An exact solution to the gravity of Bakry-Émery-Ricci

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Contents

Some problems of General Relativity

There are some difficulties in GR for example :

1. The energy of the gravitational field.
2. Gravitational singularities (UV).
3. Dark energy (late acceleration, infrared).
4. The new models must contain the infrared and UV as well as predict the classic tests of GR.

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Scalar-tensor theory

Brans-Dicke (1950) [1, 2]

$$G_{\mu\nu} = \frac{8\pi}{\varphi} T_{\mu\nu} + \frac{\omega}{\varphi^2}\left(\nabla_\mu\varphi\nabla_\nu\varphi - \frac{1}{2}g_{\mu\nu}\Box\varphi\right) + \frac{1}{\varphi}\left(\nabla_\mu\varphi\nabla_\nu\varphi - g_{\mu\nu}\Box\varphi\right) \quad (1)$$

for weak-field :

$$\varphi_0 = \frac{1}{G}\left(\frac{4+2\omega}{3+2\omega}\right) \quad (2)$$

if $\omega \to \infty : \varphi_0 = \frac{1}{G}$, (GR) see others [3, 4].

Bakry-Émery-Ricci tensor in physics

$$\mathcal{R}ic_X^{\alpha}(g) := Ric(g) + \frac{1}{2}\mathfrak{L}_X g - \frac{1}{\alpha - n} X \otimes X \tag{3}$$

,

- Scalar-tensor theory : $X = df, \alpha$ $arbitrary(including\ \alpha < 0)$ [5].
- Static Einstein : $X = df, \alpha = n + 1$.
- Optical metric for static Einstein :$X = df, \alpha = 1$.
- Kaluza-Klein dilaton : $X = df, \alpha = n + k$.
- Near-horizon geometries : $\alpha = n + 2$ $(arbitrary\ X)$.
- Yang-Mills energy gap : $X = df, \alpha = \infty$. (Lichnérowicz, Moncrief-Marini-Maitra).

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Bakry-Émery-Ricci tensor

Bakry-Émery-Ricci [6]

$$\mathcal{R}_{\mu\nu} = R_{\mu\nu} - \frac{1}{\alpha}\nabla_\mu\varphi\nabla_\nu\varphi - \nabla_\mu\nabla_\nu\varphi, \tag{4}$$

with

$$\mathcal{R} = g^{\mu\nu}\mathcal{R}_{\mu\nu}. \tag{5}$$

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The action for the new field equations

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{|g|}\,\mathcal{R}, \tag{6}$$

by variation of the action with respect to inverse metric $g^{\mu\nu}$, the new field equations are obtained.

$$\mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} = 0, \tag{7}$$

this implies that

$$\mathcal{R}_{\mu\nu} = 0 \tag{8}$$

Using the ansatz :

$$ds^2 = -f(r)\,dt^2 + f(r)^{-1}\,dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\varphi^2. \tag{9}$$

Solution to Bakry-Émery-Ricci [7]

$$f(r) = \frac{1}{1+\alpha} + \frac{(-2M)^{1+\alpha}}{(1+\alpha)\, r^{1+\alpha}}, \tag{10}$$

$$\varphi(r) = A + \alpha \ln(r/r_0), \tag{11}$$

with $\alpha = 0$ the Schwarzschild solution is reproduced.

α	Horizonte	$r \to \infty$
-3	$\pm 2iM$	-∞
-2	2M	∞
-1	$\emptyset$	$\emptyset$
0	2M	1
1	$\pm 2iM$	1/2
2	2M	1/3
3	$\pm 2\sqrt{i}M$	1/4

TABLE – 1. Values of α and horizons.

General features of the solution

- It's include the event horizon of the Schwarzschild. ($\alpha = 0$)
- $r = 2M$ for $\alpha =$ even number.
- Topological defects (Global monopole).

$$\lim_{r \to \infty} f(r) = \frac{1}{1 + \alpha}$$

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What follows...

- Perform the classical tests of General Relativity (deviation of light and the perihelion shift of Mercury).
- Put some bound the value α.
- Try identify the meaning of $\varphi(r)$.

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