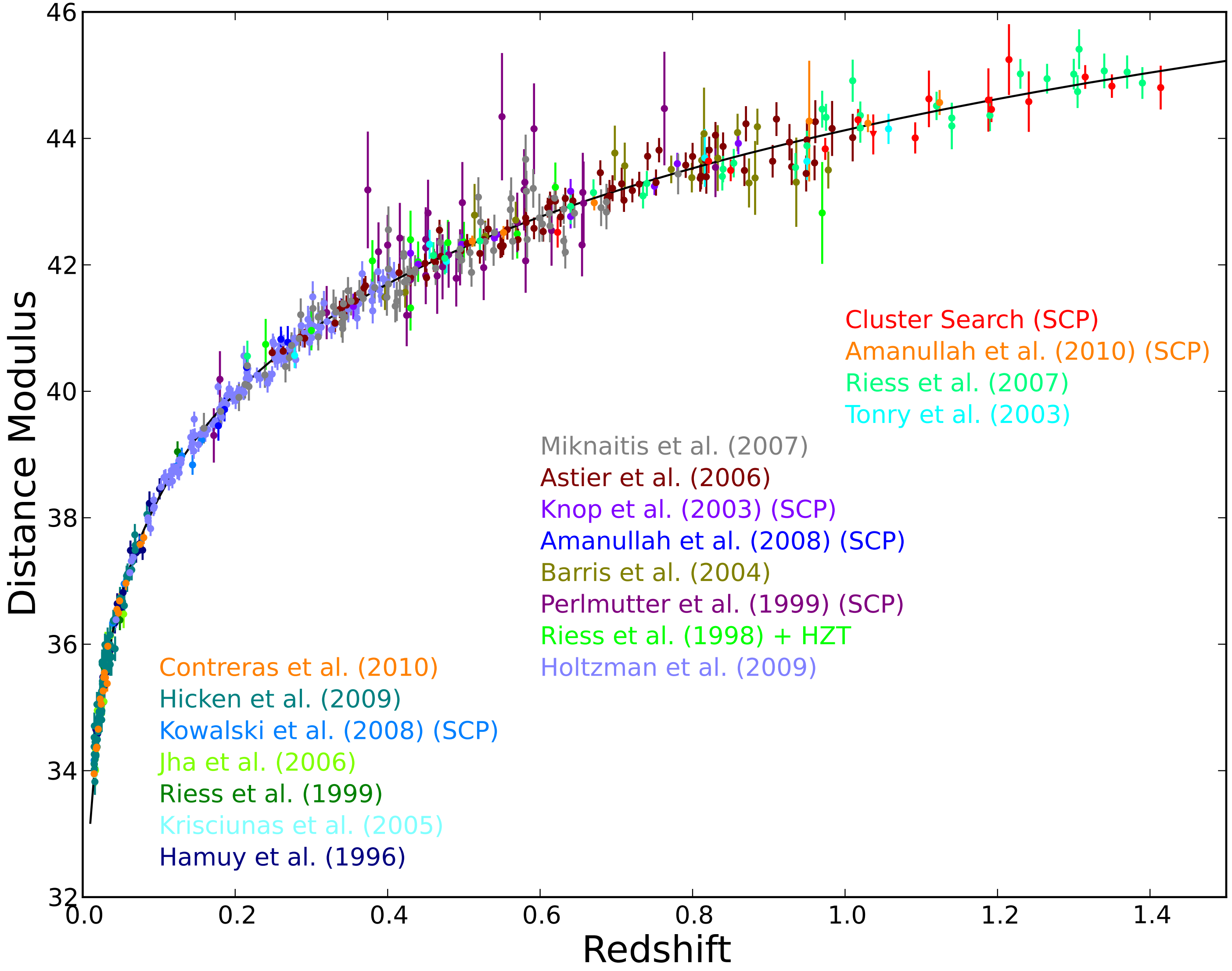
Figs.

- Other, independent measurements of ρ_Λ: cluster abundance at various z, CMB anisotropies combined with standard ruler at small redshift (baryon acoustic oscillations, BAO), etc.

Distance-redshift for different models

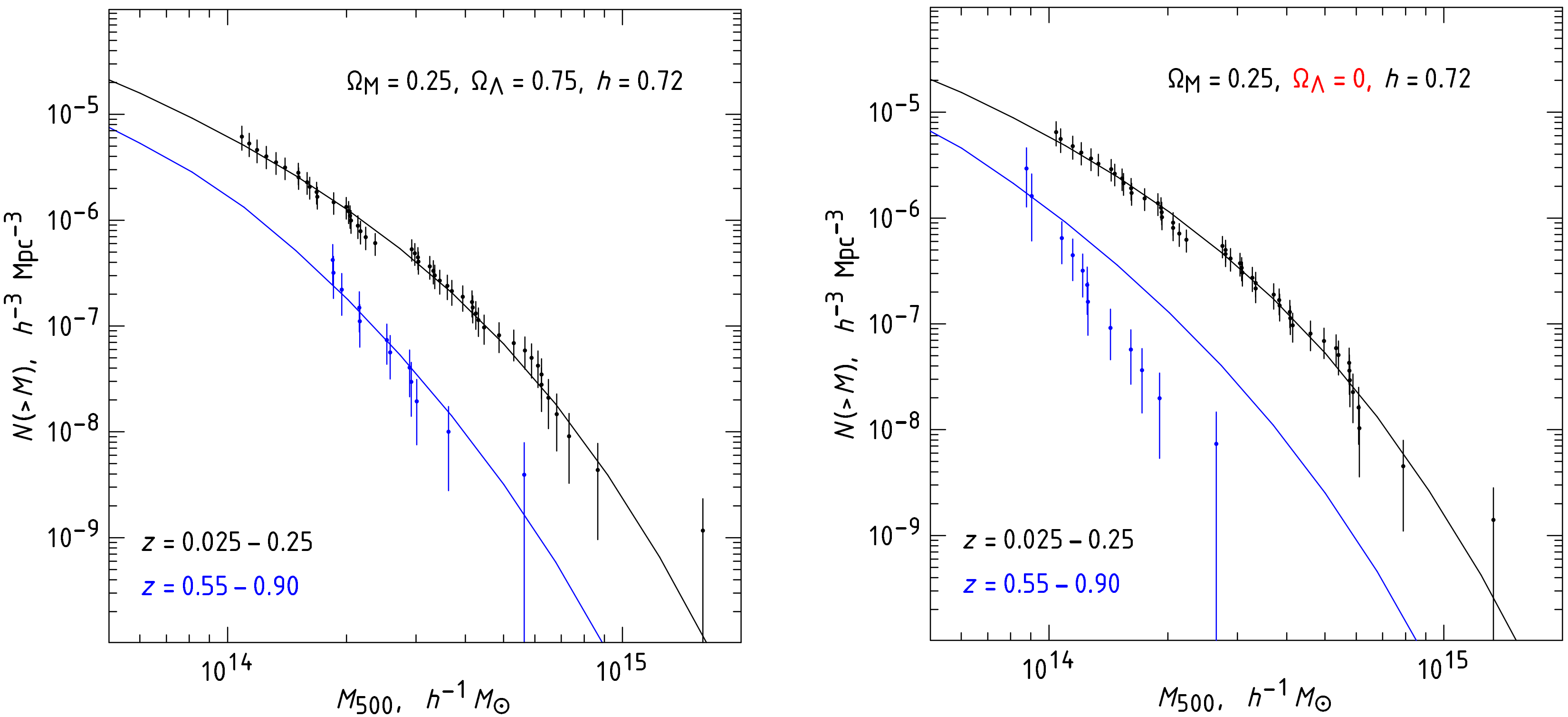


SNe1a data



Distance Modulus
Redshift
46
44
42
40
38
36
34
32
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Cluster Search (SCP)
Amanullah et al. (2010) (SCP)
Riess et al. (2007)
Tonry et al. (2003)
Miknaitis et al. (2007)
Astier et al. (2006)
Knop et al. (2003) (SCP)
Amanullah et al. (2008) (SCP)
Barris et al. (2004)
Perlmutter et al. (1999) (SCP)
Riess et al. (1998) + HZT
Holtzman et al. (2009)
Contreras et al. (2010)
Hicken et al. (2009)
Kowalski et al. (2008) (SCP)
Jha et al. (2006)
Riess et al. (1999)
Krisciunas et al. (2005)
Hamuy et al. (1996)

Cluster abundance



$\Omega_\Lambda = 0.75$

$\Omega_\Lambda = 0$, curvature domination

Who is dark energy?

- Vacuum
  By Lorentz-invariance

$$T_{\mu\nu}^{vac} = \text{const} \cdot \eta_{\mu\nu} \quad \text{Minkowski} \quad \Longrightarrow \quad T_{\mu\nu}^{vac} = \text{const} \cdot g_{\mu\nu}$$

const $= \rho_\Lambda$, fundamental constant of Nature.
$\rho_\Lambda = (2 \cdot 10^{-3}\ \text{eV})^4$: ridiculously small.
No such scales in fundamental physics.
Problem for any interpretation of dark energy

Equivalent description: cosmological constant

$$S = -\frac{1}{16\pi G}\int d^4x\, \sqrt{-g}R - \Lambda \int d^4x\, \sqrt{-g}$$

$\Lambda \equiv \rho_\Lambda = \text{const}$ = cosmological constant.

- “Old” cosmological constant problem: Why Λ is zero?
- “New” cosmological constant problem: Why Λ is very small but non-zero?

Dark energy need not be vacuum energy = cosmological constant.

- Definition of energy density and pressure:

$$T_{\mu\nu} = (\rho, p, p, p)$$

  Hence, for vacuum $p = -\rho$.

- Parametrize: $p_{DE} = w\rho_{DE} \Longrightarrow w_{vac} = -1$

w determines evolution of dark energy density:

$$dE = -pdV \Longrightarrow d(\rho a^3) = -pd(a^3) \Longrightarrow \frac{d\rho}{dt} = -3\frac{\dot{a}}{a}(p+\rho)$$

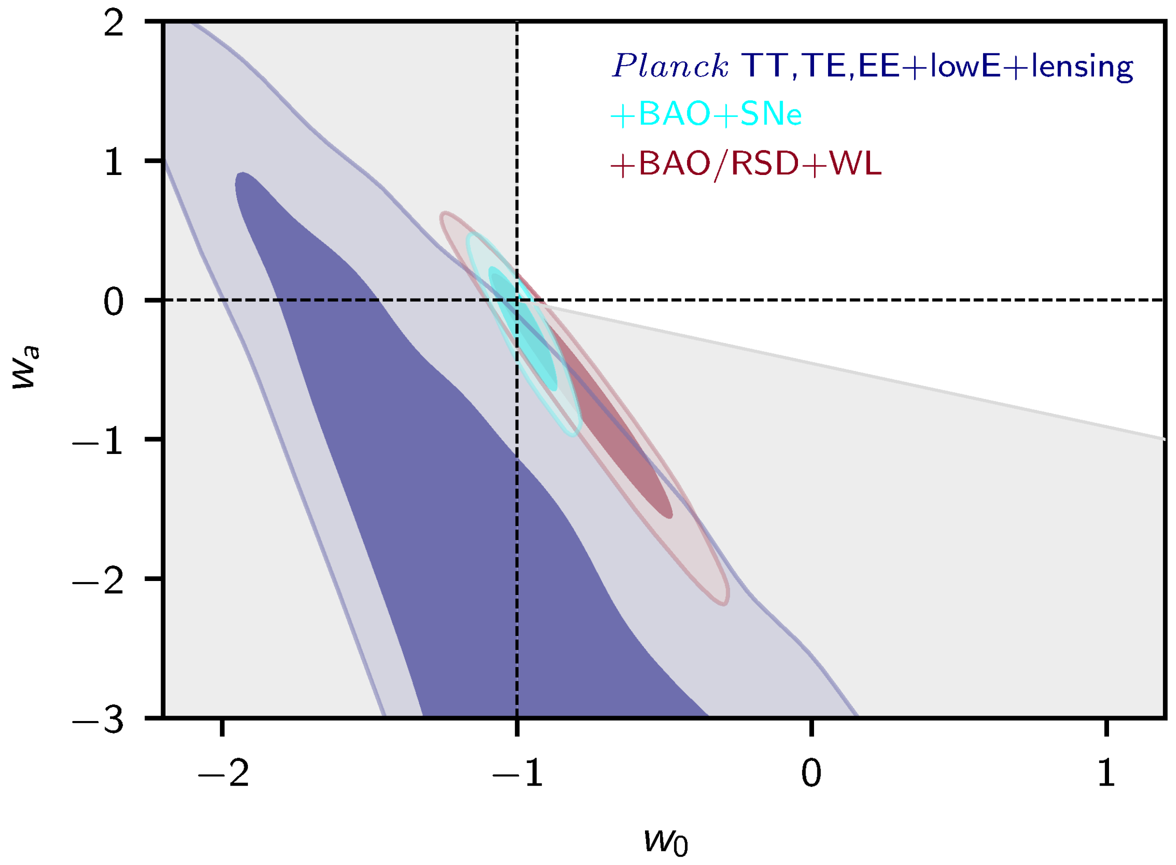
$$\frac{\dot{\rho}_\Lambda}{\rho_\Lambda} = -3(w+1)\frac{\dot{a}}{a}$$

Options:

- Vacuum: $w = -1$, ρ_Λ constant in time.
- Quintessence, "usual" field (modulo energy scale): $w > -1$, ρ_Λ decays in time.
- Phantom: $w < -1$, ρ_Λ grows in time; typically has instabilities
- General relativity modified at cosmological scales. Effective dark energy depends on time.

Present situation

$$p_\Lambda = w(a)\rho_\Lambda \, , \qquad w(a) = w_0 + w_a(1-a) = w_0 + w_a \frac{z}{1+z}$$



Changing geers

Anthropic principle/Environmentalism

Cosmology may be telling us something different — and unpleasant

- Friendly fine-tunings
  - Cosmological constant $\sim (10^{-3}\ eV)^4$

    Just right for galaxies to get formed
  - Primordial density perturbations $\frac{\delta\rho}{\rho} \sim 10^{-5}$

    Just right to form stars
    but not supermassive galaxies w/o planets
  - Dark matter sufficient to produce structure

Also

  - Light quark masses and α_{EM}

    Just right for $m_n > m_p$
    but stable nuclei
  - Many more...

Is the electroweak scale a friendly fine-tuning?

Anthropic principle/environmentalism

"Our location in the Universe
is neccessarily priviledged to
the extent of being compatible
with our existence as observers"
Brandon Carter'1974

Fig

Recent support from "string landscape"

We exist where couplings/masses are right

Problem: never know which parameters are environmental and which derive from underlying physics

Disappointing, but may be true

May gain support from the LHC, if not enough new physics

横山　操「グランド キャニオン」1961年